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Optimality of Error Dynamics in Missile Guidance Problems

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I. Introduction

The well justified proportional navigation guidance (PNG) law [1, 2] and its variants have been widely used in missile guidance during the last few decades. The long history of successful application of PNG proves its simple implementation and efficiency for various missile systems. The fundamental idea behind PNG is that it issues a normal acceleration command to nullify the line-of-sight (LOS) rate, thereby forcing the interceptor to maintain on the collision triangle. Recent developments in nonlinear control theory have also led to many elegant solutions of advanced missile guidance laws, such as sliding mode control (SMC) [3–7], H_∞ control [8], Lyapunov function theory [9], predictive control [10], feedback linearization control [11, 12] and references therein.

In essence, missile guidance law design is a kind of finite-time tracking problem depending on the operational objective. If only target interception is considered, the tracking error is defined as the zero-effort-miss (ZEM) distance [6, 7, 13, 14] or LOS rate [3, 4, 15]. Nullifying the ZEM or LOS rate results in perfect interception. For some modern missions, the final impact angle or approach angle is required to satisfy certain constraints to enhance the kill probability of the warhead [16–18]. In such scenarios, the impact angle error is considered as the tracking error in guidance law design [19–23]. In order to enhance the survivability of the missiles against advanced close-in weapon system of battleships, the concept of salvo attack [24] is introduced to achieve simultaneous attack among all interceptors. One typical implementation of salvo attack is the impact time guidance, in which the missile is forced to intercept the target at desired time and hence the final impact time error is considered as the tracking error in guidance law design [24–28]. In exo-atmospheric interception engagement, maneuvers are obtained by employing thrust in the required direction. Obviously, it is desirable to drive the missile toward to the target in a straight line along a given collision triangle [29, 30]. This way enables the missile to fully exploit its maximum maneuver capability to increase the final impact energy. In this regard, the tracking error of missile guidance problem can be defined as the heading angle error, which is the angle difference between the current flight path angle and the desired flight path angle determined by the collision triangle.

After defining the tracking error for a specific missile guidance problem, many systematic nonlinear control theories can then be utilized to drive the tracking error to zero asymptotically or in finite time. More specifically, after selecting the desired error dynamics to be achieved, the control input is derived to force the system trajectory to converge to the

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desired error dynamics. This is a general procedure to establish new missile guidance laws using nonlinear control methodologies. However, most previous studies only care about how to force the tracking error to converge to zero and disregard what is the optimal error dynamics in terms of a meaningful performance index.

Inspired by the above observations, this Note aims to examine the optimal convergence pattern of the tracking error and propose an optimal error dynamics with a meaningful performance index for missile guidance problems. To realize this, we solve the linear quadratic optimal control problem for a generalized tracking problem that is often observed in missile guidance law design using Schwarz's inequality approach. The potential significance of the proposed results is clear. On one hand, under the proposed optimal error dynamics, the existing nonlinear guidance laws can be further improved by converting them to their optimal version and the characteristics of existing nonlinear guidance laws can be easily explained. On the other hand, various new missile guidance laws can be simply developed by using the proposed results since our solution is formulated on the basis of a generalized tracking problem.

For illustration, we present four examples to show how to apply the proposed optimal error dynamics to missile guidance law design. The presented examples include zero ZEM guidance, impact angle guidance, impact time guidance as well as guidance-to-collision for exo-atmospheric interception. It turns out to be that the resulting ZEM guidance law is the well-known augmented PNG law and the impact angle guidance law is a generalized form of previous optimal impact angle guidance laws [20, 31–34]. The new impact time guidance law, which is a generalized formulation of [24, 26], is a combination of PNG and impact time error feedback term. As for guidance-to-collision in exo-atmospheric interception, the uniqueness of the new guidance law lies in its ability to enable steering the missile to intercept the target along a straight line. Comparison results show that the proposed guidance-to-collision law requires less control effort than previous SMC guidance-to-collision law [29].

The rest of the paper is organized as follows. Sec. II presents some preliminaries and the motivations of this Note. Sec. III provides the details the proposed generalized optimal error dynamics, followed by the illustration examples shown in Sec. IV. Finally, some simulation results and conclusions are offered.

II. Preliminaries and Motivations

This section states some preliminary backgrounds on the kinematics model in missile guidance problems and also presents the motivations that why we want to suggest a general optimal error dynamics for guidance law design. Before presenting the results, we make the following general assumptions for convenience.

Assumption 1. The missile dynamics is assumed to be ideal, i.e., no autopilot lag.

Assumption 2. Gravity is ignored in guidance law design.

Note that these two assumptions are widely-accepted in guidance law design for missile systems. (Assumption 1) The control loop or the autopilot is usually much faster than the guidance loop and the performance degradation due to dynamic lag effect is negligible if the flight time is greater than 10 to 20 times the time constant of dynamic lag.

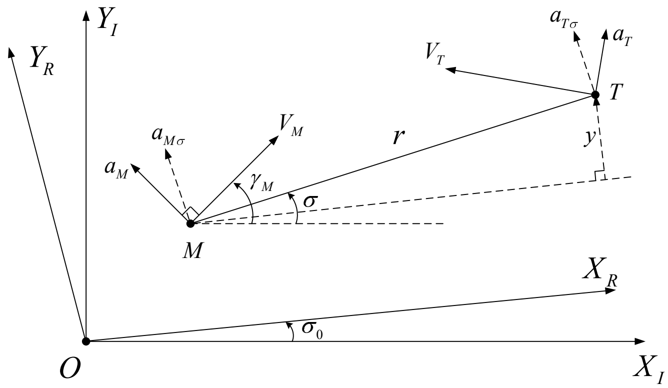

Fig. 1 The homing engagement geometry and parameter definitions.

Therefore the autopilot dynamics can be ignored in guidance law design. Also, (Assumption 2) After guidance law design, the gravitational effect can be directly compensated by a biased term $g \cos \gamma_M$ in practical implementation.

A. Model Derivation

It is assumed that the interceptor employs an ideal attitude control system that provides roll stabilization such that the homing guidance problem can be treated in two separate channels. Figure 1 shows the planar homing engagement geometry in this study, where M and T denote the missile and target, respectively. The notation of (X_I, Y_I) represents the inertial frame. For the purpose of introducing the linearized kinematics, a new frame called the reference frame (X_R, Y_R) is also defined. This frame is rotated from the inertial frame by σ_0, which is the reference angle. The variables of σ and γ stand for the LOS angle and flight path angle, respectively. r denotes the relative distance between the target and the missile. y is the relative distance between the target and the missile perpendicular to X_R-direction. a_M and a_T are the missile and target accelerations normal to the velocity vectors, respectively. The variables of a_{M_σ} and a_{T_σ} denote the missile and target accelerations normal to the LOS direction, respectively.

In the reference frame, the engagement kinematics can be expressed as

$$\begin{aligned} \dot{y} &= v \\ \dot{v} &= a_{T_\sigma} \cos(\sigma - \sigma_0) - a_{M_\sigma} \cos(\sigma - \sigma_0) \end{aligned} \tag{1}$$

where v is the relative velocity between the target and the missile perpendicular to X_R-direction.

The complementary equation that describing the relationship between a_{M_σ} and a_M is given by

$$a_{M_\sigma} = a_M \cos(\gamma_M - \sigma) \tag{2}$$

Under the small angle assumption of $\sigma - \sigma_0$, one can derive the linearized engagement kinematics as

$$\begin{aligned} \dot{y} &= v \\ \dot{v} &= a_{T_\sigma} - a_{M_\sigma} \end{aligned} \tag{3}$$

From Fig. 1, the LOS rate in the linearized engagement kinematics can be approximated as

$$\dot{\sigma} = \frac{y + v t_{go}}{V_c t_{go}^2} \tag{4}$$

where V_c denotes the closing velocity.

B. Motivations

As stated earlier, missile guidance law design is a kind of finite-time tracking problem, in which we want to regulate the tracking error to zero in finite time. The general form of the tracking problem that is often observed in missile guidance law design is

$$\dot{\varepsilon}(t) = g(t)\, u(t) \tag{5}$$

where $\varepsilon(t)$ represents the tracking error, $g(t)$ a known time-varying function, and $u(t)$ the control input.

According to missile guidance problems under consideration, the tracking error can be ZEM, LOS rate, impact angle error, impact time error, heading error, etc. Also, the form of $g(t)$ depends on guidance problems and it is invertible since the above tracking problem is controllable, i.e. $g(t) \neq 0$.

In order to regulate the tracking error to be zero, various control theories such as SMC, Lyapunov function, and feedback linearization, have been applied to system (5) in previous works. In these approaches, the desired error dynamics is first selected, and then appropriate control input is calculated to ensure that the system equation shown in (5) follows the selected error dynamics. Most of the previous works regarding guidance law design adopted the desired error dynamics as

$$\dot{\varepsilon}(t) + k\varepsilon(t) = 0 \tag{6}$$

where $k > 0$ is the guidance gain to regulate the convergence rate of tracking error.

It is easy to verify that the closed-form solution of differential equation (6) is given by

$$\varepsilon(t) = \varepsilon(t_0)\, e^{-kt} \tag{7}$$

where $\varepsilon(t_0)$ denotes the initial tracking error.

The preceding equation reveals that the tracking error converges to zero asymptotically with an exponential rate governed by the guidance gain k. Accordingly, this desired error dynamics has two major drawbacks: (1) the finite-time convergence is not strictly guaranteed; and (2) it only focuses on how to drive the tracking error to zero and never considers how to achieve zero tracking error optimally with respect to a meaningful performance index. To guarantee finite-time convergence, some approaches such as second-order SMC, terminal SMC, nonsingular terminal SMC [21, 22, 28] have been applied to missile guidance law design. However, the optimal convergence pattern of the tracking error was not taken in to account in these approaches, and the resultant guidance law was formulated in a complex form since these approaches used complex desired error dynamics at the design stage.

Motivated by these observations, this Note aims to investigate the optimal convergence pattern of the tracking error and propose an optimal error dynamics that achieves this optimal pattern as well as guarantees the finite-time convergence for guidance law design.

III. Optimal Error Dynamics

In this section, we first derive the proposed optimal error dynamics by using Schwarz's inequality and then analyze the properties as well as the potential significance of the proposed approach.

A. Derivation of the Proposed Optimal Error Dynamics

In this subsection, we discuss the optimal error dynamics for missile guidance law design. Suppose that the general tracking problem of missile guidance law design is given by (5). Then the optimal error dynamics is provided by Theorem 1.

Theorem 1. Suppose that the system equation is given by (5) and the desired error dynamics is chosen as

$$\dot{\varepsilon}(t) + \frac{\Gamma(t)}{t_{go}}\varepsilon(t) = 0 \tag{8}$$

where

$$\Gamma(t) = \frac{t_{go} R^{-1}(t) g^2(t)}{\int_t^{t_f} R^{-1}(\tau) g^2(\tau) d\tau} \tag{9}$$

Then, the resulting control input minimizes the performance index

$$J = \frac{1}{2}\int_t^{t_f} R(\tau) u^2(\tau) d\tau \tag{10}$$

where t_f represents the final time, $R(t) > 0$ the arbitrary weighting function.

Proof. The control input to achieve the selected error dynamics (8) is determined by substituting (8) into (5) as

$$u(t) = -\frac{\Gamma(t)}{g(t) t_{go}}\varepsilon(t) = -\frac{R^{-1}(t) g(t)}{\int_t^{t_f} R^{-1}(\tau) g^2(\tau) d\tau}\varepsilon(t) \tag{11}$$

Next, we seek to prove that control input (11) is the optimal solution of optimal control problem

$$\min_{u} \ J = \frac{1}{2} \int_{t}^{t_f} R(\tau) u^2(\tau) d\tau \tag{12}$$

subject to

$$\dot{\varepsilon}(t) = g(t) u(t), \quad \varepsilon\left(t_f\right) = 0 \tag{13}$$

Imposing the integration from t to t_f on both sides of (5) gives

$$\varepsilon\left(t_f\right) - \varepsilon(t) = \int_{t}^{t_f} g(\tau) u(\tau) d\tau \tag{14}$$

Since $\varepsilon\left(t_f\right) = 0$, introducing a slack variable $R(t)$ renders (14) to

$$-\varepsilon(t) = \int_{t}^{t_f} g(\tau) R^{-1/2}(\tau) R^{1/2}(\tau) u(\tau) d\tau \tag{15}$$

Applying Schwarz's inequality to preceding equation yields

$$[-\varepsilon(t)]^2 \leq \left[\int_{t}^{t_f} R^{-1}(\tau) g^2(\tau) d\tau\right]\left[\int_{t}^{t_f} R(\tau) u^2(\tau) d\tau\right] \tag{16}$$

Rewriting inequality (16) as

$$\frac{1}{2} \int_{t}^{t_f} R(\tau) u^2(\tau) d\tau \geq \frac{[-\varepsilon(t)]^2}{2\left[\int_{t}^{t_f} R^{-1}(\tau) g^2(\tau) d\tau\right]} \tag{17}$$

which gives a lower bound of the performance index.

According to Schwarz's inequality, the equality of (17) holds if and only if there exists a constant C such that

$$u(t) = C R^{-1}(t) g(t) \tag{18}$$

Substitution of (18) into (14) results in

$$-\varepsilon(t) = C \int_{t}^{t_f} R^{-1}(\tau) g^2(\tau) d\tau \tag{19}$$

Solving (19) for C gives

$$C = \frac{-\varepsilon(t)}{\int_{t}^{t_f} R^{-1}(\tau) g^2(\tau) d\tau} \tag{20}$$

Substituting (20) into (18) gives the optimal control input as

$$u(t) = -\frac{R^{-1}(t)\,g(t)}{\int_t^{t_f} R^{-1}(\tau)\,g^2(\tau)\,d\tau}\varepsilon(t) \tag{21}$$

which is identical with (11). QED.

Remark 1. In our derivation, the ideal missile dynamics assumption is utilized for simplicity. However, the proposed approach can be extended to the case when considering non-ideal missile dynamics in guidance law design. For example, assume that the autopilot is a first-order system and let τ be the time constant, the system equation can then be obtained as

$$\begin{aligned} \dot{\varepsilon}(t) &= g(t)\,u(t) \\ \dot{u} &= -\tfrac{1}{\tau}u + \tfrac{1}{\tau}u_c \end{aligned} \tag{22}$$

where u_c denotes the guidance command signal. Then, following similar procedures shown above, one can derive the optimal error dynamics for system (22). Additional guidance synthesis to minimize the miss distance due to autopilot lag without using autopilot system in guidance law design can also be found in [35–37].

B. Discussion of the Proposed Optimal Error Dynamics

One of the interesting points of the proposed optimal error dynamics is that it guarantees finite-time convergence since it is obtained by directly solving the finite-time optimal tracking problem. Also, the optimal error dynamics (8) is given in a similar form to the existing desired error dynamics (6). The only difference is the proportional gain: a constant term k in the existing method and a time-varying term $\Gamma(t)/t_{go}$ in the proposed method. Unlike the usual one, the proposed proportional gain changes from an initial small value to a final infinite value as the time-to-go goes to zero, that is

$$\lim_{t_{go}\to 0}\frac{\Gamma(t)}{t_{go}} = \infty \tag{23}$$

According to the values of $\Gamma(t)$, the increasing pattern of the proposed proportional gain further changes. Hereafter, let us discuss the characteristics of $\Gamma(t)$. From (9), $\Gamma(t)$ can be rearranged as

$$\Gamma(t) = \frac{\phi(t)}{\left(\int_t^{t_f}\phi(\tau)\,d\tau/t_{go}\right)} \tag{24}$$

where $\phi(t) = R^{-1}(t)\,g^2(t)$ for convenience.

The function of $g(t)$ is given by the missile guidance problem under consideration and the weighting function $R(t)$ is the design parameter. According to different selections of $R(t)$, the time-varying term $\Gamma(t)$ changes differently. In the following, we reveal that $\Gamma(t) > 0$ for all the time during the homing engagement in Proposition 1. Proposition 2

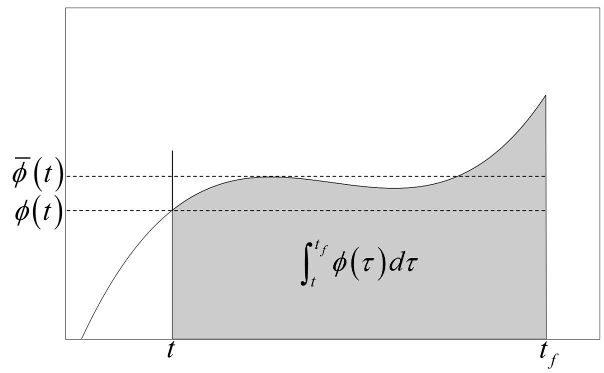

Fig. 2 The graphical interpretation of $\Gamma(t)$.

provides the limiting values of $\Gamma(t)$ for different $\phi(t)$.

Proposition 1. For given $g(t)$ and $R(t)$, $\Gamma(t)$ is always greater than zero as $\Gamma(t) > 0$.

Proof. By definition, $R(t) > 0$ and $g(t) \neq 0$. From (24), it is obvious that the numerator is positive because of $\phi(t) = R^{-1}(t) g^2(t) > 0$. Also, the denominator is positive since the integration of positive function gives positive value. Accordingly, $\Gamma(t) > 0$. QED.

Proposition 2. When the pattern of $\phi(t)$ keeps a constant, the equality $\Gamma(t) = 1$ holds. If the pattern of $\phi(t)$ decreases as $t \to t_f$, then $\Gamma(t) > 1$. If the pattern of $\phi(t)$ increases as $t \to t_f$, we have $\Gamma(t) < 1$.

Proof. In (24), the denominator can be considered as the average value of $\phi(t)$, denoted by $\bar{\phi}(t)$, during the remaining time of interception (i.e. t_{go}), as shown in Fig. 2. Accordingly, the time-varying term $\Gamma(t)$ is a ratio of the current value $\phi(t)$ to the average value $\bar{\phi}(t)$. If the term of $\phi(t)$ is given by a constant value, then the $\Gamma(t)$ is unity as $\Gamma(t) = 1$ since $\phi(t) = \bar{\phi}(t)$. If the term $\phi(t)$ decreases as $t \to t_f$, $\Gamma(t)$ is greater than unity due to $\phi(t) > \bar{\phi}(t)$. Conversely, if the term of $\phi(t)$ has an increasing pattern, then $\Gamma(t)$ is less than unity because of $\phi(t) < \bar{\phi}(t)$. QED.

It follows from (8) that $\Gamma(t) \geq 1$ is desirable to ensure a stable and a fast convergence rate at the initial time. Accordingly, Proposition 2 reveals that it is desirable to impose a constant value or a decreasing pattern on the function $\phi(t)$. As shown in (9), the computation of $\Gamma(t)$ contains of the integration of $\phi(t)$, which is given as a function of $g(t)$ and $R(t)$. If $\phi(t)$ is given by a closed-form function, $\Gamma(t)$ can be obtained analytically or numerically. Even though $g(t)$ is not given by a closed-form function due to the nature of the guidance problem, we can make $\phi(t)$ to be a closed-form function through appropriate choice of $R(t)$. Under this condition, since the physical meaning of the performance index shown in (10) changes according to choice of $R(t)$, the chosen optimal error dynamics only considers the minimization of a specific performance index.

For a specific mission, the function $g(t)$ is fixed. Thus, by properly choosing the weighting function $R(t)$ for different objectives, the error dynamics is determined by Theorem 1. Here, we provide two special cases. In the case of

$R(t) = 1$, the term $\Gamma(t)$ is given by

$$\Gamma(t) = \frac{t_{go} g^2(t)}{\int_t^{t_f} g^2(\tau)\, d\tau} \tag{25}$$

The optimal error dynamics with $\Gamma(t)$ shown in (25) minimizes the performance index

$$J = \frac{1}{2} \int_t^{t_f} u^2(\tau)\, d\tau \tag{26}$$

which provides an energy optimal guidance law.

In addition, if we choose the weighting function as $R(t) = g^2(t)/t_{go}^{K-1}$ with $K \geq 1$, then $\phi(t)$ is given by a function of time-to-go, regardless of $g(t)$, as

$$\phi(t) = t_{go}^{K-1} \tag{27}$$

In this case, the term $\Gamma(t)$ is a constant value as

$$\Gamma(t) = K \tag{28}$$

And the desired error dynamics minimizes the performance index

$$J = \frac{1}{2} \int_t^{t_f} \frac{g^2(\tau)}{\left(t_f - \tau\right)^{K-1}} u^2(\tau)\, d\tau \tag{29}$$

It is obvious that performance index (29) with $g(t) = 1$ and $K = 1$ becomes the energy minimization one. The most interesting point under this desired error dynamics is that the closed-form solution of the tracking error is readily obtained since the desired error dynamics in such a special case is given by the Cauchy-Euler equation as

$$\varepsilon(t) = \varepsilon(t_0)\, t_{go}^K \tag{30}$$

In practice, this optimal error dynamics is useful due to the fact that the decreasing pattern of the tracking error is predictable as a function of time-to-go and the desired error dynamics is given by a simplified form. Therefore, in the next section, we will apply this desired error dynamics to various missile guidance problems for simplicity and practicality.

C. Potential Significance of the Proposed Optimal Error Dynamics

In this subsection, we discuss the potential significance of the findings in this Note. First, the proposed results provide a way to improve existing missile guidance laws based on nonlinear controls. For example, existing missile guidance laws using SMC are generally composed of two terms as

$$a_M = a_M^{eq} + a_M^{dis} \tag{31}$$

where a_M^{eq} and a_M^{dis} are the equivalent control part for disturbance-free dynamics and the SMC control part for compensating external uncertainties, respectively.

Using (31), some existing missile guidance laws based on SMC can be improved by converting the equivalent control parts to their optimal forms using the proposed optimal error dynamics and compensating the undesired disturbances by add-on SMC control part.

Second, the proposed results present a theoretical background to provide a meaningful performance index of existing nonlinear missile guidance laws, which has not been clearly explained so far. For example, suppose that there is a missile guidance law using desired error dynamics (6). Comparing (6) with (8), we can easily verify that the existing error dynamics is a special case of the proposed optimal error dynamics with $\Gamma(t) = kt_{go}$. Then, imposing this condition on (9) gives

$$k = \frac{R^{-1}(t)\, g^2(t)}{\int_t^{t_f} R^{-1}(\tau)\, g^2(\tau)\, d\tau} \tag{32}$$

For $t_f \to \infty$, we can obtain that the weighting function $R(t)$ satisfies

$$R(t) = g^2(t)\, e^{kt} \tag{33}$$

Substituting (33) into (10) with $t_f \to \infty$ gives the performance index of existing missile nonlinear guidance laws using (6) as

$$J = \frac{1}{2}\int_t^{\infty} g^2(\tau)\, e^{k\tau} u^2(\tau)\, d\tau \tag{34}$$

In a similar way, we can also examine the performance index of existing missile nonlinear guidance laws based on different desired error dynamics. In addition, if a nonlinear missile guidance law is designed by using the proposed optimal error dynamics, the performance index of the designed guidance law is given by (11) and its physical meaning is clear.

Third, provided that the tracking error is suitably defined, the proposed results can be applied to different missile guidance problems for various operational objectives since we suggest a unified approach of guidance law design for a generalized tracking problem. Therefore, the proposed error dynamics provides the possibility of developing various new missile guidance laws for different practical problems.

IV. Illustrative Examples

In this section, we provide some examples of how to apply the proposed optimal error dynamics in missile guidance law design. As illustrative examples, we consider guidance law design for homing, impact angle control, impact time control, and guidance to collision.

A. Zeroing Zero-Effort-Miss

In missile guidance problem, it is well-known that ZEM must be zero to guarantee a perfect interception. Let z denote the ZEM, then, one can derive the dynamics of ZEM in the linearized engagement kinematics as

$$z = y + vt_{go} + \frac{1}{2}a_{T\sigma}t_{go}^2 \tag{35}$$

Differentiating (35) with respect to time yields

$$\dot{z} = -t_{go}a_{M\sigma} \tag{36}$$

In order to achieve zero ZEM, it is natural to choose the tracking error as $\varepsilon_z = z$. In this case, we have $g(t) = -t_{go}$ and $u(t) = a_{M\sigma}$. We select the optimal error dynamics with respect to ε_z as

$$\dot{\varepsilon}_z + \frac{N}{t_{go}}\varepsilon_z = 0 \tag{37}$$

where N is a positive constant.

It follows from (10) that the corresponding performance index is obtained as

$$J = \frac{1}{2}\int_t^{t_f} \frac{1}{\left(t_f - \tau\right)^{N-3}}u^2(\tau)\,d\tau \tag{38}$$

The preceding index can be viewed as the energy minimization weighted by $1/t_{go}^{N-3}$. Accordingly, if the value of N is chosen as 3, the resulting optimal error dynamics ensures energy minimization. Additionally, we can safely predict that the designed guidance law using (37) with $N > 3$ guarantees zero terminal acceleration command since the weighting function with $N > 3$ becomes infinite as $t_{go} \to 0$. This property provides the missile with guaranteed operational margins to cope with the undesired disturbances when the missile approaches the target.

Substituting (37) into (36) gives the optimal solution as

$$a_{M\sigma} = N\frac{z}{t_{go}^2} \tag{39}$$

Based on (4) and (35), one can derive the final guidance command as

$$a_{M\sigma} = NV_c\dot{\sigma} + \frac{N}{2}a_{T\sigma} \tag{40}$$

The obtained guidance command shown in (40) is the well-known augmented PNG [2].

Remark 2. For the considered example, the time-to-go power weighting function is utilized and the resulting guidance gain N is constant. Obviously, one can choose other weighting functions to accomplish different mission objectives. For example, in order to reduce the sensitivity with respect to the initial heading error to avoid an abrupt change during handover, a weighting function that provides a larger initial value can be designed as

$$R(t) = \frac{1}{t_{go}} + bt_{go}, \quad b > 0 \tag{41}$$

It is clear that the term bt_{go} dominate over $1/t_{go}$ during the initial time and thus can help to reduce or alleviate the sensitivity to the initial heading error. Following Theorem 1, the resulting optimal error dynamics in this case is given by

$$\dot{\varepsilon}_z + \frac{N_1(t)}{t_{go}}\varepsilon_z = 0 \tag{42}$$

where the time-varying guidance gain $N_1(t)$ is determined by

$$N_1(t) = \frac{2b^2 t_{go}^4}{\left(1 + bt_{go}^2\right)\left[bt_{go}^2 - \ln\left(1 + bt_{go}^2\right)\right]} \tag{43}$$

For surface-to-air missiles, the aerodynamic maneuverability exponentially decreases with the increasing of the flight altitude due to air density. In order to shape the guidance command against the loss of maneuverability, the weighting function can then be designed as

$$R(t) = \frac{1}{e^{mt_{go}} + n} \tag{44}$$

where m and n are constants. According to Theorem 1, the resulting optimal error dynamics in this case in formulated as

$$\dot{\varepsilon}_z + \frac{N_2(t)}{t_{go}}\varepsilon_z = 0 \tag{45}$$

where the time-varying guidance gain $N_2(t)$ is determined by

$$N_2(t) = \frac{3m^3 t_{go}^3\left(e^{mt_{go}} + n\right)}{\left(3m^2 t_{go}^2 - 6mt_{go} + 6\right)e^{mt_{go}} + m^3 n t_{go}^3 - 6} \tag{46}$$

B. Impact Angle Control

Constraining the impact angle is often desirable in terms of increasing the warhead effectiveness as well as the kill probability for both anti-ship and anti-tank missiles since it enables the interceptor to attack a vulnerable spot on a target. To achieve this, additional constraint on zero impact angle error needs to be satisfied. Therefore, when controlling the impact angle, a guidance law providing zero ZEM in conjunction with an additional command term nullifying the impact angle error is considered as $a_M = a_{M_0} + a_{IA}$. For convenience, we use the optimal zero ZEM guidance law shown in (40) as a_{M_0} for illustration.

Compared with missiles, both tanks and ships are slowly moving targets and thus can be assumed to be stationary in guidance law design, e.g. $a_{T\sigma} = 0$, $V_c \approx V_M$ in (40). This assumption is widely-accepted in impact angle control guidance law design in previous works. For stationary targets, the optimal guidance law for zeroing ZEM is given by

$$a_{M_0} = NV_M\dot{\sigma} \tag{47}$$

According to [20], the final flight path angle governed by (47) is

$$\gamma_{M_f} = \frac{N}{N-1}\sigma - \frac{1}{N-1}\gamma_M \tag{48}$$

Let γ_f be the desired final flight path angle. Then, the impact angle error can be defined as $\varepsilon_\gamma = \gamma_f - \gamma_{M_f}$. In order to achieve zero impact angle error, consider ε_γ as the tracking error, which gives error dynamics as

$$\begin{aligned}\dot{\varepsilon}_\gamma &= -\dot{\gamma}_{M_f}\\ &= -\frac{N}{N-1}\dot{\sigma} + \frac{1}{N-1}\dot{\gamma}_M\\ &= -\frac{N}{N-1}\dot{\sigma} + \frac{1}{N-1}\frac{a_M}{V_M}\end{aligned} \tag{49}$$

Substituting $a_M = a_{M_0} + a_{IA}$ into (49) gives

$$\dot{\varepsilon}_\gamma = \frac{a_{IA}}{(N-1)\,V_M} \tag{50}$$

In this example, the control input $u\,(t)$ and the function $g\,(t)$ match with a_{IA} and $1/(N-1)\,V_M$, respectively. For the impact angle error ε_γ, the optimal error dynamics is selected in a similar way as

$$\dot{\varepsilon}_\gamma + \frac{K}{t_{go}}\varepsilon_\gamma = 0 \tag{51}$$

where $K \geq 1$ is the guidance gain to be designed.

The corresponding performance index is obtained as

$$J = \frac{1}{2}\int_{t}^{t_f} \frac{1}{(N-1)^2 V_M^2 \left(t_f - \tau\right)^{K-1}} u^2(\tau)\, d\tau \tag{52}$$

Since the constant terms in the performance index do not affect the optimal pattern, the above performance index is identical to

$$J = \frac{1}{2}\int_{t}^{t_f} \frac{1}{\left(t_f - \tau\right)^{K-1}} u^2(\tau)\, d\tau \tag{53}$$

We can clearly observe from the preceding performance index that the weighting function with $K > 1$ becomes infinite as $t_{go} \to 0$. Therefore, one can imply that the optimal error dynamics with $K > 1$ guarantees zero impact angle guidance command at the final time. Additionally, if $K = 1$, the above performance index coincides with the energy optimal case.

From (50) and (51), the optimal solution of a_{IA} is easily obtained as

$$a_{IA} = -\frac{K(N-1)V_M}{t_{go}}\varepsilon_\gamma \tag{54}$$

Combining (47) with (54), one can derive the final guidance command for impact angle control as

$$a_M = NV_M\dot{\sigma} - \frac{K(N-1)V_M}{t_{go}}\varepsilon_\gamma \tag{55}$$

Using the kinematics equations and consider the reference frame as the impact angle frame [20], guidance law (55) can also be rewritten in a form as

$$a_M = -\frac{N(K+1)}{t_{go}^2}y - \frac{(N+K)}{t_{go}}v \tag{56}$$

If we choose $N = 3$ and $K = 1$, guidance law (56) becomes optimal guidance law (OGL) for impact angle control [31]. If the guidance gains satisfy $N \geq 3$ and $K = 1$, guidance law (56) reduces to interception angle control guidance (IACG) law [20]. If one enforces $N = K + 2$ and $K \geq 1$, guidance law (56) is identical with time-to-go weighted optimal guidance law (TWOGL) [32, 33]. Finally, if one selects $N > K + 1$ and $K \geq 1$, guidance law (56) turns out to be time-to-go polynomial guidance (TPG) law [34]. Consequently, using the proposed approach generates a generalized optimal impact angle control guidance law that covers several previous results.

C. Impact Time Control

Recently, salvo attack has been proved to be an effective way when against naval defense systems such as close-in weapon systems in many-to-one engagement scenarios. One typical solution of salvo attack is impact time control guidance. In a similar way, the impact time guidance law requires the guidance command to provide zero ZEM and nullify the impact time error as $a_M = a_{M_0} + a_{IT}$ [38]. In this case, an alternative form of optimal zero ZEM guidance law is adopted for a_{M_0}. In the linearized engagement kinematics, the relationship between the LOS rate and the lead angle is approximated by $\dot{\sigma} = -\eta/t_{go}$, with $\eta = \gamma_M - \sigma$. Also, the time-to-go is approximated as $t_{go} = r/V_M$. Applying these expressions to (47) gives

$$a_{M_0} = -\frac{NV_M\eta}{t_{go}} \tag{57}$$

Note that guidance command (57) is an alternative form of PNG for a stationary target. As stated in [24], the estimated final interception time under a_{M_0} is given by

$$t_f = t + \frac{r}{V_M}\left(1 + \frac{\eta^2}{2\left(2N-1\right)}\right) \tag{58}$$

Let t_d be the desired impact time. Then, the impact time error can be defined as $\varepsilon_t = t_d - t_f$. In order to achieve zero impact time error, consider ε_t as the tracking error, which gives error dynamics as

$$\begin{aligned}\dot{\varepsilon}_t &= -\dot{t}_f \\ &= -\frac{\dot{r}}{V_M}\left(1 + \frac{\eta^2}{2\left(2N-1\right)}\right) - \frac{r\eta\dot{\eta}}{\left(2N-1\right)V_M} - 1 \\ &= \cos\eta\left(1 + \frac{\eta^2}{2\left(2N-1\right)}\right) - \frac{r\eta\left(\frac{a_{M_0}+a_{IT}}{V_M} - \dot{\sigma}\right)}{\left(2N-1\right)V_M} \\ &= \cos\eta\left(1 + \frac{\eta^2}{2\left(2N-1\right)}\right) + \frac{\left(N-1\right)\eta^2}{2N-1} - \frac{r\eta}{\left(2N-1\right)V_M^2}a_{IT} - 1\end{aligned} \tag{59}$$

In practical flight, the lead angle $\eta = \gamma_M - \sigma$ is small and thus $\sin\eta \approx \eta$, $\cos\eta \approx 1 - \eta^2/2$. Using these approximations and neglecting the higher order term of η, (59) can be reduced to

$$\dot{\varepsilon}_t = -\frac{r\eta}{\left(2N-1\right)V_M^2}a_{IT} \tag{60}$$

In this example, we have $g\left(t\right) = -r\eta/(2N-1)\,V_M^2$ and $u\left(t\right) = a_{IT}$. The optimal error dynamics with respect to the impact time error ε_t is selected as

$$\dot{\varepsilon}_t + \frac{K}{t_{go}}\varepsilon_t = 0 \tag{61}$$

Substituting (61) into (60) gives the guidance command to nullify the impact time error as

$$a_{IT} = \frac{K\left(2N-1\right)V_M^2}{r\eta t_{go}}\varepsilon_t \tag{62}$$

Combining (57) with (62) leads to the impact time control guidance law as

$$a_M = -\frac{NV_M \eta}{t_{go}} + \frac{K(2N-1)V_M^2}{r\eta t_{go}}\varepsilon_t \tag{63}$$

It is obvious that the derived impact time guidance law consists of the PNG command and the impact time error feedback term. Using the relationships $\dot{\sigma} = -\eta/t_{go}$ and $t_{go} = r/V_M$, the above guidance command can also be rewritten as

$$a_M = NV_M\dot{\sigma} - \frac{KN(2N-1)V_M^5}{NV_M\dot{\sigma}r^3}\varepsilon_t \tag{64}$$

Note that if we choose $K = 4$ and $N = 3$, then guidance command (63) is identical to the impact time guidance law [24]. Also, in the case of $K = N + 1$, the obtained impact time guidance law becomes the guidance law proposed in [26]. Therefore, the obtained result is a generalized form of previous impact time guidance laws [24, 26].

In addition, it follows from (10) that the corresponding performance index of (61) is given by

$$J = \frac{1}{2}\int_t^{t_f} \frac{r^2\eta^2}{(2N-1)^2 V_M^4 \left(t_f-\tau\right)^{K-1}} u^2(\tau)\, d\tau \tag{65}$$

By neglecting the constant terms in the performance index, (65) is further reduced to

$$J = \frac{1}{2}\int_t^{t_f} \frac{r^2\eta^2}{\left(t_f-\tau\right)^{K-1}} u^2(\tau)\, d\tau \tag{66}$$

It follows from (66) that the weighting function gradually decreases as r and η decrease. This means that the acceleration command tends to increase as r and η decrease. This phenomena is not desirable for guaranteeing a finite guidance command. In order to increase the overall weighting value as $t_{go} \to 0$, it is clear that the order of time-to-go in function t_{go}^{K-1} should be set large enough to converge faster than the decrease rate of r and η. From the observation of previous result [26], it is desirable to set the guidance gain as $K \geq N + 1$ to guarantee a finite guidance command.

D. Guidance-to-Collision for Exo-Atmospheric Interception

In the case of exo-atmospheric interception, there is no aerodynamic force and the interception trajectory is only controlled by an instantaneous rotation of the missile's body. Furthermore, as the kinetic kill vehicle, which adopts the way of direct collision to destroy the target, is becoming both technically and economically feasible for exo-atmospheric scenario, the concept of nullifying the LOS rate is not preferred due to its lower speed and delayed interception [29, 30]. Recently, the concept of guidance-to-collision was proposed in [29] to accomplish direct collision for exo-atmospheric engagement. Guidance-to-collision is achieved by nullifying the heading error to follow a straight line towards the expected collision point. In this subsection, we will show that how to apply the proposed optimal error dynamics to

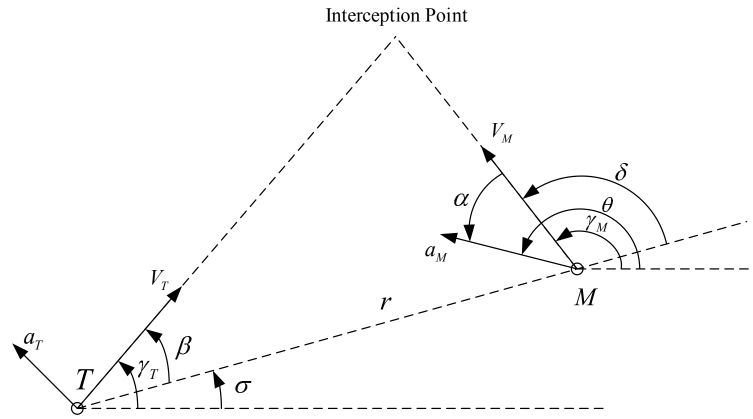

Fig. 3 Planar engagement geometry of exo-atmospheric interception.

guidance-to-collision law design.

Figure 3 depicts a schematic view of planar homing geometry for exo-atmospheric engagement. The notation θ is the interceptor body angle. The target acceleration, denoted by a_T, is perpendicular to its velocity vector while the missile axial acceleration a_M is constant provided by a mounted rocket motor along the body axis. The missile angle-of-attack is denoted by α. Other notations are the same as shown in Fig. 1.

In order to maintain the collision triangle, it is necessary to equalize between the distances traveled by the interceptor and the target perpendicular to the LOS as

$$\int_0^{t_{go}} V_M(t) \sin\delta dt = \int_0^{t_{go}} V_T(t) \sin\beta dt \tag{67}$$

At any time instant during the interception, we assume that the target performs no maneuvers. Then, the preceding equation reduces to

$$\left(V_M t_{go} + a_M t_{go}^2/2\right) \sin\delta = V_T \sin\beta t_{go} \tag{68}$$

Once a collision triangle is reached and maintained, we have

$$r = V_T t_{go} \cos\beta + V_M t_{go} \cos(\pi - \delta) + \frac{1}{2} a_M t_{go}^2 \cos(\pi - \delta) \tag{69}$$

which results in the quadratic equation

$$t_{go}^2 + \frac{2\left(V_M t_{go} \cos\delta + V_T t_{go} \cos\beta\right)}{a_M \cos\delta} - \frac{2r}{a_M \cos\delta} = 0, \quad \delta \neq \frac{\pi}{2} \tag{70}$$

To obtain the required angle δ_r of δ, we need to solve the coupled equations (68) and (70). After obtaining δ_r, the desired flight path angle that ensures the collision triangle is given by

$$\gamma_M^* = \delta_r + \sigma \tag{71}$$

Then, the heading error can be defined as

$$\varepsilon_h = \gamma_M^* - \gamma_M \tag{72}$$

In order to achieve guidance-to-collision, our objective is to nullify the heading error. Since the desired flight path angle is slowly varying, we can assume that the variation of γ_M^* is negligible as $\dot{\gamma}_M^* \approx 0$. Based on this approximation, the error dynamics of heading error is given by

$$\dot{\varepsilon}_h = -\dot{\gamma}_M = -\frac{a_M \sin\alpha}{V_M(t)} \tag{73}$$

As shown in (73), we have $g(t) = -1/V_M(t)$ and $u(t) = a_M \sin\alpha$ in this case. Here, we choose the optimal error dynamics with respect to ε_h as

$$\dot{\varepsilon}_h + \frac{K}{t_{go}}\varepsilon_h = 0 \tag{74}$$

Applying (74) to (73) gives the guidance command as

$$a_M \sin\alpha = \frac{K V_M(t)\,\varepsilon_h}{t_{go}} \tag{75}$$

From (10), the performance index of this error dynamics is given by

$$J = \frac{1}{2}\int_t^{t_f} \frac{1}{V_M^2(\tau)\left(t_f - \tau\right)^{K-1}} u^2(\tau)\, d\tau \tag{76}$$

Here, the velocity of interceptor can be approximated as $V_M(t) \approx V_M(t_0) + a_M t$ since the angle-of-attack is usually very small for guidance-to-collision interception. From performance index (76), we can readily observe that increasing the flight velocity serves to reduce the weighting function. Accordingly, it can be predicted that the acceleration command tends to increase gradually as the velocity increases. This property is not desirable for guaranteeing a finite guidance command. In order to compensate for the decreasing of the weighting function due to the velocity term, it is obvious that the guidance gain K should be set greater than the unity as $K > 1$.

If we introduce the concept of average velocity [39, 40], which is assumed to be a constant during the flight, then the guidance command can also be approximated by using the average velocity instead of the current velocity as

$$a_M \sin\alpha = \frac{K\bar{V}_M \varepsilon_h}{t_{go}} \tag{77}$$

Table 1 Initial conditions for homing engagement.

Parameters	Values
Missile-target initial relative range, $r(0)$	50 km
Initial LOS angle, $\sigma(0)$	0 deg
Missile initial velocity, $V_M(0)$	2500 m/s
Missile initial flight path angle, $\gamma_M(0)$	160 deg
Target velocity, V_T	3000 m/s
Target initial flight path angle, $\gamma_T(0)$	20 deg

where the average velocity is determined by

$$\bar{V}_M = \frac{V_M(t_0) + V_M(t_f)}{2} = V_M(t_0) + \frac{1}{2} a_M t_f \tag{78}$$

In this case, since a constant term in the performance index does not affect the optimal result, it follows from (76) that

$$J \approx \frac{1}{2} \int_t^{t_f} \frac{1}{\left(t_f - \tau\right)^{K-1}} u^2(\tau)\, d\tau \tag{79}$$

The preceding performance index with $K = 1$ can be considered as the approximated energy optimal case.

V. Simulation Results

In this section, we provide some nonlinear simulations to validate the examples shown in Sec. IV. Since example A is the well-known augmented PNG, example B is the generalized form of optimal impact angle control guidance law, and example C is the generalized form of optimal impact time guidance law, we only perform simulations for example D here. The required initial conditions are summarized in Table 1.

The simulation results, including interception trajectory, angle-of-attack command, flight path angle tracking error and control effort, with various guidance gains $K = 2, 4, 6, 8, 10$ are presented in Fig. 4, where the control effort is defined as $\int_{t_0}^{t} u^2(\tau)\, d\tau$. The results in this figure clearly reveal that guidance law (75) with larger guidance gain K leads to faster convergence speed of the heading angle tracking error but, in turn, also requires higher angle-of-attack command during the initial flight period, thereby generating more control effort. When the heading angle tracking error is close to zero, the control effort almost remains the same for all guidance gain cases, meaning that the interceptor is maintained on the collision triangle.

To further demonstrate the superiority of the proposed approach, we compare guidance law (75) with PNG and SMC guidance-to-collision law [29] in simulations. The comparison results, including interception trajectory, angle-of-attack command and control effort, obtained by these three different guidance laws are shown in Fig. 5. From this figure, it

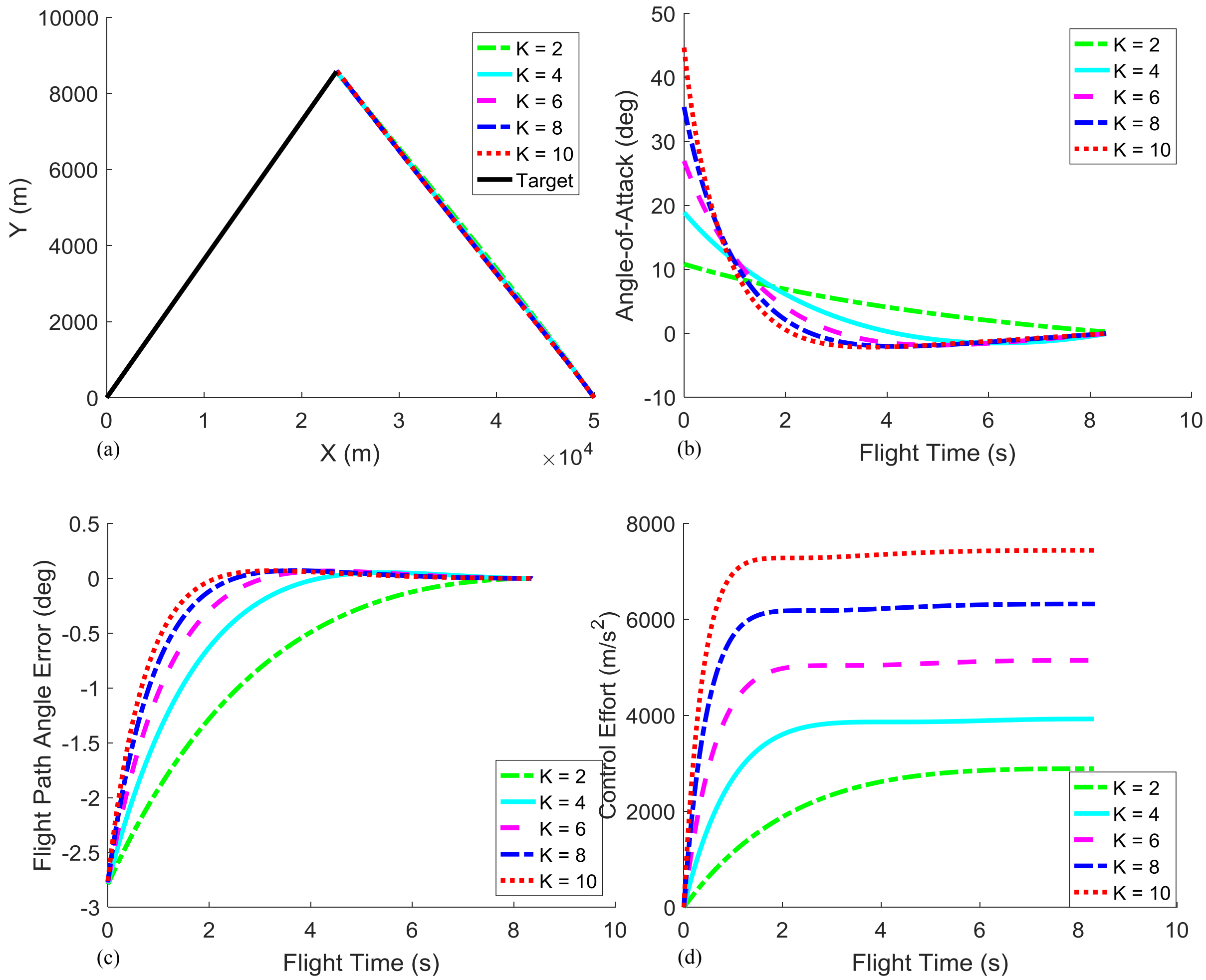

Fig. 4 Simulation results of guidance-to-collision law with various guidance gains K: (a) interception trajectory; (b) angle-of-attack command; (c) flight path angle tracking error; and (d) control effort.

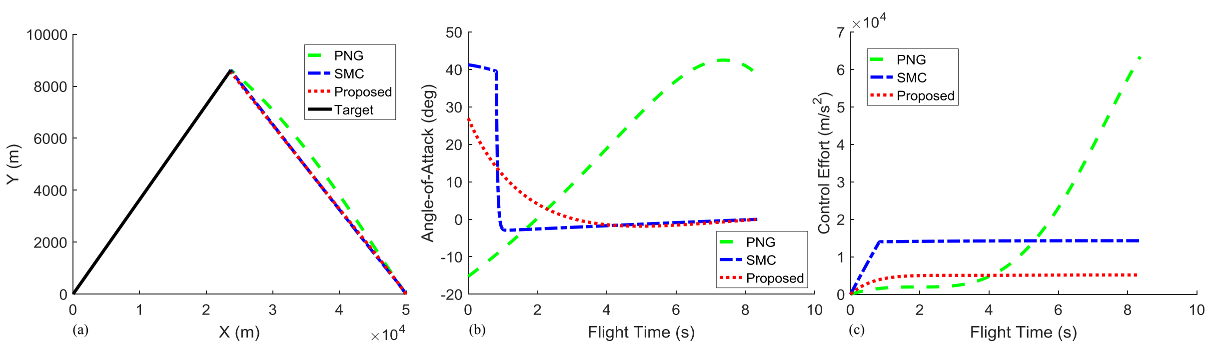

Fig. 5 Simulation results of different guidance laws: (a) interception trajectory; (b) angle-of-attack command; and (c) control effort.

can be noted that the axial acceleration of PNG does not align with the velocity vector, thereby forcing the missile to fly along a curved path to intercept the target. Unlike PNG, both SMC and the proposed guidance law hit the target following a straight line after the initial heading error is nullified. It is evident from Fig. 5 (b) that the time evolution of angle-of-attack of guidance-to-collision law remains in a small region around zero after the interceptor is maintained on the collision triangle. Therefore, guidance-to-collision laws exhibit more energetic interception than PNG as more energy is used to increase the missile flying speed. Compared to SMC guidance-to-collision law, Fig. 5 (c) reveals that the proposed guidance law (75) requires less control effort.

VI. Conclusions

This Note proposes a novel optimal error dynamics for missile guidance problems. The uniqueness of the proposed approach is that it can be applied to any operational objectives if the tracking error of missile guidance problems is suitably defined. We first reveal that the essence of missile guidance law design is a kind of finite-time tracking problem in control theory. Based on this result, a generalized error dynamics for unified missile guidance law design is proposed by solving a linear quadratic optimal control problem through Schwarz's inequality. As illustrative examples, we provide the applications of the proposed results to four well-known missile guidance problems, including zero ZEM guidance, impact angle guidance, impact time guidance and guidance-to-collision.

The proposed results are believed to have an academic significance as well as a practical one. By using the proposed results, we can exploit advantages of both approaches when designing missile guidance laws since any existing nonlinear guidance laws can be extended to their corresponding optimal version. The proposed results also provide the theoretical background to evaluate the performance index of existing nonlinear missile guidance laws. As we suggest a unified approach for guidance law design, various new missile guidance laws are expected to be developed by applying the proposed approach to practical problems.

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Optimality of error dynamics in missile guidance problems

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