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Optimum Stiffness Inflated Mattress Beams

- by -

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Inflated Structures Report C1206/2

Summary

This report considers primarily the relatively short cantilever type of inflated matresss beam, currently being employed as sidebody structure on some hovercraft.

The rib pitch for optimum stiffness/weight of such beams is determined, and it is shown that thin ribs, closely pitched, with relatively thick skins provide the most efficient structures.

Notation

V	shear force/cell
a	rib pitch
h	beam depth
L	beam length
s, $\bar{s}$	shear stiffness/cell ($\bar{s}$ nondimensional)
m, $\bar{m}$	bending stiffness/cell ($\bar{m}$ nondimensional)
w, $\bar{w}$	weight/unit length/cell ($\bar{w}$ nondimensional)
K	bending stiffness const.
f	$\bar{s}/\bar{w}$ and $\bar{m}/\bar{w}$ (non dimensional stiffness/weight)
t_r	rib material weight/unit surface area
t_s	skin material weight/unit surface area
k	t_r/t_s
I	2nd moment of area of skin/cell
y	$\sin^{-1} a/h$
A	cross section area/cell
p	internal pressure
γ	shear angle
δ_s	shear deflection

1.0 Introduction

A common example of inflated structure is the 'mattress' or diaphragm form of beam, shown in Figs. 1 and 2. In their cantilever form, (Fig. 1), these beams have been used as sidebodies on hovercraft, where they have typically been made from coated single ply fabric materials.

To assist the designer when deciding on the cross section geometry of such beams, in this report the rib pitch/beam depth ratio for optimum stiffness/weight ratio is determined.

The work assumes:-

(a) the contribution by the ribs in resisting shear by in-plane shearing of the rib material is small.

(b) the 'mattress' consists of a large number of cells.

(c) the contribution by the ribs to resistance in bending is small.

With regard to point (a), if the rib material is of lightly coated single ply fabric, the ribs are very flexible in shear. Also, it is common practice to not join the ribs to the tip end cap of cantilever type beams, in which case very little shear is transmitted into the rib.

2.0 Analysis

2.1 Shear Stiffness

Since the ribs have negligible shear atiffness in their plane, shear must be carried by pressure only.

Using Fig.s 2 and 3, it may be shown that shear force and shear deflection are given by

$$V = pA\gamma \quad \text{and} \quad \delta_s = L\gamma$$

The shear stiffness/cell for unit length of beam is therefore

$$\delta = \frac{V}{\delta_s} = pA$$

From Fig. 2, cross section area/cell is

$$A = \frac{h}{2}\left\{h \sin^{-1}\left(\frac{a}{h}\right) + a \cos\left(\sin^{-1}\frac{a}{h}\right)\right\}$$

$$= \frac{h}{2}\left\{hy + a \cos y\right\}$$

where $y = \sin^{-1}\left(\frac{a}{h}\right)$

$$\therefore \quad s = \frac{ph}{2}\left\{hy + a\cos y\right\} -$$

$$= \frac{ph^2}{2}\left\{y + \sin y \cos y\right\} \qquad (1)$$

2.2 Bending Stiffness

Since the ribs are considered to make a negligible contribution to bending resistance, from Fig. 2

$$I/\text{cell} = \frac{h^3}{8}t_s\ [\sin 2y + 2y].$$

For a given beam, initial bending stiffness is M = KEI

where K is a constant determined from load position on beam, length of beam, and beam support conditions (e.g. for the tip loaded cantilever of Fig. 1, $K = \frac{3}{L^3}$).

E is here the slope of the stress strain curve for the beam wall material in the immediate vicinity of the stress level produced by inflation loads

$$\therefore \quad m = KEI = KE\frac{h^3 t_s}{8}[\sin 2y + 2y]$$

$$= KE\frac{h^3 t_s}{4}\ [y + \sin y \cos y] \qquad (2)$$

Note that equations (1) and (2) are dependent on the same variable function.

2.3 Surface Weight

Using Fig. 2, the surface weight per unit length per cell is given by

$$w = 2hyt_s + ht_r\cos y$$

$$= t_s h\ \{2y + k\cos y\} \qquad \text{where } k = \frac{t_r}{t_s}$$

(Note that in a complete 'n' celled beam there are (n-1) ribs plus two cell walls; the above expression is therefore only suitable for a beam with a large number of cells).

2.4 Stiffness/Surface Weight

Using the results of 2.1, 2.2, and 2.3 and non dimensionalising,

$$\bar{s} = \frac{s}{ph^2} = \frac{1}{2}\left\{y + \sin y \cos y\right\}$$

$$\bar{m} = \frac{2m}{KEh^3 t_s} = \frac{1}{2}\left\{y + \sin y \cos y\right\}$$

$$\bar{w} = \frac{w}{ht_s} = (2y + k \cos y).$$

$\therefore$ Stiffness/surface weight (nondimensional) is

$$f = \frac{\bar{s}}{\bar{w}} = \frac{\bar{m}}{\bar{w}} = \frac{1}{2}\left\{\frac{y + \sin y \cos y}{2y + k \cos y}\right\} \qquad (3)$$

Using equation (3) the function $f \sim \frac{a}{h}$ is plotted in Fig. 4.

2.5 Rib pitch for optimum stiffness/weight

For optimum rib pitch, equation (3) must be maximised. Differentiating (3) gives

$$k \cos y_o (1+\cos^2 y_o + y_o \tan y)$$
$$+ 2(y_o \cos 2y_o - \sin y_o \cos y_o) = 0 \qquad (4)$$

where $y_o = \sin^{-1}(\frac{a}{h})_o$; $(\frac{a}{h})_o$ = optimum rib spacing ratio. Fig. 5 shows $(\frac{a}{h})_o$ plotted as a function of the rib thickness parameter K, together with the corresponding value of optimum stiffness/weight ratio, f_o.

Note that the most efficient designs are obtained when rib material thickness is small. However, for these designs rib pitch is required to be small so that joint weight is likely to become an important penalty.

Conclusions

The rib pitch for optimum stiffness/weight has been determined for a range of rib/skin thickness weight ratios (Fig. 5).

The results show clearly that for a given weight of beam, the stiffest beams (in both bending and shear) are obtained when the rib material is thin (low weight/unit area) compared with that of the skin, with the ribs closely pitched.

The results obtained for K = 1.0 (ribs and skin of similar material) are of interest since a large number of practical designs fall into this category. For this configuration, the ratio $\frac{a}{h}$ is seen to be 0.81 for maximum stiffness/weight, and this stiffness/weight is 57% of the maximum theoretically obtainable.

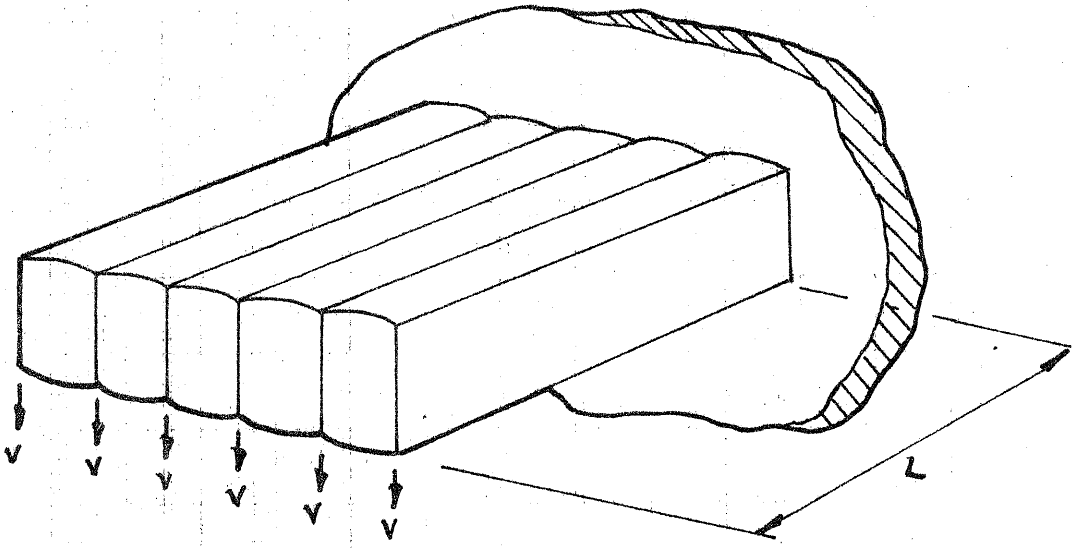

FIG 1
TYPICAL MULTICELL CANTILEVER MATTRESS OR DIAPHRAGM INFLATED BEAM

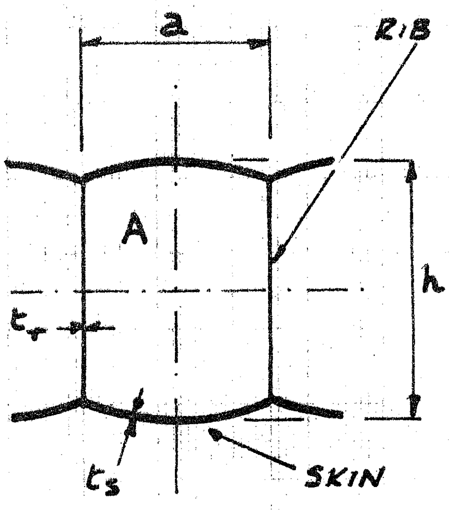

FIG 2. DETAILS OF CELL CROSS SECTION

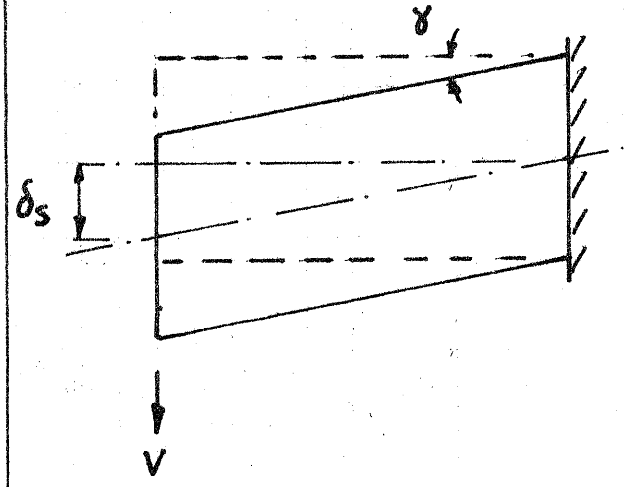

FIG 3. SHEAR DEFLECTION NOTATION

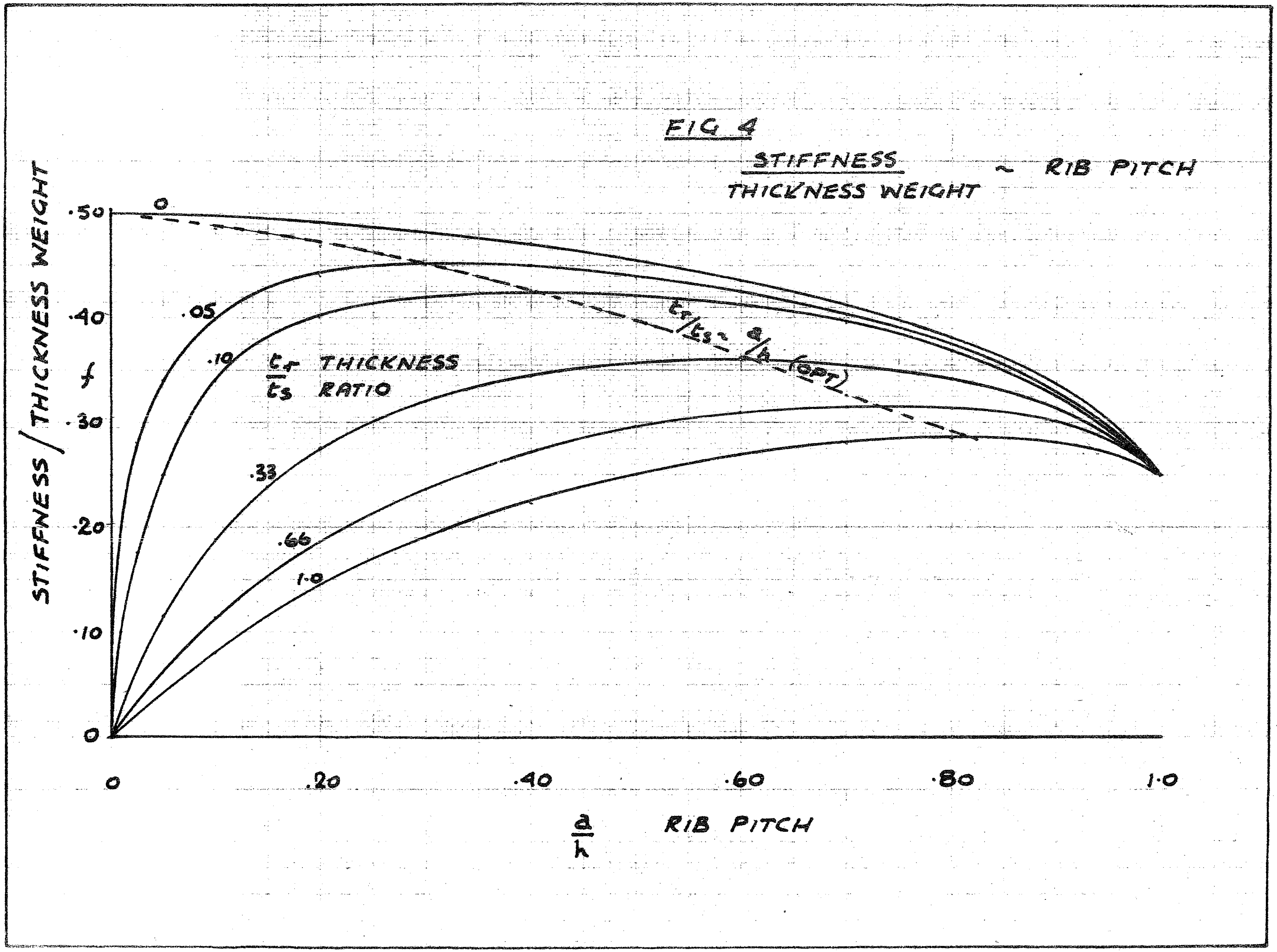

FIG 4
STIFFNESS / THICKNESS WEIGHT ~ RIB PITCH
STIFFNESS / THICKNESS WEIGHT
f
·50
·40
·30
·20
·10
0
0
.05
.10
.33
.66
1.0
t_r/t_s THICKNESS RATIO
t_r/t_s ~ a/h (OPT)
0
.20
.40
.60
.80
1.0
$\frac{a}{h}$ RIB PITCH

FIG 5

OPTIMUM RIB PITCH &
RIB/SKIN THICKNESS WEIGHT
FOR OPTIMUM STIFFNESS/WEIGHT
DESIGNS