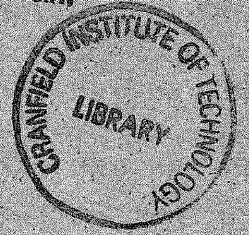


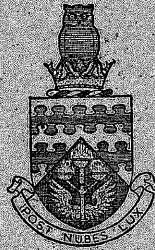
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THE COLLEGE OF AERONAUTICS
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THE CALCULATION OF THE WAVE DRAG OF
A FAMILY OF LOW-DRAG AXI-SYMMETRIC
NOSE SHAPES OF FINENESS RATIO 4.5 AT
ZERO INCIDENCE AT SUPERSONIC SPEEDS

by

G. B. MARSON, B.Sc.



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The calculation of the wave drag of a family of low-drag
axi-symmetric nose shapes of fineness ratio 4.5. at zero
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-by-

G.B. Marson, B.Sc.

S U M M A R Y

The pressure drag coefficients of a particular family
of convex logarithmic projectile nose shapes in which the nose
angle is an important parameter have been calculated over a range
of supersonic Mach numbers using a rapid approximate method due
to Zienkiewicz.⁵

The optimum nose angle for minimum wave drag of these
profiles for each Mach number has been obtained. It is shown
that above $M = 1.5$, approximately, the optimum shape is similar
to the hypersonic optimum profile and has the same or less wave
drag than this profile. However for values of M/F , where F
is the fineness ratio, below 0.5, both the hypersonic and the
logarithmic optimum profiles have a higher drag than the so-called
cubic profile (Ref. 9).

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§2. List of Symbols

$$C_D \quad \text{drag coefficient} = \frac{\text{Drag}}{\frac{1}{2}\rho_o u_o^2 \pi R^2} = \int_0^1 C_p d\left(\frac{y}{R}\right)^2$$

$$C_p \quad \text{pressure coefficient} = \frac{p - p_o}{\frac{1}{2}\rho_o u_o^2}$$

D diameter of body at shoulder

$$F \quad \text{fineness ratio} = \frac{L}{D}$$

L length of head

M free stream Mach number

p static pressure

p_o free stream static pressure

R radius of head at shoulder

S(x) cross sectional area at distance x from nose

$$t \quad \text{thickness ratio} = \frac{R}{L}$$

u_o free stream velocity

x distance from nose

y radius of body at distance x from nose

$$y' \quad \frac{dy}{dx}$$

θ_s nose semi-angle.

§3. Introduction

The rapid approximate methods described in Ref. 1 for calculating the pressure distribution on convex axi-symmetric projectile head shapes at supersonic speeds have been used by Bolton-Shaw² to determine low drag head shapes. He applied the derivative formula and the log p-θ methods to determine the type of slope distribution required for minimum drag profiles. It was noted from the pressure distributions along these optimum profiles, given by the ogive of curvature method, that the wave drag decreased with increase in nose angle. It appeared that the minimum drag was obtained for a head profile

having a nose semi-angle greater than 24° . Since the ogive of curvature method was known to become less accurate as the nose angle was increased this result was suspect. A series of wind tunnel tests^{3,4} was therefore performed in order to check the range of applicability of this method, and to find independently the nose angle for minimum drag over a range of Mach numbers. The results of these tests demonstrated in fact that the original ogive of curvature method underestimated the drag coefficient for profiles having large nose angles and that the nose angle for minimum drag was considerably less than 24° . These investigations led however to the development of a modified form of the ogive of curvature method⁵ which predicts with acceptable accuracy the pressure distribution and the drag of head shapes having nose semi-angles up to 30° .

This report describes the results of the application of this modified ogive of curvature method to calculate the drag coefficients of the series of head shapes developed by Bolton Shaw. The results are compared with those for other well known minimum drag shapes, and the optimum shapes for different Mach numbers are deduced.

§4. Brief discussion on the derivation of the family of low-drag head shapes

By the use of the derivative formula and the $\log p-\theta$ method^{1*}, Bolton Shaw² derived integral expressions for the wave drag of a projectile of arbitrary shape. Application of Euler's condition for a minimum value of the drag integral in each case gave a differential equation from which minimum drag profiles could be obtained.

Because of the considerable difficulty in solving the differential equation in the case of the derivative formula, only one suitable profile was obtained. The $\log p-\theta$ law gave a

* In Ref. 5, these methods have been revised where necessary, and their range of applicability discussed.

profile which represented minimum drag shapes for all fineness ratios collapsed into one curve by suitably scaling the axial and lateral ordinates. As this profile had infinite slope at the nose, it was impossible to apply the methods of Ref. 1 to find the pressure distribution on this profile. However, except at the nose the type of slope distribution required for an optimum shape projectile was indicated. It was found that this was very similar to that given by

$$\frac{dy}{dx} = a + \frac{b}{\frac{x}{L} + c} \quad \dots\dots\dots(1)$$

and on integration

$$\frac{y}{L} = a \frac{x}{L} + b \log_e \left(1 + \frac{x}{cL} \right) \quad \dots\dots\dots(2)$$

Equation (2) was used to describe the shape of the family of low drag profiles, referred to in this report as logarithmic profiles.

At the base of the profile, $x = L$,

$$\frac{1}{2F} = a + b \log_e \left(1 + \frac{1}{c} \right)$$

where F is the fineness ratio L/D , and D is the body diameter at $x = L$. a , b , and c may then be determined from the boundary conditions

$$(i) \quad [y']_0 = \frac{dy}{dx} \text{ at } x = 0$$

$$(ii) \quad [y']_1 = \frac{dy}{dx} \text{ at } x = L.$$

At a fineness ratio of 4.5 and a constant Mach number of 2.0, Bolton Shaw determined by means of the original ogive of curvature method the drag of head shapes with nose angles of 16° , 18.4° and 21° , for various values of the slope at the base, $[y']_1$. For each nose angle, a minimum drag was found for $[y']_1$ equal to 0.077 approximately. The drag decreased with increase in the nose semi-angle θ_s , and it appeared that a minimum drag, if present, would occur at a value of θ_s greater than 24° .

§5. Discussion of wind tunnel tests on the logarithmic head shapes

Since the ogive of curvature method was known to become less accurate with increase in nose angle, wind tunnel tests were performed with the logarithmic head shapes in order to assess its applicability and to determine the nose angle for optimum profiles over a range of Mach numbers.

The results for a profile with a nose angle of 21° at $M = 1.8$ are described in Ref. 4, and those for 21° , 24° , and 30° profiles at $M = 2.45$ and $M = 3.19$ in Ref. 3. These results showed that the original ogive of curvature method considerably underestimated the pressures over the rear portion of all these bodies, and that the discrepancy increased with nose angles. In addition, it was found that the nose semi-angle for minimum drag was about 15° over the range of Mach numbers tested.

It follows that the conclusion of Ref. 2, namely, that the drag of a head shape decreases as the nose angle is increased, giving a minimum drag at a nose semi-angle greater than 24° , is not correct. The error of course derives from the use of the original ogive of curvature method outside its range of applicability.

However, since the modified ogive of curvature has been shown to be sufficiently accurate for engineering purposes over a wide range of Mach numbers and head shapes having nose semi-angles up to at least 30° (see Table II), it has been possible to tackle again the problem of Ref. 2 on a more satisfactory basis. The next section describes this investigation and the results.

§6. The calculation of the drag of the logarithmic profiles by the use of the modified ogive of curvature method

The calculation of the pressure distribution and wave drag of arbitrary head shapes at supersonic speeds by the use of the modified ogive of curvature method is described in Refs. 3 and 5. The method depends on the fact that the ratio of the static to the stagnation pressure at a point P on the surface of an arbitrary head shape at a free stream Mach number M_0 , is practically the same as at P on the equivalent ogive of

curvature at P, for the same value of M_0 . The static pressure on the equivalent ogive of curvature can be calculated from an equation giving the decrease in pressure from the nose to the point P on the ogive surface as a function of the decrease in pressure from the leading edge of a two dimensional aerofoil having the same profile as the ogive to a corresponding point P on the aerofoil; provided that the Mach number and pressure just downstream of the nose and leading edge are the same.¹² This equation may be written

$$(p_N - p_P)_{A-S} = \lambda(p_N - p_P)_{2-D}$$

where λ depends only on M_0 and the nose angle α of the equivalent ogive of curvature. (Values of λ are given by Zienkiewicz in Refs. 1 and 5). The modified ogive of curvature method differs from the original method in that the stagnation pressure used in calculating the pressure on the local ogive of curvature at each station is taken as that behind the bow shock wave at the nose of the equivalent ogive of curvature at that station. In the original method the stagnation pressure behind the bow shock wave at the nose of the actual body was used throughout.

This modified method gives results which agree well with the experimental pressure distributions described in Refs. 3 and 4, and also with calculations made for a few particular cases by the Method of Characteristics and by Van Dyke's second order theory (Table II). The values of the pressure coefficients C_p , obtained from experiment, could be predicted generally with a possible error of at most ± 6 per cent, and the calculated drag coefficients, C_D , within ± 3 per cent. Since the results below have been calculated by the modified ogive of curvature method with a possible error of at most $\pm 2\frac{1}{2}$ per cent, comparisons between values of C_D for different profiles of the family should be possible to this order of accuracy.

In order to check the value of the end slope, $[y']_1$, as determined in Ref. 2 for the optimum profile, the wave drag was computed by means of the modified ogive of curvature method

for a number of profiles designed for different values of $[y']_1$, but each having a nose semi-angle of 12° , fineness ratio of 4.5, and at a Mach number of 2.0. The results are shown in Fig. 1. It can be seen that the drag coefficient varies very slowly over this range of $[y']_1$. It appears reasonable to assume, therefore, that the value of $[y']_1 = 0.077$ used in Ref. 2, may be retained as an optimum value without serious error. With this value for the end slope, a series of logarithmic profiles of fineness ratio 4.5 were derived having nose semi-angles of 9.75° , 12° , 15° , 21° , 24° and 30° . Their equations are given in Table I.

For each of these profiles the drag coefficient was calculated, using the modified ogive of curvature method, at Mach numbers of 1.5, 1.8, 2.0, 2.45, 3.19 and 4.0. The results are plotted in Fig. 2. It can be seen that the minimum drag head shape has a nose semi-angle of about 15° at $M = 3$, falling to a value of 12° approximately at $M = 1.5$.

In Fig. 3 the minimum values of C_D at each Mach number are plotted against Mach number. These results indicate that the minimum value of C_D decreases with increase in Mach number. The scatter of the computed values about a smooth curve in Fig. 3 does not exceed $\pm 2\frac{1}{2}$ per cent which is within the estimated order of possible error of the calculations. The drag coefficients of the inscribed cone (nose semi-angle $6^\circ 21'$) over the same range of Mach numbers are also given in Fig. 3. It is found that the drag coefficient of the optimum logarithmic profile is 76 per cent of that of the inscribed cone at $M = 1.5$, and 82 per cent at $M = 3.5$.

§7. Discussion of the results and comparison with the drag of other nose shapes

The results of these calculations show that the minimum drag logarithmic profile has an optimum nose semi-angle which varies from 12° at $M = 1.5$ to about 15° at $M = 4$. Between $M = 2.0$ and $M = 4.0$, the minimum is not sharply defined and changes in nose angle of $\pm 2^\circ$ do not affect the drag by more than ± 2 per cent. It will be noted from Fig. 1 that the drag

is not critically dependent on the value of the end slope, $[y_1]_1$, at least at a Mach number of 2.0. For the purpose of comparison with other nose shapes the logarithmic profile having a nose semi-angle of 13.5° and an end slope of 0.077 will therefore be taken as a suitable standard in the Mach number range between 2.0 and 4.0.

Although a number of so-called optimum head shapes have been derived, only the Von Karman, the Lighthill, the hypersonic optimum and the cubic profiles will be compared with the logarithmic profile.^z The results quoted, however, can be regarded as typical for all low drag head shapes. These four profiles are shown in Fig. 4, together with the logarithmic profiles of 12° , 13.5° and 15° nose semi-angle. It will be seen that the Von Karman, Lighthill and cubic profiles, which fair smoothly into a cylindrical body at $x = L$, lie close together, but considerably above the profiles having finite slope at $x = L$. Near the nose ($x/L < 0.2$), the hypersonic optimum curve is almost identical with the logarithmic profile of 13.5° nose semi-angle. For values of $x/L > 0.2$, the shape of the hypersonic optimum profile lies slightly above the logarithmic profiles. In Table II, values of C_D calculated by Zienkiewicz using the modified ogive of curvature method are compared with the values given in references 7 and 9 for the Von Karman, linear, and cubic profiles. It will be seen that the errors due to the approximate method do not exceed about 2 per cent, so that differences in C_D greater than this may be regarded as significant.

In order to compare directly, the drag results for nose shapes with different fineness ratios, the hypersonic similarity law has been used. This states that the ratio of local static to free stream static pressure at corresponding points on geometrically similar ogival heads is a function only of the hypersonic similarity parameter M/F , where F is the fineness ratio. This law has been found to hold for all the shapes considered in this report at Mach numbers above about 1.8. It

^z A brief discussion of each of these profiles is given in the appendix.

enables us to represent all the available drag data on a single diagram of $M^2 C_D$ plotted against M/F . The results are shown in figure 5. It will be seen that within the accuracy of the calculations, the logarithmic profile with a nose semi-angle of 13.5° has the same drag as the hypersonic optimum shape for $M/F > 0.5$ (e.g. $M > 2.2$ for our fineness ratio of 4.5). This is not surprising, since as noted above, these two profiles are very similar near the nose, and a closer resemblance could be obtained by use of a slightly smaller value of the end slope, $[y']_1$, for the logarithmic profile. It is known (Fig. 1) that this alteration in $[y']_1$ would hardly affect the drag.

Below $M = 2$ a smaller nose angle must be used on the logarithmic profile in order to maintain its optimum shape, and in this region the logarithmic profile has less drag than the hypersonic optimum. For $M/F < 0.5$, however, the cubic profile has slightly less drag than either the logarithmic or the hypersonic optimum shape.

§8. Conclusions

The calculations of the drag of logarithmic projectile nose shapes (ref. 2) have been revised and extended using the modified ogive of curvature method (refs. 3 and 5). It is found that the drag of these head shapes at Mach numbers between 1.5 and 4.0 is equal to or slightly lower than that of the hypersonic optimum profile. There is also little difference between the shapes of the two profiles. The nose semi-angle for minimum drag varies from 12° at $M = 1.5$ to 15° at $M = 4.0$.

Although the drag of the logarithmic profile with a nose semi-angle of 12° is lower for $M < 2.0$ than that of the hypersonic optimum, it is slightly greater than that of the so-called cubic profile.

§9. Acknowledgements

The author wishes to acknowledge the supervision and advice of Mr. G.M. Lilley and the assistance of Mr. H.K. Zienkiewicz.

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APPENDIX

Brief description of certain well known low drag
axi-symmetric head shapes

1. The Von Karman optimum nose shape

For low or moderate Mach numbers, it can be shown (see Ref. 10), that the wave drag of a slender body of revolution as the thickness tends to zero, is given by

$$C_D = \frac{1}{2\pi} \int_0^1 \int_0^1 \log \frac{L}{|x-y|} S''(x) S''(y) dx dy \quad (1)$$

where $S(x)$ is the cross sectional area of the body at a distance x from the nose.

This expression was used by Von Karman to derive an optimum profile for a projectile nose of given fineness ratio which fairs smoothly into a cylindrical afterbody. (This condition, that $[S']_1 = 0$, was decided by the fact that equation (1) is finite only if $S'(x)$ is continuous.) The optimum profile, as derived by means of the calculus of variations, is given by

$$\left(\frac{y}{R}\right)^2 = \frac{1}{\pi} \left(\frac{\pi}{2} + \sin^{-1}(2x-1) + 2(2x-1) \sqrt{x-x^2} \right)$$

and its slender body theory drag is

$$C_D = 4t^2$$

where t is the thickness ratio, R/L .

Unfortunately, this profile has a blunt nose, and does not satisfy the assumptions on which slender body theory is based. Its true drag cannot therefore be calculated by any theory at present available. The drags of slightly modified Von Karman bodies, with either conical or cubic shapes near the vertex, have been computed using Van Dyke's second order theory (Refs. 7 and 8). These results are in reasonable agreement with experimental measurements described in Ref. 8.

2. Lighthill's Linear Profile

For the reasons given above, Lighthill regarded the Von Karman profile as inadmissible as a projectile shape, and recommended the profile

$$y = t \frac{x}{L} \sqrt{3 - 2 \frac{x}{L}}$$

sometimes known as the linear profile. This has a pointed nose, and fairs smoothly into a cylindrical afterbody. Its slender body theory drag is given by

$$C_D = 4\frac{1}{2}t^2.$$

3. Leslie's Cubic Profile

In Ref. 9, Leslie shows that Lighthill's profile is the first of a series of low-drag profiles, and suggests two improved shapes; the so-called cubic shapes.-

$$y = t \frac{x}{L} \left(5 - 10 \frac{x}{L} + 10 \left(\frac{x}{L} \right)^2 - 4 \left(\frac{x}{L} \right)^3 \right)^{\frac{1}{2}}$$

whose slender body theory drag is given by $C_D = 4 \frac{1}{6} t^2$, and the quintic shape, which is specified by

$$S''(x) = \frac{7\pi t^2}{4} \left[3 \left(1 - 2 \frac{x}{L} \right) - 10 \left(1 - 2 \frac{x}{L} \right)^3 + 15 \left(1 - 2 \frac{x}{L} \right)^5 \right]$$

and has a slender body theory drag coefficient $C_D = 4 \frac{1}{12} t^2$.

Using the method of characteristics, Leslie and Perry have made accurate calculations of the drag of the cubic profile at Mach numbers below 2.0, and have shown that this profile has less drag than any other known pointed body which fairs smoothly into a cylindrical afterbody. Accurate drag results have not been published for the quintic profile.

The Hypersonic optimum Nose Shape

This is the optimum shape for a non-faired body of given fineness ratio, according to Ferrari. Its derivation is based on Newton's impact theory, and the profile is closely approximated by the 3/4 power curve.- $\frac{y}{L} = \frac{1}{2F} \left(\frac{x}{L}\right)^{3/4}$.

The results of Ref. 8 show that this profile has less wave drag than any other known pointed body at Mach numbers above 1.8.

TABLE I

EQUATIONS TO THE LOGARITHMIC PROFILES

The equation to the family is

$$\frac{y}{L} = a \frac{x}{L} + b \log_{10} \left(1 + \frac{x/L}{c} \right) .$$

The values of the constants a, b, and c are given in the table below.

Profile	Nose Semi-angle	a	b	c
1	9.75°	.0398	.1455	.479
2	12°	.0404	.1033	.260
3	15°	.0555	.0562	.115
4	21°	.0620	.0372	.050
5	24°	.0636	.0329	.0374
6	30°	.0655	.0278	.0236

TABLE II

Comparison of C_D values obtained using the modified ogive of curvature method, with those given by the method of characteristics and Van Dyke's second order theory.

Profile	Method (and Ref. No.)	F	M	C_D By Given Method	C_D By Modified Ogive of Curvature	% Error
Von Karman	Van Dyke (Ref. 7)	6	2.0	0.0226	0.0230	+ 2
Linear	Character- istics (Ref. 9)	4.17	2.0	0.0511	0.0500	- 2
		2.84	1.5	0.1062	0.105	- 1
Cubic	:	4.17	1.5	0.0494	0.0497	+ 1/2
		4.17	2.0	0.0460	0.0470	+ 2

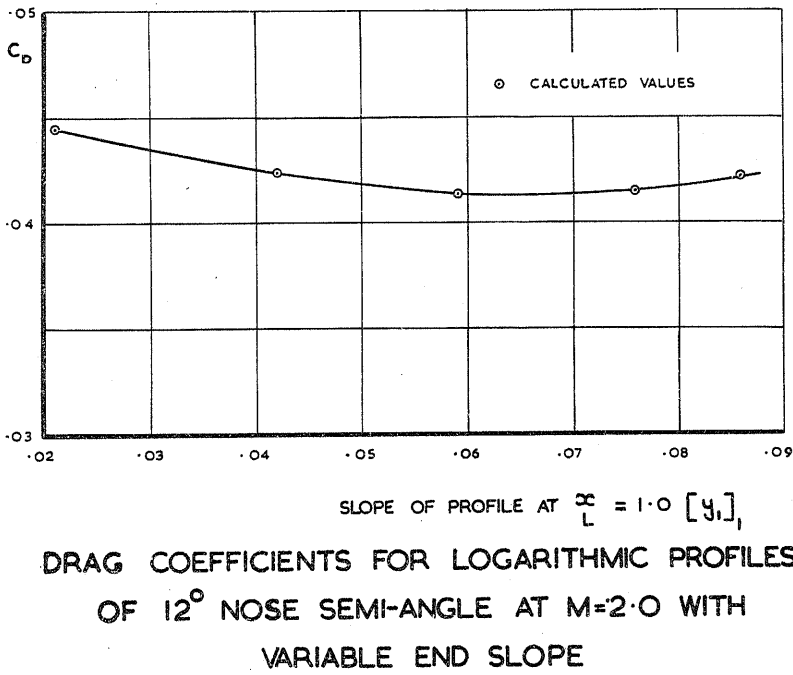
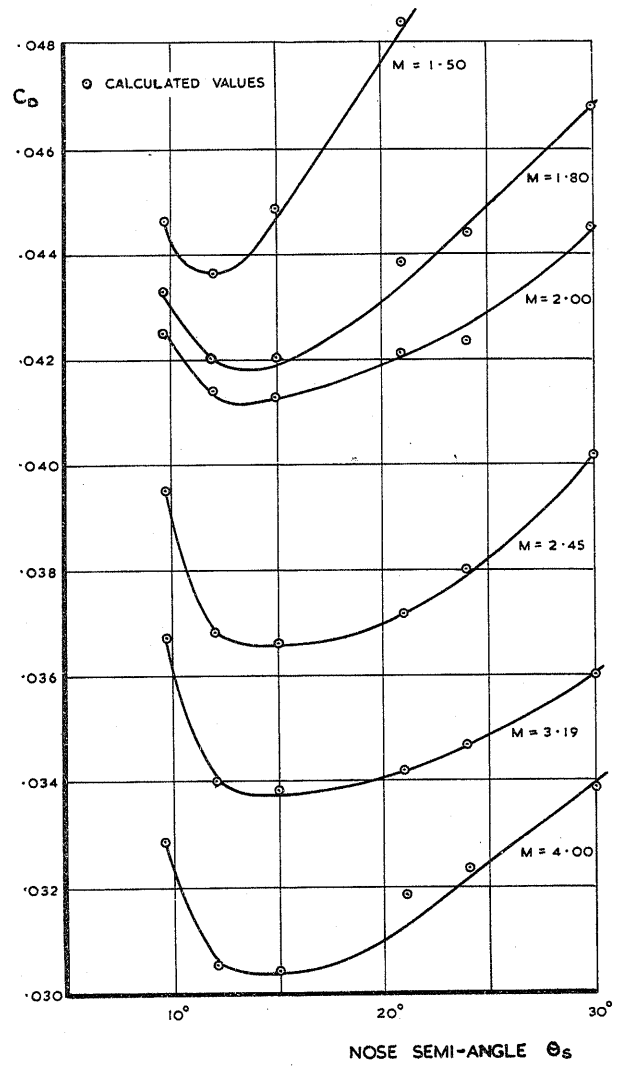
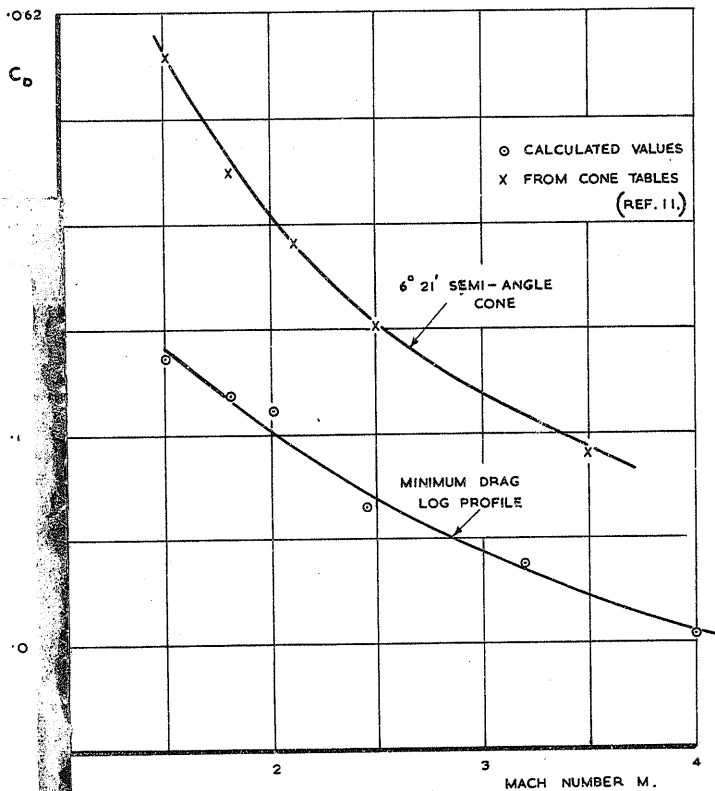


FIG. 1.



VARIATION OF C_D WITH NOSE SEMI-ANGLE FOR LOGARITHMIC PROFILES
 $[y_1] = 0.077$

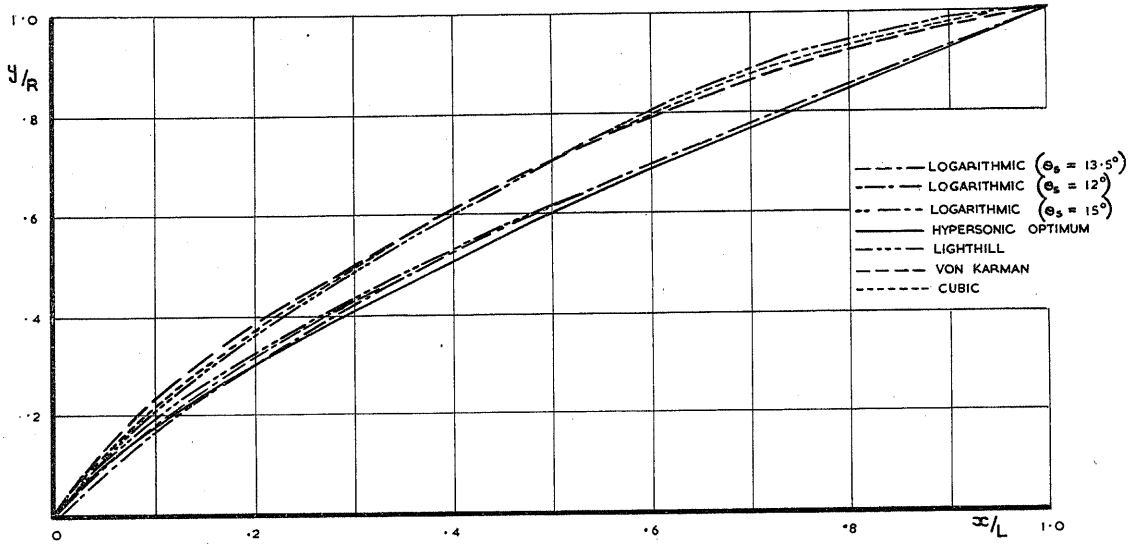
FIG. 2.



VARIATION OF C_D WITH MACH NUMBER FOR MINIMUM DRAG LOGARITHMIC PROFILE AND INSCRIBED CONE.

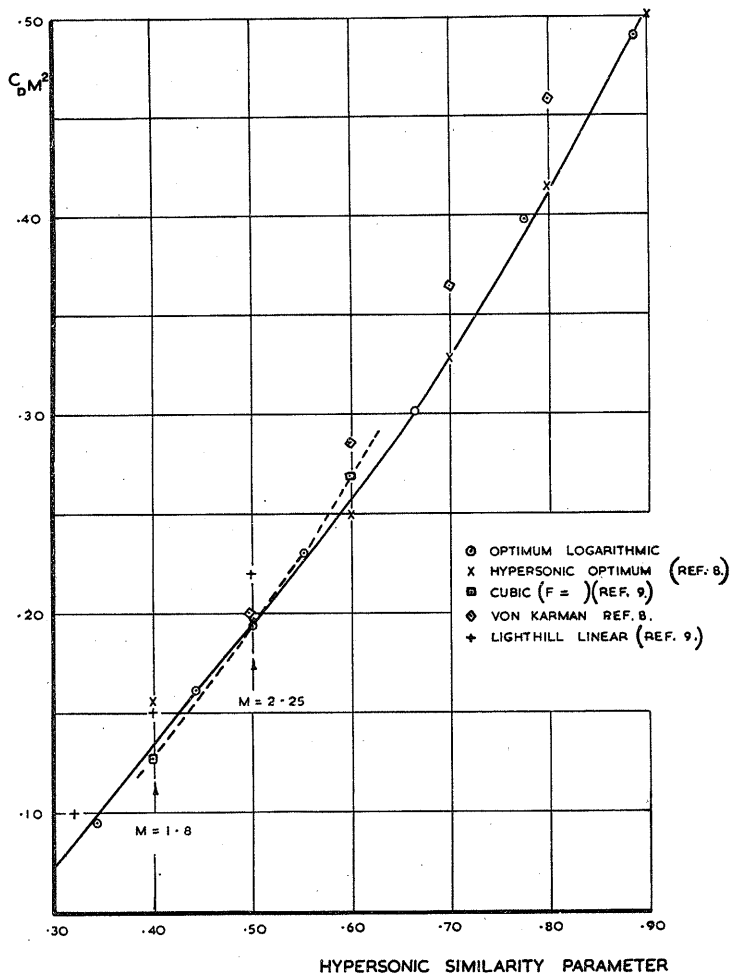
FIG. 3.

FIGS. 4. & 5.



COMPARISON OF LOGARITHMIC AND OTHER PROFILES

FIG. 4.



DRAG OF OPTIMUM LOGARITHMIC AND OTHER PROFILES

FIG. 5.