The relevance of the mechanics of metal cutting to machinability

- by -

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INTRODUCTION

The process of metal cutting is a subject of great importance to the makers and users of machine tools. Extensive research has gone into the subject but has still left most of the phenomena unexplained. Tool life is the main interest and before any real improvement in this factor can be made, the basic metallurgical factors governing the interaction between tool and workpiece must be better understood. Such improvement can be effected through control of the wear process since both tool and workpiece are metallic and machining is a process of metal flow which is associated with a serious wear problem. The absence of exact knowledge has however hampered empirical and mathematical approaches to the problem.

Basically all machining operations are considered as either oblique or orthogonal cutting, the former requiring three dimensions to specify the geometry of the cutting part of the tool and the latter two. The basic metal cutting process to be considered is that which is common, in one form or another, to all metal cutting operations using a tool, that of the wedge-shaped tool in fig. 1 (a-j) (1). Analyses of cutting have been mainly concentrated on the relatively simple case of orthogonal or two-dimensional cutting. Here the tool is so set that its cutting edge is perpendicular to the direction of relative motion between tool and workpiece and generates a plane parallel to the original work surface. In doing this the tool removes a layer of material termed the chip.

One of the major objectives of metal cutting theory is the determination of machining forces, chip geometry, tool life, energy consumption and surface finish from a knowledge of the physical properties of the workpiece and tool material and the cutting conditions alone. If this could be achieved, lengthy chip measurements, delicate dynamometry, tedious and costly tool life tests and surface finish measurements might be dispensed with.

CHIP FORMATION AND TYPES OF CHIP

Among the earliest to carry out an investigation into metal cutting were Rosenhain and Sturney (2). They differentiated between the types of chip formed and classified them into the 'tear', 'shear' and 'flow' types. Later, by the use of high speed motion pictures and photomicrographs, Ernst (3) made a basically similar exposition of three types of chip. These can be described as follows:

Type 1. The Discontinuous Chip: In this type of chip formation, segments are formed by rupture which occurs intermittently and is observed to take place ahead of the tool, leaving a rough and irregular surface. Almost without exception, a discontinuous chip is formed in all machining operations involving brittle materials such as brass or cast iron. Under certain conditions this occurs also with ductile materials.
Type 2. The Continuous Chip: Here the metal deforms plastically to form a continuous ribbon. This type of chip is obtained when cutting a ductile material such as mild steel under favourable conditions such as good lubrication between chip and tool. The resulting machined surface is smooth. Ernst concluded that this type of chip was produced through shear across a narrow zone which may be approximated to a simple plane extending from the tool edge to the work surface ahead of the tool.

Type 3. Continuous Chip with Built-up-Edge: Under certain conditions as has been shown by photomicrographs, when producing a continuous chip, a zone of highly deformed material adheres to the tool near the cutting edge. This has been named the 'built-up-edge' and is usually found welded to the tool after a machining operation. Ernst suggested that this type of chip was formed as a result of the high value of tool-chip interface friction which he therefore concluded was a deciding factor in determining the type of chip formed.

Gladman (4) suggests that the built-up-edge is work-hardened material and is one of the causes of bad finishes in machined surfaces; the built-up-edge is not stable, but periodically builds up and breaks down and some parts of it are carried away in the chip while other parts are left embedded in the surface thus marring it. He adds that this type of chip is obtained when machining ductile materials at low speeds.

A built-up-edge can also be produced when cutting a discontinuous chip (1) despite statements to the contrary. There is agreement here with Gladman (4) that the built-up-edge should not be regarded as a disordered heap of metal pressed together and welded to the tool tip but as a continuous part of the workpiece and the chip which forms it. Especially when it is large it will be unstable and will vary in size from time to time at different places along the cutting edge, parts of it sloughing off attached to the workpiece or the chip.

It is found in practice however that though these three types of chip are clearly different, small changes in the cutting conditions can cause a transition from one type to another or produce a chip that is a combination of more than one kind. For example, when machining a ductile material, a decrease in rake angle \( \alpha \), Fig. 1(a), an increase in depth of cut \( t \) or a decrease in cutting speed can cause a transition from a continuous to a discontinuous chip.

The analysis of the mechanics of metal cutting is usually restricted to orthogonal cutting where the cutting edge is perpendicular to the direction of motion of the tool. Most of the investigations are concentrated on the continuous type of chip rather than the more complex process of discontinuous chip formation.

**THE SHEAR PLANE THEORY**

The Continuous Chip. Many theories of metal cutting have used the
concept of the shear plane. This is represented in Fig. 1(a) by line AB inclined at angle $\phi$ to the direction of cutting; this angle is called the shear angle. All shear strain is assumed to be acquired as a particle crosses this line.

The significance of the shear angle lies in the fact that it indicates, to some extent, the machinability properties of the work material. The machinability of a material is described in terms of the magnitude of cutting forces, the effective cutting life of the tool and the properties of the machined surface. Machinability investigations normally take the form of cutting tests in which such parameters as cutting speed, feed and tool rake angle are varied, and the cutting forces, tool wear and surface finish are measured. The optimum tool geometry and cutting conditions for tool life, surface finish etc., can then be found. High values of shear angle are associated with large ranges of continuous chip and good surface finishes. At low values chips tend to become discontinuous and surface finish to deteriorate.

The best known of the shear angle solutions is by Ernst and Merchant (5) who, assuming that $\phi$ would attain such a value as to give a minimum expenditure of work in cutting, obtained the equation:

$$\phi = \frac{\pi}{4} + \alpha - \frac{\lambda}{2}$$

(1)

where $\lambda$ is the mean angle of friction along the tool-chip interface.

Using ideal slip-line theory, Lee and Shaffer (6) developed the relationship

$$\phi = \frac{\pi}{4} + \alpha - \lambda$$

(2)

The above equations suggest that an increase in rake angle or a decrease in friction angle will give an increase in shear angle; they also suggest a unique value for $\phi$ for a given value of $\lambda - \alpha$. Experimental values of $\phi$ by various researchers (7), (3), (9), (10), (11), (12), (13) have proved this to be untrue and also shown that $\phi$ varies with material and cutting speed, factors not taken directly into account by equations (1) and (2).

Merchant (7) modified his earlier equation by considering the properties of the work material and obtained the relationship:

$$2\phi = C + \alpha - \lambda$$

(3)

where $C$ is constant for a given material and represents the dependence of shear strength on the normal stress.

Kobayashi and Thomsen (14) introduced a quantity called 'effectiveness'
into their analysis of the shear angle problem. Thus their solution corresponds to Eqn. (1) when effectiveness is unity and gives a lower \( \phi \) for a given \( \lambda - \alpha \), when effectiveness is taken below unity. This quantity however appears to be constant for a given material and cutting speed, thus there is no apparent fundamental relationship between effectiveness and work material properties or speed.

Sata (15) developed a relationship:

\[
\phi = \cot^{-1} \left[ \frac{\cot \theta \cos \theta}{\sin (\phi + \alpha)} KL \right]
\]

where \( \theta \) is the angle between the resultant cutting force and the shear plane, \( K \) is the ratio of the shear stress on the rake face to that on the shear plane and \( L \) is the ratio of the tool-chip contact length to the depth of cut.

Another attempt at the solution of the shear plane problem was made by Oxley (16). He provided an analysis for cutting with restricted tool-chip contact and stated that for given contact length and tool rake, shear angle and friction angle can be predicted. The analysis predicts an increase in shear angle \( \phi \) and a decrease in friction angle \( \lambda \) for a decrease in contact length. By considering the stress boundary conditions at A and B, Fig. 2, the normal stress acting on the shear plane AB at these points can be expressed in terms \( k \) (yield stress along AB), \( \phi \), \( \alpha \) and \( \lambda \) as follows

\[
P_A = k \left( 1 + 2 \left( \frac{\pi}{4} - \phi \right) \right)
\]

\[
P_B = k \left( \frac{\cos 2(\phi - \alpha)}{\tan \lambda} - \sin (\phi - \alpha) \right)
\]

and by assuming a linear stress distribution between A and B,

\[
\tan \theta = \frac{P_A + P_B}{2k}
\]

where \( \theta \) is the angle of inclination of the resultant cutting force to the shear plane.

From the geometry of the figure

\[
\theta = \phi + \lambda - \alpha
\]

These values of \( \phi \) agreed reasonably well with experimental observations. In addition, the position of the resultant cutting force calculated from the shear stress distribution was shown to be consistent with the position calculated by assuming a constant state of plastic stress along the tool-chip interface.
One disadvantage of this method is that cutting tests have to be
made first to measure \( \lambda \) before \( \phi \) the shear angle can be calculated. The
most convenient way of making a quantitative comparison between theoretical
and experimental results is to compare the shear angles and friction
angles. The experimental and theoretical values were plotted against a
parameter \( \frac{h}{t} \) where \( h \) is the natural contact length. \( \frac{h}{t} \) can be calculated
by dividing the normal force by the normal stress to obtain

\[
\frac{h}{t} = \frac{\sin \lambda}{\sin \phi \cos(\phi + \lambda - \alpha) \cos 2(\phi - \alpha)}
\]  \hspace{1cm} (9)

If \( h \) is now reduced below the value it would have when cutting with
a normal tool, Fig. 3, then \( \frac{h}{t} \) is known for given \( \alpha \); \( \phi \) and \( \lambda \) can then be
calculated. By selecting \( \phi \) and \( \lambda \) to satisfy Eqns. (5), (6) and (7)
and using Eqn. (9) to calculate \( \frac{h}{t} \), graphs of \( \phi \) and \( \lambda \) against \( \frac{h}{t} \) were plotted
for each rake angle. The graphs showed that the experimental values
were of the same order as the theoretical though the actual differences
were rather large.

The foregoing are unique-valued solutions of the shear angle. Other
single-valued solutions are attributed to Christopherson (17) and Zorev (18).

Hill (19) criticised these shear plane solutions on the grounds that
they were directed towards determining a unique steady state configuration
for given tool rake angle and suggested that many steady states of the single
shear plane type were possible for different initial conditions. He
considered some of the main reasons for the disagreement between theory and
practice to be:

(a) unrealistic basic assumptions, e.g. isotropy, absence of work-hardening,
infinite shear modulus, constant friction angle, no thermal effects.

(b) inadequate experimental procedure, the geometry of the distribution
being difficult to determine during motion and under plane strain
conditions.

(c) unsound theory, even within its self-imposed limits.

He suggested that there may be infinitely many steady state config-
urations of a given type and provided a plot showing a region of permissible
values of \( \phi \), partly bounded by the Ernst and Merchant (5) and the Lee and
Shaffer (6) solutions. Hill's conclusions which follow from a train of
reasoning based on a non-workhardening material have been substantiated
by Enahoro (13), using work-material whose properties approached those of a
rigid non-workhardening material, he found that the shear angle values are
dependent on rake angle, depth of cut and cutting speed; the values of shear
angle fell within Hill's permissible region in the case of cold rolled mild
steel and aluminium alloy HE-10-WP whose properties approached those of a rigid non-workhardening material, Fig. 4.

The Discontinuous Chip: The shear plane concept has also been used extensively to investigate discontinuous chip formation. Cook, Finnie and Shaw (20) classified the discontinuous chip into two types; they distinguished between the cracks that are just visible under the microscope in machining ductile materials and the completely discontinuous chip where the material is removed in the form of separate segments.

Field and Merchant (21) noted that the discontinuous chip was formed during the machining of brittle materials like cast iron or when cutting ductile materials at low speeds without cutting fluids. As in the Merchant continuous chip analysis, they applied the principle of minimum energy to give

$$\phi_2 = \phi_1 - \lambda + \alpha + C$$  

where $\phi_1$ is the shear angle at rupture, $\phi_2$ is the shear angle when the shear plane meets the surface left by the previous segmented chip and $C$ is analogous to that in the continuous chip analysis.

The same objections to the application of the shear plane theory to the solution of continuous chips are relevant with respect to discontinuous chip formation.

**THE SHEAR ZONE THEORY**

The Continuous Chip: There is substantial experimental evidence from direct observation of chip formation to show that during cutting deformation does not take place on a single shear plane but over a finite zone. The concept of a flow region used by Okushima and Hitomi was reported by Hitomi (22). It is a region which exists between the rigid zone of the workpiece and the plastic region of the chip. The flow region is shown in Fig. 5. It was assumed for simplicity that the boundary lines are straight lines extending from the tip of the cutting tool to the starting and end points of the flow region on the free surface. Considering the flow region and chip, theoretical equations for angles of inclination of the boundary lines were deduced in the above paper.

$$\phi_1 = \frac{K_1}{2} - \frac{\lambda}{2} + \frac{\alpha}{2}$$  

$$\phi_2 = \frac{K_2}{2} - \frac{\lambda}{2} + \frac{\alpha}{2}$$  

where

$$K_1 = \sin^{-1} \left( \frac{2}{k_1} \sin \lambda + \sin (\lambda - \alpha) \right)$$

$$K_2 = \cos^{-1} \left( \frac{2}{k_1} \sin \lambda - \cos \lambda \right)$$
and

\[ k_1 = \frac{L}{t}; \quad k_2 = \frac{L}{T} \]

Sector angle or size of flow region

\[ \Phi = \phi_2 - \phi_1 = \frac{\alpha}{2} - \frac{K_1}{2} + \frac{K_2}{2} \]  \hspace{1cm} (13)

Two other Japanese workers Takeyama and Usui, using this flow region concept discovered the importance of tool-chip contact area, they employed a special cutting tool with an artificially controlled contact area, and discovered that friction force was directly proportional to tool-chip contact area. They deduced an equation for the shear angle \( \phi \) in terms of rake angle \( \alpha \) and contact area \( A_0 \)

\[ \phi = \frac{1 - \sin \alpha}{\cos \alpha} + \frac{k_0}{r_0} \frac{A_0}{A'_0 \cos \alpha} \]  \hspace{1cm} (12)

where \( A'_0 \) is the area of uncut chip, \( k_0 \) is a constant nearly equal to the shearing strength of the material and \( r_0 \) is the shearing stress in the shear plane.

Further experimental evidence (8), (23) suggests that cutting takes place over a plastic zone of the shape shown in Fig. 6. One of the assumptions made in the shear plane type of analysis is that the work material is an ideal plastic-rigid material, that is, the shear flow stress of the material is assumed constant.

In a detailed analysis of an experimentally observed shear zone Palmer and Oxley (23) found that neither the shape of the shear zone nor the position of the resultant cutting force were consistent with the concept of an ideal plastic-rigid material and could only be explained if the shear flow stress of the work material was considered to vary.

Implicit in all shear plane solutions is that the normal (hydrostatic) stress is constant along the length of the shear plane. That this is not so for a material having a variable flow stress can be seen by considering an element of a shear zone lying between two adjacent slip lines, Fig. 7. As the material passes through the shear zone its flow stress will alter as a result of work-hardening, temperature, strain rate, etc. If the shear flow stress along CD is taken as \( k - \frac{\Delta k}{2} \) and along EF as \( k + \frac{\Delta k}{2} \) so that the change in flow stress across the element is \( \Delta k \), by resolving forces parallel to CD, it can be shown that

\[ \Delta p = \frac{\Delta k}{\Delta S_1} \cdot \Delta S_2 \]  \hspace{1cm} (13)

where \( \Delta p \) is the change in hydrostatic stress across the element, \( \Delta S_1 \) is
the width of the element and $\Delta S_2$ is its length.

In cutting, it is found that the chip is harder than the work material so that the term $\frac{\Delta k}{\Delta S_1}$ is positive, indicating that the normal stress along the shear zone has its highest value at the outer free surface and becomes less compressive towards the tool point.

If, as in a recent analysis by Oxley and Welsh (24), the shear zone is idealised into a parallel sided zone, inclined at angle $\phi$ to the direction of cutting, as shown in Fig. 8, then Eqn. (13) can be integrated along the slip-line AB to give the expression

$$p_A - p_B = \frac{\Delta k}{\Delta S_1} \cdot \frac{t}{\sin \phi} \quad (14)$$

$\frac{\Delta k}{\Delta S_1}$ is assumed constant, $p_A$ is the hydrostatic stress at A (the free surface), $p_B$ is the hydrostatic stress at B (the tool point) and $\frac{t}{\sin \phi}$ is the length of AB.

For a constant value of $\frac{\Delta k}{\Delta S_1}$ along AB, the forces per unit area tangential ($F_t$) to and normal ($F_n$) to AB, are given by

$$F_t = k \cdot \frac{t}{\sin \phi} \quad (15)$$

and

$$F_n = \frac{p_A + p_B}{2} \cdot \frac{t}{\sin \phi} \quad (16)$$

From the above, the angle $\theta$ made by the resultant cutting force with AB is given, as in the shear plane analysis by Oxley (16), by

$$\tan \theta = \frac{F_n}{F_t} = \frac{p_A + p_B}{2k} \quad (7)$$

It is convenient to find the hydrostatic stress at A from the condition of the free surface between C and A. In Fig. 9 a shear line $A_1A_2A_3B_1$ is shown adjacent to AB with $A_2B_1$ parallel to AB. In order to satisfy equilibrium $A_1A_2A_3B_1$, as a line of maximum shear stress, must meet the free surface at $45^\circ$ and the hydrostatic stress in the zone $A_1A_2A_3$ must be equal to the shear flow stress ($k$). By taking moments about A, the equilibrium of the element $A_1A_2A_3$ is given, as in the shear plane analysis (16), by

$$p_A = k \left[ 1 + 2 \left( \frac{\pi}{4} - \phi \right) \right] \quad (5)$$

The change in hydrostatic stress along the shear zone can now be found by substitution in Eqn. 14. If the stress strain curve of the work
material is idealised into a straight line, as shown in Fig. 10, then

$$\Delta k = m\gamma$$

(17)

where \( m \) is the slope of the stress-strain curve at the mean strain rate and \( \gamma \) is the mean strain given by the change in velocity tangential to AB divided by the velocity normal to AB across the zone.

i.e. \( \gamma = \cot \phi - \tan (\phi - \alpha) \)

(18)

From the geometry of Fig. 8 the angle \( \theta \) can be expressed in terms of the frictional condition along the tool-chip interface and if this is described by a mean angle of friction \( \lambda \), then as in the shear plane analysis (16), again

$$\theta = \phi + \lambda - \alpha$$

(8)

For a given value of \( \lambda \) and \( \alpha \) the angle \( \phi \) can be calculated from Eqns. (14) - (18), (5), (7) and (8) such that the stress distribution along AB and the tool-chip interface are consistent for the direction of the resultant cutting force.

It is evident from Eqns. (14), (7) and (17) that the less a material work hardens (smaller values of \( m \)), the larger will be the value of the angle \( \theta \). Similarly the larger the shear flow stress \( k \), the larger will be the angle \( \theta \), with a correspondingly smaller difference in hydrostatic stress between A and B.

It is generally considered that as strain rate increases the initial flow stress increases and stress-strain curves tend to flatten off, Drucker (25), as in Fig. 11. In cutting, the mean strain rate \( \dot{\gamma} \) can be calculated by dividing the mean strain (Eqn. (18)) by the time taken for a particle to cross the shear zone, and is given by the expression

$$\dot{\gamma} = \frac{U \cos \alpha}{\Delta S_1 \cos (\phi - \alpha)}$$

(19)

where \( U \) is the cutting velocity. Thus strain rate is increased by an increase in cutting speed, an increase in rake angle, or a decrease in the depth of cut (assuming geometrical similarity, \( \Delta S_1 \) will be proportional to the depth of cut).

As an illustration of this, Fig. 12 has been prepared; it shows theoretical values of \( \phi \) plotted against \( \lambda - \alpha \) for a material in which the shear flow stress and work-hardening parameter have been considered to vary with strain rate.

The Discontinuous Chip. It is known in practice that decreasing the cutting speed or increasing the depth of cut leads to a transition from a
continuous to a discontinuous chip with the initiation of cracks in the region of the tool point.

Using the strain-hardening slip-line theory to examine discontinuous chip formation, Enahoro and Oxley (26) carried out an investigation into the hydrostatic stress distribution in the plastic zone. Various depths of cut were taken covering the range of chips from continuous to discontinuous and keeping all other conditions constant. The hydrostatic stress was found to vary from compressive at the outer free surface to tension near the cutting edge. They suggested that the occurrence of a discontinuous chip depended on the magnitude of the tensile stress at the tool point. Further work on these lines by Foot (27) has since confirmed the above.

It is not possible from a knowledge of a given state of stress, strain and strain rate whether a material will crack or not but it seems reasonable to assume that the higher the compressive stress in a material, the more likely it is to crack. In Fig. 13 the hydrostatic stress (\(\sigma_h\)) at the tool point calculated for the theory of Oxley and Welsh (24) for two values of \(\phi\) over a range of depths of cut are shown, illustrating the way in which the stress is predicted to fall with an increase in depth of cut. A similar effect, in agreement with practice, is found when the cutting speed is varied.

It can be seen from the above that the inclusion into the analysis of variable flow stress and strain rate dependent properties of the work material enable a qualitative explanation to be given for the observed effects of speed and depth of cut hitherto unexplained by the simpler shear plane model of cutting.

Machinability. In the foregoing theory it was shown that a small value of the ratio \(\frac{m}{k}\) leads to the formation of thin chips (large values of \(\phi\)) and a large compressive stress at the tool point suggests the likelihood of cracking. The thinner the chips formed in cutting, the lower the cutting forces which should give longer tool life and the less the cracking that occurs in the material in the region of the tool point the better should be the surface finish.

It would appear, therefore that the ratio of the fundamental work material properties \(\frac{m}{k}\) should give some indication of the machinability of the material. Unfortunately the strain rates involved in metal cutting are very high, being of the order \(10^6\) per sec., and there is yet no experimental technique of obtaining values of \(\frac{m}{k}\) at these strain rates. As a first approximation, however, it seems reasonable to assume that a material having a high value of \(\frac{m}{k}\) measured by a conventional materials test should have a correspondingly high value of \(\frac{m}{k}\) at a high strain rate.

Fig. 14 shows effective-stress/effective-strain curves obtained by Kobayashi, Hertzog et al (28) for four conditions of SAE 4135 steel with a
hardness of Rc35, a hardness of Rc26, as received, and annealed, their corresponding \( \mu \) values being 0.06, 0.09, 0.21 and 0.21 respectively. In estimating these values of \( \mu \), \( k \) was taken at an effective strain of 0.5, and \( m \) as the mean slope above an effective strain of 0.2. The corresponding experimental values of \( \phi \) were plotted against \( \lambda - \alpha \) given in Fig. 15; they confirm that the higher the values of \( \frac{m}{k} \) the lower is the value of \( \phi \).

In machining, the cutting force against which work is done is in the direction parallel to the cutting velocity and is given by the expression

\[
F_c = \frac{twk \cos (\lambda - \alpha)}{\sin \phi \cos \theta}
\]

\[ (21) \]

where \( w \) is the width of cut.

This equation shows that besides variations in \( \phi \), the cutting force is dependent on the flow stress \( k \) of the work-material, and thus from the point of view of power consumption and stress acting on the tools, it is necessary to consider the magnitude of \( k \) as well as \( \frac{m}{k} \).

Discussion. The analysis of the shear zone presented above, taking into account variable flow stress and strain rate effects, has explained in qualitative terms the effects of changes of speed and depth of cut on chip thickness and the formation of discontinuous chips. In addition good agreement has been shown between values of \( \frac{m}{k} \) estimated from a conventional materials test and the shear angles obtained during cutting.

It was suggested therefore, by Oxley and Welsh (29), that the value of the ratio of \( \frac{m}{k} \) together with the value of \( k \), which are fundamental material properties, should be taken as an indication of the way in which a material machines. Materials having high values of \( \frac{m}{k} \) and \( k \) are expected to machine with large cutting forces, thick chips and result in poor surface finishes.

As both \( \frac{m}{k} \) and \( k \) are bulk properties of a material, they would not indicate machining characteristics such as hard inclusions or free machining additives affecting tool-chip interface friction, nor do they take into account any metallurgical or chemical interaction between tool and work material. But being directly related to the cutting process, \( \frac{m}{k} \) and \( k \) would be expected to give better indications of the way in which a material machines than such parameters as hardness or Ultimate Tensile Strength.

CONCLUSION

The shear angle has been used as a factor in estimating machinability. High values of shear angle indicate large ranges of continuous chips, good surface finishes and low cutting forces. Low shear angle values are associated with small ranges of continuous chips, poor surface finishes and
large cutting forces. The shear plane type of deformation is however not encountered in normal metal cutting processes. Shear tends to occur over a finite zone hence the shear angle is not an accurate practical test for machinability since the shear plane solutions do not generally take account of such important factors as material properties and cutting speed.

The use of $\frac{m}{k}$ and $k$ gives a better guide to the machinability properties of a material. Large values of $\frac{m}{k}$ result in thick discontinuous chips and poor surface finishes while, if $k$ is also large, the forces and therefore the stresses acting on the tool are large and will tend to give a short tool life. Small values of both $\frac{m}{k}$ and $k$ are however consistent with continuous chips, good surface finishes and low cutting forces. Hence Oxley and Welsh (29) suggest that a compression test, carried out to evaluate $\frac{m}{k}$ and $k$, should become an important addition to any investigation on machinability.

REFERENCES


SHEAR ANGLE - $\phi^\circ$ v. $(\lambda - \alpha)^\circ$

**Copper**

- $\lambda = 30^\circ$
- $\alpha = 30^\circ$
- $\lambda = 40^\circ$

$V < 4 \text{ i.p.m.} > 8 \text{ i.p.m.}$

**Pure Aluminium**

- $\lambda = 35^\circ$
- $\alpha = 40^\circ$
- $\lambda = 45^\circ$

$V < 4 \text{ i.p.m.} > 8 \text{ i.p.m.}$

**M.S.**

- $\lambda = 30^\circ$
- $\alpha = 30^\circ$
- $\lambda = 40^\circ$

$0^\circ < 4 \text{ i.p.m.} > 8 \text{ i.p.m.}$

**HE-10-WP**

- $\lambda = 05^\circ$
- $\alpha = 30^\circ$
- $\lambda = 40^\circ$

$0^\circ < 4 \text{ i.p.m.} > 8 \text{ i.p.m.}$

**FIG. 5**
FIG. 6

FIG. 7
SHEAR ZONE ELEMENT

FIG. 8
SHEAR ZONE MODEL OF CHIP FORMATION
FIG. 9

SHEAR ZONE ELEMENT AT FREE SURFACE

FIG. 10

IDEALIZED STRESS-STRAIN CURVE

FIG. 11

INCREASING STRAIN RATE
Fig. 12

Influence of Cutting Speed on Shear Angle

Broken lines: theoretical curves for 0.010 in. depth of cut, cutting speed 1000 ft/min, rake angles from -10° to 40°. Unbroken lines: theoretical curves for similar cutting conditions but 1 in./min cutting speed.

Fig. 13

Influence of Depth of Cut on the Hydrostatic Stress $p_B$

At the tool cutting edge.
FIG. 14

EFFECTIVE STRESS-STRAIN CURVES (KOBAYASHI ET AL.)

1. SAE 4135 RC-35
2. SAE 4135 RC-26
3. SAE 4135 AS REC.
4. SAE 4135 Annealed

FIG. 15

SHEAR ANGLE VALUES FOR MATERIALS SHOWN IN FIG. 14.

\[ \alpha = 0^\circ \]

\( U = 334 \) f.p.m.

- SAE 4135 RC-35 \( \Delta \)
- SAE 4135 RC-26 \( \times \)
- SAE 4135 AS REC \( o \)
- SAE 4135 Annealed \( \circ \)