

CRANFIELD UNIVERSITY

SCHOOL OF AEROSPACE, TRANSPORT AND
MANUFACTURING

ADVANCED VEHICLE ENGINEERING CENTRE

MARCIN STRYSZOWSKI

A Self-enforced, Connected Cooperative Traffic Framework

THESIS FOR THE DOCTORATE OF PHILOSOPHY

PhD full time

Academic Years: 2016 – 2019

Initial Registration: 2nd November 2016

Submission Date: 2nd November 2019

Author

Marcin STRYSZOWSKI

Supervisors

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Abstract

This doctoral thesis proposes a novel approach to road traffic de-conflicting. It comes as a framework consisting of a user-tailored, multi-objective cost function and a negotiation algorithm, in which traffic conflicts are defined within game theoretic formulation, based on side-payment to fairly distribute the benefits, thereby ensuring feasibility within a distributed, intelligent system. The algorithm is then applied to two-agent conflict resolution in a simulated intersection and platooning/overtake scenarios. Energy consumption and loss of time are compared, indicating a threefold improvement in theoretical efficiency of the framework in relation to a noncooperative solution. It occurs when agents are the most heterogeneous. The intersection and platooning algorithms are then further developed to handle multi-agent scenarios, where complexity is the greatest challenge. A formulation based on graph theory is proposed, estimating the complexity to be no smaller than that of complete graph sequence, with time of calculation infeasibly long above 10 agents, calling for implementation specific heuristics.

The last chapter of this work considers the framework's future paths of development. It features extended cost function formulations, incorporating, among others, ancillary energy use or battery wear. System's sensitivity cheating or market penetration is also studied, proposing human-in-the-loop architecture as means to ease the adoption process.

Dedykacja / Acknowledgement

Temu, który Jest, za Słowo,
Światłość, która w ciemności świeci
i Wolę, by wybierać pośród nich,
moim Przodkom za stworzenie tej, jakże
bogatej kultury, i ich najbliższym powiernikom
moim Rodzicom, za wysiłek włożony w
wychowanie i stworzenie możliwości do nauki,
mojej Żonie, która nigdy nie odmówiła
zrozumienia i wsparcia, a które były nieod-
zowne w chwilach zwątpienia,
moim Nauczycielom, którzy ukształtowali
mnie jako inżyniera i badacza, za ich cierpli-
wość,
moim Towarzyszom, którzy nie zdają sobie
sprawy w jak wielkim stopniu ich historie mi
pomagają,
wszystkim tym, którzy nieustają w tworze-
niu bogactwa różnorodności tego Świata, gdyż
więcej jest do odkrywania niż czasu w życiu,
chciałbym powiedzieć serdeczne dziękuję
oraz zadedykować tę pracę doktorską, wierząc,
że ludzkie ograniczenia w rozumie czy języku
nie przyćmią tego dzieła.

To the One that Is, for the Word,
the Light which shines in the darkness
and the Autonomy to choose either,
to my Ancestors for creation of this opulent
culture, and their most imminent trustees,
my Parents, for their effort in my upbringing
and creation of opportunities to learn,
to my Wife, whose understanding and sup-
port has been indispensable in moments of
doubt,
to my Teachers, whom shaped me as an en-
gineer and researcher, for their excellence and
patience,
to all the Companions who do not realize to
what extent their stories have influenced me,
to all those who press to create the varied
riches of this World, as there is more to explore
than time in a lifetime,
I would like to say sincere thank You, for
your part in this work and dedicate this doc-
toral thesis, believing that my human limita-
tions of reason, ability or language shall not
shadow the intention to produce this thesis.

List of Publications

Book chapter

M. Ehsani, Y. Gao, S. Longo, K. Ebrahimi.

Modern electric, hybrid electric, and fuel cell vehicles, chapter 20. CRC Press, 2018

ISBN: 978-1-4987-6177-2.

Write-up of the User Guide of Multiobjective Optimisation Toolbox

Conference paper I

M. Stryszowski, S. Longo, E. Velenis, M. Shah.

Energy and Time-Optimal Connected Autonomous Vehicle Interaction: Cruising and Overtaking.

European Control Conference, 2018

Journal paper I

M. Stryszowski, S. Longo, D. D'Alessandro, E. Velenis, G. Forostovsky, S. Manfredi.

A Framework for Self-Enforced Optimal Interaction between Connected Vehicles.

Transactions on Intelligent Transportation Systems.

Journal paper II

M. Stryszowski, S. Longo, E. Velenis, G. Forostovsky.

A Framework for Self-Enforced Optimal Interaction between Connected Vehicles: Intersection Negotiation.

Transactions on Intelligent Transportation Systems. Under Review

Contents

1	Introduction	1
1.1	Objectives and contribution	2
1.2	Thesis Contribution	6
1.3	Outline of the thesis	7
2	Review of the literature	8
2.1	Optimisation of powertrains	9
2.2	Connected Autonomous Vehicles (CAV)	10
2.2.1	CAV safety	12
2.2.2	Security	14
2.3	Traffic management schemes	14
2.3.1	Cruising, Platooning and Overtaking	15
2.3.2	Intersections	17
2.4	Game Theory	21
2.5	Further literature	24
2.5.1	Game Theory as a view on economy	24
2.5.2	Air Traffic Control	24
2.5.3	Communication in GT and self-enforcement	26
2.5.4	Human driving and CAV	26
2.5.5	Future of road mobility	29
2.6	Summary of the literature	31
3	Formulation of the traffic conflict problems	32
3.1	Vehicle model	32
3.2	Game-theoretic solution concept	34
3.3	Topology of traffic conflicts	36
3.4	Input to calculations	39
3.5	Calculation assumptions	40
3.6	Cruising, the cost function	42
3.7	Conclusions	46

4	In-line conflict resolution	47
4.1	Cooperative conflict resolution	47
4.1.1	Strategic mode selector	50
4.2	Platooning negotiation	51
4.3	Optimal overtaking manoeuvre	52
4.3.1	Overtake model	52
4.3.2	Optimal overtake	55
4.4	Decision rule	56
4.5	Results	57
4.5.1	Platooning	57
4.5.2	Overtake solution	59
4.5.3	Sensitivity of overtake cost to variation of the distance available	61
4.6	Conclusions	62
5	Intersection conflict	63
5.1	Conflict resolution formulation	63
5.1.1	Simplified intersection decision algorithm	69
5.2	Intersection conflict simulation	70
5.2.1	Performance evaluation	73
5.3	Multi-agent intersection conflict formulation	74
5.4	Complexity of n-agent platooning problem	77
5.5	Discussion of the results	79
6	Hardware validation	81
6.1	Raspberry Pi implementation	81
6.2	Scaled, automated vehicle implementation	83
6.3	Conclusions	86
7	Framework scalability analysis	87
7.1	Cost function development	87
7.1.1	Ancillary energy loads	87
7.1.2	Battery wear model	88
7.1.3	Variable energy recovery fraction	89
7.1.4	Consideration of externalities	89

7.2	Agent honesty	91
7.2.1	Platooning	91
7.2.2	Overtake and intersection	92
7.3	System's performance and traffic implementation	93
7.3.1	Simulation and results	93
7.4	Human in the loop	95
7.4.1	Problem formulation	97
7.4.2	Controller architecture	99
7.4.3	Simulation results	100
7.5	Discussion	101
7.5.1	Human vs Machine decisionmakers	102
7.5.2	Communication and enforcement	102
8	Conclusions	104
8.1	Further work recommendation	106
8.1.1	Algorithm's structure	128

List of Figures

1	Relationship between various aspects of Connected Autonomous Vehicle driving.	2
2	Slider for interfacing user's intentions.	3
3	The intersection conflict.	4
4	The electric island.	8
5	Powertrain opt	10
6	Nuclear Safety Framework and CAVs.	13
7	Three phase traffic theory.	15
8	State-of-the-art cooperative traffic algorithm.	17
9	Intersection management based on vehicle connectivity.	18
10	Traffic light enabled ACC.	20
11	The division of the possible road vehicle manoeuvre, with respect to the order of the spatial dimension. Encircled are the manoeuvres relevant to energy and time optimisation.	36
12	Types of traffic conflicts.	37
13	Assumed map of all the scenarios a vehicle can encounter.	39
14	The cycle of self-enforced cooperation.	41
15	Components of the cost function.	44
16	Cost function's sensitivity to C_T variation.	45
17	Comparison of the CF and traditional QP.	46
18	Activity diagram of the communication between agents.	49
19	Finite-state machine model of the decision routine.	50
20	Overtake speed profile.	53
21	Platooning negotiation result.	57
22	Platooning negotiation comparison for various scenarios.	58
23	Example overtake cost solutionspace.	60
24	A projection of the above figure from above.	60
25	Overtake cost as a function of the overtake gap.	61
26	Intersection conflict topology.	64
27	Intersection conflict in time domain.	65
28	Intersection conflict's feasible solutionspace.	68

29	Intersection cost function and its derivative.	69
30	Complete formulation of the intersection.	71
31	Constraints on the intersection’s reduced solutionspace.	72
32	Monte Carlo simulation result.	74
33	Conflict propagation visualisation.	76
34	Platoon reconfiguration.	78
35	Multi-agent platoon algorithm.	79
36	Raspberry Pi implementation.	82
37	The scaled autonomous vehicle for HiL tests.	83
38	Steering Control Algorithm	84
39	Hardware-in-the-Loop experimental set-up.	85
40	Anciliary loads.	88
41	Modular cost function: battery wear model involved.	89
42	Sensitivity to energy recovery fraction.	90
43	Possible cost function hacking.	91
44	Price of Anarchy for varying traffic penetration.	94
45	3D plot of the PoA for varying traffic penetration.	95
46	Cruise speed and the gap between vehicles.	96
47	Human-in-the-Loop controller architecture.	99
48	Step response.	100
49	Intersection conflict with human driver model.	101

List of Tables

1	Vehicle types and their differentiating parameters.	40
2	Agent semantics.	48
3	Input parameters.	48
4	Control variables.	48
5	Platooning payoff matrix.	51
6	Overtake payoff matrix.	55
7	Platooning example input.	58
8	Platooning efficiency evaluation.	58
9	Overtake cost R-square fit.	61
10	Payoff matrix expressed in simplified formulation.	70
11	Intersection - the results.	72
12	Randomized input parameter ranges.	74
13	Multi-agent platooning example.	78
14	Raspberry Pi results	82
15	Calculation times for functions on Raspberry Pi.	83
16	Ancilliary energy load results.	88
17	Randomized input parameter ranges.	94

Glossary

Abbreviations

ACC	Adaptive Cruise Control
ANPR	Automatic Number Plate Recognition
ACI	Airline Cost Index
ATC	Air Traffic Control
ARTEMIS	Drivecycle type
CAV	Connected Autonomous Vehicle
CF	Cost Function
co-co	cooperative-competitive
CZ	Conflict Zone
DP	Dynamic Programming
ECU	Engine Control Unit
ESS	Energy Storage System
EV	Ego Vehicle, Electric Veh.
FCFS	First Come First Served
GT	Game Theory
HDV	Human-Driven Vehicle
HESS	Hybrid Energy Storage System
I ² C	Inter-Integrated Circuit
HMI	Human-Machine Interface
HVAC	Heat, Ventilation, Air Conditioning
IAEA	International Atomic Energy Agency
iCACC	intelligent Cooperative Cruise Control
ICE	Internal Combustion Engine
LCF	Local Cost Function
MIMO	Multiple Inputs Multiple Outputs
MHV	Mechanical Hybrid Electric Veh.
ML	Machine Learning
MPC	Model Predictive Control
NE	Nash Equilibrium
NEDC	Drivecycle type
OV	Obstacle / Other Vehicle
OEM	Original Equipment Manufacturer
PE	Pareto Equilibrium
PHEV	Plug-in Hybrid Electric Veh.
PoA	Price of Anarchy
PWM	Pulse Width Modulation
QoS	Quality of Service
RMS	Root-Mean-Square deviation
SAE	Society of Automotive Engineers
SoC	battery State of Charge
SoH	battery State of Health
TBS	Time Based Separation
UART	Universal Asynchronous Receiver-Transmitter
V2I	Vehicle to Infrastructure
V2V	Vehicle to Vehicle
VTTS	Value of Travel Time Saving
WLTP	Drivecycle type

Nomenclature

Vehicle model	
T_a	duration of vel. change maneuver
t_{step}	timestep
x	position
v	velocity
a	acceleration
V_n^m	manoeuvre velocity
V_n^*	optimal cruise velocity
\hat{F}	maximal force
η	energy efficiency
A	aerodynamic reference area
η	energy efficiency
ρ_{air}	atmospheric air density
C_d	aerodynamic drag coefficient
μ_{roll}	rolling resistance coefficient
g	gravitational acceleration constant
Game Theory	
#	cooperative solution
*	Nash solution
†	cheated solution
b	enforcing value solution
J	cost function
j	local cost function
\mathcal{A}	action set
N	number of entities/iterations
Agents	
N	set of agents
$-n$	the other agent
VP	vehicle parameters
Manoeuvres	
C	manoeuvre cost
M	Pay-off matrix
p	enforcement side-payment
\mathcal{A}	action set
N	number of entities/iterations
Platooning & Overtake	
\check{C}_{ovt}	minimal overtake cost
C_P	cost of platooning
V_{gap}	velocity of the oncoming veh.
S_{onc}	distance to oncoming traffic
S_{gap}	length of the overtake gap
S_a	distance available for overtake
T_a	Time alongside during overt.
\check{X}	minimal safety distance
l	vehicle length

Nomenclature cont.

Intersections

A, B	intersection participants
T_I	time of arrival at the intersection
T_{II}	manoeuvre execution time
T_{CZ}	CZ occupancy time interval
Δt_{fix}	time difference to de-conflict
$V[t]$	parametrized velocity profile
α	solution blending coefficient
a_{CZ}	conflict zone length
Θ	Time difference domain

Other

γ_b	battery wear coefficient
β	energy recovery fraction
\dot{m}	mass flux
\mathcal{W}	cheating multiplier
\mathcal{B}_\dagger	benefit of cheating
n	cheating penalty
$P_{\mathcal{D}}$	fraud detection probability
P'	fraction of vehicles capable of V2V
R	random number
K_n	recurrent series parameter
\mathcal{K}	sensor data
Γ	Adversary's execution control corrector
q_T	timidness parameter
P_E	Expected chance of failure to cooperate

1 Introduction

The powertrains employed in the cars we drive are approaching their theoretical thermodynamic limit. The year 2017 was the first, when a Formula 1 car achieved 50% powertrain efficiency, beating . There is little more mechanical energy to be extracted, while the need for efficiency is still growing. The vehicle automation and soon even autonomy are being intensively developed, thus the focus on the optimality of the interaction between them is the natural next focal point.

What empowered *Homo Sapiens* to shape its environment, e.g. build roads or power plants, was the ability to exchange ideas and organize into complex social structures, enabled by language [1]. It is particularly noticeable when observing how the broadband internet and mobile electronics have changed the landscape of public life in the last decade, as companies who manage private information are wealthier than some governments [2].

Despite being intertwined with the fabric of modern day economy and society, road traffic is one of the last holdouts of the information age. The oldest Highway Code will soon be a century old, while traffic lights and speed cameras are the only social electronic aids, as SatNav is not a networked device. The limited breadth of the information channel between drivers and complex ownership & liability structure are the main causes for this resistance. Today's traffic is a self-organized distributed system, whereby complex conflicts are resolved with minimum information by means of predefined rules and social norms, which research only begins to realize [3], which autonomous cars have to learn to pass the *driver-Turing* test [4] and be accepted to the driver community.

The body of research already offers optimal, centralised, traffic organisation algorithms, but their mainstream implementation may not be possible until all vehicles are fully automated, rendering our driving skill and traffic norms obsolete, which may not be feasible in the oncoming decades, if at all, based on the Connected Autonomos Vehicle (CAV) implementation predictions [5]. On the other hand, multi-agent systems of rational and autonomous agents and negotiation dynamics between them have been thoroughly examined by classical economists, game-theory mathematicians [6] [7], social scientists and even lawyers, but are somewhat overlooked by the engineers racing towards a fully autonomous car. This doctoral thesis is an attempt to bridge these distant fields

of research, thereby enabling a framework in which a synergic road traffic paradigm may be developed, where the today's norms are not replaced but capitalized upon and harmoniously augmented creating a distributed yet efficient, synchronised yet resilient traffic system within one generation. Fig. 1 outlines the topology of the CAV ecosystem, the overlaps between its various aspects and the scope of this thesis.

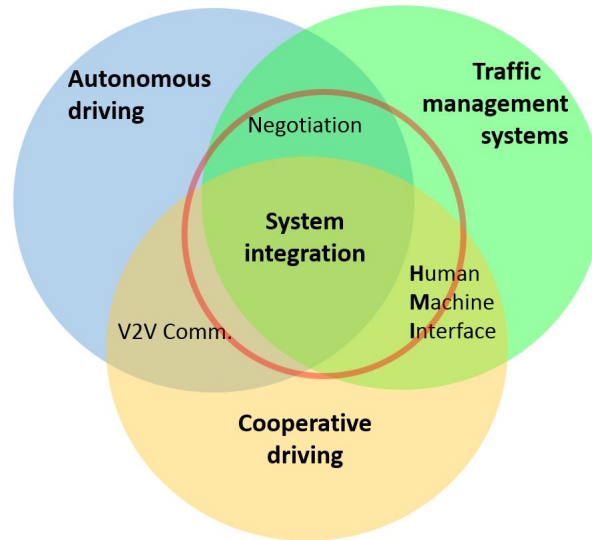


Figure 1: A Venn diagram, outlining the relationship between various aspects of Connected Autonomous Vehicle driving. The scope of the thesis is encircled with red.

1.1 Objectives and contribution

The intention behind this thesis is to explore the areas of research which promise room for further energy optimality in the road vehicle powertrain design, considering that the individual components of the powertrain and the dynamics of their interaction have been researched throughout. However, the way a vehicle as a whole interacts with others on road is only receiving attention in the recent decade. On the other hand, there is literature proposing, applying and testing various centralized traffic management schemes, such as reservation-based intersection algorithms. These publications, however, are assuming strong integration between infrastructure and vehicles, as well as full automation and trust. This may hinder a potential industrial deployment, making the way from laboratory to public roads, the main challenge for the oncoming decades. The key objective of this thesis is to propose how to design the decision functions of the individual road traffic

agents, so that the resultant behaviour of the collective minimises the cost [8]. Thus it aims to offer a novel, information-based algorithm for efficient traffic de-conflicting. Given that the further the agents' objectives are apart, the further the conflict is from being a purely competitive zero-sum game, this framework aims to capture the heterogeneity of user intention and vehicle parameters to serve as differentiators to ensure the payoff matrix defining the conflict is non-zero sum, and thus poses a distinct, optimal solution. It stands in contrast to the majority of publications, which homogenise the agents to ensure computational efficiency.

This intention-based negotiation algorithm starts with a novel cost function, which considers the user's value of time to offset the energy expense, finding the optimal cruise velocity. Today, as the user defines their objective in their navigation system, an expected time of arrival is returned. Rather than being a mere estimate, this value can serve as an estimator of the value of time. The user would be presented with a slider, with energy-optimal and time-optimal solutions on either ends, as presented in Fig. 2, having a choice of their preferred duration of the journey and estimated cost of energy to serve as an input to the cost function, based on which optimal vehicle operation strategies are found. A human driver could use it as an advanced cruise control/driving advisory, while an automated vehicle may employ it as a strategic reference for its longitudinal control, just as an airline does with the Airline Cost Index whilst choosing how to operate its airfleet.

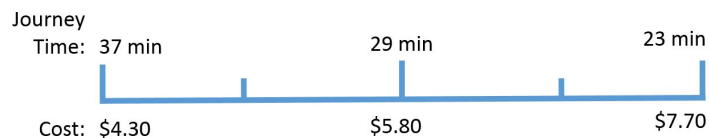


Figure 2: Proposed slider, interfacing user's value of time by preferred duration of the journey and associated cost. The diminishing journey time cost increasingly more energy. The values of the expected cruise time and costs presented serve as example.

Then, as a conflict on road occurs, the agents share their cost functions via V2V to find the optimal solution. To clarify the way optimisation is executed from the user's perspective. Consider a scenario, where a truck A is to turn left, intersecting with a light passenger vehicle B, as visualized in Fig. 3. According to the traffic code, B yields to A. However, the energy optimal solution is the reverse of traffic code rules. The small, lighter vehicle B slows down, so the heavier one A can retain its kinetic energy. The

self-enforcement of the framework originates from the B's question: *why should I yield when having right of way?*. B's energy expense and time delay are compensated with a side-payment [9], guaranteeing that every agent is always better off cooperating, and thus requiring no external oversight.

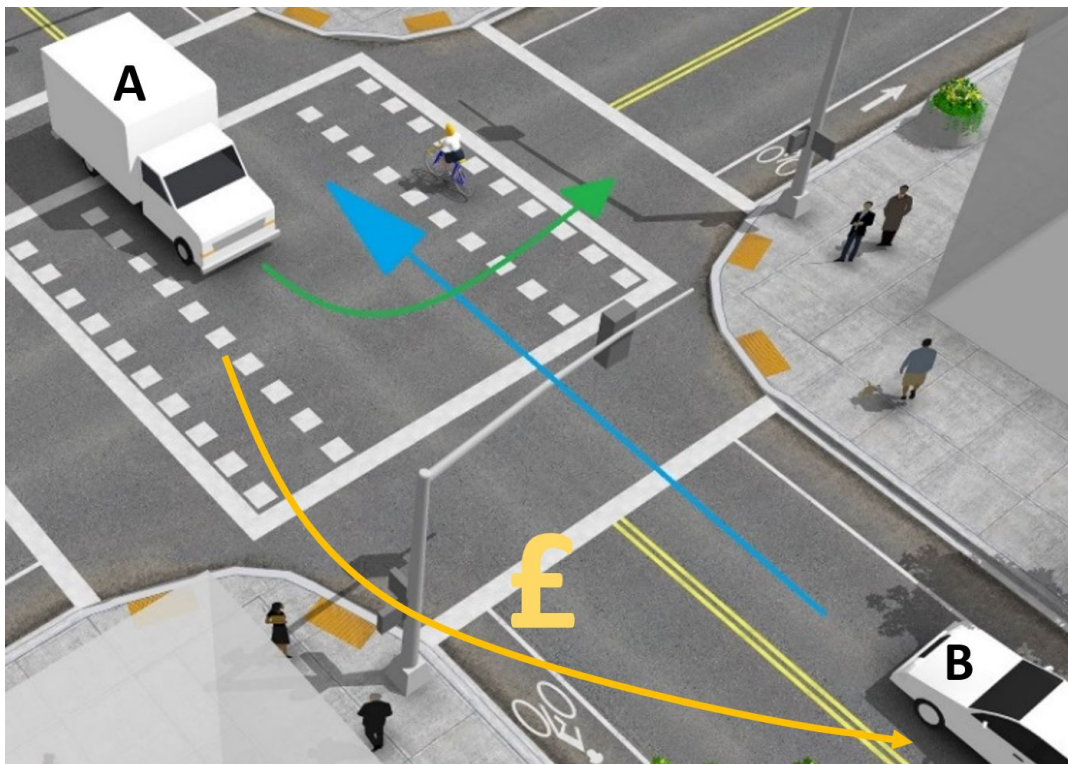


Figure 3: The cooperative intersection problem. Today, the sequence is B,A. The time/energy optimal solution reverses it to preserve A's momentum, relying on a micro-payment to incentivises B to yield, offsetting her costs.

The research hypothesis for this work then follows: the information sharing, and the cooperative-competitive solution concept offer an increased efficiency in resolving traffic conflicts, thereby confirming the Aumann's Conjecture (see Section 2.5.3) for the traffic conflict scenarios.

The objectives of this research are divided as follows:

- Define the cost function, serving primarily as vehicle's strategic decision reference. It is thus to serve as a game design rule, to define the conflicts between agents, while satisfying the self-enforcement requirement of being acceptable to an individual.
- Offer the formulation describing traffic conflicts, both intersection and in-line, solved by platooning or overtake. The game design is to originate from the predefined

cost functions, aiming to arrive at a formulation, which guarantees optimal conflict solution.

- Propose an expanded, multi-agent formulation and explore its complexity scaling, to estimate the computational power needed for real-time implementation.
- Validate the framework by hardware implementation. Scaled, automated vehicles and a communication channel are set up to evaluate framework's feasibility under physical measurement uncertainty.
- Evaluate the possible further directions of the framework's development, considering its scalability and challenges to implementation and challenging the research assumptions:
 - Further expansion of the cost function, selecting not only the velocity, but also higher order parameters, acceleration and jerk, being subject to e.g. component or battery wear or user's comfort or safety.
 - Signal the resilience to malicious intention, analysing the incentives to cheat and vulnerabilities of the self-enforcement assumption.
 - Study the implementation challenges, as the distributed nature of a self-enforced framework shall require a critical mass to be operational, calling for additional, early-adopter functionalities.
 - Consideration of the possibility of Human-Machine Interaction. Given the above challenge, a capability for CAVs to cooperate with human drivers with retrofitted V2V communication tool could ease the path to mainstream traffic cooperation paradigm.

In addition to the tangible objectives above, this work aims to assemble the literature from the fields of research relevant to the implementation of autonomous mobility, from powertrain design, through economy to social norm formation, utilizing a consistent linguistic toolbox, enabling communication regardless if one is a software engineer working on a new car or a city council executive, who strives to minimise pollution.

1.2 Thesis Contribution

The work towards the above objectives has led to the following novel contributions:

- Proposition of intention-defined value of time, serving as a trade-off to the cost of energy, in the context of road transport optimisation.
- Use of payment mechanism as a self-enforcement mechanism in a distributed traffic setting.
- Application of cooperative-competitive solution concept as a method of finding Pareto solutions in traffic conflicts.
- Evidence to support Aumann's Conjecture on pre-play communication as means to equilibrium seeking.
- Identification of legal obstacle to optimal traffic mechanism, as energy optimal conflict solutions do not coincide with traffic code regulations.

1.3 Outline of the thesis

This doctoral thesis begins with literature review in Chapter 2, which positions it in relation to the advancements in intersection management schemes, but also in neighbouring areas, such as powertrain optimisation. It is followed with a more speculative analysis of literature addressing non-engineering challenges to implementation of autonomous vehicles, touching subjects as sociology and economy, but also air traffic control, which meets similar challenges.

Moving on to the core of the thesis, Chapter 3 features the model, the solution concept and the assumptions which shape the workspace. It is concluded with the optimal cruise velocity selection algorithm as the first application of the cost function. Then, Chapter 4 outlines the conflict resolution algorithm for the in-line problems, while chapter 5 presents the two agent intersection conflict formulation. Multi-agent implementation follows. Having outlined the framework, in chapter 6 it is validated by hardware implementation. There scaled, automated and autonomous vehicles are deployed to communicate and form a cooperative platoon, demonstrating framework's feasibility.

The core of the work is then supported with Chapter 7, the further considerations, where additional perspective of the framework is considered. Various assumptions are challenged in subsequent subchapter, including the cost function's scalability, sensitivity to agent honesty, or challenges to the industrial application. The thesis is concluded in Chapter 8, with a summary of the findings and discussion of recommendations, touching on the future work.

2 Review of the literature

With the emergence of electrified powertrains, and soon complete autonomous vehicles [5], calls for research on optimal road vehicle designs based on these new powertrain technologies [10]. The 'electric island' of any powertrain, as marked in Fig. 4 always features a motor, power electronics and Energy Storage System (ESS). Both the electromechanical motor design and the electrochemistry of the ESS have extensive research to them [11].

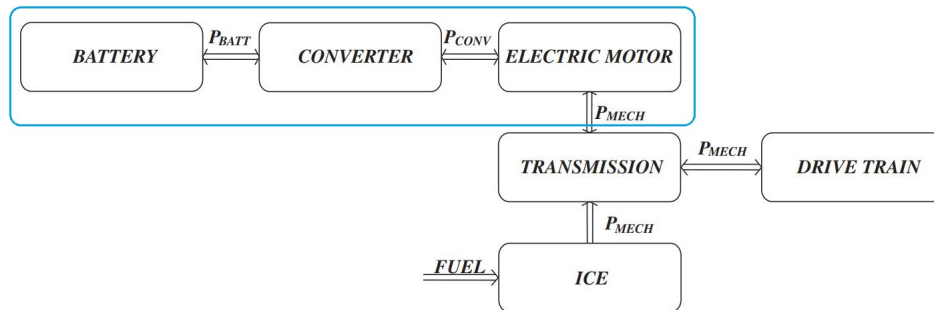


Figure 4: Topology of a parallel hybrid electric powertrain, with its electric component encircled [10].

Just as the emergence of hybrid powertrains introduced an additional level of control [12], the Connected Autonomous Vehicle (CAV), brings a new layer of strategic control, that is the traffic conflict resolution between drivers. While centralized, mass-point intersection management schemes are offered [13], so are agent to agent, game theory formulations [14] dedicated to distributed architectures. This chapter outlines these fields, to provide background for an algorithm facilitating V2V-enabled negotiation of conflicts between traffic agents.

Further, centralized optimisation algorithms for various applications are presented, followed by basic concepts of game theory as an introduction to distributed control applied to micro- or macroscopically modelled traffic systems. The research question of this Thesis is: How should one initialize/update the payoff utility functions of the individual processes so that the ensuing behavior of the entire collective achieves large values of the provided world utility? Since such technology would directly engage the driver, the economic and behavioural aspects of the proposed framework are touched.

2.1 Optimisation of powertrains

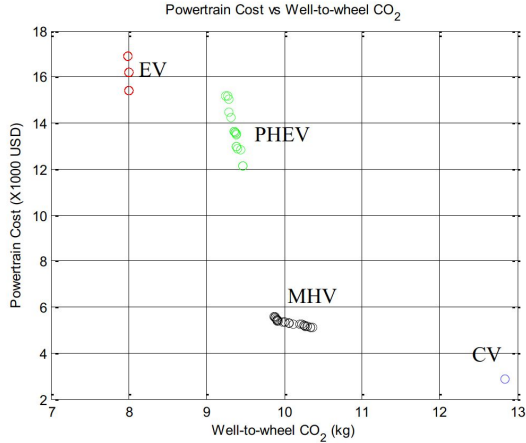
The electric powertrains have always competed with the Internal Combustion Engine (ICE). In recent years they are being revived, to diversify the energy mix in transport industry, contributing to economic stability [15]. With a growing number of powertrain topologies to choose from, a growing amount of work is done in the pursuit of the most optimal powertrain for a given application. Apart from well studied ICE and electric motors, control engineering is worked on to enable the construction of optimal hybrid topologies.

Hence a powertrain topology optimisation toolbox has been offered [16], whereby various powertrain technologies: internal combustion engine, mechanical hybrid, electric hybrid, both in-line and parallel, or fully electric with various storage technologies are scaled and dimensioned to find an optimal powertrain size for a particular drivecycle, using a genetic algorithm to optimise for vehicle's fuel efficiency and the well-to-wheel fuel efficiency, with the whole energy supply chain accounted for. The results are presented in Fig. 5a, challenging the Plug-In Hybrid Electric powertrain's economic feasibility, as the electric powertrain offers roughly 60% in Well-to-Wheel CO_2 emission cut in exchange for almost sevenfold increase in powertrain cost in relation to an ICE. These findings could e.g. help define the fair economic price of CO_2 emissions in a Pay-as-You-Pollute scheme, one of components of the developed cost function.

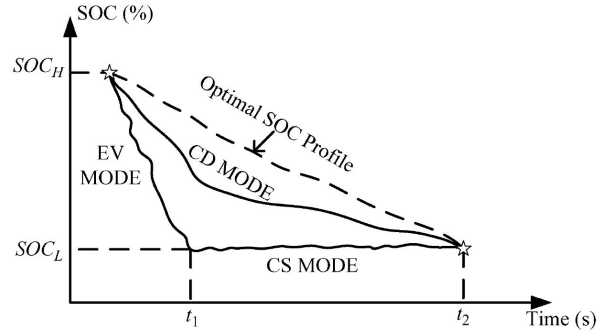
Fully electric vehicle may also feature a hybrid component. A Hybrid Energy Storage System (HESS) consists of an energy source, such as battery and power source, such as ultra-capacitor covers for power peaks, which cooperating to achieve desired energy storage characteristics [17].

The more complex the powertrain, the more refined its controller has to be in order to capitalize upon its potential. Since, currently, the supervisory powertrain controller is the human driver, effort has been made to predict their behaviour, so that the powertrain controller can adapt to driver's behaviour. Thus predictive methods of estimating driver's behaviour are proposed [18] [19] [20].

In a similar manner a neural network has been deployed in receding horizon formulation to predict tactical velocity change [21]. The same approach has been combined with traffic data to manage a hybrid powertrain, to optimise the battery input for a given journey [22]. Optimisation of battery State of Charge (SoC) for given journey, has been



(a) Multi-objective optimisation of powertrain topology for an ARTEMIS drivecycle is a trade-off between cost and energy efficiency of the powertrain. Electric Vehicle is most fuel efficient but most costly, whereas ICE emits most CO_2 [16].



(b) Dynamic programming approach to plug-in hybrid optimisation. Prior knowledge of the length of the journey allows to find an optimal battery depletion profile, with respect to energy and air pollution [12].

Figure 5: Powertrain optimisation, the topology selection and an optimal control strategy for a hybrid one.

also considered with more deliberative Dynamic Programming (DP) algorithm, whereby battery depletion profile are outlined: fully electric and charge sustaining modes, with an optimal profile being a trade-off found considering the powertrain topology, journey distance, elevation change, etc. [12].

Interestingly, game theory, described in Chapter 2.4, has been applied to minimize hybrid vehicle's emissions. The powertrain controller is set up as a decision agent, playing against the human driver, attempting to predict their actions so that vehicle can be operated in a way which minimises NO_x emissions [23].

Numerous optimisation strategies are being employed in the automotive industry, e.g. a vehicle-based traffic light assistant [24] or predictive gear shifting [25].

2.2 Connected Autonomous Vehicles (CAV)

In 2018 the Society of Autonomous Engineers (SAE) published the J3016 standard. It defines a terminological framework for describing the levels of vehicle autonomy and outlines five stages between human-driven and autonomous vehicle. However, the idea of self-driving cars emerged much earlier, as soon as defence research has enabled the aerospace guidance systems. The vision of an automated, self-driving vehicle soon fol-

lowed with first mentions as early as 1956.

Always sought for, the self-driving cars implementation prediction date has always drifted ahead, never becoming reality, as we did not realize how complex navigation through a public road is. The Project ARIADNE (Application of a Real-time Intelligent Aid for Driving and Navigation Enhancement), as described in *The Times* article from 13th Aug 1993 is the earliest traced popular literature describing a real implementation of an automated vehicle [26], where the role of the driver is that of a supervisor of the ultrasound and radar-based control system.

There is extensive literature studying navigation for automated vehicles [27] [28] [29]. However, it is theoretic, while the engineering applications performed by companies recently are trade secrets. An Open Source autonomous vehicle operating system: *Apollo.auto* has been offered, among others, by Baidu [30], offering an open GitHub repository and a dedicated hardware set. There are also numerous companies, both established and start-ups, attempting to commercialize autonomous driving as quickly as possible [31] [32].

In general, the vehicle operation is performed in four levels: perception, prediction, planning and control. Before we reach to the point where vehicles are autonomous, parallel driving is proposed as a transition, where driver and machine operate the vehicle in parallel, learning from each other and supporting each other [33].

Unlike airspace, public roads are full of environmental hard constraints and other autonomous decision-makers: drivers, cyclists, pedestrians at various stages of life or even belonging to different species, who may or may not be defined as rational. Only recently, the advances in machine learning have enabled navigation through these complex systems of public roads, but still having problems blending in, failing the *driver-Turing* test, being easily identified as an automated vehicle by other drivers. The scientific literature addressing the social interactions between autonomous agents is very general [34] [35]. Its application to public roads is primarily focused on policy and urban planning [36] [37]. However there is a number of popular articles on the matter, which observe that human drivers may 'bully' overly defensive AVs [38] [39] [40] [41], with driver's non-rational aggression as a common denominator of all above papers. On the other hand, the recently retired CEO of Daimler admitted that while today's autonomous vehicles are designed with maximum safety, in future they also need to modulate it, to assert

their space in order to prevent being 'bullied' by other road users [42]. In addition overly defensive AVs behave unpredictably, what results in other drivers rear-ending them [43].

2.2.1 CAV safety

With the road to vehicle autonomy outlined, the most important issue, leaving behind energy efficiency and comfort, is safety. While today it is in the hands of the driver, allowing little insight into it other than behavioural psychology research [44] and road safety statistics [45]. This shall change with the CAV. The supplier of the autonomous technology is the one responsible and who is most familiar with their technology, being aware of the weaknesses and failure rates. Taking an analogy from the nuclear power industry, the power plant operator is supervised by the government regulatory body, who ensure that the operator follows International Atomic Energy Agency (IAEA) safety requirements and ISO 9001/14001 standards. This features risk estimates for every operating conditions the facility can experience, integrating them in a general nuclear Core Damage Frequency estimate [46] [47]. Mitigation strategies are also developed to protect the population. Similarly, in aviation regulatory oversight is present to protect the travellers [48]. Thus, an honest seller of CAVs, should make the customer aware of the risk, for example providing an estimate of reliability of their autonomous/automated vehicle. Whilst in the case of a nuclear core the unit of risk is core damage events per year [y^{-1}], in case of transport it would be incidents, per road users per year [$d/u/y$] [49], however there is no regulation enforcing risk-informed vehicle advertisement.

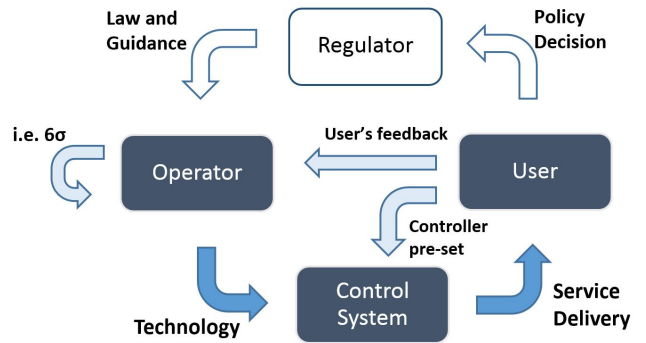
Another alternative is to allow each user to manage their risk individually, either by an explicit definition of their maximal accepted risk, or by a mechanism which would explicitly infer it based on user's feedback.

Fig. 6 offers a comparison of the nuclear safety framework and the contemporary approach to CAV design. It has been inspired by a seeming lack of strategic direction to develop a CAV vehicle safety framework. Comparing the state of the art of autonomous vehicle design framework discussion with the nuclear activities such as e.g. US Nuclear Regulatory Office proposition from 2012 [50], there is vast experience on protecting the public from engineering mistakes, which is to be learned from the nuclear and reapplied in the automotive industry.

Currently engineers attempt to overcome the technical problems by feeding more



(a) An outline of the International Agency of Atomic Energy Nuclear Safety Framework [51].



(b) A proposition of the topology of user-oriented Autonomous Vehicle Safety Framework. Intensity of colour defines the level of institutionalization. There is no CAV regulatory body.

Figure 6: A glance on similarities and differences between well established Nuclear Safety Framework and a possible, emerging, user-oriented AV Safety Framework.

learning data to the same, machine learning systems, hoping that the *Artificial Intelligence* offers a solution, instead of a structured analysis to find an informed, optimal solution. In the author's view, the literature on social science, behavioural psychology and economy may be re-purposed for the design of traffic Human Machine Interface (HMI) systems, and is therefore outlined later in this chapter. Currently, in the curriculum of both automotive and software engineering courses there is little dedicated safety oriented HMI design training, what could have hopefully prevent accidents such as the one of an experimental Uber Inc. vehicle in March 2018 [52] [53], whereby an autonomous vehicle has killed a pedestrian. While the victim had been detected by the radar, the unfortunate lack of illumination prevented the camera from supplying the data, which would allow the definition of the type of the object. The architecture of the decision system did not account for this event and no predictive threat level has been assigned to the unidentified object, leading to collision [54]. A safety analysis would have likely detected this flaw and prevented the accident which costed a by-stander's life but was not present.

Rather than relying only on the on-board sensors, in future, connected vehicles may share their data. Then, not only the present location would be more precise, but also

users could define their objectives on individual basis, so that the future actions are accounted for, and later even negotiated for an optimal resource utilisation.

2.2.2 Security

As numerous connectivity-based optimisation schemes already developed and tested, apart from safety, the main obstacle is the security and robustness of any connected system [55]. There is also literature studying possible cyber-attacks CAVs can experience [56], offering i.e. intrusion detection methods [57] or cybersecurity testbeds being put forward [58].

Given the negotiation aspect of this thesis, the proposed algorithm may be particularly vulnerable to fraud or deception attacks [59]. While their mitigation in CAV scenarios has not been put forward yet, there is general literature addressing this issue [60].

2.3 Traffic management schemes

With the number of miles driven using Adaptive Cruise Control (ACC) on public roads increasing and an increasing number of both established companies and start-ups attempting to bring autonomous driving to roads [31] [32], an opportunity emerges to seek for further energy efficiency and pollution reduction, for scenarios such vehicles may encounter. The largest contributors to the energy inefficiency are braking and aerodynamic drag. The elimination of braking is a matter of information, enabled by the connectivity. The drag, however, is roughly proportional to the square of the velocity and thus can be addressed by careful selection of the cruising velocity.

This subchapter focuses on the energy perspective opened by V2X connectivity. It neglects the technological requirements to achieve it, as there is an extensive body of literature focusing on this technology [61] [62], and its possible safety improvements [63] [64]. A survey on vehicle coordination offers deeper insight [65]. The literature outlined here could be divided by two criteria. Most of the literature features centralised, top-down algorithms, as they perform best in heavy traffic conditions. As the traffic intensity diminishes, the adaptive, distributed approaches are scarce, leaving the development of user-oriented, infrastructure-free approach as a research gap, which this thesis aims to challenge.

2.3.1 Cruising, Platooning and Overtaking

Energy consumption depends on the square of the speed, therefore cruising e.g. at 30 $[\frac{m}{s}]$ rather than 35 $[\frac{m}{s}]$ theoretically requires 26 % less energy. The selection of the optimal cruising speed can then be an important aspect of energy optimisation in the future. Very narrow scope of research, however, explores this. Since the pure minimisation of energy consumption leads to optimal speed equal to zero, researchers add an arbitrary coefficient to define the cruising speed [66] [67]. The subjective, abstract parameter does prevents this approach from being easily integrable with other systems. Clearly a deeper approach may be required. This brings up the economic purpose of the journey, which could allow to incorporate the value of user's time as a decision-factor. It is discussed further in section 2.5.4.

Expanding on the cruising scenario, a strong proposition to bridge macro-scale effects and microscopic traffic simulation is the three-phase traffic theory [68], which was developed while studying the relationship between vehicle density k [veh/km] and flowrate q [veh/h] on a single motorway. The theory proposes that there are three modes of traffic: Free flow (F), Saturated flow (S) and Jammed flow (J), which are visualised in Fig. 7. As k grows, we arrive at maximum throughput. There, any disturbance triggers the $F \rightarrow S$ transition. Then, the throughput is diminished, due to proximity between vehicles and if the upstream flow is not throttled it may trigger the congestion. An application of Cellular Automata simulation [69] to test this theory has yielded positive results at reasonable computational complexity.

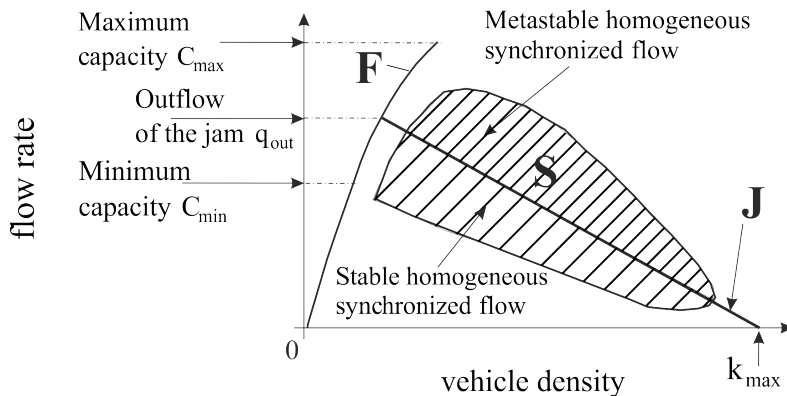


Figure 7: Three phase traffic theory. We can distinguish the Free flow, whereby vehicle density and flowrate are proportional and Saturated flow, where vehicles are synchronized, where increase in density diminishes the throughput [68].

Platooning, where vehicles follow one another has a potential to bring 15 % energy saving, by virtue of aerodynamic drag reduction [70]. Additional benefits of a connected platoon are cooperative sensing [64] [71], relevant from a safety perspective, and indirectly the decrease of complexity of intersections, since the whole platoon can be treated as a single agent. Given the risk of collision, the main challenge towards the implementation of platooning technology, apart from the need for a shared communication standard, is the controller stability [72] [73] [74]. The closer the vehicles are to each other, the greater the road throughput is, the sharper controller setting is required, making the interaction between agents a tradeoff between congestion and individual user’s comfort, what further complicates the platoon stability. This also shifts the characteristics of traffic from being the average of user’s driving norm to engineering design. The general research on multi-agent flocking dynamics is mature [75]. While it is being recently applied to aerial vehicle formation [76], the traffic implementation of it is not developed past its white paper [77]. A notion of traffic cooperation has been proposed [78], but the research redirected towards more imminent 802.11p V2V connectivity [79]. There is no research specifically addressing the consensus on the conflicting objectives among participants of a platoon.

Autonomous overtaking has received attention already. Research focuses on the theoretical background to guide further development and harmonization of the lateral and longitudinal controls or the technical requirements to handle it [80]. More recent studies propose a division of the maneuver into three phases, acceleration, overtake-cruise and deceleration, to apply adaptive control algorithm [81], or application of spacecraft rendezvous algorithms to approach the problem [82]. Most importantly, studies the feasibility of autonomous overtaking have been performed, utilising Model Predictive Control (MPC), taking the safety and comfort as objectives [83]. The cost function is defined to penalize deviation from the reference velocity and trajectory, taking into account the distance to the oncoming vehicle. However, the formulated method assumes no cooperation, as agents optimise for themselves, and does not track energy consumption nor time.

A noteworthy publication on the matter to be summarized is a hardware-in-the-loop agent-based cooperation algorithm, whereby scaled CAVs cooperate by slowing down to let another car ahead, in an in-line problem. The implementation is presented in Fig. 8, which is a screen from a video [84], which refers to [85].

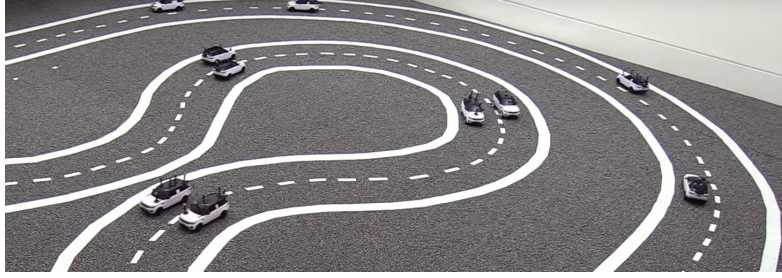


Figure 8: A glimpse on the state-of-the-art cooperative traffic algorithm. Scaled Autonomous cars cooperate by voluntarily yielding to in a lane change conflict to improve road throughput by 30% [85].

2.3.2 Intersections

While Section 2.1 considered a vehicle as standalone complex control system, the traffic management schemes consider many vehicles at once, treating them as simple mass points, aiming to minimise the total time delay and energy consumption. The main differentiating factor splits the literature into centralized and distributed schemes. The centralized approach assumes there is a single decision-maker providing velocity profiles to all traffic agents, who execute it selflessly. The Intelligent Cooperative Adaptive Cruise Control (iCACC) concept [86] provides velocity profiles ahead of the intersection's Conflict Zone (CZ) in order to correct time of arrival of vehicles in such a way that the conflicts are mitigated and sum of the time delay is minimised. This work is visualised in Fig. 9. In such arrangement, the only on-board control is the execution of the velocity profile. It is safety-critical, with no room for autonomous decisions, but offers threefold improvement mitigating intersection delays in comparison to traffic lights. This work has greatly inspired the intersection resolution algorithm developed herein. The main difference, however, is that the cost function there considers only time delay, albeit across a range of traffic densities. There are several similar publications [87] [88], where for computational simplicity, the vehicles are all kept as anonymous agents. On the other hand, in this thesis the decision is a result of multi-objective optimisation between time and energy, in order to provide a general framework for formulation of conflicts, which considers users heterogeneity. The computational complexity is allowed to increase, relying on the growing CAV's computational power to handle it. The safety-criticality of the intersections is being addressed by a design of robust intersection scheme, where an optimal trajectory is followed by a safe one, should the situation require it [89].

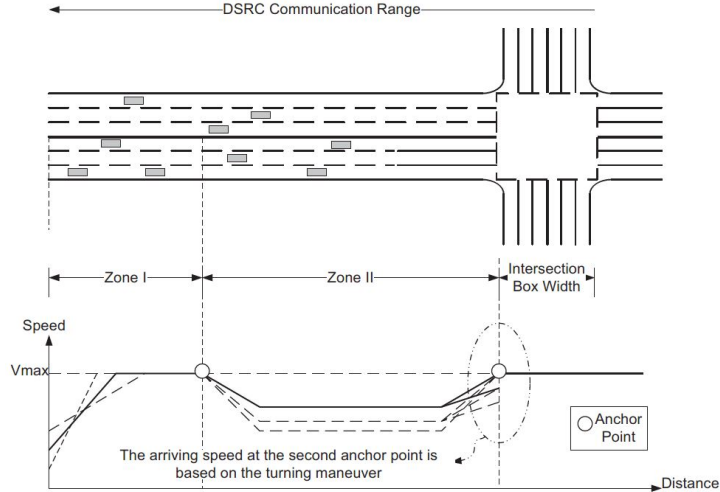


Figure 9: Intersection management based on vehicle connectivity. The conflicts are resolved by adjusting the velocity profile ahead [86] with an objective to minimize the time delay.

Elimination of the uncertainty, enables information-driven optimisation of hybrid powertrains. It has been proposed to finely adjust hybrid powertrain controls by means of a central computer [90], as opposed to on-board algorithms as in Chapter 2.1. An elegant approach to the elimination of uncertainty is a traffic light assistant system, which suggests the time to green light and is being developed into an energy-optimal adaptive cruise control [24].

There is also a number of distributed approaches to intersection management, where calculations are shared between agents, with an emphasis on system resilience [91] [92] [93] [94] [95]. A block-chain based approach to intersection occupancy scheduling has also been proposed [96]. In general, all these schemes put forward a notion of a schedule, whereby each vehicle reserves a time window of the intersection’s CZ, by means of an automated algorithm. The key difference among publications, apart from details of the algorithm formulations, is the processing unit architecture. It is either a central road-side unit, which could, for instance, operate together with traffic lights, or distributed architecture, whereby every participant contributes their processing power. All the proposed schemes, however, utilise a First-Come-First-Served (FCFS) policy. While it is fundamentally fair, it prevents refined optimisation, resulting in scenarios of paradoxically degraded performance of advanced intersection, e.g. due to platoon breakup [97] [98].

An energy-oriented approach to intersection control has been proposed as well [99] [100]. The objective of minimising energy, however, results in optimal solution being

stationary. A time delay penalty, with an artificial biasing parameter, has been thus introduced to offset it.

In order to refine the optimality of intersections, apart from time delay, the fluency of traffic, understood as the departure from the cruising speed, as well as the vehicle mass are considered to minimize energy consumption [101] [102] [103]. In such case, heavier vehicles are prioritised, as it would be more costly for them to detour. While this promises significant energy-efficiency improvement and the performance of the framework is unchallenged, no consideration is given to the enforcement mechanism. A rational decision-agent, be it a human driver or an autonomous controller, of a lighter vehicle has no incentive to adopt a strategy which does not favour them [104].

The consequence of the energy oriented optimisation is that some agents are worse-off. The enforcement of the mechanism and the economic incentive for the journey are becoming relevant. Thus an auction-based negotiation algorithm has been proposed, whereby vehicles queuing to the traffic-light regulated intersection jointly bid for the priority, with the winner having to pay for their right of way from their *onboard wallet* [105]. More generally, a market-inspired approach to urban road networks has been theorised, whereby a number of payment-focused algorithms are proposed, offering a dynamic intersection pricing algorithm or auctioning system. It is to be coupled with a driver information system as a remedy to urban congestion [106]. A block-chain payment systems has also already been implemented [107]. These publications are a voice for a micro-payment based, pay-as-you-drive system. Although the literature is not extensive, in the year 2019 Jaguar Land Rover has partnered with a cryptocurrency, to enable their cars to autonomously perform transactions [108]. As a first use case the vehicle is to be paid for information on the road quality data, but in future it is to serve as a payment mechanism, e.g. for parking or congestion charges.

Since the ownership structure of CAVs is speculated to shift, as a taxi-like, driverless pay-as-you-go mobility service may become a feasible operation model [109]. The privacy aspect of such business model, whereby intelligent agent negotiates for the user, has also been considered [104].

An Adaptive Cruise Control system has been coupled with an intelligent traffic light scheduling [24]. An auto manufacturer has implemented a first-generation implementation of this technology in their vehicles already [110], albeit only in a single country, where

traffic light systems has countrywide infrastructure standard. It is claimed that optimal traffic light approach trajectory can save 20% energy at no loss of time, as visualised in Fig. 10 [111]. The second generation of this system is consider traffic light schedule input to adopt both the velocity profile and the powertrain control strategy in a manner similar to [22].

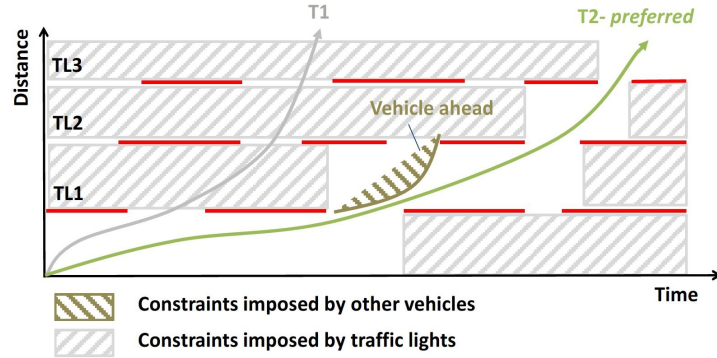


Figure 10: Traffic light enabled ACC. The red line symbolizes the traffic light constraint. Vehicle ahead is also a constraint. The data is then used to optimise the battery SOC profile [111].

Finally, vehicles can also change lanes. It is not a challenge, from a control perspective, as such a manoeuvre is a special case of both a platoon formation and an unconstrained, Y-shaped intersection. A simplistic, static game-theoretic approach to lane merging conflict has been offered, although the game design features arbitrary payoff definition [112]. A multi-lane motorway optimal lane selector algorithm has been proposed as well, featuring a complex cost function, considering multiple decision factors, including travel efficiency, safety or control effort required [113]. There, parameters as safety, equilibrium, travel efficiency, control effort, selection of the route and subjective preference are the decision factors. The output is a reference number of the lane, which the vehicle should follow. This algorithm has been also applied in Hardware-in-the-Loop [85], yielding road throughput increase of 35%.

Summarising, there is a number of publications proposing various conflict resolution schemes optimising for time, or energy consumption. All of them provide strong, promising results, but there is no effort towards providing a background for individual economic incentives for users participating in such system. Given the distributed ownership structure of the public traffic agents, the issue gains importance when considering that a traffic cooperation scheme needs for voluntary stakeholder opt-in, in order to become feasible. The user-oriented consideration to traffic may suggest a path, whereby economic intention

is a factor in optimal traffic de-conflicting, a notion this thesis intends to further.

2.4 Game Theory

While an optimisation algorithm acts on a single control system, consisting of sensors, a single decision unit, and actuators. Even with Multiple Input Multiple Output systems, there always exists one point in the system, where all information streams converge. In addition the sensors are honest, giving a complete information on the state of the system and when a decision is forwarded to the actuators, it is always selflessly executed. Whenever either of the above conditions fail, we deal with more than one decision agent. Then the decision process must account for actions of other agents too. There is no central authority, and nobody is guaranteed to have a whole picture of the system.

While designing a controller the goal is to design a strategy to optimize a given system. In case of game design the choice of control strategies is left at the discretion of players, instead game designer chooses rules of the game to elicit certain social behaviours, e.g. favouring one equilibrium over another [114].

Game Theory (GT) is a branch of mathematics, which considers systems with multiple, autonomous decision-makers. Games can be played once, as one-shot events, e.g. when taking tactical decisions on a battlefield. Then, it is difficult to conceptualize any equilibrium beyond level-k reasoning, that is attempting to estimate what would the adversary do assuming what the player does (level 1), what would they do if they accounted for what I do in response to what I do (level 2), and so on (level k) [115]. More often, games are iterative, with the same mechanism reoccurring in a set of agents. In such system, we cannot define an explicit solution, therefore various solution concepts are proposed.

A strategic model of interactions defines a game consisting of agents $\mathcal{N} = \{1, \dots, n\}$, where $n \geq 2$ and $i \in \mathcal{N}$. The agents have a set of choices $a_i \in \mathcal{A}_i$. The possible actions agents take are assigned as $\mathcal{A} \Rightarrow \mathcal{N}$ for each, i^{th} agent. The joint actions $a = (a_1, \dots, a_n) \in \mathcal{A}_1 \times \dots \times \mathcal{A}_n =: \mathcal{A}$ define the payoff function

$$J_i(a) = f(a_i, a_{-i}), \tag{1}$$

where the a_{-1} is the actions of all agents except the i -th.

The Nash Equilibrium (NE) is a state of an iterative game, where no player has an incentive to deviate unilaterally. If we mark an equilibrium strategy as a^* , a Nash Equilibrium is a state when

$$\forall i \quad a^* = \arg \max_{a \in \mathcal{A}_i} (a_i^*, a_{-i}^*), \quad (2)$$

meaning the optimal response to an equilibrium strategy is an equilibrium strategy.

The Nash theorem states that every strategic game, with finitely many players and actions, always has a mixed strategy Nash equilibrium (NE) [116]. However, finding an NE in a continuous solution space belongs to NP-hard complexity class [117]. The other equilibria, inspired by the NE and the most relevant to this thesis are:

- Correlated equilibrium [118] is one where an action is not only the best response ($>$), but also equal (\geq) to another.
- Pareto equilibrium (PE) [119] exists if no player can benefit more without it costing others. This is not relevant to zero-sum games.
- Bayesian equilibrium [120] is a NE played in a Bayesian game, that is where players do not know their and adversary's payoff functions, but need to learn them from observation. In a Bayesian game, the objective is not only to choose the best strategy, but also to deceive the opponent's situational awareness.
- Stackelberg equilibrium [121] is thought of as an asymmetric situation, where one player knows the adversary's strategy in advance. The order of the players is relevant, and thus decision-tree form is relevant. In this thesis, however, only the matrix form suffices.

A game can also be designed as a zero-sum, where the sum of all payoffs is constant and one agent's loss is another's gain. Such games are purely competitive. Non-zero-sum are games where the payoff's do not add up to a constant number. As a consequence, a form of coordination is possible to beat the environment. Therefore every non-zero sum game is partially competitive and partially cooperative [122]. It is further explained in Section 3.2. In experimental application a cooperative game, however, is only cooperative as long as the players are incentivised to cooperate. It can be provided either by an external authority, e.g. law, or be incorporated into the game design. The self-enforcement is

understood as a capability of the framework to operate without external oversight. It is achieved by consideration given to sharing of benefits enabled by the cooperation. A side-payment method of enforcing cooperative games has been proposed recently [9].

The complexity and ambiguity of a solution results in game theory being an aid to reasoning about a multi-agent problem, as opposed to an explicit solution. As such, there is a number of example games, with certain distinct research characteristics. As an example, we consider a Stag Hunt game. Defined is a game with two players: *ROW* and *COL*. Each can either attempt to hunt a Stag (*S*), where both are better off cooperating, or a Hare (*H*), a smaller payoff, but independent of the adversary. We can then create a payoff bi-matrix, which considers all combinations of actions assigning value to each case, for both players.

		<i>COL</i>	
		S	H
<i>ROW</i>	S	3,3	0,2
	H	2,0	2,2

We can then observe, that there are two Nash equilibria. In a one-shot game, we don't have any prior to draw conclusions from. Given a random chance of 50 – 50, if we go for a Stag, the expected payoff E for either player is $0.5 \cdot 0 + 0.5 \cdot 3 = 1.5$. Compared to a certain payoff of 2 for a Hare, the optimal solution is Hare. We can consider it as a risk-dominant equilibrium. In an iterated game, however, the selection of a strategy is also an information on the player's intention. One can play Stag, the locally suboptimal strategy to mark the readiness to cooperate. An intelligent adversary recognizes it and plays Stag too, arriving at the payoff-dominant equilibrium, with no incentive to deviate. It is a Pareto equilibrium.

Engineering applications of GT involve distributed controller design used to facilitate drone formations [123] or distributed electric power generator control. It is relevant for wind turbine farms [124]. GT algorithms to foster cooperative driving are also present [125].

GT is also extensively used to model traffic conflicts already offering multiple approaches, employing advanced methods of finding solutions in multi-agent systems, however, again, the game design features homogeneous agents [86] [14] [126]. GT is also employed to find numerous unconventional problems. As such we can find GT approaches to the Poker card game [127], and to the political decision making scenarios [128].

2.5 Further literature

This section contains any literature which is not essential for the positioning of this thesis in relation to research on the traffic optimisation. It provides, however, a high level view on some non-engineering aspects of the traffic optimisation. It is an introduction to the Chapter 7, which focuses on the Framework Scalability Analysis, where the core novelty of this work is tested against its assumptions.

2.5.1 Game Theory as a view on economy

Game theory finds application in several fields of research, such as economy, where it serves to help understand decisions humans make [129] or design auction schemes [130]. Another recent example is the bounded rationality approach to economy [131] [132]. It strives to explain the deviations of neoclassical economy by means of cognitive science, model decisions under the finite computational power constraint.

One possible approach to GT is to treat it as a mathematical method of framing conflictual multi-agent optimal control or decision problems, to aid in the design of given system environment. Games, however, have a natural tendency of converge towards some equilibria, which tend to be summarised with literary parallels. As such we have the stag-hunt game, described above, but other examples involve: the chicken game, beauty contest, congestion game, cake cutting game, etc. All of them serve as metaphor to reflect some game dynamics, offering a frame for reasoning and understanding of societal interactions or methods of arriving at the consensus [133]. GT also allows definition of bargaining or auction algorithms. These negotiation games can be methodologically formulated to design fair auction schemes or to frame historical events, such as the Cuban Missile Crisis, in a numeric decision scheme [6]. It also has been employed to study political negotiation scenarios, exemplified with a model of the dilemma faced by a voter in a secret ballot election scheme, used in every western democracy, or parliamentary veto bargaining models [128].

2.5.2 Air Traffic Control

Consideration to road traffic management cannot be given without mentioning Air Traffic Control (ATC). The optimisation problem seems similar: to manage conflicts between agents, minimising delay and energy use. The environment, however, is quite

different. The very selection of words: Air Traffic *Control* and Road Traffic *Management*, mark different liability structures. Whilst airborne the sole responsible for any damage is the government appointed airspace operator [134], with the aircrew merely executing ATC operator's instructions. By design it is a centralised system, with clearly defined hierarchy, as any potential incident would be disastrous. On the other hand, on the ground, where road collisions yield little risk of fatality, the drivers are the held liable. By design, it is a distributed system.

The clearly defined liability has driven research on ATC de-conflicting for the previous several decades [135] [136]. The cruising conflicts occur only in most congested areas and are usually mitigated by vertical separation. The biggest challenge, however, is the approach to the descend and landing phases, where all aircraft converge to the same point, with the wake turbulence forcing a time separation. Additionally, each aircraft type follows an individual descent path, which is sensitive to weather conditions and the wait time is constrained by fuel burn. The Time-Based Separation (TBS) system [115], an automated aircraft approach scheduling algorithm is recently being implemented in the largest airports to assist the ATC controllers. It is a step towards automation of ATC, where the key obstacle is not the implementation strategy, as it is on roads, but computational complexity, system resilience and robustness.

The Airline Cost Index (ACI) is a function that defines the aircraft's operating cost, accounting for operating time and fuel burn as a function of velocity. Should it be public and known to the ATC, complexity of de-conflicting would decrease vastly. However, the ACI is the key operating cost of an airline and thus the deciding factor in its business strategy decision. Because of high competitiveness of the air travel market the ACI is thus always a trade secret. On the ground, however, automated taxiing is being developed [137].

As one can notice, while the ATC and road traffic management are similar in their core idea, the liability structure and nature of constraints have resulted in very different approaches. An aircraft operates in three dimensions, mitigating conflict in vertical and lateral axes, but can change its speed only in a very narrow window. Cars, on the other hand, are constrained to their lane, obviously cannot fly, but can easily accelerate or even stop. Therefore, with ATC research being several decades ahead, traffic designers can draw inspiration from their colleagues, but must do their own due diligence to understand

the specificity of the traffic optimisation problem.

2.5.3 Communication in GT and self-enforcement

The main challenge of GT is that the formulation does not capture the contextual information of the game. An example to visualise it is the ultimatum game [138], where one player decides how to split the resource. The second player may reject it, then nobody gains anything. The calculations and experimental results differ. While mathematically, the second player's best response is to accept any offer, human players reject unjust offers, with the threshold varying across culture. These unwritten social norms convey expectations into the game, but there has been no formulation proposed, which would enable capturing this phenomenon. The solution concept employed in this work, the cooperative-competitive value [122] relies on the notion that agents can exchange information and side-payments.

The Aumann's Conjecture observes that, in non-cooperative games, the communication between players allows to achieve higher overall welfare, even with no enforcement mechanism. The only value of the agreement is to convey information [139]. It is the payoff-dominance of the coordinated solution, that provides the incentive to honour the agreement. However, the 'pre-play cheap talk' is also considered to come at no cost, while the negotiation algorithm promising the PE, proposed here, comes at a measurable computational effort, which should not be neglected in the game design.

2.5.4 Human driving and CAV

Human drivers and machine-based decision systems *reason* in different ways [140]. Humans understand the situation and its context, but are subject to fatigue. While an 'AI' never gets tired, it merely emulates reasoning and cannot guarantee its performance.

Economy and sociology are broad areas of research which, instead of considering tangible objects like physics and engineering, deals with large systems consisting of human actors. As a result it lacks tangible, measurable quantities and relies to a larger extent on theory, for its metrologic aspect.

Engineers are focusing on maximizing the energy efficiency of transportation. The most efficient way of saving energy is not to travel at all. However, a methodological consideration of the reason for a journey would be impossible without violation of the

user's freedom [141]. So far the financial ability has always served as the decision factor deciding the distribution of goods or services, leaving the fairness of the system to be a function of the distribution of wealth. The Austrian School of Economics proposes the cost of opportunity as the cost of the gain that could have been achieved if an action was not taken [142]. Applying this notion to mobility, the driver would assign a value of time to justify their journey. To support this concept, the Value of Travel Time Savings (VTTS) is a dominant economic factor for user benefits estimates, which are conducted when deciding infrastructure development directions [143]. However, in this thesis the Cost of Time evaluation, denoted C_T , is applied to vehicle control. It serves as one of the optimisation criteria offsetting the cost of vehicle operation, which is one of the main contributions of this work.

2.5.4.1 Cognitive aspect of driving

The cognitive processes enabling us to drive are still unknown. Even automated racing cars, in a seemingly much simpler environment than traffic, are still far from matching human drivers [144]. There is also literature attempting to measure the driver's distraction by means of eye movement tracking [145], or studying the effect of visual disruptions on driver safety [146]. In the previous decade an effort has been made to develop a prediction system which would aid in detecting potential collisions with other objects based on intention prediction [147]. More recently a sensor suite dedicated to emotion detection is being set up to analyse the relationship between emotions and cognitive load [148]. However, there is little research attempting to explore the phenomena occurring in the very human brain during driving, e.g. by utilising brain imaging techniques to track thought activity, with [149] being the major publication.

Surprisingly, there are no legal regulations or research on the Human-Machine Interface (HMI) between the driver and their car, resulting in today's cars having the driving data feed morphed with entertainment for passengers, resulting in an 'infotainment' system. It is a strong contrast with the aerospace industry, where consideration is given to the cognitive processes undertaken by pilots while executing tasks, resulting in carefully optimised layout of flight instruments and innovations such as head-up display systems, which are proven to improve pilot's precision in aircraft control [150]. Currently, research focuses on enabling single-seat aircraft to be manageable from task load perspective by

a single pilot [151].

2.5.4.2 Social norms

Society in which we live in consists of autonomous individuals. The space for arguments always follows The free will. Every culture, however, develops a set of social norms that provide behavioural patterns to ease these tensions. The simple example being a formation of queue in a shop, or letting people to pass through a door. These everyday processes escape our attention, but where performance of socio-technical system matters the most, the military, the role of cultural norms has been identified as a single most important performance predictor [152].

Research on how such norms emerge indicate that heterogeneity of individuals fosters creation of new norms, as stronger agents volunteer to challenge the status quo [3] [153]. The emergence of cooperation in the biosphere, a seemingly purely noncooperative world is also easily observable and understood [154]. Currently attempts to capture social network behaviours by means of GT algorithms take place [155] [156]. Their application varies from improving urban space to advertisement targeting. In work on cooperation between human and robotic agents, some findings indicate that adding noise to the system helps improve the overall wellness [157].

Just as we refer to behavioural norms exploring cuisine abroad, in the same manner traffic norms are organized. Driving an automobile features a language formulated to inform (road signs), signal intentions (indicator and stop lights), or resolve conflicts (traffic lights or the law). The written, formal rules: The Highway Code also provides the rules on how to share liability should an incident occur. There are also informal norms such as using emergency lights to signal a traffic jam on a motorway, flashing one's lights to signal a voluntary yield, or tailgating to signal hurrying. These norms may be very subtle and account for the driving experience varying between countries. The emergence of such particular unwritten rules is not known.

The literature on social norms in traffic focuses on safety. The road accident predictors are being sought for [158] and the road aggression on is one of focal points [159]. However, the emergence or control of informal traffic norms described above is forgone. If researched, however, it could provide a deliberative framework for fine-tuning of a CAV

traffic behaviour to match local driving norms with precision unmatched by adaptive learning algorithms.

2.5.4.3 Ethics of autonomous systems

Another social challenge of traffic automation is ethics. With the exception of autonomous weapon systems on a battlefield [160], roads are the first occurrence of autonomous systems to perform safety-critical decisions considering human life. Since it is the OEM who takes responsibility, there is little clarity as to how to progress. There is a number of papers considering the CAV ethics issue in critical conditions [161] [162] [163], albeit superficial, reluctant to draw conclusions. Most interestingly, an article studying moral decisions humans make in a simulation of an event where it is too late to break and one can either hit pedestrians or a wall indicate the heterogeneity of decisions across various cultures [164]. None of the above, however considers safety of CAVs as an inherent feature, ensuring fail-safe operation of vehicles, just as it takes place with aircraft or nuclear systems, as it has been touched in chapter 2.2.1.

It is relevant to notice that consideration is given to the question whether autonomous military aircrafts' *kill-decisions* are to be performed in deliberative architectures, where decision parameters are ultimately human-decided. The alternative is an adaptive architecture, e.g. using Machine Learning (ML) [165], whereby an abstract decision rule is tested in the process, to empirically match the model to observation. ML is the dominant method for deploying decision frameworks autonomous road vehicles'. The intelligent architecture design for negotiations in multi-agent environments is outlined in [166].

2.5.5 Future of road mobility

The research on traffic management and the vision of industrial actors converge on a vision for the future of road transport. The abundance of information, V2X connectivity and autonomous control systems promise a highly synchronised, cooperative, safe mode of mobility. The current paradigm, however, with a clearly defined liability is well established in a very distributed environment. Multiple automotive manufacturers compete with one another, attempting to predict the will of the consumers, while complying with the governmental regulations. Consumers, in turn, comply with regulations, with the

insurance services cushioning the costs. The multitude of actors results in the transition period being difficult to forecast.

As of the date of writing this work, there are OEMs, who provide V2V communication on their vehicles to inform of the dangers on the road ahead [167]. The data sharing, however, is limited only to vehicles of the same brand, since the trustworthiness of the third party data is inestimable. However, today we put the safety of our lives in the hands of others daily and, despite the car being everpresent, transport accidents are only the 9th mode of death worldwide [168]. This discrepancy suggests there is a gap in understanding of human social trust dynamics, from which information sharing paradigm among road users could emerge.

The main change during the transition is in the liability structure. In an autonomous system it is the service provider who is responsible, not the driver themselves. This causes an ethical challenge. The safety decisions, which are currently made by drivers in the heat of the moment, will now have to be formally predefined in software.

A possible solution of this stalemate, tackling both the safety and ethics problems is shifting the role of the drivers to Open-Source programmers, who work together on creating a global traffic cooperation framework in the same way as Apache server software has been created, that is by hobbyists, driven by passion [169]. The social structure of Open-Source software developers and their motivations have been a subject of several studies [170] [171]. This would require the driver, despite possibly not following the situation on road as required by lower SAE autonomy levels, to be familiar with their vehicle's decision architecture, and therefore being able to anticipate its failure points rather than waiting for the vehicle to request a take-over.

2.6 Summary of the literature

The literature review has outlined the body of research focused on traffic optimisation. The first focus, the literature on powertrain optimisation (2.1) offers strong, well structured research with no gap for novelty. Moving on to the control of vehicles, the history of Connected Autonomous Vehicles (2.2) and state-of-the-art challenges are outlined, identifying safety and interactions with human drivers as key challenges. The traffic management research (2.3) offers methods of fighting congestion. There is extensive literature on both macroscopic traffic algorithms and cooperative agent-to-agent systems, applied to the relevant scenarios: intersections, overtakes and platooning. Novel business models are explored and the security of V2V communication is studied. But the consideration to the user's economic incentive to opt-in into either of these technologies remains a gap. In addition, the heterogeneity of agents, otherwise neglected, may serve as the key enabler in finding energy-optimal solutions to traffic conflicts.

Having outlined the direction of applied research, the mathematical tools needed to conceptualise de-conflicting, the game theory, is introduced (2.4). The extended literature provides a step back, to offer a distant look on the economic and sociological relevance of the traffic optimisation problems, such as ethics and cognitive science. Air Traffic Control (2.5.2) is also mentioned as a good example of successful implementation of optimisation algorithms to de-conflicting. The differences between aircraft and cars are so vast, however, that the solutions applied are not compatible. Finally, the literature on human-machine interactions 2.5.4 in distributed systems is gathered, to consider challenges to CAV implementation.

3 Formulation of the traffic conflict problems

This chapter outlines the mathematical formulation of the optimal resolution of the traffic problems. Firstly, the vehicle model is outlined. It is then followed by the definition of game-theoretic solution concept assumed and the geometric model of conflicts. All the assumptions taken in order to formulate the problem and control its complexity are also listed. Finally the negotiation algorithms for all considered cases are presented.

3.1 Vehicle model

While more refined vehicle models improve result precision, the main objective of this work is to propose a fundamental understanding of the decision and negotiation algorithm. Simplistic vehicle models are thus applied for clarity, as there is little novelty in already researched optimality of powertrain selection [16].

Powertrains can be modelled either as a forward-facing or backward-facing model. The first one employs a driver model and a closed-loop controller with a timestep under 0.1 second, allowing to capture the whole of longitudinal dynamics of the system and its physical limits [172]. It is deemed to be unnecessarily complex, however, for the scope of this work, which focuses in inter-vehicle conflict-resolution. In the later, backward-facing model, the drivecycle provides reference to the powertrain model, which operates on a quasi-static basis.

For the purpose of this work, a simplified, backward-facing powertrain model is employed and outlined here. The time is discretised into $T = 1, 2, \dots, i, \dots, N$ elements with a timestep t_{step} in order to employ Euler forward method to numerically resolve the vehicle model. The n^{th} vehicle's state is defined by its acceleration a_n , speed $v_{i,n}$ and position $s_{i,n}$ as

$$a_n = \frac{F_{net,i,n}}{m_i}, \quad (3)$$

$$v_{i+1,n} = v_{i,n} + a_{i,n}t_{step}, \quad (4)$$

$$s_{i+1,n} = s_{i,n} + v_{i,n}t_{step}. \quad (5)$$

The control variable is the acceleration a_n , being assumed constant, as dynamics are

neglected for simplicity, to focus on the strategy selection. Then the time of given velocity change manoeuvre T_a is found from the boundary velocities V_n^* and V_n^{ovt}

$$T_a = \frac{(V_n^{ovt} - V_n^*)}{a_n}. \quad (6)$$

The propulsive force required to execute is found by relating to the balance of forces defined as

$$F_{net,i,n} = F_{W,i,n} - F_{roll,i,n} - F_{drag,i,n} \quad (7)$$

where the $F_{W,i,n}$ is the propulsive force, assumed to represent torque of the motor acting on the tyre-ground interface. It is constrained by \hat{F}_n , to limit the powertrain power output, as

$$F_{W,i,n} \leq \hat{F}_n. \quad (8)$$

Regarding the resisting forces, the rolling resistance is defined as

$$F_{roll,i,n} = \mu_{roll,i} m g v_{i,n}, \quad (9)$$

where m is vehicle's mass and g gravitational constant, and the aerodynamic drag force is defined as

$$F_{drag,i,n} = \frac{1}{2} A_n \rho_{air} C_{i,d} v_{i,n}^2. \quad (10)$$

The powertrain's unit energy consumption is calculated as

$$E_{i,n} = \frac{1}{\eta_P} F_{W,i,n} v_{i,n} t_{step} \quad (11)$$

where the powertrain efficiency η_P is a product of all powertrain components, in the case of electric system $\eta_{i,M}$, $\eta_{i,PE}$, $\eta_{i,B}$, being respectively, electric motor, power electronics and battery.

Finally, the energy expense of a manoeuvre is found as

$$E_n = \sum_N E_{i,n}. \quad (12)$$

Given that friction brakes convert kinetic energy to heat, any its use is wasteful. Since this work focuses on the strategic selection of optimal vehicle control strategies friction

braking is neglected. Apart from energy dissipation, the deceleration manoeuvres may be performed with electric motor, recovering some of the energy. It is capped at a fixed maximum retrograde force

$$\hat{F}_{rec} = -\hat{F}_{W,i,n}\beta, \quad (13)$$

where β is the energy recovery limit.

3.2 Game-theoretic solution concept

Game Theory is a method of formulating optimisation problems, as explained in Section 2.4, where more than a single decision agents are present. On a road it could be two vehicles approaching an intersection on a collision course. They are self-optimizing agents, whose objectives are different, but not mutually exclusive. Thanks to the properties described here, they may agree on a globally optimal solution and share the benefit, according to their negotiative power. Equilibrium seeking approaches to GT do not require enforcement, but are computationally expensive and cannot guarantee the payoff-dominant solution [117]. Incentivised by a promise of an optimal solution, a self-enforced cooperation can be achieved by a cooperative-competitive (co-co) solution concept, which is predicated upon agents being able to communicate [173], but guarantees the Pareto-optimal equilibrium at low computational cost, under requirement of agent honesty. The threat of cost wasted by a manoeuvre, which is interrupted by the adversary's agreeability constraint, together with the promise of efficiency are assumed to suffice to justify the choice of the solution concept. The problem of the system's sensitivity to agent honesty, however, is further discussed in Section 7.2.

This work offers an optimisation framework based on cooperative Game Theory, whereby CAV agents cooperate by sharing their intentions to find the optimal solution, and the payment to enforce it, limiting the communication to four messages.

The information abundance in a traffic conflict anti-coordination game enabled by the communication allows to shift the strategic solution of traffic conflicts from some NE to the Pareto Equilibrium, whereby resources: energy and time, are allocated optimally with respect to users' intention. The difference between an individual's locally optimal solution and the globally optimal PE solution is covered by a side-payment.

If agents are heterogeneous, their interaction is by definition a non-zero sum game. If

they are willing to communicate, they can adopt a pair of strategies $(u_1^\#, u_2^\#)$ [122] which minimise the combined, cooperative cost $J_{sum}^\#$ as

$$J_{sum}^\# = \min_{u_1, u_2} \left(J_1(u_1, u_2) + J_2(u_1, u_2) \right) \quad (14)$$

where $J_{sum}^\#$ is the total, combined cost of cooperative strategies, which by definition of non-zero sum games is a PE.

In the pair of strategies yielding $J_{sum}^\#$, one of the agent's noncooperative strategy may, however, return greater payoff than a cooperative one

$$J_n^* > J_n^\#. \quad (15)$$

Then, a rational agent will never cooperate, unless an incentive is provided. However, given the definition of PE [119] we observe that if the cooperative solution is globally more optimal than NE, meaning that the cooperation is enforceable, then at least one agent's payoff in a cooperative solution shall be greater than its Nash payoff, that is

$$\exists J_{sum}^\# > J_{sum}^* \Rightarrow J_{-n}^\# > J_n^* > J_n^\#. \quad (16)$$

where $-n$ refers to *the other* agent.

The benefiting agent then can afford, and needs to provide a side payment $p = J^b$ to compensate for the inequality (15) guaranteeing, under threat of rejection, that

$$J_n^\# + J_n^b > J_n^*, \quad (17)$$

satisfying the requirement of agreeability.

As proposed in [174], the payment should be proportional to the players' power, understood as relative values of J^* , and contribution to the common achievement. The payoffs are thus divided as follows:

$$J_{split}^\# = \frac{J_1(u_1, u_2) + J_2(u_1, u_2)}{2}, \quad (18)$$

$$J_{split}^b = \frac{J_1(u_1, u_2) - J_2(u_1, u_2)}{2}. \quad (19)$$

With such division, we can formally split the game as a sum of a purely cooperative game, where players have payoffs $J_n^\#$, and a purely competitive, zero-sum game, where players have opposite payoffs $J_n^p = -J_{-n}^p$. Then, the result of a co-co game is defined as

$$J_1 = \frac{J^\#}{2} + J^p, \quad J_2 = \frac{J^\#}{2} - J^p. \quad (20)$$

3.3 Topology of traffic conflicts

This subchapter defines the boundaries of the proposed framework. Given that the purpose of transportation is to generate value by the movement of goods or users, at the expense of time and energy. The loss of time is inversely proportional to the speed, while the largest contributors to the energy inefficiency are braking and aerodynamic drag. The elimination of braking is a matter of data, enabled by the connectivity. The drag, however, is roughly proportional to the square of the speed and thus can be addressed by careful selection of the cruising speed.

The possible manoeuvres a car can perform are proposed in Fig. 11. They are sorted with respect to the order of the main reference parameter. For example, in case of parking or passing a stationary obstacle, the limiting constraint is the position, while on a corner it is the lateral acceleration. The Intersection, Overtake and Lane Change are special cases where both position and velocity are important.

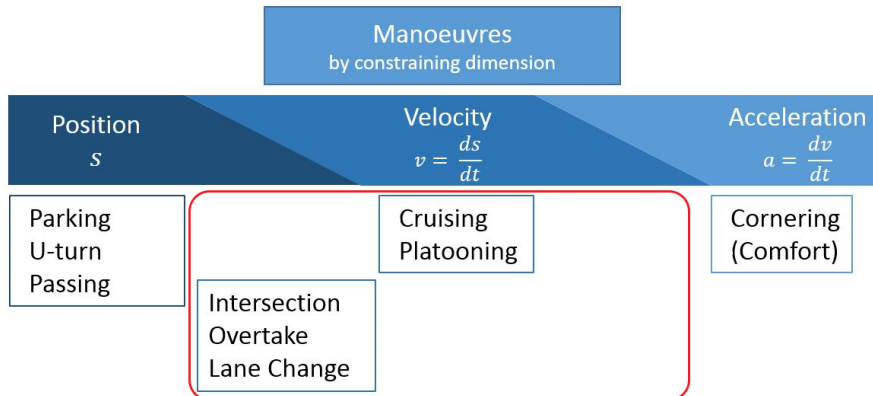


Figure 11: The division of the possible road vehicle manoeuvre, with respect to the order of the spatial dimension. Encircled are the manoeuvres relevant to energy and time optimisation.

Given the above, the most basic manoeuvre subject to time/energy optimisation is free cruising. With an optimal speed V^* selected and adhered to, interruptions occur forcing departure from the V^* .

We propose that all the energy/time relevant traffic conflicts can be defined with respect to the temporal and spatial domains, as visualised in Fig. 12. Time-wise, the conflicts occur either as a discrete event, or continuously, over-time. Space-wise, the conflicts can be of fixed location, or can be relative and delocalized. Of course the more constrained and time-critical a conflict, the more challenging the implementation is. As such human drivers can be observed attempting to communicate, be it by eye contact or relative positioning in Platooning and Lane Change scenarios, as they allow enough time to do it. The constrained and event-critical Intersection and Overtake, on the other hand, are strongly regulated, with the Highway Code, road markings or traffic lights. The growing computational power at hand, may soon enable a feasible optimal conflict resolution paradigm for all traffic conflicts.

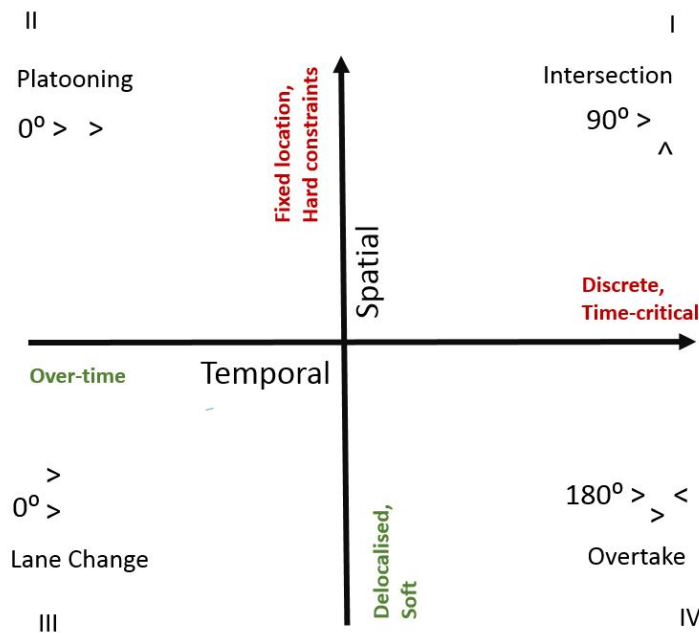


Figure 12: A graphical representation of the types of traffic conflicts across the temporal and spatial domains. On time domain we distinguish time-critical and continuous events. Similarly, in spatial domain, the conflict can be constrained by the environment or delocalised.

All the interactions between vehicles, where the optimal strategy is a tradeoff between energy and time are assumed to be platooning, overtaking, lane change and negotiating an intersection. The Fig. 13 outlines the topological representations of the considered

problems. Other scenarios, which the reader can think of, cannot be defined with respect to this cost function, as comfort or safety are much more relevant. For example in cornering, the energy loss is related to differential friction and the tyre slip. Therefore in this work, the manoeuvres are defined only by the longitudinal dimension, neglecting the lateral component.

The lane change is a special case, carrying both the easiest features of the Overtake and Platooning. While the problem is geometrically similar to a platooning, the solution will resemble a reduced overtake conflict. While it is mentioned here for the sake of completeness, it will not be further considered to avoid repetitions. Definitions are as follows.

- **Cruising** is defined as a steady state operation at constant speed V .
- **Intersection** is a traffic conflict occurring between agents on separate routes, which are intersecting. The overlap of the roads is the Conflict Zone (CZ). The problem is defined as a CZ occupancy schedule.
- **In-line conflict** occurs as an agent encounters a slower vehicle ahead, preventing it from cruising freely, costing time. It can be resolved by:
 - **Overtake**, which occurs when the road conditions allow. While a free overtake, where no speed change is needed as agents have enough space to remain alongside, opposing traffic or road topology may call for speeding up, consuming more energy. An overtake is a single energy expense, which mitigates the over-time loss of time.
 - **Platooning** or convoy, occurs when an overtake is not feasible. There the agents agree on a new, consensus speed and proceed together.

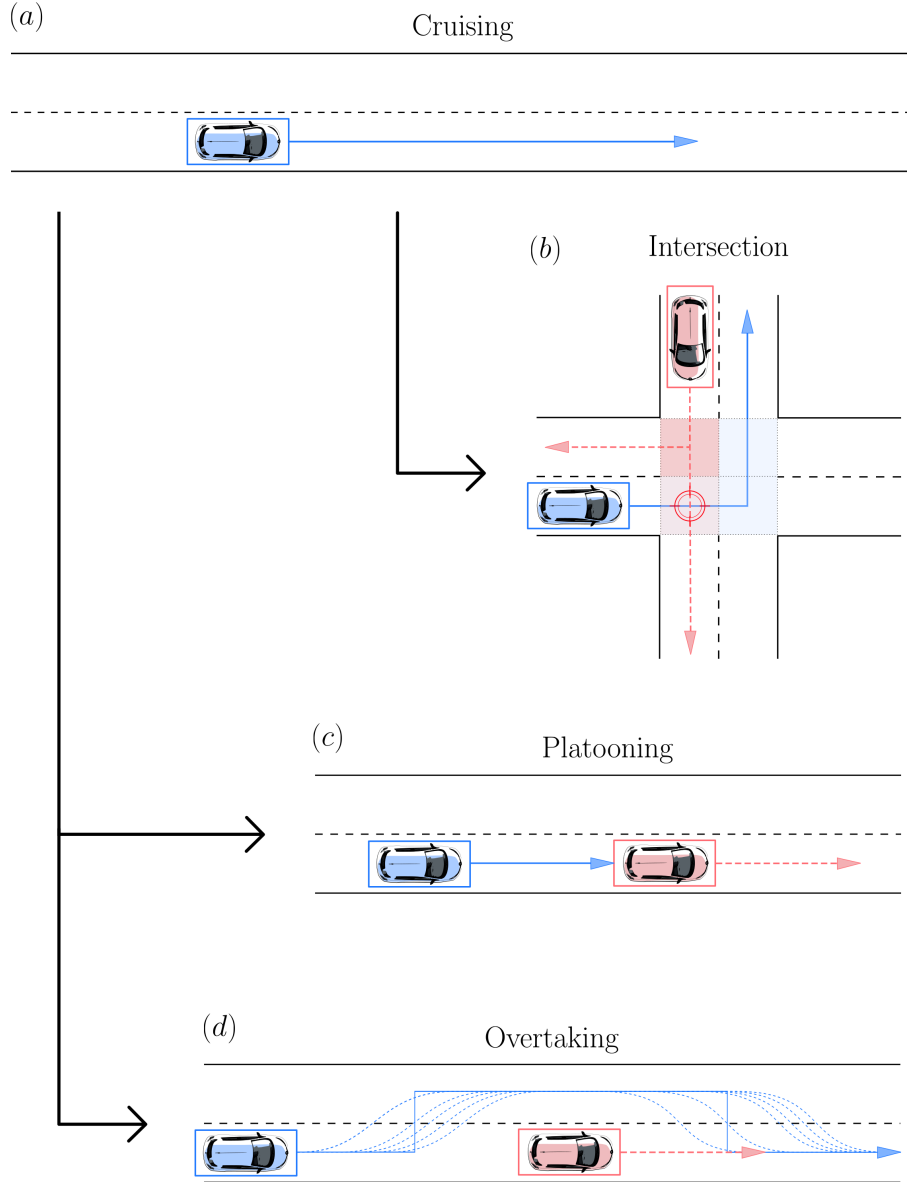


Figure 13: A map of all the scenarios a vehicle can encounter accounting for the assumptions, which may be optimised with respect to energy and time decision variables

3.4 Input to calculations

In order to verify the performance of the framework, all the scenarios outlined above are simulated and analysed. Because the optimality of the solution is predicated upon heterogeneity of agents, two vehicle types are considered: a Car and a Truck. Their individual parameters are listed in Tab. 1. Common parameters are: powertrain efficiency, assumed $\eta_P = 0.82$, rolling resistance $\mu_{roll} = 0.005$, and energy weight, corresponding to its market value $C_E = 0.12$ [$\text{£}/\text{kWh}$] [175]. In case of the dynamic scenarios, that is overtake and intersection problems, the acceleration is kept fixed at $a_n = 2$ [m/s^2] and the

model dynamics is discretised with a timestep $t_{step} = 0.1$ [s]. The requirement to precisely model the energy consumption, in case of the dynamic scenarios, calls for integration of velocity profile at every iteration. This slows down the numerical optimization algorithm. The intended embedded implementation of the algorithm is to be performed in Python or Objective-C languages. These factors have led to select an exhaustive search for optimization. Later, in Further Considerations, Section 6.3, dynamic calculations employ MATLAB’s optimisation toolbox function *fmincon*.

Table 1: Vehicle types and their differentiating parameters. The Car is a generic small passenger vehicle weighing 1.4 t, whereas the Truck is a small cargo vehicle with 10 t of mass.

Veh. type Symbol	mass m [kg]	aerodyn. area A [m ²]	drag coef. C_d [-]	length ΔX [m]	max force \hat{F}_n [kN]
Car	1400	2	0.3	10	4
Truck	10^4	4	0.5	25	10

3.5 Calculation assumptions

The algorithm presented in this work will be developed and modelled in a software environment. With all the prerequisites to outline the algorithm mentioned in this Section, in order to present the results, as obtained in Sections 4.5 and 5.2, all the simulations are performed based on the following assumptions. Some of them are challenged in Sections 6 and 7.

- **No information loss:** the Quality of connectivity Service (QoS) is assumed to be perfect. There is no latency and no data packets lost. All agents can communicate.
- **Honesty:** the agents are always truthful when sharing their objective functions. This eliminates a substantial amount of complexity, and is feasible under the self-enforcement assumption and is further discussed in Section 7.2.
- **Full traffic penetration:** all of vehicles on roads are capable of communication, negotiation and execution of predefined speed profiles. This notion is discussed in Section 7.3.

- **Self-Enforcement:** ideally, optimal solutions should be agreed upon without external enforcement. Thus, for rational, autonomous agents to be willing to participate, the participation must always be beneficial. It is feasible, given that cooperation yields optimality and a side-payment mechanism exists to share the benefit, as it is outlined in Fig. 14.

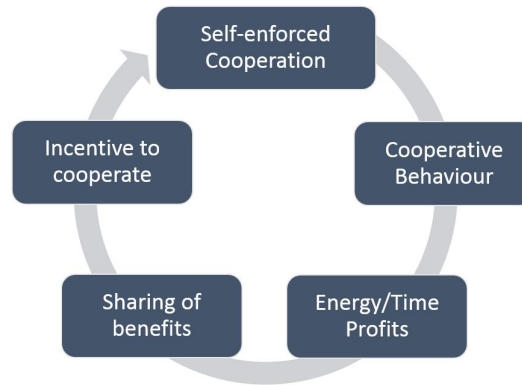


Figure 14: The cycle of self-enforced cooperation.

- **Longitudinal positioning error** is neglected, as this causes safety problem in cooperative scenarios. In validation its mitigation is emulated by introduction of a distance margin.
- **Lateral movement** is neglected, since vehicles do not operate at the limit of handling, tyre slip is negligible and energy loss minimal.
- **Vehicle dynamics** of higher order is neglected. The acceleration changes are instantaneous.
- **Anciliary loads** such as HVAC or headlights are neglected. This assumption is challenged in Section 7.1.1.

3.6 Cruising, the cost function

In the case of uninterrupted cruising, most often only the energy is considered [99] [176], as the other operating costs are either proportional to it or negligible. However, if we optimise just for the cost of operation, the optimal solution is to stand still. Hence, a component of the cost function defining the purpose of the journey is required to offset the cost.

Founders of the Austrian School of Economics [141] suggest that any economic phenomenon, such as commuting, is a result of human individual intention. Furthermore, as an individual chooses between the available, yet exclusive options, the choice of either carries an additional cost of not being able to pursue the alternatives, called the Cost of Opportunity. We may then account for the cost of time a journey takes defining it as a value of user's time C_T [\$/h]. It is intended to be selected by the user of the vehicle, as a mean of expressing their intention as to how much do they hurry. As the user begins their journey, currently they define the destination in the navigation system, which predicts the duration of the journey. The novelty here is to collect not only the destination but also the context of the journey, i.e. how much does one hurry. This could be interfaced by offering a choice on the time of arrival to the destination, providing the cost-benefit estimation by a slider as in Fig. 2.

If we then consider that the intention is to move a distance ΔS , in some steps δS , and v is the speed at given time. Then, since it cannot be infinite, for given speed there is always a time elapsed $\Delta t(v)$ defined as

$$\Delta t(v) = \frac{\Delta S}{v}. \quad (21)$$

Since the loss of time occurs over the distance of the journey

$$\Delta T(v) = \int_0^S \Delta t(v) dS := \frac{1}{v}. \quad (22)$$

The energy is found in a similar manner from the propulsive force necessary to maintain a steady state speed v , considering the powertrain efficiency η_P , aerodynamic characteristics

$C_d A$ and rolling resistance coefficient μ_{roll} .

$$E(v) = \frac{1}{\eta_P} \int_0^S \left(\frac{1}{2} \rho_{air} C_d A v^2 + g \mu_{roll} v \right) dS. \quad (23)$$

Then the value of the unit of energy consumed by the powertrain is defined as C_E , and the overall cost function of each agent n is then a balance between time and energy.

$$J_n(v) = C_E E(v) + C_T \Delta T, \quad (24)$$

or in more structured form

$$J_n(v) = Av^2 + Bv + C + \frac{D}{v}, \quad (25)$$

where:

$$A = \frac{C_E}{\eta_P} \frac{1}{2} \rho_{air} C_d A \quad (26a)$$

$$B = \frac{C_E}{\eta_P} g \mu_{roll} \quad (26b)$$

$$C = 0 \quad D = C_T. \quad (26c)$$

Consideration of extensions to the cost function is presented in Section 7.1. Apart from time and energy, there are also other factors which may be accounted for, including ancilliary energy consumption, users' comfort, or wear model of the sensitive components of the vehicle, such as the battery [177], to control vehicle's operating cost as a whole, or manage pollution. The ride comfort and perception of safety are the most imminent candidates [178] [179], but the main control inputs for them, are relative position and its higher temporal derivatives, acceleration and jerk. Accounting for them would thus require much greater computational power. This, along with the subjectivity of safety and comfort, results in a focus on two main decision factors.

The optimal cruise speed V^* which minimizes the cost function is then just a function of time and energy costs and is found as

$$V^* = \underset{v}{\operatorname{argmin}} \left(J_n(v) \right). \quad (27)$$

The minimal cost per unit distance, i.e. the cruising cost C_C , corresponding to V^* , is

defined as

$$C_{C,n} = J_n(V_n^*) = \min_v J_n(v). \quad (28)$$

The derivative of the cost function (24) with respect to velocity assumes the form

$$\frac{dJ_n}{dv} = 2Av + B - \frac{D}{v^2} \quad (29)$$

which, when solved for $V \in \mathcal{R}^+$ has only one root, guaranteeing a unique minimum.

The sensitivity of optimal cruising speed V^* to user's objective C_T is examined herein, as it is the only variable that is not fixed, for a given vehicle.

Fig. 15 visualises the components of the CF. The cost of time delay term is a hyperbola, while the energy cost is as a parabola. The CF minimiser refers to the optimal speed V^* . Going slower is time inefficient, while going faster than needed wastes energy.

Then, cost functions, for the vehicle parameters VP of Car type, and the trajectory of the minimizer for C_T varying between \$5 and \$55 are presented in Fig. 16, to showcase the CF's dynamics. The marginal decrease of C_T naturally offers diminishing gain in optimal speed V^* , and the CF minimizer follows a parabola itself.

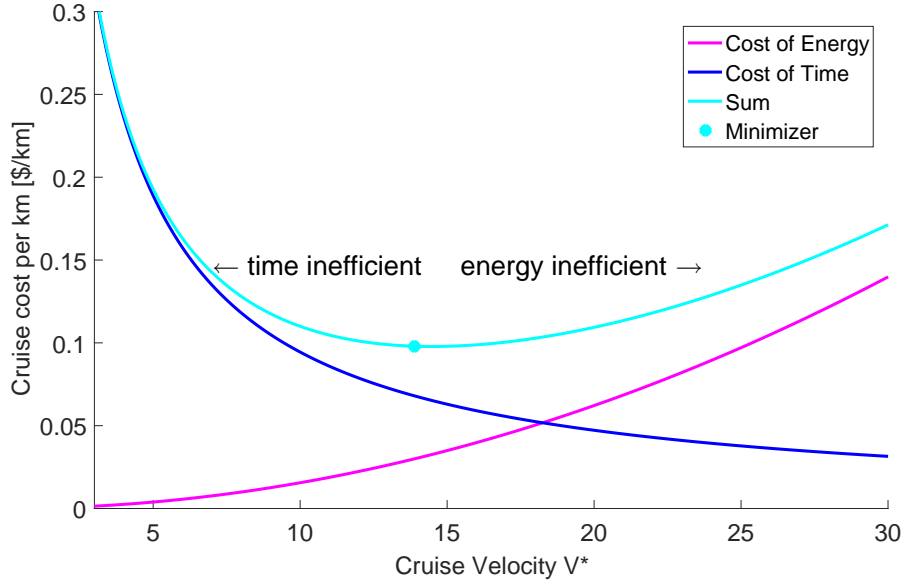


Figure 15: The cost function and its components, the hyperbolic Cost of Time and the parabolic Cost of Energy. The CF minimizer refers to the optimal speed V^* . Going slower is time inefficient, while going too fast wastes energy.

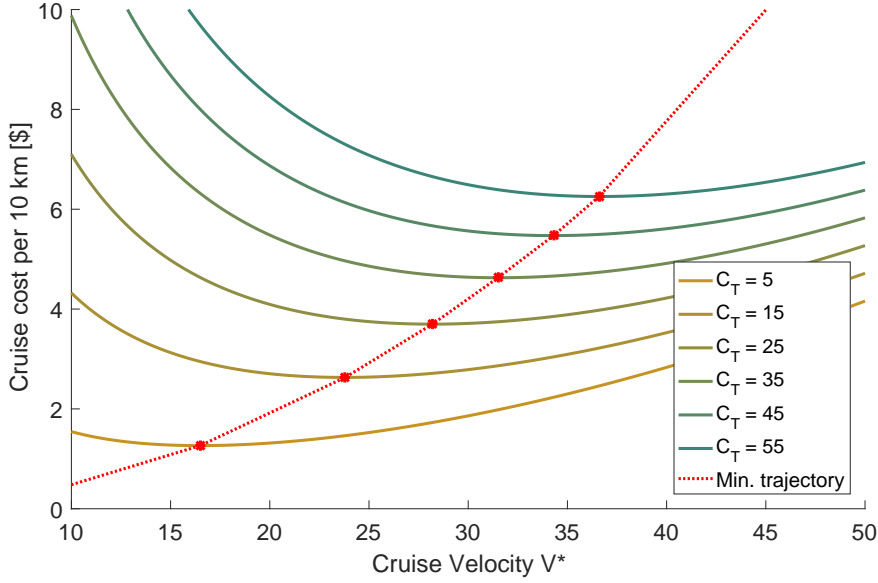


Figure 16: The cost functions sensitivity to varying cost of time C_T . The red curve indicates the trajectory of the functions' minima. It follows a parabola, as the increasing speed results in the drag increased as its square.

In inter-vehicle communication, the CF serves as a cost of departure estimator. Observing the dynamics of the CF, we can observe that the slope is smaller for $\Delta V+$, since the hyperbolic component approaches infinity when stopping, meaning speed change by the same value may incur less cost whilst accelerating. the Cost Function's gradient differs, depending on the direction, as opposed to a common quadratic cost function, where the cost function is defined with respect to a fixed weight and a quadratic expression $(x - x_0)^2$. There, the hyperbolic behaviour on the time-inefficient, slow-down side is not present. Implementation of a traffic negotiation, with a hyperbolic term approaching infinity as the speed approaches zero,

$$\lim_{v \rightarrow 0} (J_n(v)) = \infty \quad (30)$$

may result in the algorithm appreciating the cost of delay appropriately. Fig. 17 visualises the difference between the hyperbolic cost function developed here and a quadratic one.

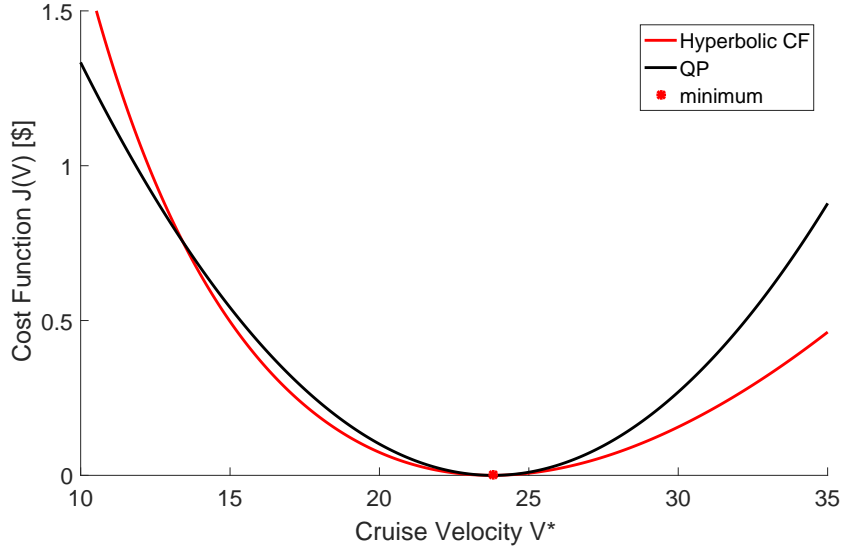


Figure 17: The difference between the hyperbolic CF presented herein and a standard quadratic cost function $(x - x_0)^2$. Extended, asymmetric CF captures the heavy penalty on the jammed, low speed journey, and is more lenient towards speeding up in relation to the quadratic CF.

3.7 Conclusions

The backbone of the V2V-enabled cooperative traffic de-conflicting framework has been outlined in this chapter. It begins with the definition of the vehicle model (3.1) and the GT solution concept applied (3.2). Then it explains the concepts related to vehicle control, the topology of the conflicts, and other assumptions made to perform calculations (3.3). Finally, the cost function is proposed, introducing a notion that vehicle's objective is derived from its parameters and the user's value of time cost of opportunity (3.6). The CF's minimum indicates the optimal cruise speed and its dynamics is compared to a quadratic CF, observing a bias against standing still, as it approaches infinity when close to zero.

The definition of the key building blocks of the framework in this chapter allows to move on to its application on intersections and in-line traffic conflicts in the following chapters.

4 In-line conflict resolution

The Cost Function (CF), which allows to find the optimal cruise speed V^* , is defined in the previous chapter, together with the topology of possible conflicts between cars. This section outlines the negotiation algorithm. It utilises the cooperative-competitive solution concept, outlined in Chapter 3.2, capitalising on the dynamics of the CFs to find the globally optimal solution to conflicts, along with the side-payment needed to enforce it. The algorithm is then applied to it to in-line conflicts, that is a scenario where a freely-cruising vehicle encounters a slower one ahead. It can overtake it, or propose to form a platoon.

4.1 Cooperative conflict resolution

The conflict resolution requires communication, compliance and cooperation. This section defines the communication algorithm. Its architecture, where calculations take place, and semantics, that is the messages exchanged [180]. The Tab. 2 lists the communication occurring as the Ego Vehicle (EV) meets an Obstacle Vehicle (OV). It requests the adversary's vehicle parameters VP , to execute the algorithm as outlined in Section 8.1.1, and to find the optimal solution to the conflict. The manoeuvre is then proposed and rejected or agreed upon or, limiting the negotiation communication to four messages. The architecture of the negotiation algorithm is presented in an activity diagram visualised in the Fig. 18. It has been inspired by the N^* negotiation algorithm [181]. In addition, the input data to the algorithm is listed in the Tab. 3, and the decision variables for each event are listed in Tab. 4. Given that vehicles are to interact with each other, \check{X}_n defines the n^{th} car's minimal distance at which other vehicles can approach it.

Table 2: The table of possible messages to be exchanged between agents.

Message	Content
Request parameters	-
Share VP	$\{V^*, A, B, C, D, \check{X}_n\}$ see eq. (25)
Propose a solution	see Tab. 4
Decide	Yes/No

Table 3: The input parameters for the traffic conflicts, defining the environmental constraints relevant to given topologies.

Scenario	Input	Symbol
Cruise	User's value of time	C_T
Platooning	expected platooning distance	S_S [m]
Overtake	dynamic overtake gap	S_{max} [m]
Intersection	communication range	S_I [m]

Table 4: The control variables which define the strategies for each topology. The output of the decision algorithm.

Scenario	Decision variable	Symbol
Cruise	Equilibrium speed	V^*
Platooning	Consensus vel., payment	$V_P^\#$ [m/s], p [\$]
Overtake	Cons. vel. profiles, payment	$V_{EV}^{out}, V_{OV}^{out}, p$
Intersection	Phasing vel. profiles, payment	V_P, p

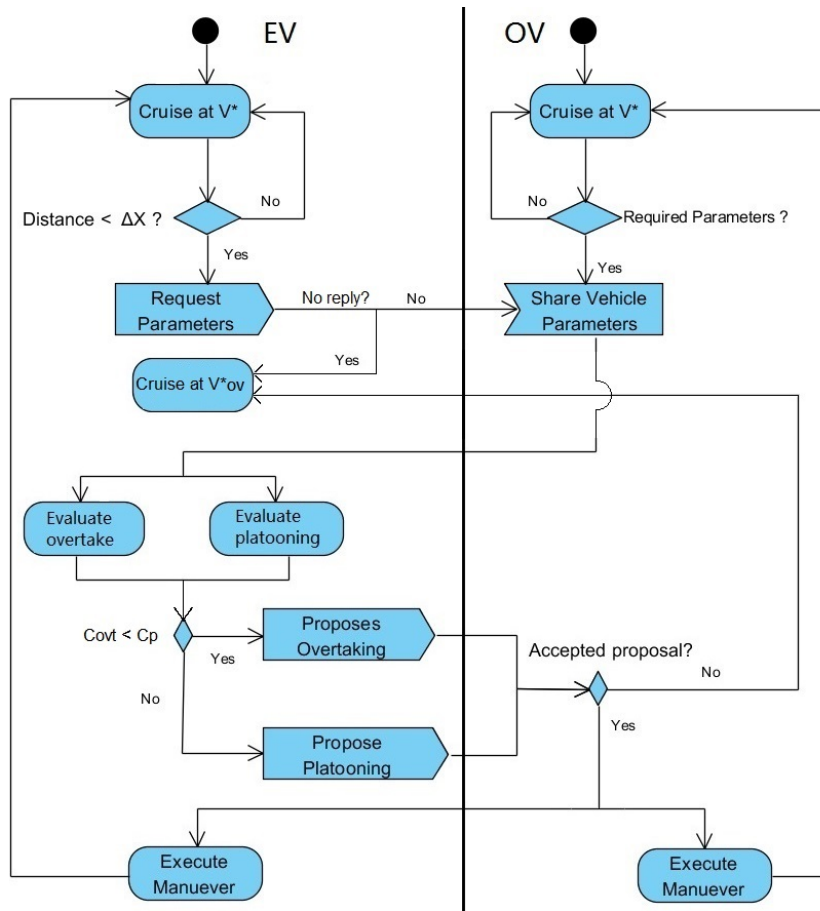


Figure 18: Activity diagram of the communication between agents needed to find an agreeable strategy. The EV, who has the biggest incentive to cooperate takes the initiative, requests the OV's parameters and assumes the computational burden.

4.1.1 Strategic mode selector

The possible scenarios a CAV can encounter are outlined in Section 3.3, however, since the individual scenarios require specific functions, and a communication protocol. Therefore, a strategic finite-state automaton is visualised in Fig. 19. It allows for navigation between the scenarios presented. Either vehicle can Cruise (C) and as a conflict occurs the agents employ GT resolution algorithm for given scenario. Then, as Cruising is not possible due to obstacles, the transitions to Intersection (I) or the in-line conflicts: Platooning (P) and Overtake (O) occurs. The former one, however, can transition to the latter, as when the road conditions change an efficient overtaking manoeuvre may emerge as a new possibility. The set of all possible manoeuvres \mathcal{X} is such that $\mathcal{X} \in \{\mathcal{C} \mathcal{P} \mathcal{O} \mathcal{I}\}$. Again, the lane change here is a special case of the P.

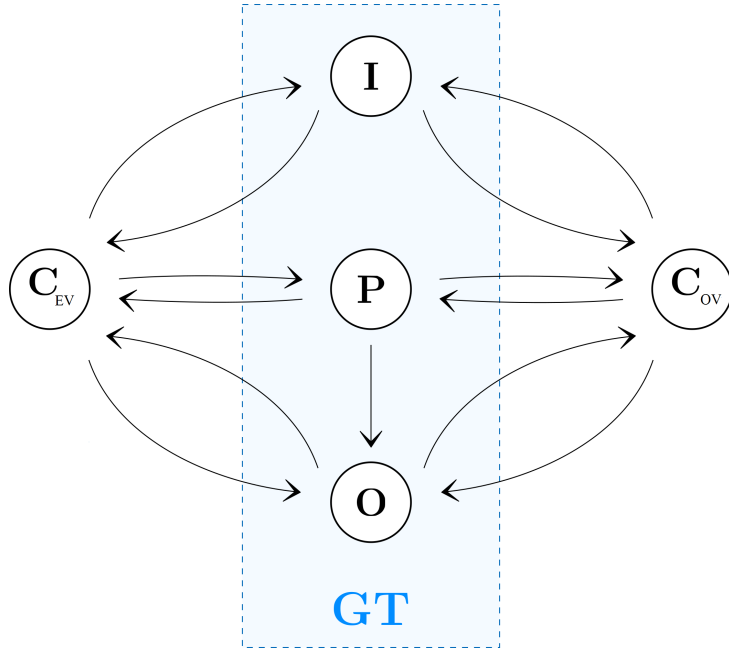


Figure 19: Finite-state map of two automata navigating their interactions. While C can transition to any other state and back, but P can be resolved either by agents parting their ways or an overtake, should the road situation change.

4.2 Platooning negotiation

Suppose that as a vehicle cruises freely, it encounters a constraint ahead, in a form of a slower vehicle or a platoon ahead, one of the solutions is to request an increase of the speed, incentivising it with a payment. The proposed approach, assumes that each, n^{th} vehicle shares their cost function $j_n(v)$, albeit normalized by subtracting the cruise cost $C_{C,n}$, and only between participating agents V^* , as due to convexity the solution lies between them. Then the CF defines only the cost of departure from V^* and is defined as

$$j_n(v)|_{V_{OV}^{EV}} = J_n(v) - C_{C,n}, \quad (31)$$

or by assigning the negative cruise cost to the constant C in the CF (26c) $C = -C_{C,n}$. Because this normalized CF is valid only for given journey, it is further called the Local Cost Function (LCF). It is defined as $j_{EV}(v)$ and $j_{OV}(v)$, for platooning speeds between V_{EV}^* and V_{OV}^* , and discretised with a step ΔV . Because the GT uses payoff matrices to define conflicts, the Tab. 5 represents the LCF expressed as a payoff matrix $M_{P,n}$. It defines the influence of agents' strategies on each other. Since the information loss is neglected and any set of strategies outside of the diagonal of the matrix would be infeasible from a safety perspective, the values not on the diagonal are rejected. The solution concept is based on complete information, eliminating the equilibrium seeking.

Table 5: Platooning payoff matrix $M_{P,n}$. Only the diagonal is saturated, as any other combination, by definition of the manoeuvre, is not feasible.

V_P	V_{OV}^*	$v_{OV}^* + \Delta V$	\dots	V_{EV}^*
V_{OV}^*	$j_n(V_{OV}^*)$	NaN	\cdot	\cdot
$v_{OV}^* + \Delta V$	NaN	$j_n(V_{OV}^* + \Delta V)$	NaN	\cdot
\vdots	\cdot	NaN	\ddots	NaN
V_{EV}^*	\cdot	\cdot	NaN	$j_n(V_{EV}^*)$

Since the promise of payment turns the game into a cooperative one, the payoffs from diagonals of $M_{P,EV}$ and $M_{P,OV}$ are then summed as

$$j_P(v) = j_{EV}(v) + j_{OV}(v) \quad (32)$$

and the optimal platooning speed V_P is found as

$$C_P = \min_{V_P} j_P(v). \quad (33)$$

Then, the initiating agent proposes a platooning speed V_P . The cost of the OV's departure from V_{OV}^* is covered by the EV's payment $p_P = j_{OV}(V_P)$. Note that this formulation defines the specific cost of platooning with the unit $[\$/km]$.

To evaluate the performance of the framework, the reference, that is the noncooperative solution, is defined as

$$C_P = \min_{V_P^{NC}} j_P(V_P^{NC}) \quad (34)$$

where V_P^{NC} is the noncooperative platooning speed. The Price of Anarchy (PoA), defining the loss of optimality of the GT solution in relation to the globally optimal, is then

$$PoA = \frac{j_P(V_P)}{j_P(V_P^{NC})}. \quad (35)$$

4.3 Optimal overtaking manoeuvre

An overtake is an alternative to the platooning strategy. The reasoning behind this is to spend energy in order to mitigate the continuous loss of time when platooning. An energy-focused overtake model is thus proposed below.

4.3.1 Overtake model

As mentioned above, the lateral movement is neglected as there is substantial work addressing it [182] [83]. The model is thus focused only on longitudinal dimension and energy. While the $X_{i,n}$ defines given vehicle's distance to the reference frame, the ΔX_i is the relative, longitudinal distance between the EV and OV at given timestep. The event-related safety distance \check{X}_s is the largest of either vehicle's safety distance \check{X}_n (see 4.1), but accounted for vehicles' length l_n , since they are modelled as mass points.

$$X_s = \max_{EV,OV} \check{X}_n + \frac{1}{2} (l_{EV} + l_{OV}). \quad (36)$$

Then, the overtake manoeuvre is defined to occur when relative distance ΔX_i is within the absolute value of the safety distance

$$2X_s < |\Delta X_i|, \quad (37)$$

from $-X_s$ with the OV in the lead, to $+X_s$ with the EV in the lead.

The frame of reference for an overtake is considered to be the moment vehicles begin an overtake. Then the longitudinal distance of the manoeuvre is constrained by the maximal distance to be spend alongside, S_a . Since during the overtake the EV has to drive two vehicle lengths further, the distance of the OV is constrained to

$$S_{a,OV} = S_a - (2X_s). \quad (38)$$

The positions during the overtake are defined using the moment vehicles begin to overlap as a reference. It is assumed that the manoeuvre is performed either at a constant speed $v_{ovt,n}$ or constant acceleration a_n . The selection of manoeuvre accelerations depends on e.g. battery wear or user comfort. To focus on the negotiation algorithm, a_n is fixed. Thereby each agent's decision variable is the overtake speed only. Example speed profiles for an overtake are presented in Fig. 20. The overtake then consists of three phases: acceleration at a_n from V_n^* to the preselected overtake speed $V_{ovt,n}$, cruising at constant $V_{ovt,n}$ and deceleration back to V_n^* at $-a_n$. Acceleration rates are assumed constant, leaving V_{ovt} the only decision variable.

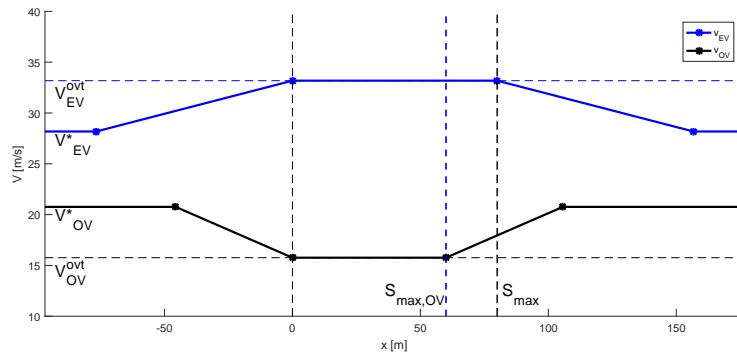


Figure 20: Velocity profiles of vehicles performing an overtake, visualised in space domain. The velocities are assumed to be adjusted before the overlap, as to minimize the time on the opposite lane.

Then, the powertrain model returns energy consumption to evaluate the energy ex-

pense $C_{E,n}^{ovt}$. The overtake cost is found by adding the time cost as

$$J_{ovt}(V_{EV}^{ovt}, V_{OV}^{ovt}) = \sum_N C_{E,n}^{ovt} + C_{\Delta T,n}^{ovt}. \quad (39)$$

The manoeuvre is constrained by the oncoming traffic

$$V_{EV}^{ovt} - V_{OV}^{ovt} > 2X_s \frac{V_{EV}^{ovt}}{S_{max}}. \quad (40)$$

Since the overtake gap is never stationary, a maximal distance to perform an overtake is derived from dynamics of the oncoming vehicle. Defining S_{gap} as a distance between two oncoming vehicles, between which the overtake is to occur, which are moving from the opposite direction at V_{gap} ,

$$S_a = S_{gap} \left(1 - \frac{V_{gap}}{V_{EV}^{ovt}}\right). \quad (41)$$

This can be viewed as relative speed being sufficient to pass before the EV's distance exceeds S_a . It is assumed that vehicles coming from the opposite direction have a known, constant speed, so the S_a is constant.

Equation (41) for the dynamic overtake gap length S_a has been derived from the real length of the gap S_{gap} , and the velocity of the gap V_{gap} . After factoring the S_{gap} into the parentheses the equation (41) is

$$S_a = S_{gap} - \frac{S_{gap}}{V_{EV}^{ovt}} V_{gap}, \quad (42)$$

where overtake time is defined by a fraction

$$T_{ovt} = \frac{S_{gap}}{V_{EV}^{ovt}}, \quad (43)$$

which then allows to find the distance the oncoming traffic travels during the overtake

$$S_{gap} = -V_{gap} T_{ovt} \quad (44)$$

which is subtracted from the static gap, returning to (42).

4.3.2 Optimal overtake

With the overtake model as described above, a matrix of possible solutions to the conflict $M_{V_{ovt}}$ is created. Given a speed set from V_n^* to the highest modelled overtake speed \hat{V}_n , discretised with a step ΔV

$$V_n^{ovt} = \{V_n^*, V_n^* + \Delta V, \dots, \hat{V}_n\}, \quad (45)$$

the $M_{V_{ovt}}$ assumes the form as in Tab. 6. In reverse to the EV, the OV's speed is counted from the minimal \check{V}_{OV} to V_{OV}^* .

Table 6: Payoff matrix $M_{V_{ovt}}$ for an overtake. The last position in the first row, $M_{1,N}$, is presented as infeasible. The notation is simplified for clarity.

$V_{EV}^{ovt} \backslash V_{OV}^{ovt}$	\check{V}_{OV}	\dots	V_{OV}^*
V_{EV}^*	$J_{ovt}(V_{EV}^*, \check{V}_{OV})$	$J_{ovt}(V_{EV}^{ovt}, V_{OV}^{ovt})$	NaN
\vdots	\vdots	$J_{ovt}(V_{EV}^{ovt}, V_{OV}^{ovt})$	$J_{ovt}(V_{EV}^{ovt}, V_{OV}^{ovt})$
\hat{V}_{EV}	$J_{ovt}(\hat{V}_{EV}, \check{V}_{OV})$	\dots	$J_{ovt}(\hat{V}_{EV}, V_{OV}^*)$

Then the optimal overtaking manoeuvre is a vector of V_n^{ovt} which minimizes the $M_{V_{ovt}}$

$$C_{ovt}(V_{EV}, V_{OV}) = \min_{V_n^{ovt}} \left(M_{V_{ovt}}(V_{EV}, V_{OV}) \right). \quad (46)$$

Given the convexity of the CF, the V_n^{ovt} is found by finding a root of cost function's gradient

$$(V_{EV}^{ovt}, V_{OV}^{ovt}) \ni \nabla J_{ovt}(V_{EV}^{ovt}, V_{OV}^{ovt}) = 0, \quad (47)$$

subject to feasibility constraint

$$V_{OV}^{ovt} < -\frac{2X_s}{S_{max}(1 - \frac{V_{gap}}{V_{EV}^{ovt}})} V_{EV}^{ovt} + V_{EV}^{ovt}. \quad (48)$$

and powertrain force constraint

$$F_{net,n} < \hat{F}_n, \quad (49)$$

where \hat{F}_n is the maximal propulsive force.

Finally, as agreeability condition, payment $p_{ovt} = C_{OV}^{ovt}$, covering all costs of the manoeuvre on OV's side is issued, as the EV would be free to cruise unimpeded.

4.4 Decision rule

Having evaluated the specific cost of platooning C_P and the cost of overtake C_{ovt} , the decision to overtake is selected if

$$C_P S_S \geq C_{ovt} \quad (50)$$

and platooning otherwise. The EV proposes to OV a feasible strategy and a payment, which, to ensure enforcement, must satisfy (17). The strategy selected by the EV, be it platooning or an overtake, is accepted by the OV if

$$C_x - p \geq C_C. \quad (51)$$

4.5 Results

Since the platooning and overtake models and optimisation routines have been outlined, this Section features the calculation results.

4.5.1 Platooning

In a scenario where a freely cruising vehicle encounters a slower one ahead, which cannot be overtaken, the platooning negotiation is performed. The costs of departure from optimal speed for each vehicle is compared and the possible solution points are found. The results, for two Cars with various C_T , are visualized in Fig. 21. Point 1 marks the cost of a baseline, non-cooperative manoeuvre, where the EV merely follows the OV at OV's V^* . The point 2 marks the cooperative function's minimum, where agents' combined costs of departure from optimum are minimised.

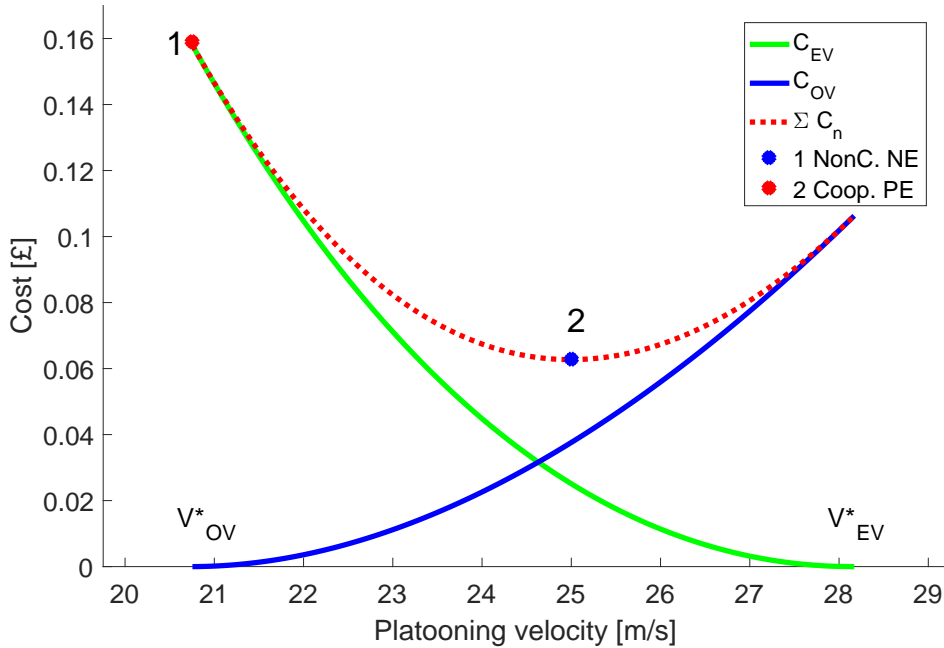


Figure 21: A result of the platooning negotiation algorithm. The blue point marks the cooperative, Pareto equilibrium solution, while the red one marks the noncooperative, Nash solution.

The same calculation has been performed for various vehicle types, to explore how the difference between agents changes the solution. The scenarios are: a Car following a Truck, again two Cars and a Truck following a Car. For clarity, the C_T has been selected to match V^* for different vehicle types, as in Tab. 7. The results are presented in Fig. 22

and Tab. 8. One can observe how the Pareto-optimal solution shifts towards the larger vehicle's V^* , reflecting its negotiative power. $PoA = 1$ means the negotiated solution is equal to the non-cooperative. Because the objective is to minimise the cost, higher PoA values indicate higher the gain in efficiency.

Table 7: Vehicle types and preferences for several cases of platooning.

Feat.\Case	C1		C2		C3	
	EV	OV	EV	OV	OV	EV
Veh. type	Car	Truck	Car	Car	Truck	Car
C_T [£/h]	25	67	25	10	167	10
V^* [m/s]	28.4	21	28.4	21	28.4	21

Table 8: Platooning solution costs and Prices of Anarchy (35), a comparison between the three cases. The larger the difference between the EV and OV, the greater the PoA . The growth is driven by the large cost of the noncooperative solution.

Case	Cost [£ 10^{-2}]		Price of Anarchy
	NonC.	Pareto	
1	15.7	12.7	1.23
2	15.7	6.26	2.5
3	105	9.6	10.9

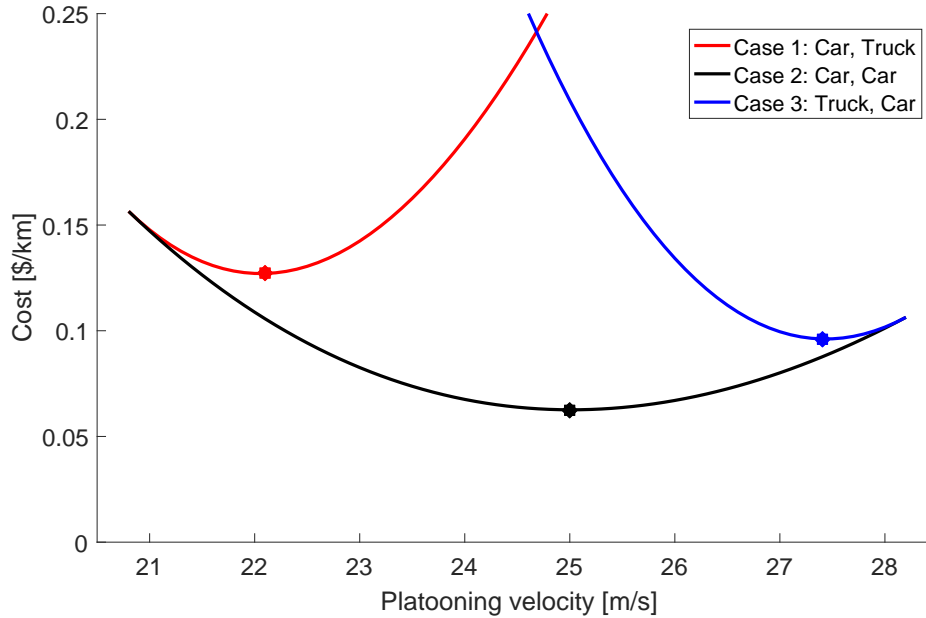


Figure 22: Sum of platooning cost for various vehicle types. The red is a Car following a Truck and blue a Truck following a Car. The differences in negotiative power are easily noticeable. Points mark the functions' minima.

4.5.2 Overtake solution

The scenario whereby the EV encounters a slower vehicle is continued, now with the overtake possible. While in platooning the cost is incurred over time, as proportional loss of time $\Delta T/T$. The alternative, event-based resolution to an in-line conflict is possible, where the cost is incurred once, drawn from the energy component of the cost function, as an overtake features quick speed changes.

The overtake calculation utilises the same algorithm, using the same vehicles as in the platooning example and the model defined in Section 3.1. The vehicle parameters are listed in Tab. 7 and the values of $M_{V_n^{ovt}}$ assume the distribution presented in Fig. 23. Additionally, all calculations are computed for a fixed overtake gap S_{gap} of 150 [m]. For small overtake speed differences ($V_{EV}^{ovt} - V_{OV}^{ovt}$), the manoeuvres are rejected as infeasible, due to physical constraint - another object approaching from the opposite. The cost increases quickly, leaving the feasibility boundary as location of the optimal solution. The constraint on propulsive force rules out solutions for which required acceleration is not achievable at given V^{ovt} . Fig. 24 outlines the same example, viewed from above, to visualise the surface of feasible solutions. The powertrain constraint is at $V_{EV}^{ovt} = 36.9$ [m/s]. The optimal solution, for an overtake gap of 80 [m], is found to be, in this example, $V_{EV}^{ovt} = 34.7$ [m/s] and $V_{OV}^{ovt} = 14.3$ [m/s].

The computation time to find this solution, with the precision of 0.01 [m/s] is on average 5.2 [s]. However, assuming that the solution is on the feasibility constraint, what is justified by the convexity of the cost function, it takes only 0.57 [s] with an exhaustive search algorithm. The Fig. 23 depicts with blue the isolated curve.

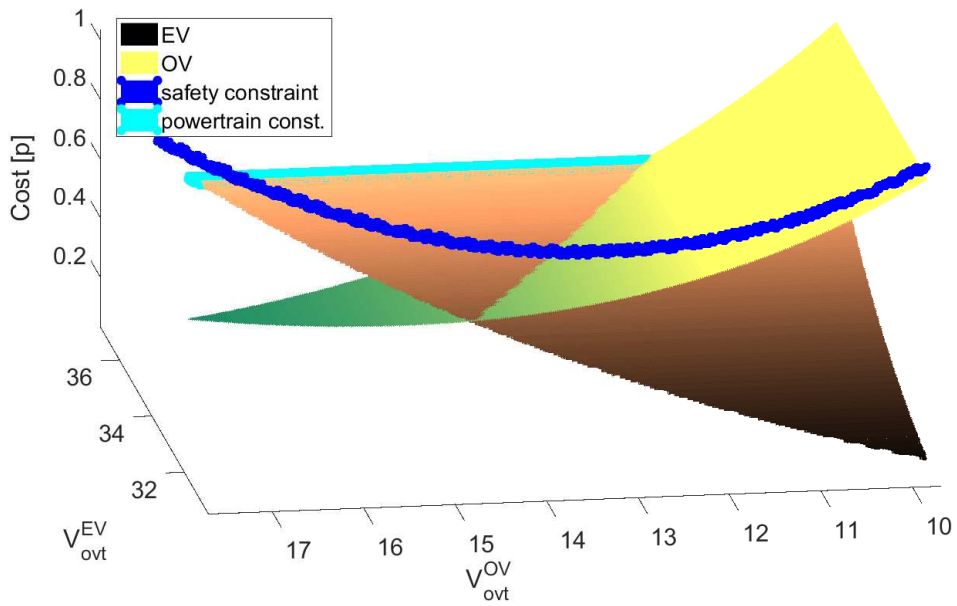


Figure 23: An example of the overtake costs surfaces and the feasibility front of their sum. Given the convexity of the CF, the optimal manoeuvre lies on the feasibility constraint, which reflects the size of the gap ahead of the oncoming traffic. The powertrain power constraint is also presented on the EV's cost function.

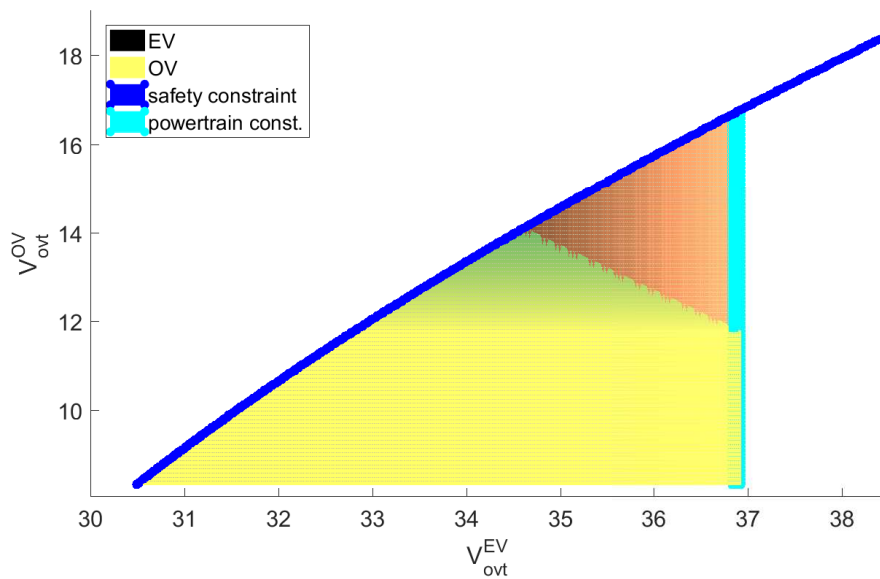


Figure 24: A projection of the above figure from above, showcasing the constrained solutionspace. Both the oncoming traffic constraint and the powertrain power constraint are presented.

4.5.3 Sensitivity of overtake cost to variation of the distance available

So far, the above calculations have considered a fixed overtaking distance. This analysis tracks the cost of an optimal overtake \check{C}_{ovt} as a function of the available space S_a . Three cases from Tab. 7 are being considered. The results are presented in Fig. 25. The overtake velocities needed to execute the manoeuvre within the S_{max} constraint are plotted, together with the total cost of manoeuvre. The cost decreases hyperbolically with overtake gap available, as a smaller gap requires a higher speed difference. Tab 9 lists the results of an R-square fit to a hyperbola $R^2_{C_{ovt}(S_a)} = \frac{a}{(x-b)} + c$.

Table 9: The results for the cost of overtake as a function of distance $C_{ovt}(S_a)$ fitting to hyperbola using R-square fit method.

Case	C1	C2	C3
R^2 [%]	99.31	99.67	99.85

Upon encountering a slower vehicle the EV would platoon behind the EV, await for an opportunity to overtake. As a gap large enough emerges on the opposite line, it triggers the recalculated overtake, according to the decision rule outlined in chapter 4.4, as the EV would be free to cruise unimpeded afterwards.

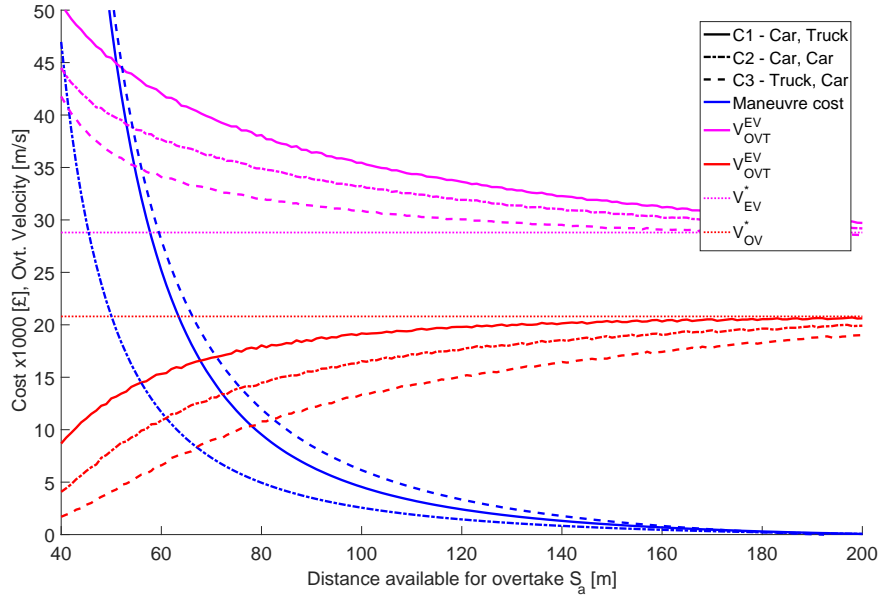


Figure 25: Overtake cost, and payment required for performing an overtake is here plotted as a function of overtake gape space available for the maneuver S_{gap} . The range is from 40 to 200 [m]. The oncoming vehicle's velocity $V_{gap} = -20$ [m/s]. Note that while vehicles cooperate, that is both change their velocity, the OV is more prone to slow down. This phenomenon is a result of the energy being proportional to speed squared.

4.6 Conclusions

This Section presents the results returned by the algorithm, when applied to the in-line conflicts, that is platooning and overtaking. Both possible solutions are calculated and compared, selecting the less costly one.

In the platooning scenario the calculation results allow to observe how the difference in vehicle mass and drag changes the dynamics of the solution, as the optimum shifts towards the heavier vehicle. The bigger the EV is in relation to the OV, the larger the benefit brought by the algorithm.

In the overtake scenario the Fig. 23 represents the solution space well. For a fixed overtake gap, the same manoeuvre can be performed with varying overtake speed differences, but it is certain that optimal overtake always features OV's deceleration, since speeding up is always more costly. In addition, the sensitivity of the overtake cost to the distance available suggests the C_{ovt} is a hyperbolic function with RMS fit with R^2 above 99.3% for all cases, prompting further work direction towards analytical definition of the overtake cost variability, to reduce complexity.

5 Intersection conflict

The intersection formulation addresses a conflict between agents whose trajectories collide at a road junction. The priority is considered based on the current traffic rules: yielding to the vehicle on the right. An optimal, cooperative solution can be different, however, but the enforcement payment must cover the cost of deviation from primary rules.

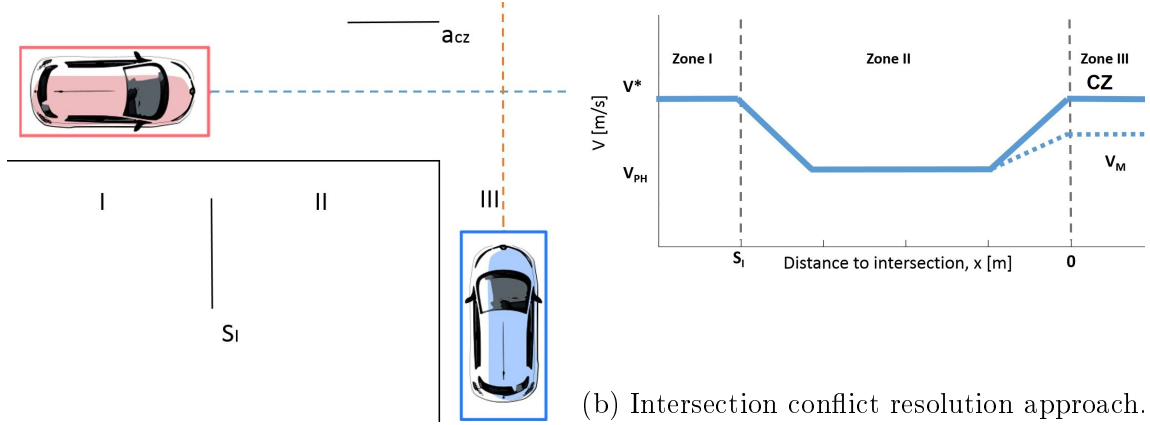
The activity diagram of the algorithm is the same as the Platooning/Overtake problem. The computational effort of the algorithm is to be performed by the agent with greater potential benefit, further called the Ego Vehicle (EV), the adversary is the Obstacle Vehicle (OV). As an agent meets an obstacle, it requests adversary's preference parameters and finds the best manoeuvre, which is proposed and agreed upon. The problem of system's sensitivity to agent honesty is discussed below, in Chapter 7.2.

To clarify the way optimisation is executed from the user's perspective. Consider a scenario, where a Truck is to turn left, but according to the traffic code, to mitigate a collision it needs to yield to a small vehicle approaching from ahead. The scenario is introduced in Fig. 3 in Chapter 1. The solution of the algorithm is the opposite of traffic code rules. The small, lighter vehicle yields, so the heavier one can retain its kinetic energy, receiving a side payment for its effort.

While the traffic code violation poses a liability challenge, there is no way around the fact that an optimal drive cycle minimises the used powertrain power.

5.1 Conflict resolution formulation

The mitigation of conflict is performed by adjusting speed before the manoeuvre, ensuring that n^{th} agent's time of arrival $T_{I,n}$ at the CZ of agents is sufficiently separated from others. The speed profile is parametrized, keeping the number of decision variables, and thus complexity, to minimum. As a vehicle approaches the intersection, firstly Zone I is where vehicles establish communication. Its length must suffice to solve the described algorithm before entering Zone II, where the phasing manoeuvre occurs. The length



(a) The geometry of the intersection problems, along with example speed profile in Zone II. The width of the lane

(b) Intersection conflict resolution approach. The speed profile, shifting from V^* to the phasing speed V_{PH} , in this case for a slow-down scenario.

Figure 26: The intersection conflict topology is visualised marking the approach zones. In Zone I connectivity is established and a solution found. Beyond the S_I distance, in Zone II, the phasing manoeuvre occurs at a phasing speed. Zone III is the intersection conflict zone, the CZ.

of the Zone II is defined as S_I . Finally, Zone III is the Conflict Zone, of length a_{CZ} where agents manoeuvre at V_{man} . Fig. 26a outlines the geometry and example of the intersection problem.

An example vehicle speed profile across the approach to the CZ is visualised in Fig. 26b for a slow-down, yielding scenario. This method of formulating traffic conflicts is similar to the intelligent Cooperative Adaptive Cruise Control (iCACC) algorithm [86]. There, however, the cost function is defined only with respect to time delay. The main novelty and distinguishing factor of this algorithm, is the multi-objective optimisation built upon the framework.

It is assumed that the vehicle follows a constant speed $V_{ph,n}$, and acceleration is performed at a constant a_n , positive when speeding up and negative when decelerating. Varying the selected $V_{ph,n}$ changes the length of speed change manoeuvre.

Let us define the time domain as Θ . With the geometry of the problem and the single control variable $V_{ph,n}$, the time of arrival at intersection $T_{I,n}$ and the CZ occupation time for each vehicle is calculated, given that

$$T_{CZ,n} = \frac{2L_n + a_{CZ}}{V_{M,n}}. \quad (52)$$

L_n is the length of the n^{th} vehicle and $V_{M,n}$ is the speed at which the CZ is traversed.

For notation simplicity we define CZ exit time as

$$T_{E,n} = T_{I,n} + T_{CZ,n}. \quad (53)$$

Then, the interval of time a vehicle spends occupying the CZ, \mathcal{T}_{CZ} is defined as

$$\mathcal{T}_{CZ,n} \in \{\Theta : T_{I,n} \leq T_n \leq T_{E,n}\}. \quad (54)$$

A conflict occurs if

$$\Delta t_{fix} =: \mathcal{T}_{CZ,EV} \cap \mathcal{T}_{CZ,OV} \neq \emptyset, \quad (55)$$

defining Δt_{fix} as the time overlap, but also as minimal correction required to mitigate the conflict. It is visualised in Fig. 27.

The cost of the traversing the intersection $C_{int}(v)$, for given a speed profile is

$$V_{ph,n}[t] = [a_n, V_{ph,n}, -a_n, V_M]. \quad (56)$$

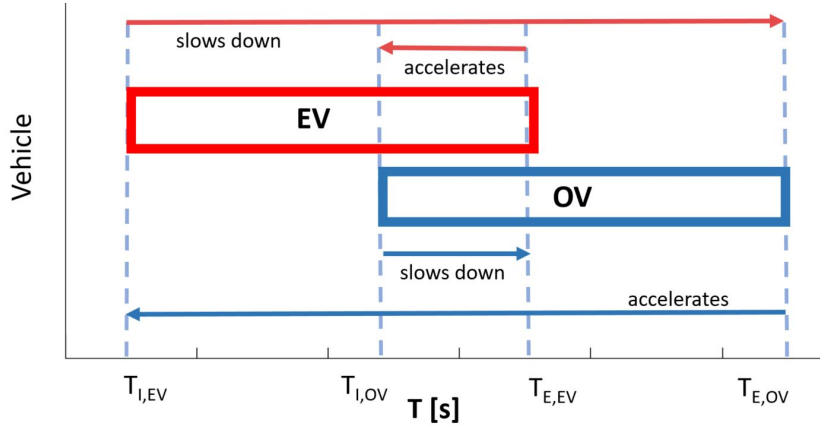


Figure 27: Intersection conflict in time domain and all possible conflict resolutions. It can be mitigated by either agent accelerating or slowing down. A - acceleration, S - slowing down.

It is then found as in equation (24), by integration of the cost function over the distance of Zone II, where vehicles adapt their speed to mitigate the conflict

$$C_{int}(V_{ph,n}[t]) = \int_{S_I}^0 C_E E(v) + C_T \Delta T dx. \quad (57)$$

Since a speed profile results affects the time of arrival at the intersection, the intersection traversing cost as function of time change $C_{int}(\Delta T)$ is found by means of an expression

$$\Delta T(V_P) = f(V_{P,n}[t]). \quad (58)$$

It allows to form a function of manoeuvre cost in relation to the time correction, given the parametrized phasing speed profile $V_{ph,n}[t]$

$$C_{P,n}(\Delta T) = \int_{S_I}^0 C_E E(V_{P,n}[t]) + C_T[t] \Delta T \, dx. \quad (59)$$

An example of the $C_P(\Delta T)$ result is outlined in Section 5.2. Interestingly, the convexity of cost function and powertrain energy inefficiency lead to a steep cusp around the function's root, which is always at zero. It suggests that an optimal solution is on one of the axes. Note that the negative ΔT indicates an acceleration manoeuvre.

We can then note that to each conflict there are four distinct solutions, with each of both agents either slowing down or accelerating. We can then define all the minimal correction values required to mitigate the conflict

$$t_n^{fix+} = T_{I,n} - (T_{I,-n} + T_{CZ,-n}), \quad (60a)$$

$$t_n^{fix-} = (T_{I,n} + T_{CZ,n}) - T_{I,-n}, \quad (60b)$$

defining $-n$ as the other player and $+/-$ depicting the conflict from ahead, usually mitigated by slowing down, or from behind, mitigated by speeding up. We can then combine them in a 2 by 2 matrix ΔT^{fix} , with A as a solution where an agent accelerates and S slows down,

$$\Delta T_{min}^{fix} = \begin{bmatrix} \Delta t_{EV,A}^{fix} & \Delta t_{OV,A}^{fix} \\ \Delta t_{EV,S}^{fix} & \Delta t_{OV,S}^{fix} \end{bmatrix}, \quad (61)$$

observing that

$$\Delta t_{EV,A}^{fix} = -\Delta t_{OV,S}^{fix} \quad (62a)$$

$$\Delta t_{EV,S}^{fix} = -\Delta t_{OV,A}^{fix} \quad (62b)$$

and simplifying, given that $-a$ is the alternative action,

$$\Delta t_{n,a}^{fix} = \Delta t_{-n,-a}^{fix} \quad (63)$$

This allows to distinguish two correction values: Δt_a^{fix} and Δt_b^{fix} , characterized by the inequality

$$\Delta t_a^{fix} \leq \Delta t_b^{fix}. \quad (64)$$

A conflict can be also resolved by combination of two neighbouring solutions, with both agents yielding partially. A blending coefficient $\alpha = [0, 1]$ is introduced, such that

$$\Delta t^{fix} = (1 - \alpha)\Delta t_n^{fix} + \alpha\Delta t_{-n}^{fix}. \quad (65)$$

Note, that while naturally the time domain is the same for all agents, the control of the problem features two temporal parameters, one for each agent, as seen in eq. (55) above. In order to detach these two and be able to define a function between them, consider a two-dimensional time domain Θ^2 such that

$$\Theta^2 =: \Theta_{EV} \times \Theta_{OV} \ni T_{EV}^{fix}, T_{OV}^{fix}. \quad (66)$$

The defined coordinate system is presented in Fig. 28. There, the quarters *I* and *III* do not belong to feasible region, due to alike signs on axes. The time in which the conflict occurs there is changed, but not trivially resolved. The solutions with α defined partial yields (65) mark the boundary of the feasible set in the quarters *II* and *IV* of the coordinate system. The axes have opposite signs, what bears a physical meaning of one agent accelerating and the other yielding, resulting in the conflict mitigation. This results in a feasible set Θ_F^2 being constrained by

$$\begin{aligned} \Theta_F^2 := \{ \Theta_{EV}^-, \Theta_{OV}^+ \} &\geq \langle -\infty, T_{EV,A}^{fix}, \\ (1 - \alpha)\Delta t_{EV,A}^{fix} + \alpha\Delta T_{OV,S}^{fix}, T_{OV,S}^{fix}, \infty \rangle \end{aligned} \quad (67a)$$

$$\begin{aligned} \Theta_F^2 := \{ \Theta_{OV}^-, \Theta_{EV}^+ \} &\geq \langle -\infty, T_{OV,A}^{fix}, \\ (1 - \alpha)\Delta t_{OV,A}^{fix} + \alpha\Delta T_{OV,A}^{fix}, T_{EV,S}^{fix}, \infty \rangle, \end{aligned} \quad (67b)$$

where $\alpha = [0, 1]$. The angle brackets $\langle \rangle$ denote endpoint inclusion in the set. The topology of the feasible region is visualised in Fig. 28. Finally, the optimization problem is formally defined as

$$C_{ph}(\Delta t_{EV}, \Delta t_{OV}) = \min_{\Delta t_n \in \Theta^2} (\Delta t_{EV}^{fix}, \Delta t_{OV}^{fix}), \quad (68)$$

subject to constraint

$$\{t_{EV}^{fix}, t_{OV}^{fix}\} \in \Theta_F^2. \quad (69)$$

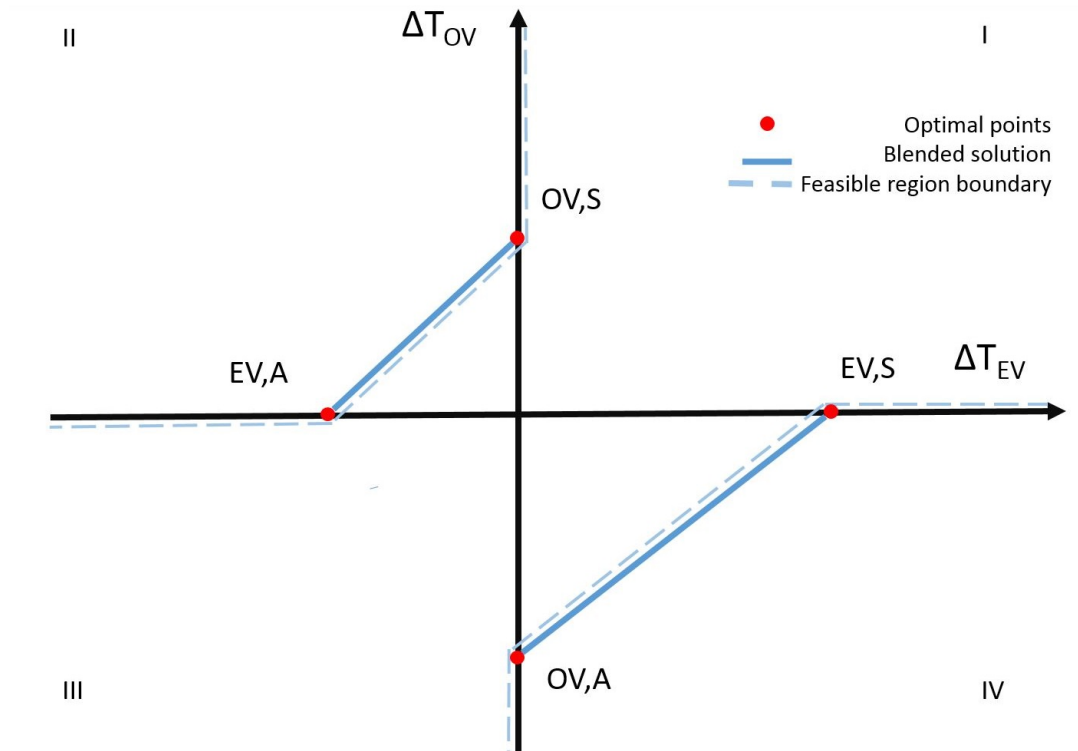


Figure 28: Intersection conflict in time domain. The dashed line outlines the boundaries of the feasible region. I to IV name the quarters. A - acceleration, S - slowing down. Note that acceleration results in negative ΔT .

5.1.1 Simplified intersection decision algorithm

As mentioned in Literature Review, in Section 2.3.1, one of challenges in traffic optimisation is the complexity and ensuing computational time required. For a number of players n growing, the computational time increases rapidly. While the Section 5.4 discusses the complexity of the platooning problem, Section 5.3 provides an estimation of the intersection problem complexity. However, this Section contains a naive, simplified optimisation algorithm for multi-agent intersection problem is presented in the Section 5.3. It is based on the assumption that the solution is always pure, meaning only one agent is required to adjust their strategy. We offer, thus, a simplified formulation, that allows a computationally lean method of finding the candidate minimizers Δt_{min}^{fix} (61).

We observe that the apparent cusp around the root of the cost function indicates that mixed strategies, that is where α varies, are characterised by a concave function. The derivative of the cost function is presented in Fig. 29, showing negative curvature on the $\Re+$ side what allows to assume that the optimal solutions are pure, that is on either of axes of the system, what is defined as

$$\operatorname{argmin}_{\alpha} \{0, 1\}. \quad (70)$$

This assumption enables us to reduce the Θ^2 solution space and project both agent's cost functions to a single dimension, to find the potential solution points. We can now also define the three possible actions agents can undertake $\mathcal{A}_n = \{A, 0, S\}$, where 0 corresponds to no action.

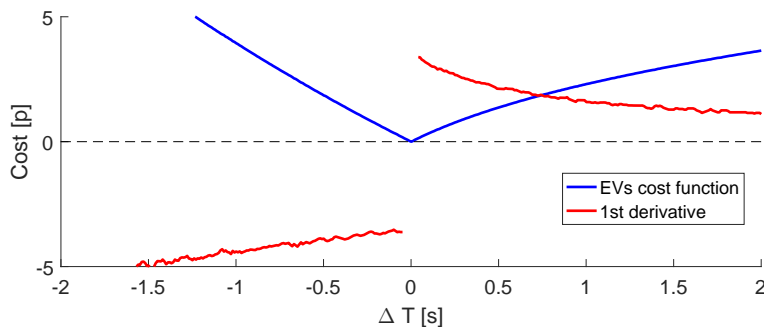


Figure 29: An example of the integrated cost function and its 1st derivative are presented. Plot of the integrated cost function $C_{P,n}(\Delta T)$ (59). It allows to find the cost for the given time correction. Note the steep cusp around zero and the linear function as the ΔT increases. The derivative's slope decreases asymptotically, suggesting that the summarised cost function between the optimal points are concave.

We then find the costs of manoeuvres $C_P(\Delta t^{fix})$ and express them as game-theoretic payoff bi-matrix $\mathcal{M}_{EV,OV}$ as in Tab. 10, rejecting the infeasible strategies. The result is presented in Fig. 31, marking the constraints and points where they intersect with the CF, that is the candidate points.

Table 10: The payoff bi-matrix M . Strategies marked with $*$ are infeasible, while the ones marked with \ddagger are feasible but rejected based on the assumption (70).

$\mathcal{M}_{EV\setminus OV}$	S	0	A
A	$*$	$C_P(\Delta t_{EV,S}^{fix}), 0$	\ddagger
0	$0, C_P(\Delta t_{OV,A}^{fix})$	$*$	$0, C_P(\Delta t_{OV,A}^{fix})$
S	\ddagger	$C_P(\Delta t_{EV,S}^{fix}), 0$	$*$

Thence, the conflict between agents is reduced to a cooperative 3x3 game, whereby the cost function solutions for the candidate points $C_P(\Delta t_{min}^{fix})$ are considered

$$\mathcal{A} = \underset{\mathcal{A}_n \in \{A,0,S\}}{\operatorname{argmin}} C_P(\Delta t_{\mathcal{A}}^{fix}). \quad (71)$$

An enforcement payment p covers the cost of the manoeuvre's cost, should the solution differ from the traffic regulations.

The Price of Anarchy (PoA) is defined as a proportion of the GT solution in relation to the globally optimal one

$$PoA = \frac{\min C_P(\Delta T)}{C_{NC}^*}, \quad (72)$$

where C_{NC}^* is the cost of the subgame noncooperative, equilibrium solution, regulated by the traffic code. Evaluation of the loss of efficiency caused by the requirement of agreeability is then possible.

5.2 Intersection conflict simulation

To visualise the way the outlined algorithm operates, a simulation of an example is performed. As explained in Section 1.1 on Fig. 3, the intersection scheduling operates by finding optimal correction times for vehicles, so that the sequence of vehicles is optimised by prioritising e.g. massive vehicles, to minimise combined powertrain effort. Firstly the integrated cost function is visualised. It is a function of cost of arrival at a defined time $\int_{S_t}^0 C_P(V_P : T_{arr})dx$ and is presented above in Fig. 29. Any departure from the

cruising speed, which corresponds to the unadjusted time of arrival, corresponds to no cost. However, increasing the speed, that is decreasing the ΔT increases the cost rapidly, while the delay has shallower dynamic.

This cost function is then compared with the other agents', to complete the intersection optimisation algorithm. The optimal solution is found by projecting the functions on the Θ^2 domain. The combined cost function is visualised in Fig. 30, presenting two $C_P(\Delta T)$ functions projected onto the feasible region as defined by the equation (67) and visualised above, in Fig. 28, both located in Ch. 5.1.

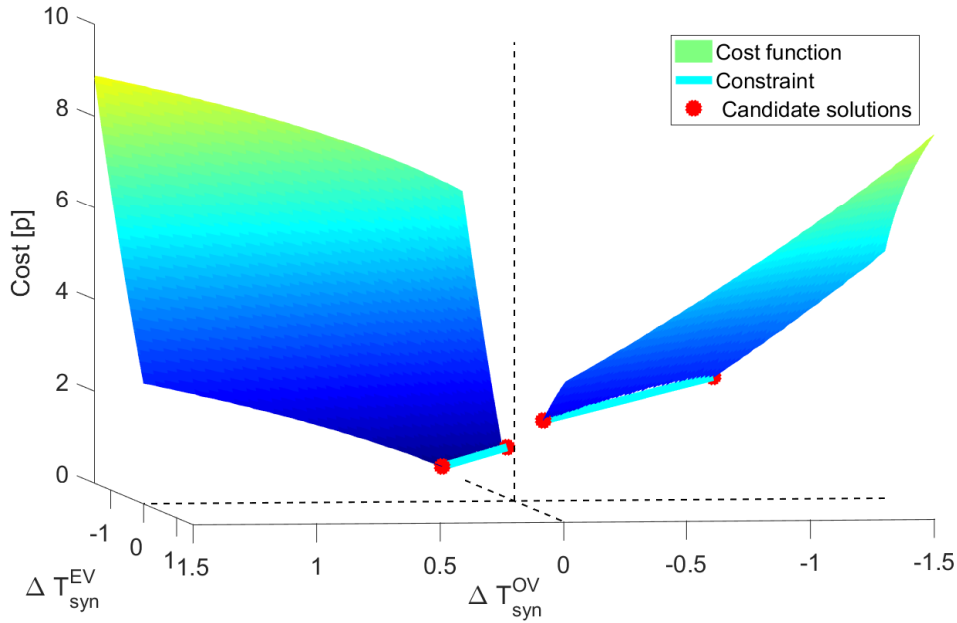


Figure 30: The solution of the intersection problem employing the complete formulation. It showcases the three dimensional model of the combined cost functions for a collision solution between agents. The convexity of the CF allows finding candidate optimal points, marked with red. The axes of the coordinate system are marked, to aid with spatial visualisation.

Then, by executing the simplified algorithm, we find the edges of the cost functions. Given the convexity of the function, the search can be limited to there, as the optimal solution is on the intersection of the constraint and the CF. The cost function edges are thus projected on a single, combined ΔT domain and visualised in Fig. 31. Out of four points where the CFs intersect with constraints, one of which is out of the plot's scale, one can easily find the smallest of the four points. In the presented example it is when the OV yields by speeding up, allowing the EV to cruise unimpeded, just behind it. For numerical comparison Tab. 11 lists the simulation results. In this scenario, the combined

cost of manoeuvre is decreased from $\pounds 2.02 \cdot 10^{-2}$ to $\pounds 0.85 \cdot 10^{-2}$, what corresponds to $PoA = 2.38$.

While the traffic code resolves conflicts based on the position of the vehicles, the proposed algorithm disregards it, focusing solely on the cost minimisation, also neglecting safety. As such, for around 50% of cases, the optimal solution is in violation of the traffic code, calling for more complete, optimality focused legislation.

Table 11: A comparison of the intersection conflict resolution results. A noncooperative case - with the EV slowing down following traffic code, and an optimized, cooperative case, where the heavier EV incentives the OV with a micro-payment to accelerate.

	Noncoop.	Cooperative
Solution	<i>EV, S</i>	<i>OV, A</i>
Time corr. [s]	+0.84	-0.84
Cost [10^{-2} £]	2.02	0.85

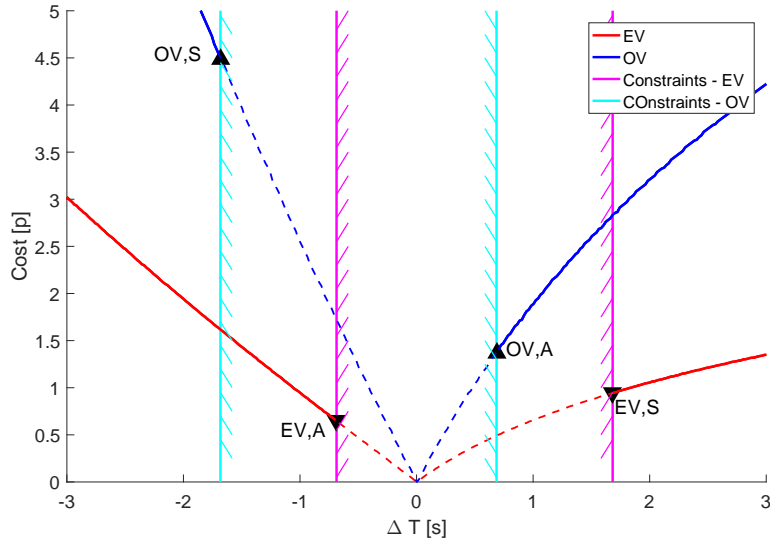


Figure 31: The solution of the intersection problem employing simplified formulation. It showcases cost functions and their constraints of the example case reduced to two dimensions. The candidate optimal points, on constraints, are marked. The minimal point corresponds to a scenario whereby the EV yields by speeding up, allowing the OV to cruise unimpeded. The dashed line refers to the CF within the infeasible zone.

5.2.1 Performance evaluation

The optimality of the proposed algorithm is measured by evaluation of the PoA , that is loss of optimality in relation to the centralized, unenforced solution, which is incurred by the agreeability constraint. The baseline for the comparison is the solution regulated by the traffic code, with the vehicle on the left to yield.

While normally a simulation involving powertrain components would involve cyclic drive cycle runs, e.g. employing NEDC, ARTEMIS or more recent WLTP load cycles [183], this framework tackles the optimality of the very cycle itself. Thus its evaluation calls for estimations of its efficiency, applied to particular road scenarios. In order to give reliable numbers, randomized Monte Carlo simulations are being performed to give Price of Anarchy estimates. Since the co-co solution concept brings value only for games between heterogeneous agents, PoA is evaluated as a function of the traffic heterogeneity, that is the proportion of trucks in the traffic.

Monte Carlo simulation is performed using MATLAB, with $n = 10^4$ samples for each data point, randomizing input data as in Tab. 17, as uniform distributions. Thereby randomness of traffic events and the user's choice of their personal cost of time delay $C_{T,n}$ is captured.

As a key test, to evaluate the level of energy and time savings enabled by V2V cooperation, Fig. 32 presents the PoA as a function of traffic heterogeneity. One can note that the PoA is always above 2 with a peak of 3.12 whenever the fraction of trucks in relation to cars is between 20% and 40%, a number similar to the findings in the literature. The results suggests that from both time and energy perspective the optimality of First-Come-First-Served (FCFS) schemes may be further optimised, by reorganizing the order of the vehicles, as the management of the order of vehicles on the intersection can promise efficiency improvement. It is worth reiterating, however, the subjectivity of the time parameter C_T and the fact that the simulated model did not consider overlaps between multiple vehicles interactions introduce some subjectivity to the result. It is expected that the performance of the algorithm degrades, as the traffic density grows, but certainly offering an operating window, for low and medium density traffic, which may offer measurable benefits.

Table 12: The randomized input parameter ranges for the Monte Carlo intersection simulation.

Veh. type	unit	min	max
C_T (Car)	\$	5	50
C_T (Truck)	\$	33.5	368
Maneuver distance S_I	m	150	250

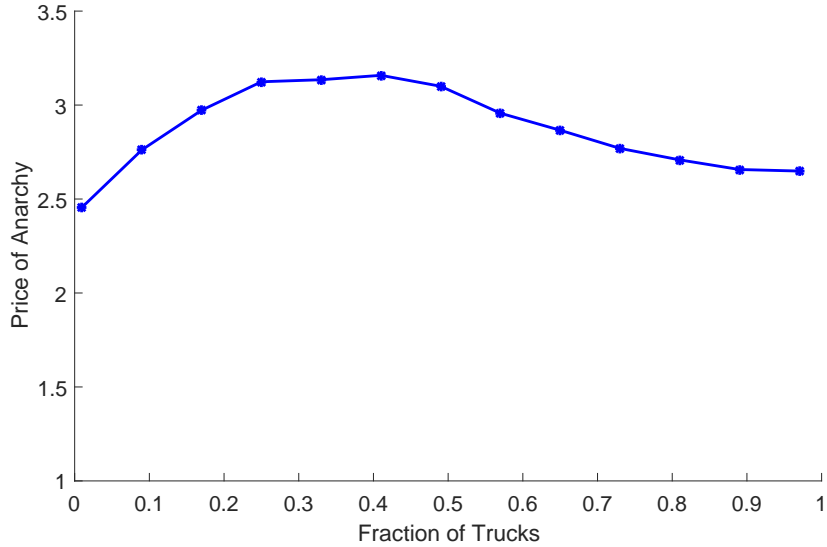


Figure 32: Results of a Monte Carlo simulation with 10^4 samples for each data point. Price of Anarchy as a function of the fraction of all vehicles is presented. Decisions based on traffic code are considered as a baseline. The PoA is always above 2.5 with a peak of 3.12 when 20% to 40% of vehicles are trucks. This indicates that from both time and energy perspective, the optimisation of the order of vehicles on the intersection can promise vast optimality, reiterating however the subjectivity of the time parameter C_T .

5.3 Multi-agent intersection conflict formulation

Until now, the algorithm, for simplicity, defines interactions between two agents, as this allows for a clear understanding of the problem. This subchapter considers a multi-agent scenario, where n agents's trajectories are intersection. A generalised formulation is put forward, based on the intersection derivation from ch. 5.1. While most research work leaves the solution of multi-agent conflicts to centralised optimisation algorithms [184], as presented in Section 2.3.2, this is a proposition of a deliberative and distributed definition of the problem. Given the safety-critical assumption of the complete information, the relations between n agents conflicting on an intersections is assumed to be defined by the

complete graph topology, with number of links defined as K_n [185] [186]. The number of conflicts is defined as $K_n = \frac{1}{2}n(n-1)$ [187]. However, since each conflict may be mitigated by either of action of either agent, the complexity is expected to double. The further increase in complexity originating from the interactions between multiple agents is yet to be estimated.

Given an intersection conflict between n agents, defined by a set $N = \{1, 2, \dots, i, \dots, n\}$ of agents, sorted ascending with respect to their time of arrival $T_{I,n}$. We can then define an adjacency bi-matrix, defining the Δt required to mitigate the given conflict (60a). For clarity, the Δ symbol is dropped. The T_S thus denotes the solutions where a agent slows down,

$$T_S = \begin{bmatrix} - & t_{1,j} & \cdots & t_{1,n} \\ t_{i,1} & - & \cdot & t_{2,n} \\ \vdots & \cdot & \ddots & \cdot \\ t_{n,1} & t_{n,2} & \cdot & - \end{bmatrix}. \quad (73)$$

Of course, since an agent cannot be in conflict with oneself, the positions on the diagonal are not feasible and marked as $-$. Similarly, the T_A refers to the acceleration side of the resolution, where the correction values are negative. The positioning in the matrix can be interpreted as: the i^{th} agent's obstacle is resolved by j^{th} agent adapting its time of arrival at the intersection by $\Delta t_{i,j}$.

The matrices still consider all interactions, even those where no conflict occurs, consuming the computational power. As a conflict detection measure, we find all the ΔT signs not matching with the matrix they belong to and reject them. All interactions which do not pose a conflict are equalised to 0

$$[T_S^+, T_A^-] = \max(T_S, 0) \min(T_A, 0). \quad (74)$$

Then, by analogy to (62) we can observe that the sum of the obtained matrices transposed is zero

$$T_S^+ + T_A^{-T} = 0. \quad (75)$$

Since the adjacency matrices are symmetric along its diagonal, is customary to represent only one half to avoid repetition. We may then merge the bi-matrix so that all relations are captured in a single matrix. We could then define a single matrix $\Delta T_0 = T_S^+$ and

to access the acceleration-resolved side of conflicts, we could find $T_A^- = -\Delta T_0^T$. Then, just as in (59), the matrix of phasing manoeuvre cost $C_P(\Delta T_0)$ is found by solving the integrated cost function over the assumed phasing speed profile, for each position in the matrix

$$C_P(\Delta T_0) = \begin{bmatrix} c_{i,j}^P(t_{i,j}) & \cdots \\ \vdots & \ddots \end{bmatrix}. \quad (76)$$

With the conflicts detected and their mitigation time corrections defined, the evaluation of costs follows. It requires, however, to capture solutions involving more than two agents. Suppose that while an agent conflicts with two adversaries, where one solution is S while the other A . Then, in either direction, the third vehicle's solution is affected by both conflicts. Consider the scenario of three vehicles, A , B and C , as presented in Fig. 33. In such instance, while there is no AC conflict at the moment, however, the S -solution to the AB conflict, where B would slow down, creates a new BC conflict. This conflict propagation may be approached by stepping back, and considering also the non-conflictual interactions, otherwise lost in (75).

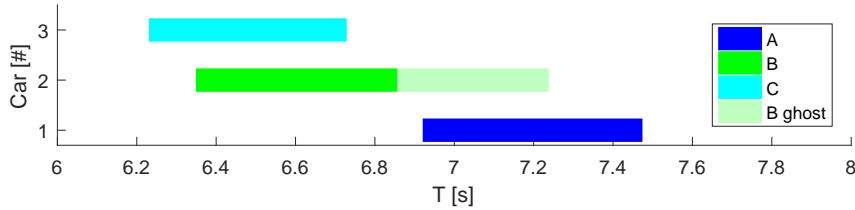


Figure 33: An example visualisation of a propagated conflict. While there is no BC conflict, an S -solution to AB overlaps with C 's CZ occupancy. This greatly complexifies the n -agent formulation.

Finally, having identified the conflicts in ΔT_0 , we propagate the $\Delta t_{i,j}$ on the original $[T_S, T_A]$ bi-matrix, respectively downstream for S and upstream for A , obtaining

$$T_S := \Delta t'_{i+1,j+1} = \Delta t_{i+1,j+1} + \Delta t_{i,j} \quad (77a)$$

$$T_A := \Delta t'_{i-1,j-1} = \Delta t_{i-1,j-1} + \Delta t_{i,j}. \quad (77b)$$

Then, again, to find each conflict resolution cost, we solve the equation (59), that is the CF integrated over the phasing speed profile (59), albeit accounting for the recursion

defined in equation (77), to give

$$C_P(\Delta T_S) := c_{i,j}^P(t_{i,j}) = \sum_{k=1}^n \sum_{l=1}^n c_{k,l}^P(t_{k,l}) \quad (78a)$$

$$C_P(\Delta T_A) := c_{i,j}^P(t_{i,j}) = \sum_{k=1}^n \sum_{l=1}^n c_{k,l}^P(t_{k,l}) - \sum_{k=1}^i \sum_{k=l}^j c_{k,l}^P(t_{k,l}). \quad (78b)$$

The optimization problem is then formally defined as

$$C_P(\Delta T_A, \Delta T_S) = \min \left(C_P(\Delta T_S), C_P(\Delta T_A) \right). \quad (79)$$

While the simulation using Monte Carlo was stable, the estimation of complexity was inconclusive. While the n is similar to the complete graph series [187], the exact complexity depends on more constraints than the model currently captures. Development of an experiment with real traffic data may be needed for further work.

5.4 Complexity of n-agent platooning problem

So far, in this thesis, the Game Theory Optimisation problem has been defined as a conflict between two agents, where based on mutual trust, the agents agree on a Pareto Optimal solution. But suppose there is a road-side unit [90] alongside larger intersections. In the Chapter 5.3, we have introduced first a 3rd and then n^{th} agent, with the matrix T_S (73) where an Euler explicit method to model the n -agent intersection, to evaluate the rate at which the complexity scales with the number of agents, yielding observations but no conclusion, as the conflict propagation is as a source of additional complexity layer.

Considering the above, this Chapter is a step back, to correlate the above findings with another numeric validation, this time using platooning, Section 4.2 as an example. Because the Platooning is indifferent to position, x_n , only dependent on its difference, $\Delta x_{EV,OV}$, and it is a two dimensional problem, it is substantially quicker to compute.

Here we consider a scenario where a platoon \mathcal{P}_m consisting of m -agents meets a slower n -agent platoon, \mathcal{P}_n . Then, as visualised in Fig. 34, vehicles can be reconfigured to find a new, re-optimized platoon.

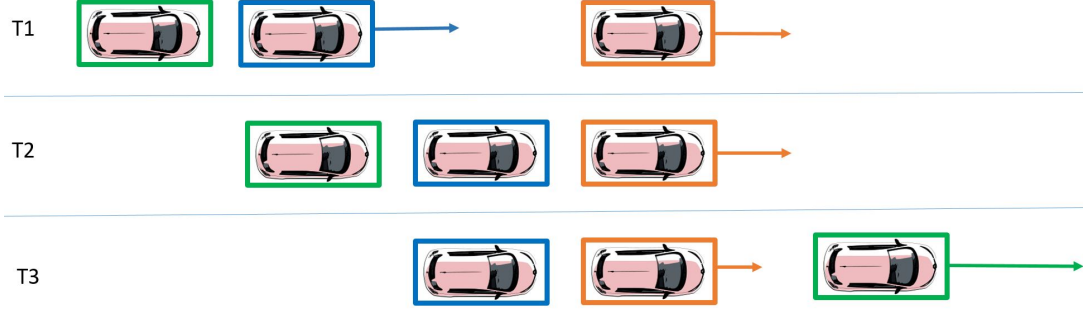


Figure 34: Visualisation of the reconfiguration of two platoons. In this example, a faster platoon approaches a slower, single vehicle platoon. As they merge, the multi-agent platoon is re-optimised, finding that, the optimal solution is to allow the green vehicle to overtake the remaining ones, as the additional cost it would have to pay to the orange agent outweighs the overtake cost.

All configurations of platoons are considered, with Tab. 13 presenting an example i^{th} membership array for $i \in N : \{1, \dots, (n + m - 1)\}$. Fig.35 outlines the algorithm to find the optimal $m-n$ -agent platoon optimal reconfiguration. It has been developed with help of MSc student Kritikesh Ravishankar [188].

Table 13: Example i^{th} membership array for multi-agent platooning scenarios.

Car no.	m_1	m_2	m_3	n_1	n_2
in i^{th} platoon?	1	0	1	0	0

Given that participation in a platoon is expressed binarily, the rate of growth of the platooning problem's complexity follows a 2^N sequence [189]. The N is here the sum of all vehicles, except the first one, as by definition of platooning it is the slowest of all, thus $N = n + m - 1$. The time to compute the time to find the optimal solution for two platoons with n and m agents exceeds 10 [s], as $N > 6$, too long to be relevant.

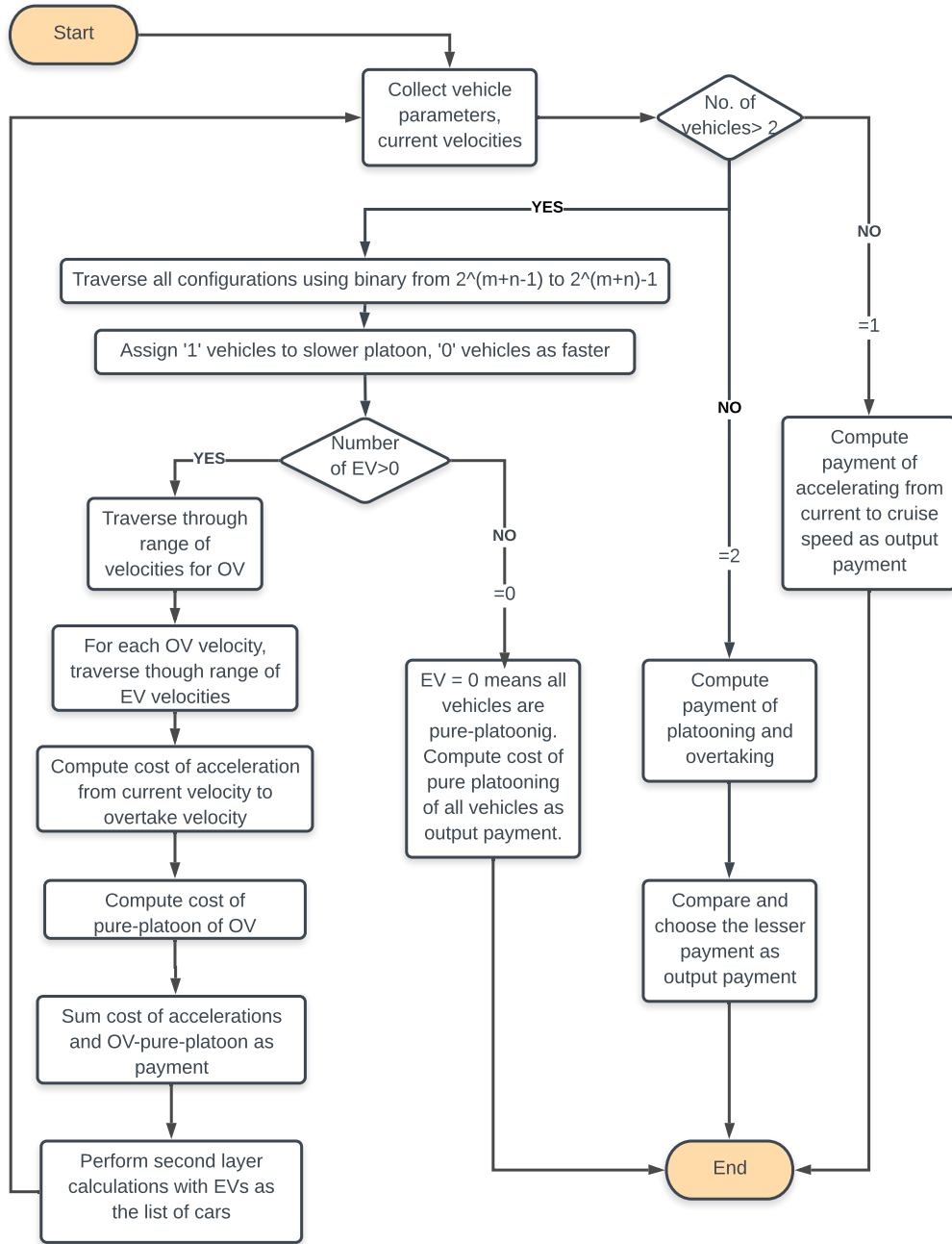


Figure 35: The flow diagram of the n - m -agent platoon re-optimisation algorithm.

5.5 Discussion of the results

Naturally, the information exchange allows to greatly improve the energy efficiency of the conflict resolution in relation to the traffic code regulations. The $PoA = 2.5$ indicates a 2.5-fold improvement in time and energy efficiency of the negotiated traffic, in relation to today's traffic, with predefined, fixed right-of-way rule. The result is theoretical only,

considering full system penetration, that is all cars being able to communicate, and for conflicts among no more than 2 agents. Although a platoon of vehicles can be reduced to single agent [190].

It is important to observe that in roughly 50% of cases the optimal solution is the very opposite of what a non-cooperative, law-regulated solution dictates. This poses a challenge to the framework’s implementation, as the initial confusion and lack of liability protection to the users may pose a barrier too large to overcome without the road operator’s support. The platooning is the only scenario, whereby the conflict occurs continuously over time with minimal speed differences, as opposed to time-critical separation required by the intersection and overtake problems. It would then be the first scenario to be feasibly deployed. It would require, however, apart from precise relative positioning, a notion of mutual trust between machine and human drivers, which is addressed in Section 7.4.

The multi-agent implementation however, while guarantees a Pareto optimal solution, it worsens. As the solutions This additional layer of complexity is challenging to compute. Firstly the analytical operations are performed on numerical solutions, since (59) is not easily computed analytically. In addition the recurrence within the matrix occurs only on conflictual interactions, likely ruling out the possibility of a formal derivation of the complexity at all, calling for a numerical simulation to settle it. The issue is further considered, albeit with a simpler to model. The Platooning model, Section 4.2, is studied for n agents in Section 5.4.

The platooning implementation, however, carries complexity characterised by 2^N power series for the platooning problem, and still undefined complexity for the intersection problem. It has been established that it follows complete graph complexity K_N , excluding the conflict propagation. Regardless of the result, the computational time for $N < 5$ is not likely to exceed 20 seconds, retaining feasibility for medium-intensity traffic application. It can also be further minimised by a platoon merging heuristic.

Summarizing, while the benefits of the data-enabled negotiation of traffic conflicts offers vast benefits, its deployment poses numerous obstacles, from multi-agent complexity, through need for new legal regulation, finishing at vulnerability to users opting out.

6 Hardware validation

The results in the above, Chapter 5 demonstrate the theoretical performance of the framework proposed herein. However, they were obtained in simulations, for which a number of assumptions were made, as described in Section 3.3. These assumptions separate the simulation from the physical domain. Thus, the validation of this work requires a Hardware-in-the-Loop (HiL) implementation, so that some real-world challenges are demonstrated to be overcome. Thus, this chapter features hardware implementation of the algorithm. In addition, just as the Section 5.3 above offered an approach to multi-agent intersection problem, here the complexity of the platooning between multiple vehicles is considered.

After every bit of theory comes test in practice. A control strategy or an algorithm that is proven to work in simulation is tested in a hardware set-up. There are a number of Hardware-in-the-Loop (HiL) testbeds to test various configurations and control strategies for hybrid powertrains [191] [192]. In similar manner, HiL testing of autonomous vehicle control systems is being researched. However, this area is dominated by commercial testbeds [193] [194]. While the above testbeds focus on the optimisation of the powertrain itself, hardware testbeds for social interactions in cooperative driving are also present [195] [196].

6.1 Raspberry Pi implementation

As a first step, to demonstrate the practical applicability of the negotiation procedure, as presented in the communication activity diagram from Fig. 18 in Section 4.1, a simple platooning scenario is considered, where a freely cruising Ego Vehicle (EV) encounters a slower vehicle (OV) on its way, as outlined in Chapter 4. The overtake and platooning scenarios are simulated and the less costly one is selected. The C++ code is provided in the Appendix.

Individual agents are simulated on standalone devices, with only LAN connections to negotiate. In addition to the negotiation algorithm, the boards are equipped with connectivity, synchronisation and display handling functions. The algorithm has been

implemented on two Raspberry Pi 3 Model B chips. They are equipped with a 64bit ARM Cortex-A53 quad-core CPU with 1.2 GHz frequency and 1 GB RAM. The boards were connected by Ethernet cable and equipped with displays. The Fig. 36 presents the results. The photographs of the screens, the EV is on the left and the OV on the right. It is presented during the following manoeuvres: A: cruising, B: Platooning and C an overtake. For further explanation, the Tab. 14 features an explanation of symbols used. The computations were performed on the EV, with the OV deciding to accept the offer.

Table 14: The meanings of symbols as displayed on the Raspberry Pi displays below.

X	V	D	P
reference position	speed	distance to the vehicle ahead	payment incurred



Figure 36: Raspberry Pi displays, respectively, during the approach, cruising freely, platooning and overtake. As the EV, denoted as 1, approaches, it executes the algorithm as presented in Fig. 18 in Section 4.1. The OV may reject it, but accepts, as the payment offsets the cost, and executes the requested speed profile.

To assess the computational burden, the execution times of the algorithm were measured using the Raspberry Pi's `clock_gettime()` function. Twenty observations were made for platooning and overtake scenarios. The time values obtained are shown in Tab. 15. The platooning evaluation function takes less than 0.001 [s] to execute, while the overtake calculation takes consistently less than 0.4 [s]. Because the algorithm is to be executed once per manoeuvre, this example demonstrates its feasibility as a real-time application.

Table 15: Calculation times for the distributed platooning and overtake negotiation implemented on Raspberry Pi. For a single 1.2 GHz core, the platooning calculation is negligibly fast, while the overtake takes less than 0.4 s.

Function	Average [s]	Std. Dev.	Coef. of Variation
<i>EvalPlat</i>	$5.79 \cdot 10^{-4}$	$2.2 \cdot 10^{-5}$	$3.79 \cdot 10^{-2}$
<i>EvalOvt</i>	0.38	$7.5 \cdot 10^{-3}$	$1.9 \cdot 10^{-2}$

6.2 Scaled, automated vehicle implementation

As the next step in validation, the algorithm has been implemented on a scaled, automated vehicle platform, which is visualised in Fig. 37. This validation approach is very similar to [85], where a fleet of cooperative vehicles is made to cooperate on a two lane road. However, the main difference from this work is, again, the emphasis on the heterogeneity of agents. In this validation work, the connected vehicles perform negotiations in real time, thanks to MQTT [197] communication protocol in, to run the hardware-implemented model of platooning and overtake algorithms. This work has been implemented with the help of MSc student Ashwil Joseph [198]. The distributed control architecture has been designed by a visiting student, Mario Pavan.

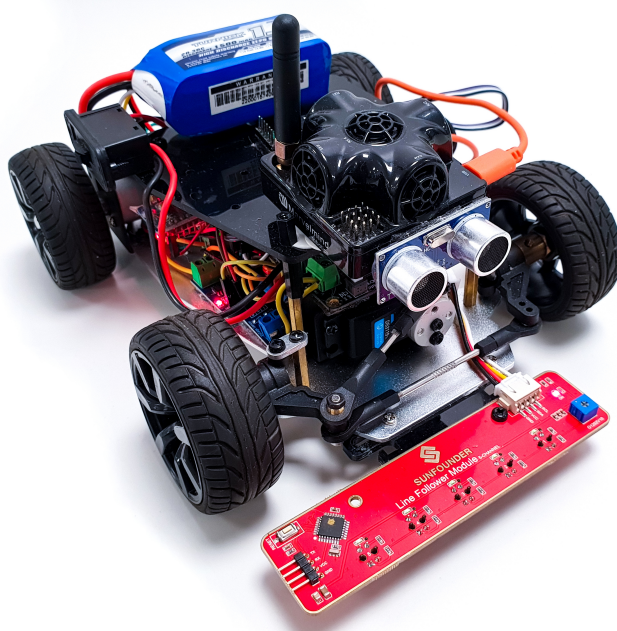


Figure 37: The scaled automated vehicle employed of the hardware validation. The sensors visible are the ultrasound positioning beacon, line follower module and the standard Arduino’s ultrasonic range finder, necessary for the platoon formation.

The hardware composition of these scaled cars is outlined in Fig. 38. It consists of a PWM Module, which controls the motors’ rpm and the steering servomotor, Line Follower

Module and IMU, which all communicate to Raspberry Pi through I2C protocol. The motor encoder, voltage and current sensor reading are then received by Arduino Nano which communicates to Raspberry Pi using the USB bus. The indoor positioning is provided by the Marvelmind Mobile Beacon, which communicates with the Raspberry Pi using virtual universal asynchronous receiver-transmitter (UART) emulated on the USB interface. Additionally, in the platooning mode, a forward ultrasound sensor serves as additional speed difference sensor, to allow precise manoeuvres. The Inter-Vehicle communication is managed by a cloud-server based MQTT protocol [199], facilitated by a Personal Computer. Wireless 802.11bgn protocol, that is Wi-Fi, serves as the physical layer, and the MQTT is used as a subscription-publication based protocol on data link layer.

Having constructed the vehicles and the the connectivity connectivity between them, the deployment of the automated intersection negotiation is mainly focused on the tuning of the PID controllers. The most challenging PID to tune was the fine speed control. It regulates the relative position of the vehicles, during the platooning, with the motor torque.

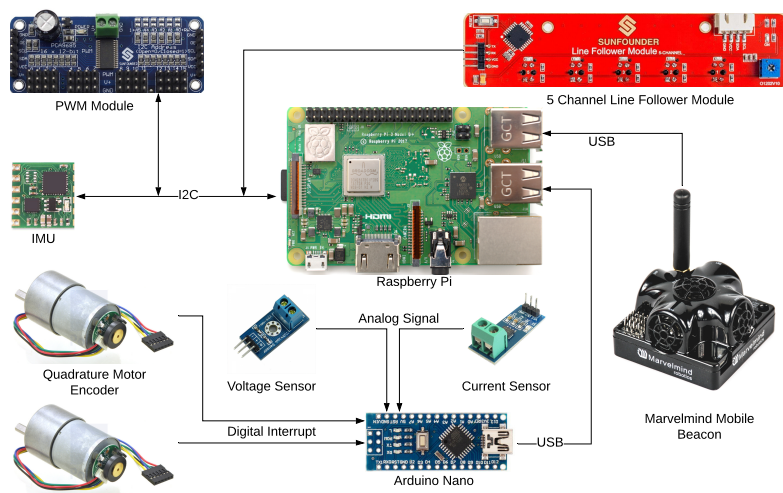
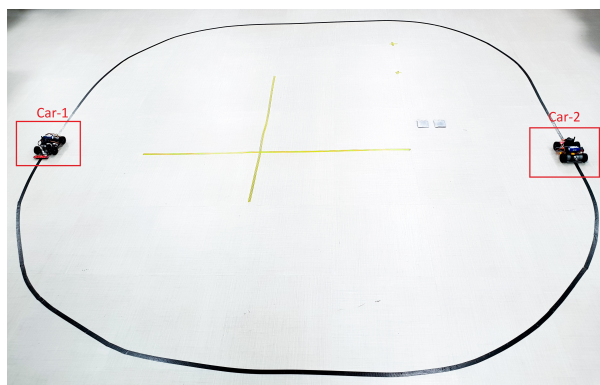


Figure 38: Vehicle's hardware control system topology. The Arduino is used separately to control the motors, to ensure motor phase alignment.

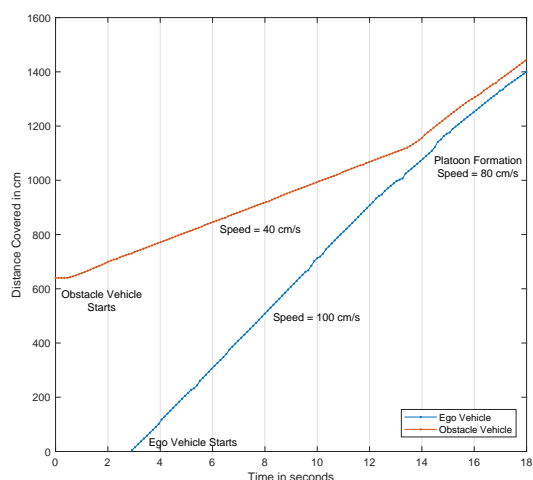
The set-up to validate the negotiation algorithm is a closed-loop track as presented in Fig. 39a. It was observed in Section 5.5 that the easiest to implement is the platooning scenario. Two vehicles with various cost functions are cruising freely on the track until the ultrasonic sensor detects a conflict. The communication is managed through a server,

but the negotiation algorithm is run on the faster vehicle, who has the incentive to cooperate. The slower one agrees to cooperate if, and only if the agreement benefits them. It is visualised in the activity diagram in Fig. 18 in Chapter 4.1. A new, cooperative platooning speed is agreed upon and executed, as plotted in Fig. 39b. The platooning control is performed by the ultrasound sensor and a PID controller, achieving a stable platoon at a new, consensus speed V_P .

Observation of the hardware deployment has offered an insight into the influence on the PID controller tuning on traffic stability. When the control was insufficient, the vehicles collided. When control was overshoot, they vehicle was jerky. It is thus a trade off between safety and comfort. Today, the distance control is performed by human drivers, thence the road throughput is a result of the average of drivers' preference, regulated by the cultural perception of safety. As automated platooning emerges [72] the trade-off between congestion and safety becomes an engineering choice, whereby the engineer chooses between user comfort and road throughput, just as it is with aircraft autopilot [200].



(a) The track with the automated cars located at opposite sides. The beginning of an experiment.



(b) Distance of each vehicle elapsed from the beginning of the simulation, plotted as a function of time. The transition to platooning and the new, consensus speed are observable.

Figure 39: The Hardware-in-the-Loop set-up for the negotiation algorithm. Two vehicles with various cost functions cruising freely until they detect a conflict. All the communication is performed in a distributed manner just as vehicles detect they are on collision course

6.3 Conclusions

This Chapter has outlined the application of the game-theoretic de-conflicting algorithm applied on hardware. It serves as a validation of the framework, as it demonstrates its operability in a physical, V2V system. Firstly, the algorithm was run on embedded Raspberry Pi platforms. It then expanded to become a scaled automated vehicle implementation. The objective was to demonstrate that the execution of the platooning scenario on the automated vehicles, in real time was achieved. The vehicles perform calculations in real time and cooperatively execute a velocity transient. The development of multi-agent intersection/overtake/platooning scenario is possible as the most imminent next step.

7 Framework scalability analysis

In the core of the framework, as derived and validated in the Chapters above, the main novelty is in the game theoretic method of negotiation. It has also been implemented as a Hardware-in-the-Loop application, but there are several assumptions. Thus, for further validation of this work, in the following chapter various assumptions are challenged, to offer additional viewpoints on the behaviour of the algorithm proposed. Unless stated otherwise, the input from Section 3.4 and assumptions from Section 3.5 hold.

Firstly, a study of the cost function modularity is proposed and a tests of the honesty assumption. It is followed by a sensitivity analysis of the algorithm's efficiency to the fraction of agents present on a public road, who can communicate. The chapter is concluded with an exploratory notion, whereby human drivers could cooperate with CAVs.

7.1 Cost function development

So far, the cost function relates the energy and time costs to find optimal actions, neglecting other, higher order parameters, which affect the overall experience of the user. These include comfort, component wear or perception of safety [178] [179]. The main control input for these are relative position and its higher temporal derivatives, acceleration and jerk, which call for a more complex formulation. This feature, along with metrological challenges associated with the subjectivity of safety and comfort, resulted in the pursuit for a more comprehensive cost function.

7.1.1 Ancillary energy loads

Thus far, this work only considere the energy consumption originating from the vehicle motion, as assumed in Section 3.5. In this Section, however, the ancillary load is considered as well, be it cabin heating, lights, etc. It is introduced as an additional term, E_{anc} . The cost function assumes the form

$$J(v) = C_E E(v) + C_T \Delta T + C_E E_{anc}. \quad (80)$$

It is modelled on a Car type vehicle to examine the sensitivity of the optimal speed V^* to variation in E_{anc} . The value of the cost of energy C_E is taken from Section 3.4 [175]. Three values are considered, 1, 2 and 3 [kW]. Results are presented in the Tab. 16 and the cost function visualised in Fig. 40. The gradual increase in the V^* reflects the effort to minimise the duration of the fixed ancillary load.

Table 16: The optimal cruise speed as presented in Section 3.4, enriched with ancillary energy consumption, independent of time.

Ancillary power [kW]	0	1	2	3
V^* [m/s]	14.49	16.13	17.5	18.68

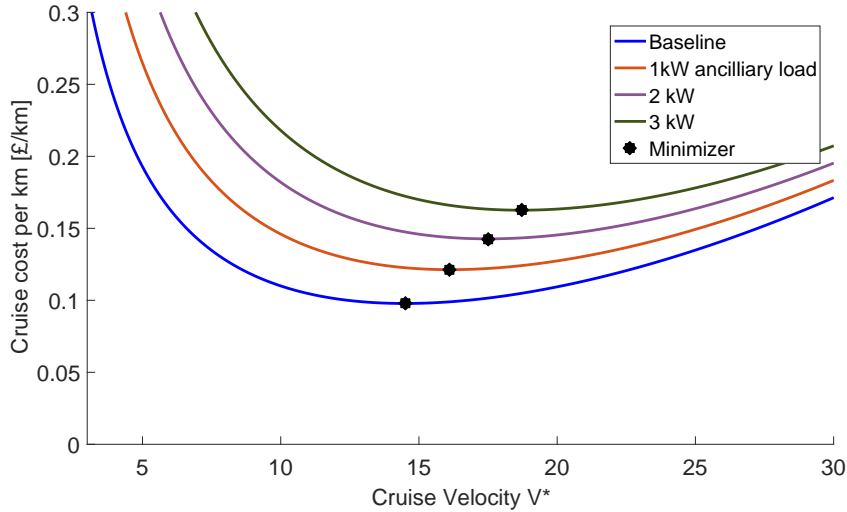


Figure 40: The cost function’s sensitivity to ancillary load. Increasing regenerative braking obviously flattens the cost function, further proving the elasticity of the CF.

7.1.2 Battery wear model

While so far the CF considers only energy and time consumption, this Section features an additional cost, the cost of component wear. The weakest link of an electric powertrain is the battery. A simulation of an overtake manoeuvre is performed, using MATLAB’s *fmincon* optimisation routine to find optimal speed profiles $v_n(t)$ [201]. The time domain is discretised with step of 0.1 [s] and no overtake parametrisation is used. There, the throttle traces of both agents were optimised together cooperatively, using the CF enriched with a simplistic, linear battery State of Health (SoH) estimator model J_{batt} [202] defined as

$$J_{batt} = \gamma_b \int_0^t |i_b(t)| dt \quad (81)$$

where γ_b is an arbitrary wear coefficient, selected to represent the sensitivity of the battery. This is a first approximation of the wear. The result of the simulation, is presented in Fig. 41. One can observe that the penalty on the absolute value of the current $|i_b|$ reduces the input while elongating the manoeuvre. Furthermore, while in the control sample, without battery wear, the throttle traces are symmetric, the solution accounting for the battery wear is biased towards moderation of the control effort.

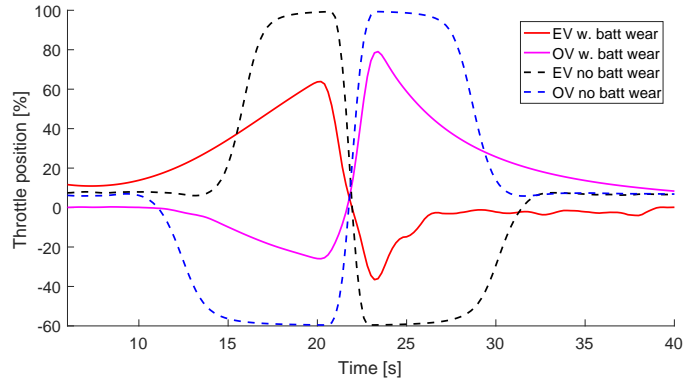


Figure 41: The cooperative overtake speed profile optimisation with no parametrisation. Two original cost function is compared with an extended one, featuring a linear battery wear model. The later one features asymmetric throttle traces, preferring moderation of the control effort during the deceleration phase, actually reducing the energy recovery.

7.1.3 Variable energy recovery fraction

The electric motors allow for energy recovery. It is varied however, to manage the battery wear. Because of that, the cost function's sensitivity to varying electric powertrain energy recovery settings is examined [11]. So far no energy recovery have been considered, with the energy loss originating from either powertrain inefficiency, drag or energy dissipation from breaks. In this simulation, upon deceleration a fraction of kinetic energy is redirected to the energy storage, defined as $\frac{E_{rec}}{E_W}$, according to the definitions (11) from ch. 3.1. The Fig. 42 presents the CF for varying energy recovery fractions.

7.1.4 Consideration of externalities

An externality is an effect of an action, possibly unintended, which affects an another agent. For example, the noise originating from the rolling tyres affects the neighbours,

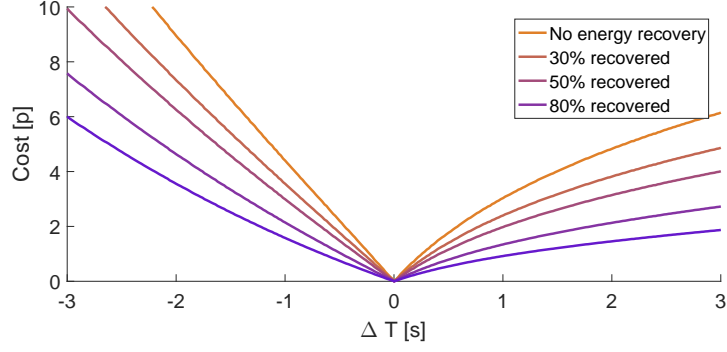


Figure 42: The cost function’s sensitivity to varying fraction of energy recovery. Increasing regenerative braking obviously flattens the cost function, further proving the dynamics of the CF.

it is a negative externality. In fact, any societal externality cost [203] could potentially be introduced to the CF as well. Apart from air pollution from engine, even brake dust or tyre wear pollution and their influence on the biosphere could be also defined and quantified.

Offering an example, the mass flux of rubber (R) from tyre wear deposited in the environment (E), could be performed with an equation:

$$\dot{m}_{R \rightarrow E} = \bar{q} \bar{S}_R \frac{m_0}{m_{T_R}}, \quad (82)$$

where the \bar{q} is the average vehicle throughput, \bar{S}_R average length of life of a rubber tire, m_0 the initial mass and m_{T_R} at the end of life. The unit of $\dot{m}_{R \rightarrow E}$ is $[kg/(km \cdot y)]$. Gathering such statistical data, in synchronism with research on e.g. health effect of heavy metal pollution in urban dust from tyre and brakes [204] [205], could enable deeper, real time awareness on how transport really affects the citizen’s health [205].

7.2 Agent honesty

As the proposed framework features a payment system, there exists an incentive to tinker with the device, or simply strategically manipulate one's cost function in order to game the system. Thus to understand the system's sensitivity to the agent's honesty, thus section examines the possibility of exploitation of the system rules.

7.2.1 Platooning

Given that agents request a payment for their change in speed, the system could be cheated by increasing the local CF's (31) slope, while maintaining the same minimizer. It is achievable if some parameters of the cost function are swayed by a constant cheating multiplier \mathcal{W} , arriving at a local, platooning cost function with the same minimum, but with steeper slopes, as presented in Fig. 43, requiring much higher payment. The dynamics of the negotiation, however, would result in a speed much closer to the cheater's V^* , negating a fraction of the gain.

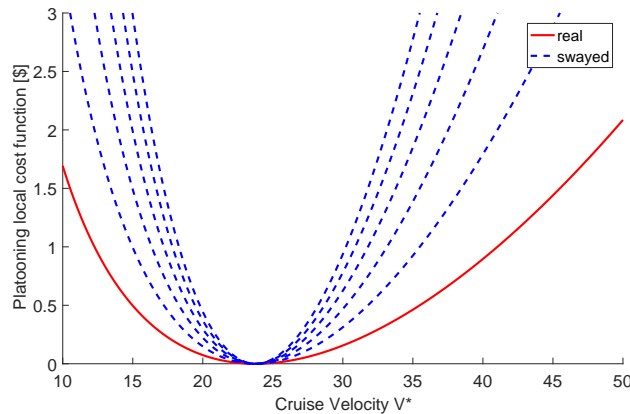


Figure 43: The truthful local platooning cost function compared with its cheated versions, with a swaying factor $\mathcal{W} = \{2, 3, 4, 5, 6\}$.

This vulnerability can be mitigated, however, by sharing of the cruise cost $C_{C,n}$ as a control value. Just as today there are ANPR equipped law compliance systems i.e. enforcing vehicle road tax [206], in future, there could be mobile connectivity stations, which check on the vehicles within their V2V range.

7.2.2 Overtake and intersection

The overtake and intersection are dynamic events. The cost of such manoeuvre is sensitive to vehicle mass, which is variable and cannot be inferred from observation. Therefore agents may be inclined to overvalue it as the payment is proportional to the agent's mass. However, since vehicles' behaviour can be monitored over time, a cheater would exhibit inconsistency in strategy selection between uninterrupted cruise and negotiation. A networked approach to cheating detection could thus be developed and incorporated into a reputation mechanism, drawing on examples as [57]. The following conjecture is a preliminary derivation of the penalty to be incurred by the system to a dishonest user. It assumes that there is only a chance of detection. It is a starting point for definition of rule design in a multi-agent system.

Conjecture: We may define the benefit of a single cheated interaction as

$$\mathcal{B}^\dagger = C^\dagger - C^b \quad (83)$$

where b corresponds to a cost of an honest, cooperative agreement, while \dagger to a cheated agreement. Then, let the penalty for cheating be an exclusion from the n next cooperative manoeuvres, assuming

$$\mathcal{P} = n (C^b - C^*). \quad (84)$$

With $P_{\mathcal{D}}$ as probability of discovery, we define the risk [46] associated with cheating as

$$\mathcal{R} = P_{\mathcal{D}} \mathcal{P}. \quad (85)$$

Given that the agent is rational [207], meaning always maximizes their payoff, an agent is honest if, and only if the risk outweighs the benefit $\mathcal{B}^\dagger < \mathcal{R}$. In order to evaluate the level of enforcement of honesty, associated with a minimal chance of detection $P_{\mathcal{D}}$, which guarantees honesty

$$P_{\mathcal{D}} = \frac{C^\dagger - C^b}{n(C^b - C^*)} = \frac{\mathcal{B}^\dagger}{\mathcal{P}}. \quad (86)$$

The n parameter can be used to control the severity for the penalty, to deter agents from cheating despite $P_{\mathcal{D}}$ being low, to minimise the cost of the detection system.

7.3 System's performance and traffic implementation

So far in this Thesis, it has been assumed that all cars are capable of communication, as stated in Section 3.5. Because the cooperation can occur only when both interacting agents can communicate, it is hypothesised that the system's efficiency depends on its traffic penetration. The probability of cooperative interaction thus is a function of probability for either agent. In similar fashion as random interactions in a bimolecular chemical reaction, referred to as a second order reaction kinetics [208]. Defining P' as the macroscopic system's penetration, or microscopic probability of a given agent being equipped with the communication capability, we thus expect the Price of Anarchy to be second order proportional to P'

$$PoA(P') \propto P'^2. \quad (87)$$

This would suggest that the real-world implementation of efficient, autonomous traffic consists of two phases. At a low ratio of CAVs there is a very little marginal gain for each new participant. However, as the number of cooperative agents eclipses some critical mass and the chance of success in traffic increases, and the marginal gain of $\Delta PoA(P')$ increases. Then each next user feels a growing incentive to join the trend, accelerating the convergence to complete traffic saturation.

7.3.1 Simulation and results

Monte Carlo simulation is performed using MATLAB, with $n = 10^4$ samples for each data point. The input data is randomised as in Tab. 17, as uniform distributions. Note that Trucks' C_T is increased so that V^* ranges are same for both types. To model heterogeneity of traffic, vehicle type is chosen depending on the macroscopic fraction of trucks P_T . The traffic' penetration with the system is defined as a fraction of vehicles $P_S \equiv P'$, which are capable to negotiate. Defining a random value from the interval $R = [0, 1]$, cooperative solution is chosen if $\{P_S < R \wedge P_S < R\}$ and noncooperative otherwise. The system's sensitivity to the fraction of vehicles equipped with compatible communication technology is then examined. If at least one of the interacting agents were not equipped, the negotiation algorithm is suppressed and the noncooperative solution is executed.

Results are presented in Fig. 44. To evaluate the hypothesis from (87) the second order dynamics curve is added. The root-mean-square (RMS) error of the fit to the parabola is 0.082, confirming the hypothesis. The result of the simulation, projected on the PoA curve is visualised in Fig. 45. This indicates, that until the number of users reaches 30 %, the marginal gain to the user is minimal. This poses an obstacle to the implementation of the technology proposed in CAVs, and calls for a method of retrofitting it to the existing vehicles. This is addressed in the following subchapter.

Table 17: The randomized input parameter ranges for the Monte Carlo intersection simulation for the traffic penetration estimation.

Veh. type	unit	min	max
C_T (Car)	\$	20	40
C_T (Truck)	\$	133	267
phasing distance S_{II}	km	100	250

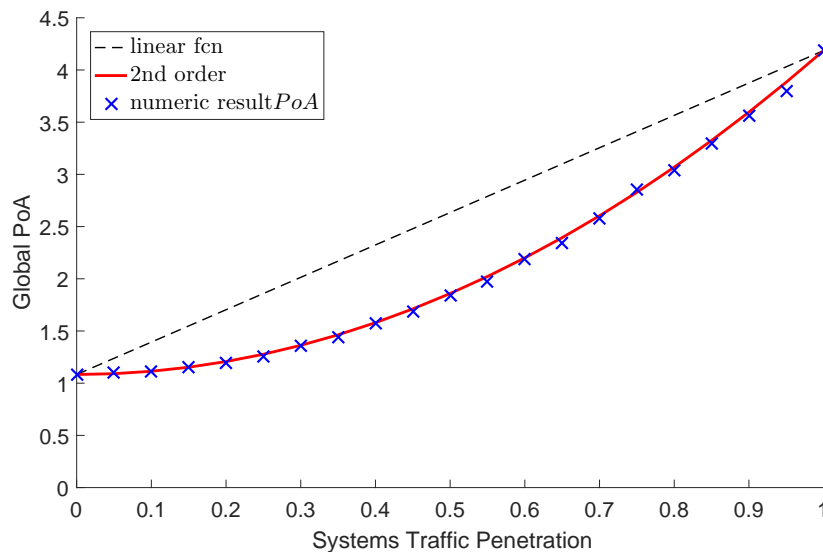


Figure 44: PoA as a function of system's traffic penetration. Linear and 2^{nd} order functions are added for reference. The RMS error of the fit to the parabola is 0.082, confirming the hypothesis.

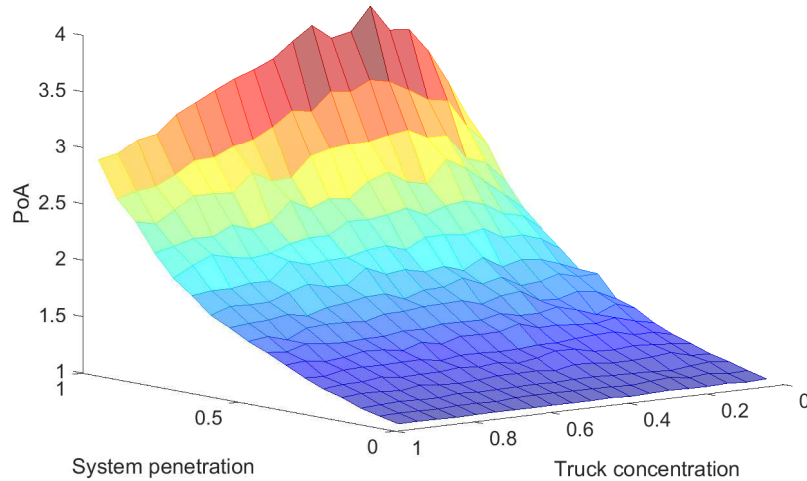


Figure 45: A surface of PoA as a function of System’s penetration and truck concentration. While the pattern of the main result of the efficiency correlated with heterogeneity is visible, the PoA marginal gain at low traffic penetration is minimal, posing a challenge to the connected traffic implementation.

7.4 Human in the loop

The above calculation casts a shadow on the feasibility of the implementation, as it indicates that a certain critical mass of users is required to provide sufficient marginal gain to incentivise growth. This obstacle, however, could be mitigated if CAVs were able to communicate and cooperate with Human Driven Vehicles (HDV).

Until now in this thesis, much like in chess, the key assumption is that all information was known. However, in a real application, some data in the system may be subject to noise, sensor false positives or even some forms of deception [60]. In our case it may also be human error, such as delayed response or the user ‘changing mind’ in the process [89]. A scenario where a Human-Driven Vehicle (HDV) encounters an Autonomous Vehicle (AV) on an intersection is here considered. Suppose that the HDV is equipped with hardware for communication and negotiation, along with a Human-Machine Interface (HMI), which feeds the system’s state data to the driver, and which is trusted by the driver [209]. Then provision of false information, either as a malfunction or malevolence, could be mitigated with belief-based methods of evaluating information with on-board sensors [210]. This was mentioned also in ch. 7.2.2. It is worth noting that similar

GT approach to interaction between mistrustful agents is implemented as an 'AI' for a strategic computer game *StarCraft* [211].

The main challenge to implementation of such traffic cooperation framework is the safety and security [56]. Just as in the case of guided weaponry, the precision can be ensured a function assigning risk to the distance between cars with some distance $\underline{x}_{A,B}$, guaranteeing separation, corresponding to required level of safety. The smallest achievable distance CAVs can get to each other is the key factor limiting the throughput of roads, according to the three-phase traffic model, as minimising the gap triggers safety deceleration as visualised in Fig. 46 [68]. Since the diminishing gap reduces the safety of passengers, the road throughput and user safety are conflicting objectives. Today it is moderated in a distributed, cultural manner by driver's perception of safety. Therefore in the years to come when ACC controlled platoons will become mainstream, road congestion and user safety shall become a decision factors for the electronics engineers, as well as in the economic calculation the trade-off between economic cost of delays and insurance and health expenditure [212].

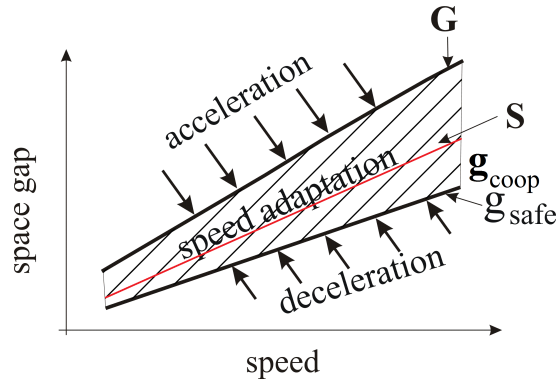


Figure 46: The relationship between the cruise speed and the gap between vehicles, according to the three-phase traffic model. It is the key factor limiting the road throughput [68].

The implementation of such a concept in road traffic arrangement also requires driver's readiness and capability to cooperate with the recommendation system on a more intuitive basis than in the case of aircraft. The self-enforcement only guarantees compliance of a rational agent, whilst human decision-makers deviate from rationality [213]. Secondly, a control algorithm for the correction of the human-introduced error is required.

7.4.1 Problem formulation

With the prerequisites defined above, the proposed controller consists of the negotiation algorithm, being the core novelty of this thesis, along with a simplistic error correction mechanism, sensor and trust evaluator. The agents play a chicken game [14] on an intersection, gathering evidence from sensor data and comparing it with pre-negotiated reference to evaluate the *trust* and *safety*. Then we suppose that on the HDV's side human error is introduced, which the CAV's controller must account for [89]. The vehicle interaction is modelled as continuous time, what is realized by randomisation of the order of action at every timestep.

7.4.1.1 Error correction

The execution error of the other vehicle is estimated by comparing the adversary's speed profile agreed on during negotiation $v_{ref}^{-n}(t)$ and the real, measured speed $v_{real}^{-n}(t)$, finding the execution error $\Delta v_{ref,e}^{-n}(t)$ defined as

$$\Delta v_{ref,e}^{-n}(t) = v_{ref}^{-n}(t) - v_{real}^{-n}(t), \quad (88)$$

where the superscript $-n$ refers to the 'other' agent.

Conjecture: for given t the $v_{real}^{-n}(t)$ is found using maximum likelihood estimation as $v_{real}^{-n}(t) = \hat{V}_{-n}$, based on the vector of measurements \vec{V}_k from k sensors [214], defined as

$$\hat{V}_{-n} = \arg \max_{v \in R^+} \mathcal{L}(V|\vec{V}_k(t)). \quad (89)$$

The notion $\mathcal{L}(V|\vec{V}_i)$ is understood as the V which has the highest chance of being the peak of the distribution of measurements $V_k(t)$. Further development of the quality of the solution here, may involve Bayesian rare events analysis [215].

In this application, the above error estimation is applied to estimate a control input to correct the error introduced by a human. The corrector $\Gamma_n(t)$ is found

$$\Gamma_n(t) = \alpha \Delta v_{ref,e}^{-n}(t) + \beta \int_0^t \Delta v_{ref,e}^{-n}(t) dt, \quad (90)$$

where α and β are weights used to regulate the response characteristics, which could be interpreted as *timidness*.

Then, the vehicle's control variable, that is acceleration, is found as

$$a_n(t) = a_{max}^n \cdot erf\left(v_{ref}^n(t) - v_n(t) + \Gamma_n(t)\right), \quad (91)$$

resembling a modified PI controller. *erf* denotes an error function. Note that the correction coefficient refers to the data defining the other vehicle's observation.

7.4.1.2 Trust model proposition

The decision whether to engage in a cooperative behaviour is found from a model-based decision agent [216]. It decides whether the cooperation is worth the risk, based on the observation of the adversary's speed.

The history of past interactions and given agent's execution error is defined as mean correction cost C_C , considering the cost function in (24) it is found as

$$C_{i,n}^C = \frac{1}{\mathcal{K}_n} \sum_{k=1}^{\mathcal{K}_n} J\left(v, (\Gamma_{n,k}(t))\right). \quad (92)$$

\mathcal{K} is the history of measurements of the observed n -th agent. Proposed parameter may either be local to an agent, or can be shared within a population, arriving a problem of a reputation studied in [217]. Then, the agent chooses to engage in the cooperative manoeuvre if the expected value outweighs correction error costs

$$C^b - C^* \geq C_{i,n}^C. \quad (93)$$

Should the required error be too large to correct, the safety game module considers the trajectory and adversary's trust, to decide to continue the cooperative manoeuvre or to defect and follow a safe trajectory. It is based on the chicken-game [14] [218] type decision.

The decision to continue cooperation is selected as long as the benefits of cooperation outweigh the risk of incurring cost of departure to safety $P_E \cdot C_{Saf}$. The C_{EM} is defined as the cost of emergency, that is the cost of transition from cooperative to the safe speed profile. The probability of failed cooperation P_E is found as

$$P_E = exp\left(-q_T C_{i,n}^C\right), \quad (94)$$

where q_T is some arbitrary input of *trust* gain used to moderate the timidness of an agent. Then decision to cooperate is taken if

$$C^b - C^* > P_E C_{Saf}. \quad (95)$$

7.4.2 Controller architecture

With all the components described above, they are integrated into the agent architecture [166], presented in Fig. 47. The negotiation algorithm feeds reference values to the controller, adversary's behaviour is measured and the correction is found. Finally, the correction is quantified, to track adversary's trustworthiness and the manoeuvre is executed through the vehicle model, moving on the execution of the other vehicle's

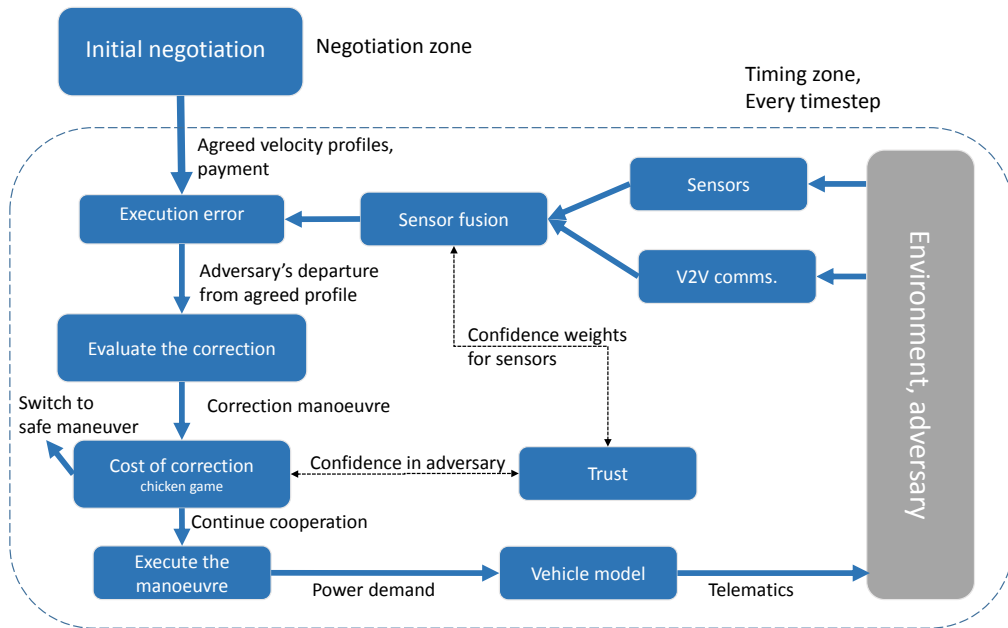


Figure 47: The control agent architecture conceptualised, intended for the interaction with human drivers. The AV corrects for adversary's error and updates its belief on their trustworthiness.

7.4.3 Simulation results

In order to verify the behaviour of the human-in-the-loop framework proposed, an intersection scenario has been simulated. The vehicles are modelled, as defined in chapter 3.1. Since the interaction between agents occurs over time, the problem is formulated as a differential game. Continuous time effect is accounted for by randomizing the order of agents [219] as they increment their time in steps of $t_{step} = 0.01$ [s].

7.4.3.1 Step response

At $t = 0$ the reference of the human driven OV is incremented, the EV is observing and following the OV. The resulting speed plots are presented in Fig. 48.

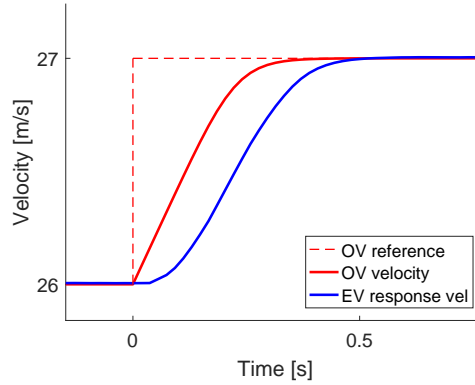


Figure 48: Velocity profile of the human-driven vehicle (OV) as a response to a step reference. The autonomous (EV) vehicle, in this example follows the OV as its reference.

7.4.3.2 Intersection Scenario

As two vehicles meet, in an intersections scenario, they agree on speed profiles and begin to execute the speed profile at $t = 0$ [s]. The OV is to yield but features an execution error of reference undershoot. The EV ideally should not deviate from its V_{EV}^* , but corrects for the OV's error, to ensure sufficient separation when arriving at the conflict zone. In the middle of the manoeuvre a disturbance in the OV's speed is further introduced. Then, as the cost of correction required is too large, according to (95), the trust between agents is broken and the EV accelerates to a safe trajectory, ensuring sufficient separation at the CZ. The results are presented in Fig. 49. Observe that the EV uses the difference between OV's reference speed, and the real speed.

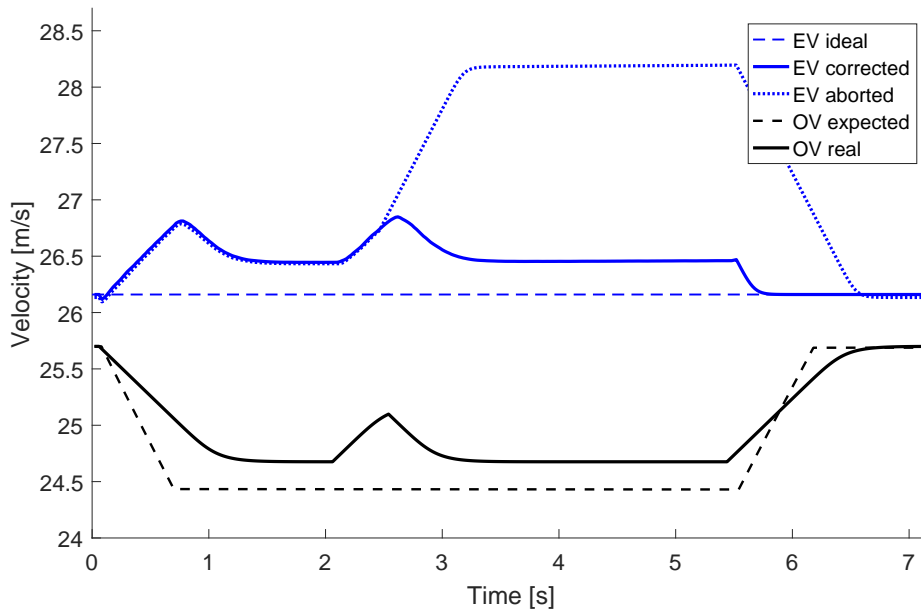


Figure 49: Speed profile of the human driven vehicle (OV) following a step reference and autonomous (EV) vehicle following with OV as reference. The EV accounts for the adversary’s undershoot error, and as the error achieves a trust breaking threshold it aborts the cooperative manoeuvre escaping to a safe speed profile.

7.5 Discussion

This Chapter featured analyses to provide an additional view on the traffic framework proposed in this thesis, to allow further understanding of its dynamics. The concept of cost function modularity has been tested by adding a new component to the cost function. Then the sensitivity of the framework to its market share is considered, yielding a pessimistic result of minor marginal gain at an early stage of potential implementation, prompting a scenario where retrofitted connectivity is to feature cooperation between connected HDVs and CAVs.

The notion of putting the humans into the loop requires an assumption that agents act as rational decision-makers. While this would greatly soften the barrier to implementation of information-abundant, connected traffic, the behavioural norm change would require powerful regulatory action to foster or enforce it.

7.5.1 Human vs Machine decisionmakers

The above Chapters 3 to 5 present an automated framework to find optimal solutions to traffic conflicts. It is based entirely on mathematically described decision rules, whereby any output is calculated deterministically. It stands in opposition to today's Machine Learning approach, where decisions may contain a random or arbitrarily assigned component. This poses a challenge to autonomous vehicle ethics, as randomness needs to be ruled out when the user's and public health are at stake [212].

Additionally, as yet, human capability to intuitively find unconventional, efficient tactics in complex systems is unchallenged by machine-based models [140]. Whether it is a political party, a businessman, or a car driver, human decision-agents provide examples of surprising tactics, which seem counter-productive for a simple convex optimiser. A great example of this is the 2019 Italian Formula 1 Grand Prix qualifying session. There, as result of safety incidents, a group of drivers was released together towards the end of the last session to attempt last time attack. The high speed nature of the track allows drivers to gain time by closely following the one ahead. The first one to speed up, thus, would be the one to lose the slipstream advantage. This prompted an emergence of a 'bubble', where all drivers would slow down to otherwise unsafely low speeds. Furthermore, coincidentally three drivers at the front were happy with the current standing and slowed down even further, so those behind would run out out of time. This coalition of drivers from competing teams emerged spontaneously, with no communication. The situation occurred as a response to the specific, unprecedented circumstance. This gamesmanship has earned the coalition some positions on the race grid, at a cost of a mere reprimand [220]. A counter-intuitive coalition has emerged as a tactical response to the specific, unique circumstance. The distributed, non-cooperative system was optimally solved by humans forming a coalition at a time substantially shorter than a centralized optimisation would take.

The above also exemplifies a specific case where the optimal solution is to do the very opposite of the current equilibrium strategy, with a transient of disorganisation following. Returning to the public road scenarios, where currently the traffic code regulations apply, an optimal solution in half of the cases may be very opposite of today's and in violation of the traffic code. This indicates that the optimal traffic paradigm is as much a challenge for engineering as it is for the public liability regulator. A phenomenon which requires multi- analysis in the research to come.

7.5.2 Communication and enforcement

The main challenge of both the *bi-matrix form* and *decision tree form* game formulations is the inability to capture the contextual information, which is the main reason for the Aumann's

Conjecture still being unproven [221] [222] [223].

The Aumann's Conjecture, as described in [139] and in Section 2.5.3, observes that pre-play communication of virtually zero computational effort controls the equilibrium selection, speeding up convergence or even enabling arrival at PE. In our connected car an example message is *OV will choose V_{OV}^* or EV will yield*. However, the automated V2X communication brings computational power to convey more complex information, the complete cost function. The negotiation algorithm then finds the solution, which is closest to Pareto Optimal but still self-enforced, thanks to the calculated side-payment. The computational complexity, however, means that the pre-play communication is not free [224] and might need to be accounted for in the game design. The Monte Carlo simulations from ch. 5.2.1 also suggests that heterogeneity of agents maximises the performance of the negotiation algorithm. This suggests a correlation between cooperative-competitive solution concept and Aumann's Conjecture, calling for further study of the matter at hand.

As an example, hypergame theory has been offered to formulate conflicts, where agents can have variable or uncertain number of actions [225]. In similar way, Aumann's Conjecture observed that a-priori information affects the equilibrium to which the game converges [139], without the change of the pay-offs. Just as hypergame formulation captures the existence of strategies unknown to some agents, in similar fashion, a new game formulation is needed, whereby the abundance of information and its entropy [226] are captured into the mathematical formulation of the conflict.

8 Conclusions

This thesis offers a framework for cooperative traffic de-conflicting, with implementation as a strategic Connected Autonomous Vehicle controller. It comes as a multi-objective cost function, where each agent defines their objective and a negotiation algorithm optimally resolves conflicts between them. While the literature review returns a number of traffic management schemes already, on a strategic control level they all assume that all agents are homogeneous. The key novelty of this work is the cost function, which features user-defined parameter, fostering heterogeneity among agents. It is the source of both for the optimality and self-enforcement of the framework. It is then followed by the negotiation algorithm facilitating optimal cooperation in all time/energy relevant scenarios. That is cruising, intersections and in-line conflicts: platooning and overtake. The results indicate vast improvement in the efficiency of traffic, albeit the measured result is partially subjective. The study of the negotiations among heterogeneous agents also indicates that the *PoA* grows as the mass of the initiating agent increases, as the information sharing allows the smaller agent to move aside at minimal costs. This negotiation algorithm is thus biased towards heavy goods vehicles.

This work's main novelty is in the game theoretic method of negotiation, such that agents may trust that adversaries join to cooperate, just as today drivers trust each other. The algorithm formulates the road traffic conflicts with respect to agent's individual cost functions, employing game theory to enable definition of a Pareto optimal solutions. While the system can be distributed in low density scenarios, up to 6 agents on a single PC core, in high density traffic, not only a centralized computation unit could be located next to busy intersections, but there is a number of heuristics, which may bring down complexity by several orders of magnitude. The reduction of energy consumption and pollution would offset the cost of the road-side infrastructure. With all the potential benefits, it is impossible to engage all stakeholders of road traffic: drivers, operator, OEMs and insurance to fairly share data and benefits of its use using current business models.

In addition, the platooning occurs on a straight road. The three-phase traffic theory suggests which existence of a breaking point in the traffic level build-up, as outlined in Section 2.3.1. The control of platoon [70] thus may affect when the will the traffic stream deteriorate into a traffic jam. We observe, that the distance between vehicles becomes a trade-off when minimising between the Value of Travel Time savings [143] and the collision rate. The flammability of battery-electric cars has been identified as a health, or life risk [227]. A responsible technology

supplier should notify the regulator body, much like in nuclear facilities, the risk associated with the use of an autonomous vehicle, to protect the citizens and passengers' health, as referred to in ch, 2.5.4. Given electrification as a factor [15], and the changing automotive industry, we should begin discussing the correlation between society's health health, when following the Key Performance Indicators of the Transport Sector. Summarizing, as the number of vehicles with Adaptive Cruise Controllers grows, being de-facto autonomous vehicles from energy consumption perspective, cooperative platooning may become a key congestion mitigation method. In addition, the research on mixed human-machine systems indicates that even a small fraction of coordinated robotic agents in the network may affect the populations wellness factor significantly [157].

As the framework's mathematical formulation has been put forward, it has been theoretically tested for platooning and overtake conflicts. A Monte Carlo simulation has also been employed to test the most complex, intersection scenario. The framework simulation results indicate up to threefold increase in optimality in relation to noncooperative solution occurring when agent heterogeneity is maximal. It is noteworthy, however, that while part of the gain is in the subjectively defined cost of time, it still offers substantial gain in energy efficiency.

Then, platooning, the easiest to implement, has served as a scenario for a Hardware-in-the-Loop validation, achieving a successful hardware demonstrator. The multi-agent conflicts tend to scale badly, so the n-agent complexity analysis follows, concluding the computational effort growth following the complete graph links sequence.

Finally, some aspects of the framework's possible challenges to deployment are touched. Namely, the modularity of the cost function is considered, by adding i.e. a linear battery wear model. The system's sensitivity to traffic penetration with communication technology is confirmed to follow second order reaction dynamics, what results in a low initial gain in efficiency, offering little incentive to early adopters. This finding, however, neglects the multi-agent interactions. As a method of overcoming it, a retrofitting technology enabler has been attempted. The possibility of cooperation between human-driven and automated vehicles is tested, putting forward a chicken-game reputation/trust dynamics model. It is integrated into an intelligent architecture, whereby an agent controls their speed profile accounting for the human-like errors in the pre-negotiated strategy execution. If experimentally validated, the proposed approach may enable further development of connectivity-based cooperation paradigm for both human-driven and autonomous vehicles, which could speed up the CAV implementation significantly.

Departing from the tangible deliverables, as of the date of writing this thesis, there is no

debate on the safety regulation for CAVs. Interaction with the industry, while laconic due to IP reasons, hints a discrepancy between the language of engineers and that used by game theorists used to model conflicts. Thus, while the real implementation of the proposed algorithm might be distant in time, the most imminent objective of this thesis is to provide a terminological toolbox for automotive engineers, to conceptualise the problems encountered on the way to computerize the automobile interactions, with all the socio-engineering challenges it brings to the table.

8.1 Further work recommendation

The core of the contribution is a complete negotiation algorithm with a focus on its simplicity, to promote understanding. In the process of development it was further tested by the additional analyses, ch. 6.3. While the results are promising, a significant amount of further work is required to complete the technology.

From the scientific research perspective, the results of the simulations, so far, prove that the pre-play communication improves the welfare of agents. However, the key assumption was the completeness of the information. As such, the following subjects to consider in further research.

- Elimination of the perfect Quality of Service and honesty assumptions in a laboratory environment, to evaluate the framework's resilience and the trust dynamics among agents.
- Development of the robust model for decision-making under uncertainty based on a Bayesian game formulation. It would feature a form of reputation algorithm as well as security countermeasures.
- Addressing the issue of the formal definition of conflict propagation, in order to arrive at robust mathematical definition of n -agent conflict resolution formulation.
- While GT offers models for e.g. incomplete information games, the contextual information not captured neither by bi-matrix, nor decision tree form games. The autonomous vehicles may serve as example to develop method to formally capture the contextual pre-play communication.
- As a measure of success of the above, the Aumann's Conjecture could finally be formally confirmed or disproved.

- To provide economic validation for the cost of opportunity concept embedded in the cost function, the relationship between national hourly productivity and average free cruise speed could be examined.

The alternative glance on the further work is to continue the engineering development of the traffic cooperation framework. Further work in this aspect would feature the following.

- Development of a complete Hardware-in-the-Loop testbed for the framework presented. It would feature multiple scaled vehicles on an 8-shaped track, where all considered scenarios would take place.
- The above would allow to experimentally study the non-convexity of the multi-agent conflict formulation, as to explore the possibility of complexity reduction in the multi-agent problem. Alternatively, a heuristic formulation is to be developed.
- Extension of the cost function. Apart from time and energy, the decision factors would feature user comfort, component wear, safety, etc. These factors, however, are subjective, making the engineering objectives hard to define.
- Setting up a Human-in-the-Loop experiment to evaluate the possibility of human-machine cooperation, establishing advanced human driver characteristics.
- Building a regulatory guidance framework, translating the nuclear regulatory framework or airspace safety framework to the specific application of safe engineering of autonomous vehicles.

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title

C++ code for HiL negotiation

The following code has been implemented on Raspberry Pi devices. Please observe it is not complex, highlighting only the key functions. The main body of the environment and other functions not mentioned below, such as display control are available upon request.

```
#include "header.h"

int main()
{
    rCC;
    vehicle *vehicle2 = (struct vehicle* )malloc(sizeof(struct vehicle));
    int rc;
    pthread_t threads [NUM_THREADS];

    /*      key_t    queue = ftok (FTOK_PATH_Q, FTOK_CHAR_Q);
    int id_queue    =      msgget (queue , IPC_CREAT |      0664);
    if (id_queue < 0){ perror ("msgget fallita ");      exit (1); }
    synchronize (id_queue);*/

    *vehicle2 = Init_communication_V1_V2 ();
    *vehicle2 = Init_communication_V2_V1 ();

    synchronize ((void*) vehicle2 );

    *vehicle2 = InitializeVeh2 ((void*) vehicle2 );
    sleep (5);

    rCC = EvalCruise ((void*) vehicle2 );
    sleep (5);
    vehicle2->V[0] = rCC.V_Target;
    vehicle2->CC[0] = rCC.CE;
    vehicle2->CC[1] = rCC.CT;

    cout << "Creazione thread 0" << endl;
    rc = pthread_create (&threads [0] ,NULL, MainProcess , (void*) vehicle2 );

    if (rc){ cout << "errore" << endl; exit (-1); }

    //msgctl (id_queue , IPC_RMID, 0);
```

```

for(int k = 0;          k < NUM_THREADS; k++)
{
pthread_join(threads[ k], NULL);
cout << "Vehicle" << k+2 << " has finished" << endl;
SaveOnCsv((void*) vehicle2);
}

```

```
pthread_exit(NULL);
```

```
void VehMdl( float dt, void *veh1, int time){
```

```

vehicle* vehicle1 = (struct vehicle*)veh1;
float a, wheelF, v_mean,sumF, v, x, dEn, Evthrust;

```

```

wheelF = (vehicle1->wheelFmax)*vehicle1->U[0];
sumF = VehForce((void*)vehicle1, wheelF, time);

```

```

a = 0;//sumF/vehicle1->VP1[0];
v = vehicle1->V[time] + a*dt;

```

```

v_mean = (0.5)*(vehicle1->V[time]);
x      =      vehicle1->X[time] + v_mean*dt;

```

```

dEn = wheelF*(x-vehicle1->X[time]) /vehicle1->VP1[5] ;
Evthrust = vehicle1->Ethrust[time] + dEn;

```

```

vehicle1->X[time+1] = x;
vehicle1->A[time+1] = a;
vehicle1->V[time+1] = v;
vehicle1->Ethrust[time+1] = dEn;
}

```

```
float VehForce( void *veh1, float wheelF, int time){
```

```

vehicle* vehicle1 = (struct vehicle*)veh1;

```

```

float rollF = vehicle1->V[time]*vehicle1->VP1[1];
float dragF = 0.5*vehicle1->VP1[2]*1.18*vehicle1->VP1[3]*(vehicle1->V[time]*vehic
float sumF = wheelF - rollF - dragF;

```

```

return sumF;
}

```

```

V_C EvalPlat(void *veh1, void *veh2, int dist, float V2, int time){

struct J_arrV rJ_arrV_OV, rJ_arrV_EV; float V_Star_EV, V_Star_OV;
vehicle* vehicle1 = (struct vehicle*)veh1; vehicle* vehicle2 = (struct vehicle*)veh2;
int V_OV = V2-2; int V_EV = vehicle1->V[time] + 2;

rJ_arrV_EV = PrefVec((void*)vehicle1, V_OV, V_EV);
rJ_arrV_OV = PrefVec((void*)vehicle2, V_OV, V_EV);

V_Star_EV = vStar((V_EV - V_OV)/0.1+1, rJ_arrV_EV);
V_Star_OV = vStar((V_EV - V_OV)/0.1+1, rJ_arrV_OV);

cout << endl;
cout << "-----PLATOONING-----" << endl;
cout << "V_Star_EV: " << V_Star_EV; cout << "V_Star_OV: " << V_Star_OV
<< endl;
cout << endl;

return Plattooning((void*)vehicle1, (void*)vehicle2, V_Star_EV, V_Star_OV);
}

```

```

V_C EvalOvertake(void *veh1, void *veh2, int dist, float V2, int time){

struct J_arrV rJ_arrV_OV, rJ_arrV_EV;
vehicle* vehicle1 = (struct vehicle*)veh1;
vehicle* vehicle2 = (struct vehicle*)veh2;
float V_Star_EV, V_Star_OV, Smaxdyn, deltaV_ovt, feasibility;
int i, b, row, column, lenght = 0;
float M_E_EV, M_E_OV, M_T_EV, M_T_OV, CO_sum, CO;
Delta_En_T rDelta_En_T;
struct V_C rV_C;
int V_OV = V2-2; int V_EV = vehicle1->V[time] + 2;

rJ_arrV_EV = PrefVec((void*)vehicle1, V_OV, V_EV);
rJ_arrV_OV = PrefVec((void*)vehicle2, V_OV, V_EV);

V_Star_EV = vStar((V_EV-V_OV)/0.1+1, rJ_arrV_EV);
V_Star_OV = vStar((V_EV-V_OV)/0.1+1, rJ_arrV_OV);

cout << endl;

```

```

cout << "-----OVERTAKING-----" << endl;
cout << "V_Star_EV: " << V_Star_EV << "   V_Star_OV: " << V_Star_OV << endl;
cout << endl;

column = (int) ((1.4*V_Star_EV) - V_Star_EV )/0.2 + 1;
row = (int)(V_Star_OV - (0.2*V_Star_OV) )/0.2+1;

float matV_EV[row][column], matV_OV[row][column], CO_pareto [4][column] ;
float M_EV[row][column], M_OV[row][column], M_sum [row][column];

//----Meshgrid-----//
for(int idy = 0; idy < row; idy++){
matV_EV[idy][0] = V_Star_EV;
for(int idx = 1; idx < column; idx++){matV_EV[idy][idx] = matV_EV[idy][idx-1] + 0.2;
}

for(int idx = 0; idx < column ; idx++){
matV_OV[0][idx] = 0.2*V_Star_OV;
for(int idy = 1; idy < row; idy++){matV_OV[idy][idx] = matV_OV[idy-1][idx] + 0.2;
}

//----Init M_EV, M_OV, CO_pareto----//
for(int idy = 0; idy < row; idy++){for(int idx = 0; idx < column; idx++){M_EV[idy][idx] = 0;
for(int idy = 0; idy < row; idy++){for(int idx = 0; idx < column; idx++){M_OV[idy][idx] = 0;
for(int idy = 0; idy < 4; idy++){for(int idx = 0; idx < column; idx++){CO_pareto[idy][idx] = 0;

//----Feasibility ----//
for(int k = 0; k < row; k++){
for(int l = 0; l < column; l++){

Smaxdyn = (vehicle1->EnvP[1] * (1-(vehicle1->EnvP[4] / matV_EV[k][l] ) ));
deltaV_ovt = 2*vehicle1->EnvP[0]* (matV_EV[1][l]/Smxodyn);
feasibility = (matV_EV[1][l] - matV_OV[k][l]) - deltaV_ovt;

if(feasibility > 0){
length = l;
rDelta_En_T = single_ovt((void*)vehicle1 , (void*)vehicle2 , matV_EV[1][l] , matV_OV[k][l]);

M_E_EV = vehicle1->Cost[0] * rDelta_En_T.deltaE_EV;      M_T_EV = vehicle1->Cost[0] * rDelta_En_T.deltaE_EV;
M_E_OV = vehicle2->Cost[0] * rDelta_En_T.deltaE_OV;      M_T_OV = vehicle2->Cost[0] * rDelta_En_T.deltaE_OV;

M_EV[k][l] = M_E_EV + M_T_EV;
M_OV[k][l] = M_E_OV + M_T_OV;

CO_pareto[0][l] = M_EV[k][l]; CO_pareto[1][l] = M_OV[k][l];
CO_pareto[2][l] = matV_EV[1][l] ; CO_pareto[3][l] = matV_OV[k][l] ;

break ;
}
else{ M_EV[k][l] = NAN;      M_OV[k][l] = NAN;}
}
}
}
}

```

```

}

//----M_sum, CO_pareto----//
for(int idy = 0; idy < row ; idy++){for(int idx = 0;   idx < lenght
; idx++){M_sum [idy][idx] = M_EV[idy][idx] + M_OV[idy][idx];}}
for(int y = 0;   y < 4 ;           y++){for(int x = 0;   x < lenght -1;
x++){CO_pareto[y][x] = CO_pareto[y][x+1];}}

vector<float> CO_par_sum (lenght-1), M_CO_sum(row*lenght);
int mm, nn, idz =0;

//----CO_sum----//
for(int j= 0;           j < lenght -1;   j++){CO_par_sum[j] = CO_pareto[1][j] +
vector<float>::iterator result = min_element(begin(CO_par_sum), end(CO_par_sum))
int p = distance(begin(CO_par_sum), result);
CO_sum = (CO_par_sum[p]);

CO = CO_pareto[1][p];

//----M_CO_sum----//
for(int idx = 0; idx < lenght; idx++){for(int idy = 0; idy < row; idy++){
M_CO_sum[idz]= M_sum[idy][idx] - CO_sum;
idz = idz+1;}
}
vector<float>::iterator r = min_element(begin(M_CO_sum), end(M_CO_sum));
int ii = distance(begin(M_CO_sum), r);

mm = ii/lenght;
nn = ii%lenght;

rV_C.payment = 100*CO;
rV_C.velocityEV = matV_EV[1][nn]; rV_C.velocityOV = matV_OV[mm][1];

cout << "V_Overtacking_EV      = " << rV_C.velocityEV<< endl;
cout << "V_Overtacking_OV      = " << rV_C.velocityOV << endl;
cout << endl;
cout << "Payment per all distance = " << 100*CO << endl;

return rV_C;
}

```

8.1.1 Algorithm's structure

and has the following structure.

Algorithm 8.1: (*lower, upper*)

procedure CELSIUSTOFAHRENHEIT(*c*)

$f \leftarrow 9c/5 + 32$

return (*f*)

main

$x \leftarrow lower$

while $x \leq upper$

do $\left\{ \begin{array}{l} \mathbf{output} (x, \text{CELSIUSTOFAHRENHEIT}(x)) \\ x \leftarrow x + 1 \end{array} \right.$