Instant dynamic response measurements for crack monitoring in metallic beams

Behzad A. Zai¹. Muhammad A. Khan². Sohaib Z. Khan¹,³. Kamran A. Khan⁴

¹Dept. of Engineering Sciences, National University of Sciences and Technology, Karachi, Pakistan
²School of Aerospace, Transport and Manufacturing, Cranfield University, Cranfield, United Kingdom
³Dept. of Mechanical Engineering, Islamic University of Madinah, Madinah, Saudi Arabia
⁴Dept. of Aerospace Engineering, Khalifa University of Science, Technology and Research (KUSTAR), Abu Dhabi, UAE

Corresponding author:
Muhammad A. Khan, Cranfield University, Cranfield, United Kingdom.

Email: muhammad.a.khan@cranfield.ac.uk

Abstract
This paper investigates the interdependencies of a cantilever beam modal behavior, its dynamic response and crack growth. A methodology is proposed which can predict the crack growth in a metallic beam by using its dynamic response only. Analytical and numerical relationships are formulated in between the fundamental mode and the crack growth by using the existing literature and finite element analysis (FEA) software respectively. Relationship in between the dynamic response and modal behavior is formulated empirically. All three relationships are further used to predict crack growth and propagation. The load conditions are considered same in all the experiments for model development as well as for the model validation. Predicted crack growth is
compared with the visual observations. The overall error is within acceptable limits in all comparisons. The obtained results demonstrate the possibility to diagnose the crack growth in metallic beams at any instant within the operational conditions and environment.

**Keywords**
Dynamic response, crack growth, catastrophic failure, modal behavior, mathematical models

**Nomenclature**

$A = \text{Cross section of the beam}$

$a = \text{RMS value of acceleration}$

$B = \text{Width of the beam}$

$E = \text{Modulus of Elasticity}$

$F = \text{Corrective function}$

$H = \text{Depth of the beam}$

$L = \text{Length of the beam}$

$P, Q, R \text{ and } S = \text{Polynomial coefficients}$

$t_c = \text{Crack depth}$

$u = \text{total strain energy}$

$\Delta u = \text{change in strain energy}$

$x = \text{Longitudinal location of the crack on the beam from the clamped end}$

$\alpha = \text{Dimensionless depth of the transverse crack}$

$\beta l = \text{Factor of boundary conditions}$
\( \rho = \text{Density of the beam material} \)

\( \omega_n = \text{Natural frequency} \)

\( \omega_{nc} = \text{Natural frequency of a cracked beam} \)

\( \Delta \omega_{nc} = \text{Difference between the natural frequencies of normal and cracked beam} \)

1. Introduction

An adequate measure of the failure life of a material is a useful tool for many purposes, like timely replacement, liability, durability and endurance of the product. Estimation of the material fatigue and monitoring is always challenging. Lack of analytical solutions makes it more difficult to predict the cause of fatigue crack [1]. The path and the growth of the crack are also complex to determine analytically. However, comprehensive research has been done to analyze the crack at micro structure level by destructive evaluation methodologies involving micro scale imaging and material surface preparation with chemicals [2]. But still research methods are required that can evaluate the behavior of crack mechanics without disassembling the machine component under normal operating conditions.

It can be safely estimated that the cracks may have effects on: the section modulus of the area around the crack, the deflection curve or energy content of the beam, mode values and mode shapes [3-15]. This can be verified from Timoshenko beam solution (as shown in Eq. 1) as provided in Rao to calculate the fundamental modal frequency of a cantilever beam [16].

\[
\omega_n = (\beta l)^2 \sqrt{EI/\rho AL^4}
\]

(1)
In past studies, numerical and experimental approaches were used to perform free vibration analysis of a beam with an open edge crack. The results demonstrated the changes in natural frequencies due to crack at various locations [17]. A systematic approach was used to study and analyze the crack in cantilever beams by using the inverse problem. The results discussed the impact of crack location and its size. The characteristic equation obtained from the vibration analysis of beam was also used to relate the beam stiffness and location of crack [18]. A similar relationship between the damage and the dynamic response was developed numerically in a commercial FEA software [19]. Most of the past research modeled the cracks in a beam with the help of a mass-less spring. The changes in the global characteristics of structure vibrations were analyzed by using different perturbation methods [20-21].

Thato et al. developed an approach for multiple cracked cantilever beams to obtain eigenvalues and eigenvectors using Finite Element Method (FEM) and MATLAB® that was validated by the experiments [22]. Researchers were also successful in finding out the severity (i.e., effective towards catastrophic failure) of the concerned crack. Chen used FEA to calculate the strain frequencies in order to predict the crack severity and location [22]. Agarwallaa [23] presented the changes in the dynamic behavior of the cantilever beam due to the presence of crack, i.e., the natural frequency, the stiffness of the beam and the dynamic stability. Douka [24] observed the non-linear behavior of the cantilever beam with a breathing crack. The experimental and simulated responses were examined and an instantaneous frequency was obtained. This frequency was used to determine the physical process of the breathing crack and the crack size. Results showed that the instantaneous frequency increased as the crack depth increased.

Crack initiation and propagation have also been studied with numerical and experimental approaches. The relation in between the relative length (i.e., crack distance from the fixed end with
respect to the length of the specimen) to the depth ratio (i.e., crack depth with respect to the thickness of the specimen) was discussed in detail [25]. Mostly linear methods are used to detect crack behavior which is a non-linear phenomenon [26].

Past research has emphasized on the dynamic responses of the structure. However, instant values of the dynamic response of a structure are still difficult to use in the analysis and prediction of the crack growth. This paper investigates the interdependencies of structure’s modal behavior, its dynamic response and crack growth in a quantitative manner. A methodology is proposed which can predict the crack growth in a metallic cantilever beam by using its dynamic response only. It is based on an analytical formulation, experimental data and numerical results. In validation, predicted crack growth is compared with the actual observations. The obtained results demonstrate the possibility to diagnose the crack growth in metallic beams within the operational conditions and environment.

The proposed methodology has used preliminary experiments to establish the base of mathematical relationships for crack growth prediction and hence it is necessary to describe the details of the test specimen and the experimental setup in the start of this paper. Later methodology is explained in detail. Results, discussion and conclusion are provided at the end.

2. Specimen preparation
Aluminum (Al 1050) is the selected material of the specimen. The thickness of the specimen is 3.3 mm and the length is 190 mm respectively. These two dimensions are kept constant throughout the experiments. The specimen is manufactured by CNC wire cut to maintain the dimensional accuracy for the different specimens. The dimensions of the specimen are shown in Fig-1. These dimensions are selected to get the maximum stress concentration at the fillet area of the specimen. A pre-defined crack is induced in all the specimens with a constant width of 0.2mm. The variation
of depths in crack ranges from 0.5mm to 1.5 mm with an increment of 0.25mm. Crack depths are selected on the criteria that it starts from a small value of 0.5mm which shows that the crack is in its initial phase. Its maximum value goes up to 1.5 mm and signifies the point where the crack has travelled up to half of the specimen thickness.

3. Experimental Setup
The experimental setup consists of a power amplifier (modal LA-200), a signal generator (TENLEE 9200), a data acquisition card (NI-9174) and a modal exciter (MS-100). The complete setup is shown in Fig-2. Time domain measurements are obtained on National Instrument® Signal Express as shown in Fig-3. The analysis modules of ‘Power Spectrum’ and ‘Amplitude and Levels’ were selected in the Signal Express. The former was used to identify the actual response frequency value, while the latter was used to obtain the actual amplitude of the response frequency.

The beam specimens with pre-selected crack depth were mounted on a modal exciter in a fixed-free condition as shown in Fig-2. An accelerometer was attached at the fixed end of the modal exciter to measure the dynamic response available in a frequency spectrum. As the exciter and the specimen were firmly attached so any measured response can provide a cumulative amplitude of the dynamic response of the whole system which was largely dominated by the specimen displacement at the free end due to the resonance phenomenon. This amplitude was continuously monitored on Signal Express ‘Power Spectrum Module’ with a data sampling rate of 25.4 kHz.

Each experiment was started with a fresh specimen. The signal generator was used to provide a constant peak to peak value of 5 volts in a sine waveform, which consequently provides a constant displacement loading of ± 5 mm to the specimen with the help of the power amplifier.
4. Methodology
4.1 Analytical formulation between the natural frequency and the crack depth
Structures demonstrate oscillatory (i.e. dynamic) response if an external excitation (i.e. force) interacts with them [16]. The characteristic of this response in terms of amplitude and frequency is dependent on the stiffness of the structure which is a direct measurement of the elastic properties of the structural material [29].

In this section, analytical modeling is presented which describes the effect of crack depth in terms of stiffness reduction of a model spring and its possible impact on the overall dynamic response of the structure under an external excitation. Beam is assumed to have Clamped-free boundary conditions and the crack is modeled as a mass less torsional spring as shown in Fig-4. The relationship between the beam natural frequencies and crack depth is obtained using the Rayleigh quotient and the governing equations are solved using Newton Raphson method [27-30].

Consider the beam as shown in Fig-4. The crack is modeled as mass less torsional spring; the stiffness of torsional spring $k_t$ is given by Ostachowicz et al. [29].

$$k_t = \frac{E Bh^2}{72\pi F(\alpha)}$$  \hspace{1cm} (2)

Where,

$$f(\alpha) = 0.638\alpha^2 - 1.035\alpha^3 + 3.720\alpha^4 - 5.177\alpha^5 + 7.553\alpha^6 - 7.332\alpha^7 + 2.491\alpha^8$$

and $\alpha = \frac{t_c}{H}$

Free bending vibration of a uniform beam is identified by following well known differential equation.

$$EI \frac{\partial^4 y}{\partial x^4} + \rho A \frac{\partial^2 y}{\partial t^2} = 0$$  \hspace{1cm} (3)
Applying boundary conditions to find mode shape and bending moment

\[ y(0) = 0 \quad y'|_{x=0} = 0 \quad y''|_{x=L} = 0 \]

\[ y(x) = y_0 \left[ 1 - \cos \left( \frac{\pi x}{2L} \right) \right] \] (4)

\[ y' = y_0 \left( \frac{\pi}{2L} \right) \sin \left( \frac{\pi x}{2L} \right) \] (5)

\[ y'' = y_0 \left( \frac{\pi}{2L} \right)^2 \cos \left( \frac{\pi x}{2L} \right) \] (6)

Curvature of the beam is the second derivative of the beam deflection; therefore, the bending moment can be derived from beam curvature and flexural rigidity (EI) as shown in Eq. (7).

\[ M = EI y'' \] (7)

The total strain energy can be derived from direct strain and strain energy due to bending as shown in Eq. 8-9.

\[ u = \frac{EI}{2} \int_0^L (y'')^2 \, dx \] (8)

\[ u = \frac{EI}{64 \pi^4} \left[ \frac{1}{L^3} \right] (y_0)^2 \] (9)

Reduction in strain energy and change in natural frequency due to crack can be found out using Eq. 10-11, presented by Majid et. Al [28]

\[ \Delta u = \frac{M^2}{2k_c} \] (10)

\[ \Delta \omega_{nc} = \frac{\Delta u}{2u} \omega_n \] (11)

Finally, the natural frequency of cracked beam can be calculated using Eq. 12

\[ \omega_{nc} = \omega_n - \Delta \omega_{nc} \] (12)
Eq. 12 can be used to analytically calculate the natural frequencies of cracked beam for various crack depths. These equations can be merged to be written in a generalized form.

\[
\omega_{nc} = \left[ 1 - \left\{ \frac{72\pi I F(\alpha)}{BH^2L} \left( \cos \left( \frac{\pi x}{2L} \right) \right)^2 \right\} \right] \omega_n
\]  

(13)

Eq. 13 is the proposed modified equation which summarizes all the variables used in the referred models. It can be noted that the natural frequency of cracked beam is independent of the initially assumed value of mode shape and material properties. However, it depends on the natural frequency of uncracked beam (or at the previous crack depth), crack location, geometrical parameters of the specimen and the crack.

4.2 Prediction of crack depth

4.2.1 Correlation between dynamic response and natural frequency
The proposed methodology consists an empirical relationship between the dynamic response and the natural frequency of the selected metallic beam. This relationship is established on the results of the preliminary experiments as described here.

In the start of each experiment, a fresh specimen with pre-defined crack depth was mounted on the test rig and the accelerometer was installed at the top of the modal exciter knob. The setup was capable of analyzing and recording the in-situ dynamic response of the specimen while vibrating at any frequency. An impact test was carried out to determine the fundamental frequency of a fresh specimen experimentally. The specimen was set to run at an operating frequency using the signal generator. Initially, this operating frequency was equal to the fundamental frequency obtained from the impact test. Simultaneously, the root mean square (RMS) value of the acceleration was also monitored. The drop in this value was used as a sign of change in the natural frequency of the specimen. The impact test was carried out again with a light wooden mallet to find the new modal
frequency. This procedure was repeated until the catastrophic failure of the specimen. We have defined this failure of the specimen as when the specimen can no longer show amplitude at the free end.

The RMS value of acceleration was observed against the frequency drop of the specimen until failure. A graph was plotted and a relationship was developed using curve fitting methods in MATLAB©.

The dynamic response (i.e. RMS value of acceleration) was recorded for each modal frequency of the specimen available during the test until its catastrophic failure. The results of the experiments were compiled and graphs were plotted for each of the specimens with a predefined crack values ranges between 0.5 and 1.5 mm. Seven identical specimens were tested in each of these ranges and results are shown in Figs-5–9.

Curve fitting was performed on the plotted data with a 3rd degree polynomial as shown with the bold line in the Figs-5-9. Using Matlab©, the mathematical equation of these plots were extracted. The general relationship for each of the plot was obtained as Eq. (14):

$$\omega_{nc} = Pa^3 + Qa^2 + Ra + S$$  \hspace{1cm} (14)

$P$, $Q$, $R$ and $S$ are the coefficients as determined by the Matlab© for predefined crack depth as shown in Table 1

4.2.2 Correlation between the crack depth and the natural frequency

Both the analytical formulation (Eq.13) and a numerical relationship are used in this paper to correlate crack depth with cracked beam natural frequency. The description about the numerical approach is described here.
In establishing numerical relationship between the values of fundamental mode vs. crack depth, the finite element modal analysis was carried out on the modeled specimen using Ansys© v14.0 as shown in Fig-10. Modal analysis module of Ansys© workbench was used to obtain the natural frequency of the specimen at a crack depth ranging from 0.1mm to 3.29mm with increments of 0.05mm as shown in Fig-11. The geometry of the crack surface was considered as a rectangle with a constant width of 0.2 mm which was according to the original test specimen dimensions.

The frequency and crack depth were plotted and a possible numerical correlation was formed. A 3rd degree polynomial was used to draw the plot on the obtained numerical natural frequency values as shown in Fig-12. Matlab© curve fitting toolbox was used to determine this plot and its representative mathematical model is shown in Eq.15.

\[
\omega_{nc} = -3.02(t_c^3) + 4.376(t_c^2) - 3.869(t_c) + 87.24
\]  
(15)

Eq.13 and Eq.15 are compared in Fig-13 for an initial crack value of 0.5 mm. The comparison shows the prediction of crack depths vs. natural frequencies from both equations is in close proximity.

The Eqs. 13, 14 and 15 can correlate RMS of acceleration, modal frequency and crack depth together. The RMS value is used to find specimen’s modal frequency at any instant by using Eq.14. This modal frequency can be used to determine the crack depth by using Eqs.13 and 15 and hence accomplishes the aim of this research by diagnosing the crack growth while the metallic beam is within the operational conditions and environment.

5. Results and discussion
5.1 Crack depth prediction and its validation
The validation experiments were performed on ten fresh specimens with different pre-defined crack depths. The specimens were tested on the modal frequency for an arbitrary time segment
and its dynamic response was observed and recorded. This was done to eliminate the time dependency on the value of the dynamic response. The value of the observed dynamic response was used to calculate the possible modal frequency by using Eq.14, while Eqs.13 and 15 were used to obtain the crack depth subsequently. After a stoppage, the test was started on the value of fundamental mode as obtained from the Eq.14. This process was carried out until the specimen reached to catastrophic failure. At this time of failure, the specimen was visually inspected and the crack depth was measured by a digital microscope with a magnification of 200x as shown in Fig-14 (separately with their respective predefined initial crack depth values). These visually measured crack depth values were compared with the values obtained from Eq.13 and Eq. 15. Percentage error of less than 8.11% was obtained in all tested ten specimens presented in Table 2.

5.2 Possible prediction of crack growth behavior
This section provides a discussion on the possible crack propagation behavior and the comparative error as mentioned in Table 2. It is to be noted that the crack depth values from visual inspection are only available at the time of start and end (i.e. catastrophic failure).

In Fig-15, the diamond shape point in red color representing an actual catastrophic failure for an initial 0.5 mm deep crack specimen. With this initial crack depth specimen, Eq. 13 is used for the analytical crack growth prediction from the natural frequencies as obtained during the experiments. The error of 3.60%, as shown in Table-2, is also adjusted in this prediction and plotted as shown in Fig-15. Similar results are obtained for other specimens (i.e. initial crack depth ranges from 0.75mm to 1.5mm) as shown in Fig-16.

The first value of natural frequency in Fig-15 is the fundamental frequency of specimen with the initially seeded crack of 0.5mm. This crack will start propagating once the load is applied and this specimen is forced to vibrate on its natural frequency. This propagation will reduce the stiffness
ultimately causing a decrease in the natural frequency as depicted by the curve. The lowest value is achieved till its catastrophic failure. In Eq.13, during propagation, all the variables are same except the corrective function (i.e. $F(\alpha)$) which is increasing with the crack depth and causes a decrease in the cracked beam natural frequency. A similar trend is observed with other initially crack depth as shown in Fig-16.

Analytical modeling can be used to get the crack propagation from initiation to failure. Only actual initial and final crack depth can be observed and measured visually to get possible crack propagation plot as shown in Fig-15 and Fig-16. The remaining data (between crack initiation and final failure) of possible crack propagation plot is achieved via values obtained analytically and estimated percentage error (analytical vs. visual) as mentioned in Table 2.

6. Conclusion

A methodology is proposed to predict the crack depth in Al 1050 beam operating at a modal frequency by its dynamic response values. The methodology avoids any unnecessary dismantling of the beam structure while it is under testing even in determining the possible depth of a propagating crack. Simple mathematical relationships are proposed to determine the increase in crack propagation while just the dynamic response of the beam is required. The validation experimentation showed the effectiveness of the methodology. The error in actual and predicting depths is under the acceptable limit (i.e., a maximum of 8.11%). An extension of this work will focus on different shapes of the predefined cracks and their behavior in failure prediction. Further, this procedure can also be used to analyze the crack propagation path and its rates without dismantling the structural element from its routine operations.
References:

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Figure-1: (a) The dimensions of the specimen (in mm) and, (b) The manufactured specimen.

Figure-2: Experimental setup with schematic.
Figure-3: Signal Express Monitoring and Recording Display

Figure-4: Cantilever beam with crack analyzed as mass less rotational spring
Figure-5: Dynamic response of specimen with crack depth - 0.5 mm

Figure-6: Dynamic response of specimen with crack depth - 0.75 mm
Figure-7: Dynamic response of specimen with crack depth - 1.0 mm

Figure-8: Dynamic response of specimen with crack depth - 1.25 mm
Figure-9: Dynamic response of specimen with crack depth - 1.5 mm

Figure-10: Ansys© model showing location and depth of predefined crack
Figure 11: Result of modal analysis on ANSYS

Figure- 12: Natural frequency vs. Crack depth
Figure- 13: Natural Frequency vs. Crack depth with two approaches-initial crack of 0.5mm

Figure- 14: Crack depths visual inspection at the time of catastrophic failure
Figure- 15: Possible crack propagation for initial crack of 0.5 mm

Figure- 16: Possible crack propagation for initial crack of 0.75 mm – 1.5 mm
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