CoA Note No. 99

THE COLLEGE OF AERONAUTICS

CRANFIELD

DYNAMICS OF FUEL IN TANKS

by

J. W. ADDINGTON

NOTE NO. 99

January, 1960

THE COLLEGE OF AERONAUTICS

CRANFIELD

Dynamics of Fuel in Tanks

- by -

J. W. Addington, B.Sc., D.C.Ae.

SUMMARY

The validity of the available theory in determining the natural frequency of the fundamental fuel sloshing mode in a rectangular tank was experimentally investigated. The results were found to agree within 5% over the whole range, and were quite accurate in the extreme "shallow and deep water" regions.

A qualitative investigation into baffle types and positioning indicates that baffles should be positioned in a region of highest velocity such that the surface waves are trapped and the energy dissipated at the shallowest depth possible. This suggests that, for a tank with varying liquid depth, a multi-perforated baffle should be employed. A total orifice area of one quarter the baffle area is suggested.

CONTENTS

1. Introduction

A large mass of fuel with a free surface in an aircraft tank may have serious dynamic effects on the aircraft, especially if a resonant mode of oscillation is allowed to develop.

The development of thinner wings has led to the removal of the tanks from the wings, where structural members acted as baffles, to the fuselage where large unbroken tank volumes exist. At the same time, higher ratios of fuel weight to aircraft gross weight have become customary. These two factors have given rise to increasing interest in aircraft dynamical problems affected by the motion of fuel. In particular, the design of aircraft (or missile) autopilots requires an accurate estimate of the natural frequencies of the sloshing modes which lie within the autopilot effective control frequency band.

Several reports (Refs. 2, 3, and 4) delve into the theoretical implications of fuel sloshing, based on the original work given in Ref. 1, but there appears to be no information regarding practical experiments to show the validity of this theory and its limitations.

From the theory for surface waves are derived the formulae given in Appendix A, and it was decided that initial testing (Phase I. of the research programme) should be carried out in an attempt to verify them. These formulae are derived from the assumption of sinusoidal motion of the free surface, and photography of surface waves (Phase II) was undertaken to check this assumption. To complete a preliminary investigation into the problems of fuel sloshing and their methods of solution some work (Phase III) on baffle types and positioning was carried out.

In his report E. W. Graham (Ref. 2) develops formulae for the forces produced by the fuel oscillation in a rectangular tank, and it would be in the logical sequence of any experimental work to verify practically the results he obtained. However, to do this, further tankage as well as a low force-measuring transducer would have had to be designed and developed, and this was impractical for this investigation.

In all the work, only the first and fundamental mode of oscillation was considered, as this mode is the one which causes largest surface waves, C of G movement, and tank forces. In even modes, standing waves are formed but these are not of high amplitude, and in higher odd modes the surface waves are quite small.

2. Apparatus and Testing Procedure

2.1. Apparatus

The test rig was primarily a moving table on which was mounted a tank; the table was oscillated in a horizontal plane by a sinsusoidal input. The layout of the rig is shown diagrammatically in Fig. 1, with photographs of the complete installation in Figs. 2 and 3.

The motion input consisted of an infinitely variable hydraulic drive (Vickers Double Power Unit) driven by an A.C. electric motor (2 H.P., 1400 R.P.M.). This was connected to the table through a variable amplitude eccentric.

The hydraulic drive was fitted with a worm gear control to give accurate speed control (within 0.1 R.P.M. from zero to 180 R.P.M.).

To achieve a simple low friction mounting for the table, a three point suspension of ball bearings running in "V" grooves was used.

To measure the frequency of oscillation, a low speed tachometer was designed and coupled to the drive shaft of the eccentric. As this piece of equipment was developed for this test rig a complete description of it is given in Appendix B.

The tank used for all testing was made of Perspex, and was 9 in. wide, 18 in. long and 9 in. deep. The walls and bottom were grooved to accomodate solid baffles which effectively vary the tank length from 6 in. to 18 in. The grooves would also accept the perforated baffles used in Phase III testing. Water was used as the liquid medium throughout the tests.

2.2. Testing Procedure

Phase I. The tank was set to a required length and depth, and oscillated through a range of frequencies. The resonant frequency was apparent by the build up of the surface wave to a maintained maximum. The test series consisted of variations of tank length from 6 in. to 18 in., and depth from 2 in. to 7 in. - in addition to this a few further variations of dimensions were used to enable a complete coverage of the depth/length ratio to be obtained.

Throughout all testing the amplitudes of oscillation were kept to a minimum to give satisfactory determination of the natural frequency, and were generally limited 0.10 in.

Phase II. Photography at 16 and 32 frames a second of selected tank sizes was carried out, while the tank was oscillated at the natural frequency determined in Phase I.

Phase III. Baffles as shown in Figs. 4 and 14 were inserted in the mid-length position of the 18 in. tank. Baffle (a) had a variable orifice to show the effect of size of aperture. The maximum orifice size obtainable was $8\frac{1}{4}$ in. by $1\frac{1}{4}$ in. The diameters of the individual apertures in Fig. 4, (b, c and d) were 2.5, 1.77 and 1.25 in. respectively.

3. Presentation of Results

Phase I.

The natural frequencies of oscillation measured on test for various tank lengths and depths are given in Table 1, and plotted in Fig. 8, alongside those calculated from the theoretical formula :

$$f = \frac{1}{2\pi} \sqrt{\frac{\pi g}{L} \tanh \frac{\pi h}{L}}$$

It will be seen from the above equation that the non-dimensional form $f\sqrt{L/g}$ can be derived as a function of (h/L).

The actual values of $f\sqrt{L/g}$ are plotted in Fig. 9, together with the theoretical values of

$$F\left(\frac{h}{L}\right) = \sqrt{\frac{1}{4\pi} \tanh \left(\frac{\pi h}{L}\right)}$$

Table 2 gives some additional results which were evaluated to ensure complete coverage of h/L range available.

Phase II.

Some selected still photographs taken from the cine film of the wave motion are produced in Fig. 10, and further diagrams produced from the cine film data are produced as Figs. 11 and 12. These diagrams were obtained using a 'single shot' projector and show the wave form at various times throughout the cycle.

Phase III.

(a) Using a variable baffle

The opening in the baffle was varied from zero to maximum, and, as an attempt to obtain some quantitative data, measurements were made of the amplitude of wave in the tank at each setting. These readings obtained for various mean depths of baffle orifice are given in Table 3, and are shown plotted in Fig. 13.

(b) Using constant baffle orifice area

By using the baffles shown in Fig. 4 (b), (c) and (d), the total orifice area was kept constant. Three similar sets of baffles were manufactured to give three different mean depths in a 4 in. depth of liquid.

Little variation could be observed between the various types and depths of the distributed areas; all satisfactorily eliminated the build up of large surface waves at the resonant frequency of both baffled and unbaffled tank lengths. Maximum damping was obtained when the one large hole was replaced by several smaller holes.

(c) Various other baffles

From the results obtained in (a) and (b) above, it was decided to construct the perforated baffle shown in Fig. 4(e). This baffle has 14 holes of 1.25 in. diameter, and it successfully kept the surface wave amplitude down to the region of 0.25 in. even for large tank oscillatory amplitudes. The tank was oscillated at all frequencies in the operating range without the build up of waves.

Other types of baffle tested consisted of a weir type baffle shown in Fig. 14(a), which reduced the tank depth instantaneously, and that shown in Fig. 14(b) which reduced the width instantaneously. Both these baffles disturbed the flow, that shown in Fig. 14(b) produced surface eddies, which inhibited the build up of the resonant wave. The weir type baffle became ineffective when its height was less than one third of the depth of liquid.

4. Discussion

Phase I.

When the test rig was designed, no knowledge was available of the effectiveness of horizontal motion in producing surface waves. Initial testing showed resonance build up to be quite critical, and occurring over a small frequency band width. With the satisfactory development of the low speed tachometer and experience in the use of the rig, the natural frequencies of oscillation could be determined to within $\pm$ 2%.

The difference between the practical and theoretical results is clearly seen in both Figs. 8 and 9. The variation is not a scattered one, the test results varying from theory in an orderly manner. This variation is not large, however, and the theoretical results are in almost exact agreement when h/L is less than 0.25, and greater than unity.

Phase II.

We see immediately from Figs. 10 - 12 that oscillations at the resonant frequency rapidly build up into something other than sine waves for short tanks, and obviously take the form of travelling waves in a large shallow tank, as would be expected.

The theory used throughout is based on the assumption that "Small Oscillations" exist, such that the square of the velocity can be neglected in the derivation of the velocity potential equations. Thus it is most probable that the realm of 'small oscillations' is not applicable to this resonant mode oscillation, and we obtain the wave form illustrated here. There is no apparent change in frequency with the change in wave form, although if this took place gradually it would be hard to verify. If there were a change of frequency amounting to a maximum of 5%, this would undoubtedly pass unnoticed in the Phase I testing, and could probably readily account for the variation of practical to theoretical results obtained in the transition region between the shallow and deep water regions, within both of which the theory is more exact.

If this 'small oscillation' theory is causing the discrepancy in the results, the error would be amplified in any determination of force using this theory, and clearly indicates that test work to investigate this would be valuable.

Phase III.

The results from tests using the variable baffle clearly show that the baffle orifice should be near the surface, but not protruding above the surface, such that surface waves are not trapped. The frequency of the oscillation thus approximates to that of the lengths of tank between baffle and end, and the flow through the orifice tends to dampen out the oscillation, as the waves on opposite sides of the baffles are 180° out of phase. This is an attempt to reduce the amplitude of the baffled tank length surface waves. It thus becomes clear, that as a solid baffle divides the tank into two, the surface waves at the natural frequency of the long tank are reduced, only to suffer increased disturbances at the natural frequency of the baffled tank length, and when designing a tank baffle both these frequencies must be borne in mind. The baffle shown in Fig. 4(e) was thus experimented with and found very satisfactory in dissipating surface waves over the complete range of frequencies encountered in previous testing. In this baffle, the orifice area measures about one-quarter the immersed baffle area and is reasonably evenly distributed.

In all testing the baffles were placed at the centre of the tank, where they would be most effective, because in this system of oscillation the highest horizontal velocity of fluid flow occurs here. When we consider the weir type baffle and that of instantaneous contractional baffling (Fig. 14) we rely on the dissipation of energy by eddy build up, and this is dependent on the velocity of flow. In Ref. 1, it is shown that motion diminishes rapidly from the surface downwards, hence the diminishing effect of this type of baffle with depth. However, a small disturbance will stop the build up of the resonant wave to an excessively large oscillation.

Considering baffles in regions at high velocity, high vertical velocities are produced at the end of the tank, and it could possibly be a contribution towards improved fuel motions if horizontal baffles could be fitted at the tank ends. This, however, is not generally convenient in an aircraft structure and it has not been considered further here.

General

In considering the theoretical formulae used it may be noticed that no account is taken of fuel density and viscosity. In all testing carried out in this work, water was the liquid under test. Future tests could readily disclose the effect of variation of these physical factors, although the effects of viscosity are generally considered to be small for unbaffled tanks of the size considered in aircraft. If baffles were present this certainly would not be the case, because of the large amount of damping introduced. There is little interest in investigating the theoretical potential flow pattern when baffles are present, since the actual flow pattern can be radically different. Hence the scope of theory is limited primarily to determining whether or not baffling is necessary.

A final point of interest is that the natural frequency of tank oscillation is dependent on the gravity constant 'g', and thus if the fuel tank experiences an acceleration normal to the mean free surface the value of effective 'g' is changed: e.g. in zero 'g' flight the natural frequencies approach zero. Such effects might be important in considering aircraft manoeuvres and in missile dynamics where a normal acceleration differing from 1 'g' is maintained for a considerable period.

5. Conclusions

Available theory is sufficiently accurate to allow a reasonable estimate of the natural frequency of rectangular tanks to be obtained.

Although the theory is based on 'small oscillations', and oscillations at the resonant frequency cannot be classed as small, the frequency does not appear to change. However, further tests should be done to determine this effect on theoretical force formulae.

Baffles should be situated at regions of highest velocity so that they trap surface waves, but allow dissipation of energy at as shallow a depth below the free surface as possible. A total orifice area about one quarter of the submerged baffle area seems satisfactory, and this should be distributed as evenly as possible over the submerged baffle area.

6. References

1. Lamb, H. — Hydrodynamics. Chapter IX. 6th Ed. Cambridge University Press, 1932.

2. Graham, E.W. — The Forces produced by Fuel Oscillation in a Rectangular Tank. Douglas Aircraft Co. Report SM.13748.

3. Luskin, H. and Lapin, E. — An analytical approach to the fuel sloshing and buffet problems of aircraft. Jnl. Aero. Sciences Vol.19, 1952 pp 217-228.

4. Graham, E.W. and Rodriguez, A.M. — The characteristics of fuel motion which affect airplane dynamics. Jnl. App. Mech. Vol.19, 1952 pp 381-388.

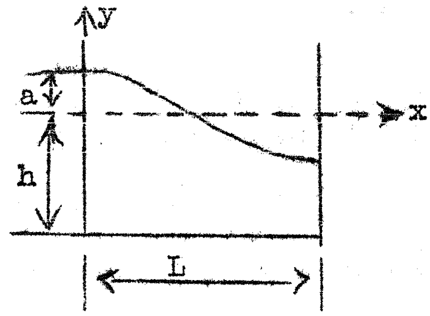
APPENDIX A

Theory

Considering a rectangular tank of unit width with small amplitude fuel oscillations in the x - y plane only. (This is a two dimensional problem).

The velocity potential is given by Ref. 1 as:-

$$\phi = \frac{-ga \cosh\left[K(y+h)\right] \cos(Kx)\cos(\sigma t + e)}{\sigma \cos(Kh)}$$



where:

$K = \frac{m\pi}{L}$ (m an integer)

$\sigma^2 = gK \tanh(Kh)$

a = Maximum amplitude of the wave

g = Acceleration due to gravity

h = Depth of undisturbed liquid

L = Length of tank

x = Horizontal displacement

y = Vertical displacement

t = Time

e = Phase angle

The period of the fundamental oscillation is given by :-

$$P = \frac{2\pi}{\sigma} = 2\pi\sqrt{\frac{L}{\pi g} \coth\left(\frac{\pi h}{L}\right)} \text{ secs.}$$

$$\text{or Frequency } f = \frac{1}{2\pi}\sqrt{\frac{\pi g}{L} \tanh\left(\frac{\pi h}{L}\right)} \text{ cycles/sec.}$$

$$\text{or } \frac{P\sqrt{gh}}{2L} = \sqrt{\frac{(\pi h)}{L} \coth\left(\frac{\pi h}{L}\right)} = \frac{\text{Period}}{\text{"Shallow water" period}}$$

As, for h/L << 1/2 $\coth\left(\frac{\pi h}{L}\right) \simeq \frac{1}{\left(\frac{\pi h}{L}\right)}$

$$\therefore \quad P \simeq \frac{2L}{\sqrt{gh}} = \text{"Shallow water" period}$$

and for h/L >>1/2 $\coth\left(\frac{\pi h}{L}\right) \simeq 1$

$$\therefore \quad P \simeq 2\sqrt{\frac{\pi L}{g}} = \text{"Deep water" period.}$$

In Ref. 4, it is shown that for a tank subjected to horizontal motion parallel to the x-axis, if the displacement relative to the co-ordinates is:-

$$x(t) = A \sin wt$$

The resonant frequency is again given by :-

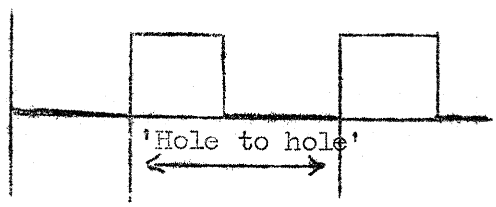
$$\omega^2 = \frac{\pi g}{L} \tanh \frac{\pi h}{L}$$

APPENDIX B

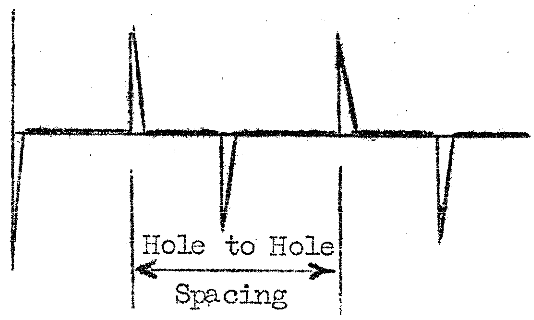
DESCRIPTION OF THE LOW SPEED TACHOMETER

The low speed tachometer was designed to give accurate readings of speed of revolution between 0.6 and 3.0 revolutions per second. The method used a disc mounted on the eccentric drive shaft, in which was drilled 50 holes, the holes being arranged to pass between a light source and a photo-sensitive transistor. The disc, the light source, and transistor mounting are shown in Fig. 7.

A circuit diagram of the control box is given in Fig. 5. The light impulses on the photo sensitive transistor X_1, cause voltage impulses to appear at the transistor X_2 base, when the switch is in the "USE" position. These pulses are then amplified in order to increase the rate of change of the signal, and are suitably cut off at the maximum output level of the transistor to give an approximate square wave output :-

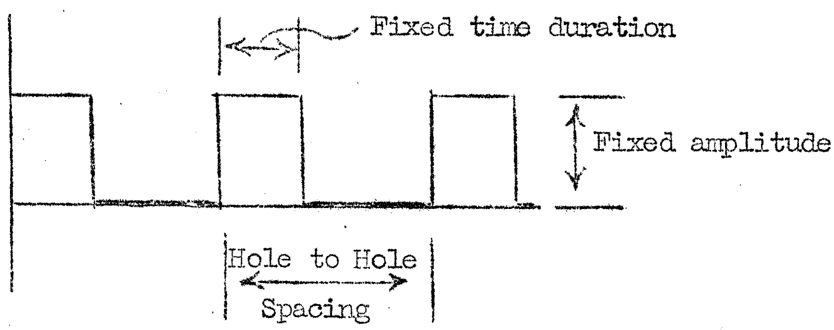


The signals are then differentiated, which produces pulses thus :-



The positive instantaneous pulses are then cut off by the MR1 Rectifier, and the negative pulses trigger a monovibrator which generates rectangular pulses of fixed amplitude and time duration.

Thus each input pulse produces one accurately determined signal :-



These signals are passed through a moving coil meter which automatically integrates this wave form to give a reading proportional to the mean level, and which is thus proportional to the mean level and the speed of revolution of the shaft.

(The range of the tachometer can be easily varied by changes in the circuit constants or the monovibrator, and thus the duration and amplitude of the output pulses).

A calibration circuit is included in the device which can give either half wave or full wave rectification to a 50 c/s mains input, and when this replaced the output from the photo-sensitive transistor, 50 or 100 pulses per second are fed through the general circuit. A variable resistance is put into the monovibrator to allow adjustment of the meter reading to 1 c/s with half wave, or 2 c/s if full wave rectification is selected. This is to allow for the slight though inevitable changes in the circuit component constants.

A photograph of the control box is included as Fig. 6.

TABLE 1

VARIATION OF FREQUENCY WITH TANK LENGTH AND DEPTH

(Frequencies given in Cycles/Sec.)

TANK LENGTH L IN INS.		FLUID DEPTH 'h' IN INS.					
		2	3	4	5	6	7
6	Test Result	1.94	2.04	2.10	2.12	2.14	2.18
	Theoretical	2.00	2.17	2.23	2.25	2.26	2.26
8	Test Result	1.60	1.71	1.78	1.82	1.87	1.91
	Theoretical	1.59	1.78	1.88	1.92	1.94	1.95
10	Test Result	1.34	1.47	1.54	1.58	1.64	1.69
	Theoretical	1.31	1.51	1.62	1.68	1.71	1.73
12	Test Result	1.17	1.30	1.37	1.42	1.47	1.52
	Theoretical	1.11	1.30	1.41	1.49	1.53	1.56
14	Test Result	1.01	1.15	1.22	1.28	1.33	1.39
	Theoretical	.96	1.14	1.25	1.33	1.39	1.42
16	Test Result	.90	1.04	1.12	1.17	1.21	1.27
	Theoretical	.85	1.01	1.12	1.20	1.26	1.30
18	Test Result	.78	.94	1.03	1.08	1.12	1.18
	Theoretical	.76	.91	1.02	1.10	1.16	1.20

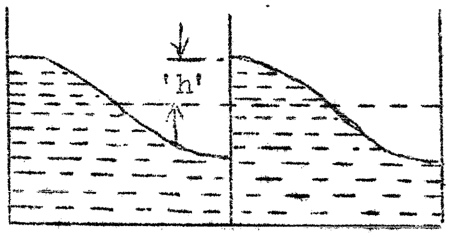
TABLE 2

ADDITIONAL TEST RESULTS TO COMPLETE h/L RANGE

TANK LENGTH L IN INS.	DEPTH h IN INS.	NATURAL FREQUENCY 'f' CYCLES/SEC.	h/L	f√L/g
2.92	3.0	> 3.0	1.02	-
2.92	4.0	> 3.0	1.36	-
3.92	5.0	2.8	1.28	.282
3.92	6.0	2.8	1.52	.282
4.92	6.0	2.43	1.22	.274
4.92	7.0	2.43	1.43	.274
10.0	1.0	.96	.100	.155
12.0	1.0	.79	.083	.139
14.0	1.0	.69	.071	.131
14.0	1.5	.83	.107	.158
16.0	1.0	.60	.062	.122
18.0	1.0	< .60	.056	-
18.0	1.5	.69	.084	.149

TABLE 3

WAVE AMPLITUDE USING VARIABLE BAFFLE



THICKNESS of slot	OVERALL WAVE HEIGHT. (2h)		
	MEAN DEPTH		
	1"	2"	3"
.25	1.75	3.25	> 4.0
.50	.75	1.75	3.0
.75	.50	1.00	2.25
1.00	.25	.60	1.75
1.25	.15	.30	1.50

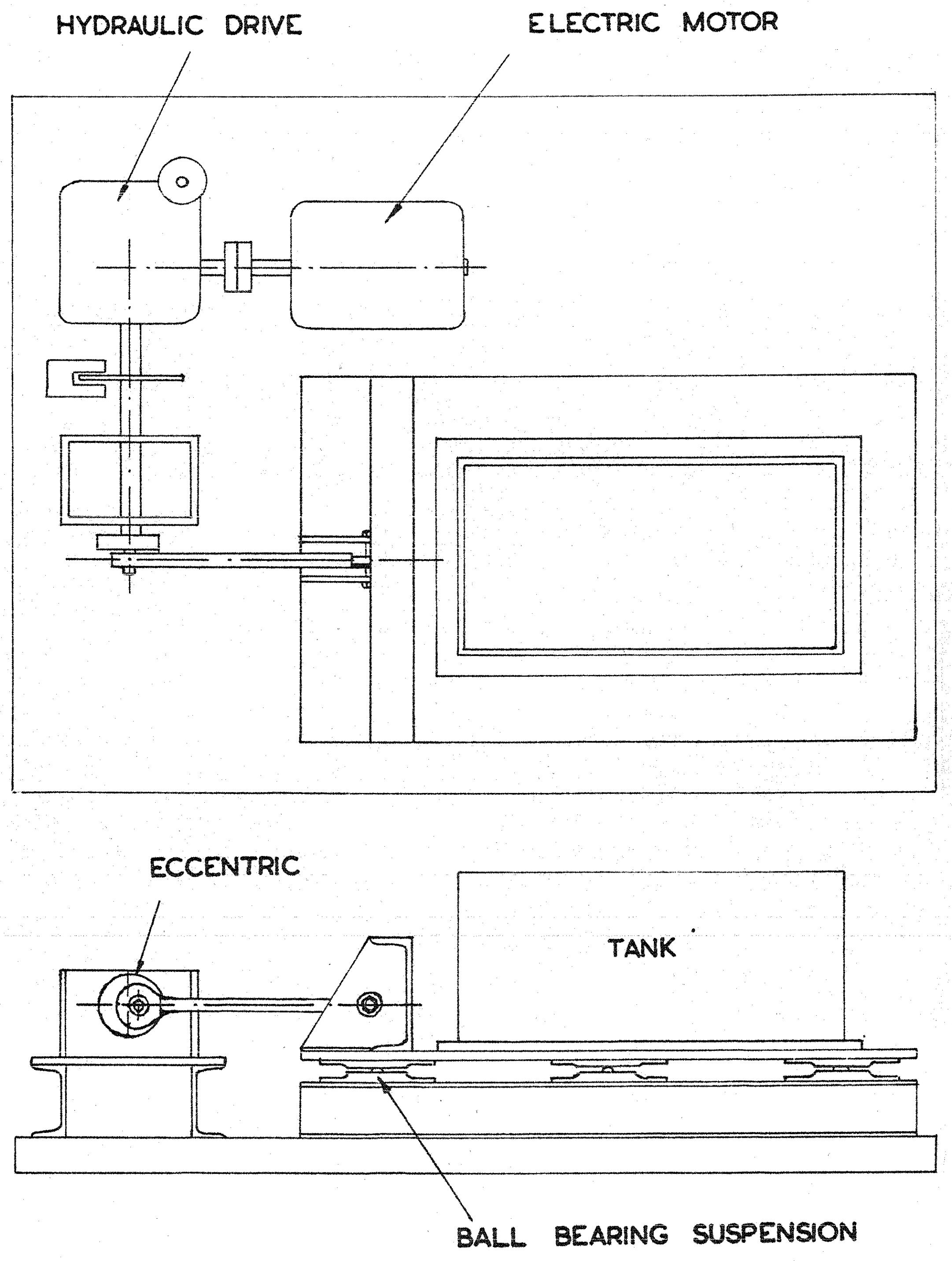


FIG. I. DIAGRAMMATIC LAYOUT OF THE TEST RIG

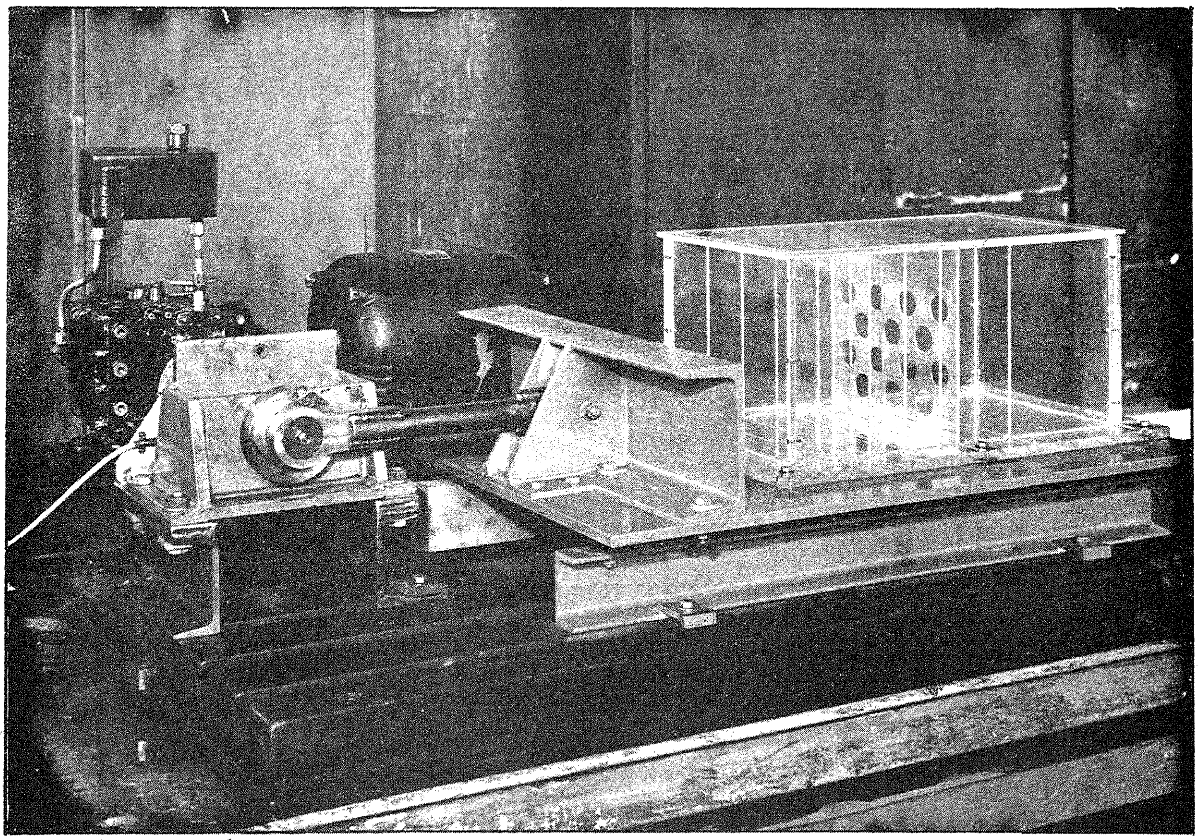


FIG. 2. GENERAL VIEW OF TEST RIG ARRANGEMENT

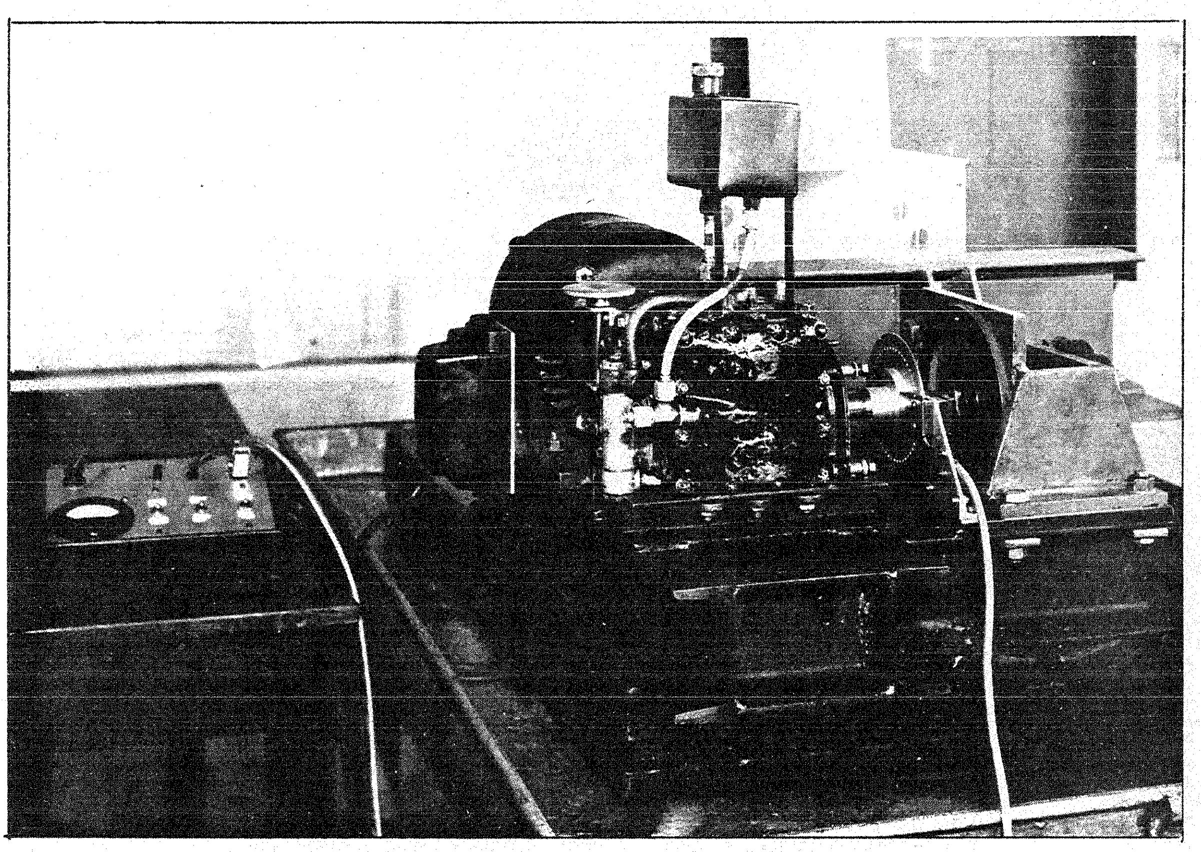


FIG. 3 VIEW OF TEST RIG

SHOWING DRIVE AND TACHOMETER

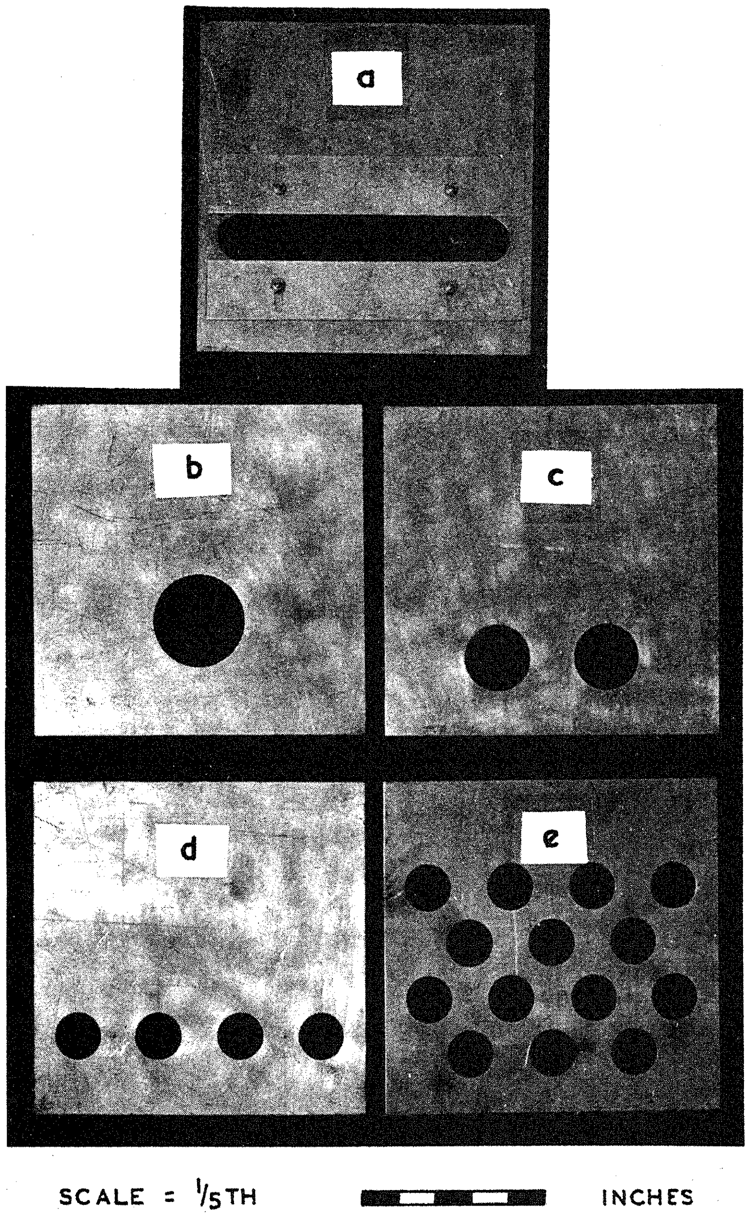
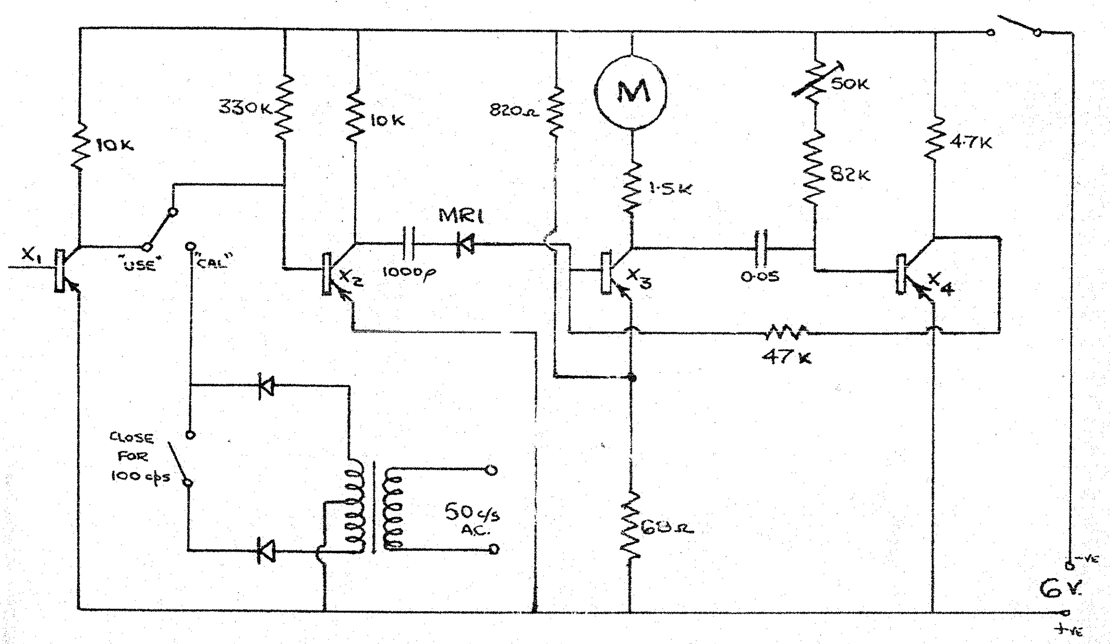


FIG. 4. BAFFLES USED IN PHASE III TESTING



X_1 - OCP71 PHOTO-SENSITIVE TRANSISTOR

X_2 - V1050B

X_3 AND X_4 - V1030A

FIG. 5. LOW SPEED TACHOMETER: CONTROL BOX CIRCUIT DIAGRAM

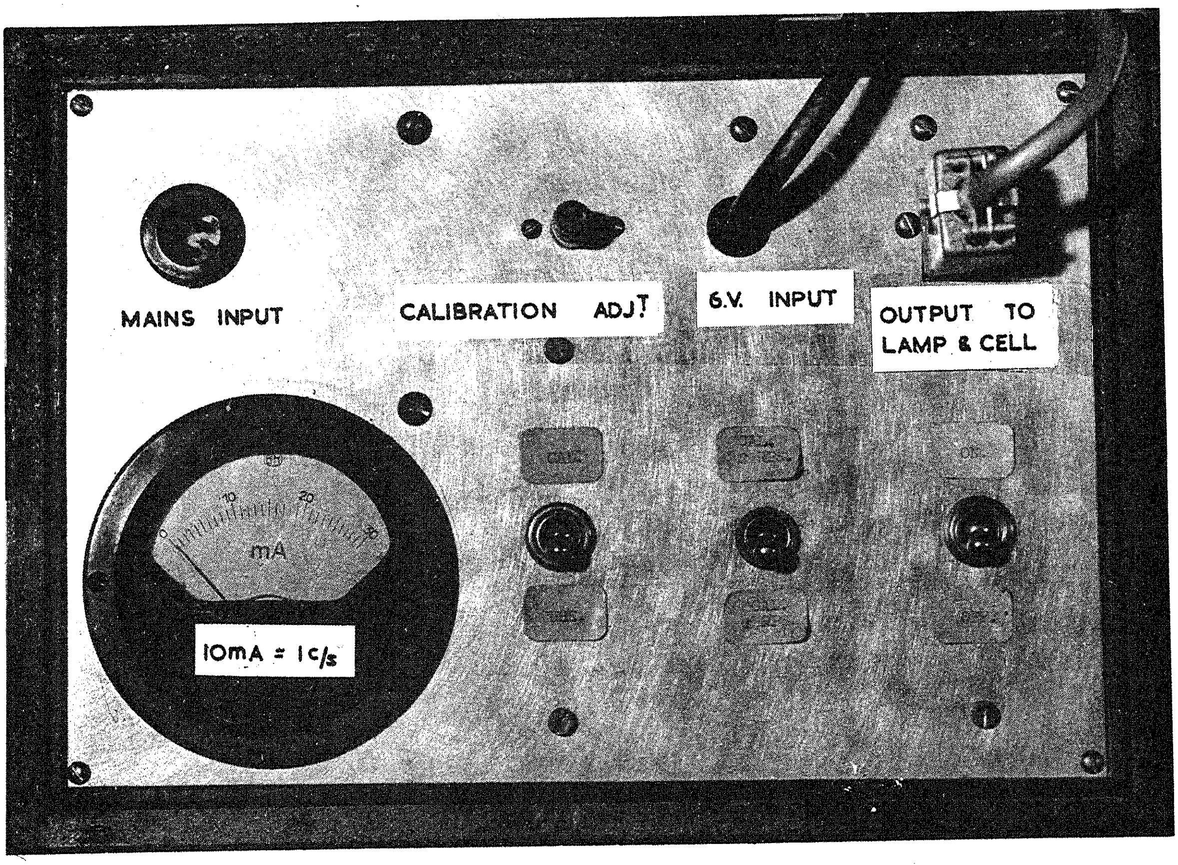


FIG. 6. LOW SPEED TACHOMETER: CONTROL BOX.

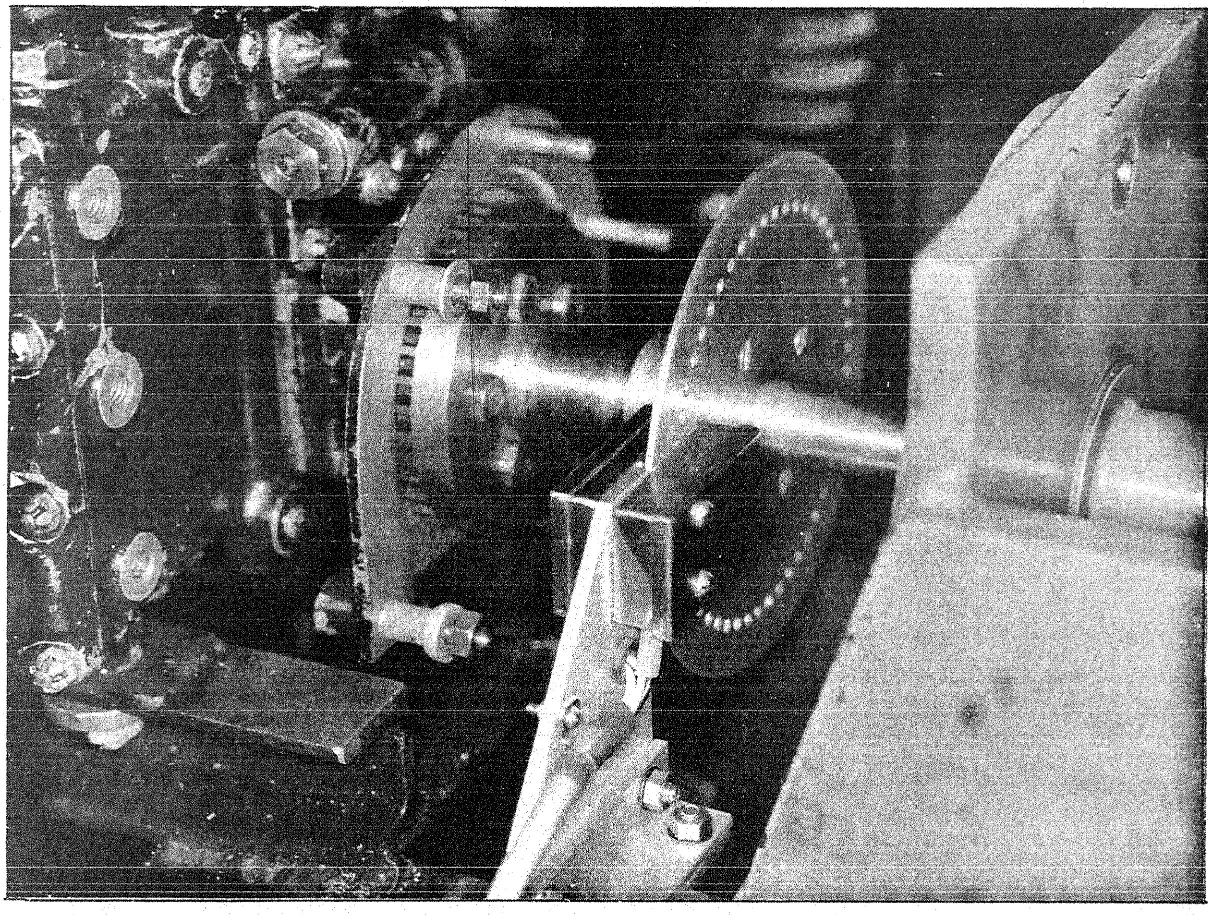
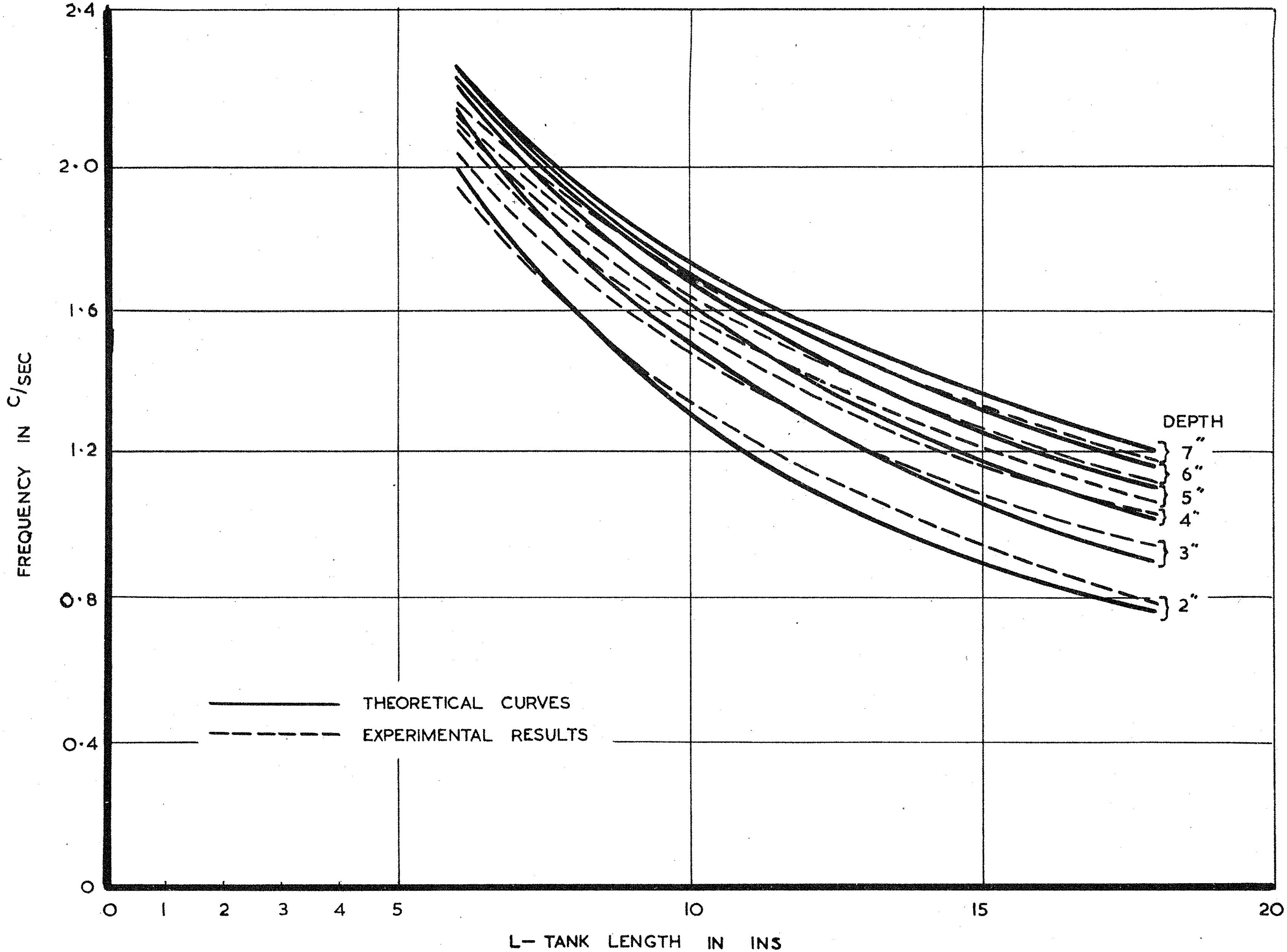
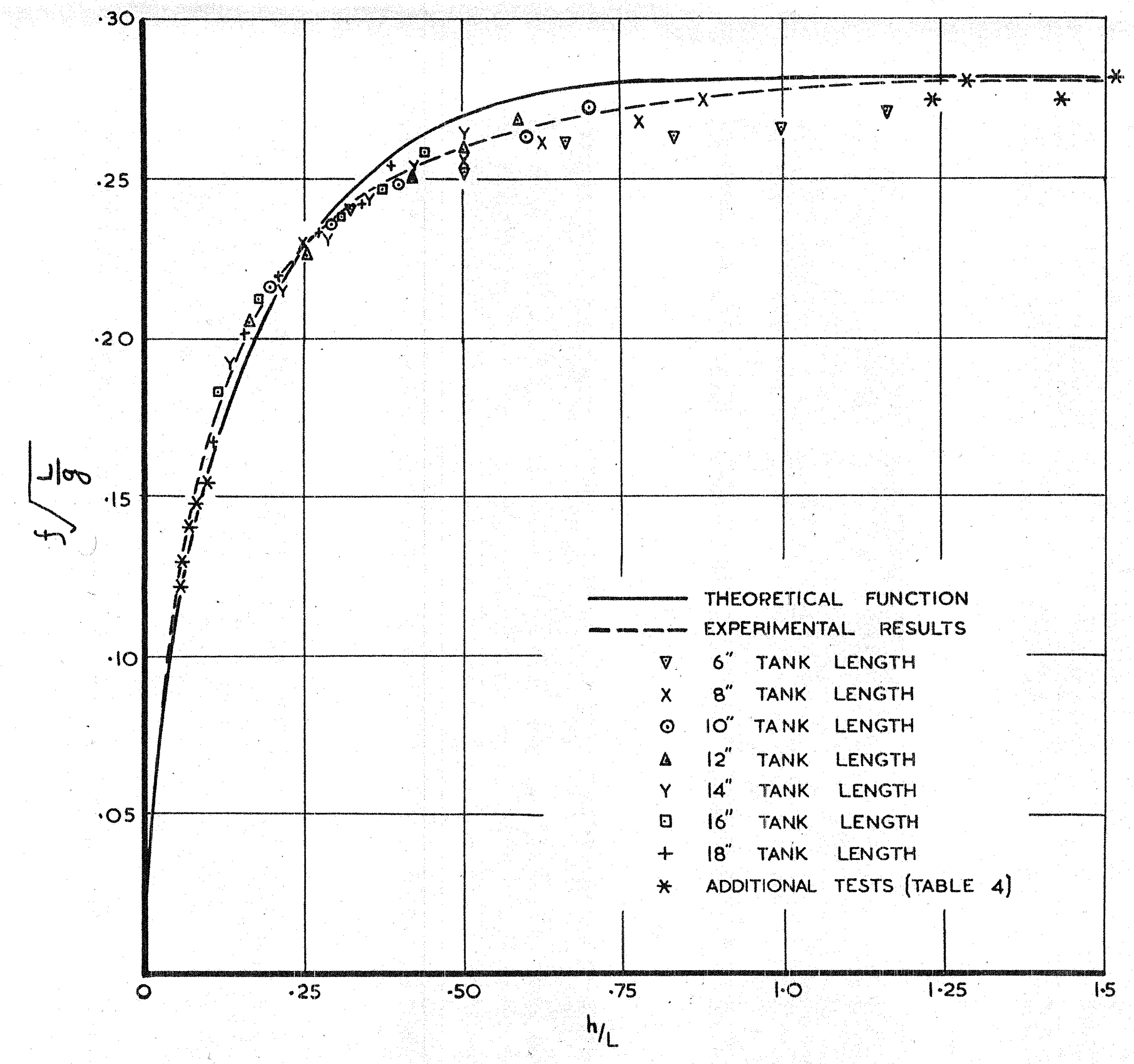


FIG. 7. LOW SPEED TACHOMETER : LAMP AND TRANSISTOR MTG.



"DYNAMICS OF FUEL IN TANKS"

FIG. 8. MEASUREMENTS OF NATURAL FREQUENCY



"DYNAMICS OF FUEL IN TANKS"

FIG. 9. MEASUREMENT OF NATURAL FREQENCY

(NON-DIMENSIONAL FORM)

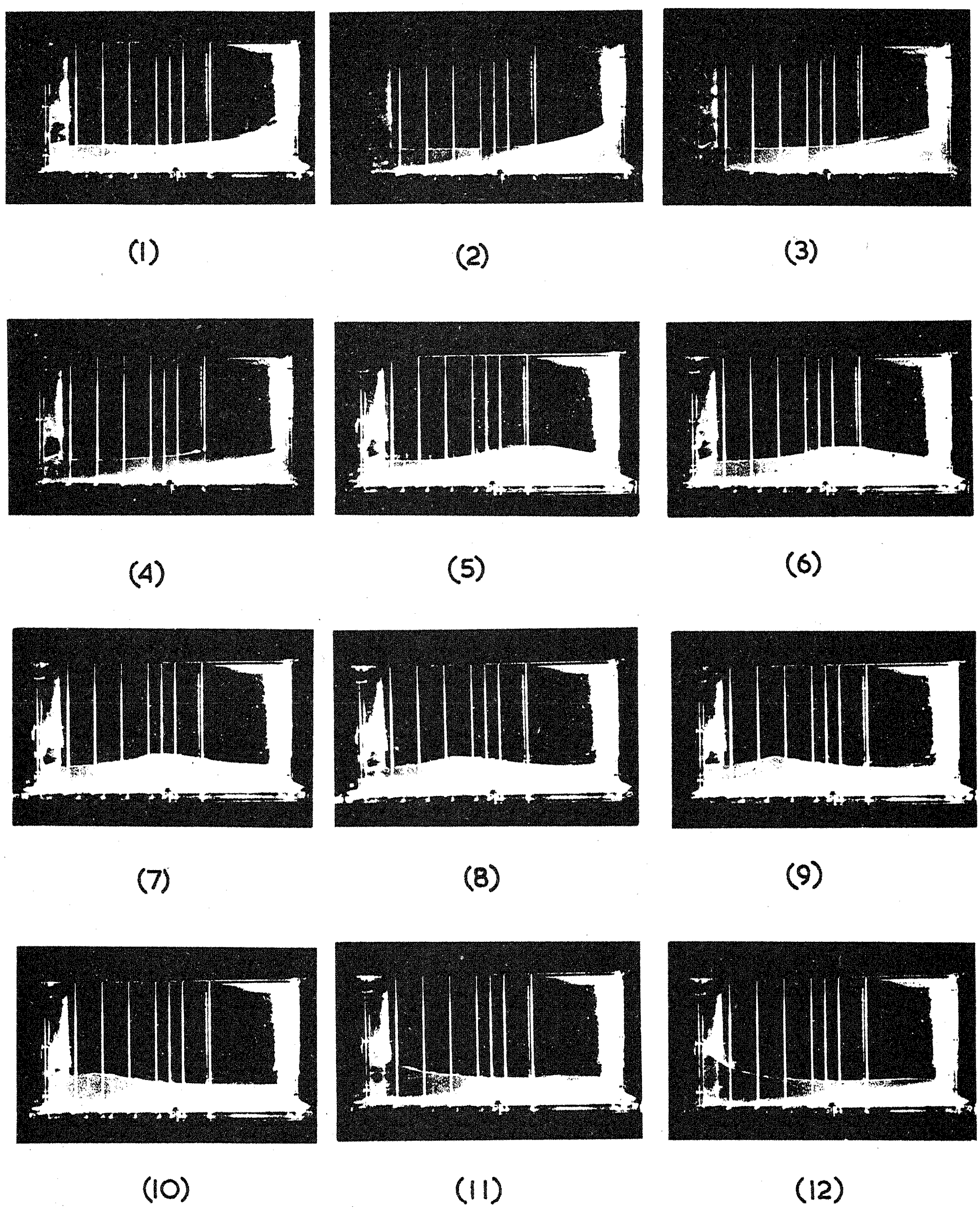


FIG. 10. SURFACE WAVE FORM OF OSCILLATION IN 18" TANK WITH 2" DEPTH. PHOTOGRAPHS ARE AT ·06 SECS INTERVALS

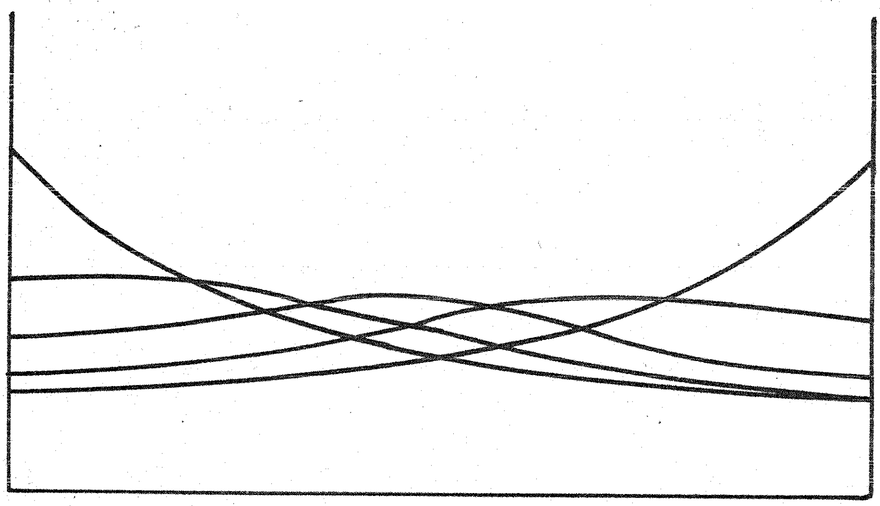


FIG. 11. SHOWING SURFACE WAVE FORM OF OSCILLATION IN 9" TANK WITH 2" DEPTH OF LIQUID (1/2 FULL SCALE) SUCCESSIVE TRACES ARE AT ·06 SEC INTERVALS

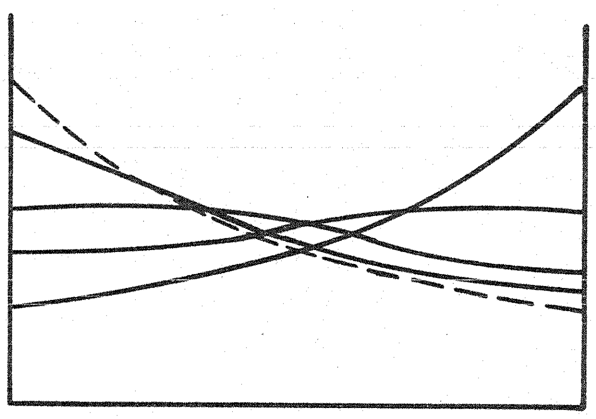
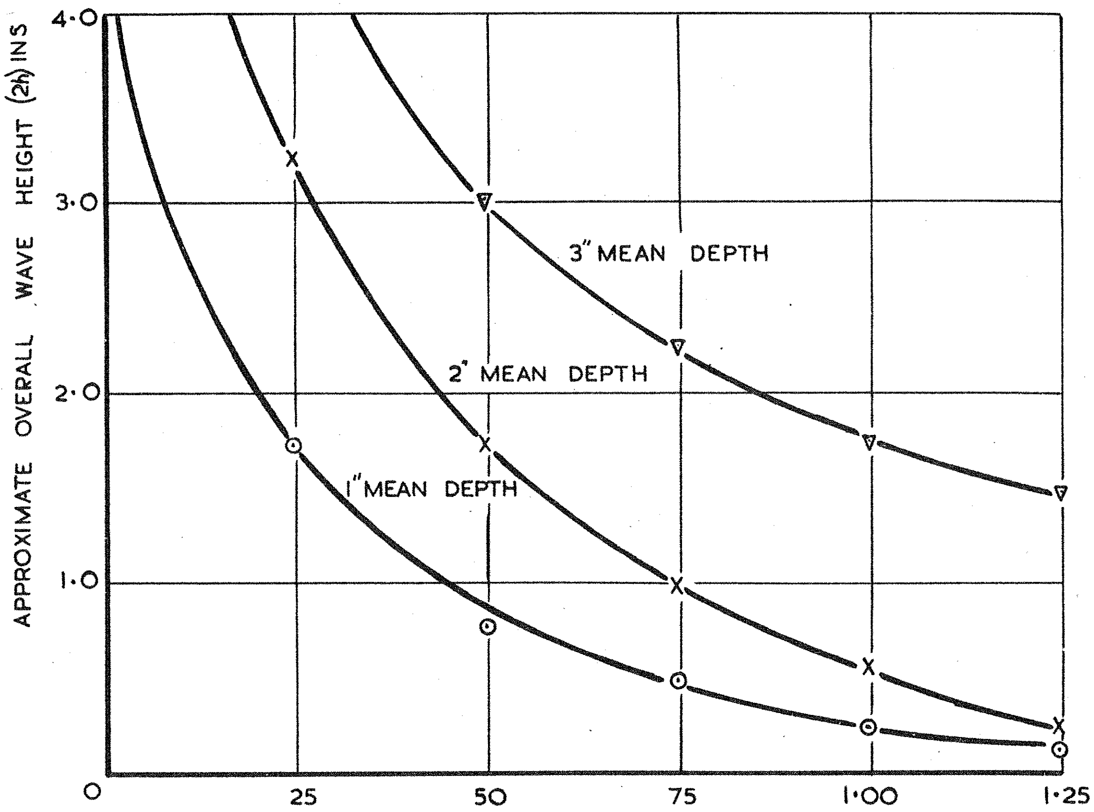


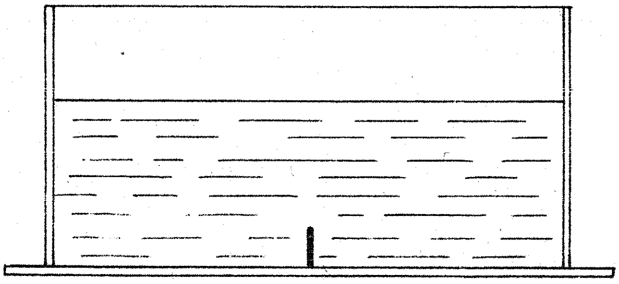
FIG. 12. SHOWING SURFACE WAVE FORM OF OSCILLATION IN 6" TANK WITH 2" DEPTH OF LIQUID (1/2 FULL SCALE) SUCCESSIVE TRACES ARE AT .06 SEC INTERVALS

"DYNAMICS OF FUEL IN TANKS"

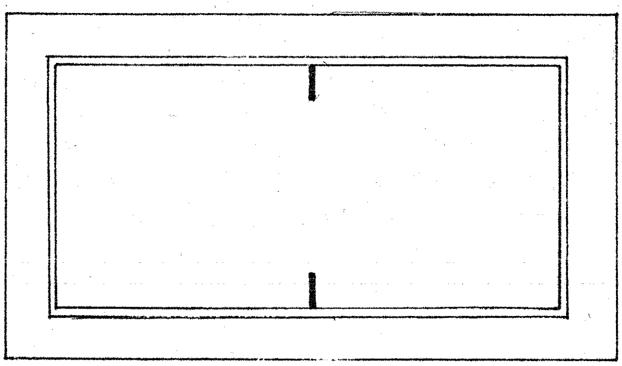


"DYNAMICS OF FUEL IN TANKS"

FIG. 13. TEST USING VARIABLE BAFFLE



(a) BAFFLE INSTANTANEOUSLY REDUCING DEPTH



(b) BAFFLE INSTANTANEOUSLY REDUCING WIDTH

FIG 14