Simplified Generalised Drift Velocity Correlation for Elongated Bubbles in Liquid in Pipes

Aniefiok Livinus, (corresponding author)
Oil and Gas Engineering Centre, School of Water, Energy and Environment, Cranfield University, MK43 0AL.
E-mail: a.livinus@cranfield.ac.uk
Tel.: +447466965390
Patrick G. Verdin,
Oil and Gas Engineering Centre, School of Water, Energy and Environment, Cranfield University, MK43 0AL.
E-mail: p.verdin@cranfield.ac.uk
Tel: +441234750111 x5214
Liyun Lao,
Oil and Gas Engineering Centre, School of Water, Energy and Environment, Cranfield University, MK43 0AL.
E-mail: l.lao@cranfield.ac.uk
Tel: +44123475011 x4696
Jan Nossen,
Department of Process and Fluid Flow Technology,
Institute for Energy Technology,
P.O. Box 40, NO-2027 Kjeller, Norway.
E-mail: Jan.Nossen@ife.no
Morten Langsholt,
Department of Process and Fluid Flow Technology,
Institute for Energy Technology,
P.O. Box 40, NO-2027 Kjeller, Norway.
E-mail: Morten.Langsholt@ife.no
Hans-Gunnar Sleipnæs,
Department of Process and Fluid Flow Technology,
Institute for Energy Technology,
P.O. Box 40, NO-2027 Kjeller, Norway.
E-mail: Hans-Gunnar.Sleipnes@ife.no

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Aniefiok Livinus[1], Patrick Verdin[1], Liyun Lao[1], Jan Nossen[2], Morten Lagsholt[2], Hans-Gunnar Sleipnæs[2],

1. Oil and Gas Engineering Centre, School of Water, Energy and Environment, Cranfield University, MK43 0AL.
2. Department of Process and Fluid Flow Technology, Institute for Energy Technology, P.O. Box 40, NO-2027 Kjeller, Norway.

ABSTRACT

Most of the existing drift velocity models have limitations, and sometimes low predictive capabilities, primarily because they are derived from experimental data which scarcely account for the combined effect of viscosity, surface tension and pipe inclination. Published data of drift velocity of elongated bubbles in pipes have been extracted from the open literature, and new data have been generated from Taylor bubble experiments conducted in a low pressure flow loop using nominal oil viscosities of 160cP and 1140cP in 0.099m and 0.057m internal diameter inclined pipes (1.0 to 7.5 degrees from horizontal). These data have been processed and a simplified generalised drift velocity correlation established. The evaluation of some existing elongated bubble rise velocity has also been carried out. The prediction of the drift velocity of a single elongated gas bubble in liquid in pipes can sometimes be over-estimated by 20% or more, and sometimes be under-estimated by 20% or more. It is shown that the new proposed simplified generalised correlation has an improved predictive capability when used to estimate the drift velocity of a bubble in stagnant liquid in a pipe.

Keywords: Taylor bubble, rise velocity, generalised drift velocity, elongated bubble.

Nomenclature and Abbreviation

C_o	Constant (in translational velocity)	
D	Pipe diameter	m
Eo	Eötvös Number	
Fr	Froude Number	
Fr_θ	Froude Number at pipe inclination	
g	Acceleration due to gravity	m/s^2
Mo	Morton number	
Re	Reynolds number	
N_{vis}	Viscosity number	
v_d	Drift velocity	m/s
v_d^h	Drift velocity for horizontal flow	m/s
v_d^v	Drift velocity for vertical flow	m/s
v_m	Mixture velocity	m/s
v_t	Translational velocity	m/s
Vso	Superficial velocity of oil	m/s

ρ	Density	kg/m^3
μ	Viscosity	kgm^{-1}s^{-1}
σ	Surface tension	N/m
θ	Pipe inclination to horizontal	Degree
R	Buoyancy Reynolds number	
LTT	Long Tapered Tail	
PDF	Probability Density Function	
STT	Short Tapered Tail	
STwtB	Short Tapered with detached tiny Bubbles	

INTRODUCTION

In the production and transportation of oil in pipes, intermittent plug/slug flows are multiphase flow regimes often encountered which can create significant pressure fluctuations. To better understand the phenomena and to design equipment for the production and transportation of oil resources, multiphase flow models are essential. Several of these models, for example the slug flow models, apply a number of closure relationships to link gas and liquid phases in a one-dimensional two-fluid model approach. One such closure relationship is the translational velocity for long gas bubbles in liquid flow in pipes. The slug translational velocity is the sum of the bubble velocity in stagnant liquid (i.e. drift velocity) and the maximum velocity in the slug body. Nicklin (1962) proposed the following equation:

$$v_t = C_o v_m + v_d \qquad (1)$$

where C_o is approximately 1.2 for turbulent flows and 2.0 for laminar flows, v_m is the mixture velocity (the sum of the superficial liquid and gas velocities), and v_d is the drift velocity.

By considering the potential and kinetic energy only, and ignoring the frictional and capillary effects of the falling liquid around a bubble in a vertical pipe, Dumitrescu (1943) and Davies & Taylor (1950) evaluated the bubble velocity in a liquid in a vertical tube as:

$$v_d = Fr[(gD)(1 - {\rho_g}/{\rho_l})]^{1/2} \qquad (2)$$

where, D is the pipe diameter, g is the acceleration due to gravity, and Fr is the Froude number, which represents the ratio of inertial to gravitational forces ($Fr = v_d/[(gD)(1 - {\rho_g}/{\rho_l})]^{1/2}$). These authors derived the same dimensionless group (Froude number) as 0.351 and 0.328 respectively. By applying the inviscid potential flow theory to steady gravity currents and analysing the problem of an empty cavity advancing along a horizontal pipe filled with liquid, Benjamin (1968) established the Froude number as 0.54. His study, however, ignored the effects of viscosity and surface tension.

Other known set of dimensionless groups that have been applied to estimate the rise velocity of a single bubble moving in liquid in a pipe under the influence of gravitational, inertial, viscous and interfacial forces:

Reynolds number, $$Re = \frac{\rho v_d D}{\mu} \qquad (3)$$

Eötvös number, $$Eo = \frac{(\rho_l - \rho_g)gD^2}{\sigma} \qquad (4)$$

Viscosity number, $$N_{vis} = \mu\big(gD^3(\rho_l - \rho_g)\rho_l\big)^{-0.5} \qquad (5)$$

Buoyancy Reynolds number, $$R = \frac{\big(D^3 g(\rho_l - \rho_g)\rho_l\big)^{0.5}}{\mu} \qquad (6)$$

When inertia dominates, Eo is large and Fr = 0.351, 0.328 for vertical tubes (Dumitrescu 1943 and Davies and Taylor 1950 respectively), and for horizontal tubes, Fr = 0.54 (Zukoski, 1966; Benjamin 1968). When the surface tension dominates, the bubbles do not move (Bretherton 1961; White and Beardmore 1962; Masica et al. 1964; Weber 1981). Where viscosity is essential, relationships between Fr, Eo and Morton number have been adopted.

Following the pioneer works of Dumitrescu (1943) and Davies & Taylor (1950), many prediction formulae of drift velocity of elongated bubbles in stagnant liquid have been developed (Benjamin 1961; White & Beardmore 1962; Brown 1965; Wallis 1969; Tung and Parlange 1976; Weber 1981; Bendiksen 1984; Weber et al. 1986; Hasan and Kabir 1988; Viana et al. 2003; Gokcal et al. 2008; Jeyachandra et al. 2012; and Moreiras et al. 2014). Unfortunately most of the available drift velocity models have applicability limitations and low predictive capabilities, either because they were established using a limited number of experimental data that scarcely account for the combined effects of viscosity, surface tension, and pipe inclination or, because of their formulation. In this study, the drift velocities of elongated bubbles have been gathered from numerous sources and from recent experiments conducted in a low pressure flow loop for a stagnant oil of viscosity 160cP and 1140cP for 0.099m and 0.057m internal diameter pipes inclined at angles between 1.0 and 7.5 degrees from horizontal. These data have been used to develop a simplified generalised drift velocity model with high predictive capability. The performance analysis of some of the existing models from the literature is presented first.

TAYLOR BUBBLE EXPERIMENTAL DATA

The characteristic shape of an elongated bubble has been suggested by Fagundes et al. (1999) as a means to access the transition between the sub-regimes, slug and plug flow, of intermittent flows. From the recent Taylor bubble experiments conducted in a low pressure flow loop in stagnant and flowing liquid, observed characteristics shapes of the bubbles recorded using high-speed camera are presented on Figures 1 and 2. In the stagnant liquid (Figure 1), the Taylor bubble nose always seems to be prolate spheroid (or bell-shaped) and tends to be off the centre, and close to the top of the pipe. However, for the flowing case (Figure 2), the nose tends to be closer to its own centre. Depending on the pipe inclination, oil viscosities, volume of gas injected or the size of the bubble, the bubble's tail is

either 'short-tapered with/without detached tiny bubbles' (STT and STwtB), or 'long-tapered' (LTT). For a pipe inclination below 2.5^o and as the bubble length increases, its body exhibits a wavy pattern with decreasing amplitude and its tail tends to be long tapered.

Recently, researchers (see Table 1) have conducted Taylor bubble experiments in stagnant oil viscosities up to 1000cP for pipe inclination ranging from 0° to 90°, in 10° intervals. However, particularly for moderately inclined pipes (1-8 degrees), it is believed that the new data of drift velocity presented here can contribute to improve the knowledge of the inclination dependency in drift velocity correlations.

(a) 160cp_0.099m ID pipe_1degree, (LTT)

(b) 160cp_0.099m ID pipe_2.5degree, (STT)
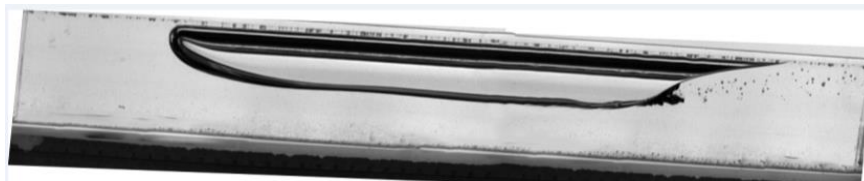
(c) 160cp_0.057m ID pipe_5degree

Figure 1: Some selected bubbles in stagnant liquid conditions.

(a) 160cp_0.099m ID pipe_2.5degree_Vso = 0.11m/s
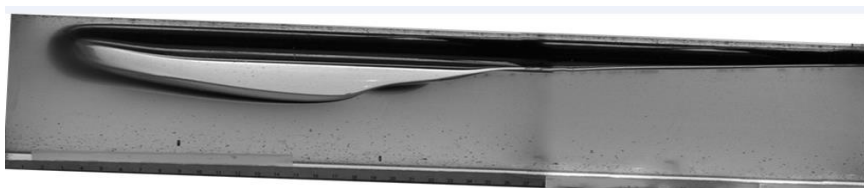
(b) 160cp_0.099m ID pipe_2.5degree_Vso = 0.21m/s

(c) 160cp_0.099m ID pipe_2.5degree_Vso = 0.31m/s

Figure 2: Some selected bubbles in flowing liquid conditions

The data used in this study were extracted from various Taylor bubble experiments available in the open literature, in addition to the data from the recent experiments conducted in a low pressure flow loop, see Table 1.

Table 1: Summary of Taylor bubble experimental data

s/No	Sources	Pipe ID (m)	Pipe angle	Liquid density, ρ_l (kg/m^3)	Liquid viscosity, μ_l (cP)	Surface Tension, σ, (N/m)
1.	(Own Data)	0.099, 0.057	$0^o - 7.5^o$	870, 960	160, 1140	0.027, 0.03
2.	Losi and Poesio (2016)	0.022	$0^o - 5^o$	860, 875, 886, 998	1, 37.5, 195.5, 804	0.0717, 0.0263, 0.0267, 0.0151
3.	Moreiras et al (2014)	0.0508	$0^o - 90^o$	873	1, 39, 66, 108, 166	0.072, 0.0275
4.	Jeyachandra et al (2012)	0.0508, 0.0762, 0.1524	$0^o - 90^o$	998, 889	1, 154, 256, 378, 574	0.072, 0.029
5.	Gokcal et al (2008)	0.0508	$0^o - 90^o$	998, 889	1, 104, 185, 296, 412, 645, 934, 1287	0.072, 0.029
6.	Sosho and Ryan (2001)	0.0127, 0.0381	$5^o - 90^o$	998, 1057, 1149, 1195, 1320, 1241, 1510	1, 3, 7.3, 36, 191, 883, 7210	0.072, 0.07, 0.026, 0.049, 0.051, 0.063, 0.066
7.	Cook and Behnia (2001)	0.032, 0.0445, 0.05	$5^o - 30^o$, 90°	998, 1113	1, 21	0.0736, 0.048
8.	Viana et al (2001)	0.0762	90°	878	424.4	0.0315
9.	Weber (1986)	0.0373, 0.0135, 0.0221		787, 1280, 1330, 1340, 1410	0.544, 51.1, 194, 518, 1830, 6120	0.022, 0.0791, 0.081, 0.087, 0.0775
10.	Brown (1965)	0.0264	90°	998, 787, 865, 881	0.942, 1, 19.42, 142.3	0.0727, 0.0244, 0.0295, 0.0305

11.	Goldsmith and Mason (1962)	0.004, 0.006, 0.008	90°	967 ,972, 974, 986, 998, 1219	91, 110, 130, 841, 1054, 5280	0.0202, 0.0204, 0.0306, 0.0311, 0.0653

PERFORMANCE OF SOME EXISTING DRIFT VELOCITY CORRELATIONS

In 1969, Wallis (1969) indicated three independent dimensionless numbers: the Froude number (Fr), the Eötvös number (Eo) and the Reynolds number (Re), and defined three regions of influence: the inertia dominant region, the viscosity dominant region, and the surface tension dominant region. He proposed a general correlation for Taylor bubble rise velocity that considers the Eötvös number and buoyancy number as presented from Equation 7 to Equation 9.

$$v_d = k \left[\frac{Dg(\rho_l - \rho_g)}{\rho_l} \right]^{0.5} \tag{7}$$

where

$$k = 0.345 \left(1 - e^{-0.01R/0.345} \right) \left(1 - e^{(3.37 - Eo)/m} \right) \tag{8}$$

$$R = \frac{\left[D^3 g (\rho_l - \rho_g) \rho_l \right]^{0.5}}{\mu} \tag{9}$$

$$\begin{cases} m = 10, & R > 250 \\ m = 69R^{-0.35} & 18 < R < 250 \\ m = 25 & R < 18 \end{cases}$$

These correlations were formulated for vertical flows only. Viana et al (2003) showed that Wallis' correlations can predict the bubble rise velocity for vertical flows with a relative error of ±20%. The performance of Wallis' (1969) model has also been checked using the data gathered for vertical flows only. Figure 3 (a) shows that the calculated Froude numbers do compare well with the majority of the measured Froude numbers. The estimated Froude numbers that do not match reasonably well are mostly from Goldsmith and Mason (1962), Sosho and Ryan (2001), Weber (1986), and Jeyachandra et al (2012). A probability density function as presented in Figure 3 (c) has been generated to clearly show the distribution of the percentage error. The majority of the data lies within a [-20%; +30%] error bandwidth. Thus, an error range of about -20% to +30% is likely to be obtained in the prediction of the Froude number using the Wallis (1969) model for vertical flows.
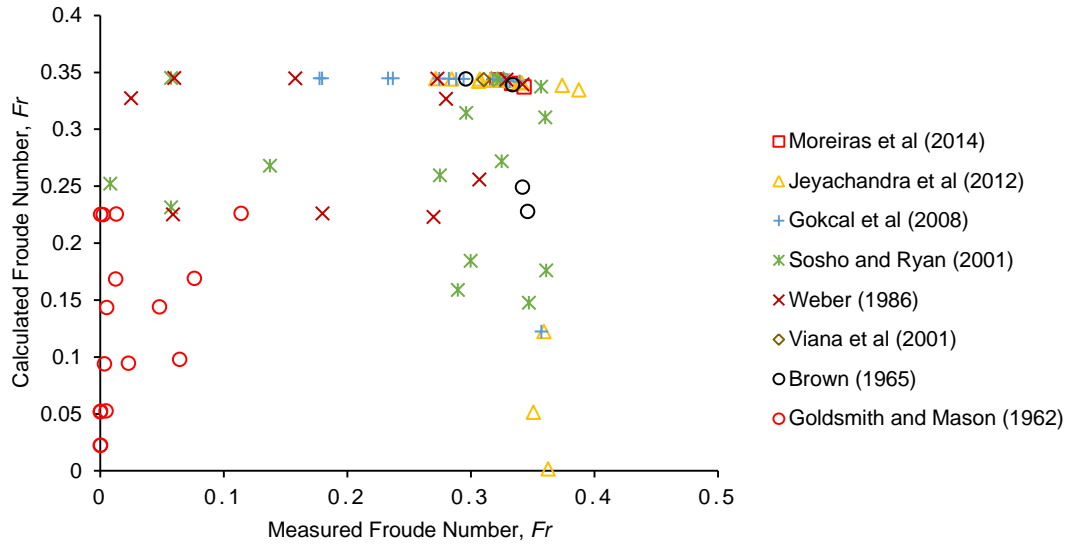
(a) Measured Froude number vs calculated Froude number using Wallis (1969) model
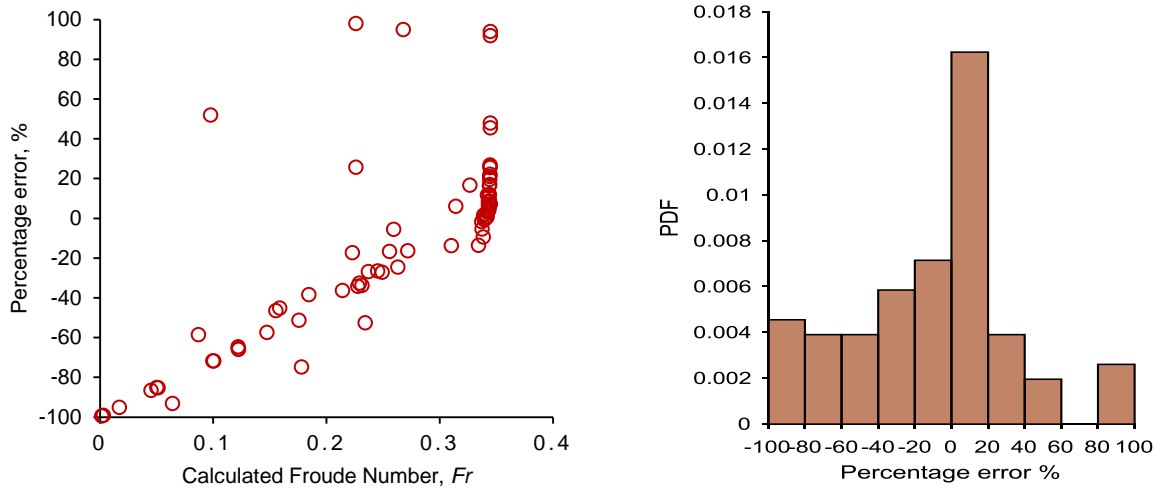
(b) Percentage error vs calculated Froude number

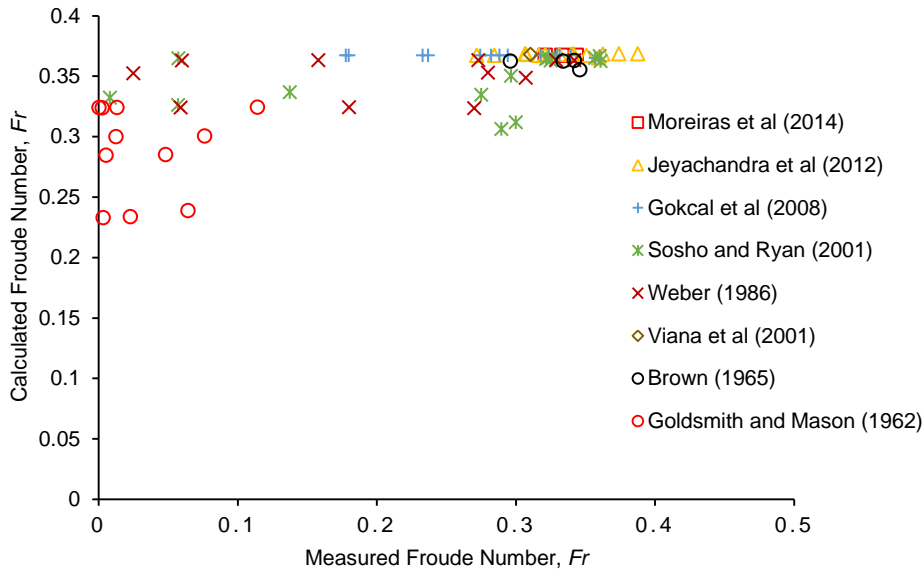(c) Probability density function of the % error

Figure 3: Performance of Wallis (1969) drift velocity correlation on gathered experimental data

Tung and Parlange (1976) expressed the rise velocity of long gas bubbles as a function of surface tension, gravity, pipe diameter and liquid density, as shown in Equation 10. Their correlation was formulated for vertical flows only.

$$Fr = \frac{v_d}{\sqrt{gD}} = \left(0.136 - 0.944\frac{\sigma}{\rho g D^2}\right)^{0.5} \qquad (10)$$

The evaluation of the performance of their correlation has been carried out using the data gathered for vertical flows only. Figure 4 (a) shows that the predicted Froude numbers do compare reasonably well with most of the experimental-based Froude numbers. The estimated Froude numbers that do not match reasonably well are mostly from Goldsmith and Mason (1962), Sosho and Ryan (2001), and Weber (1986). The percentage error bandwidth is mostly -10 to +20%, see Figure 4 (b). From the

probability density function plot shown in Figure 4 (c), it appears that there is a high chance of obtaining very good estimates of the Froude number when the Tung & Parlange (1976) model is used to predict the drift velocity of an elongated bubble in vertical flows.

(a) Measured Froude number vs calculated Froude number for Tung & Parlange (1976) model
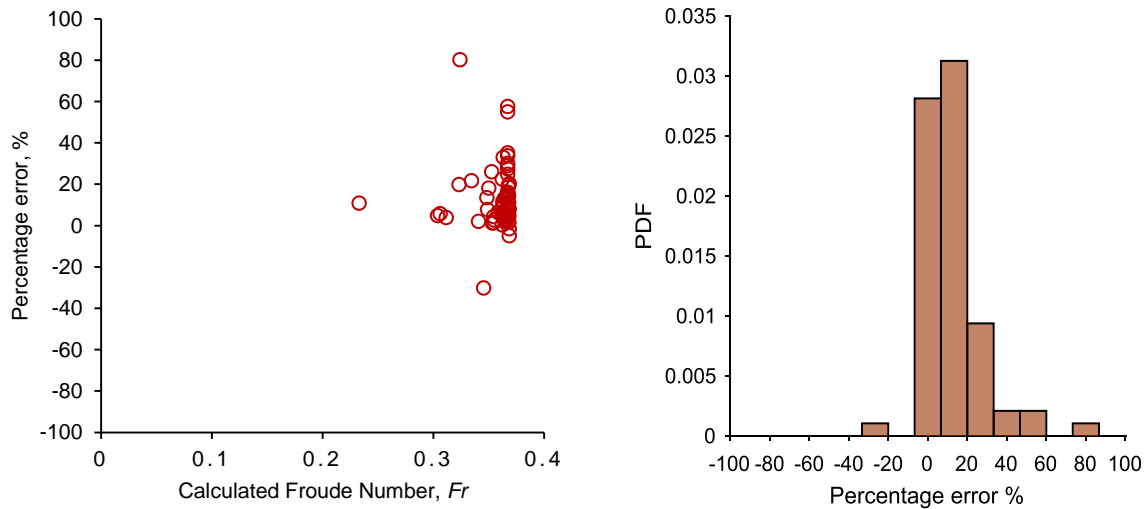
(b) Percentage error vs calculated Froude number   (c) Probability density function of the % error

**Figure 4**: Performance of Tung & Parlange (1976) drift velocity correlation on gathered experimental data

Based on the experimental data from Zukoski (1966) for liquid of low viscosities, Weber (1981) formulated a correlation for drift velocity in horizontal pipes in terms of Eötvös number:

$$\frac{v_d}{\sqrt{gD}} = 0.54 - 1.76 Eo^{-0.56} \tag{11}$$

where

$$Eo = \frac{(\rho_l - \rho_g)D^2 g}{\sigma} \qquad (12)$$

In terms of performance, this model does not provide a better match of the experimental data. As can be seen in Figure 5 (a), most predictions tend to be around a Froude number of 0.51. The errors "correlation versus measurements" range from -20% to 60%, as shown in Figure 5 (b, c). Therefore, when this model is used for the prediction of the rise velocity of bubble in stagnant liquid in pipes, there is a high chance of over-prediction of the drift velocity around 20% and sometimes as high as 40%.
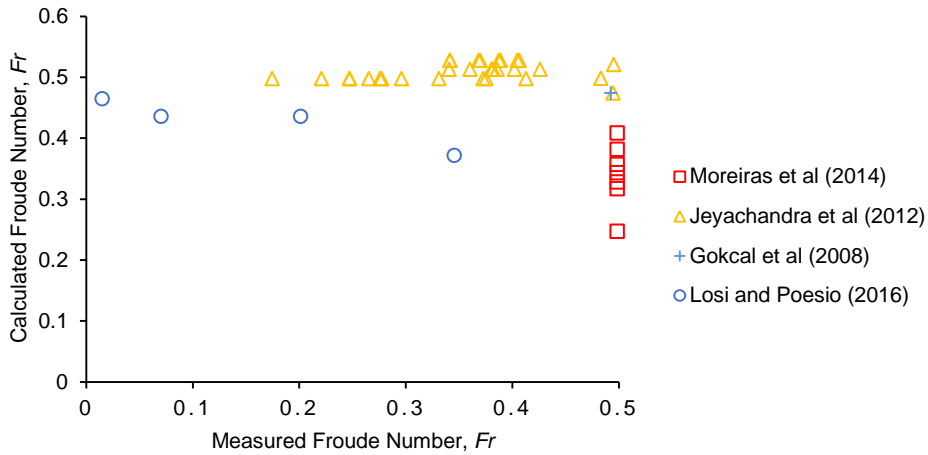
(a) Measured Froude number vs calculated Froude number using Weber (1981) model
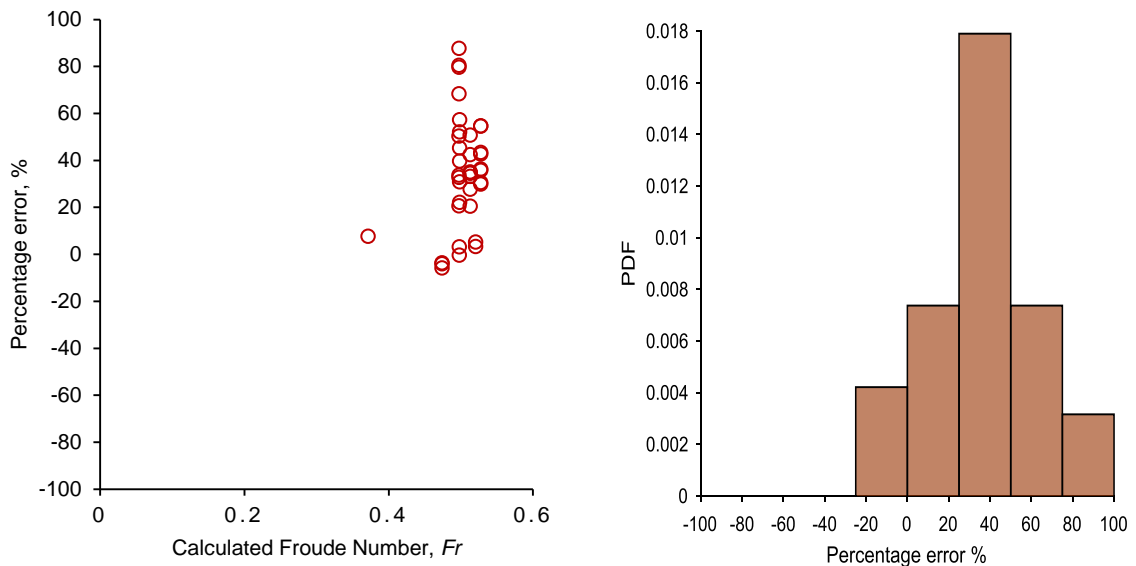
(b) Percentage error vs calculated Froude number (c) Probability density function of the % error

Figure 5: Performance of Weber (1981) drift velocity correlation on gathered experimental data

The most notable drift velocity model developed for a liquid viscosity of 1cP for any pipe inclination is the Bendiksen (1984) model. Bendiksen conducted an experimental study of single elongated bubbles in flowing water, at different inclination angles and then correlated the drift velocities for

horizontal and vertical flows. To take into account the effects of inclination, he presented a correlation for the drift velocity, in terms of Fr, at all inclination angles. The correlation combined the Froude number of the two limit cases, horizontal and vertical flows, by means of the cosine and the sine of the inclination angles:

$$v_d = v_d^h cos\theta + v_d^v sin\theta \quad (13)$$

where $v_d^h = 0.542\sqrt{gD}$ and $v_d^v = 0.351\sqrt{gD}$

As the above correlation does not account for the effects of viscosity, the comparison of the predicted Froude numbers using this model has been restricted to the gathered data for liquid viscosity of 1cP. As can be seen in Figure 6 (b, c), the performance of this correlation is quite good when using data from the literature, with percentage of errors between -20% and +30%. It appears clearly from Figure 6 (c), that a percentage error of ±20% error is likely to be present in the estimation of the rise velocity of bubble in stagnant liquid of 1cP in a pipe when the Bendiksen (1984) correlation is used.
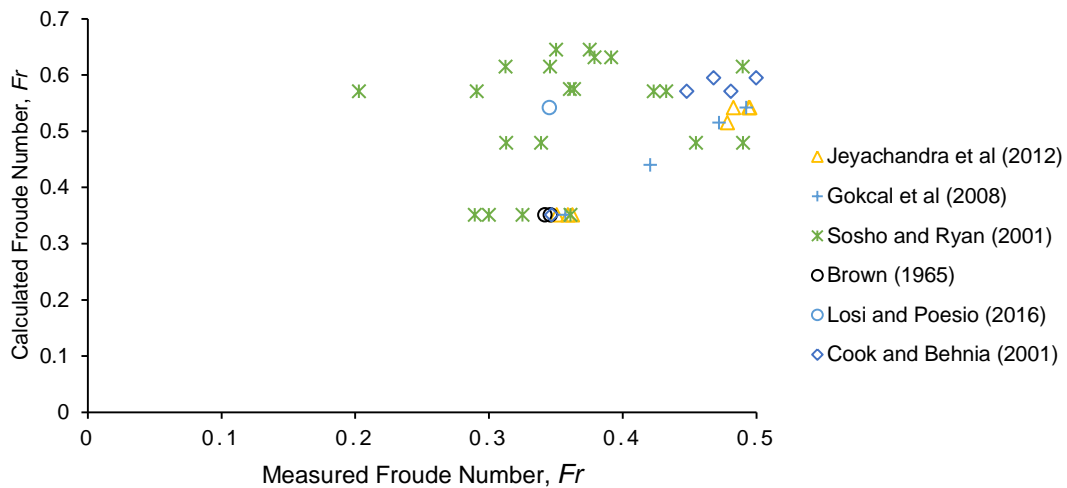
(a)   Measured Froude number vs calculated Froude number using Bendiksen (1984) model
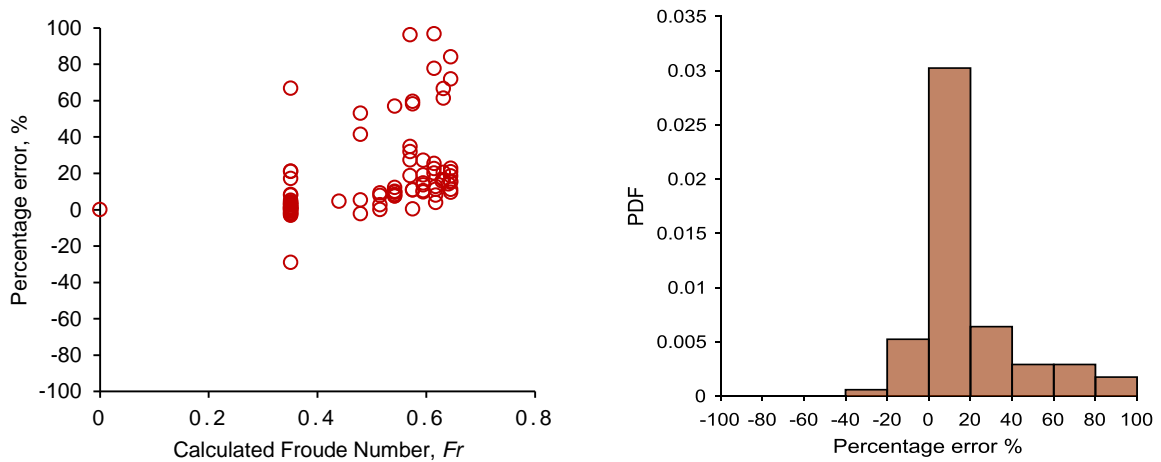
(b) Percentage error vs calculated Froude number   (c) Probability density function of the % error

Figure 6: Performance of Bendiksen (1984) drift velocity correlation on gathered experimental data

Weber et al., (1986) experimentally investigated the effects of liquid viscosity (51.1-6120 cP) on the drift velocity for inclined pipes. Their studies revealed that, depending on liquid viscosity, the drift velocity for horizontal flows can be smaller or larger than the drift velocity for vertical flows. Equations 14 and 15 represent the proposed drift velocity correlation by Weber et al., (1986).

$$v_d = v_d^h cos\theta + v_d^v sin\theta + 1.37(\Delta v_d)^{2/3} sin(\theta)(1 - sin(\theta)) \tag{14}$$

when $\Delta v_d = v_d^v - v_d^h > 0$;

if $\Delta v_d \leq 0$, then

$$v_d = v_d^h cos\theta + v_d^v sin\theta \tag{15}$$

The performance of this correlation using the gathered data shows a percentage error bandwidth between -10% and 60%, as shown in Figure 7 (b, c). The estimated Froude numbers not matching well the measured-based ones are mostly derived from data from Goldsmith and Mason (1962), Sosho and Ryan (2001), Losi and Poesio (2016), Weber (1986) and also the data generated experimentally by the authors (labelled as "own data" in Figure 7 (a)).
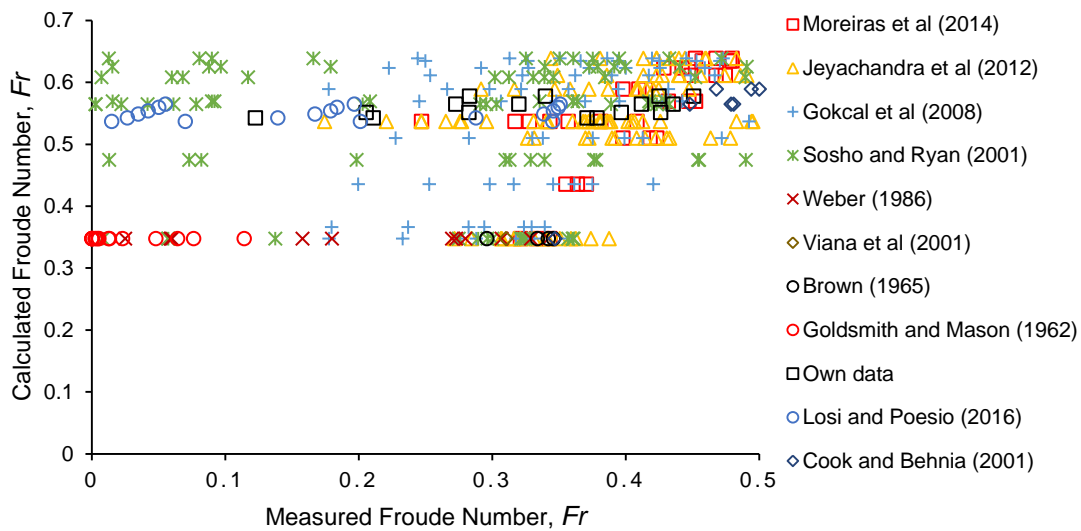
(a)   Measured Froude number vs calculated Froude number using Weber et al. (1986) model
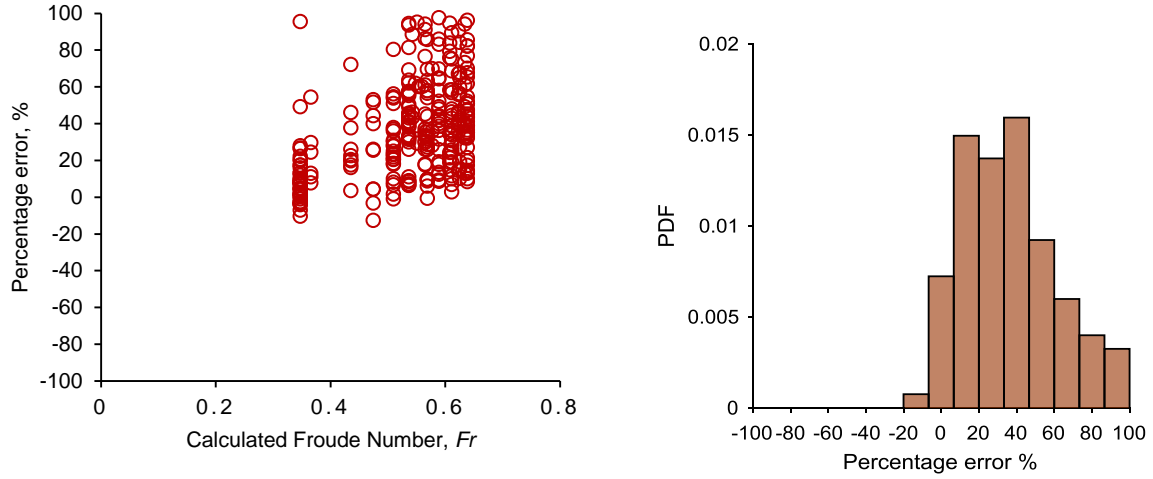
(b) Percentage error vs calculated Froude number   (c) Probability density function of the % error

Figure 7: Performance of Weber et al. (1986) drift velocity correlation on gathered experimental data

More recently, several experimental campaigns have been performed by Gokcal et al. (2008), Jeyachandra (2012), Moreiras et al. (2014). They all show that the viscosity has a large impact on the value of the drift velocity. In 2011, Jeyachandra et al. (2012) conducted experiments to determine the drift velocity of gas bubbles in stagnant liquid, with relatively high viscosities: 155 to 574cP and surface tension values 0.029-0.030N/m, in large acrylic pipes (0.0508m, 0.0762m, and 0.1524m diameters) for inclination angles ranging from 0 to 90°. They formulated a drift velocity correlation for any pipe inclination, similar to Bendiksen (1984)'s model, but in terms of Froude number, see Equation 16. The drift velocity for vertical flows was determined using the Joseph (2003) model (Equation 18). For horizontal flow, the Froude number was expressed in terms of viscosity number and Eötvös number, see Equation 17.

$$Fr_\theta = Fr^h cos\theta + Fr^v sin\theta \tag{16}$$

$$Fr^h = 0.53 e^{-13.7 N_\mu^{0.46} Eo^{-0.1}} \tag{17}$$

$$v_d^v = -\frac{8}{3}\frac{\mu}{\rho D} + \sqrt{\frac{2}{9}gD + \frac{64}{9}\frac{\mu^2}{(\rho D)^2}} \tag{18}$$

Figure 8 (a) shows that the calculated Froude number using the Jeyachandra et al. (2012) model tends to compare well with the gathered measured Froude number. This model agreed fairly well with the experimental Froude number with percentage error ranging from -20% to 40%. This is shown in Figure 8 (b, c). From the probability density function presented in Figure 8 (c), it can be seen that a 20% error is likely to be obtained in the prediction of the Froude number when using the Jeyachandra et al. (2012) model to estimate the drift velocity of bubble in stagnant liquid in pipe.
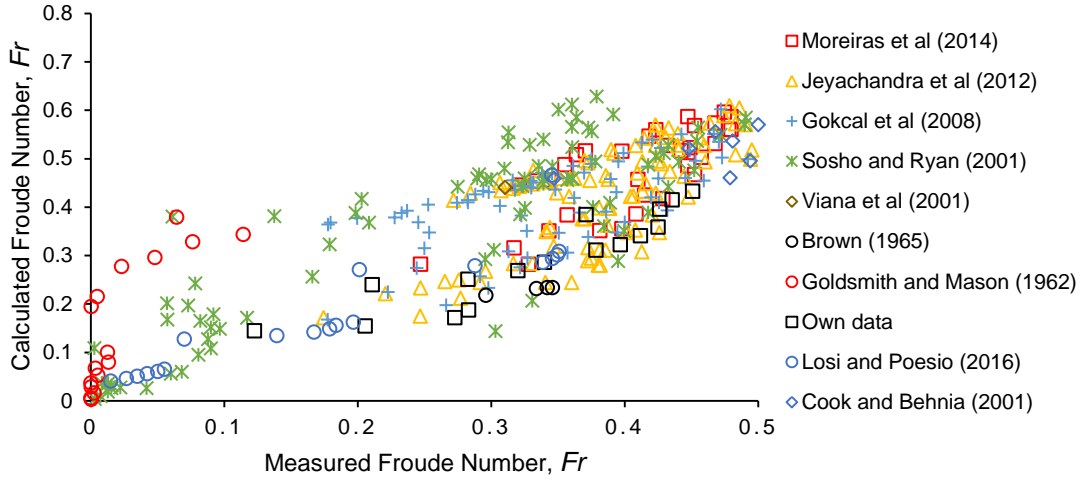
(a)   Measured Froude number vs calculated Froude number for Jeyachandra et al. (2012) model
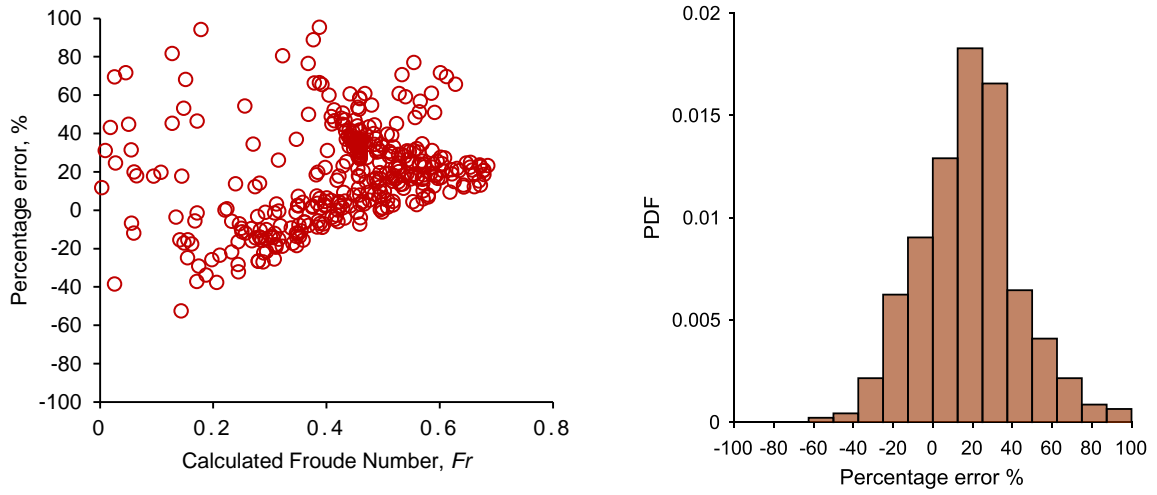
(b) Percentage error vs calculated Froude number   (c) Probability density function of the % error

Figure 8: Performance of Jeyachandra et al. (2012) drift velocity correlation on gathered experimental data

Moreiras et al. (2014) developed a new approach to model the horizontal drift velocity and, in general, the drift velocity in inclined pipes (see Equations 19 to 21). These authors used their own experimental data and limited data from the literature for pipe diameters between 0.0373m and 0.178m. Their correlation is valid for pipe internal diameter greater than or equal to 0.0373m.

$$Fr_H = 0.54 - \frac{N_{vis}}{1.886 + 0.01443 N_{vis}} \qquad (19)$$

$$Fr_V = -\frac{8}{3} N_{vis} + \sqrt{\frac{2}{9}\frac{\rho_L}{\rho_L - \rho_G} + \frac{64}{9}N_{vis}^2} - \left(\frac{\sqrt{2}}{3} - 0.35\right)\sqrt{\frac{\rho_L}{\rho_L - \rho_G}} \qquad (20)$$
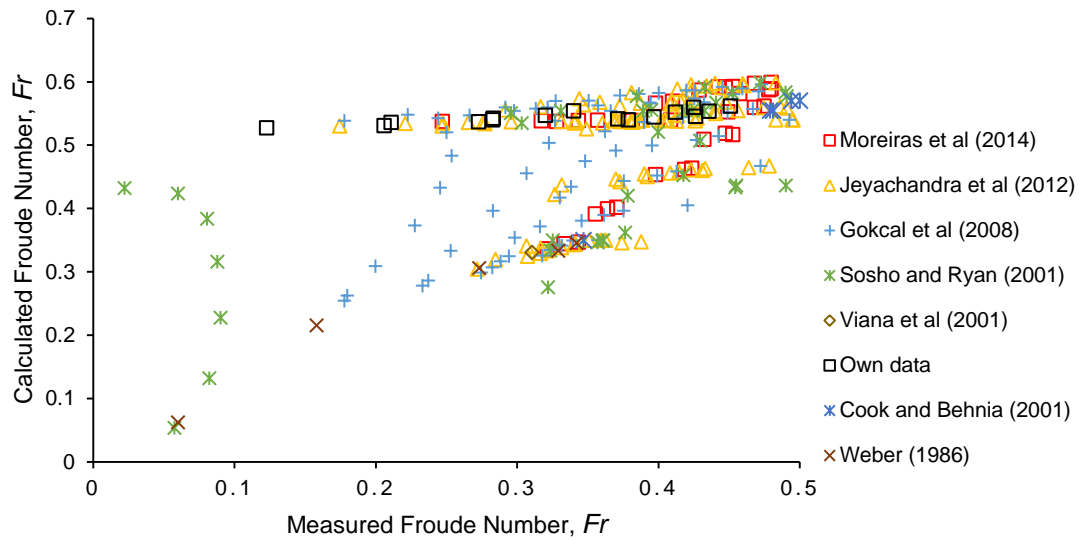
$$Fr = Fr_H cos(\theta)^a + Fr_V sin(\theta)^b + Q \qquad (21)$$

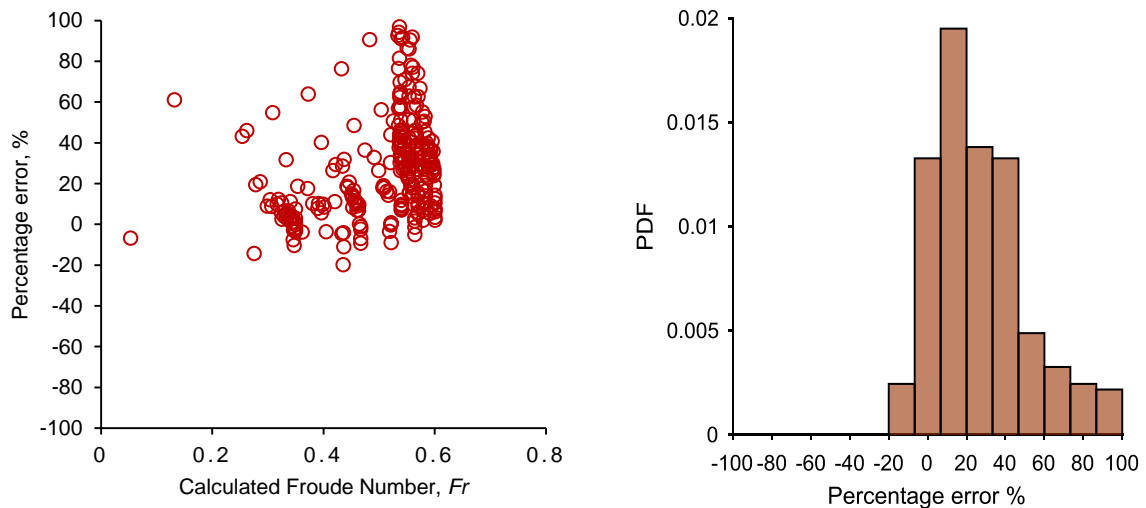$$Q = 0 \quad \text{if} \quad Fr_V - Fr_H < 0 \qquad (22)$$

$$Q = c(Fr_V - Fr_H)^d sin\theta(1 - sin\theta) \qquad \text{otherwise} \qquad (23)$$

with $\quad a = 1.2391, b = 1.2315, c = 2.1589, d = 0.70412$

The Moreiras et al. (2014) correlation using the data gathered for the specified internal pipe diameter validity shows a percentage error ranging from -20% to 40%, as shown in Figure 9 (b, c).

(a)   Measured Froude number vs calculated Froude number for Moreiras et al. (2014) model

(b) Percentage error vs calculated Froude number   (c) Probability density function of the % error

Figure 9: Performance of Moreiras et al. (2014) drift velocity correlation on gathered experimental data

Several of the non-complex existing drift velocity models have shown wide applicability limitations, and sometimes low predictive capabilities, either because they were derived from data with a narrow

range of experimental parameters or because of their formulation. The development of a simplified generalised drift velocity model with improved predictive capability from a large pool of Taylor bubble experimental data has therefore been carried out, this is described in the next section.

FORMULATON OF A NEW SIMPLIFIED DRIFT VELOCITY CORRELATION

A large number of Taylor bubble experimental data available from the literature was gathered and processed. For the sake of clarity, they are not all reported here and only the recent experimental data generated by the authors are summarised in Table 2. The Eötvös numbers, viscosity numbers and buoyancy Reynolds numbers were then calculated. Dimensionless numbers were used for the formulation of the correlation: the Froude number, the ratio of the buoyancy Reynolds number and the Eötvös number. These numbers have been considered to have one group representing the ratio of the inertial to gravitational forces (i.e. the Froude number), and another group containing the properties of the fluid.

Table 2: Experimental data

Pipe ID	Pipe angle	Liq_density	Liq_viscosity	Surf. Ten.	Drift_vel.	Froude
(m)	(deg)	(kg/m^3)	(cP)	(N/m)	(m/s)	
0.099	1.0	870	160	0.027	0.36656	0.3711
0.099	2.5	870	160	0.027	0.421	0.42621
0.099	5.0	870	160	0.027	0.4303	0.43563
0.099	7.5	870	160	0.027	0.44542	0.45093
0.057	1.0	870	160	0.027	0.2837	0.37851
0.057	2.5	870	160	0.027	0.29741	0.39681
0.057	5.0	870	160	0.027	0.30868	0.41184
0.057	7.5	870	160	0.027	0.31848	0.42492
0.099	1.0	960	1140	0.037	0.1976	0.21081
0.099	2.5	960	1140	0.037	0.27904	0.28247
0.099	5.0	960	1140	0.037	0.316	0.31989
0.099	7.5	960	1140	0.037	0.3358	0.33993
0.057	1.0	960	1140	0.037	0.092	0.12274
0.057	2.5	960	1140	0.037	0.1542	0.20572
0.057	5.0	960	1140	0.037	0.2044	0.27269
0.057	7.5	960	1140	0.037	0.2122	0.28310

Experiments were carried out for a narrow range of relatively small pipe inclinations. However, the results obtained show that the drift velocity increases with the increase of pipe inclination and pipe diameter but decreases with the increase of oil viscosity. These observations are in agreement with the findings of other researchers (e.g. Gokcal et al. (2008), Jeyachandra (2012), Moreiras et al. (2014)).
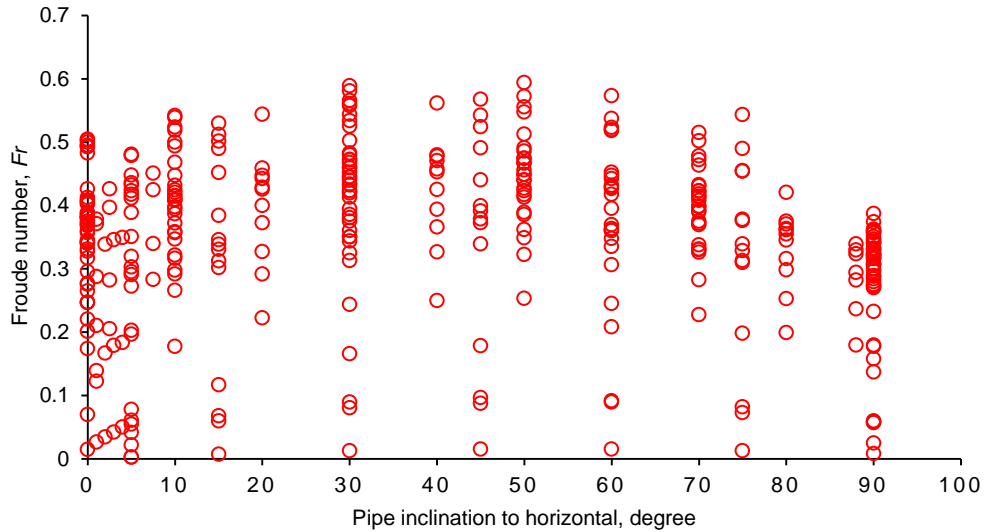
Figure 10: Froude number at different pipe inclination for the gathered data.

Figure 10 shows the Froude number, Fr, plotted against the angle of inclination from the horizontal. It appears that the Froude numbers show different levels of curve patterns: a typical curve increases from zero-degree (representing horizontal flow), reaches a wide maximum for angles of inclination between 30° and 60°, and then decreases to 90° (representing vertical flow). This is more apparent as the Froude number increases. Figure 11 clearly shows these different levels of curve patterns under various range of the ratio of Buoyancy Reynolds numbers (R) and Eötvös numbers (Eo). As the value of R/Eo decreases, Fr decreases. This is largely due to the influence of fluid viscosity and pipe diameter.
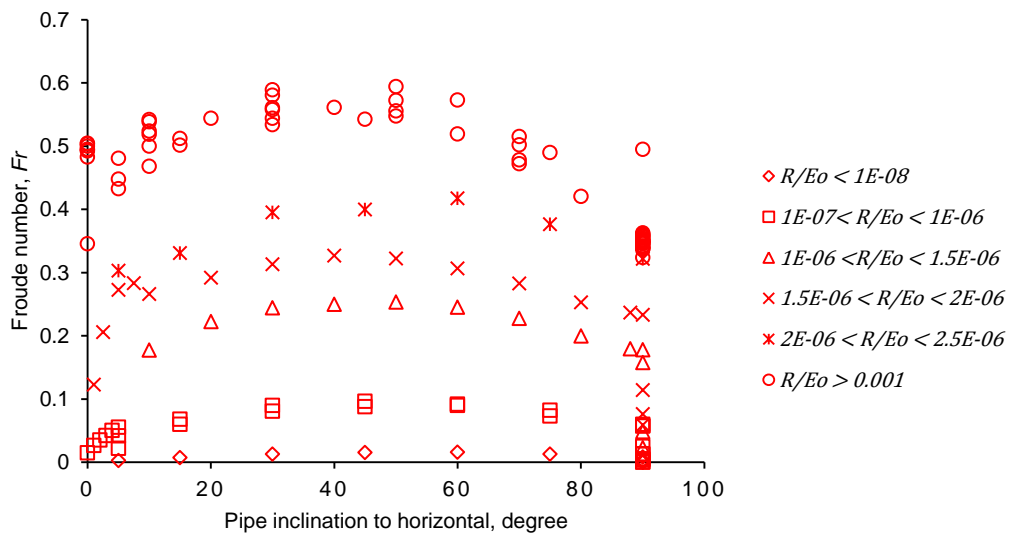
Figure 11: Froude number vs Pipe inclination at various R/Eo

A simplified generalised empirical drift velocity model was created, based on a curve fitting of the log-log relationships of the Froude number against the combination of Eötvös number and buoyancy

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Reynolds numbers. This has been established with a power law model, using a Gaussian-Newtonian algorithm in MATLAB®. Figure 12 shows the data obtained from a large pool of Taylor bubble experiments plotted in the log Fr vs log (R/Eo) formulation. The relationship shows an exponentially increasing pattern. For some data, a high deviation from the observed exponential pattern is obtained. These data are however mainly from small tubes obtained from Goldsmith and Mason (1962), and data for non-Newtonian fluids in smaller tubes (pipe diameter less than 0.012m) from Sosho and Ryan (2001). The deviation observed might be due to the dominance of capillary and viscous forces over gravity. White & Beardmore (1962) postulated that bubbles will not rise when the Eötvös number is less than 4. Therefore, the predictive capability of the new model will be very poor for fluid and pipe conditions with Eötvös number approaching this value, and also for non-Newtonian fluids in pipe diameters less than 0.012m.

A power law model, $y = ax^b + c$, was used to fit the data using the non-linear least squares regression function in MATLAB®, with the variables being a=7.928E-07, b=7.443 and c=0.3276. The best fit obtained has an R-squared value around 0.73. As mentioned previously, this is due to the few points showing a large deviation from the other measured data.
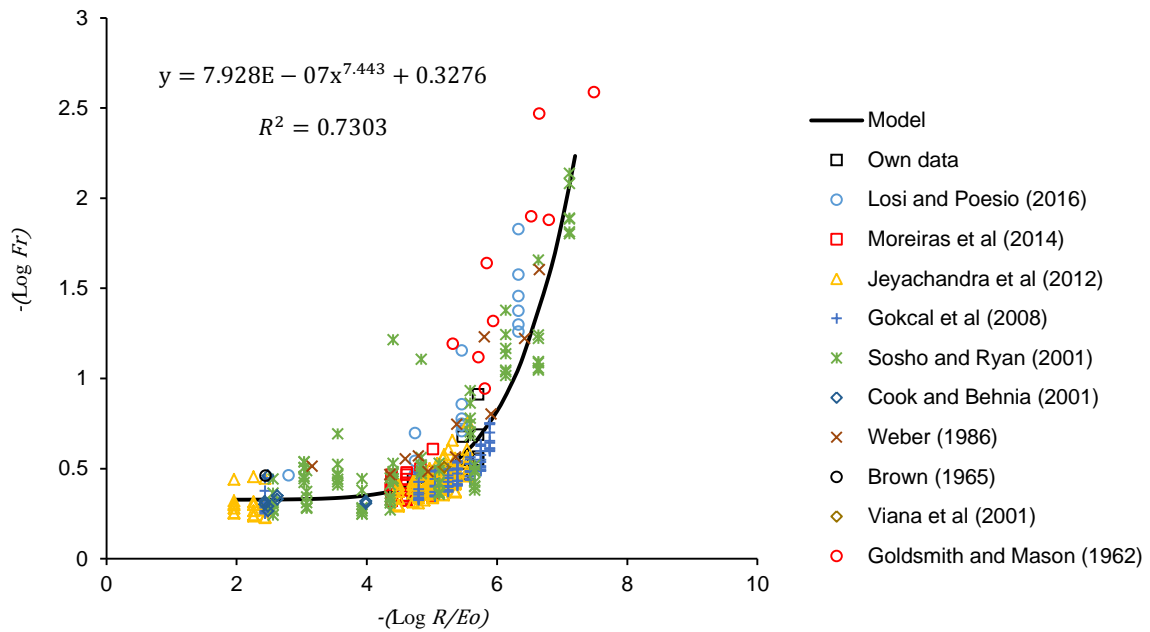
Figure 12: Curve fitting of the plot of log Fr vs log (R/Eo)

The Froude number for any liquid, Fr_l, can therefore be calculated using the equation below:

$$Fr_l = 10^{-m} \tag{24}$$

where

$$m = 7.928E - 07\left(-log_{10}\frac{R}{Eo}\right)^{7.443} + 0.3276 \tag{25}$$

To include the effect of any pipe inclination for any liquid, the Froude number is combined by means of the cosine and the sine of the inclination angles of the pipe to horizontal. This approach was first adopted by Bendiksen (1984):

$$Fr_l^\theta = Fr_l(cos\theta + sin\theta) \tag{26}$$

The drift velocity can therefore be calculated from Equation 2 using the estimated Froude number, Fr_l^θ:

$$v_d = Fr_l^\theta[(gD)(1 - {}^{\rho_g}/_{\rho_l})]^{1/2} \tag{27}$$

Equations 24 to 27 form the new models required to estimate the Froude number and the drift velocity for a single elongated gas bubble in liquid in pipe.

ASSESSMENT OF THE DEVELOPED CORRELATION

The predictive capability of the new correlation is assessed using all the data gathered. As can be seen from Figure 13 (a), there is an improved agreement with the experimental data. Approximately 80% of the data are congested within the ±20% error bandwidth. This is also represented using the probability density function as given in Figure 13 (c).
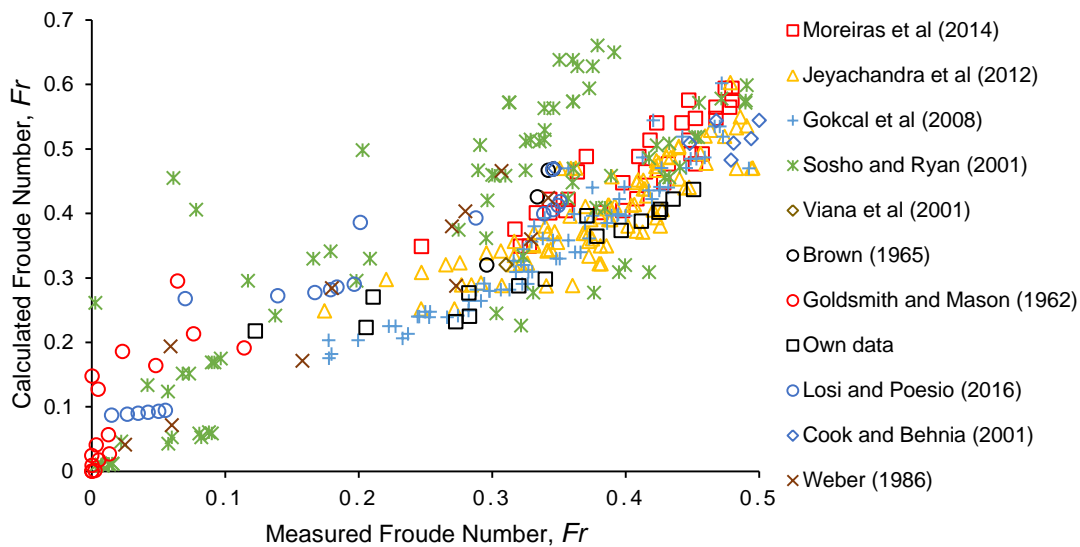
(a) Measured Froude number vs calculated Froude number using the new model
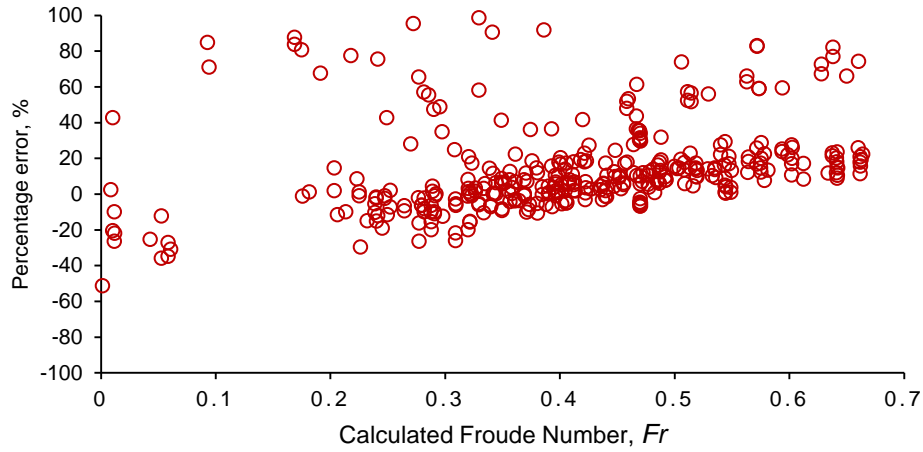
(b) Percentage error vs calculated Froude number
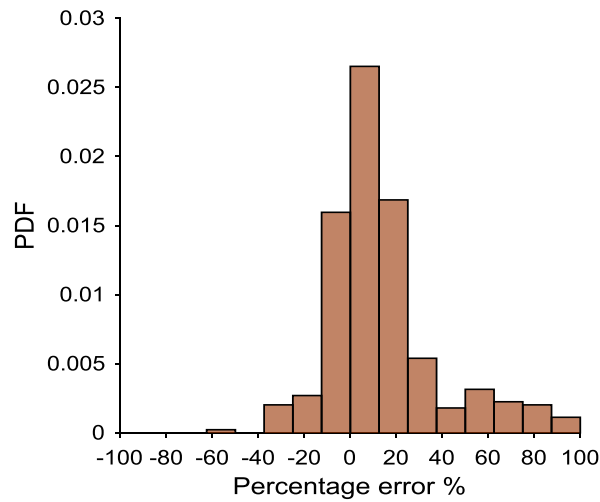
(c) Probability density function of the % error

Figure 13: Performance of the new drift velocity correlation on gathered experimental data

From this probability density function presented in Figure 13 (c), it can be seen that an error in the range [-10%, 20%] is likely to be obtained in the prediction of the Froude number when the new model is used to estimate the drift velocity of a bubble in a stagnant liquid in a pipe.

The performance of the new developed correlation was compared with the performance of each of the other correlations used in this study in their respective ranges of validity. As can be seen from the probability density function of the percentage error plots in Figure 14(a-g), the new correlation matches reasonably well with the other correlations, and sometimes performs better. However, for a liquid viscosity of 1cP, the Bendiksen (1984) model performs better than the new model, see Figure 14(e).
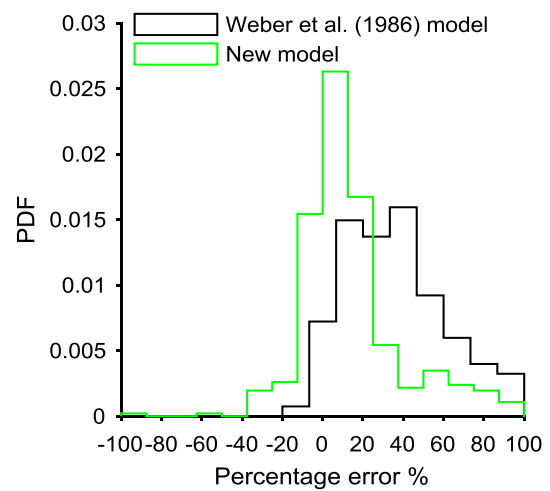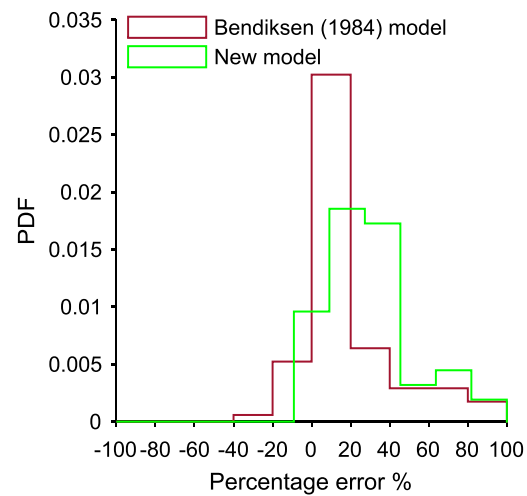
20
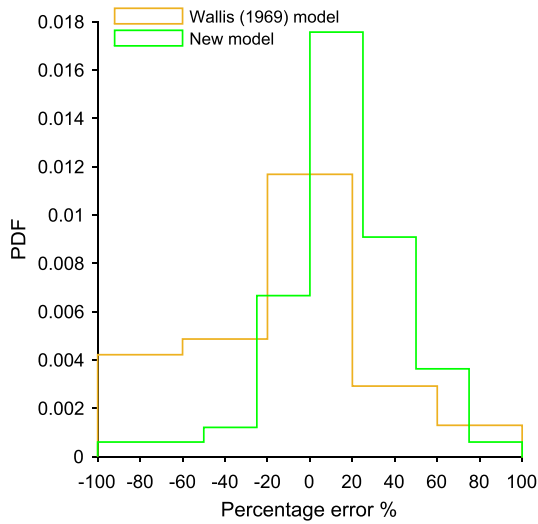
(a) Wallis (1969) model vs New model
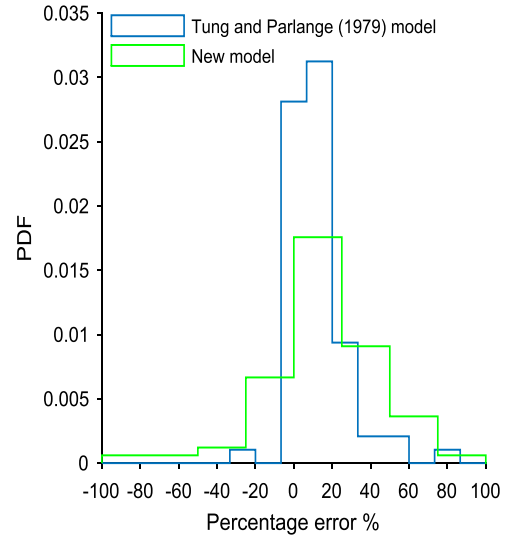
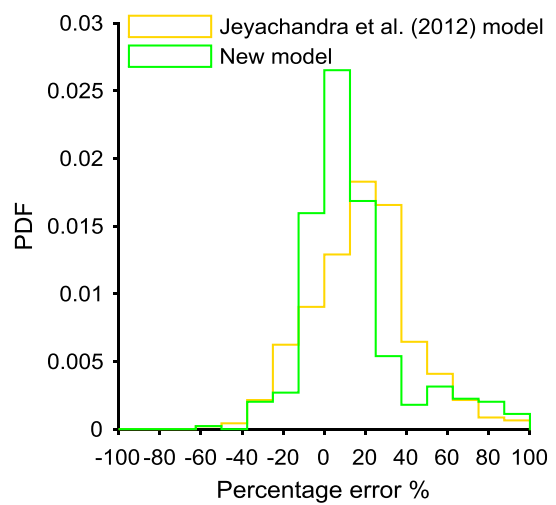(b) Tung and Parlange (1979) model vs New model
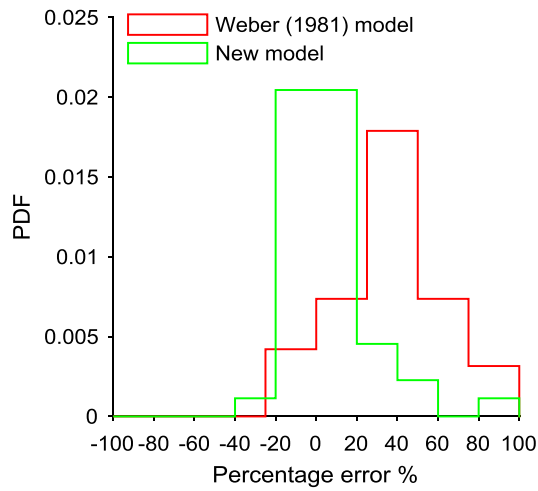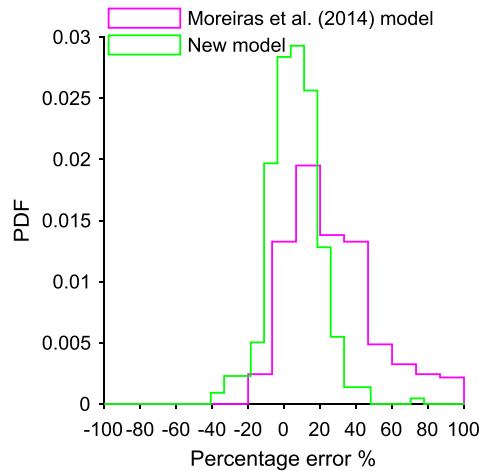
(c) Weber (1981) model vs New model

(d) Weber et al (1986) model vs New model

(e) Bendiksen (1984) model vs New model

(f) Jeyachandra et al (2012) model vs New model

(g) Moreiras et al (2014) model vs New model

Figure 14: Comparisons of the New model with other models under study in their respective ranges of validity

CONCLUSION

Several existing drift velocity models show limitations, and sometimes low predictive capabilities. This can be explained either because they were obtained with a too narrow range of experimental parameters or simply because of their formulation. The performance analyses of these models showed that the prediction of the drift velocity of a single elongated gas bubble in liquid in pipes could sometimes be over-estimated by 20% and above, and sometimes be under-estimated by 20% and above.

Taylor bubble experiments were conducted in a low pressure flow loop. The measured data, in addition to published data of drift velocity of elongated bubbles in pipes were used to develop a new simplified generalised drift velocity correlation. This new proposed formulation showed a good predictive capability under the conditions specified in this study, with approximately 80% of the experimental data located within the ±20% error bandwidth when the novel formulation was used. There is thus a high likelihood of obtaining a percentage of error in the range [-10%, 20%] when the new model is used to estimate the drift velocity of a bubble in stagnant liquid in pipe. This outperforms the existing models discussed here, apart from the Bendiksen (1984) model applied for liquid viscosity of 1cP.

ACKNOWLEDGEMENTS

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