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Cooperative Control for Multiple Interceptors to Maximize Collateral Damage

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Abstract: In this paper, a cooperative control method to satisfy the relative interception angle constraints in multi-to-one engagement case is proposed. In this study, we consider the relative interception angle constraints of the multiple interceptors, which is intended to enhance the survivability of the multiple interceptors against a defense system of high value target as well as to maximize the collateral target damage. The proposed cooperation control can reduce the total control energy required while satisfying the given interception angle constraints. This characteristic allows to increase the change of mission in the multi-to-on engagement scenario. In this paper, the numerical simulations are conducted to verify the feasibility of proposed concept.

Keywords: Cooperative control, multiple interceptors, network system, interception angle control, collateral damage.

1. INTRODUCTION

Recently, it is a common that a high value target such a a battle ship has an own defense system to protect itself against incoming anti-ship interceptors. Accordingly, from the interceptor standpoint, the probability of mission success in the traditional one-to-one engagement scenario is dramatically degraded because an interceptor is frequently destroyed by the formidable defense systems of the target. In order to increase the chance of mission success, one way is that the interceptor is designed to allow extreme evasive maneuvering capabilities against the target defense system as studied in Kim et al. (2010). However, this approach has an issue that the development cost for such interceptor is usually high. Instead, a cost effective way is to perform a cooperative interception using multiple interceptors which are developed at low cost relatively. The cooperative interception can be performed by controlling the flight times Jeon et al. (2006) or the interception angles Lee et al. (2007) of the multiple interceptors. The rationale of this approach is that in this way we can saturate the reaction ability of the defense system of target against the incoming interceptors so that the survivability of multiple interceptors can be significantly improved. In addition, under the circumstance of cooperative interception with approaching angle control, the multiple interceptors can approach the high value target with a specific interception angle interval in order to maximize attacking effect (i.e., the collateral target damage).

In general, the cooperative control can be classified into two ways Shaferman and Shima (2015). One thing is the implicit cooperation and another one is the explicit cooperation. In the concept of implicit cooperation, each interceptor (i.e., agent) is guided by one-to-one engagement guidance law Jeon et al. (2006); Lee et al. (2007); Ryoo et al. (2010); Erer and Merttopçuoglu (2012) with a cooperation parameter such as a common interception time or a relative interception angle. In that cooperation concept, obviously, it is difficult to cope with a change of engagement situation and reconfigure the flight trajectories of multiple interceptors because the cooperation parameter is manually pre-programmed into all interceptors before launching them. Thus, according to changes of engagement situation, the multiple interceptors may unnecessarily use a lot of control energy or take detours to maintain ineffective flight formations. Unlike the implicit cooperation, the explicit cooperation can communicate among the multiple interceptors and reconfigure the desired value of cooperation parameter during the flight. Therefore, this cooperation method can increase the chance of mission success even in the presence of unexpected event. In previous studies, the explicit cooperation for impact time control already has been reported in the references Jeon et al. (2010); Shiyu and Rui (2008). However, the explicit cooperation intended to control interceptions angle of multiple interceptors is not studied yet. Accordingly, in this study, we propose a cooperative control method of multiple interceptors with the relative interception angle constraints.

This study is motivated from our previous work on one-to-one controller for interception angle control Lee et al. (2013). This paper aims to extend this controller to the multi-to-one engagement case with the relative interception angle constraints of multiple interceptors intended to maximize the collateral target damage and minimization of the overall control energy intended to increase the chance of their mission success. The control command in Lee et al. (2013) is consists of the pure proportional navigation (PPN) command Zarchan (2012) for zeroing miss distance and the additional command for interception angle control as desired. According to reference Bryson (1975); Zarchan (2012), it is well-known that PPN command with $N = 3$ is considered as the energy optimal control. Therefore, the minimization of additional command term can lead to the minimization of overall control energy. Since the additional command term is given by the function of the desired interception angle, the amount of control energy required highly depends on the selection of this parameter. In the proposed method, the multiple interceptors try to find out their desired interception angles, through information sharing of their flight parameters, that minimize the overall control energy as well as maximize the collateral target damage. The numerical simulations are conducted to show the validity of proposed approach.

This paper is structured as follows. In Section 2, the problem to consider in this study is stated. The proposed method is provided in Section 3. The simulation results are provided in Section 4. Finally, we conclude our study in Section 5.

2. PROBLEM FORMULATION

2.1 Planar Engagement Kinematics

In this section, the engagement kinematics for cooperative attack of multiple interceptors is explained. Before deriving the engagement kinematics, the following assumptions are made:

(1) Each interceptor is cruising.
(2) Each interceptor is sea-skimming.

Under these assumptions, we consider the constant speed interceptor and the planar engagement kinematics as shown in Fig. 1. The coordinate system (X_I, Y_I) is the inertial reference frame. The parameters T represents a stationary target such as a battle ship. The parameter I_i represents the i-th interceptor which is modeled as a point-mass. The parameters R_i and σ_i represent the range and the line-of-sight (LOS) angle between the target and the i-th interceptor. The parameters γ_i and $\gamma_{f,i}$ are the flight path angle and the desired impact angle of the i-th interceptor. Next, the parameter V_i and a_i denote the speed and the normal acceleration of i-th interceptor. By imposing the normal acceleration, the flight path angle is altered as:

$$\dot{\gamma}_i = \frac{a_i}{V_i} \tag{1}$$

For the stationary target, the relative kinematics of the i-th interceptor can be written as

$$\dot{R}_i = -V_i \cos(\gamma_i - \sigma_i) \tag{2}$$

$$\dot{\sigma}_i = -\frac{V_i \sin(\gamma_i - \sigma_i)}{R_i} \tag{3}$$

Then, the equations (1), (2), and (3) constitute the system equation of engagement kinematics. Here, the normal acceleration of each interceptor can be considered as the input of system equation. The normal acceleration should

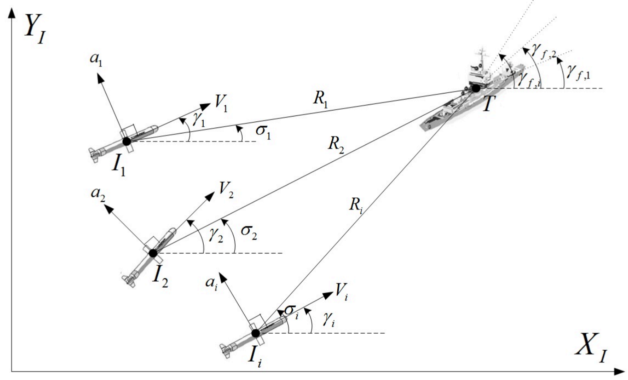

Fig. 1. Engagement kinematics for cooperative attacking of multiple interceptors.

be issued to satisfy the following condition in order to hit the target.

$$\gamma_i = \sigma_i \tag{4}$$

where t_f is the final time. For the stationary target, the remaining time of interception so called the time-to-go for the i-th interceptor can be simply approximated as

$$t_{go,i} = \frac{R_i}{V_i} \tag{5}$$

2.2 Interceptor Numbering

Before getting into the problem statement, let us define the index of each interceptor for convenience. As shown in Fig. 1, the leftmost one is defined as the first interceptor, i.e., $i = 1$. Let us number remaining interceptors in a counterclockwise direction from the first interceptor.

2.3 Problem Statement

In this section, we explain the cooperative control problem to consider in this paper. The basic premise of this study is as follows. When the multiple interceptors engage to destroy a high-value target, it is more effective to hit other spots evenly rather than to hit the same spot again in order to maximize the collateral damage. Based on this aspect, therefore, it is desirable to approach the target at regular intervals with adjacent interceptors in Fig. 1. To this end, interception angle control should be performed at each interceptor while intercepting the target. The constraints on interception angles intended to maximize the collateral damage are given by

$$\begin{aligned} |\gamma_{f,1} - \gamma_{f,2}| &= \Delta\gamma \\ &\vdots \\ |\gamma_{f,i-1} - \gamma_{f,i}| &= \Delta\gamma \\ &\vdots \\ |\gamma_{f,n-1} - \gamma_{f,n}| &= \Delta\gamma \end{aligned} \tag{6}$$

where the parameter n represents the total number of interceptors in the group. The parameter $\Delta\gamma$ is the desired relative interception angle between adjacent interceptors, which is predetermined. Note that since the interception angle constraints are given by relative values between adjacent interceptors shown in equation (6), if one of desired interception angles, for convenience $\gamma_{f,1}$, is determined, then the other desired interception angles are automatically decided by equation (6). In the conventional approach, one of desired interception angles $\gamma_{f,1}$ can be programmed as an arbitrary value between 0 to 2π, by an operator.

$$\gamma_{f,1} \in [0, 2\pi] \tag{7}$$

In this approach, however, depending on the value programmed by the operator, the multiple interceptors may take a long detour unnecessarily in order to satisfy the terminal condition with the programmed value, thus a lot of control energy may be required. This phenomena is not desirable for intercepting the target effectively because of the following reasons. First, increasing mission time due to detour may degrade the survivability of the multiple interceptors because it leads to increasing of reaction time of target's defense system against the multiple interceptors. Second, usage of a lot of control energy may reduce the change of mission success. Additionally, the conventional approach is less flexible to counteract an unexpected situation during the mission.

Therefore, this study aims to propose a cooperative control approach to tackle the above issues. Basically, it is assumed that the multiple interceptors can share their parameters such as speed, LOS angle, range, and flight path angle, through their communication systems. In the proposed approach, based on consensus among multiple interceptors, the final formation (i.e., the selection of $\gamma_{f,1}$) that minimizes the detour and the usage of control energy while satisfying equation (6) will be decided by themselves without the operator.

3. PROPOSED METHOD

In this section, the proposed cooperative control method is provided. First, the local controller of each interceptor, which can provide the desired interception angle and the zero miss distance is explained. And then, we discuss the cooperative method to determine the desired interception angles that satisfy equation (6) as well as minimize the overall control effort of multiple interceptors.

3.1 Local Controller

This section discuss a local controller of each interceptor. In order to control interception angle as well as provide zero miss distance, the interception angle control guidance law Lee et al. (2013) is adopted for the local controller. For the i-th interceptor, this controller is composed of two control commands as

$$a_i = a_{A,i} + a_{B,i} \tag{8}$$

where $a_{A,i}$ is the control commend to ensure the interception of the i-th interceptor. It is given by the form of pure proportional navigation (PPN) Zarchan (2012) command as

$$a_{A,i} = N_i V_i \dot{\sigma}_i \tag{9}$$

where N_i represents the navigation constant which is usually chosen between from 3 to 5. Additionally, the command $a_{B,i}$ is the control command to provide the desired interception angle $\gamma_{f,i}$ for the i-th interceptor, which is given by

$$a_{B,i} = -\frac{V_i\left(N_i - 1\right)}{t_{go,i}}\left(\gamma_{f,i} - \hat{\gamma}_{f,i}\right) \tag{10}$$

where $\hat{\gamma}_{f,i}$ represents the predicted interception angle by the control command $a_{A,i}$, which is given by

$$\hat{\gamma}_{f,i} = -\frac{1}{N_i - 1}\gamma_i + \frac{N_i}{N_i - 1}\sigma_i \tag{11}$$

3.2 Cooperative Determination of Interception Angles

In this subsection, we explain how to decide interception angles of multiple interceptors, which minimizes the required control energy as well as satisfies the relative angle constraints shown in equation (6). As shown in equation (8), the local controller is consists of two command terms. According to reference Zarchan (2012), it has been revealed that the first command with $N_i = 3$ is the optimal control command in the term of control energy minimization. Thus, if we consider the interception only, employing the local controller with the first term only can guarantee the optimal trajectory that minimizes the overall energy consumption. However, if we consider the interception angle control as well, then the additional control command (i.e., the second term given in equation (8)) is required. This command term is given by the function of the time-to-go, the speed, the navigation constant, the predicted interception angle by the first control command, and the desired interception angle for the i-th interceptor. Therefore, it is obvious that the required control effort for the interception angle control varies depending on these parameters. Here, the time-to-go, the speed, and the predicted interception angle are given by the geometry condition. The navigation constant and the desired interception angle are only the design parameter. If the navigation constant is selected as $N_i = 3$ intended to achieve the energy optimal trajectory for the interception, then the design parameter that determines the required control effort is only the desired interception angle. Thus, in the proposed cooperative control of multiple interceptors, the main goal is to determine the set of desired interception angles that require less control effort to achieve them and satisfy the relative angle constraints shown in equation (6) to maximize the collateral target damage.

Thus, we can address the optimal problem to achieve the preceding goal as

$$\min_{\gamma^*_{f,1},\ldots,\gamma^*_{f,n}} \quad J = \sum_{i=1}^{n}\left(\frac{4V_i^2}{t_{go,i}^2}\left(\gamma_{f,i} - \hat{\gamma}_{f,i}\right)^2\right) \tag{12}$$

subject to equation (6).

where the magnitude of $a_{B,i}$ with imposing $N_i = 3$ is used in this formulation. The set of parameters $\gamma^*_{f,1}, \ldots, \gamma^*_{f,n}$ represent the set of optimal interception angles that minimize the overall magnitude of control effort at the current time. From equation (6), without change of formula, this condition can be rearranged with respect to the desired interception angle of the first interceptor (i.e., $\gamma_{f,1}$) as

$$\begin{aligned}\gamma_{f,2} &= \gamma_{f,1} + \Delta\gamma \\ \gamma_{f,3} &= \gamma_{f,1} + 2\Delta\gamma \\ &\vdots \\ \gamma_{f,n} &= \gamma_{f,1} + (n-1)\,\Delta\gamma\end{aligned} \tag{13}$$

By using equation (13), the original optimal problem can be simplified as

$$\min_{\gamma^*_{f,1}} \quad J = \sum_{i=1}^{n} \left(\frac{4V_i^2}{t_{go,i}^2} \left(\gamma_{f,1} + (i-1)\,\Delta\gamma - \hat{\gamma}_{f,i}\right)^2 \right) \tag{14}$$

In this setting, the number of decision variables are reduced compared to equation (12). If the variable $\gamma^*_{f,1}$ is decided, then other decision variables can be determined by using equation (13).

Hereafter, we explain how to solve the predetermined optimal solution, that is $\gamma^*_{f,1}$. According to the well-known optimal control theory Bryson (1975), the optimality condition is given by

$$\frac{\partial J}{\partial \gamma_{f,1}} = 0 \tag{15}$$

Imposing this condition to equation (14) yields

$$\gamma^*_{f,1} = -\frac{\sum\limits_{i=1}^{n} \frac{V_i^4}{t_{go,i}^4} \left((i-1)\,\Delta\gamma - \hat{\gamma}_{f,i}\right)}{\sum\limits_{i=1}^{n} \frac{V_i^4}{t_{go,i}^4}} \tag{16}$$

Additionally, if all interceptors in the group are cruising with the same speed as $V_i = \ldots = V_n = V$, then the preceding equation can be further simplified as

$$\gamma^*_{f,1} = -\frac{\sum\limits_{i=1}^{n} \frac{1}{t_{go,i}^4} \left((i-1)\,\Delta\gamma - \hat{\gamma}_{f,i}\right)}{\sum\limits_{i=1}^{n} \frac{1}{t_{go,i}^4}} \tag{17}$$

As shown in equations (11) and (16), we can readily observe that the computation of $\gamma^*_{f,1}$ requires the information on the speeds, the flight path angles, the LOS angles, and the remaining time of interceptions for all interceptors in the group. Therefore, a cooperation of multiple interceptors is required to decide the set of optimal desired interception angles. In this study, the decentralized cooperation concept is utilized. In this cooperation concept, there is no coordinator and each interceptor has own local controller shown in equation (8). Each interceptor collects the required information from neighbor interceptors in order to determine the optimal value of $\gamma^*_{f,1}$. Once $\gamma^*_{f,1}$ is determined locally, each interceptor explicitly decides own desired interception angle by using $\gamma^*_{f,1}$ with the rule shown in equation (13). The basic premises of this approach is that if all the information matches, the obtained $\gamma^*_{f,1}$ at each interceptor is identical to each other. This process can be considered as the explicit cooperation as shown in Fig. 2.

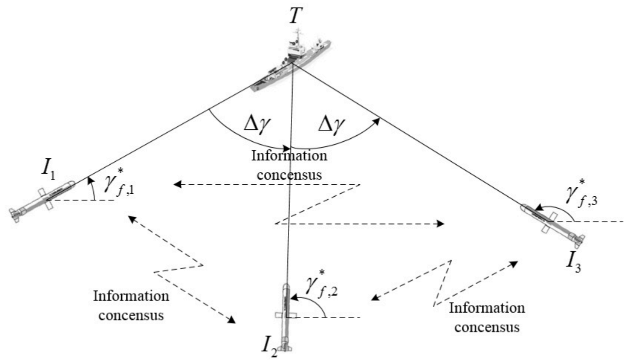

Fig. 2. The illustration of decentralized cooperation concept used in this study.

4. SIMULATION RESULTS

In this section, we perform the numerical simulations to verify the proposed concept compared to the conventional method. In this simulation study, we consider three interceptors as $n = 3$. The engagement conditions for cooperative attack of multiple interceptors are given by Table 1. The relative interception angle between each interceptor is selected as $\Delta\gamma = -30\,\mathrm{deg}$ in order to maximize the collateral target damage. In the conventional approach, it is assumed that the set of desired interception angles as given in Table 1 are assigned to each interceptor by the operator. Then, each interceptor independently performs the engagement by using own local controller with the predetermined desired interception angle.

Table 1. The engagement conditions for cooperative attack of multiple interceptors.

	X_0 $[km]$	Y_0 $[km]$	V $[m/s]$	γ_0 $[deg]$	γ_f $[deg]$
Target	0.0	0.0	-	-	-
Interceptor 1	-10.0	0.5	250	30.0	70.0
Interceptor 2	-6.0	6.0	250	50.0	40.0
Interceptor 3	-3.0	10.0	250	-20.0	10.0

Figs. 3 and 4 show the flight trajectories and the flight path angles under both approaches. The solid lines and the dotted lines represent the results of proposed method and the results of conventional approach, respectively. Table. 2 provides the interception angles achieved. From the results obtained, we can readily observe that the relative interception angle separation, which is required to maximize the collateral damage, is satisfied under both cases. However, as shown in Fig. 3, in the conventional approach, the multiple interceptors unnecessarily take detours to achieve the pre-programmed desired interception angles, compared to the proposed method. These results imply that a lot of control energy is required in the conventional approach.

Table 2. The interception angles achieved.

	$\gamma_1\left(t_f\right)$ $[deg]$	$\gamma_2\left(t_f\right)$ $[deg]$	$\gamma_3\left(t_f\right)$ $[deg]$
Conventional Approach	70.01	40.01	9.99
Proposed Approach	-0.002	-30.00	-60.00

For comparisons, we determine the average control efforts of two control commands under the both approaches as

$$J_1 = \sum_{i=1}^{n} \int_{t_0}^{t_{f,i}} a_{A,i}^2\left(\tau\right) d\tau \tag{18}$$

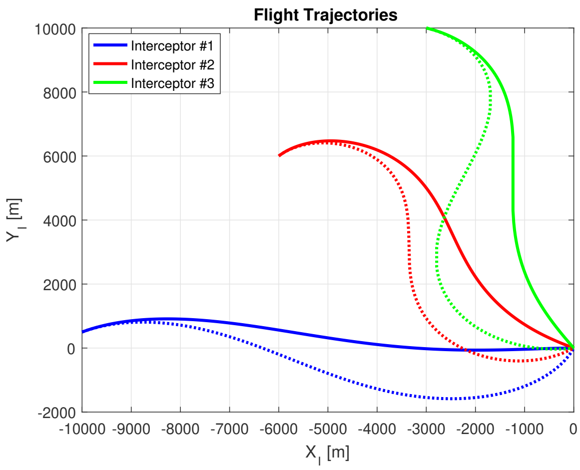

Fig. 3. Flight trajectories under both approaches.

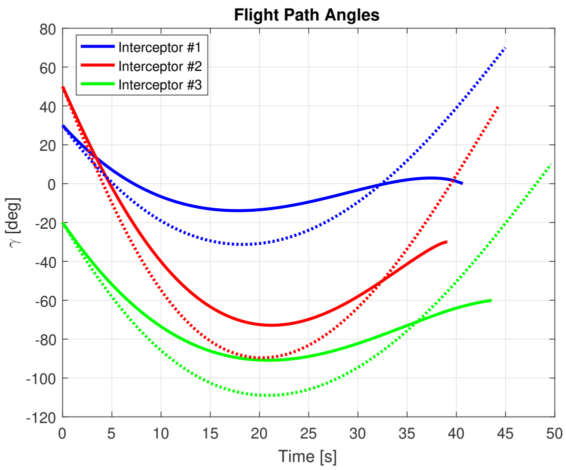

Fig. 4. Flight path angles under both approaches.

$$J_2 = \sum_{i=1}^{n} \int_{t_0}^{t_{f,i}} a_{B,i}^2 (\tau)\, d\tau \tag{19}$$

J_1 is the total control effort required for providing zero miss distance. And, J_2 represents the total control effort required for interception angle control. Figs. 5 and 6 show the responses of total control effort required for two control commands under both approaches. Table. 3 provides the final value of control energy required of multiple interceptors. These values are recorded as $14,560.8 m^2/s^3$ for $a_{A,i}$ and $14,898.1 m^2/s^3$ for $a_{B,i}$ in the proposed method. Under the conventional approach, these values are given as $128,887.1 m^2/s^3$ for $a_{A,i}$ and $63,906.4 m^2/s^3$ for $a_{B,i}$. These results clearly show that the proposed method can significantly reduce the control effort required compared to the conventional approach.

Table 3. The total control energy required.

	$J_1 \left[m^2/s^3\right]$	$J_2 \left[m^2/s^3\right]$
Conventional Approach	128,887.1	63,906.4
Proposed Approach	14,560.8	14,898.1

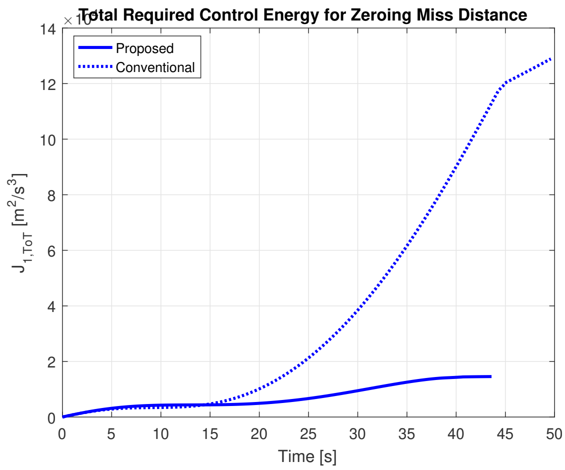

Fig. 5. Total control energy required for zeroing miss distance under both approaches.

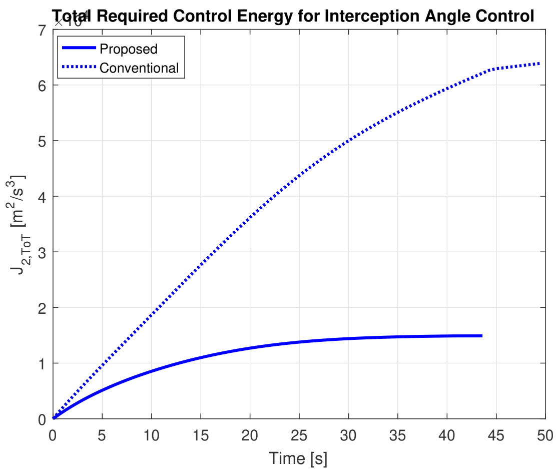

Fig. 6. Total control energy required for interception angle control under both approaches.

5. CONCLUSION

In this study, we propose a cooperative control approach of multiple interceptors using the concept of biased pure proportional navigation. In order to saturate the defense system of target and maximize collateral target damage, a flight formation that keeps the relative interception angle separation is considered. Additionally, a cooperative way that ensures the control energy minimization is also proposed to increase the probability of mission success. Through numerical simulations, we show that the proposed approach can be applicable to challenge of multiple interceptors intended to intercept a high value target such as battle ship.

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