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Preliminary Design Framework for the Power Gearbox in a Contra-Rotating Open Rotor

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Abstract

Geared contra-rotating open rotors have the potential to further reduce fuel consumption relative to geared turbofans, but require a more-complex speed-reducing transmission system to drive the propellers. Hitherto, the preliminary design for such transmission systems has been reported independently of the overall engine modelling or has been limited by many pre-determined engine constraints. This has restricted the feasible design space of the transmission systems. A simple transmissions preliminary design approach is needed that does not involve complicated mechanical assessments and can be integrated with engine preliminary design studies. This paper presents a novel design framework for sizing a double-helical differential planetary gearbox for a contra-rotating open rotor. An up-to-date methodology is proposed for the design of maximum load capacity gears for this application based on the power transmitted, durability and space envelope requirements. The methodology was validated with published data, demonstrating only very minor differences in geometry dimensions. Parametric analyses have been made to assess the impact of the design assumptions on gearbox dimensions. The framework also enables the identification of feasible torque ratios between the output shafts of the speed-reducer and the contra-rotating propellers driven by them. The impact of the torque ratio on the size of the gearbox has been analysed for equal propeller rotational speeds and different speed ratios between the output shafts. This study shows that potential torque ratios lie between 1.1 and 1.33, with the higher ratios enabling more compact gearboxes having four or five planet gears.

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1 Introduction

The sustainable economic growth of the aerospace industry requires innovative solutions. There have been numerous studies to investigate engine architectures optimized for low emissions, noise and weight, by increasing engine thermal and propulsive efficiencies [1]–[7]. The Contra-Rotating Open Rotor (CROR) is one of these architectures with the potential for improving propulsive efficiency as it benefits from low specific thrust and presents the capability to reduce the propeller speed and loading to maintain a high stage efficiency. The CROR is a gas turbine with unducted propellers driven through a transmission system or directly-driven by contra-rotating turbines.

The need for compact engines has led the designers to introduce speed-reducing transmission systems to avoid excessively-long multistage turbines [4]. Some key challenges of geared engine arrangements are the weight, efficiency, physical size and mechanical integrity of the transmission systems. The use of a power gearbox in large commercial turbofans over 100 kN take-off (T/O) thrust, to replace the direct link between the low-pressure turbine (LPT) and the fan is relatively new to the aerospace industry [8]. Entering service in 2013, the P&W PG1000G was the first high-thrust turbofan with a gearbox demonstrator rating of 30,000 hp (22.4 MW), three to four times higher than previous geared turbofans like the ALF 502 [9] and more than twice the power of the TP400-D6 turboprop propeller reduction gearbox [10].

The potential performance benefits of a CROR relative to single propeller engines and conventional turbofans have been extensively discussed in the literature, from acoustic [11]–[19], aerodynamic [11], [15], [20]–[23] and operability [13], [20]

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perspectives. However, the speed-reducer in a geared CROR is a critical component and a technology enabler for the engine architecture [24], [25]. Published research has tackled the preliminary design of the transmission system independently of the overall CROR modelling ([24]–[30]), without assessing its impact on the general arrangement.

The contribution of this paper is the development of a design framework for the planetary gears in a CROR, incorporating the engine performance requirements into the preliminary design of the transmission system. The transmission system and the CROR share a common preliminary design variable that is the torque ratio (*TR*) between the contra-rotating propellers, i.e. between the output shafts of the speed-reducer. The framework developed provides a means to determine the range of practical torque ratios for the smallest feasible gearset compatible with the power transmitted, durability and space-envelope requirements of the application. The study formed an initial stage of project DEMOS (Developing advanced Engine Multi-disciplinary Optimization Simulations [31]).

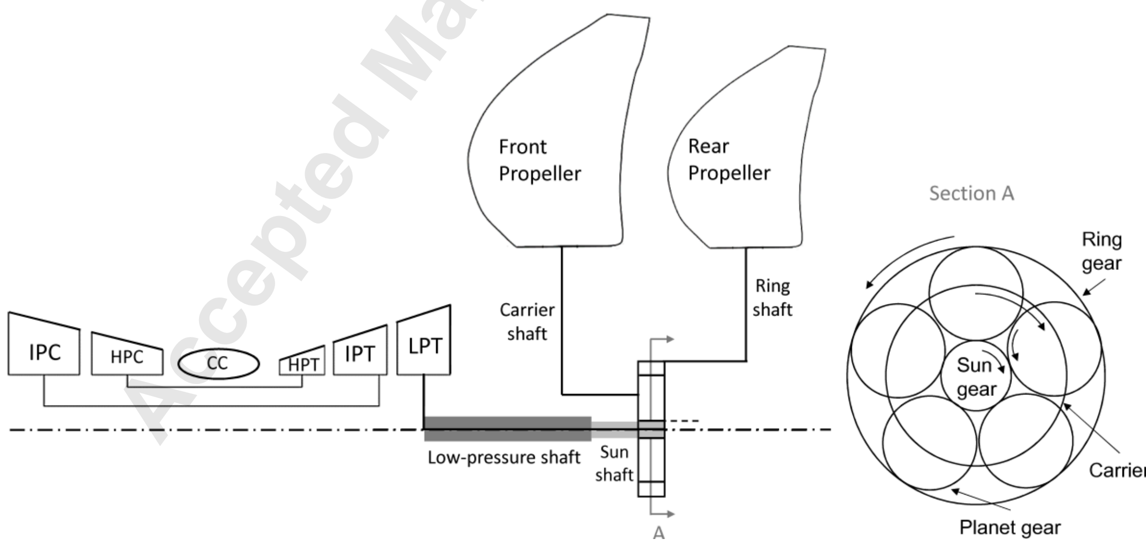

Figure 1 Sketch of a CROR and cross-section of a differential planetary gearbox with kinematics

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The transmission system application evaluated in this paper is a geared CROR pusher arrangement (Figure 1). The engine core includes a two-spool gas generator and a low-pressure system (the propulsor) comprising a free-power LPT driving co-axially mounted contra-rotating propellers through a transmission system that enables the LPT and the propellers to run at their optimum speeds and efficiencies. In the CROR, the propellers are the main source of thrust and determine the power to be transferred through the transmission system from the LPT. The design rotational speed of each propeller is limited in practice by compressibility losses and noise from the high-subsonic helical Mach numbers at the blade tips. Therefore, propeller rotational speeds are a design parameter of the engine model and are inputs to the preliminary design of the transmission system.

2 Transmission Systems in Geared Contra-Rotating Open Rotors

The CROR entails a more complex mechanical design than geared turbofans due to the contra-rotation of its output shafts. A planetary gear arrangement favours compact transmission designs as it enables load distribution between multiple load paths, corresponding to the number of planets [32]. Contra-rotation of the output shafts can be obtained by differential action of the gears (Figure 1), which can carry more power than a simple planetary configuration by virtue of a higher input rotational speed [33]. In the CROR, the sun shaft transmits the power and rotational speed from the low-pressure shaft to the sun gear, the carrier shaft links the centres of all the planets to one of the propellers and the ring shaft connects the annulus (exterior ring) gear to the other

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propeller. The gears are designed at take-off power level, where the torques are maximum [26], [27].

In the early design phases of a new transmission system, none of the gear dimensions is known and the selection of design inputs has a direct impact on the outcome. The main parameters affecting the size of the transmission system are the torque requirements, the relative rotational speeds of its shafts (linked to the ratio between gear diameters) and the design durability of the assembly. The relative diameters of the gears in a differential planetary gearbox (DPGB) determine the gear ratio and the achievable speed reduction between the input and output shafts. The rotational speeds of two out of the three shafts can be independently selected and determine the rotational speed of the third shaft. In the CROR, the propeller rotational speeds define the rotational speed of the LPT for a given ratio of gear diameters. However, the overall radial dimensions of the gearbox can only be calculated if the output (ring) gear diameter, or that of the input (sun) gear and the relationship between them, is known. The axial dimensions of the gears are also a function of the gear diameter ratio.

The volume of the DPGB constrains the possible locations of the transmission system in the CROR. The most common connection arrangements are sketched in Figure 2, where the DPGB could be located between the LPT and the front propeller, or between propellers.

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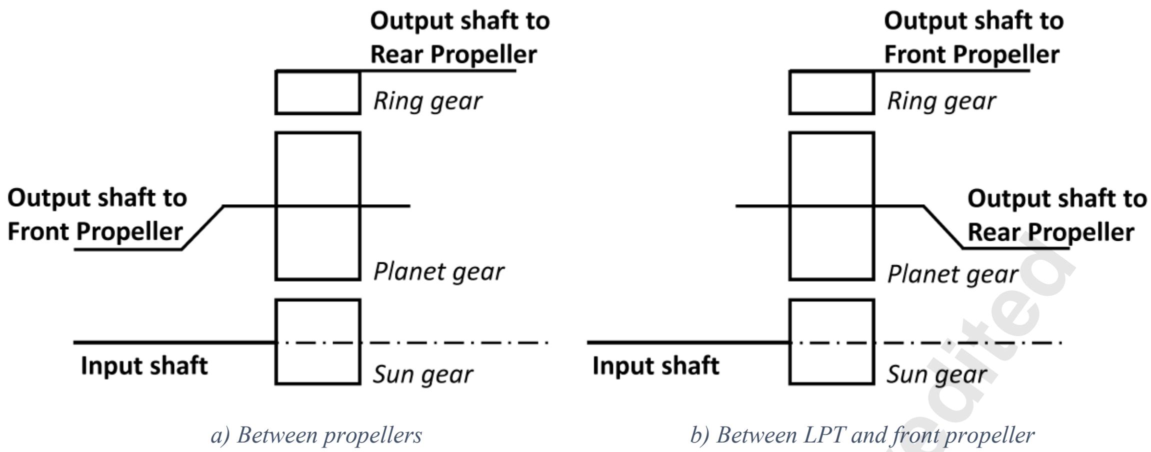

Figure 2 Common location and connection arrangements of the differential planetary gearbox in a CROR

Even though the design of a power-train for a CROR has been addressed in past research projects, publications are scarce and few specify the design methodology.

Table 1 Design investigations and parameters of differential planetary gearboxes for CROR application

Research Project	Design Level	DPGB Position in Engine	Power Class [hp]	Speed Ratio (SR)	Front/ Rear Prop RPM	Number of Planets in DPGB	DPGB Design Durability (MTBUR) [h]	Sun Gear Diameter [cm (in)]	Type of Gears	Planet Bearings
APET [Allison] [24]	Conceptual	Between gas generator and propellers	10,000	8.19:1	1329	4	30,000	11.6 (4.6)	Single helical	Tapered roller
APET [P&W] [25]	Conceptual	Between gas generator and propellers	12,000	8.23:1	1233	5	30,000	17.78 (7)	High contact ratio (HCR)	Single-row spherical roller
AGBT [P&W] [27]	Conceptual Detailed Testing	Between LPT and front prop	12,000	8.32:1	1235	5	30,000	18 (7)	High contact ratio (HCR)	Single-row spherical roller
AGBT [Allison] [26]	Conceptual Detailed Testing	Between LPT and front prop	13,000	8.33:1	1140	4	30,000	14.53 (5.72)	Double helical	Integral double-row cylindrical roller
DREAM [4]	Conceptual	Between propellers	20,000 (estimated value)	7.06:1	860	NA	NA	NA	NA	NA
KHI [28]–[30]	Detailed Testing	Between propellers	20,000	NA	NA	5	50,000	NA	Double helical	NA

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Table 1 provides a summary of the main gearbox design characteristics used in previous projects and enables the identification of the type of components to be used in advanced gearboxes and the feasible design margins for modern CROR applications. Double helical gears for the DPGB have been selected in this study to favour a compact design enabled by their load distribution capability and the cancellation of thrust forces. The lower stress levels in these gears contribute to the durability targets of a high-power CROR gearbox. These targets are defined through the Mean Time Between Unscheduled Removals (MTBUR), which is the standard measure of design life used for gear systems [34].

Gearbox technologies for a geared CROR were proposed during the 1980s in the Advanced Prop-Fan Engine Technology (APET) programme [24], [25]. The technologies were matured during the Advanced Gearbox Technology (AGBT) programme for high-power transmission and long design life at high shaft rotational speeds [26], [27]. The DPGB presented an unrivalled power-to-weight ratio for the CROR compared to other gearbox architectures as well as higher reliability associated with the smaller part-count and reduced size, which are linked to weight and cost reduction [24]–[27].

The design goals and life requirement of the gearbox were achieved in the AGBT programme by keeping the bearing and gear contact stresses low [26]. Of the bearings in the transmission system, the planet bearings experience the highest loads and would be custom-designed based on the stringent space-constraints imposed by the planet gears. The diameters of the planet gears are a function of the sun gear diameter and the gear ratio. In the DPGB for a CROR, it has been assumed that the smallest gear (the pinion) is

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the sun gear, which accumulates the largest number of fatigue cycles [35], and hence determines the dimensions of the CROR gears for the input power, torque and speed reduction ratio range [26]. Therefore, the gear dimensions provide a preliminary estimate of the volumetric requirements of the DPGB, subject to the availability of sufficiently-compact and adequate bearing technology.

Another research activity aimed at designing a compact, light and efficient DPGB transmission system for a CROR engine is described in reference [36]. The focus of these assessments was on the design selection of the transmission system and its performance evaluation. The in-line transmission was designed for a 15,000 hp (11.2 MW) power class engine with equal and opposite rotational speed propellers [36], [37]. The study provided a kinematic analysis of the gear system and a representation of the gearbox efficiency characteristics relative to input torque and rotational speed [37].

In the early 2000s, the dramatic rise in fuel prices and the urge for cleaner engine technologies rejuvenated the industry's interest in open rotor technology as one potential solution to those problems [38]. In DREAM (valiDation of Radical Engine Architecture systeMs) the performance and mechanical aspects of a DPGB were incorporated to the engine design setup for a geared CROR [4]. The unavailability of a mechanical preliminary design methodology for the transmission system was highlighted and a methodology was proposed to estimate its dimensions and weight based on the preliminary design of the gearset. The DPGB preliminary design was based on the mechanical strength calculation of the gears to comply with pitting and tooth breakage criteria, which were not specified. The final geometry of the gears in DREAM as per [4] was obtained iteratively only using

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standard modules (i.e. ratio of pitch diameter to number of teeth [39]) with specified design requirements. The width of the gears was also pre-selected. The most compact gear design may not have been obtained with the design constraints reported in [4], as identified below:

- In [4], the DPGB speed ratio was selected based on an estimated upper limit of the permissible stress levels for a three-stage LPT. While this limit must be taken into account in the design process, it should be used as a constraint rather than as an input for the DPGB sizing.
- The research reported in [4] considered the sun shaft diameter as an input to the design (obtained from the mechanical model of the LPT shaft) and set the value for the minimum sun gear diameter. However, as described in [26], the sun shaft is a quill shaft whose diameter may differ from the LPT shaft and can be adjusted to the gear requirements. This facilitates the DPGB design, as the vibration effects of a long, thin shaft operating at high rotational speeds, i.e. the LPT shaft, do not have to be taken into account for the preliminary volumetric assessment of the transmission system. If the constraint on the minimum sun gear diameter is removed and the sun pitch diameter is not known, the gearbox dimensions are undefined.
- The study reported in [4] considered the LPT last-stage hub diameter to be an upper limit for the gearbox outer diameter. However, if the correct area ratio is preserved between the outlet of the LPT and the engine core nozzle, the maximum radial dimensions of the transmission system would not need to fall within this limit.

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A schematic representation of the resultant differences in the available design space for the transmission system can be seen in Figure 3, based on the design approaches followed in DREAM, as per [4], and DEMOS.

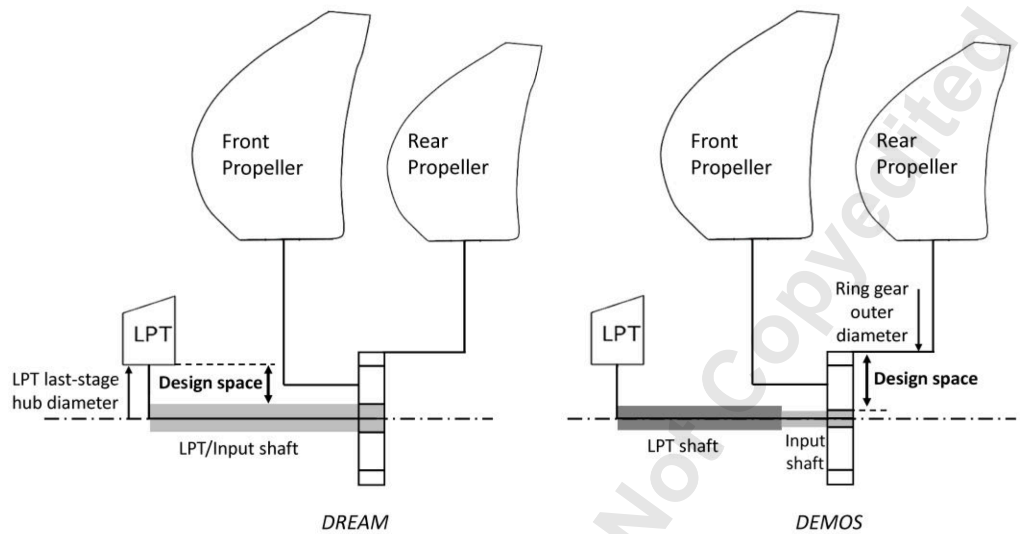

Figure 3 Increase in design space between DREAM [4] and DEMOS favoured by higher constraint flexibility

The development of a gearbox pre-design tool for aero-engines is reported in [40]. The gearbox geometry was obtained from engine specifications, with missing data obtained from correlations. The geometry of the gears was calculated iteratively, assessing the stress levels on the gears for a given number of teeth, tooth size, typical pressure and helix angles, and material, to comply with specified safety factors. Planet bearing sizes and shaft dimensions were estimated as a function of the planet gear design and user inputs. A range of gearbox designs was produced, varying the module and the number of teeth, and the changes in gearbox mass with compactness were evaluated. The tool was validated against internal and published data (NASA's AGBT gearbox [26]), with a maximum geometrical deviation of 3.54% stemming from the differences in normal

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module. No information is reported on the modelling of a gearbox for a CROR other than the validation case using information from the AGBT project ([26], [27]).

Kawasaki Heavy Industries (KHI) investigated a geared CROR engine and the design of its transmission system with a prototype of a DPGB developed to demonstrate a technology readiness level (TRL) appropriate for whole engine development [28]–[30]. Limited information is provided on the design of the DPGB prototype. The reduction ratio and the rotational speeds of the propellers and LPT are not specified and the design process is not described. However, these publications indicate the expected DPGB durability for modern CRORs.

Table 2 shows the areas of research addressed by previous projects. There is a clear need for an alternative approach to estimating the size of the DPGB in an open rotor application, which does not involve convoluted mechanical assessments. The mechanical design of the gearbox has previously either been treated in isolation ([28]–[30]) or not used to inform the engine model ([4], [26], [27]).

Table 2 Overview of methodology and scope of differential planetary gearbox research projects for CROR application

Project	Torque Ratio (*TR*) selection based on		Trade-off between CROR performance and DPGB size
	Performance	Mechanics	
AGBT [26], [27]	✓	✗	✗
DREAM [4]	✓	✗	✗
KHI [28]–[30]	–	✓	✗
DEMOS (this study)	✓	✓	✓

3 Gear Design Methodologies for Gearbox Sizing

Based on the literature reviewed, the gearset in a DPGB is assumed to determine the volumetric requirements of the power gearbox, which can comply with the power

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and durability specifications of the CROR. The size of the gears is a function of the pinion pitch diameter and the tooth count [25]. Two commonly-used preliminary design methods to estimate basic gear geometrical characteristics are the Q-factor method and applying gear manufacturing standards [35]. The Q-factor method is used to start the preliminary design of a gearset if previous experience exists for the specific application. Details of the method can be found in any mechanical design book (e.g. [35]).

If no experimental data are available, the estimation of stresses acting on the gears is one of the main challenges of preliminary gear design. Several assumptions have to be made to account for the effects of stress concentrations, and gear misalignment or tooth errors [35]. An alternative procedure for calculating the pinion pitch diameter for optimum load capacity is presented in [41]. The method is based on the classical Hertz equation for surface compressive stress on two curved surfaces [35]. It provides a means of estimating the preliminary gear design for a novel gearset based on allowable stresses when no previous experimental data are available. However, scoring factors and gear misalignment are not considered [25], resulting in optimistic designs.

The second parameter to consider for gear sizing is the tooth-count estimation. A closed-form procedure for designing minimum-weight spur and helical gearsets is presented in [42]. The pitch diameter is calculated based on surface fatigue following the approach in [41] and the optimum number of teeth is obtained simultaneously satisfying the surface fatigue and bending constraints in the pinion.

The understanding of gears has evolved since the formulation of these methods, as reflected in gear manufacturing standards [43]. The contribution of this paper to gear

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preliminary design is the updating of the methodologies presented in [41] and [42] and their incorporation to the design framework of the transmission system in a geared CROR pusher arrangement.

4 Novel Design Framework for Preliminary Design of a Differential Planetary Gearbox for Contra-Rotating Open Rotors

This study presents a new framework to calculate the space envelope of the transmission system suitable for a CROR application. The required inputs for the preliminary design of the transmission system are the transmitted-power from the LPT, the prescribed design life, the rotational speed of two out of the three rotating shafts and an initial value for the torque ratio (*TR*). The gearset is designed at end-of-runway (EOR) take-off condition (sea level, M_0 = 0.25, ISA+15 K).

There are two key elements of the proposed framework to size the DPGB: the development of the preliminary gear sizing methodology and the torque ratio (*TR*) selection between DPGB output shafts, i.e. between CROR propellers. In turn, the gear sizing methodology has two main steps:

- Sizing of the pinion pitch diameter based on torque, material selection, gear ratio, total derate factor, contact stress, normal pressure angle, helix angle and profile contact ratio of the gears.
- Calculation of the gear number of teeth based on pitting resistance and bending strength calculations.

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4.1 Development of the Preliminary Gear Sizing Methodology

4.1.1 Calculation of the Pinion Pitch Diameter

The minimum pitch diameter for the pinion gear is a function of the applied load and the allowable surface compressive stress and can be obtained for maximum load capacity from the classical Hertz equation [44]. Therefore, the pitch diameter for the pinion helical gear in a planetary gearset can be calculated from Eq. (1) [41].

$$d \geq \left[\frac{0.7 \dfrac{Q}{q} E (m_G + 1) \cos^2 \Psi \, K_d}{S_c^2 \left(\dfrac{F}{d} \right) m_G \sin \varphi_n \cos \varphi_n m_p} \right]^{\frac{1}{3}} \tag{1}$$

Q is the torque on the pinion gear, *q* is the number of power paths (i.e. the number of planets), *E* is the modulus of elasticity, m_G is the gear ratio, Ψ is the helix angle, K_d is the total derate factor [44], S_c is the contact stress experienced by the pinion gear, *(F/d)* is the gear aspect ratio, φ_n is the normal pressure angle and m_p is the profile contact ratio. The constant 0.7 derives from the elastic coefficient (C_p) relating the material properties of mating gears, assuming both are made of the same material and have a Poisson ratio of 0.3 as for most steels [43]. A further explanation of these terms is provided in the following sections.

The substitution of each of these parameters in Eq. (1) determines the minimum diameter of the sun (pinion) gear capable of complying with the power transmission and durability requirements of the application. Subsequently, the radial and axial dimensions of all the gears in the gearset, i.e. planet gears and external gear, can be calculated. The pitch radii of the planet gears equals the product of the gear ratio (m_G) and sun radius

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(R_S). The radius of the external gear or ring gear (R_R) results from the addition of the sun radius and the planet radii (R_P) as shown in Eq. (2).

$$\mathrm{R}_R = R_S + 2R_P \qquad (2)$$

Finally, the face width (F) of the gearset is approximated from the product of the resultant pinion pitch diameter (d) and the gear aspect ratio (F/d) assumed appropriate for the aerospace sector.

This process represents a first step to assessing whether a transmission system is suitable for a specific CROR application. The following sections explain the parameters defining the pinion pitch diameter (d) in Eq. (1) and present the selection process for the appropriate values.

4.1.1.1 Torque (Q)

The propeller loads of the CROR (thrust, weight and pitch couples) are absorbed by the gearbox housing and the carrier and ring shafts, substantially isolating the gearset to ensure the desired design durability [26]. Consequently, the torque transmitted from the LPT to the sun gear is the only load considered for the preliminary mechanical design of the pinion gear.

4.1.1.2 Material Selection

The type of material determines the behaviour of the gear under loading. It has a direct impact on the allowable stress of the gears and ultimately on their dimensions. Aerospace gears used in primary drive systems, such as the power gearbox in a CROR, are manufactured from heat-treated steel alloy materials and are surface-hardened to ensure resistance to pitting and to provide tooth bending strength [45]. In DEMOS, a generic

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carburized and hardened Grade 3 (premium-quality) steel is assumed to be representative of high-grade aerospace gears [46]. The choice of this type of carbon steel leads to maximum allowable contact and bending stress values of 275,000 psi (1896 MPa) and 75,000 psi (517 MPa) respectively [43] and an elastic constant (E) of 30 Mpsi (207 GPa) [47].

4.1.1.3 Gear Ratio (m_G)

The gear ratio m_G is defined as the ratio between the larger and the smaller number of teeth (N) in a pair of gears, proportional to the ratio of their pitch-circle radii (R), which is never less than 1.0 [39], and is a function of the pinion pitch-circle diameter (d) and the centre distance at which the gear pair operates. Therefore, the gear ratio, m_G, for the CROR application can be defined as the ratio of the planet (P) to sun (S) pitch-circle radii. Following this definition, the gear ratio (m_G) can also be expressed as a function of the external gear radius (R_R) through the gearbox ratio ($R_{DPGB} = R_R/R_S$).

However, in the preliminary design of the DPGB for the open rotor engine, there might not be enough information regarding the dimensions of the gears. In the absence of this information, the gear ratio m_G can be expressed as a function of an engine-related parameter such as the torque ratio between the propellers (TR) as presented in Eq. (3).

$$m_G = \frac{N_P}{N_S} = \frac{R_P}{R_S} = \frac{R_{DPGB} - 1}{2} = \frac{TR - 2}{2(1 - TR)} \quad (3)$$

The torque ratio (TR) between the output shafts is defined in Eq. (4) as the nominal ratio between the torque in the carrier (Q_C) and ring (Q_R) shafts, connected to the front (FP) and rear (RP) propellers respectively. This definition means the TR is always greater

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than one, and the propeller connected to the carrier shaft always experiences the highest torque.

$$TR = \left|\frac{Q_C}{Q_R}\right| = \left|\frac{Q_{FP}}{Q_{RP}}\right| = \frac{2(1+m_G)}{1+2m_G} = \frac{R_{DPGB}+1}{R_{DPGB}} \quad (4)$$

Due to the geometrical relations of the gears in a DPGB, an even torque split between the two propellers is not possible. An initial value of 1.33 is selected for *TR*, which corresponds to an m_G of 1.01 as per Eq. (3), as in previous DPGB assessments for open rotors [26]. Higher *TR* figures are possible, but are not considered here as the planet gears rather than the sun gear may then become the life-limiting gear component. The value of m_G is linked to the torque ratio between the output shafts as shown in Figure 4.

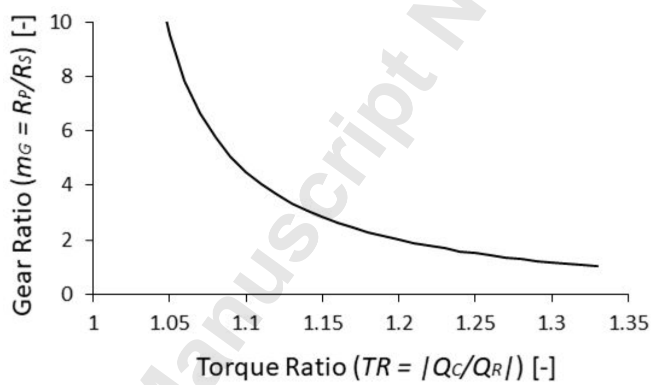

Figure 4 Gear ratio (m_G) against torque ratio (TR) for the differential planetary gearbox

4.1.1.4 Number of Planets (q)

The number of planets in the DPGB is a function of the gear ratio (m_G) and the number of teeth in the meshing gears [48]. In the absence of information on the number of teeth, a factor K_q is introduced in Eq. (5) to calculate the number of planets (q) and prevent possible meshing interference between planets. A sensitivity analysis on K_q indicates that a value of 0.96 results in a big number of small teeth and a value of 0.92 leads to a small number of larger teeth. Therefore, a suitable value of K_q for the CROR

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application would be 0.94. The maximum number of planets (q) in the DPGB will be the next lower integer from Eq. (5), which can also be expressed as a function of the torque ratio (TR).

$$q = \frac{K_q \pi}{\operatorname{asin}\left(\frac{m_G}{1 + m_G}\right)} = \frac{K_q \pi}{\operatorname{asin}\left(\frac{2 - TR}{TR}\right)} \quad (5)$$

4.1.1.5 Total Derate Factor (K_d)

The total derate factor [44] accounts for the design imperfections in the gear design and is defined in Eq. (6). There are different sources of derate, which can be grouped into the overload, dynamic, size, and load distribution factors (K_o, K_v, K_s, and K_m respectively). These factors are commonly selected based on the analysis of actual operating conditions and the aerospace industry will have established proprietary values based on experience. Therefore, the values proposed in this paper are estimates of factors that could be used for gears in a modern contra-rotating open rotor application.

$$\mathrm{K}_d = \mathrm{K}_o \mathrm{K}_v \mathrm{K}_s \mathrm{K}_m \quad (6)$$

The selected values are for a highly loaded, high-speed aero gear application and are defined as follows:

- The overload factor (K_o) considers the effect of all externally applied loads in excess of the nominal tangential load [43]. The power source to the DPGB in this study is considered to be a gas turbine with a free power turbine running at constant-speed (at take-off, i.e. design point) and the driven elements are contra-rotating propellers, which are assumed to cause only a light shock to the operating conditions, leading to a value of K_o = 1.25 [49].

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- The dynamic factor (K_v) makes allowance for internally generated gear tooth loads, which are induced by non-conjugate meshing action of the gear teeth, with values ranging between 1.09 and 1.15 for higher to lower accuracy grades, i.e. tolerance levels [50]. This study assumes maximum precision of the gear design, hence a value of K_v = 1.09.
- The size factor (K_s) accounts for the unevenness in material properties [43]. For aerospace helical gears, the gear size is not considered to have a detrimental effect, hence K_s has a value of unity [45].
- The load distribution factor (K_m) modifies the rating equations to reflect the non-uniform distribution of the load along the lines of contact [43]. Following the calculation procedure in [43], a value of K_m = 1.4 is used in this study.

The resultant total derate factor can be calculated substituting the different factors in Eq. (6). The selected values apply to any novel aero-engine gear designs with similar technology and application characteristics. Therefore, a resultant value of K_d = 1.9 is used to calculate the pinion pitch diameter from Eq. (1).

4.1.1.6 Contact Stress (S_c)

If the size of the pinion gear were known, the contact stress (S_c) would be calculated using the information on the material properties, the geometry of the gear and the transmitted torque [43]. In the absence of the geometry, S_c is assumed to equal a derated value of the allowable stress (S_{ac}) of the material as per Eq. (7). The derate factors for contact stress account for uncertainties in the design and are commonly referred to as "safety factors". However, some of these uncertainty factors have been removed from

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the unknown area of the pitting safety factor (S_H) and are considered to be predictable within the design method [43].

$$S_c \leq \frac{S_{ac} Z_N C_H}{S_H K_T K_R} \tag{7}$$

The hardness factor (C_H), temperature factor (K_T) and reliability factor (K_R) have a value of unity for the application under analysis (refer to [43] for further details). The pitting resistance factor (Z_N) adjusts the values of S_{ac} for gear design lives other than 10^7 cycles and accounts for the relationship between the stress and the number of cycles (S-N characteristics) for the gear material. In high-speed helical gears, the value of Z_N, as expressed in Eq. (8), should not exceed 0.68 [50] when the design life exceeds 10^{10} cycles as would be the case in modern CRORs.

$$Z_N = 2.466 n_L^{-0.056} \leq 0.68 \tag{8}$$

The number of cycles (n_L) is a function of the gear design life (L) in hours, the rotational speed of the pinion (n_S) in RPM and the number of contact paths (q), equivalent to the number of planets in the DPGB. It is apparent from the expression in Eq. (9) that the higher the rotational speed of the pinion and the design life requirements, the bigger the number of cycles experienced by the gear.

$$n_L = 60 L n_S q \tag{9}$$

4.1.1.7 Gear Aspect Ratio (F/d)

The gear aspect ratio represents the relationship between the face width (F) and the pitch diameter (d) of the pinion gear. The width of the groove or gap between the helical gears is not included in the face width that makes contact with a mating gear, so the axial length of the double-helical gears will be greater than F. For industrial gearsets,

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an initial approximation of the gear aspect ratio (*F/d*) for external double-helical gears, as a function of the gear ratio (m_G) is proposed by AGMA in [51], as shown in Eq. (10). This approximation favours gearbox designs with face widths larger than the pitch diameter of the pinion gear.

$$\frac{F}{d} = \frac{2m_G}{m_G + 1} \tag{10}$$

Wider gears would offer a higher resistance to pitting fatigue and result in smaller overall radial dimensions for the DPGB for the same gear ratio (m_G), as indicated by the squared markers in Figure 5.

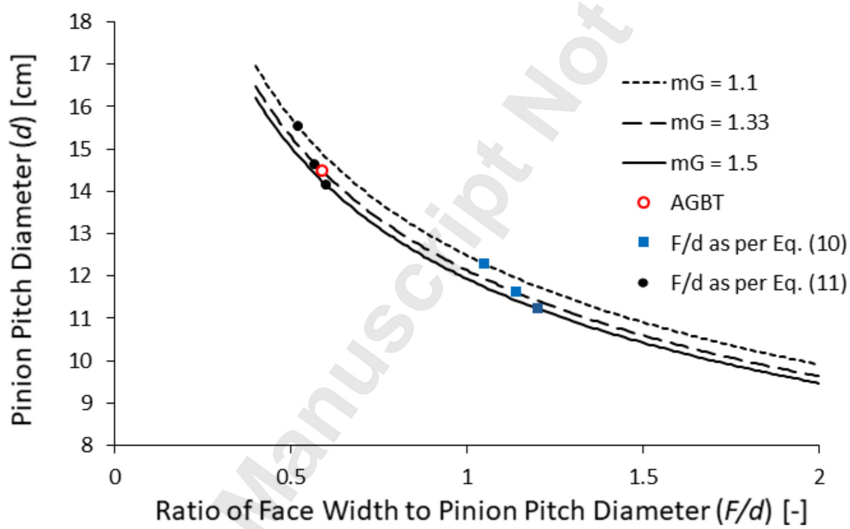

Figure 5 Variation of pinion pitch diameter (d) for DEMOS with gear aspect ratio (F/d), showing F/d values for the AGBT gearbox [26] *and as per Eqs. (10) and (11)* [51]

However, the trend followed by the aerospace industry favours narrower face widths, with values of the gear aspect ratio (*F/d*) around 0.6. Examples of this trend are the power gearbox in the PW1100G geared turbofan (approximately 0.62 [52], [53]) or the DPGB in the NASA AGBT project [26] (approximately 0.6) shown in Figure 5. Equation (11) is adopted in this study instead to better correlate the gear aspect ratio with the gear

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ratio (m_G) for double-helical aerospace gears, which corresponds to the estimation proposed by AGMA in [51] for a single helical gear.

$$\frac{F}{d} = \frac{m_G}{m_G + 1} \tag{11}$$

One reason behind this choice for aero gas turbines stems from the need for compact engine designs, where the length and space between components are minimised because longer engines result in weight, performance and installation penalties.

4.1.1.8 Normal Pressure Angle (φ_n)

The pressure angle in the normal plane of a helical tooth is referred to as φ_n [39]. The normal pressure angle represents the angle at which the load is transmitted from one tooth to another one in mesh. High values of φ_n can provide higher load capacity, but also worse smoothness and noisier gear meshing [26]. For high-speed helical gears, the value of φ_n should not be greater than 25° [50]. For this study, a value of 22.5°, as proposed in [26], is selected. The combination of φ_n and helix angle (Ψ) of the gears has an impact on the profile contact ratio (m_p), as shown in Figure 6, which will also affect the pitch diameter of the pinion gear (d) according to Eq. (*1*).

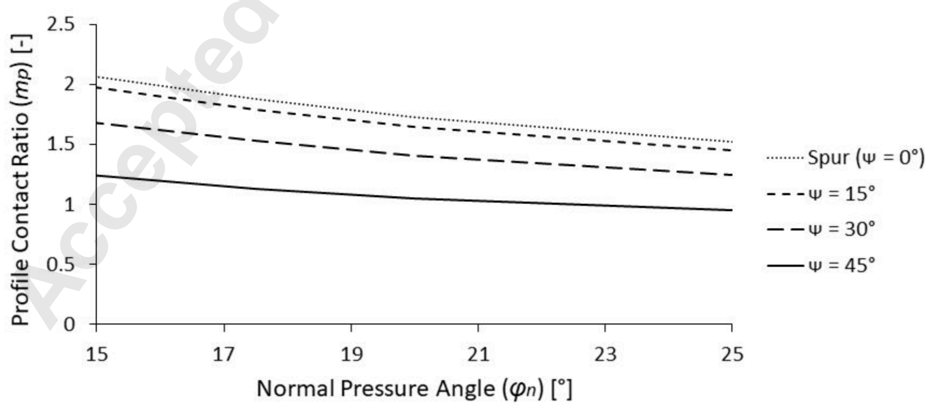

Figure 6 Profile contact ratio (m_p) against normal pressure angle (φ_n) for different helix angles (Ψ) (adapted from Table 4.5 in [35]*)*

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4.1.1.9 Helix Angle (ψ)

The helix angle represents the tooth tilt in the lengthwise direction of the gear [35]. High helix angles increase the load sharing capacity of the gear and must be of the same degree at each mesh. The selection of the helix angle is a compromise between the strength of the teeth and smooth operation of the gears. Double-helical gears cancel out the thrust loads and generally allow higher helix angles than single-helical gears. For advanced high-speed helical gear units, helix angles typically range from 30° to a maximum of 45° [35]. A high helix angle, $\psi = 42.5°$ was selected in this study to achieve a compact design for the CROR application.

4.1.1.10 Profile Contact Ratio (m_p)

The profile contact ratio represents the number of teeth in contact at each mesh cycle in the plane tangent to the tooth surface. It has a big impact on gear design as it determines the teeth excitation, hence vibration level and noise [44]. A higher number of teeth in contact reduces the load carried by each pair of teeth and reduces the dynamic forces during engagement and disengagement. The relationship between the profile contact ratio and the normal pressure angle on the teeth is presented in Figure 6 for different helix values.

4.1.2 Calculation of the Number of Gear Teeth

The calculation of the number of teeth in a gearset must simultaneously comply with tooth resistance to contact and bending stress requirements. It is common practice to use standards such as AGMA 2001-D04 [43] to rate gear teeth, i.e. determine the allowable transmitted power for pitting resistance and bending strength. For each gear

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mesh, the power rating expressions in [43] can be re-arranged to define constants for pitting resistance (K_c), in Eq. (12), and bending strength (K_t), in Eq. (13) [42]. These constants are proportional to the product of face width (F) and pitch diameter (d) squared, which is a simplified representation of the solid gear volume and is indicative of the gear weight [54].

$$K_c = Fd^2 = \frac{Q}{q}\frac{1}{I}K_d C_f \left(\frac{C_p}{S_c}\right)^2 \tag{12}$$

$$K_t = \frac{Fd^2}{N} = \frac{Q}{q}\frac{1}{J}K_d\frac{1}{S_t} \tag{13}$$

N is the number of gear teeth, Q is the torque on the gear, q is the number of load paths (number of planets in the DPGB), K_d is the total derate factor, C_f is the surface condition factor for pitting resistance, which equals unity assuming appropriate surface conditions [43], C_p is the elastic coefficient [43], S_c is the allowable contact stress, and S_t is the allowable bending strength. I and J are the geometry factors for pitting and bending resistance respectively. For external helical gears, such as the sun gear in the DPGB, [51] indicates a value of 0.5 for J and the approximation of I as per Eq. (14).

$$I = \frac{1 + 0.00682\varphi_n}{4.0584}\frac{m_G}{m_G + 1} \tag{14}$$

φ_n is the normal pressure angle in degrees and m_G is the gear ratio. If the size of the pinion gear were known, the bending stress (S_t), defined in Eq. (15), would be calculated using the information on the material properties, the geometry of the gear and the transmitted torque [43]. In the absence of the geometry, S_t is assumed to equal a derated value of the allowable stress (S_{at}) of the material. Some of the uncertainty factors

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(i.e. derate factors) have been removed from the unknown area of the bending safety factor (S_F) and are considered to be predictable within the design method [43].

$$S_t \leq \frac{S_{at} Y_N}{S_F K_T K_R} \tag{15}$$

The temperature factor (K_T) and the reliability factor (K_R) have a value of unity for the application under analysis [43]. The bending resistance factor (Y_N) adjusts the values of S_{at} for gear design lives other than 10^7 cycles and accounts for the relationship between the stress and the number of cycles (S-N characteristics) for the gear material. In high-speed helical gears, the value of Y_N, defined in Eq. (16), should not exceed 0.8 [50] when the design life exceeds 10^{10} cycles, where n_L is the number of cycles as defined in Eq. (9).

$$Y_N = 1.6831 n_L^{-0.0323} \leq 0.8 \tag{16}$$

The approach proposed in [42] defines the optimum number of teeth in the pinion gear (N_S), for optimum load capacity, as the ratio between the constants for pitting resistance (K_c) and bending strength (K_t), resulting in Eq. (17).

$$N_S = \frac{K_c}{K_t} = \frac{J}{I}\left(\frac{C_p}{S_c}\right)^2 S_t \tag{17}$$

There is a potential issue in service, mainly relating to tooth bending stress, if the same pairs of teeth keep meeting too regularly. This is often avoided by choosing the tooth count of one of the gears as a prime number. The remaining gears in the set require tooth numbers that are not multiples of the chosen prime to avoid pairs of teeth on different wheels continually meshing with each other. This would not be a problem if every tooth were perfect and equally spaced to its neighbours, but in practice no tooth is perfect, so the aim of the designer should be to reduce the potential for uneven tooth

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wear. However, this practice restricts designs to a more limited range of discrete torque ratios and, as in the case of the AGBT [26], is not always applied. Small numbers of teeth should be avoided to minimise undercutting of the teeth on external gears, and large numbers should be avoided because smaller teeth are more sensitive to tolerances.

The number of teeth in the planet gears can be calculated by rearranging Eq. (3), and from Eq. (18) for the ring gears.

$$\mathrm{N_R} = \mathrm{N_S}(2\mathrm{m_G} + 1) \tag{18}$$

At this point, the preliminary gear design can be determined based on the calculated pinion pitch diameter (*d*) and its tooth number (N_S).

4.2 Output Shafts Torque Ratio Selection

The torque ratio (*TR*) is the ratio of torque in the carrier, connected to the front propeller, to torque in the ring shaft, connected to the rear propeller. The approach proposed in this study for the selection of the torque ratio combines the mechanical requirements from the gearbox design with the engine performance specifications.

To understand how the torque ratio relates to the rotational speed of the output shafts and the geometry of the gearset, it is necessary to introduce the basic equations defining the operation of a differential planetary gearbox, Eqs. (19) to (23). The power (*P*) balance in a DPGB defined in Eq. (19) is expressed in Eq. (20) as a function of the torque (*Q*), and rotational speed (*n*) of the sun (*S*), ring (*R*) and carrier (*C*) shafts and the mechanical efficiency of the gearset (η_{DPGB}).

$$\eta_{DPGB} P_S + P_C + P_R = 0 \tag{19}$$

$$\eta_{DPGB} n_S Q_S = n_R Q_R + n_C Q_C \tag{20}$$

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When the carrier rotates coaxially with the input (sun) shaft, the planets not only revolve about their axes but also orbit and rotate about the centre of the DPGB assembly, as seen from the kinematics of the system shown in Figure 1. Assuming the same force is experienced by the teeth in all the gears in the transmission system, Eq. (20) can be written in terms of the gear pitch radii (R_S, R_P and R_R for the sun, planet and ring gears respectively) as shown in Eq. (21).

$$\eta_{DPGB} n_S R_S = n_R R_R + n_C (R_S + R_R) \tag{21}$$

Therefore, the torque on the carrier (Q_C) and ring (Q_R) shafts are expressed, in Eq. (22) and Eq. (23) respectively, as a function of the input torque (Q_S), the gear ratio (m_G) and the mechanical efficiency of the gearset (η_{DPGB}).

$$Q_C = 2\eta_{DPGB} Q_S (1 + m_G) \tag{22}$$

$$Q_R = \eta_{DPGB} Q_S (1 + 2m_G) \tag{23}$$

The term gear ratio is used in the researched literature as the ratio between the rotational speed of the input and output shafts, assuming that both propellers rotate at the same nominal speeds. However, the definition of gear ratio (m_G) in Eq. (3) has been adopted for this study, as proposed in gear standards [39]. The speed reduction in the DPGB is, therefore, expressed through the speed ratio (SR) defined in Eq. (24) as the ratio between the nominal rotational speed of the sun and ring shafts.

$$SR = \left|\frac{n_S}{n_R}\right| \tag{24}$$

If both propellers rotate at the same nominal speed ($|n_C| = |n_R|$), Eq. (25) represents the relationship between the torque ratio and the speed ratio (SR).

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$$TR = \frac{SR + 1}{SR - 1} \tag{25}$$

However, the definition of torque ratio presented in Eq. (25) does not contemplate the possibility of having different rotational speeds for the propellers in the CROR ($|n_C| \neq |n_R|$). Therefore, Eq. (26) presents a more generic expression of the torque ratio (TR).

$$TR = \frac{SR + 1}{SR - \left|\frac{n_C}{n_R}\right|} \tag{26}$$

5 Results

5.1 Validation of the Gear Sizing Methodology

The predictive capability of the proposed gear-sizing methodology has been validated against the geometrical data provided in [26] for the AGBT project. The double-helical gears in [26] presented a pressure angle (φ_n) of 22.5°, a helix angle (ψ) of 26°, a profile contact ratio (m_p) of 1.31, and an approximate gear aspect ratio (F/d) of 0.6. The power requirements, speed ratio (*SR*) and rotational speed of the propellers are the inputs to the kinematic model. The substitution of *SR* in Eq. (25) allows the calculation of the torque ratio (*TR*) and subsequently the gear ratio (m_G) from Eq. (3). The material selected in [26] for the pinion (sun) gear was CBS 600, whose properties have been approximated in this study as a Grade 2 case-hardened, carburized steel alloy. The choice of this type of carbon steel led to values of maximum allowable contact and bending stress of 225,000 psi (1,551 MPa) and 65,000 psi (448 MPa) respectively [43] and an elastic constant (E) of 30 Mpsi (207 GPa). The derate factor (K_d) is calculated using the values

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presented in previous sections of this paper. Finally, the pitting safety factor (S_H) is calculated to match the pinion pitch diameter (d) of the gearset in [26]. Subsequently, the bending safety factor (S_F) is calculated to match the number of teeth in the pinion gear (N_S). The outcome of these calculations is a 0.6 % difference in the calculated pinion pitch diameter compared to the AGBT pinion gear as per [26]. Based on these results, the upgraded methodology proposed in this paper for the sizing of the DPGB is deemed to be validated and appropriate for the CROR application using a gearbox arrangement similar to the one in reference [26].

5.2 Parametric Assessments of the Gearbox Preliminary Design

Sensitivity analyses of the parameters affecting the DPGB preliminary design have been performed to overcome the limitation of having only a single validation case and to assess the impact of the assumptions on the design. These assessments have been performed on the DPGB for the contra-rotating open rotor in DEMOS [31] with a gearbox design power requirement at take-off of 20,000 hp (14.9 MW), and both propellers rotating at the same nominal speed (similar to [4]) unless otherwise specified. The size variation of the gearset has been evaluated for the different parameters defining the size of the pinion pitch diameter (d) in Eq. (1), and a description of their effect on the preliminary gearbox design is provided below.

5.2.1 Gear Design Life and Number of Planets

The design life (durability) selected for the DPGB, together with the number of planets and the rotational speed of the input shaft, directly affects the number of load cycles that the pinion gear experiences, as expressed in Eq. (9). Longer times between

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unscheduled removals (i.e. higher gear design lives) lead to bulkier gearbox arrangements, as shown in Figure 7.

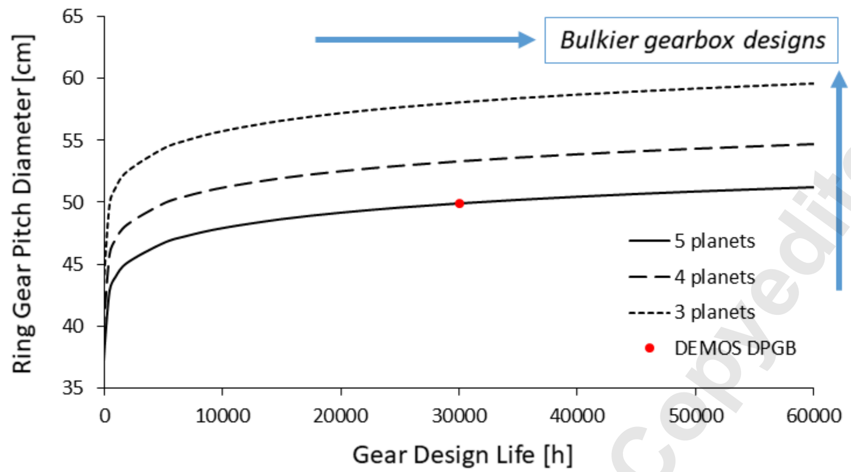

Figure 7 Variation of the pinion (sun) gear diameter for DEMOS with the prescribed gear design life

The design life of transmission systems reported in the literature for open rotor applications ranges from 30,000 h to 50,000 h for older and more modern designs respectively. A prescribed life of 30,000 h has been adopted, as indicated in Figure 7. It is also apparent that the presence of a higher number of planets results in more compact gearbox designs. For the gear design life of 30,000 h, a DPGB with 5 planets is 7 % smaller in pitch diameter than one with 4 planets and 16 % smaller than one with 3 planets.

5.2.2 Torque and Material Selection

The transmitted power, proportional to the transmitted torque (Q), and the selected gear material also have a direct impact on the size of the gearbox. The torque in the sun shaft is an output of the engine performance calculations. It is apparent from Eq. (1) that higher torque values lead to larger gear designs. Additionally, assuming all gears are made of the same steel, the use of materials with a higher surface hardness and modulus of elasticity (E), i.e. stiffer materials, results in higher allowable contact stresses

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on the gears. Consequently, more compact designs can be obtained if advanced steel materials (e.g. Grade 3) are used in future aero applications.

5.2.3 Normal Pressure Angle, Helix Angle and Profile Contact Ratio

The variation of the normal pressure angle (φ_n), helix angle (ψ), and profile contact ratio (m_p) of the gears needs to be assessed in combination. The profile contact ratio reduces with the increase of the gear tooth normal pressure angle and helix angle (Figure 6). The impact of this variation on gear diameter can be seen in Figure 8, where it is apparent that higher values of φ_n and ψ favour more compact designs.

A normal pressure angle of 22.5° and a helix angle of 45° have been selected for the DPGB preliminary design in DEMOS to achieve a compact design. However, high helix angles will impose a high degree of twist on the ring section, which presents a slender gear for its diameter. This could be countered with an axially-offset ring outer mounting and increasing the stiffness of the ring gear, but this will affect the system's ability to minimise external-load-induced imposed structural distortion. Higher helix angles will also result in higher heat generation stemming from the more highly-loaded sliding contact for a given pressure angle. The presented methodology, therefore, provides a means of assessing the impact of variations in normal pressure angle, helix angle and profile contact ratio, on the preliminary gearbox design, but further refinements in the selection of these parameters would require the use of more specific tools.

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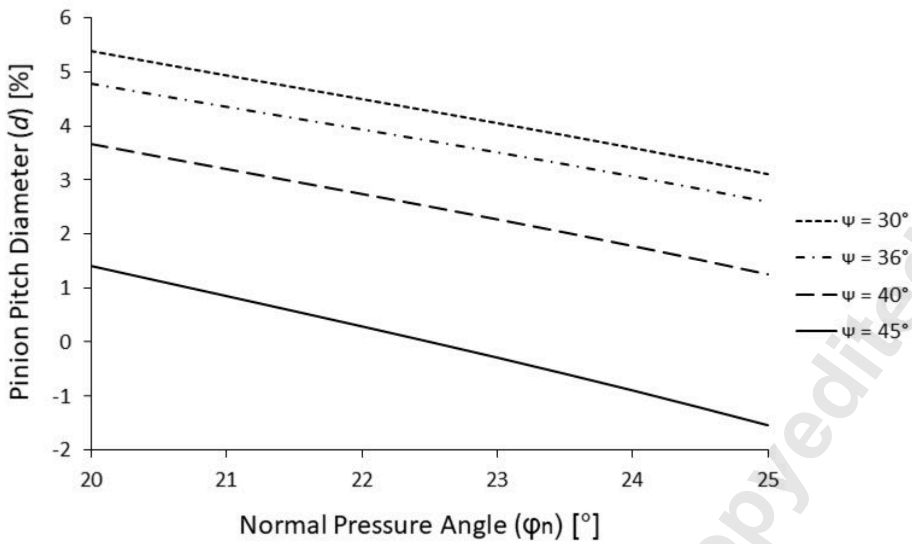

Figure 8 Variation in pinion (sun) pitch diameter with normal pressure angle (φ_n) and helix angle (ψ)

5.2.4 Gear Ratio and Torque Ratio

The gear ratio (m_G) of the gearset determines the power or torque split between the gears, as well as the speed reduction that can be achieved with the transmission system. The size variation of the differential planetary gearbox has been assessed as a function of these parameters for two main cases: (1) output shafts with equal rotational speeds ($|n_C| = |n_R|$), and (2) output shafts with different rotational speeds ($|n_C| \neq |n_R|$). An evaluation of the effects of torque ratio (*TR*) on the gearbox preliminary design for the case of unequal rotational speeds of the propellers has not been found in the open literature. Therefore, the presented results could be informative for future performance and noise studies, where the torque ratio design objectives may be conflicting.

5.2.4.1 Output Shafts with Equal Rotational Speeds ($|n_C| = |n_R|$)

The torque ratio (*TR*) between the output shafts cannot be higher than 1.33 if the sun gear is the pinion as deduced from Eq. (3). The characteristics of the DPGB also

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preclude an equal torque split between the output shafts, so the *TR* must be higher than unity.

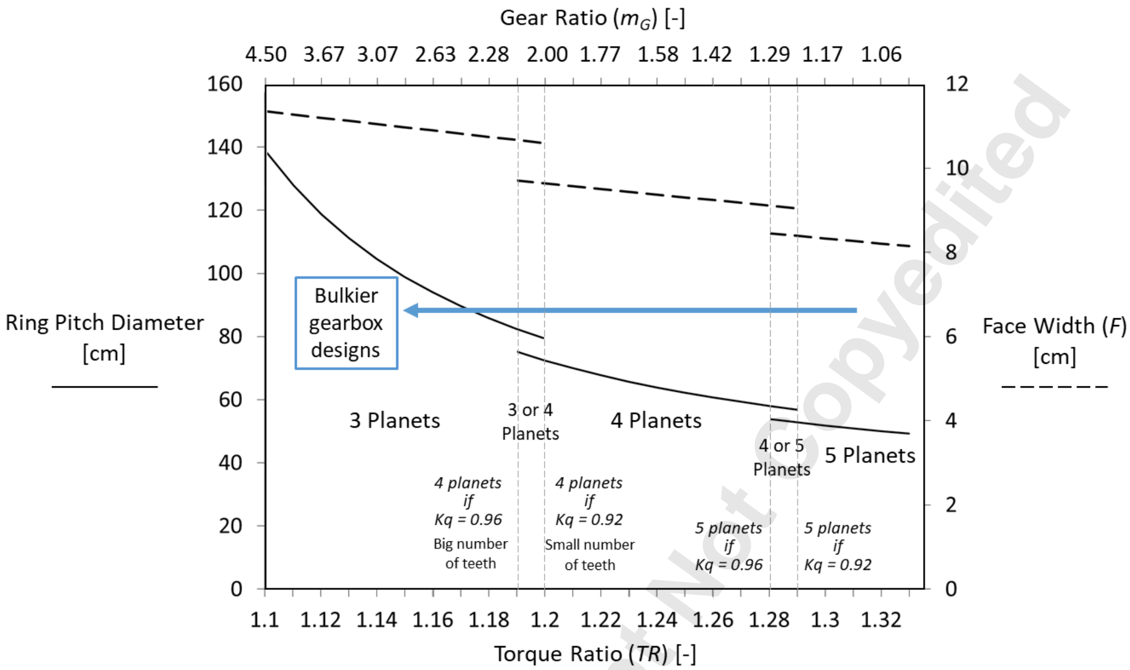

Figure 9 Variation of the ring pitch diameter and face width (F) in DEMOS with torque ratio (TR)

Figure 9 shows the variation of radial (ring pitch diameter) and axial (face width) dimensions of the DPGB with gear ratio (m_G) and torque ratio (*TR*) between the output shafts connected to the propellers. The radial dimensions of the DPGB, considerably higher than the axial ones, may determine the possible locations of the gear train in the CROR. Higher torque ratios result in more compact designs, where the sun is the pinion in the gearset up to a value of *TR* = 1.33. For higher values of *TR* the planets in the DPGB would be the smallest gears and the methodology proposed in this paper should be applied to them instead.

From the perspective of the transmission system pre-design, a practical lower limit for the torque ratio could only be obtained if a more detailed analysis of its components

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would be performed with more advanced tools. The speed reduction capability of the gearbox increases with decreasing values of *TR*, increasing the rotational speed of the LPT shaft if the rotational speed of the propellers is kept constant. However, in the CROR, the centripetal stresses generated in the LPT would also impact the mechanical feasibility of the propulsor. A possible upper limit in the speed ratio (*SR*) of 10 has been proposed in [55] for the DPGB in a CROR application "for questions of mechanical feasibility". This corresponds to a minimum *TR* of 1.22 as per Eq. (25), i.e. a maximum gear ratio (m_G) of 1.75 and a gearbox ratio (R_{DPGB}) of 4.5.

5.2.4.2 Output Shafts with Different Rotational Speeds ($|n_C| \neq |n_R|$)

If the carrier shaft is connected to the front propeller and rotates faster than the ring shaft, a given speed ratio (*SR*) can be achieved at a higher torque ratio (*TR*) resulting in more compact gearbox designs (Figure 10). Higher torque ratios correspond to lower gear ratios (m_G) and a reducing difference between the diameters of the sun and planet gears as m_G approaches unity. The resultant smaller planet gears would experience higher pitch-line and orbital velocities with higher tooth bending stress and higher centripetal loads imposed on the planet bearings, which could be detrimental to their life.

Figure 10 shows that the higher the speed ratio (*SR*), the smaller the torque difference between the propellers. A specified gear ratio (m_G) can only be obtained with one ratio of output rotational speeds for each gearbox speed ratio (*SR*). The *SR* of 10, can be achieved with four or five planets (Figure 9) when the rotational speed of the carrier shaft will be higher than or equal to the ring shaft, i.e. for m_G = 1.015-1.73, as shown in Figure 10. It is also apparent from Figure 10 that an *SR* close to 7 would be the minimum

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speed reduction attainable with the DPGB if both output shafts rotate at the same speed and the sun gear is preserved as the pinion gear, i.e. $m_G > 1$.

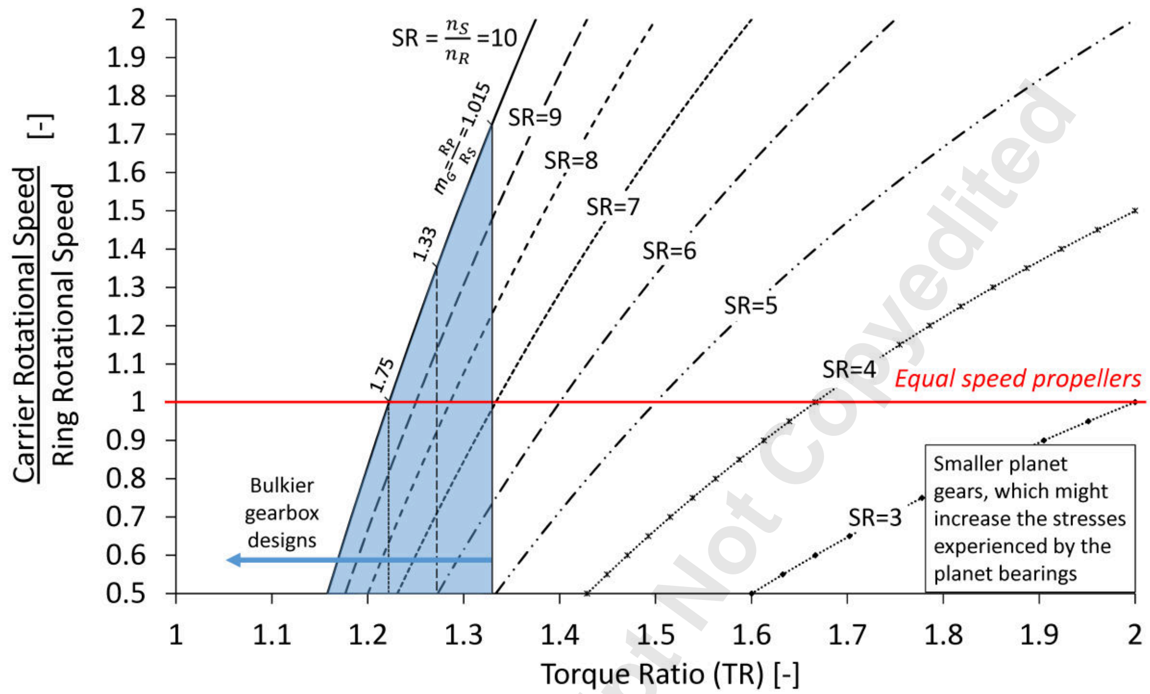

Figure 10 Variation of torque ratio (TR) and gear ratio (m_G) with speed ratio (SR) and rotational speed ratio between the output shafts for the DPGB in a CROR, with highlighted feasible design area for the sun as pinion gear

6 Conclusions

This paper provides a framework to calculate the dimensions of a differential planetary gearbox to comply with the transmitted power and durability requirements of a contra-rotating open rotor. The framework developed enables the identification of the feasible region for the preliminary design of the transmission system to produce compact gearbox arrangements and high speed-reduction ratios between the input and output shafts.

Sensitivity analyses have been performed on key parameters affecting the size estimation of the transmission system, to evaluate the impact of the assumptions on the design. The integration of the up-to-date gearbox sizing methodology into an engine

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preliminary design framework would inform the design-space exploration assessments. In such evaluations, the mechanical constraints of the transmission system would be considered in the engine preliminary design including the LPT.

The framework presented in this paper enables the identification of the feasible range of torque ratios that comply with the mechanical integrity of the transmission system. Higher torque ratios between the output shafts, i.e. lower gear ratios (m_G), result in more compact gearbox designs. A feasible range of torque ratios between 1.1 and 1.33 has been identified for the differential planetary gearbox, based on the assumptions made.

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Nomenclature

AGBT	Advanced Gearbox Technology
AGMA	American Gear Manufacturers Association
C_f	Surface condition factor for pitting resistance [-]
C_H	Hardness factor [-]
C_p	Elastic coefficient $[(MPa)^2]$ or $[(psi)^{0.5}]$
CROR	Contra-Rotating Open Rotor
DEMOS	Developing advanced Engine Multi-disciplinary Optimization Simulations
DPGB	Differential Planetary Gearbox
DREAM	valiDation of Radical Engine Architecture systeMs
d	Pinion pitch diameter [*cm*] or [*in*]
E	Modulus of elasticity [*MPa*] or [*psi*]
EOR	End of Runway
F	Gear Face Width [*cm*] or [*in*]
F/d	Gear aspect ratio [-]
FP	Front Propeller
I	Geometry factor for pitting resistance [-]
J	Geometry factor for bending resistance [-]
KHI	Kawasaki Heavy Industries

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K_c	Pitting resistance constant [cm^3] or [in^3]
K_d	Total derate factor [-]
K_m	Load distribution derate factor [-]
K_O	Overload derate factor [-]
K_q	Planet interferance factor [-]
K_S	Size derate factor [-]
K_R	Reliability factor [-]
K_t	Bending strength constant [cm^3] or [in^3]
K_T	Temperature factor [-]
K_v	Dynamic derate factor [-]
L	Gear design life [h]
LPT	Low-Pressure Turbine
M_0	Flight Mach Number [-]
m_G	Gear Ratio (R_P/R_S) [-]
m_p	Profile Contact Ratio [-]
MTBUR	Mean Time Between Unscheduled Removal [h]
N	Number of teeth [-]
n	Rotational speed [RPM]
n_L	Number of cycles [-]

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NA	Not Available
Q	Torque [*Nm*] or [*lb in*]
q	Number of load paths (number of planet gears) [-]
P	Power [*W*] or [*hp*]
R	Radius [*cm*] or [*in*]
R_{DPGB}	Gearbox Ratio (R_R/R_S) [-]
RP	Rear Propeller
S	Stress or strength [*MPa*] or [*psi*]
S_F	Bending safety factor [-]
S_H	Pitting safety factor [-]
SR	Speed Ratio ($SR = \|n_S/n_R\|$) [-]
T/O	Take-off
TR	Torque Ratio (Q_C/Q_R) [-]
TRL	Technology Readiness Level
Y_N	Bending resistance factor [-]
Z_N	Pitting resistance factor [-]

Greek

ψ	Helix Angle [°]

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φ_n	Normal Pressure Angle [°]
η_{DPGB}	Mechanical Efficiency of the gearset [-]

Subscripts

ac	Allowable contact
at	Allowable bending
C	Carrier
c	Contact
FP	Front Propeller
P	Planet
R	Ring
RP	Rear Propeller
S	Sun
t	Bending

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Preliminary design framework for the power gearbox in a contra-rotating open rotor

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