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THE COLLEGE OF AERONAUTICS

CRANFIELD

RESEARCH INTO THE DYNAMIC LOADS ON GEAR S

by

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SUMMARY

The existing methods of estimating the actual loads on the teeth of gears at speed are by no means satisfactory. Discrepancies between the methods are illustrated by calculations made for typical conditions to be used in the tests. A discussion on the possible factors that may influence the dynamic load and on the practical value of an exact knowledge is included.

A power circulating rig that is being built at the College for the measurement of the dynamic loads is described together with the proposed method of instrumentation. The main design features of this rig are its accuracy under loading conditions, its slip free loading coupling and its adjustable centre distance.

The proposed research programme includes tests on spur and helical gears both with and without profile modification. Investigations will be carried out on these gears at pitch circle velocities over the range 0 - 20,000 ft. /min. and tangential tooth loads of 0 - 3,000 lb. /in. face.

CONTENTS

INTRODUCTION

In the design of gears three main types of tooth failure have to be considered. These are tooth fracture in bending and surface failure due, either to surface fatigue (pitting) or to lubrication breakdown (scuffing). The life to all three modes of failure will depend on the dynamic load experienced by the teeth. This dynamic load arises from the external torque plus a dynamic increment primarily due to effective errors in the teeth and the rotational speed of the gear. Various semi-mathematical approaches have been made to the solution of this dynamic load, but in the main they have been contradictory and there has been very little experimental or practical evidence to support them.

It is the object of this research programme to investigate the effect of the many factors which may influence this dynamic load. From the results of the experiments it should be possible either to substantiate an existing theory or establish some empirical laws in their place. At the very least there will result some practical evidence as to which factors are the more important.

Since many of the troubles experienced with high speed gears are of a vibrational nature, it is also intended that a concurrent investigation will be made into the effect of the factors on the torsional vibrations of the gears.

In the first section of this note the problem of the dynamic load on gears is discussed more fully. The disparity between existing methods of estimating the dynamic effect is shown and the various factors influencing this effect are discussed. An attempt is also made to indicate the practical value of an exact knowledge of the dynamic load. A brief description of the test rig built at the College of Aeronautics for these experiments follows, together with the proposed methods of instrumentation. An outline of the proposed research programme is given in Section III.

SECTION I

1. EXISTING METHODS OF DETERMINING THE DYNAMIC LOAD

The use of an empirical speed factor to take account of the dynamic effect as in the British Standard method (Ref. 12) has received much criticism in the past. Of the various methods that have been proposed to determine the dynamic load and so set the problem on a more logical basis, those by Earle Buckingham (Ref. 1) and Professor W. A. Tuplin (Refs. 2 and 3) have received wide interest. The approach to the problem and the results obtained by these two methods are completely different.

To illustrate this difference, calculations have been carried out for the proposed test gears over the range of test speeds. The dynamic load increments so found have been plotted in Fig. 3 for an error of 0.0005 in. on one gear (i. e. equivalent to 0.00025 in. error on each gear) at tangential loads of 500, 1,000 and 1,500 lbs. It will be noted that the results from the Tuplin method are quite independent of tooth loading. Although these calculations have only been carried out for "solid" test gears, the effect of altering the rim thickness of the gear with both methods can easily be seen. Both take account of the alteration in effective mass, but only Tuplin makes allowance for the effect of rim thickness on the stiffness of the teeth. This decrease in stiffness has the effect of reducing the dynamic load as given by the Tuplin method and the disparity between the two is increased. This is well illustrated by the calculations made by Armstrong Siddeley Limited (Ref. 4), using both methods on the Double Mamba and Python reduction gears. With the thin rimmed gears in this case, Buckingham's method gives dynamic increments some twenty-five times that obtained by the Tuplin method.

It would be very difficult to say without supporting experimental or practical evidence which of the two methods is the more correct. From careful consideration of the problem, it appears that mathematical analysis alone is unlikely to provide a satisfactory design method which can be universal to all gear applications. In fact it is felt that this is the root of all the differences of opinion on the various methods in use. The British Standard method which at the time of its conception was based on a very wide range of experience has been very satisfactory in many gear applications and it may well be that the methods of Buckingham and Tuplin are also satisfactory for certain gear conditions. The wide range of variables and the great differences of opinion on their importance indicates that the problem will only be finally solved by experimental evidence resulting from extremely careful isolation of all the factors which may influence the dynamic load.

2. FACTORS INFLUENCING THE DYNAMIC LOAD

In the design of the test rig for this proposed research programme, it was necessary to ensure control of all the factors which may influence the amount of the dynamic increment. The factors considered are enumerated below together with some comments on their possible influence. It will be noted that these factors form the basis of the research programme detailed in Section III.

(a). Pitch Circle Velocity

Whatever else affects the dynamic load, pitch circle velocity will be a major factor. Buckingham (Ref. 1) and others suggest that with increasing speed there is a limiting value to the dynamic load for given loading conditions. Correlation or otherwise of this point should readily come from the proposed experiments.

(b). Profile and Pitch Errors

In order that some fundamental information can be obtained, it is intended that research will be first carried out into the effect of single errors in an otherwise "perfect" gear. With normal production gears this condition will not arise and the possible effect of accumulative errors must be considered. Should this effect be important, then the problem of correlation between test results and a reliable procedure for design studies is increased considerably. It appears that the only solution to this problem will be by the estimation of the worst possible conditions based on some statistical knowledge of errors probable in production gears.

(c). Static Tooth Loading

The disparity in the Tuplin and Buckingham methods on the dependence of the dynamic increment on static tooth loading has already been shown (Fig. 3). This question should be readily answered from the type of tests envisaged.

(d). Flexibility of teeth, rim and web

There appears to be very little evidence on the effect of rim and web thickness on the stiffness of gear teeth. Tuplin's method takes into account the stiffness of the rim but not the effect of face width/web thickness ratio. Limited experiments by Armstrong Siddeley Ltd. (Ref. 4) have shown that with a change of web ratio from 6. 0 to 3. 4 the tooth stiffness increased by some 12%. Correlation of any test results obtained to empirical laws may well require a more exact knowledge of the effects of rim and web thickness on the flexibility of gear teeth. It is hoped that some useful information on this aspect will result from the calibration of the dynamic load pick-ups during the experiments.

(e). Mass Effect

The inertia of the gear wheel and any rigidly attached mass will undoubtedly have some effect on the dynamic load. Mass effect of bodies connected by normal shafting will probably have little bearing on the dynamic load.

(f). Misalignment

In any normal engineering application there is almost certainly to be some misalignment of mating gears. This may either be an initial error or due to deflections of the shaft, bearings and mounting under load. The possibility of this misalignment affecting the dynamic load cannot be ignored, since it may well give rise to cross bedding vibrations.

(g). Pitch Circle Errors

Eccentricity of a gear pitch circle can give rise to vibrations at rotational speed frequency and ovality will produce vibrations at twice this frequency. Of the two forms of error, eccentricity would seem to be the more likely, especially as it is liable to be aggravated by location errors.

(h). Diametral Pitch

The fact that fine pitch teeth on gears give smooth running is well known. Buckingham (Ref. 1) also shows that there can be a reduction of the expected dynamic load on fine pitch gears; there being insufficient time for the load to build up in the manner explained by his general theory. Investigation into this aspect will be complicated by the difficulties of instrumenting small teeth.

(i). Lubrication

The possibility of the type, condition and quantity of the lubricant affecting the dynamic load cannot be entirely neglected and some experimental evidence on possible damping effects is required.

3. PRACTICAL VALUE OF THE PROPOSED RESEARCH PROGRAMME

An exact knowledge of the amount of the dynamic load in a gear train will only be of practical value if it enables the life of the gears to be assessed more accurately. Because of the nature of the bending fatigue life curves, small inaccuracies in the fatigue limit or the calculated stresses can result in quite large errors in the estimation of gear life. As far as tooth bending failures are concerned, the problem of fillet stress concentration has received some attention (particularly Jacobson, Ref. 9), and attempts have been made to establish information on the characteristics of various gear materials (Refs. 7 and 8). Similarly, in the case of surface fatigue, work has been carried out on the pitting characteristics of gear materials (Ref. 6) and a limited amount on the effects of lubrication, propagation of cracks, etc. Whilst the results from this work are far from complete, particularly on the material characteristic side, e. g. effect of case thickness, residual stresses etc. , very little experimental effort has been given to what would appear to be a fundamental problem of knowing the exact loading conditions on gear teeth. This may well be due to the fact that errors arising from an insufficient knowledge of the dynamic effects have been small compared with errors due to other factors. Even so, as the research work already mentioned on these other factors progresses, a knowledge of the mechanics of the dynamic load becomes more important for a complete understanding of the gear problem.

A large number of troubles and failures, particularly in aircraft engine spur gears, have arisen from resonant vibrations. The greater use of helical gearing in these engines has resulted in a lessening of the difficulties, but undoubtedly they will still arise. The avoidance of resonance in the initial design is difficult, since the possible modes are numerous and complex. In general, however, small changes in design have greatly affected the modes of resonant vibration. With engines having a wide range of operating speeds, such as marine installations, this form of solution is not so easy. The intended investigations into the torsional vibrations arising from the experiments on the dynamic load may well give some indication as to which forms of excitation are the more important.

SECTION II

DESCRIPTION OF THE TEST RIG

It is intended to carry out the research programme on a power circulating test rig which has been specially designed with these tests in view, and an attempt has been made to control all the factors which may influence the dynamic load. This has necessitated a large amount of precision work of the type normally associated with the manufacture of very accurate machine tools. The general arrangement of the rig is shown in Fig. 1. It was felt that the rig should be capable of producing loading conditions of the order normally met in aircraft engine gears. To this end the rig will produce a pitch line velocity of 20,000 ft. /min. and a tangential tooth loading of 3,000 lbs/in.

Shaft and Bearing Arrangement

Four separate bearing housings are mounted in pairs on two small bedplates, which in turn are mounted with the remainder of the equipment on one large bedplate. Each housing contains one double roller bearing and two deep groove ball bearings as shown in Fig. 2. These are ultra precision bearings produced by SKF. Limited, Sweden. By suitable alteration of the split spacing ring length, the radial clearance of the roller bearing can be adjusted to any amount (or completely removed) by forcing it up its tapered seat. Axial thrust is taken by the two ball bearings which can be preloaded by suitable adjustment of the shim under the end cover. This bearing arrangement will give very accurate location of the shaft axis with the minimum of radial and axial movement. Each pair of housings are separated by two distance pieces mounted on reference lugs and held together by four one-inch diameter bolts, facilitating easy and accurate adjustment of the centre distance. At present the gearcase arrangement is such as to allow a change of centre distance of 0.4 in. In order that the shaft deflections should be less than the tooth errors being investigated, it was necessary to use a nominal shaft diameter of 4 ins. The overhung shaft arrangement gives good accessibility to the test gear and duplication of this bearing arrangement with the slave gears, simplified manufacture.

To isolate the test gears from torsional vibrations arising from the slave gears, the bearing shafts are joined by long torsion shafts. It is estimated that this arrangement will sufficiently damp all vibrations except those of low frequency arising from eccentricity of the pitch circle at comparatively low R. P. M.

Test and Slave Gears

It is intended that the gears will be precision ground to the highest accuracy possible and errors introduced afterwards as required. The comparatively large diameter of the bearing shafts has necessitated the use of gears of 9 ins. pitch circle diameter. Test gears of 4 D. P. are being used to give a reasonable size tooth for instrumentation. Both gears have the same number of teeth so that the nature of the mating teeth is always known. The slave gears are of 8 D. P. and three times the width of the test gears so as to reduce the amount of vibration arising from them. To ensure accurate location, the gears are mounted on plain spigots and driven by two dowels. The dowel holes being jig machined to ensure interchangeability.

Loading and Drive Arrangements

Torque is fed into the test rig by means of a loading arm acting on one half of the loading coupling, the other half being rigidly locked. To avoid the possibility of slip on the rejoining of the coupling a vernier dowel arrangement has been designed. This makes use of the high flexibility of the shaft system and the fact that only fixed increments of load will be required in the tests. Twenty holes in one half of the coupling and twenty-one in the other allows the two halves to be locked together at eighteen loading positions over the working range.

The power circulating rig is driven by a direct current motor of 50 h. p., through a 3:1 pulley drive using a high speed "Meteor" belt. Variation of the D. C. generator voltage and the motor controls allows any speed from zero to maximum to be obtained.

Lubrication

All bearings on the rig are lubricated by oil mist from one lubricator and the gears are supplied with oil from a combined pressure/scavenge pump through a Micronic oil filter. To provide a good protection against scuffing it is intended to use Castrol Hypoy oil for the gears. Provision has been made in the oil tank for a heater and thermostat control so that the equilibrium temperature can be established rapidly.

Instrumentation

The measurement of the loads on a gear tooth whilst in motion is by no means easy. It is possible, however, to measure the deflections of the tooth relative to the gear hub and so establish the history of the tooth loading cycle by suitable calibration. The difficulty lies in the measurement of these small deflections without materially affecting the loading cycle.

At first it was felt that a resistance strain gauge arrangement would be suitable and provision was made on one of the slave gear shafts for the connection of high speed slip rings. The position of these rings complicated the electrical connections but left both test shafts free for the fitting of torsiographs. Experiments carried out on a mock-up of this arrangement (Fig. 5) using perspex gears showed it to have poor sensitivity, and high amplification with all its associated problems was therefore, required. Further investigation with a capacity gauge arrangement (Fig. 6) gave good sensitivity but a foreseen difficulty was capacity changes in the circuit, particularly at the slip rings. To over-come this problem, preliminary work has been carried out using the grid dip principle for the transfer of the loading information from the rotating shaft. This method has been successfully used by D. Napier and Son Limited for the measurement of blade vibrations. A block diagram of the circuit is shown in Fig. 4. Results so far have been encouraging and work is proceeding with this system.

It is intended that the dynamic load information will be displayed simultaneously with torsiograph information from both test gears on a multichannel oscilloscope. The whole display will be recorded by a high speed drum camera. Also, if the loading cycle is sufficiently repetitive, it may be possible by the use of a suitable time base to display the information from the test tooth as a deflection curve.

SECTION III

PROPOSED TEST PROGRAMME

A large amount of preliminary running will be required to develop the test technique and to check on the instrumentation. All test will be carried out at a series of pitch circle velocities through the range 0 - 20,000 ft. /min. and at various tangential tooth loads from 0 - 3,000 lb. /in. face. It is proposed to carry out the tests in four stages -

- (i) On spur gears without profile modification
- (ii) On spur gears with profile modification
- (iii) On helical gears without profile modification
- (iv) On helical gears with profile modification.

Investigations at each stage will be carried out as given below. It is more likely that some of the factors investigated in stage (i) will show to have little effect and it should be possible to shorten the tests considerably in the later stages.

(a) Tests on "Perfect" Gear

To establish a datum set of results, the range of tests will be carried out on an extremely accurate gear.

(b) Effect of Profile Error

Test will be carried out on profile errors introduced as flats at the pitch circle, up to a maximum error of 0.0015 inch. Depending on the results of these tests further investigation may be carried out with the flats in different positions, e. g. root and tip.

(c) Effect of Pitch Error

It is felt that suitable small pitch error (i. e. with profile correct) will be found by inspection. Large errors will have to be ground into the gear as required. In this investigation and in (a) and (b) above, care must be taken to ensure that the teeth before and after the test teeth are accurate. The mating teeth must also be "perfect".

(d) Effect of Accumulative Pitch Errors

Various forms of accumulative error will be investigated. That is, errors will be introduced on the teeth preceding and following the test tooth. The exact form of the errors will largely depend on the results of (b) and (c).

(e) Effect of Rim and Wed Thickness

These tests will be carried out for a range of rim thicknesses with various web thicknesses. It is felt that the number of gears required for these tests may be considerably reduced if existing gears can be machined back. Some investigation will be necessary to check the effect of these operations on the accuracy of the gear.

(f) Mass Effect

Although the equivalent mass at the pitch circle will be varying in test (e), it is felt that there is a case for observing this effect alone. This can be achieved by substantially altering the inertia of the strain gauge plate which is rigidly connected to the gear wheel.

(g) Other Factors

A limited range of tests will be carried out on the following factors. The results obtained deciding whether it will be useful to investigate further.

(i) Misalignment of shaft
(ii) Lubrication
(iii) Eccentricity of gears
(iv) Centre distance
(v) Tooth pitch.

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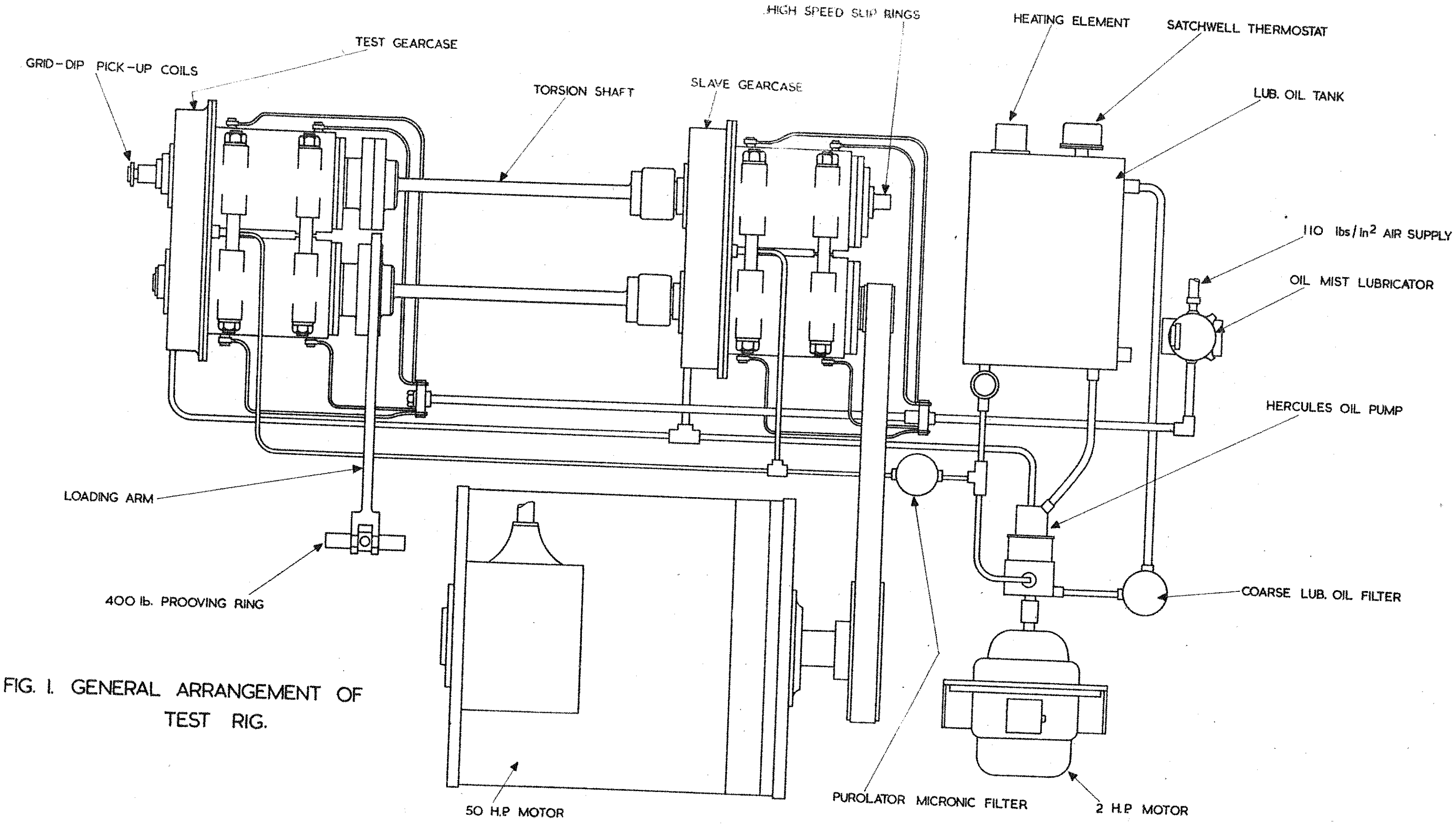

FIG. 1. GENERAL ARRANGEMENT OF TEST RIG.

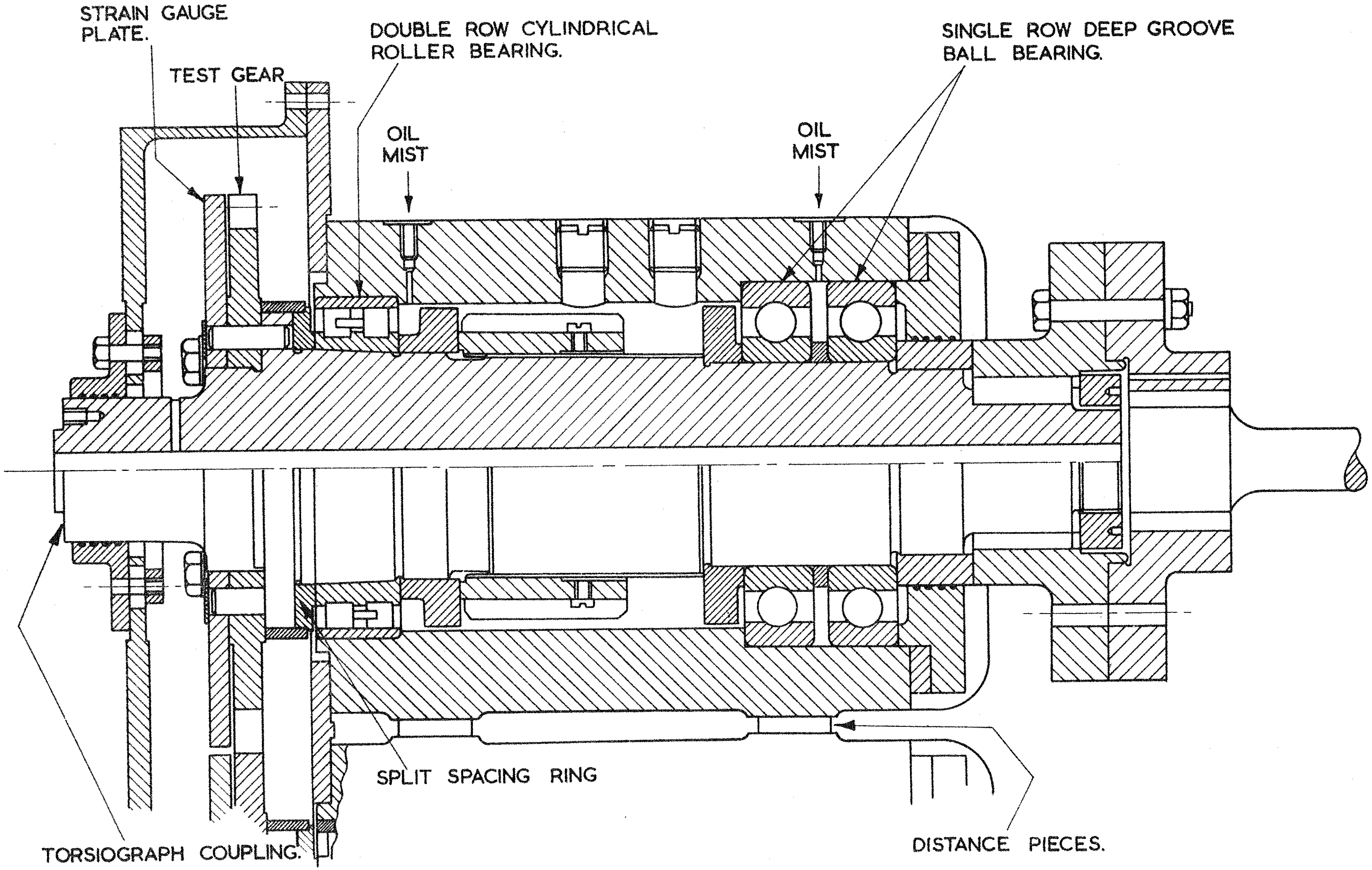

FIG. 2. TYPICAL BEARING ARRANGEMENT

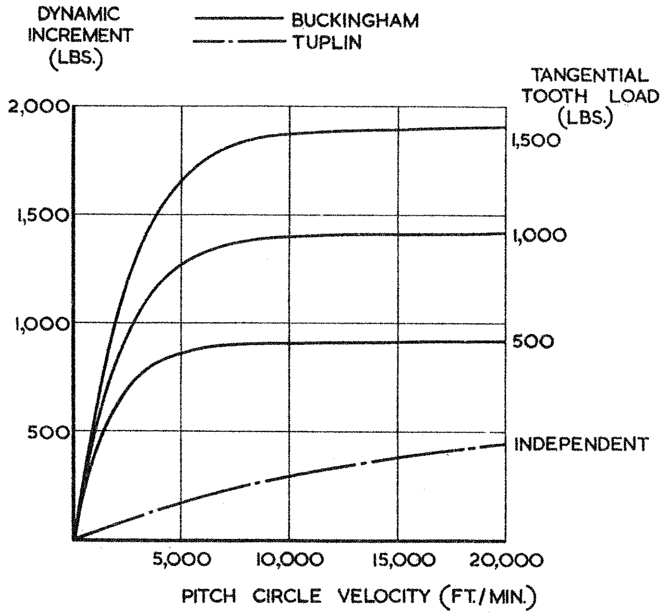

FIG. 3. SHOWING VARIATION OF DYNAMIC INCREMENT WITH PITCH CIRCLE VELOCITY AS CALCULATED BY THE BUCKINGHAM AND TUPLIN METHODS—FOR 0·0005 IN. MEASURED ERROR ON ONE TEST SPUR GEAR.

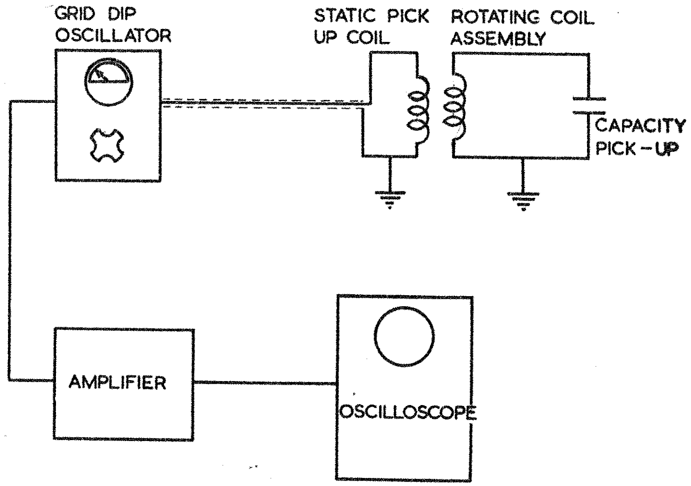

FIG. 4. BLOCK CIRCUIT DIAGRAM OF GRID DIP METHOD FOR MEASURING TOOTH DEFLECTIONS.

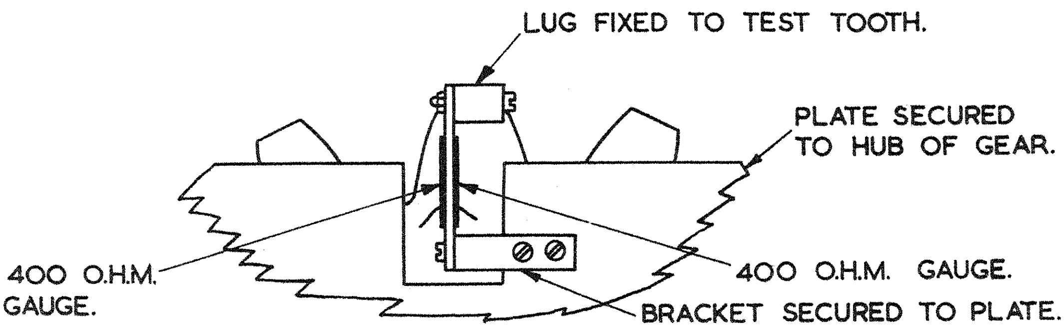

FIG. 5. SHOWING RESISTANCE STRAIN GAUGE ARRANGEMENT USED IN INSTRUMENTATION TESTS.

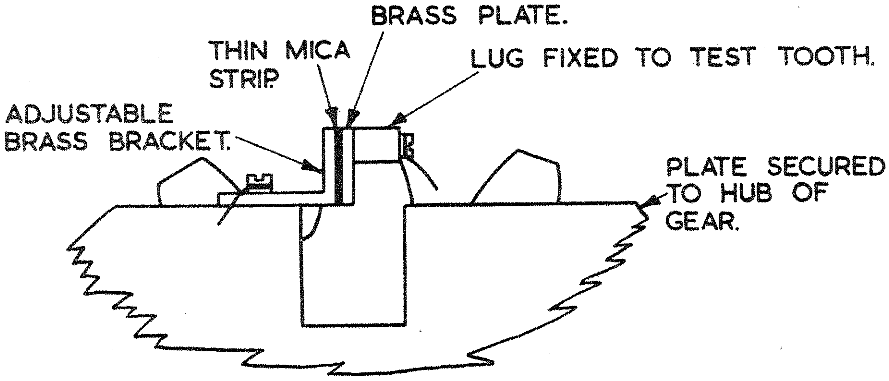

FIG. 6. SHOWING CAPACITY ARRANGEMENT USED IN INSTRUMENTATION TESTS.