

## Research Article

# Introducing a Chaotic Component in the Control System of Soil Respiration

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Chaos theory has been proved to be of great significance in a series of critical applications although, until now, its applications in analyzing soil respiration have not been addressed. This study aims to introduce a chaotic component in the control system of soil respiration and explain control complexity of this nonlinear chaotic system. This also presents a theoretical framework for better understanding chaotic components of soil respiration in arid land. A concept model of processes and mechanisms associated with subterranean  $CO_2$  evolution are developed, and dynamics of the chaotic system is characterized as an extended Riccati equation. Controls of soil respiration and kinetics of the chaotic system are interpreted and as a first attempt, control complexity of this nonlinear chaotic in partitioning components of soil respiration.

#### 1. Introduction

Chaos is a kind of external, complex, and seemingly irregular motion in the deterministic system due to randomness [1]. The sensitivity of the chaotic system to the initial value makes the input changes of the chaotic system be reflected in the output rapidly, so the chaos theory provides a more realistic nonlinear modeling method [2]. Chaos theory has been proved to be of great significance in a series of critical applications [3–6]. The basic idea of chaos theory with complex nonlinear dynamics originated in the early 20th century, formed in the 1960s, and developed more in the 1970s–1980s [7–10]. Chaos is a complex nonlinear dynamic behavior. This theory reveals the unity of order and disorder, certainty, and randomness. It is regarded as the third most creative revolution in the field of science in the 20th century after relativity and quantum mechanics.

Because the chaos system can produce "unpredictable" pseudo-random orbits, many research studies focus on the related algorithms and performance analysis of constructing pseudo-random number generators utilizing chaos systems. For continuous chaotic systems, many chaotic pseudorandom sequences have been proved to have excellent statistical properties. However, until now, applications of chaos theory in analyzing soil respiration have not been addressed. It is necessary to introduce a chaotic component in the control system of soil respiration and explain control complexity of this nonlinear chaotic system. In previous studies, we have found that soil respiration ( $R_s$ ) estimate in arid regions should not have neglected the contribution of abiotic exchange [11]. Neglecting the contribution of inorganic component has resulted in overestimates of soil respiration in arid regions, which partly explains the truth of the well-known missing CO<sub>2</sub> sink [12]. The inorganic component of soil respiration ( $R_{io}$ ) is therefore necessary to be taken into account for a more reliable estimate of soil respiration in arid regions [11, 12]. This study will further reanalyze the concept, kinetics, and data of  $R_{io}$  and show that it is a chaotic component of soil respiration in arid regions.

Objectives of this study are (1) to show that  $R_{io}$  is a chaotic component of soil respiration in arid land and present a theoretical framework for a better understanding of this chaotic component, (2) to interpret the chaotic system on controls of soil respiration and kinetics of the chaotic system, and (3) to reduce the control complexity of this nonlinear chaotic system by introducing a period regulator.

#### 2. Theory and Kinetics

2.1. A Concept Model. We hypothesize that the underground CO<sub>2</sub> assignment in arid and semiarid regions has been regulated by a hidden loop in groundwater cycle. In brief, groundwater discharge and recharge have regulated the components of soil respiration. Based on this hypothesis, subsurface CO<sub>2</sub> transportation, dissolution, sequestration, and other reassignment processes in the soil-groundwater system are largely driven by precipitation, evaporation, irrigation, dew deposition, etc. These are hydrologic processes associated with the chaotic component  $R_{io}$  of soil respiration. Such processes regulate the storage and turnover rates of inorganic carbon and its dissolvable part in the profile of soils [11]. In arid regions with saline and sodic soils, apart from precipitation in the form of rain or snow, dew and fog also play a vital role in providing an essential source of water for soil [13].  $CO_2$  in soil can react with dew and then dissolve carbonate or even migrate into saline aquifer [14, 15].

Influenced by the hidden loop, soil respiration in arid regions is no longer a definite system. It becomes a nonlinear chaotic system. In order to describe the nonlinear chaotic system, the conceptual framework of known and unknown processes associated with the hidden loop in groundwater cycle, along with the possible mechanisms, is shown in Figure 1.

2.2. Kinetics of the Control System. The hypothesized hidden loop can explain particularity of  $CO_2$  assignment in arid and semiarid regions. Differential, difference, and dynamic equations are used for modeling many problems arising in engineering and natural sciences [16, 17]. This suggests us to develop a differential equation to describe the hypothetical system kinetics. Since the absorbed  $CO_2$  is hypothetically dissolved in saline aquifers, we characterized the dynamics of  $CO_2$  concentration in the groundwater-soil system in [18, 19] as a simple form of Riccati equation. Analytic solutions of the equation under some necessary and sufficient conditions were also presented.

However, there are still considerable uncertainties and difficulties in fully understanding the underlining mechanisms and critical factors driving such a hidden loop. One major challenge is how to characterize the structure of the soil-groundwater system [15]. It is natural to conjecture that the underlining groundwater cycling processes associated with subsurface  $CO_2$  sequestration in different layers should be different. The whole story is shown in Figure 2.

#### 3. Chaos and Control Complexity

3.1. Further Evidence for Being Chaotic. In previous publications, it was demonstrated that the variations of  $R_{io}$ originate from the physical forcing of abiotic factors such as soil salinity (EC), alkalinity (pH), temperature  $(T_s)$ , and water content (WC<sub>s</sub>) and their linear relationships with its daily mean intensity appear to be valid within a seasonal cycle as a whole. However, in diurnal cycles, taking into account the complicated and undetermined processes associated with the chaotic component  $R_{io}$ , the soil respiration system in arid land is a nonlinear chaotic system. Variability in the data of  $R_{io}$  presents further evidence for  $R_{io}$  being chaotic. Before the chaos theory was proposed, scientists had thought that there are only two kinds of phenomena-the phenomena which act strictly according to a rule and the phenomena which happen stochastically [20]. As seen in Figure 3, we construct a constant vector for the period control (CVPC) in variation of R<sub>io</sub> (Figure 3(a)), but environmental controls of R<sub>io</sub> are seen to interact (Figures 3(c)-3(f)). Practical variability of  $R_{io}$  looks stochastic (Figure 3(b)). CVPC for hourly variations of  $R_{io}$  in diurnal cycles is an exponent-sine coupled normalization transformation of time sequence (TSN), as follows:

CVPC = 
$$e^{\sin(TSN)}$$
,  
TSN =  $\frac{\pi (x - \min(x))}{12 (\max(x) - \min(x))}$ ,  $x = [1, 2, ..., 24]$ .  
(1)

3.2. Control Complexity of the System. Since soil respiration in arid land is a nonlinear chaotic system, the resulted control complexity is naturally reconciled [21]. A wellknown index to characterize the control complexity is temperature sensitivities (i.e.,  $Q_{10}$ ) of  $R_s$ . Analyses on data collected from previous studies revealed diel turbulence in  $Q_{10}$  values even if excluding the negative  $R_s$  data. On the basis of utilizing the basic and reanalyzed data collected from [21], we found that the variability of  $Q_{10}$  values is far from certain. All the  $Q_{10}$  values used in the analysis were calculated utilizing the simple model of  $R_s$  (the derivative of the exponential chemical reaction-temperature equation originally developed by Van't Hoff) [18, 19, 21], and for consistence, the negative values of  $R_s$  were not included in calculations of  $Q_{10}$ . Controls of T on  $Q_{10}$  at each

#### Complexity



FIGURE 1: Story of the hidden loop in the nonlinear chaotic system, including known and unknown parts.



FIGURE 2: Hypothetical system kinetics of the hidden loop: (1) three carbon pools (the atmosphere, soil and groundwater) are connected through carbon cycles and water cycles, along with the underlining processes associated with  $CO_2$  sinks (green solid circles) and sources (blue solid circles); (2) the inorganic  $CO_2$  change beyond the red rectangle (if excluding influences of groundwater) are driven by evapotranspiration and vapor condenses, while the inorganic  $CO_2$  assignment and ventilation within the red rectangle are largely driven by groundwater recharging/discharging.

site were, respectively, analyzed in linear regressions for a between-ecosystem comparison. Results from these analyses were further compared with the analyses of the variation of  $Q_{10}$  with *T*. Using  $Q_{10}$  values from both sites, the effects of WC<sub>s</sub> on the  $Q_{10}$  of  $R_s$  to  $T_s$  and the  $Q_{10}$  of  $R_s$  to  $T_a$  were analyzed in quadratic regressions. In order to



FIGURE 3: The period character of CVPC (a) almost coincides with the period character in the hourly scale variations of  $R_{io}$  (b). None of  $T_s$  (c), WC<sub>s</sub> (d), and the optimal linear combination of  $T_s$  and WC<sub>s</sub> (e) is better than CVPC (f) to describe the temporal pattern of  $R_{io}$  in diurnal cycles.

further test the role of WCs in determining Q10, four coupling models were employed to analyze coupling effects of T and WC<sub>s</sub> on  $Q_{10}$ . The front two models were established under the hypothesis that the influences of WCs and T on  $Q_{10}$  were mutually independent. The first model hypothesized that the influences of WCs and T were linearly independent; the second model hypothesized that the influences of  $WC_s$  and T were exponentially independent. The latter two models were established under the hypothesis that the influences of WC<sub>s</sub> and T on  $Q_{10}$  were not mutually independent. The third model hypothesized that  $Q_{10}$  was dominantly determined by WCs and T linearly interacted on the responses of the  $Q_{10}$  to WC<sub>s</sub>; the fourth model hypothesized that  $Q_{10}$  was dominantly determined by T and WC<sub>s</sub> linearly interacted on the responses of  $Q_{10}$  to T. Descriptive statistics were used to calculate the R-squared values (R), root mean squared error (RMSE), and F-statistics vs. constant model and p values of the data from each set of reduplicates. The data analysis was processed using MAT-LAB (Mathworks, Natick, MA, USA), and the statistical analyses were synchronously conducted.

We further examined the variability of  $Q_{10}$  values, as seen in Figures 4 and 5.

#### 4. Treating the Control Complexity

Taking into account negative  $R_s$  data in arid regions is strongly necessary to reduce uncertainties in the current global/regional carbon balance and in the predictions of future feedbacks in the coupled carbon-climate system ([15, 22–28]). Further modeling approach is advantageous to understand CO<sub>2</sub> footprints ([29–35]). For the convenience of statement, we describe the "doubly average" diurnal dynamics of  $R_{io}$  (being averaged among diverse soil sites and meanwhile averaged from different days) by the linear combination of TS, WCs, and CVPC. Let  $\alpha 1$ ,  $\alpha 2$ , and  $\alpha 3$  be regression coefficients (termed as "parent parameters," invariable within each special soil site), respectively, and let  $\varepsilon$ be the residual; then, we have

$$R_{\rm io} = \alpha_1 T_s + \alpha_2 W S_s + \alpha_3 C V P C + \varepsilon, \qquad (2)$$

where CVPC for the hourly scale variations of  $R_{io}$  can be easily extended to daily or larger scales.

Utilizing the data in Figure 3 as inputs of equation (2) for a practical simulation, performance of treating the control complexity is shown in Figure 6. According to performance of the model on the third day (a1, b1), the



FIGURE 4: Diel turbulence in temperature sensitivities  $(Q_{10})$  with soil temperature  $(T_s)$  and water content (WC<sub>s</sub>).



FIGURE 5: Diel turbulence in temperature sensitivities  $(Q_{10})$  with air temperature  $(T_s)$  and water content  $(WC_s)$ .

fifth day (a2, b2), the seventh day (a3, b3), and the eighth day (a4, b4) after 1 mm diurnal precipitation, the bias in the simulations by using equation (2) exists within a measuring period. However, this is according to performance of the model on the fifth day (c1, d1), the ninth day (c2, d2) after a 5-day continuous precipitation of

 $0.6 \sim 3.6 \text{ mm}$ , and the first day after a precipitation of 1.7mm (c3, d3). The model can even describe the variability of Rio on the days after a continuous precipitation and the day right after small-size rainfall. The model becomes invalid in the simulation on the first day after ~9.9 mm rainfall (c4, d4), when the intensity of Rio is



FIGURE 6: Treating the control complexity by the proposed model on subsequent days after rainfalls (a1-a4, c1-c4), which were modified in simulations by equation (2) (b1-b4, d1-d4).

changing too fast. Overall, the nonlinear chaotic system is simplified and can be further developed.

Due to potential overlap in environmental, temporal, and spatial components of ecological data, partitioning the variations among pure environmental controls, pure spatial controls, pure temporal controls, pure spatial component of environmental controls, pure temporal component of environmental controls, pure combined spatial and temporal component controls, combined temporal and spatial components of environmental controls, and unexplained component should be included in multivariate analysis of the chaotic system. The whole story of control complexity of this nonlinear chaotic system is therefore worthy of further investigation.

In reference [19], we have presented more details on the variations of the determining processes of  $R_{io}$  of soil respiration and characterize the dynamic of CO<sub>2</sub> concentration in the soil-groundwater system as an input-output balance equation, as follows:

$$C(nT+T) - C(nT) = \frac{(V_1q + r^n) \cdot T - \int_{nT}^{nT+T} V_1 \cdot C(s) + r_n p_n ds}{V}$$
(3)

where C(t) is CO<sub>2</sub> concentration in a considered gas room  $V_1$  in the soil-groundwater system and q is the CO<sub>2</sub> concentration in the atmosphere. For the *n*th time interval [*nT*, (n+1)T],  $r_n$  is the average ratio between the input and output of CO<sub>2</sub>.

Suppose that the input of  $CO_2$  into the soil-groundwater system was finally dissolved in the groundwater of volume V. Let D(t) be the amount of DIC at t and the growth rate of DIC is r. As hypothesized in Section 2, the determining processes of  $R_{io}$  are driven by groundwater discharge (outflow) and recharge (inflow), with volume Q. Provide that outflow = inflow and assume that outflow after the inflow is uniformly mixed with the groundwater unitV. As seen in reference [18], the quality conversation law implies that Complexity

$$D(t + \Delta t) - D(t) = Qp\Delta t + \int_{t}^{t+\Delta t} rD(t) - Q\frac{D(t)}{V+Q}dt.$$
(4)

Finally, considering the restricting effect of current DIC, which is characterized as  $R - \lambda D(t)$ , equation (4) can be further improved as [18]

$$D(t + \Delta t) - D(t) = Qp\Delta t + \int_{t}^{t+\Delta t} r(R - \lambda D(t))D(t)$$
  
-  $Q\frac{D(t)}{V+Q}dt.$  (5)

The next research priority is to analyze the characteristics of bifurcation and chaos in the inherent spatial and temporal variations of  $R_{io}$  by using Feigenbaum graphs [36] and further develop equations (2), (3), and (5). Based on this study, the natural increase of CO<sub>2</sub> is the third determining process of  $R_{io}$  besides the input and output of CO<sub>2</sub>, which involves organic components of soil respiration. This process, along with the input and output of CO<sub>2</sub>, determine the increase rate *r* of the difference between the subterranean and surficial CO<sub>2</sub> concentration and also determine the density of  $R_{io}$ .

#### 5. Conclusion

For a better understanding of how soil  $CO_2$  fluxes change with space and time, it is necessary to introduce  $R_{io}$  as a nonlinear chaotic component of soil respiration in arid land. Ecology is a study not how things but how things change with space and time, and hence, it is also necessary to interpret the control complexity of this chaotic component. In the assessment of the importance of organic and inorganic factors influencing  $R_{io}$ , inherent spatial and temporal variations in ecological data should be taken into account whenever possible. A next research priority is to analyze the characteristics of bifurcation and the chaos difference between the subterranean and surficial  $CO_2$  concentration and further understand the whole story of the control complexity of  $R_{io}$ .

#### **Data Availability**

All the data utilized to support the theory and models of the present study are available from the corresponding authors upon request.

#### **Conflicts of Interest**

The authors declare that there are no conflicts of interest regarding the publication of this article.

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