CONFIDENTIAL REPORT

Some notes on the factors influencing the possible damage to buildings due to the passage of shock waves from aircraft flying at supersonic speeds

-by-

Geoffrey M. Lilley, D.I.C., M.Sc., A.F.R.Ae.S.

S U M M A R Y

The purpose of this note is to discuss, from the limited information available, the order of magnitude of the strength and the configuration of the shock waves, close to ground level, generated from aircraft flying at supersonic speeds. The level flight case is the only one considered in detail but brief reference is also made to other manoeuvres such as the supersonic dive of limited duration. The alterations in path and strength of a shock wave in passing through the heterogeneous atmosphere are shown to be very important. For normal temperature variation with altitude the attenuation of shock strength, with distance downward from the aircraft, will be decreased. A similar effect will occur when shock waves are propagated against a wind, whose velocity increases with height.

These results should be of interest to civil engineers and others making estimates of the possible damage to buildings due to the above cause. A preliminary, mainly qualitative discussion of this part of the problem is included, which although not complete should at least give some idea of the damage to be expected. In this connection it is found that for aircraft flying at and above 5000 ft. at Mach numbers up to 2 the damage to buildings should be limited to a number of 'freak' cases of window breakage and similar minor damage. In the case of aircraft diving at supersonic speeds at a high altitude the excess pressures caused by the resulting shock waves and their time duration are so small that the chance of any damage occurring is almost negligible.

Dec 1952

§1. Introduction

The shock waves generated by aircraft and other bodies travelling at speeds close to and greater than the speed of sound are heard by ground observers as 'bangs'.* Experiments with full scale aircraft, travelling in high speed dives from high altitude at speeds slightly in excess of sonic, have shown that the intensity of the shock waves, near the ground, is generally less than 2lb. per sq.ft. It has been claimed that such shock waves give rise to minor damage to buildings, such as window breakage. The available evidence suggests, however, that in only one or two isolated cases has damage been caused in this way.

In the near future it must be expected that aircraft will operate at supersonic speeds in level flight not only at high but also at low altitudes. The strengths of the generated shock waves will probably, in such cases, exceed those observed in the high speed dive. Due to the differences in the two motions and the strong directionality of the shock wave pattern, which is altered by atmospheric changes in pressure and temperature with height, the results of existing experiments cannot be expected to give guidance as to the possible effects in the level flight case. It should be noted that because of refraction of the shock waves by temperature and wind gradients, experiments on missiles operating at supersonic speeds in climbs close to the ground may give different results from aircraft flying at level supersonic speeds at a slightly higher altitude. The available theory is reviewed below and tentative results are presented for the calculation of the strength of these shock waves near ground level when generated in a heterogeneous atmosphere.

A short account is included of the effect of shock wave reflection from the ground and walls. As a result an estimate is made of the overpressures acting on vertical walls. A comparison of these results is then made with existing data for blast damage and some tentative conclusions are drawn as to the likely damage to property.

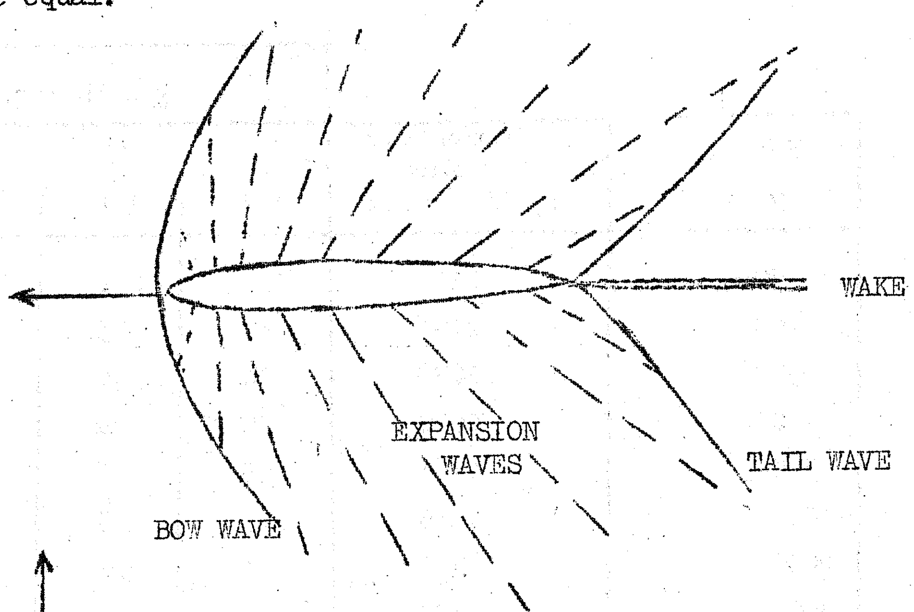
/ §2 ...

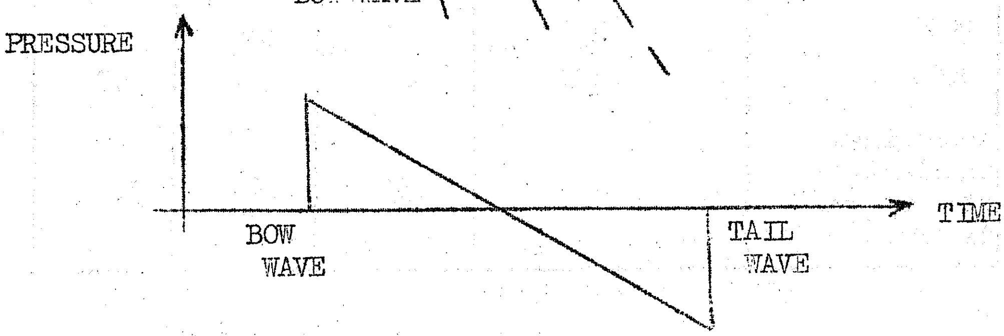
---

* See references 1 to 11.

§2. Steady supersonic motion in a uniform atmosphere

The shock wave pattern around an aircraft in steady motion at supersonic speeds can be represented approximately, at large distances from the aircraft, by the shock wave pattern around an equivalent body of revolution. This statement must not be taken to mean that in all cases at a sufficiently large distance from the aircraft the individual shock waves from the fuselage, wings and tail unit will have coalesced into two main shocks, but rather that for all practical purposes, in which ground level effects are required, differences from the assumed pattern can be neglected. Thus close to the ground the shock wave pattern consists of bow and tail waves separated by expansion waves.[8] The pressure 'signature' has the form of a distorted 'N', in which the pressure and suction phases are equal.





The magnitude of the pressure discontinuity,[12] Δp, across the bow wave, at a normal distance y from a parabolic shaped body of revolution of fineness ratio δ and length ℓ, moving through a uniform atmosphere of pressure p_o at a Mach number M_o is

$$\Delta p = 0.53 \frac{p_o \delta}{(y/\ell)^{3/4}} (M_o^2 - 1)^{1/8} \qquad \ldots\ldots\ldots\ldots (1)$$

where approximately for an aircraft

$$\delta = \sqrt{\frac{2.39\ V}{\ell^3}} \qquad \ldots\ldots\ldots\ldots (2)$$

/and V ...

and V is the total volume. Mean values of δ and ℓ for a supersonic fighter will be about 0.25 and 40ft. respectively.

The following table has been prepared using the above values for δ and ℓ. It gives Δp, at ground level, as a function of M_o and the altitude y.

TABLE 1

The pressure rise Δp, at ground level, across the bow wave from a body of revolution travelling at a Mach number M_o at an altitude y in a uniform atmosphere having the same values of pressure and temperature as in the standard atmosphere at an altitude y.

Equivalent fineness ratio $\delta = 0.25$
Equivalent length of body $\ell = 40$ft.

Height y ft.	Atmospheric pressure p_o lb./sq.ft.	Δp lb./sq.ft. M_o 1.1	1.4	2.0	3.0
50	2112	194	234	270	306
200	2101	69	83	96	112
500	2078	34	41	47	54
1000	2040	20	24	28	31
5000	1761	5	6	7	8
10000	1455	2.5	3	3.5	4
20000	973	1	1.2	1.4	1.6
30000	629	0.5	0.6	0.7	0.8
Approximate inclination of bow wave to ground		65°	46°	30°	19°

The values of Δp up to about 1000ft. cannot be regarded as reliable since the distances are too close to the aircraft for the assumptions of the theory to apply. In addition values of Δp obtained from equation (1) become very inaccurate for Mach numbers close to unity. The reason for this is that the bow shock wave is detached from the nose of the body and its strength, at least close to the body, may be greater than the theory indicates. Far away from the body the asymptotic theory[12] should apply although the size of the region close to the body, in which the theory ceases to be accurate, is not known. For these reasons the values quoted in table 1 for a Mach number of 1.1 may be too small.

/The time ...

The time interval Δt between the passage of the bow and tail shock waves at distance y from the body is given by (see table 2 below)

$$\Delta t = \frac{3.43 \, \Delta p}{a_o \, p_o} \, y \, \frac{M_o}{\sqrt{M_o^2 - 1}} \qquad \text{............(3)}$$

where a_o is the speed of sound in the uniform atmosphere at rest.

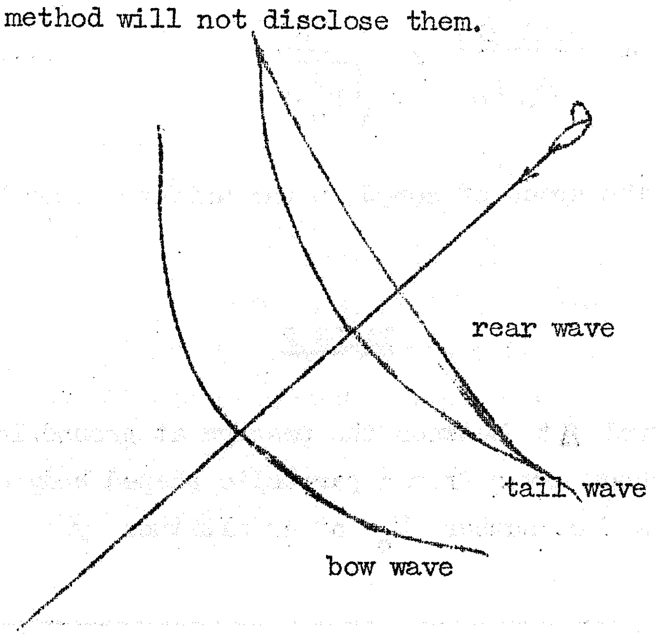
TABLE 2

The time interval Δt between the passage at ground level of the bow and tail shock waves from a parabolic shaped body of revolution travelling at a Mach number M_o at an altitude y.

Height y ft.	Δt secs. M_o 1.1	1.4	2.0	3.0
5000	0.11	0.08	0.07	0.07
10000	0.13	0.09	0.09	0.09
20000	0.16	0.12	0.11	0.11
30000	0.2	0.14	0.14	0.14

It can be seen from tables 1 and 2 that the Mach number has less significance than at first might have been thought. The above tables can also be used to obtain an approximate value for Δp at a large distance from an aircraft, whose flight path is not uniform. The reason for this is as follows. Each portion of a shock wave at a large distance from a body has been formed at some particular point on the flight path. Its initial strength is largely a function of the instantaneous speed at that point although the attenuation it suffers, whilst moving to a distance, depends (a) on interactions with shock waves and expansion waves generated from other positions along the flight path (b) on viscous and thermal effects and (c) on the spreading of the shock from its source. If therefore we assume that such attenuation is similar to that existing under uniform flight conditions, we find approximately that the shock strength at equal values of y and M_o in non-uniform motion is equal to that in uniform motion. If we apply this to the case of the supersonic dive in which the maximum Mach number reached is 1.05 and the supersonic motion finishes about

20,000 ft. a value of shock pressure rise of about 1 lb./sq.ft. is obtained across the bow and tail waves at ground level. Naturally, when more than two shock waves exist in the non-uniform flight case the above method will not disclose them.



Thus in the example quoted a third wave[8] sometimes exists, but, since it is weak, it plays a relatively unimportant role in the consideration of shock wave damage to property.

It has been suggested[3] that in certain aircraft manoeuvres a focussing of the shock waves might result in excess pressures of the order of 1000lb./sq.ft. More information is however necessary to ascertain whether these results are valid and whether they have practical application. Such manoeuvres have not been considered here.

It is of interest to compare the strengths of the shock waves in the cases discussed above with the strength of blast waves from explosions.*

T.N.T. lb.	distance from explosion ft.	Δp lb. per sq. ft.
40,000,000	12,000	216
4,000,000	6,000	216
400,000	3,000	216
40,000	1,500	216
4,000	750	216
400	375	216
40	188	216
4	94	216
20	580	22
200	1,160	22
2,000	2,320	22

/83. ...

* These figures are based on the assumption that $\Delta p \sim 1/r$ and $\frac{r}{r_o} = \frac{W}{W_o}^{1/3}$ where r is the distance from the explosion, W is the charge of T.N.T., and suffix o refers to reference values.

§3. The effects of a variable atmosphere

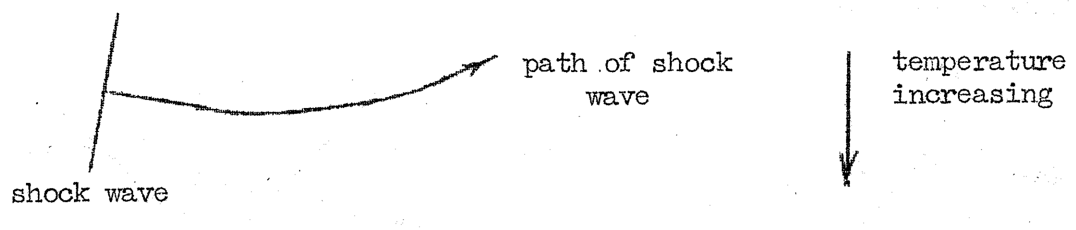
In section 2 above it has been shown that the increase in pressure across a shock wave at large distances from an aircraft is small compared with the increase in atmospheric pressure downwards. It is important to consider, therefore, as to whether or not such shock waves will suffer considerable attenuation in moving through this large pressure gradient. A simple calculation, based on acoustic approximations[13], shows that, due to the change in air density, the strength* of a weak plane wave propagated vertically downwards will be reduced by a factor

$$1 \Big/ \exp\left(\frac{\gamma g y}{2 a_o^2}\right)$$

where γ = ratio of specific heats (1.4 for air)
g = acceleration due to gravity
a_o = speed of sound
y = height.

Thus a shock wave propagated from 20,000ft. will suffer a reduction in strength of 0.6 in moving to ground level. Similar results can be expected to hold for weak oblique shock waves propagated from aircraft in level motion at high altitudes.

The effect of temperature and wind variation with height on the refraction of sound waves is well known and similar results will hold for weak shock waves. Thus in the normal state of the atmosphere a shock wave whose course is approximately horizontal will turn gradually upwards. A temperature inversion in the atmosphere will have an opposite effect.



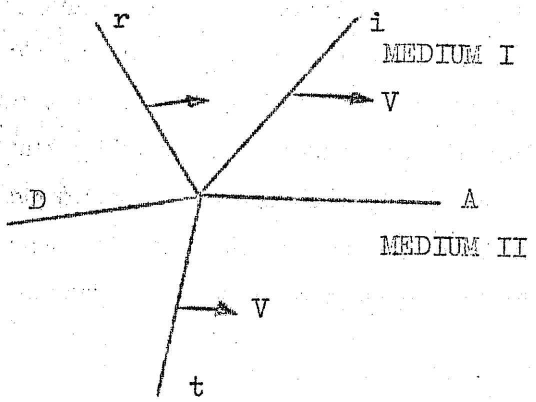
Similarly when the wind speed increases with altitude a shock wave, moving with the wind, will be bent downwards.

The importance of the refraction effect lies not only in the change in shape of the shock wave with time but also in its change in strength (measured as the pressure rise across it). This effect can be well illustrated from a consideration of the refraction of a plane shock wave incident to an interface between two separate layers of air at different temperatures.[14] Let us assume that the two media are separated by an interface A and that no mixing of the

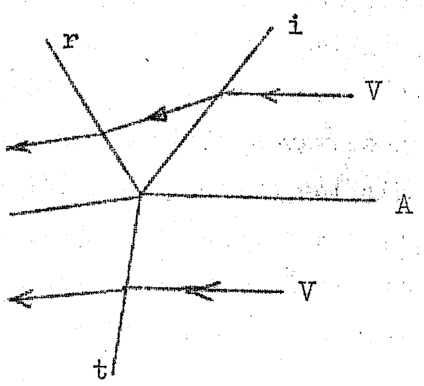
/two ...

* The strength of a shock wave is defined here as the pressure rise across it.

two fluids takes place. Let the incident shock i move with a velocity V parallel to the interface (see figure a)

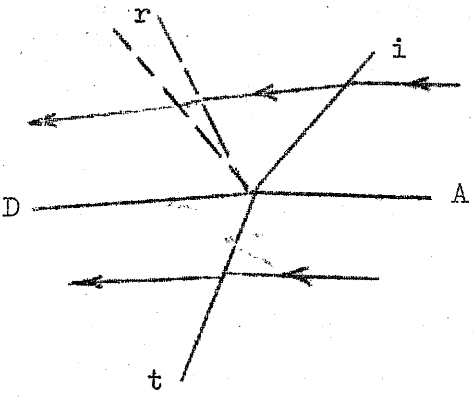


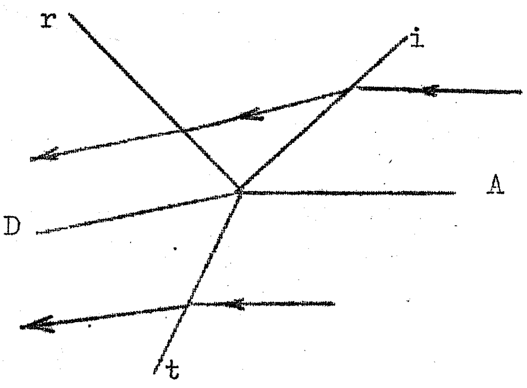
(a)



(b)

For ease of discussion we can consider the equivalent problem of the shock i at rest with both media flowing towards it at the uniform speed V. (See figure b). For equilibrium the pressures in both media upstream of i must be equal. If the speed of sound a_2 is greater in medium II than a_1 in medium I a shock wave t will be transmitted into medium II provided that V/a_2 is greater than unity. The reflected wave r may be a shock wave or an expansion and for certain values of V/a_1, V/a_2, and incident shock strength, it has zero strength. Its strength is automatically adjusted so that the flow direction and pressure are the same on both sides of the downstream interface D, which must represent a vortex sheet. Strong incident shocks give rise to reflected shocks whereas weak incident shocks may give rise to reflected expansions. For given values of V/a_1 and V/a_2 we find that with increase in the incident shock strength the Mach number of the flow downstream of t approaches unity.





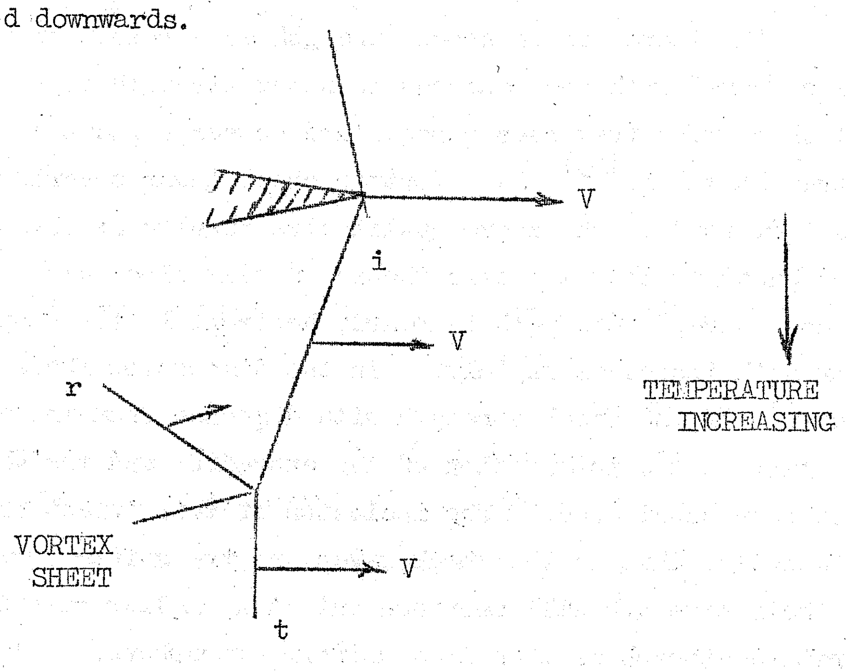
Further increase in the strength of i produces the strong type of transmitted shock t with subsonic flow behind it.

When V/a_2 is less than unity we find that although no shock wave can be transmitted into the lower medium the pressure rise across i is partially transmitted upstream in the medium II since the flow there is subsonic. In this case a continuous increase in

/pressure ...

pressure occurs along the interface A up to the position of the incident shock. Approximate calculations show that for values of V/a_1 and V/a_2 just greater than unity the transmitted shock t becomes more nearly normal and the pressure rise across it is in general greater than that across the incident shock i.

The above results can be applied qualitatively to deduce the strength and change in shape of the bow wave from an infinitely long wedge shaped body travelling at constant height and constant supersonic speed in an atmosphere having uniform pressure everywhere but continuously variable temperature (decreasing upwards). In a uniform atmosphere the bow wave would have constant inclination to the horizontal but in the variable temperature atmosphere its inclination increases with distance from the body. At a certain point regular transmission breaks down and a nearly normal shock wave is transmitted downwards.



The speed of propagation of t will equal that of the body but the ratio of its speed to the local speed of sound in the undisturbed medium will be smaller than that of the body, since the speed of sound increases downwards. A complete solution to this problem has not yet been published. It is perhaps unfortunate that in the range of Mach numbers close to unity, of direct interest here, the solution is likely to prove very difficult. However, approximate calculations indicate that the pressure rise across the transmitted waves, in this range of Mach numbers, are greater than that across the incident waves.

In order to illustrate the importance of this atmospheric refraction effect the following table has been prepared. It shows the pressure rise across normal shock waves travelling at speeds just above sonic.

/Table 3 ...

TABLE 3

The pressure rise across normal shock waves at ground level

M_o Speed of normal shock wave divided by the undisturbed local speed of sound	Δp pressure rise lb./sq.ft.
1.0	0
1.01	48
1.02	96
1.03	144
1.04	192
1.05	240

The increases in shock strength as a result of refraction can be compared with the changes in shock strength found when a normal shock wave traverses concave and convex corners formed between two straight walls.[15] As a further check towards verifying the trends indicated in the above qualitative results approximate calculations based on the acoustic theory of weak shock waves show that as a shock wave, whose path is nearly horizontal, turns upwards so its strength increases rapidly. In the discussion above the attenuation of the shock strength with distance from the aircraft due in part to the interaction of the expansion and the shock waves has not been considered. The inclusion of this effect shows, qualitatively, that as the shock waves are transmitted towards ground level their strength will decrease but at a smaller rate in a variable temperature atmosphere than in a uniform atmosphere. Similar effects will result when a shock wave moves into a wind gradient.

An overall quantitative assessment of these various atmospheric effects is difficult but some approximate calculations indicate that for low values of supersonic Mach number the strength of the shock waves close to the ground, given in Table 1, should be multiplied by a factor of about 5.

A further atmospheric effect, which may be of some importance in certain cases, results in shock waves being reflected back from very high altitudes with increased intensity. The explanation of this phenomenon, which is well known from records of ground explosions, lies in the temperature inversions which occurs at very high altitudes. Thus the 'upward going' shock wave from an aircraft in level supersonic flight will be reflected and then refracted back to the ground some time after the aircraft has passed overhead. It is unlikely, however, that the strength of this wave will exceed that of the original 'downgoing wave'.

/The conclusions ...

The conclusions that can be drawn from this section can be summarised as follows.-

(i) Shock waves transmitted towards the ground from aircraft in level supersonic flight will be attenuated slightly as they move into regions of increasing density.

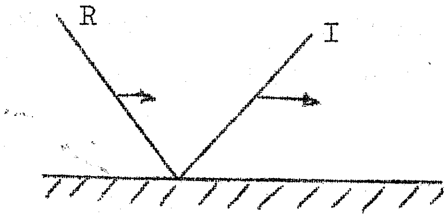
(ii) The change in shape of a shock front as it moves into regions of increasing temperature is shown qualitatively to lead to transmitted waves of increased strength.

(iii) The values of Δp in Table 1, which are only strictly valid when (a) the bow wave is attached to the body, say for M_o 1.4, (b) sufficient distance from the aircraft is taken, say for y 1000ft, must be modified to include the effects of the heterogeneous atmosphere. It is suggested that sufficient allowance for these can be made if the values of Δp in Table 1 are multiplied by a factor of about 5. (It should be noted that the quoted values of Δp are for particular values of fineness ratio and body length).

(iv) Since, as stated in (iii) above, the data in Table 1 are only very approximate for values of M_o less than 1.4, even though the calculations are based on the assumption of a uniform atmosphere, the values of Δp should be multiplied by a further factor of about 2.0 in this range only.

§4. Shock wave reflection at ground level

When a very weak plane shock wave strikes a solid surface obliquely it reflects from it and the angle of reflection is only slightly less than the angle of incidence. The pressure behind the reflected wave is approximately twice that in front of the incident wave. The angles of incidence and reflection are no longer equal when reflection of strong shock waves occurs.



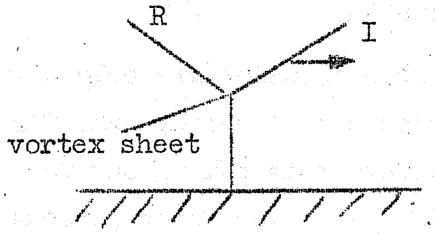
Regular reflection (a)

In the latter case the pressure behind the reflected wave is 8 times that in front of the incident wave. It is important to note that reflection of cylindrical and spherical waves causes pressure increases far in excess of eight.[16] However, in our problem since the incident shock waves, at a large distance from the aircraft, will have intensities of less than 200lb./sq.ft. they may be classed as weak shock waves, having a pressure doubling on reflection. (It must be realised, however, that for aircraft flying at altitudes of less than say 200ft.

/at speeds ...

at speeds just greater than sonic, or at heights very close to the ground at speeds just below sonic, the strength of the shock waves might well exceed a figure of 200lb./sq.ft.)

When the shock strength and angle of inclination of the incident wave exceed critical values for each Mach number the reflection can no longer be regular, or weak as it is usually called. The shock wave configuration changes from the typical 'V' to a 'Y' pattern. The latter is called Mach reflection.

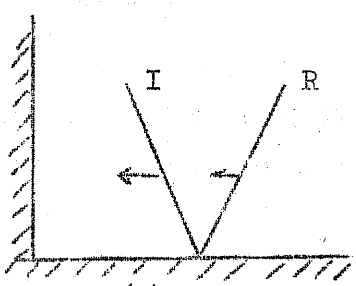


Mach reflection (b)

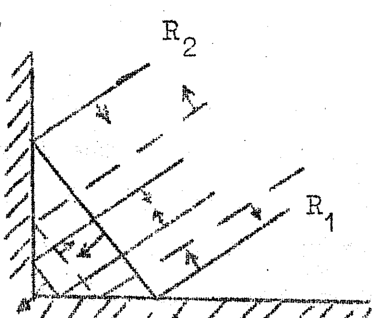
In this case the nearly normal shock wave, close to the surface, has a strength exceeding that of the incident wave. Ordinarily at low supersonic Mach numbers Mach reflection does not exist, but a similar phenomenon occurs when a temperature gradient exists ahead of the shock wave, in which one or more of the shocks are curved. It can be seen, therefore, that this modified form of Mach reflection plays an important part in both the estimation of atmospheric refraction and ground reflection effects.

As an example we can take the case of an aircraft travelling at a Mach number of 1.1 at a height of 25,000ft. under normal atmospheric conditions. On the acoustic approximation to the shape of the bow wave a cusp will be formed at ground level. In reality this cusp will be a normal shock wave travelling at a speed just greater than sonic. The pressure change across it would lie between 5 and 20lb./sq.ft.*

The treatment above for shock waves passing over horizontal ground can easily be applied to their movements along vertical walls. In this case pressure doubling also occurs between the pressure initially upstream of the incident shock and that finally downstream of the reflected shock. An interesting problem presents itself when an oblique incident shock reflects regularly from the ground and then moves towards a vertical wall.



(c)



(d)

* An estimate of the strength, on the basis of the method discussed in section 3, could be obtained as follows. From Table 1 $\Delta p \simeq 1$ lb./sq.ft. Since M_o is less than 1.4 multiply by 2 and to allow for atmospheric effects multiply by 5. Hence $\Delta p = 10$lb./sq.ft.

When the shocks reflect in the corner between the vertical wall and the ground it is found that a pressure excess of four times the original strength of the incident waves results. Subsequent reflections could increase this pressure excess still further. A limit is necessarily set in practice by dissipation of the waves and by the interaction of the shock waves with the expansion waves springing from the finite edges of the wall. The reflection of expansion waves which follow in the wake of shock waves can be treated in a similar manner. It is found that these create a negative pressure equal in magnitude to the total overpressure exerted by the incident shock and its reflections.

We can summarise the results of this section by estimating the shock pressures for two typical flight cases (i) A supersonic dive from high altitude in which the maximum Mach number reached is 1.05 and the shock waves separate from the aircraft at about 20,000ft, (ii) Level flight at Mach numbers below 2.0 at an altitude of 5,000ft. We will consider the maximum overpressure developed on a vertical wall in both cases.

In case (i) the strength of the incident wave should be about 2lb./sq.ft. Hence the maximum overpressure would be 8lb./sq.ft.

In case (ii) the strength of the incident wave, on the assumption that its value is about 5 times that quoted in Table 1, would be about 25lb./sq.ft. over the Mach number range considered. On reflection from a vertical wall this would increase to 100lb./sq.ft.

As stated previously no great accuracy can be claimed for these estimated excess pressures. From a number of different considerations it is felt that they are more likely to represent maximum rather than minimum excess pressures.

§5. The range of excess pressures on structures for which minor damage is likely to occur

The tests on atomic bombs[17] have shown that overpressures of about 200lb./sq.ft. on the ground behind the incident shock wave (exclusive of pressure rise due to reflection) causes complete window breakage, light damage to window frames and doors, and moderate plaster damage. Overpressures of about 50lb./sq.ft. cause window breakage and minor damage to plaster. The duration of the pressure and suction phase, in these tests, are 1 sec. and about 4 sec. respectively.

On the other hand in a strong gale (50 m.p.h.) when the maximum positive and negative excess pressures reach 6 and 10lb./sq.ft. respectively very minor damage to property occurs. Widespread damage

/results from ...

results from a hurricane (100m.p.h.) when these pressures reach 20 and 31 lb./sq.ft. respectively. Naturally the reason why such low excess pressures can cause damage is that they are acting for long periods and in addition the structure receives a buffet due to the gusts present.

In order to assess whether any given structure will withstand a shock load it is necessary to compare the impulse ($\int_0^t p\,dt$) of the shock with the elastic constants of the structure and the criterion for failure of the structure. In all cases it is known that the overpressure must exceed a critical value F_o, which is a function of the given structure, before a failure can occur. The value of F_o is independent of the magnitude of the impulse. For instance a typical value of F_o for a glass window 3ft x 3ft x 1/16in. thick* would be about 12lb./sq.ft. (This figure depends critically upon the thickness of the glass). Actual failure depends mainly on the displacement of the structure, and its pre-loading.

§6. Some tentative remarks on the possible minor damage to property from shock waves generated by supersonic aircraft.

It has been shown above that at present insufficient information exists on the precise configuration of shock waves, close to the ground, generated by supersonic aircraft flying at various altitudes. Published papers(8) give sufficient information to explain the character of the noise heard but give little or no information on the exact shock pattern giving rise to the noise. It is therefore of little practical value to perform extensive calculations on the resistance to shock of the different components of a building. It appears to be sufficient to compare the results obtained in sections 4 and 5 to establish at least the order of damage likely in typical flight cases. The effect consequent on reduction of the shock impulse, due to the small time duration, will be neglected.

In the case of the high speed dive (case i in section 4) it is seen that the excess pressure of about 8lb./sq.ft. is in general too small to cause even window breakage. However, freak cases, involving multi-shock reflection and favourable atmospheric refraction effects, might occur when the excess pressure was just sufficient to cause such minor damage. The number of cases, especially outside of main built up areas, would be very small indeed.

/In level flight ...

* This thickness of glass is used to illustrate the worst possible effects. Naturally the thickness of glass for a window of this size would, in general, be greater than 1/16in.

In level flight at 5000ft. (case ii section 4) the incident shock strength is also barely sufficient to cause complete window breakage and minor damage to plaster. The number of freak cases, when such damage will occur, is likely to be many times greater than in the previous example.

§7. Conclusions

A brief survey has been made of the factors involved in making a prediction of the strength of the shock waves and their configuration close to the ground from an aircraft travelling at supersonic speeds.

Attention is drawn to the following effects which hitherto have not been fully taken into account.

(i) Attenuation of the shock waves due to their receding distance from the aircraft.

(ii) Atmospheric attenuation as the shock waves move into regions of higher density.

(iii) Atmospheric amplification due to atmospheric refraction (temperature variation with height).

A qualitative assessment of these various factors has been applied to a discussion on the possible damage to property due to the passage and reflection of the shock waves.

It must be emphasised that until further evidence is available to establish the accuracy of these calculations they can only be regarded as guides to the orders of magnitude of the pressures involved. It is tentatively concluded that

(a) For aircraft in supersonic dives of finite duration at high altitudes window breakage or other very minor damager is only likely to occur in a number of 'freak' isolated cases.

(b) For aircraft in level supersonic flight at Mach numbers less than 2 at 5000ft, the strength of the incident shock wave is generally not sufficient to cause widespread minor damage but the number of 'freak' cases when such damage is likely to occur is greater than in case (a) above.

/References. ...

No.	Author	Title, Date etc.
1.	Prandtl L.	Über Schallausbreitung bei rausch bewegten Korpern - Jahrbuck Dent. Akad. Luftfahrtforschung. 1938 (pp.23-30) and R.A.E. trans. 431 (1953).
2.	Yates A.H.	Those sonic bangs, Flight, October 1952. Letter Flight, October 1953.
3.	Warren C.H.E.	Sonic bangs, R.A.E. Tech. Note Aero 2192 and 2192a (1952). More about sonic bangs, Flight, October 1952. Letter, Flight, November 1952. Letter, Nature, Vol. 171 (1953) p.214 Sonic bangs - a summing up, Flight, Sept. 1953.
4.	Hilton W.F.	High speed aerodynamics (Longmans, 1952)
5.	Ackeret J.	Akusticke phanomene bei hohen fluggeschwindigkeiten. Schweizer Aero Review No. 11, Nov. 1952.
6.	Gullstrand T.R.	Sonic bangs. Saab Sonics No. 19 (1953) p.18.
7.	Fozard J.W. and others	The sonic boom. Hawker Siddeley Review, December (1952) p.100. Letter, Flight, December (1952).
8.	Lilley G.M. and others	On some aspects of the noise propagation from supersonic aircraft. College of Aeronautics Rep. No. 71 (1953) and J.R.Ae.S. June (1953). The supersonic bang. Nature, Vol. 171 (1953) p.994.
9.	Gold T. Bastin E.W. Rothwell P. Esclagnon E.	Letter, Nature Vol. 170 (1952) p.808 ' ' Vol. 171 (1953) p.215 ' ' Vol. 171 (1953) p.215 ' ' Vol. 171 (1953) p.827
10.	Hunn B.A. Young N. Dickenson D.R.H. Misc.	Letter, Flight Oct. (1952), Nov. (1952) and Oct. (1953). ' Flight Oct. (1952) and Nov. (1952) ' ' Oct. (1952) ' ' Sept. (1952), Oct. (1952) Nov. and Dec. (1952). Aug. (1952), May (1953).
11.	Kirk A.	The apparent explosions of supersonic aircraft. Jnl. R.A.F. Technical College, Henlow 1. Sept. (1953).
12.	Whitham G.B.	Supersonic flow past a body of revolution. Proc. Roy. Soc. Series A. Vol. 201 (1950) p.89-109 and The flow pattern of a supersonic projectile. Comm. Pure and App. Maths. Vol. 5 (1952) p.301-348.
13.	Lamb H.	Hydrodynamics, 6th Ed. (1932) C.U.P. p.541-547.
14.	Polackek H. and Seeger R.J.	On shock wave phenomena. Refraction of shock waves at a gaseous interface. Proc. 7th Int. Congress App. Mech. (1948) Vol. 2, Part 2, pp.485-499.
15.	Lighthill M.J.	Diffraction of Blast I, II. Proc. Roy. Soc. A. Vol. 198 (1949) and Vol. 200 (1950).

16. Guderley G. Powerful spherical and cylindrical compression shocks in the neighbourhood of the centre of the sphere and of the cylinder axis. Luftfahrtforschung Vol. 19 No. 9 (1942) pp. 302-312, and R.T.P. trans. 1778.

17. U.S. Atomic energy commission and Dept. of Defense. The effects of Atomic Weapons. McGraw-Hill (1950).