Dynamics of Cost Uncertainty for Innovative High Value Manufacturing Products – A Geometric Phenomenon

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Abstract

In practice the forecasting of cost uncertainty for high value manufacturing products is typically a statistical exercise focused on predicting a static cost range at a future point in time. This only leads to robust forecasts if sufficient historical data is available, robust knowledge of cost estimating relationships exists and these relationships do not change in the time between creating the forecast and verifying its accuracy. The more innovative the product is the less likely it however is that these prerequisites are met. Using cost data from the U.K. Ministry of Defence Royal Air Force A400M transport aircraft from 2002 to 2014 as an example, the dynamics of cost estimating relationships over time are examined using a novel non-statistical forecasting approach. The approach considers cost uncertainty as a geometric phenomenon, does not rely on prior information and permits easy identification of patterns in changes of cost estimating relationships over time.

Keywords: Cost Uncertainty; Geometric Forecasting; Dynamic Space

1. Introduction

The shape created by time series data in a coordinate system is commonly used to describe the properties of that data and as the basis for forecasting its propagation over the time of the whole product life cycle [1]. Shapes often used for this purpose are trend lines or probability density functions applied to charts such as scatterplots, sand-charts or histograms [2,3,4,5]. When examining data from only a single point in time, which is often the case for innovative products, the trend line or probability density function methods are, however, not applicable and forecasting efforts need to rely on qualitative analogy and / or expert opinion only [6,7,8].

This paper presents a quantitative experimental method to extract sufficient information for forecasting purposes from data of a single point in time by visualizing its components as a polar force field and then examining the area, direction and magnitude of the thus created space that is declared to represent a probability field [9,10]. These attributes of cost “geometry” are considered as reflections of the living systems nature of the whole product life cycle [11,12] and hence suitable for the robust identification of relevant propagation patterns [13,14,15].

The polar force field can be visually understood as an abstraction of a radar chart and its own right represents a unique geometrical layout method for a cost estimating relationship model. The dependencies between its components can be derived by following a geometric forecasting process and then correlating the values of actual and forecast component values [15].

Components are any set of costs / cost variances which are available for a single point in time and typically aggregated as a total value that is reported on. The exemplary data
demonstrates 13 of such while different numbers can occur (i.e. 6 dimensions are used in the U.S. Department of Defence).

The polar force field is a probability field represented as a vector space where all vectors originate at the same point and are radially arranged with a constant degree of separation. If cost variance data from a specific time period is used then that time period is termed a “state space” in contrast to a “dynamic space” which could be considered to describe the change over time between two state spaces. Vector spaces can be described both graphically in pictures / images and mathematically using vector algebra.

The method finds its primary application when prior information is not available and this condition may occur at multiple times during the whole product life cycle [16,17,18].

2. Exemplary Data

For exemplary purposes this paper uses total variation from approved cost at the main gate for the Demonstration & Manufacture Phase reported on in the United Kingdom National Audit Office Major Projects Reports for the United Kingdom Ministry of Defense Royal Air Force A400M transport aircraft for the time period from 2002 to 2014 as shown in Table 1. Δ CV refers to the change in cost since the last reporting period. Figures in normal font indicate increases defined as follows

• Changed Capability Requirement (CCR): “Variations due to changes in customer’s requirement for equipment, flowing from operational reassessment rather than budgetary factors or support to current operations.”
• Budgetary Factors (BF): “Variations due to changes in the customer’s requirement for equipment, flowing from changed budgetary priorities.”
• Technical Factors (TF): “Variations which are due to changes in technical ability to deliver the project.”
• Procurement Processes (PP): “Variations due to changes associated with the contractual process including time taken in contract negotiations and placing contracts,

<table>
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<th>Year</th>
<th>Δ CV due to CCR</th>
<th>Δ CV due to BF</th>
<th>Δ CV due to TF</th>
<th>Δ CV due to PP</th>
<th>Δ CV due to CP</th>
<th>Δ CV due to I</th>
<th>Δ CV due to ER</th>
<th>Δ CV due to AAR</th>
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The visualisation of the single time period data is accomplished by converting cost variance values into vectors with corresponding magnitude, an outward direction and fixed at the centre. Invariant are also the radial degree between the axes. All data is considered at its absolute value, therefore, positive and negative cost variances are considered equally, and all values are visualised as relative percentages of their arithmetic sum. Finally all vectors are added head to tail in order to create an aggregated vector. The heads of the vectors can then be connected in a circular sequential manner in order to create the boundaries of an actual probability space. Finally the total length of the perimeter of the probability space is used to define the (maximum) reference probability space by creating edge lengths of identical length. Fig. 1 shows the 2002 data as a polar vector graph with actual and reference areas.

![Fig. 1. Polar force field visualisation of cost variance data for the United Kingdom Ministry of Defense Royal Air Force A400M in 2002 (£M)](image)

### 3. Geometry of Cost Uncertainty

Based on Fig. 1 the geometric attributes of symmetry, aggregated vector magnitude and aggregated vector direction (angle) can be calculated for every year of reported cost variance:

- Symmetry (S) is the degree to which a shape is invariant to being transformed across a reference point. These researchers consider this as the ratio between the actual area and the maximum area [16].
- The magnitude (M) and angle (A) of the aggregated vector are given by the head to tail addition of all cost variance vectors represented in the polar vector graph.
- The uncertainty range (UR) is the difference between the values of the upper (ULF) and lower limit (LLF) forecasts.
- The forecast accuracy (FA) is the difference between the actually reported cost variance in the next time period and the mode forecast (MF).
- The most likely value (ML) is the result of a Monte Carlo simulation at 80% confidence level when the upper limit, lower limit and mode forecasts are used as worst case, best case and most likely. The Monte Carlo simulation is a mathematical process applied to probabilistic problems based on repeated random sampling of a data range based on a probability density function (such as a bell curve) chosen by the investigator.

From a geometric / vector perspective the initial challenge encountered is how to treat different types of cost variance in that some increase total costs (normal font) and some decrease total cost (red cursive font). The presented method converts cost variance into vectors that share starting coordinates which are declared to be topologically invariant. The starting coordinates are a single point which represents the centre of the vector space. For these reasons each vector, by default, must share a common prefix (therefore be positive or negative) which is considered to be an attribute of the common invariant vector scale.

The presented method thus considers only the absolute value of cost variance. The method furthermore converts these absolute values into relative values (the individual cost variance values are transformed into percentages of total) in order to ease the comparison of geometrical shapes generated whereby this does not affect the value of the actual geometrical attributes evaluated.

Using the previously generated geometric attributes basic principles of vector algebra can be applied to calculate an upper limit, mode and lower limit forecast. These values can be input into a Monte Carlo simulation for calculating a most likely value [15].

The method for forecasting the mode calculates the relative % of total change represented by each cost variance vector, uses that to determine the relative proportion of the aggregated vector, adds the magnitude of that to the individual cost (variance) vector and adds the forecast value of all individual vectors in order to forecast the mode value of total future cost variance. The method for forecasting the upper limit calculates the magnitude of the component vector of the aggregated vector which has the same radial degree as the current state individual cost variance vector and adds it to the magnitude of each individual vector. For this the x- and y-coordinate differentials of the end points of the two vectors are calculated and added to the end point of the relevant individual current cost (variance) vector. The sum of the individual vector forecast is added in order to forecast upper limit of total future cost variance. The method for forecasting the lower limit multiplies the results of the upper limit algorithm and the mode algorithm. This represents the lower limit of the total future cost variance. The results are shown in Table 2.

### 4. Geometric Assessment of Single Time Period Data

The cost variance data from Table 1 and the assessment of the geometric attributes in Table 2 can now be visualized in the form of scatter-charts with linear trend-lines to highlight the basic shape of the data [20,21,22].
Table 2. Geometrical attributes of cost variance data for the United Kingdom Ministry of Defence Royal Air Force A400M.

<table>
<thead>
<tr>
<th>Year</th>
<th>Total Δ CV</th>
<th>Compounded Δ CV</th>
<th>A (%)</th>
<th>M (%)</th>
<th>SI (%)</th>
<th>UR (%)</th>
<th>ML (%)</th>
<th>MF (%)</th>
<th>FA (%)</th>
<th>ULF (%)</th>
<th>LLF (%)</th>
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<td>926</td>
<td>40.67</td>
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<td>59.52</td>
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</table>

Fig. 2 illustrates the annual value of reported cost variance with an average value of £1,064M and a standard deviation of £88M.

![Fig. 2. Value of reported cost variance (£M)](image)

\[ y = 3.0879x - 5.1365 \]
\[ R^2 = 0.0171 \]

Fig. 3 illustrates the compounded annual value of reported cost variance with an average annual growth rate of £95M and a standard deviation of £4M.

![Fig. 3. Value of compounded reported cost variance (£M)](image)

\[ y = 1.0741x - 2E-06 \]
\[ R^2 = 0.9997 \]

Fig. 4 illustrates the forecast accuracy achieved by the presented method with an average accuracy of 107% and a standard deviation of 14%.

![Fig. 4. Geometrical forecast accuracy (%)](image)

\[ y = 0.0361x + 0.7148 \]
\[ R^2 = 0.2131 \]

Fig. 5 illustrates the magnitude of the aggregated vector with an average value of 36% and a standard deviation of 9%.

![Fig. 5. Magnitude of the aggregated vector (%)](image)

\[ y = 2.1772x + 26.561 \]
\[ R^2 = 0.8231 \]

Fig. 6 illustrates the direction of the aggregated vector with an average value of 63° and a standard deviation of 15°.

![Fig. 6. Direction of the aggregated vector (°)](image)

\[ y = 1.7645x + 50.561 \]
\[ R^2 = 0.8899 \]

Fig. 7 illustrates the uncertainty range of the forecast with an average value of 13% and a standard deviation of 0.2%.

![Fig. 7. Uncertainty range of the geometrical forecasts (%)](image)

\[ y = 0.0004x + 0.1304 \]
\[ R^2 = 0.65317 \]
Fig. 8 illustrates the geometrical symmetry of the forecast with an average value of 64% and a standard deviation of 14%.

As shown in Fig. 2 and Fig. 3 annual cost variance remains relatively constant resulting in a compounded linear growth trend with high R\textsuperscript{2} value. The geometric attribute with the highest linear correlations are magnitude of the aggregated vector as a direct reflection of annual cost variance.

Symmetry and the uncertainty range have a medium linear trend line correlation while the direction of the linear vector does not.

The forecast accuracy can now be compared to the values of the various geometrical attributes. Fig. 9 illustrates the relationship between the forecast accuracy and the geometrical symmetry.

Fig. 10 illustrates the relationship between the forecast accuracy and the aggregated vector length.

Fig. 11 illustrates the relationship between the forecast accuracy and the aggregated vector angle.

5. Discussion

A previous literature review [14] failed to identify uncertainty quantification metrics suited to visualising and quantifying cost estimate -uncertainty if cost variance data from only a single time period was available. The exploration of possible visualisation approaches then led to the examination of spider charts and an investigation of whether the shape of cost variance, when visualised as a spider chart, changed in a predictable manner of time, which was affirmed in respect to their symmetry [18]. Efforts to determine why changes in symmetry occurred in a predictable manner occurring in parallel then led to an investigation of dependencies between cost variance dimensions which are traditionally visualised as cost estimating relationships, or cost dependency models [13]. The researchers then examined the degree to which the spider chart visualisation could be considered as a specific layout form of a cost estimating relationship. The approach of converting the spider chart axes into vectors then emerged as an experimental path leading to a much simpler forecasting approach compared to the previously...
chosen approach based on symmetry. The application of the polar force field method for visualising and quantifying cost variance, with the ensuing use of vector algebra to arrive at forecasting algorithms is based upon applying principles from physics to the field of cost estimation. Specifically the geometric space created by joining the vertices of cost variance dimensions, when represented as a polar force field, is considered to represent a probability space, the attributes of which, such as symmetry, provide indications as to the future shape of that space.

The consideration of the polar force field as a specific layout of a cost estimating relationship significantly eased discussion of the use of geometry / shape for exploring small cost data with stakeholders in research and industry, since the concept of dependency models is well known and widely used in the field of cost estimation. The degree to which the underlying principles of force fields, as used in physics, are applicable to the cost estimation context are the primary subject of future work, and based upon the possible consideration that cost is one attribute of the whole product life cycle which could be considered as a living system in its entirety [11,12].

6. Conclusion and recommendations for further research

The presented method is currently being implemented by a major aerospace manufacturer in order to estimate the (changing) maturity of cost estimates at iterative stages of the whole product life cycle and in order to identify estimates which have the potential of significant cost overruns. From an industry perspective “too low” forecast thus appear identifiable and in practice represents a forecast which will lead to significant overruns on budgeted costs. The highest forecast represents a forecast that may be higher than actual costs and represents a more desirable scenario than forecasting low from the perspective of planning stability. Important, however, is to recognize that any deviation of cost from plan is undesirable thus emphasizing the important of understanding when which control intervals of monitoring actual to forecast cost (including suitable methodologies) makes the greatest sense [18].

The presented method and experimental results can be understood as a novel inverse uncertainty quantification approach [23] under conditions of minimum prior data. The use of geometric attributes of cost variance data from a single time period provides an alternative path of data analysis to methods based on analogy, expert opinion, parametrics and / or regression. Further ongoing research involves examining the effect of changing invariants, such as the radial degree of the polar force field, in order to improve forecasting accuracy.

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