THE APPLICATION OF A C^* FLIGHT CONTROL LAW TO LARGE CIVIL TRANSPORT AIRCRAFT

Edmund Field

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Cranfield Institute of Technology
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Edmund Field
Flight Dynamics Group

March 1993

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## CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>ii</td>
</tr>
<tr>
<td>CONTENTS</td>
<td>iii</td>
</tr>
<tr>
<td>FIGURES</td>
<td>v</td>
</tr>
<tr>
<td>NOTATION</td>
<td>vii</td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Background to the Programme of Research</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Background to this Study</td>
<td>1</td>
</tr>
<tr>
<td>2. BACKGROUND TO THE C* CRITERION</td>
<td>3</td>
</tr>
<tr>
<td>2.1 Cornell Aeronautical Laboratory Angle of Attack Time History Envelope</td>
<td>3</td>
</tr>
<tr>
<td>2.2 Development of the C* Parameter</td>
<td>3</td>
</tr>
<tr>
<td>2.3 Development of the C* Boundaries</td>
<td>7</td>
</tr>
<tr>
<td>2.4 The Use of C* Controllers</td>
<td>8</td>
</tr>
<tr>
<td>3. PITCH RATE FEEDBACK EFFECTS</td>
<td>9</td>
</tr>
<tr>
<td>4. NORMAL ACCELERATION FEEDBACK EFFECTS</td>
<td>11</td>
</tr>
<tr>
<td>5. C* ANALYSIS</td>
<td>13</td>
</tr>
<tr>
<td>5.1 Pitch Rate Feedback</td>
<td>13</td>
</tr>
<tr>
<td>5.2 Normal Acceleration Feedback</td>
<td>14</td>
</tr>
<tr>
<td>5.3 C* Response</td>
<td>14</td>
</tr>
<tr>
<td>5.4 Comparison of Results to MIL-STD-1797A</td>
<td>18</td>
</tr>
</tbody>
</table>
6. CONSIDERATION OF THE FULL ORDER MODEL

6.1 Introduction 20
6.2 Pitch Rate Feedback Effects 21
6.3 Normal Acceleration Feedback Effects 23
6.4 C* Analysis on the Full Order Model 25
6.5 Modification of the Phugoid Roots 28

7. IDENTIFICATION OF PROBLEMS WITH THE ANALYSIS 29

7.1 The Baseline Boeing 747 is Inherently Well Behaved 29
7.2 Control Law Structure 30

8. IDENTIFICATION OF PROBLEMS WITH THE C* CRITERION 31

8.1 Direct Lift Effects 31
8.2 Long Term Effects 31
8.3 Derivation of Cross-over Velocity and Choice of $1/T_\theta$ 32
8.4 Same Boundaries for All Aircraft Types 32
8.5 Application of the C* Criterion to In-Flight Data 32
8.6 Discussion of C* Based Controllers Currently Used 33

9. CONCLUSIONS 34

REFERENCES 35
### FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Original Time History Envelope Flying Qualities Criterion</td>
</tr>
<tr>
<td>2</td>
<td>Pitch Rate Responses for Several Values of $T_{\theta 2}$</td>
</tr>
<tr>
<td>3</td>
<td>$C^*$ Time History Envelopes</td>
</tr>
<tr>
<td>4</td>
<td>$C^*$ Frequency Response Envelopes</td>
</tr>
<tr>
<td>5</td>
<td>Pitch Rate Root Locus Plot</td>
</tr>
<tr>
<td>6</td>
<td>Normalised Pitch Rate Responses for Various Pitch Rate Feedback Gains</td>
</tr>
<tr>
<td>7</td>
<td>Normal Acceleration Root Locus Plot</td>
</tr>
<tr>
<td>8</td>
<td>Normalised Normal Acceleration Responses for Various Normal Acceleration Feedback Gains</td>
</tr>
<tr>
<td>9</td>
<td>Pitch Rate Root Locus Plot</td>
</tr>
<tr>
<td>10</td>
<td>Normal Acceleration Root Locus Plot</td>
</tr>
<tr>
<td>11</td>
<td>Normalised $C^*$ Time History Response</td>
</tr>
<tr>
<td>12</td>
<td>$C^*$ Frequency Response Bode Plots</td>
</tr>
<tr>
<td>13</td>
<td>Normalised Pitch Rate Responses</td>
</tr>
<tr>
<td>14</td>
<td>Normalised Normal Velocity Responses</td>
</tr>
<tr>
<td>15</td>
<td>Normalised Normal Acceleration Responses</td>
</tr>
<tr>
<td>16</td>
<td>Normalised $C^*$ Responses</td>
</tr>
<tr>
<td>17</td>
<td>Short Period Requirements of MIL-STD-1797A, Appendix A, Figure 13</td>
</tr>
<tr>
<td>18</td>
<td>Pitch Rate Root Locus Plot</td>
</tr>
<tr>
<td>19</td>
<td>Pitch Rate Phugoid Mode Root Locus Plot</td>
</tr>
<tr>
<td>20</td>
<td>Comparison of Reduced Order and Full Order Pitch Rate Response</td>
</tr>
<tr>
<td>21</td>
<td>Normalised Pitch Rate Response for Various Pitch Rate Feedback Gains</td>
</tr>
<tr>
<td>22</td>
<td>Normal Acceleration Root Locus Plot</td>
</tr>
<tr>
<td>Figure 23</td>
<td>Comparison of Reduced Order and Full Order Normal Acceleration Response</td>
</tr>
<tr>
<td>Figure 24</td>
<td>Normalised Normal Acceleration Response for Various Pitch Rate Feedback Gains</td>
</tr>
<tr>
<td>Figure 25</td>
<td>Reduced Order C* Response from the Reduced and Full Order Analyses</td>
</tr>
<tr>
<td>Figure 26</td>
<td>Reduced and Full Order C* Responses from Full Order Analysis - First 3.5 Seconds</td>
</tr>
<tr>
<td>Figure 27</td>
<td>Reduced and Full Order C* Responses from Full Order Analysis - First 50 Seconds</td>
</tr>
<tr>
<td>Figure 28</td>
<td>Full Order Model C* Frequency Response Bode Plot</td>
</tr>
</tbody>
</table>
NOTATION

Abbreviations

CAL  Cornell Aeronautical Laboratories
CAP  Control Anticipation Parameter
FBW  Fly-By-Wire
FCS  Flight Control System
HOS  Higher Order System
NASA National Aeronautics and Space Administration
PI  Proportional plus Integral
PIO  Pilot Induced Oscillation
SAS  Stability Augmentation System
TIFS Total In-Flight Simulator
USAF United States Air Force
VMS Vertical Motion Simulator
rad  Radian
sec  Second
mph  Miles per hour

Symbols

K  Feedback gain
K_{nz}  Normal acceleration feedback gain
K_q  Pitch rate feedback gain
M_{\alpha}  Pitching Moment due to angle of attack
V  Forward velocity
V_{co}  Cross over velocity
dB  Decibels
g  Acceleration due to gravity
lb  Pound weight
n_z  Normal acceleration
q  Pitch rate
s  Laplace operator
\alpha  Angle of attack
\delta_e  Elevator deflection
\theta  Pitch attitude
1/T_{\theta 2}  Higher frequency zero in q/\delta_e transfer function
\omega_n  Natural frequency
\omega_{sp}  Short period natural frequency
\omega_p  Phugoid natural frequency
\zeta_{sp}  Short period damping ratio
\zeta_p  Phugoid damping ratio

Subscript

ss  Steady state
1. INTRODUCTION

1.1 Background to the Programme of Research

The work contained in this report is part of an on-going programme of research into handling qualities of fly-by-wire civil transport aircraft currently being undertaken within the Flight Dynamics Group of the College of Aeronautics (reference 1).

Although much work has been undertaken into handling qualities of military aircraft over the last 30 years, civil aircraft have received considerably less attention. Over the last decade civil transport aircraft incorporating fly-by-wire flight control systems have been introduced into commercial operation, the latest including some modified aerodynamic designs (reference 2). However the civil arena lacks the supporting research into handling qualities that the military side has enjoyed. More recently the civil side is beginning to receive the attention it deserves with work in Europe by Fokker (reference 3) and the Group for Aeronautical Research and Technology in EURope (GARTEUR), reference 4, for example. In the United States much work has been done by the manufacturers such as Boeing (references 5, 6 and 7) and McDonnell Douglas (references 8, 9 and 10), and as ever work supported by the US Air Force and NASA.

The primary aim of this current programme is to design flight control laws to give fly-by-wire civil transport aircraft excellent flying qualities at all flight conditions, but especially in piloted flight phases. The most critical flight phase of a civil transport is that of the landing approach, and, as with other studies of this type, this phase receives the greatest attention in this study.

This report concerns an analysis of the C* parameter. The C* criterion was one of the first handling qualities criteria designed to take account of advanced aerodynamic designs of modern aircraft and higher order systems introduced by flight control systems. Several aircraft have since employed control laws based around the C* parameter. A proportional feedback C* controller was applied to a Boeing 747-100 in landing approach configuration, and assessed against the C* criterion and the US military specification MIL-STD-1797A (reference 11).

1.2 Background to this Study

The classical longitudinal reduced order handling qualities requirements are expressed in terms of the short period natural frequency and damping ratio. These two parameters completely describe the short term angle of attack, and normal acceleration, dynamic response of a classical aircraft, and so are an effective measure of the aircraft's handling qualities.

However with the introduction of augmented aircraft it was found that aircraft that had well tailored short period dynamics exhibited deficient handling qualities. This was especially true for aircraft that employed advanced aerodynamic design and Flight Control System (FCS). While the addition of Stability Augmentation Systems (SAS) could introduce additional mode dynamics at frequencies close to the short period mode frequency, it was also postulated that the pilot may be sensitive to other motion cues in addition to the normal acceleration as described by the classical short period handling qualities requirements.
It was postulated that pilots sense attitudes, velocities and accelerations, and since time history envelopes convey information relating to all of these, it was proposed that a time history envelope is more likely to provide correlation with pilot opinion than the classical requirements. A time history envelope accounts for numerator dynamics, higher order effects, and to some extent, non-linear response, as well as the response characteristics portrayed by the \( \omega_n - \zeta \) requirements. Time history envelopes were considered necessary but not sufficient, common sense must also be used in evaluating the response. For example, an intolerable lightly damped high frequency mode superimposed on the dominant response might still be contained within the "acceptable" time history envelope. It was argued that although a step command may not represent the most common pilot input, the step response implicitly describes response to other inputs that a pilot uses, and for analysis a step input is easily repeatable for comparative testing.

Many time history envelope criteria have been developed since the 1960's, however one of the first and best known is the C* criterion. This report considers the application of a simple proportional feedback C* controller to the Boeing 747 and its assessment against the C* criterion.

As with most criteria C* only considers the reduced order aircraft model, however more recently the importance of the longer term response is being increasingly recognised. Therefore a considerable part of this report is given to consideration of the long term response.
2. BACKGROUND TO THE C* CRITERION

2.1 Cornell Aeronautical Laboratory Angle of Attack Time History Envelope

The earliest known application of a time history response envelope to specify flying qualities requirements was for the TFX (F-111) procurement, the first time a quantitative flying qualities requirement for short period dynamics was included in a procurement document. The specification was transformed into a time history envelope to take account of the higher order dynamics introduced by the flight control system, thus satisfying the intent of the specification, even if its letter was not preserved.

The criterion was developed in 1963 by Cornell Aeronautical Laboratories (CAL) reference 12, as part of a study for the Boeing Aircraft Company, from the original iso-opinion short period "bullseye" specification used during the TFX (F-111) procurement. Normalised angle of attack responses from the boundaries of the short period specification (figure 1) were plotted and encompassed by an envelope that spanned the range of satisfactory and acceptable responses for category A flight. Angle of attack was chosen as the response variable because the short period natural frequency and damping ratio completely specify the short period angle of attack response of the vehicle. Therefore, the time history envelope was directly related to the existing specification.

Reference 12 also describes the development of a general flight control system configuration for the proposed vehicle that used a pitch rate gyro and vertical accelerometer located at the pilot's station for feedback in a fixed gain feedback configuration. The rate gyro was used to increase the short period damping ratio while the accelerometer feedback increased the equivalent short period natural frequency.

2.2 Development of the C* Parameter

In 1965 Boeing published their own time history envelope criterion called C* which used a normalised linear sum of pitch rate and normal acceleration at the pilot station, reference 13. The C* envelope introduced two parameters not necessary in the CAL angle of attack criterion. Firstly the pitch rate transfer function numerator term, 1/T_{θ2}, and secondly a value of weighting or relative contribution of pitch rate and normal acceleration.

Tobie, Elliott and Malcom, reference 14, argued that the original CAL thumbprint represented combinations of short period natural frequency and damping ratios that were preferred by the pilots of several variable stability airplanes, primarily the F-94. Therefore the thumbprint is valid for the tasks required of the evaluation pilots during the F-94 flight tests, and is also valid for similar tasks in similar aircraft at similar flight conditions. However to extrapolate this criterion to other aircraft at other flight conditions assumes:

1. the predominant variable sensed by the pilot is normal acceleration;
2. the short period response may be represented by that of a linear second order system.
a) Short Period Specifications Used to Plot Time History Envelope

b) Envelope Of Acceptable Responses

Figure 1  Original Time History Envelope Flying Qualities Criterion
(From Reference 12)
They go on to argue that the pilots were also able to sense pitch cues in addition to the normal accelerations, and that therefore their opinions were also influenced by the pitch response. A simplified two-degree-of-freedom constant speed representation of the longitudinal axis of an aircraft can be represented by the following block diagram:

\[ \frac{F_p}{s^2 + 2\zeta_{sp}\omega_{sp}s + \omega_{sp}^2} \rightarrow n_x \rightarrow \frac{gT_{\theta_2}}{V(s+1/T_{\theta_2})} \rightarrow q \]  

(1)

While the short period natural frequency and damping ratio completely describe the normal acceleration response to pilot inputs, the pitch rate response, however, also contains a zero and a multiplier, both of which vary with flight condition. Thus, it was argued, if a pilot is influenced by the pitch response then regulating \( \omega_{sp} \) and \( \zeta_{sp} \) alone cannot guarantee equivalent total response from airplane to airplane and from flight condition to flight condition. The zero \( (s + 1/T_{\theta_2}) \) has a profound effect on the pitch rate transient response. Figure 2 shows the pitch rate response for a step input when the aircraft is approximated by a second order system with a finite zero. The denominator roots were chosen to fit the centre of the thumbprint as given in figure 1 (\( \omega_{sp} = 5.0 \text{ rad/sec}, \ z_{sp} = 0.7 \)), while the range of zero values is typical of a 1960's fighter. The F-94 tests were performed at a flight condition where \( T_{\theta_2} \) was approximately 0.67 seconds. The value of \( T_{\theta_2} \) for the Boeing 747 in landing configuration, as used in the analysis of this report, is about 2 seconds. It should be noted that the aerodynamic parameters defining \( T_{\theta_2} \) also define the values of \( \zeta_{sp} \) and \( \omega_{sp} \), but that these effects are neglected in this simplification.

![Pitch Rate Response Diagram]

Pitch Rate (q) Response

Time (seconds)

\( \omega_{sp} = 5 \text{ rad/s} \quad \zeta_{sp} = 0.7 \)

<table>
<thead>
<tr>
<th>Configuration</th>
<th>( T_{\theta_2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>0.667</td>
</tr>
</tbody>
</table>

Figure 2 Pitch Rate Responses for Several Values of \( T_{\theta_2} \)
The lower q transient response shown in figure 2, configuration 3, is thus representative of that experienced by the F-94 pilots, and exhibits a one hundred percent overshoot. Tobie, Elliot and Malcom suggested that a then modern 1960's high-performance fighter could have a range of \( T_{\theta_2} \) from about 0.67 to 5 or higher, and could therefore exhibit pitch rate overshoots up to 12:1 for the same \( \omega_{sp} \) and \( \zeta_{sp} \) combination. They go on to argue that "acceptance of the thumbprint therefore implies that pitch response is insignificant to airplane handling qualities".

They postulated that pilots respond to a blend of pitch rate and normal acceleration, with the ratio varying according to natural variations in the aircraft's response. At low velocities normal acceleration cues are weak, therefore the predominant cue would be pitch rate. At high velocities where slight pitching may produce large normal acceleration changes, normal acceleration cues dominate. This blend of normal acceleration and pitch rate was named \( C^* \) and is defined as:

\[
C^* = K_{nz} n_z + K_q q
\]  

(2)

where \( n_z \) is the normal acceleration at the pilot's station. The dimensionless \( C^* \) parameter can be obtained by blending the outputs of a pitch rate gyro and a linear accelerometer at the pilot's station, as suggested in reference 12. The outputs of the two sensors would be blended with a fixed ratio, however the relative contribution of \( n_z \) and \( q \) would automatically vary with the velocity as a result of the variation inherent in the \( n_z \) and \( q \) transfer functions.

The selection of the constants \( K_{nz} \) and \( K_q \) was somewhat arbitrary and has since come in for some criticism. It was argued that at high velocities the \( n_z \) cue would be dominant and at slower approach velocities \( q \) would dominate. Thus at some mid velocity, called the cross-over velocity \( (V_{co}) \) both cues would command equal pilot attention. This velocity was chosen as 400 ft/sec, and by setting the steady state contribution of \( n_z \) and \( q \) equal the ratio of \( K_{nz} \) and \( K_q \) was fixed. From the two-degree-of-freedom representation of an aircraft given in equation (1):

\[
q = \frac{n_z \ g \ T_{\theta_2}}{\sqrt{s + 1/T_{\theta_2}}}
\]  

(3)

The steady state relationship between \( n_z \) and \( q \) is given by:

\[
q_{ss} = \frac{n_z \ g}{\sqrt{g}}
\]  

(4)

When the contributions of \( n_z \) and \( q \) are equal:

\[
K_{nz} n_{zss} = K_q q_{ss}
\]  

(5)

and if \( K_{nz} \) is set equal to 1, combining equations (4) and (5) gives:

\[
K_q = \frac{V_{co}}{g}
\]  

(6)

With \( V_{co} = 400 \) feet per second and \( g = 32.2 \) ft/sec²

\[
K_q = 12.4
\]  

(7)

Therefore the ratio between the feedback gains is fixed at:

\[
\frac{K_q}{K_{nz}} = 12.4
\]  

(8)
2.3 Development of the C* Boundaries

The C* time history envelopes were developed using data from two variable stability aircraft. Data from the CAL F-94 was used to define the up-and-away boundaries in what was considered to be a representative flight condition of 400 mph and 20,000 feet. Data for the landing approach condition were taken from a handling qualities investigation into low speed landing approach flight characteristics using Boeing's own Model 367-80, the 707 prototype, which was modified to a five-degree-of-freedom variable stability aircraft.

Establishment of the envelopes followed three stages:

1. determining the total C* transfer function of the test aircraft. In addition to the natural frequency and damping ratio test points, this included the numerator terms and actuator dynamics;

2. transposing the acceptable boundary test points into unit step time histories using the above transfer function;

3. forming minimum and maximum time history boundaries from the envelope resulting from the previous step.

The lower portion of the envelopes were modified for the first few tenths of a second to account for the effect of forward transmission dynamics from sources such as actuators, mechanical linkages and shaping networks. The C* time history envelopes for the tracking and landing approach flight conditions are given in figure 3.

![C* Time History Envelopes](image)

--- Landing Approach Boundary
--- Tracking Boundary

Figure 3 C* Time History Envelopes

For consistency with other analysis techniques of the time Boeing also produced C* frequency response envelopes developed in the same way as the time history envelopes. These are given in figure 4 for the tracking and landing approach flight conditions.
2.4 The Use of C* Controllers

With the acceptance and use of the C* parameter as a handling qualities metric, it was not a surprising step that a control law based on the parameter would evolve. The parameter was based on the belief that at low velocities the pilot reacts to pitch changes while at high velocities normal acceleration cues dominate. Accepting this approach it can be proposed that at low velocities a pilot controls the flight path of his aircraft through control of the pitch attitude (and therefore pitch rate) while at higher velocities he controls the flight path through control of the normal acceleration, and hence angle of attack. Thus using a C* demand control system he is able to directly control these parameters throughout the speed range of the aircraft.

Several aircraft in current operation claim to use C* controllers (F-16, F-18, A320-340), however these do not control the "pure" C* parameter described in the previous section. Instead the term C* controller today seems to be used to cover any controller which incorporates normal acceleration and pitch rate feedbacks in any ratio, and in most cases incorporate other feedback quantities as well.

The fixed ratio of the blend of normal acceleration and pitch rate does not allow the designer to specify exact values for both the short period natural frequency and damping ratio. As proposed by CAL in section 2.1 pitch rate feedback can be used to specify the short period damping ratio, while normal acceleration feedback can be used to determine the natural frequency. Thus using appropriate feedbacks the short period dynamics can be specified using normal acceleration and pitch rate feedbacks, although not in the ratio of the originally proposed C* criterion.

Many modern FCS incorporate Proportional plus Integral (PI) controllers. These controllers themselves introduce additional dynamics and can greatly modify the aircraft response. This report considers only proportional feedback, more complicated controller structures will be considered elsewhere. Indeed the analysis in this report concerns only the C* parameter and envelopes of references 13 and 14.
3. PITCH RATE FEEDBACK EFFECTS

As introduced in section 2.1 feedback of pitch rate to elevator will augment the short period damping, while having minimal effect on the natural frequency. Figure 5 shows the root locus for the reduced order pitch rate transfer function for the Boeing 747 in landing configuration. The closed loop poles are annotated in order of increasing feedback gain.

![Pitch Rate Root Locus Plot](image)

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$K_q$</th>
<th>Short Period Roots</th>
</tr>
</thead>
<tbody>
<tr>
<td>open loop</td>
<td>-</td>
<td>$\omega_{sp} = 0.77$ $\zeta_{sp} = 0.62$</td>
</tr>
<tr>
<td>1</td>
<td>0.585</td>
<td>$\omega_{sp} = 0.83$ $\zeta_{sp} = 0.7$</td>
</tr>
<tr>
<td>2</td>
<td>2.33</td>
<td>$\omega_{sp} = 1.01$ $\zeta_{sp} = 0.9$</td>
</tr>
<tr>
<td>3</td>
<td>3.35</td>
<td>$(s + 1.1)$ $(s + 1.1)$</td>
</tr>
<tr>
<td>4</td>
<td>4.14</td>
<td>$(s + 0.8)$ $(s + 1.7)$</td>
</tr>
<tr>
<td>5</td>
<td>9.98</td>
<td>$(s + 0.6)$ $(s + 4.1)$</td>
</tr>
</tbody>
</table>

Figure 5 Pitch Rate Root Locus Plot

The open and closed loop poles are given in figure 5 while the reduced order pitch rate numerator remains unchanged and is:

$$-0.3764 \, (s + 0.50)$$

Clearly as the feedback gain is increased the closed loop short period poles follow the locus until they meet on the real axis, at which point they separate and diverge along the axis, one migrating to the open loop zero at $(s + 0.50)$, the other to minus infinity. Over the curved part of the locus the damping ratio is therefore increasing until the locus meets the real axis at which point the damping ratio becomes unity. This results in two real roots, rather than the pair of complex roots as previously. As the gain is increased further the roots diverge with the higher valued root (the one on the left of the plot) becoming increasingly dominant. With the two real roots the response of the aircraft is no longer second order, becoming more first order like as the roots diverge.
Figure 6 shows the normalised time history responses to a step elevator input for the pole locations marked on figure 5. Note that for clarity only the responses for configurations 1 and 5 are annotated.

![Graph showing normalised pitch rate responses for various pitch rate feedback gains](image)

**Figure 6** Normalised Pitch Rate Responses for Various Pitch Rate Feedback Gains

For configuration 1, with a damping ratio of 0.7, the response is clearly second order. As the feedback gain is increased the response for configuration 2 exhibits less overshoot and peaks sooner, due to the increased damping ratio and natural frequency, however is still clearly second order. This trend continues as the feedback gain is increased as can be seen by the responses for configurations 3 and 4, which are similar to that for configuration 2. Clearly as the closed loop poles become real the response remains second order like since for these configurations the poles are close to one another. As the closed loop poles diverge, configuration 5, the response becomes more first order like in appearance as the higher frequency pole at \(s + 4.1\) becomes dominant over the lower frequency pole at \(s + 0.6\). This results in a faster well damped response with a time constant of approximately 0.25 seconds.
4. NORMAL ACCELERATION FEEDBACK EFFECTS

Also introduced in section 2.1 was the concept that feeding back normal acceleration to elevator augments natural frequency, while having little effect on the damping ratio. Reference 12 also states that if the accelerometer used to measure the normal acceleration is located forward of the instantaneous center of rotation, stability of the short period mode would be retained if the pitch rate feedback was lost, thus producing a robust flight control system. Figure 7 shows the upper half of the root locus for the reduced order normal acceleration transfer function for the Boeing 747 in landing configuration, where the normal acceleration is measured at the pilot's station.

![Root Locus Plot](image)

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$K_{nz}$</th>
<th>Short Period Roots</th>
</tr>
</thead>
<tbody>
<tr>
<td>open loop</td>
<td>-</td>
<td>$\omega_{sp} = 0.77$ $\zeta_{sp} = 0.62$</td>
</tr>
<tr>
<td>1</td>
<td>0.077</td>
<td>$\omega_{sp} = 0.8$ $\zeta_{sp} = 0.57$</td>
</tr>
<tr>
<td>2</td>
<td>0.873</td>
<td>$\omega_{sp} = 1.0$ $\zeta_{sp} = 0.39$</td>
</tr>
<tr>
<td>3</td>
<td>6.03</td>
<td>$\omega_{sp} = 1.2$ $\zeta_{sp} = 0.25$</td>
</tr>
</tbody>
</table>

Figure 7 Normal Acceleration Root Locus Plot

The open and closed loop poles are given in figure 7 while the reduced order normal acceleration numerator is:

$$-0.799 \left( s^2 + 0.543s + 1.62 \right)$$

which has complex roots.

As the normal acceleration feedback gain is increased the closed loop poles migrate away from the origin. The greatest effect of this is to increase the natural frequency, although clearly the damping ratio is decreased slightly as well. For all values of feedback gain, unlike for pitch rate feedback, the roots remain complex and so the response remains oscillatory.
Figure 8  Normalised Normal Acceleration Responses for Various Normal Acceleration Feedback Gains

Clearly as the normal acceleration feedback gain is increased, the natural frequency increases causing the response to peak sooner. However the effect of the damping ratio changes is not marked.
5. C* ANALYSIS

A simple proportional C* controller was applied to a Boeing 747-100 in landing configuration. The analysis was performed on a transfer function based model, derived from the data of reference 15 using the PC based Control System Design and Simulation Software package CODAS. This consisted of separate closures of the pitch rate and normal acceleration feedback loops to the specification of the C* parameter as derived in references 13 and 14 and given in section 2.2.

5.1 Pitch Rate Feedback

As shown in section 3 pitch rate feedback may be used to augment short period damping. An ideal mid range value as specified in MIL-STD-1797A (reference 11) is \( \zeta_{sp} = 0.7 \). For the basic unaugmented 747 model in landing configuration the short period mode is described by:

\[
\begin{align*}
\omega_{sp} &= 0.760 \\
\zeta_{sp} &= 0.618
\end{align*}
\]

giving the open loop characteristic equation:

\[
s^2 + 0.9392s + 0.5778
\]

To augment the damping ratio to the desired value of 0.7 requires a pitch rate feedback gain of 0.565, yielding closed loop poles giving:

\[
\begin{align*}
\omega_{sp} &= 0.827 \\
\zeta_{sp} &= 0.696
\end{align*}
\]

as shown on figure 9, yielding a modified characteristic equation:

\[
s^2 + 1.151s + 0.684
\]
5.2 Normal Acceleration Feedback

For a controller conforming to the original definition of the C* parameter the ratio between the normal acceleration and pitch rate feedback gains is given by equation 8. Thus:

\[ K_{nz} = 0.0455 \]

The normal acceleration root locus plot is shown in figure 10.

![Normal Acceleration Root Locus Plot](image)

\[ K_{nz} = 0.0455 \quad \omega_{sp} = 0.847 \quad \zeta_{sp} = 0.667 \]

Figure 10 Normal Acceleration Root Locus Plot

The short period damping ratio has reduced slightly, but this value is still in the middle of the level 1 requirements of MIL-STD-1797A and so is perfectly acceptable. These values produce a modified characteristic equation which also forms the denominator of the C* transfer function:

\[ s^2 + 1.130s + 0.717 \]

while the reduced order C* numerator is:

\[-0.80 (s - 5.58) (s - 0.81)\]

5.3 C* Response

Figure 11 shows the normalised C* time history response for the aircraft, both unaugmented and with the pitch rate and normal acceleration feedbacks to the design of a C* proportional feedback controller. Superimposed on the plot is the C* boundary for the landing approach task.
Figure 11  Normalised C* Time History Responses

The unaugmented 747 C* response crosses the envelope's lower boundary after about 0.6 seconds and does not fully emerge until about 1.5 seconds after the step input to the elevator. The response to the 747 with the C* controller however lies within the C* envelope and so is acceptable as defined by the C* criterion.

It should be noted that the envelope only covers the first 3.4 seconds. At this limit it can be seen that the responses are close to the upper C* envelope boundary, and so this could be a possible problem. Again, however, the augmented response appears to be better behaved and can clearly be seen turning away from the boundary, suggesting that this may not be such a cause for concern.

Figure 12 shows the C* frequency response for the aircraft, both unaugmented and with the C* proportional feedback controller. Superimposed on the plot is the C* boundary for the landing approach task.

Figure 12  C* Frequency Response Bode Plots
As for the time history response the unaugmented aircraft model lies outside the lower boundary of the envelope between about 1 and 5 rad/sec. Again the augmented response is within the C* envelope confirming its acceptability by the C* criterion.

Figures 13 to 16 show the effect that the C* feedback has on the pitch rate, normal velocity, normal acceleration and C* responses respectively over the first 10 seconds. Clearly all of these responses have been speeded up slightly by the incorporation of C* feedback, as well as a lessening of the overshoot. This is evidenced in the changes in the values of natural frequency and damping of the model before and after the augmentation.

<table>
<thead>
<tr>
<th></th>
<th>$\omega_{sp}$</th>
<th>$\zeta_{sp}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unaugmented</td>
<td>0.760</td>
<td>0.618</td>
</tr>
<tr>
<td>With C* feedback</td>
<td>0.847</td>
<td>0.667</td>
</tr>
</tbody>
</table>

![Figure 13](image) Normalised Pitch Rate Responses

![Figure 14](image) Normalised Normal Velocity Responses
Figure 15  Normalised Normal Acceleration Responses

Figure 16  Normalised $C^*$ Responses
5.4 Comparison of Results to MIL-STD-1797A

The Control Anticipation Parameter (CAP) is given by:

\[ \text{CAP} = \frac{\omega_{sp}^2}{n_z/\alpha} \]  

where:

\[ n_z/\alpha = \frac{V}{gT_{\theta_2}} \]

From reference 15, the speed of the 747 in the landing approach condition considered is 221 ft/sec, giving \((n_z/\alpha) = 3.43\). The values of CAP for the unaugmented and augmented 747 are:

<table>
<thead>
<tr>
<th></th>
<th>(\omega_{sp})</th>
<th>CAP</th>
<th>(\zeta_{sp})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unaugmented</td>
<td>0.760</td>
<td>0.168</td>
<td>0.618</td>
</tr>
<tr>
<td>With C* feedback</td>
<td>0.847</td>
<td>0.209</td>
<td>0.667</td>
</tr>
</tbody>
</table>

These values are plotted on figure 17, the short period requirements of MIL-STD-1797A, Appendix A, Figure 13c (reference 11). Both the unaugmented and augmented models meet the level 1 boundaries, however the unaugmented model only just meets the lower boundary with a value of CAP of 0.168 against the boundary value of 0.16. Against the C* criterion the unaugmented aircraft failed to meet the requirements of the lower boundary, although it only slightly exceeded this boundary for a short period.

This introduces an important question on the use of time history envelopes. How serious is it for the response to exceed a boundary? Furthermore the C* response envelopes were developed from the same data used for the CAP boundaries, and yet for this example C* finds the unaugmented 747 deficient, while CAP suggests the aircraft is level 1. This brings up two interesting points.

Firstly the 747 is a very large aircraft with an inertia in excess of the model 367-80 used for the criteria development and so it is entirely possible that the lower boundaries do not adequately take account of these higher inertia aircraft. Secondly there is some question over the lower boundaries of CAP. This is currently receiving attention in the United States through an in-flight simulation program using the USAF/Calspan Total In-Flight Simulator (TIFS), and due to be augmented later in the year through a simulation program using the NASA Ames Vertical Motion Simulator (VMS).

A further point, however, to note is the fact that the 747 models used in this analysis did not include any higher order dynamic effects or other features that led to the poor correlation of pilot opinion and classical requirements, and hence the development of the C* criterion. The criterion was only just met with the basic model, what effect any additional higher order systems and time delays may have had can only be speculated upon.

To properly test these results would require an in-flight simulation since the only pilot opinion taken account of was that of the pilots of the Boeing 367-80 when collecting data for another study, which was then used in the development of the C* boundaries. No direct consideration of pilot opinion has been taken in this analysis.
Control Anticipation Parameter (CAP)

Damping Ratio, $\zeta_{sp}$

+ Unaugmented 747
* 747 With C* Feedback Controller

c) Category C Flight Phases

Figure 17 Short Period Requirements of MIL-STD-1797A. (from Appendix A, Figure 13)
6. CONSIDERATION OF THE FULL ORDER MODEL

6.1 Introduction

The C* criterion is based on the short term response of the reduced order model. This is because it was felt that the critical task for the TFX, for which the C* criterion was originally developed, was short term manoeuvring. Clearly if a pilot is continually making inputs to the controls, then only the short term response was considered necessary since the long term transients would not have time to develop. This is a fair assumption for fighter aircraft, especially if the short period and phugoid modes are well separated, the short period natural frequency is ideally designed to be at least 10 times that of the phugoid to avoid mode coupling. Hence the C* criterion can be considered a manoeuvring criterion, as are most other requirements developed since.

While this may be fair for fighter aircraft, transport aircraft are a different case. For the most part a transport pilot makes small inputs to produce small changes in the aircraft's flight path. Even in the highest gain task of a transport, the landing, the pilot will still only be making small inputs. Therefore the long term response becomes important since this will have a dominant influence on the total response of the aircraft.

The importance of the long term response of an aircraft has been recognised more recently. Boeing have introduced speed stability to their design for the 777, thus producing an aircraft with a stable phugoid, as opposed to the earlier designs of neutral speed stability. In an in-flight simulation program in support of the 7J7 flight control law development using the Calspan TIPS, pilots found the lack of speed stability in the flight path command control laws simulated to be undesirable. Boeing have learned from this and have since developed the 777 control law which provides speed stability to the pilot.

The importance of speed stability, especially during landing approach, is a recurrent theme in reference 7. This is not only the case for transport aircraft, but many in-flight simulation programs performed by Calspan show that pilots prefer configurations with speed stability for fighter aircraft in the landing approach, references 16, 17 and 18. Indeed reference 19 suggests that poor short period dynamics are not the most important factor in landing handling qualities, PIOs due to control system lags seem to be more detrimental.

Clearly the total response of the full order aircraft model is very important in assessing the handling qualities of any aircraft in the landing approach.
6.2 Pitch Rate Feedback Effects

Section 3 dealt with the effects of pitch rate feedback on the reduced order model. Figure 18 gives the pitch rate root locus plot for the full order model of the Boeing 747 in landing approach configuration. For clarity the phugoid mode root locus is enlarged and given in figure 19.

![Pitch Rate Root Locus Plot](image1)

Figure 18 Pitch Rate Root Locus Plot

![Pitch Rate Phugoid Mode Root Locus Plot](image2)

Figure 19 Pitch Rate Phugoid Mode Root Locus Plot

The effect the introduction of the phugoid mode has on the response can be seen in figure 20, a direct comparison of the reduced and full order models for the open loop response. After about 3 seconds the phugoid motion comes into effect modifying the response from that of the short period mode only. The fully developed phugoid is clearly visible, with a period of approximately 40 seconds, and can be seen to be stable.
The positions of the closed loop poles 1, 2 and 3 on figures 18 and 19 correspond to the closed loop poles 1, 3 and 5 (pole at $s + 0.6$) of the reduced order model given in figure 5. Their characteristics are tabulated below.

### Reduced Order Model

- **$K_q$**
- **$\omega_{sp} = 0.83$, $\zeta_{sp} = 0.7$**
- **$3.35$**
- **$(s+1.1)$
- **$9.98$**
- **$(s+0.6)$**

### Full Order Model

- **$K_q$**
- **$\omega_{sp} = 0.84$, $\zeta_{sp} = 0.7$**
- **$0.626$**
- **$3.34$**
- **$8.01$**
- **$\omega_p = 0.14$, $\zeta_p = 0.07$**
- **$\omega_p = 0.11$, $\zeta_p = 0.19$**
- **$\omega_p = 0.08$, $\zeta_p = 0.33$**

while the full order pitch rate numerator is:

$$-0.3764 \, s \, (s - 0.44) \, (s - 0.079)$$

The only appreciable pole position change is that of the higher frequency pole of case 3. The phugoid has had the effect of reducing its value, and hence slowing down the response and making it less first order like, as for the reduced order model. The full order time histories for the given feedback gains are given in figure 21. The differences in the phugoid modes' frequency and damping properties, as tabulated above, can clearly be seen.

Also of interest is the fact that a higher feedback gain is required for the full order model as long as the short period closed loop poles are complex. Once the poles become real, a lower feedback gain is required, for the full order model, at least to place the lower frequency pole.

It should also be noted that for all full order responses given in this report, the normalising gains used are the same as for their equivalent reduced order models. This is obviously only an approximation, but allows easier cross examination of the different responses. However it should be appreciated that the relative scaling of the responses is not accurate.
Figure 21  Normalised Pitch Rate Response for Various Pitch Rate Feedback Gains

6.3 Normal Acceleration Feedback Effects

Section 4 concerned the effects of normal acceleration feedback on the reduced order model. Figure 22 gives the equivalent upper half of the normal acceleration root locus plot for the full order model of the Boeing 747 in landing approach configuration.

Figure 22  Normal Acceleration Root Locus Plot

The effect the introduction of the phugoid mode has on the normal acceleration response can be seen in figure 23, a direct comparison of the reduced and full order
seconds the phugoid motion comes into effect modifying the response from that of the short period only mode. Again the fully developed phugoid is clearly visible, with a period of approximately 40 seconds, and can be seen to be stable.

![Graph showing reduced order and full order normal acceleration response](image)

**Figure 23** Comparison of Reduced Order and Full Order Normal Acceleration Response

The positions of the closed loop poles on figure 22 correspond to the closed loop poles of the reduced order model given in figure 7. Their characteristics are tabulated below.

<table>
<thead>
<tr>
<th>Reduced Order Model</th>
<th>Full Order Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{nz}$</td>
<td>Short Period Roots</td>
</tr>
<tr>
<td>0.077</td>
<td>$\omega_{sp} = 0.8$</td>
</tr>
<tr>
<td>0.873</td>
<td>$\zeta_{sp} = 0.57$</td>
</tr>
<tr>
<td>6.03</td>
<td>$\omega_{sp} = 1.0$</td>
</tr>
<tr>
<td></td>
<td>$\zeta_{sp} = 0.39$</td>
</tr>
<tr>
<td></td>
<td>$\omega_{sp} = 1.2$</td>
</tr>
<tr>
<td></td>
<td>$\zeta_{sp} = 0.25$</td>
</tr>
<tr>
<td></td>
<td></td>
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<td></td>
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</tbody>
</table>

while the full order normal acceleration numerator is:

$$-0.799 s (s - 0.011) (s^2 + 0.55s + 1.62)$$

The introduction of the phugoid has had very little effect upon the short period poles. The full order time histories for the given feedback gains are given in figure 24. The differences in the phugoid modes' frequency and damping properties, as tabulated above, can clearly be seen.

In all cases a lower feedback gain is required to achieve the same short period natural frequency for the full order model than for the reduced order model, however the differences are not great.
6.4 C* Analysis on the Full Order Model

The analysis of section 5 was repeated for the full order model. In order to obtain a short period damping ratio of 0.7 required a pitch rate feedback of 0.626, resulting in a normal acceleration feedback gain of 0.050. The resulting modal properties are:

<table>
<thead>
<tr>
<th>Short Period Roots</th>
<th>Phugoid Roots</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_{sp}$</td>
<td>$\zeta_{sp}$</td>
</tr>
<tr>
<td>Unaugmented a/c</td>
<td>0.770</td>
</tr>
<tr>
<td>With $q$ feedback</td>
<td>0.841</td>
</tr>
<tr>
<td>With $n_x$ feedback (i.e. C* feedback)</td>
<td>0.861</td>
</tr>
<tr>
<td>$K_q = 0.626$, $K_{nx} = 0.050$</td>
<td></td>
</tr>
</tbody>
</table>

while the full order C* numerator is:

$$-0.799 \, s \, (s - 0.047) \,(s^2 + 0.59s + 4.65)$$

The C* response of the short period mode only was compared to the response from the reduced order analysis of section 3, and is shown in figure 25. The two responses are almost identical, as are those for pitch rate and normal acceleration. Therefore the addition of the phugoid mode to this analysis has had negligible effect upon the short period closed loop pole locations.
Figure 25    Reduced Order C* Response from the Reduced and Full Order Analyses

Figure 26 shows the comparison between the C* reduced and full order responses for the full order model analysis. Superimposed on the plot are the C* boundaries for the landing approach task. Clearly for the first 3.4 seconds both responses are almost identical, and both meet the C* envelope requirements. After about 2 seconds the responses separate and the phugoid mode starts to have an influence on the response.

Figure 26    Reduced and Full Order C* Responses from Full Order Analysis - First 3.5 Seconds
A more complete picture may be obtained by considering the same responses, but for a longer time period. Figure 27 shows the same responses but for 50 seconds. Here the Phugoid mode can clearly be seen. The responses for pitch rate and normal acceleration show a similar trend.

![Normalised $C^*$ (t)](image)

**Figure 27** Reduced and Full Order $C^*$ Responses from Full Order Analysis - First 50 Seconds

As for the reduced order analysis the short period mode meets the requirements of MIL-STD-1797A with a CAP of 2.16, increased from 2.09 for the reduced order model. The value of $n_z/\alpha$ and $\zeta_p$ are unchanged. Concerning the phugoid mode MIL-STD-1797A specifies a value of $\zeta_p > 0.04$. From the table above the unaugmented 747 does not meet this requirement, however the model incorporating the $C^*$ feedback does with a value of $\zeta_p = 0.081$. Furthermore the $C^*$ feedback controller increases the ratio of the short period and phugoid natural frequencies to 6.5 from 5.1 for the unaugmented model, thus slightly increasing the separation of the two modes.

For this particular control law design at the flight condition considered of this aircraft both the reduced and full order model responses meet the $C^*$ boundaries. However it is possible that for other control law designs, other configurations and other aircraft the reduced order model response may meet the $C^*$ envelopes, while the full order model response does not, and vice versa. This may therefore be considered a deficiency of an analysis using a reduced order model only, since it does not take into account the phugoid mode.

Confirmation of the inability of the $C^*$ criterion to take account of phugoid dynamics can be seen by considering the frequency response of the full order model, figure 28. The low frequency response lies outside the envelope, therefore suggesting the configuration to be deficient, clearly not the case from the time response envelope, or MIL-STD-1797A. This therefore confirms that the $C^*$ criterion does not take into account the phugoid mode, which has been shown to be of great importance to aircraft handling qualities, especially in landing approach.
6.5 Modification of the Phugoid Roots

Many modern aircraft control laws produce responses which exhibit no appreciable phugoid mode. In this case the aircraft's response is more accurately represented by the reduced order model assumed by the $C^*$ analysis. This may be achieved in several ways, through the use of a proportional plus integral control law, through pole/zero cancellation and through pole placement. This last method is discussed here.

Sections 6.2 and 6.3 showed how feedback of pitch rate and normal acceleration not only augment the short period, but also effect the placement of the phugoid roots. The Boeing 747 model used for this analysis represents a well behaved aircraft, however for some aircraft examples exhibiting reduced static stability it is possible for the phugoid mode to be less well behaved. In these cases while specifying feedback gains to give good short period dynamics, the phugoid roots can become real, producing a non oscillatory phugoid, and thus an aircraft without speed stability.
7. IDENTIFICATION OF PROBLEMS WITH THE ANALYSIS

As suggested in section 6.5 the analysis performed did not expose several potential problems that may be encountered with the introduction of a C* based controller. The original aircraft model used is well behaved and so required little augmentation to bring it into acceptability of the C* criterion. This section addresses several of the problems encountered with this analysis.

7.1 The Baseline Boeing 747 is Inherently Well Behaved

The baseline 747 in landing approach mode is generally well behaved, as defined by the short period mode. This can be confirmed by the fact that the aircraft has been in operational service for over 20 years, and is not regarded as having especially poor handling qualities in any flight condition. There is therefore little room for improvement in such an analysis, only fine tuning the short period mode with low feedback gains, rather than dramatically altering the aircraft's response.

Modern aircraft are somewhat different, especially those that take advantage of reduced static stability. A measure of the static stability of an aircraft is the derivative \( M_{\phi} \), which has an important effect upon \( \omega_{sp} \) and \( \zeta_{sp} \) as well as the phugoid mode. As the static stability of the aircraft is reduced it therefore becomes necessary to augment the short period mode to give the aircraft acceptable handling qualities. In this case there is considerably more scope for potential handling qualities deficiencies to arise, in particular the effect of the augmentation upon the phugoid mode.

The C* criterion was also designed to take account of Higher Order effects such as FCS delays and actuator dynamics, however these have not been introduced to this model. This would be possible, however since the aircraft is heavy and slow responding, it is unlikely that delays introduced by FCS would be appreciable compared to the response of the basic aircraft. Poor choice of actuators could introduce unacceptable dynamics, but since the actuators fitted to current 747s are clearly adequate, the choice of suitable actuators should not cause a problem. These effects are however highly relevant to fighter aircraft which are ever more manoeuvrable and faster responding, the type of aircraft that the criterion was designed for.

It has been noted that the 747 is a very slow responding aircraft, even the augmented model only just meets the lower C* boundary. The model considered is an early mark 747, since when the aircraft has grown considerably, the latest version being the 747-400 which has a maximum landing weight of 630,000 lbs as opposed to the weight of the model used in this analysis of 564,000 lbs. Present transport aircraft handling qualities programmes are considering aircraft of a million pounds weight, such as the McDonnell Douglas MD-12. Clearly as aircraft become heavier their inertia increases leading to even slower responses. It is possible to speed up the response slightly, as suggested in section 2.3, by using normal acceleration feedback to specify the closed loop natural frequency. This is limited however by the open loop zeros, whose locations are affected by the inertial properties of the aircraft, since the closed loop pole must lie on the root locus, and so the response could only be speeded up slightly, the natural frequency could not be augmented much beyond 1.5 rad/sec in this particular analysis. It is therefore questionable whether the basic 747 is an appropriate aircraft for this type of analysis.
7.2 Control Law Structure

The analysis of this report has been limited to cover only proportional feedback to the design of the $C^*$ parameter. As stated in section 6.5 the response of an aircraft is greatly influenced by the control law implemented. For instance the use of an IP controller can seriously modify any apparent phugoid mode.

However in the analysis performed the phugoid remained complex and oscillatory. Although it is possible to produce real phugoid roots using such feedback techniques, it is not possible with the model considered due to the open loop pole and zero locations. These in turn are a function of the aircraft's geometric and aerodynamic properties, therefore suggesting that the 747 model considered is inherently too stable to be of any great potential to exhibit interesting characteristics.
8. IDENTIFICATION OF PROBLEMS WITH THE C* CRITERION

The previous section dealt with problems encountered with the application of a C* based controller to the Boeing 747 in landing configuration, and its assessment by the C* criterion. Since its original publication in 1965 the C* criterion has been subject to considerable assessment and various criticisms of it have been found. This section discusses some of these criticisms.

8.1 Direct Lift Effects

With the pilot forward of the center of rotation, as the aircraft pitches he is able to sense a normal acceleration due to the pitching motion. If the pilot is a long way forward of the center of rotation, as in the 747, the normal acceleration associated with the initial pitching motion is considerable, and results in a faster equivalent short period natural frequency. This in turn leads to a better pilot rating, therefore the normal acceleration cue is a better handling qualities response for these aircraft. However the C* parameter includes negligible normal acceleration at landing.

Tobie, Elliot and Malcom in reference 14 argued that "acceptance of the thumbprint therefore implies that pitch response is insignificant to airplane handling qualities" as a basis for the C* criterion. However it is now believed that pilots are not aware of pitch rate overshoots, and the significance of pitch rate as a cue has been over stressed, for example reference 17. However, pilots are aware of pitch attitude overshoots.

8.2 Long Term Effects

Rynaski in reference 17 argues that control of flight path angle implies control of the lift of the aircraft. For a conventional aircraft lift is varied through the use of the elevator to rotate the aircraft to a new angle of attack. If the angle of attack responds rapidly and smoothly to a commanded input and then remains relatively constant, the pilot is able to judge the effectiveness of his control manipulation by observing changes in attitude, which is then equal to changes in flight path. Therefore the key to good flying qualities and precision flight path control of aircraft during approach and landing is precise and accurate control of lift, or angle of attack.

Furthermore, the angle of attack response exhibits minimum phugoid visibility. Therefore there is a direct relationship between the pitch rate phugoid, visible through the pitch attitude of the aircraft, and the flight path response of the aircraft.

Aircraft that incorporate pitch rate controllers, similar to C* at low speeds, will usually produce a fast, smooth and well behaved pitch rate response. Often however, the smooth, well behaved pitch rate response results in an unacceptably sluggish angle of attack response that will grow without bound following a step input command. The pilot may no longer be able to judge flight path changes by changes in pitch attitude and his opinion of the configuration deteriorates.

In addition this report has stressed the importance of the phugoid mode to handling qualities, and has shown the C* criterion to take inadequate account of this mode. The
C* criterion is not unique in this respect, most longitudinal handling qualities criteria only take account of the short term response of the reduced order model, however it is necessary that the long term response also be considered.

8.3 Derivation of Cross-Over Velocity and Choice of 1/T_{02}

The cross-over velocity defines the ratio of pitch rate and normal acceleration feedback gains. The analysis given above incorporated the feedback gains in the ratio defined by the original C* criterion. However it has been argued that the cross-over velocity chosen had no apparent association with the existing flying qualities specification (reference 17), from where the supporting data was taken for its development, and so is open to question.

By relaxing the definition of the cross-over velocity it becomes possible to specify feedback gains independently, thus enabling more accurate specification of the short period natural frequency and damping ratio. However this will also require construction of new C* envelopes, since the existing envelopes are based upon a cross-over velocity of 400 ft/sec.

In addition to the question over the choice of cross-over velocity is the value of 1/T_{02} used. The value used was chosen to be that of the F-94 flying at 400 mph at an altitude of 20,000 feet. The use of this fixed value of 1/T_{02} again has been the source of criticism.

8.4 Same Boundaries for All Aircraft Types

The C* criterion applies the same boundaries to all aircraft types. It has been seen above that the baseline 747 does not meet the lower boundary of the landing approach envelope, and as discussed it is suggested that as aircraft become larger and heavier more aircraft with acceptable flying qualities may not meet this boundary. However this lower boundary would seem very slow for a fighter.

The use of just one set of boundaries for all aircraft types is therefore questioned, in the light of the results of this study.

8.5 Applications of the C* Criterion to In-Flight Data

In reference 14 Tobie, Elliot and Malcom stated that a flight test program to produce the desired data for the development of the C* envelopes was a large scale and costly undertaking, and so instead data already available from the CAL F-94 and Boeing model 367-80 variable stability aircraft was used. Since their development much data has been collected during other in-flight simulation studies, primarily by Calspan, for example references 19, 20, 21, 22, 23 and 24.

This data has since been applied to the C* criterion to assess its applicability as a design and evaluation tool. Most notable is the study performed by Neal and Smith in 1970, reference 21. They found correlation of pilot ratings with the C* criterion to be variable. For configurations with negligible control system dynamics, such as the 747 model considered, correlation with the C* criterion was fairly good. However the
disagreements were caused by the effects of control system dynamics, the very effects the criterion was designed to handle.

These results have been repeated in many similar analyses for example by Mooij in reference 25. As a result the $C^*$ criterion has had limited use and is seldom considered today.

8.6 Discussion of $C^*$ Based Controllers Currently Used

Section 2.3 introduced the concept of a $C^*$ controller, however went on to describe that most aircraft claiming a $C^*$ controller in fact do not conform to the original design and also use additional feedbacks. This fact itself may be taken as an indication that a pure $C^*$ controller has been found to be deficient, though it must be remembered that the $C^*$ parameter was proposed as a handling qualities criterion, not the basis of a flight control law. It should be evident, however, that an aircraft claiming a $C^*$ controller should not be taken as having deficiencies, in most cases the manufacturers are doing their designs an injustice.

The only current commercial aircraft using a $C^*$ based control law are the Airbus A320-340. The control law implemented produces a neutrally stable aircraft, which can pose problems in the landing flare, with the aircraft exhibiting a tendency to float. This was corrected with a rather crude fix in the A320, however the A340/330 has a more advanced design, reference 2. From fifty feet above the runway the ground speed is fed back in addition to the pitch rate and normal acceleration. Thus as the aircraft's speed begins to decay in the flare, the pilot has to hold a back pressure on the stick to stop the nose from dropping, and thus the aircraft handles like a conventional aircraft in the flare.

The route that Boeing have chosen for the 777 is somewhat different. Recognising the deficiencies of a $C^*$ controller Boeing investigated the use of flight path controllers for the 717, however during an in-flight simulation study the pilots found that the neutral stability was undesirable, especially in the landing approach where the pilot no longer had the benefit of speed stability cues through the effector feel system. Thus for the 777 Boeing are implementing a $C^*u$ control law. This is similar to $C^*$ except that in addition to pitch rate and normal acceleration, forward speed is also fed back. This produces an aircraft with speed stability, similar to the A340/330 in the flare, however throughout the entire "up and away" flight conditions. The flare law is a further advancement still, however the aircraft will revert to the $C^*u$ control law in the flare should the primary system fail. Both the primary flare law and reversionary $C^*u$ control laws have received very favourable responses from pilots who have flown the 777 simulator and the 757 fly-by-wire aircraft which was set up to replicate the 777 control laws through the right hand pilot's controls, references 5 and 6.

It is therefore evident that a pure $C^*$ based control law, to the specification of the original parameter, produces undesirable handling qualities.
9. CONCLUSIONS

A proportional feedback C* control law was applied to the Boeing 747-100 in landing approach configuration. Pitch rate feedback was used to augment the short period damping ratio while normal acceleration was used to augment the natural frequency.

The unaugmented aircraft response failed to comply with the lower boundary of the C* landing approach envelope, however the augmented aircraft utilising the C* proportional feedback controller complied with the same envelope. Both models met the requirements of MIL-STD-1797A.

It was concluded that the baseline 747 was not the most appropriate aircraft for this analysis as it is an inherently heavy and well behaved aircraft with little need for augmentation. In addition it does not employ any advanced aerodynamic design or higher order systems in its design, such as the C* criterion was designed to account for.

Deficiencies of the C* criterion were given. References to its deficiencies as a handling qualities metric were discussed, and it is concluded that pursuing analysis based on the C* criterion would seem to be of limited value for application to modern civil transport aircraft.

The use of C* controllers was also addressed. Pure C* controllers to the design of the C* parameter are rare, therefore indicating their deficiency for the task. Several control laws loosely based on C* are in use, however advantages and deficiencies of these systems are more a function of the control law used than of the C* parameter itself.

It is clear from this analysis that the control law structure is vitally important to good aircraft handling qualities, and should receive further attention.
REFERENCES


