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Summary

A derivation of the exact equivalent source and doublet distributions, associated with the flow around a prolate ellipsoid of revolution, is presented. This derivation provides a valuable alternative approach to the classical one, does not require a knowledge of the theory of sperical harmonics, and lends itself to a ready comparison with slender-body theory.

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1. INTRODUCTION

The theory of inviscid, incompressible, flow around ellipsoids is one of the classic problems of fluid dynamics for which exact solutions are known. See for example Ref.1, pp.139-156 or Ref.2, pp.277 - 304. Furthermore, these exact solutions for ellipsoids provide a valuable source of comparison for ascertaining the accuracy of approximate techniques such as 'panel' and axial singularity methods. As a result most students of fluid dynamics are made aware of the form of the velocity and pressure distribution graphs associated with the axi-symmetric flow around a prolate ellipsoid of revolution, even though the underlying mathematical theory may be less familiar to them. Since the topic is a classical one, and the results are well known, it is reasonable to ask why yet another paper on the subject is justified.

The present author's excuse stems from two sources. Firstly, recent studies by him, Ref.3, have shown the value of knowing the exact axial doublet distribution required to generate a prolate ellipsoid in axi-symmetric flow. Secondly, comments made by Hess, Ref. 4, and D'Sa and Dalton, Ref.5, indicate that these authors and, I suspect, many others, do not appear to be aware of Temple's derivation, Ref.6, Chapter 15, of the exact equivalent source and doublet distributions associated with the flow around a prolate ellipsoid of revolution. This derivation is a valuable alternative approach to that provided by Munk in Ref.2, pp.283-5 and pp.287-8, or Lamb in Ref.1, and does not require a knowledge of the theory of spherical harmonics.

These derivations will be re-presented in the following:

2. THE LONGITUDINAL MOTION OF A PROLATE (OR OVARY) ELLIPSOID OF REVOLUTION - SOURCE DISTRIBUTION

The geometry of the flow problem is shown in Figs. 1 & 2, the ellipsoid surface being defined by

$$\frac{x^2}{a^2} + \frac{r_b^2}{b^2} = 1\,, \quad (b \leqslant a) \tag{2.1}$$

where

$r_b = (y_b^2 + z_b^2)^{\frac{1}{2}}$ is the body radius,

a and b are the semi-major and semi-minor axes, respectively, and the foci are given by

$$c = |(a^2 - b^2)^{\frac{1}{2}} \tag{2.2}$$

Following Temple, Ref.6, Section 15.4, we are going to generate the ellipsoid by placing a linear distribution of sources between the foci.

We may write this distribution as

$$f(\xi) = -\,2\pi U\,\lambda\xi, \quad (-\,c \leqslant \xi \leqslant c) \tag{2.3}$$

where $U = U_\infty \text{Cos}\alpha$, the axial component of the onset velocity, Fig.1,
and λ is an undetermined constant.

Now the velocity potential at a general field point $P(x,r,\theta)$, due to a source of unit strength located at a position ξ on the axis of symmetry, is

$$\phi_s = -\,\frac{1}{4\pi R(x,\xi,r)}\,, \tag{2.4}$$

where

$$R(x,\xi,r) = |\{(x-\xi)^2 + r^2\}^{\frac{1}{2}}| \tag{2.5}$$

is the Euclidean distance between the source point and the field point. The Stokes stream function, ψ for an axi-symmetric flow may be obtained through the definitions of the velocity components, which are

$$\left.\begin{aligned} u_r &= \frac{\partial\phi}{\partial r} = -\frac{1}{r}\frac{\partial\psi}{\partial x} \\ u &= \frac{\partial\phi}{\partial x} = \frac{1}{r}\frac{\partial\psi}{\partial r} \end{aligned}\right\} \tag{2.6}$$

As a result the stream function for a unit source becomes

$$\psi_s = -\frac{(x-\xi)}{4\pi R(x,\xi,r)}\ , \tag{2.7}$$

whilst that arising from the distribution $f(\xi)$ will be

$$\psi_s = \frac{U\lambda}{2}\int_{-c}^{c}\frac{\xi(x-\xi)}{R(x,\xi,r)}\,d\xi \tag{2.8}$$

The total stream function of this source distribution when placed in the uniform stream U, will then be

$$\psi(x,r) = \tfrac{1}{2}Ur^2 + \frac{U\lambda}{2}\int_{-c}^{c}\frac{(x-\xi)}{R(x,\xi,r)}\,d\xi \tag{2.9}$$

*<u>Note</u>

It is to be noted that the definition of the Stokes stream function in (2.7) - (2.9) follows the more usual modern form of Ref.7, Section 4.11, and differs from that of Temple by a factor of 2π.

In order to carry out the integration in (2.9) we make the substitutions

$$\left.\begin{aligned} x-\xi &= r\,\mathrm{Sinh}\,\eta \\ d\xi &= -r\mathrm{Cosh}\,\eta\,d\eta \end{aligned}\right\} \tag{2.10}$$

and

$$\begin{aligned} R(x,\xi,r) &= |\{(x-\xi)^2 + r^2\}^{\frac{1}{2}}| = r|\{1 + \mathrm{Sinh}^2 y\}^{\frac{1}{2}}| \\ &= r\,\mathrm{Cosh}\,\eta \end{aligned} \tag{2.11}$$

As a result (2.9) becomes

$$\begin{aligned} \psi(x,r) &= \tfrac{1}{2}Ur^2 - \tfrac{1}{2}U\lambda r\int_{\eta(-c)}^{\eta(c)}(x-r\,\mathrm{Sinh}\,\eta)\mathrm{Sinh}\,\eta\,d\eta \\ &= \tfrac{1}{2}Ur^2 - \tfrac{1}{2}U\lambda r[x\mathrm{Cosh}\,\eta - \tfrac{1}{2}r(\mathrm{Sinh}\,\eta\,\mathrm{Cosh}\,\eta - \eta)]_{\eta(-c)}^{\eta(c)} \\ &= \tfrac{1}{2}Ur^2 - \tfrac{1}{4}U\lambda[(x+\xi)R(x,\xi,R) + r^2\mathrm{Sinh}\{\tfrac{x-\xi}{r}\}]_{-c}^{c} \end{aligned}$$

or

$$\psi(x,r) = \tfrac{1}{2}Ur^2 \{1 - \frac{\lambda}{2r^2} [(x+c)R(x,c,r) - (x-c)R(x,-c,r)$$

$$+ r^2\text{Sinh}^{-1} \{\frac{x-c}{r}\} - r^2\text{Sinh}^{-1}\{\frac{x+c}{r}\}]\} \qquad (2.12)$$

In order to compare(2.12) with the classical result we will express it in terms of ellipsoidal co-ordinates, μ, ζ, θ Fig.3. This system of co-ordinates is defined by

$$\left.\begin{array}{l} x = c\mu\zeta \\ r = c(1-\mu^2)^{\frac{1}{2}} (\zeta^2 - 1)^{\frac{1}{2}} \end{array}\right\}, \quad \begin{array}{r} -1 \leqslant \mu \leqslant 1 \\ 1 \leqslant \zeta \leqslant \infty \end{array} \qquad (2.13)$$

where the system of surfaces ζ = constant define a family of confocal ellipsoids of revolution, and the system of surfaces μ = constant define an orthogonal family of confocal hyperboloids of revolution, whilst θ is the cylindrical co-ordinate of Fig.1. Further description of the ellipsoidal co-ordinate system is given in Ref.2, pp.277-281, or Ref.8. pp. 370 - 3.

Using (2.13) it is not difficult to show that

$$\left.\begin{array}{l} R(x,c,r) = |\{(x - c)^2 + r^2\}^{\frac{1}{2}}| = c(\zeta - \mu) \\ \text{and} \\ R(x, -c,r) = c(\zeta+\mu) \end{array}\right\} \qquad (2.14)$$

and these are illustrated on Fig.3.

Also

$$\text{Sinh}^{-1} \{\frac{x-\xi}{r}\} = \ln \{\frac{x-\xi}{r} + [(\frac{x-\xi}{r})^2 + 1]^{\frac{1}{2}} \}$$

$$= \ln \{\frac{x-\xi}{r} + \frac{R(x,\xi,r)}{r}\}$$

from which

$$\text{Sinh}^{-1}\{\frac{x-c}{r}\} - \text{Sinh}^{-1}\{\frac{x+c}{r}\} = \ln\{\frac{x-c + R(x,c,r)}{x+c + R(x-c,r)}\} = \ln(\frac{\zeta-1}{\zeta+1}) \qquad (2.15)$$

Substituting from (2.13) - (2.15) into (2.12) we obtain

$$\psi(x,r) = \tfrac{1}{2}Ur^2\left\{1 - \frac{\lambda}{2}\left[\frac{c^2}{r^2}(\mu\zeta + 1)(\zeta - \mu) - \frac{c^2}{r^2}(\mu\zeta - 1)(\zeta + \mu) + \ln\left(\frac{\zeta - 1}{\zeta + 1}\right)\right]\right\}$$

$$= \tfrac{1}{2}Ur^2\left\{1 - \frac{\lambda}{2}\left[\frac{2\zeta}{(\zeta^2 - 1)} + \ln\left(\frac{\zeta - 1}{\zeta + 1}\right)\right]\right\}$$

or

$$\psi(\mu,\zeta) = \tfrac{1}{2}Uc^2(1 - \mu^2)(\zeta^2 - 1)\left\{1 - \lambda\left[\frac{\zeta}{(\zeta^2 - 1)} + \tfrac{1}{2}\ln\left(\frac{\zeta - 1}{\zeta + 1}\right)\right]\right\} \qquad (2.16)$$

which, after allowing for the factor 2π, agrees with Temples result.

The stagnation streamtube is defined by the values of μ,ζ,θ for which $\psi(\mu,\zeta,\theta) = 0$. From (2.16) $\psi = 0$ when

$$\text{(a)} \quad \mu = \pm 1 \quad \text{(see Fig. 3)} \qquad (2.17)$$

or when

$$\text{(b)} \quad \lambda = \left\{\frac{\zeta}{\zeta^2 - 1} + \tfrac{1}{2}\ln\left(\frac{\zeta - 1}{\zeta + 1}\right)\right\}^{-1}. \qquad (2.18)$$

The solution to (2.18) is unique and may be written

$$\zeta = \zeta_o \geqslant 1 ,$$

from which we deduce that the two stagnation points are located at $(\mu = 1,\zeta_o)$ and $(\mu = -1,\zeta_o)$, i.e. the stagnation points are at

$$(r = 0,\ x = c\zeta_o) , (r = 0,\ x = -c\zeta_o) .$$

Thus if the length of the semi-major axes are given by $x = \pm a$, then

$$a = c\zeta_o . \qquad (2.19)$$

The intersection of the ellipse $\zeta = \zeta_o$ with the r axis corresponds to $\mu = 0$, $\zeta = \zeta_o$, or from (2.13) $x = 0$, $r = c(\zeta_o^2 - 1)^{\frac{1}{2}}$. Thus if the length of the semi-minor axes of the ellipse are given by $|r| = b$, then

$$b = c|(\zeta_o^2 - 1)^{\frac{1}{2}}| \qquad (2.20)$$

It follows that the stagnation streamtube consists of the axis $r = 0$, $|x| \geq a$, together with the ellipsoid defined by $\zeta = \zeta_o$, $0 \leq \theta \leq 2\pi$.

The source distribution which generates this stagnation streamtube is, from (2.3),

$$f(\xi) = -2\pi U\left\{\frac{\zeta_o}{\zeta_o^2 - 1} + \tfrac{1}{2}\ln\left(\frac{\zeta_o - 1}{\zeta_o + 1}\right)\right\}^{-1}\xi, \tag{2.21}$$

where ξ lies between the foci $\pm c$, and for a given ellipsoid whose axes are a and b,

$$\zeta_o = \frac{a}{c} = \frac{a}{|(a^2 - b^2)^{\frac{1}{2}}|} = \frac{1}{|(1 - b^2/a^2)^{\frac{1}{2}}|} \tag{2.22}$$

Now the cross sectional area of the ellipsoid is

$$S(x) = \pi r_b^2(x) \; ,$$

where $r_b(x)$ is the body radius at x and is given by

$$r_b = b(1 - \frac{x^2}{a^2})^{\frac{1}{2}} \; .$$

Thus $$S(x) = \pi b^2(1 - \frac{x^2}{a^2}) \tag{2.23}$$

from which

$$S'(x) = \frac{dS}{dx} = -\,2\pi\frac{b^2}{a^2}\,x \tag{2.24}$$

The source distribution predicted by 'slender-body' theory is

$$f(\xi) = US'(\xi) \tag{2.25}$$

or

$$f(\xi) = -\,2\pi U(\frac{b}{a})^2\xi$$

$$= -\,2\pi U(\frac{\zeta_o^2 - 1}{\zeta_o^2})\xi \;, \quad -a \leq \xi \leq a \,. \tag{2.26}$$

Comparing (2.26) with (2.21) we see that the exact distribution and that predicted by slender-body theory have the same linear form but differ in the gradient $f'(\xi)$. In the exact solution

$$f'(\xi) = \frac{df}{d\xi} = -\,2\pi U\lambda \;,$$

where λ is given by

$$\lambda = \left\{ \frac{\zeta_o}{\zeta_o^2 - 1} + \tfrac{1}{2}\ln\left(\frac{\zeta_o - 1}{\zeta_o + 1}\right) \right\} ,$$

whilst in the slender-body result

$$f'(\xi) = - 2\pi U\left(\frac{\zeta_o^2 - 1}{\zeta_o^2}\right) = - 2\pi U\left(\frac{b}{a}\right)^2 \quad .$$

When $^b/_a < 0.1$, these gradients differ little. See Ref. 6, pp. 187-8.

3. THE LONGITUDINAL MOTION OF A PROLATE ELLIPSOID OF REVOLUTION - DOUBLET DISTRIBUTION.

We wish now to perform the analysis using an axial distribution of doublets whose axes are aligned in opposition to the stream U. The stream-function for a unit doublet is

$$\psi_D(x,r) = -\frac{r^2}{4\pi R^3(x,\xi,r)}\ , \tag{3.1}$$

whilst the total stream function for an axial distribution, $g(\xi)$, of these doublets, set in a uniform stream U, will be

$$\psi(x,r) = \tfrac{1}{2}Ur^2 - \frac{r^2}{4\pi}\int\frac{g(\xi)}{R^3(x,\xi,r)}\,d\xi\ , \tag{3.2}$$

where $g(\xi)$ and its extremities are to be decided.

As in the determination of $f(\xi)$, we may be guided by the corresponding distribution predicted by slender-body theory. This is

$$g(\xi) = US(\xi) = \pi Ur_b^2(\xi)$$

$$= \pi Ub^2(1 - \frac{\xi^2}{a^2})$$

or

$$g(\xi) = \pi U(\frac{b}{a})^2(a^2 - \xi^2)\ , \qquad -a \leqslant \xi \leqslant a\ . \tag{3.3}$$

See Ref. 3 and Ref. 9, pp. 8 - 9.

Our choice here closely follows that used by Temple in solving for the transverse motion. See Ref. 6, p. 188. Thus we take

$$g(\xi) = \pi U\lambda(c^2 - \xi^2)\ , \qquad -c \leqslant \xi \leqslant c\ , \tag{3.4}$$

where λ is an undetermined constant.

Substituting from (3.4) into (3.1) we get

$$\psi(x,r) = \tfrac{1}{2}Ur^2 - \frac{Ur^2\lambda}{4}\int_{-c}^{c}\frac{(c^2 - \xi^2)}{R^3(x,\xi,r)}\,d\xi \tag{3.5}$$

Using the substitutions of (2.10) and (2.11) this becomes

$$\psi(x,r) = \tfrac{1}{2}Ur^2 + \frac{U}{4}\lambda\int_{\eta(-c)}^{\eta(c)} \{c^2 - (x - r\mathrm{Sinh}\eta)^2\}\mathrm{Sech}^2\eta d\eta$$

$$= \tfrac{1}{2}Ur^2 + \frac{U\lambda}{4}\int_{\eta(-c)}^{\eta(c)} \{c^2 - x^2 + 2xr\mathrm{Sinh}\eta - r^2\mathrm{Sinh}^2\eta\}\mathrm{Sech}^2\eta d\eta$$

$$= \tfrac{1}{2}Ur^2 + \frac{U\lambda}{4}\Big[(c^2 - x^2)\mathrm{Tanh}\eta - 2xr\mathrm{Sech}\eta - r^2(\eta - \mathrm{Tanh}\eta)\Big]_{\eta(-c)}^{\eta(c)}$$

$$= \tfrac{1}{2}Ur^2 + \frac{U\lambda}{4}\left[\frac{(c^2 - x^2 + r^2)(x - \xi)}{R(x,\xi,r)} - \frac{2xr^2}{R(x,\xi,r)} - r^2\mathrm{Sinh}^{-1}\left\{\frac{x-\xi}{r}\right\}\right]_{-c}^{c}$$

or

$$\psi(x,r) = \tfrac{1}{2}Ur^2\left\{1 + \frac{\lambda}{2}\left[\frac{(c^2 - x^2 + r^2)(x - \xi)}{r\,R(x,\xi,r)} - \frac{2x}{R(x,\xi,r)} - \mathrm{Sinh}^{-1}\left\{\frac{x-\xi}{r}\right\}\right]_{-c}^{c}\right\} \tag{3.6}$$

From (2.13) and (2.14) we may readily demonstrate that

$$\frac{(c^2 - x^2 + r^2)}{r^2}\left\{\frac{x - c}{R(x,c,r)} - \frac{x + c}{R(x,-c,r)}\right\} = \frac{-2\zeta(\zeta^2 - 2\zeta^2\mu^2 + \mu^2)}{(\zeta^2 - 1)(\zeta^2 - \mu^2)} \tag{3.7}$$

and

$$2x\left\{\frac{1}{R(x,c,r)} - \frac{1}{R(x,-c,r)}\right\} = \frac{4\mu^2\zeta}{(\zeta^2 - \mu^2)} \tag{3.8}$$

Substituting from (3.7), (3.8) and (2.15) into (3.6) then gives

$$\psi(\mu,\zeta) = \tfrac{1}{2}Uc^2(1 - \mu^2)(\zeta^2 - 1)\left\{1 - \lambda\left[\frac{\zeta}{(\zeta^2 - 1)} + \tfrac{1}{2}\ln\left(\frac{\zeta - 1}{\zeta + 1}\right)\right]\right\} \tag{3.9}$$

which is in exact agreement with (2.16). The deductions expressed in (2.17) - (2.20) also follow.

The doublet distribution which generates the stagnation streamtube (which is the same streamtube as that described in Section 2) is, from (3.4),

$$g(\xi) = \pi U\left\{\frac{\zeta_o}{\zeta_o - 1} + \tfrac{1}{2}\ln\left(\frac{\zeta_o - 1}{\zeta_o + 1}\right)\right\}^{-1}(c^2 - \xi^2)\ , \qquad -c \leqslant \xi \leqslant c \tag{3.10}$$

whilst that predicted by slender body theory is

$$g(\xi) = \pi U\left(\frac{\zeta_o^2 - 1}{\zeta_o}\right)(a^2 - \xi^2) \qquad -a \leq \xi \leq a \qquad (3.11)$$

We note that the source distribution given by (2.21) may be obtained by differentiating (3.10) with respect to ξ, i.e.

$$f(\xi) = \frac{dg(\xi)}{d\xi} \qquad (3.12)$$

This result also holds for the slender-body distributions.

4. THE TRANSVERSE MOTION OF A PROLATE ELLIPSOID OF REVOLUTION

The transverse velocity component is $W = U_\infty \mathrm{Sin}\alpha$ and the body is to be generated by means of an axial distribution, $\sigma(\xi)$, of doublets whose individual axes are set such as to oppose W, i.e. they point in the sense z negative. Since a stream function for this situation has not been determined we work in terms of velocity potential.

As shown by Karamcheti in Ref. 7, Chapter 20, the velocity potential at a general field point (x,r,θ) due to the axial distribution, $\sigma(\xi)$, is

$$\phi_D(x,r,\theta) = \frac{r\mathrm{Sin}\theta}{4\pi}\int \frac{\sigma(\xi)}{R^3(x,\xi,r)}\, d\xi \ , \qquad (4.1)$$

the integral being taken between the forward and rearward limits of $\sigma(\xi)$. The velocity potential due to the transverse flow will be

$$\phi_S = Wz = Wr\mathrm{Sin}\theta \qquad (4.2)$$

The total velocity potential will be

$$\phi(x,r,\theta) = Wr\mathrm{Sin}\theta + \frac{r\mathrm{Sin}\theta}{4\pi}\int \frac{\sigma(\xi)}{R^3(x,\xi,r)}\, d\xi \qquad (4.3)$$

Again we may be guided in our choice of $\sigma(\xi)$ by slender-body theory. The latter predicts that

$$\sigma(\xi) = 2WS(\xi) = 2\pi W\left(\frac{b}{a}\right)^2(a^2 - \xi^2) \ , \qquad -a \leqslant \xi \leqslant a \qquad (4.4)$$

whilst we, in a closely similar way to our choice in Section 3, take

$$\sigma(\xi) = 2\pi W\lambda(c^2 - \xi^2) \ , \qquad -c \leqslant \xi \leqslant c \qquad (4.5)$$

As a result ϕ becomes

$$\phi(x,r,\theta) = Wr\mathrm{Sin}\theta + \frac{\lambda Wr\mathrm{Sin}\theta}{2}\int_{-c}^{c} \frac{(c^2 - \xi^2)}{R^3(x,\xi,r)}\, d\xi \ , \qquad (4.6)$$

where the definite integral is exactly the same as that in (3.5).

It follows that

$$\phi(x,r,\theta) = Wr\ \mathrm{Sin}\theta \left\{1 + \frac{\lambda}{2}\int_{-c}^{c} \frac{(c^2 - \xi^2)}{R^3(x,\xi,r)}\, d\xi\right\}$$

which from (3.6) and (3.9) yields

$$\phi(\mu,\zeta,\theta) = cW\sin\theta(1 - \mu^2)^{\frac{1}{2}}(\zeta^2 - 1)^{\frac{1}{2}}\left\{1 + \lambda\left[\frac{\zeta}{(\zeta^2 - 1)} + \tfrac{1}{2}\ln\left(\frac{\zeta - 1}{\zeta + 1}\right)\right]\right\} \tag{4.7}$$

Now the component of velocity normal to an ellipsoid ζ = constant is proportional to the potential gradient $\partial\phi/\partial\zeta$. In particular the velocity normal to the stream surface $\zeta = \zeta_o$, i.e. the solid surface of the ellipsoid of revolution, must be zero. It follows that by differentiating (4.7) with respect to ζ, and equating this expression to zero, we may solve for λ with $\zeta = \zeta_o$. We have, from (4.7),

$$\frac{\partial\phi/\partial\zeta}{Wc(1 - \mu^2)^{\frac{1}{2}}\sin\theta} = \frac{\zeta}{(\zeta^2 - 1)^{\frac{1}{2}}}\left\{1 + \lambda\left[\frac{\zeta}{(\zeta^2 - 1)} + \tfrac{1}{2}\ln\left(\frac{\zeta - 1}{\zeta + 1}\right)\right]\right\}$$

$$+ \lambda(\zeta^2 - 1)^{\frac{1}{2}}\left\{\frac{1}{(\zeta^2 - 1)} - \frac{2\zeta^2}{(\zeta^2 - 1)} + \tfrac{1}{2}\left[\frac{1}{\zeta - 1} - \frac{1}{\zeta + 1}\right]\right\}$$

which after further resolution reduces to

$$\frac{\partial\phi}{\partial\zeta} = \frac{Wc(1 - \mu^2)^{\frac{1}{2}}\sin\theta}{(\zeta^2 - 1)^{\frac{1}{2}}}\,\zeta\left\{1 + \lambda\left[\frac{\zeta^2 - 2}{\zeta(\zeta^2 - 1)} + \tfrac{1}{2}\ln\left(\frac{\zeta - 1}{\zeta + 1}\right)\right]\right\} \tag{4.8}$$

Imposing the surface boundary condition $\frac{\partial\phi}{\partial\zeta} = 0$, $\zeta = \zeta_0$, we obtain

$$\lambda = -\left\{\frac{\zeta_0^2 - 2}{\zeta_0(\zeta_0^2 - 1)} + \tfrac{1}{2}\ln\left(\frac{\zeta_0 - 1}{\zeta_0 + 1}\right)\right\}^{-1} \tag{4.9}$$

where ζ_0 is given by 2.22.

Again when $b/a < 0.1$, the value of λ from (4.9) differs little from $(b/a)^2$. See Ref. 6, p.189.

5. Conclusions

The exact equivalent source and doublet distributions associated with the flow around prolate ellipsoids of revolution have been derived. This approach requires no knowledge of the theory of spherical harmonics and, as shown by Temple in Ref. 6, Chapter 15, provides a valuable means of comparison with slender-body theory.

The present authors interest in these results has sprung from his own study of approximate methods for the aerodynamic characteristics of thin bodies presented in Ref. 3.

References

1. Lamb, H. 'Hydrodynamics' Sixth Edition. Cambridge University Press (1932)

2. Durand, W.F. (Ed.) 'Aerodynamic Theory' Vol. 1, Fluid Mechanics, Pt. 2 Springer (1934)

3. Christopher, P.A.T. 'Aerodynamic characteristics of thin bodies in incompressible flow' College of Aeronautics, C.I.T., Cranfield Report No. 8803 (To be published)

4. Hess, J.L. 'The unsuitability of ellipsoids as test cases for line-source methods' J. Aircraft, Vol. 22, No. 4, April 1985, pp 346-7

5. D'Sa, J.M. and Dalton, C. Reply to a letter by Kuhlmann, J.M. on a paper by the authors. J. Aircraft, Vol. 24, No. 7, p 479

6. Temple, G. 'An Introduction to Fluid Dynamics' Oxford University Press (1958)

7. Karamcheti, K. 'Principles of Ideal - Fluid Aerodynamics' John Wiley (1966)

8. Thwaites, B. (Ed.) 'Incompressible Aerodynamics' Oxford University Press (1960)

9. Landweber, L. 'The axially symmetric potential flow about elongated bodies of revolution' David Taylor Model Basin, Report 761 August, (1951)

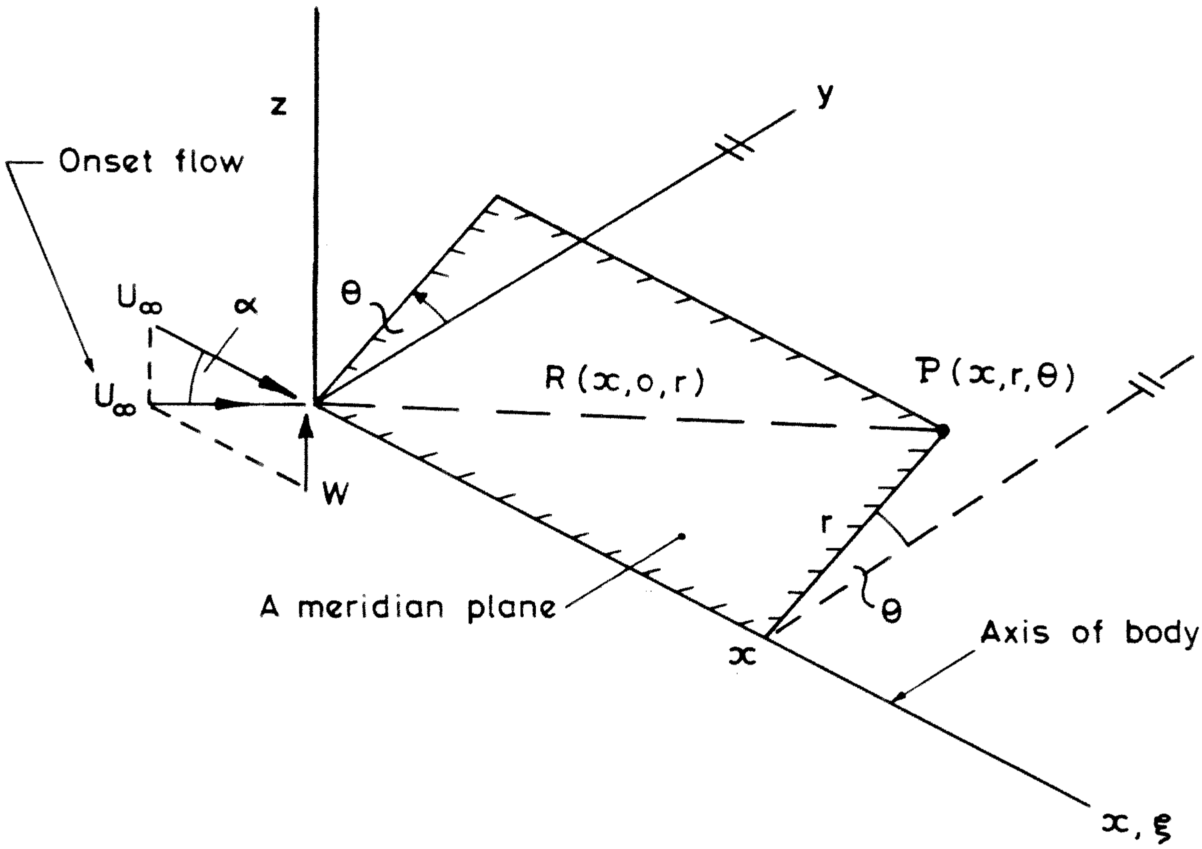

Fig.1. Geometry of Flow.

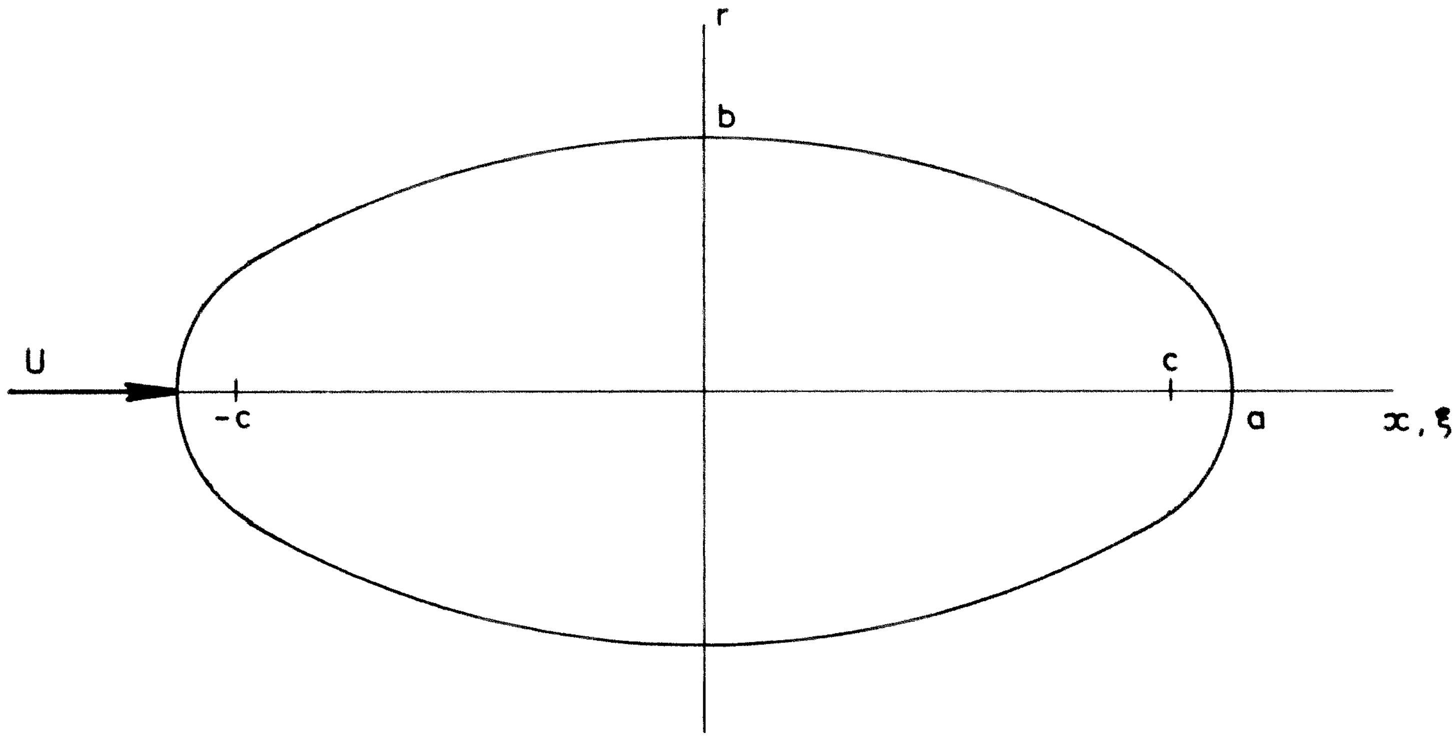

Fig. 2. Geometry of the axi-symmetric flow problem.

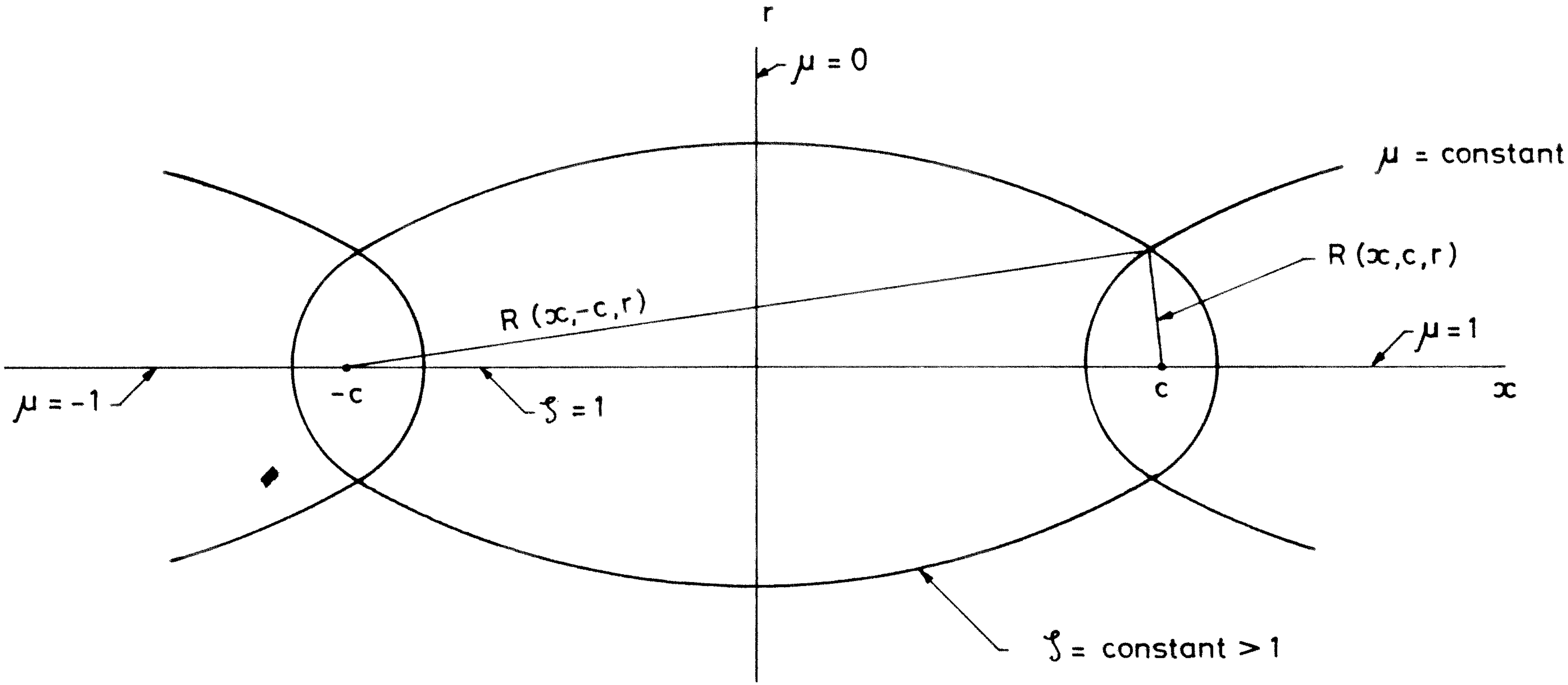

Fig. 3. The (Semi) Ellipsoidal Co-ordinate System.