

THE COLLEGE OF ACROHAUTICS

CRAFFIELD

MATERIALS MANDLING RESEARCH UNIT

THE APPLICATION OF INFORMATION THEORY TO PARCEL SORTING.

by

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1. INTRODUCTION

The work on parcel sorting was originally suggested to the Materials Handling Research Unit by British Rail which is one of the sponsoring organisations contributing to the Research Fund. This work has also formed the subject of a research thesis, which has been used as the basis of this Report. The student, Mr. B.R. Degerlund is therefore the main author, in co-operation with the staff of the Research Unit.

Quite a high proportion of commercial goods travel from supplier to customer as a 'parcal'. That is, a single package securely packed, moving from origin to a destination by various means of transport. Since most parcels originating from a single point are destined for different addresses which may be in any part of the country (we are only considering inland parcels) a sorting procedure has to take place. Theoretically, this could occur at any point after despatch, but is usually done at a point central to collection from a number of suppliers. The advent of mail order business which is still expanding rapidly in this country, has brought a vastly increased flow of parcels into the various organisations undertaking to deliver them.

It was appreciated from the beginning of the work that this problem was not peculiar to B.R. or other parcel handling organisations such as the G.P.O., but the principles involved should be capable of application over a much wider field. There are many cituations where a sorting activity takes place in order to reduce a mass of randomly mixed items into discrete categories, and it is likely that many of these could benefit by being examined by the methods suggested in this report.

One of the early difficulties was to decide what constituted a parcel, and certain rules were proposed to arrive at a definition. It was generally taken to be a package of any shape or form which travelled between two points as a separate entity. It was necessary to place some restrictions on size and weight, since articles as large and unwieldly as a double-bed size mattress regularly travel on B.R. as 'parcels traffic', but do not lend themselves very well to rapid handling. Thus a parcel was generally taken to mean a box or package of about 2 ft. cube which could readily be handled by one man. This division is loosely observed by L.R., who tend to treat separately those items which are obviously unwieldly or excessively heavy. There are three rough divisions into size and "handleability":-

- Postal parcels accepted by the G.P.O.
 under stringent size and weight limits.
- Railway parcels traffic, which takes almost any size, both parallel with the G.P.O. range and upwards, and similar services run by national or private road carriers. The upper size/weight limit tends to be set by the pricing structure rather than by imposed physical limitations.
- (iii) Larger and generally more irregular packages which usually travel by rail 'sundries' service or could similarly travel by road haulage. This category also includes multiple packages to one address.

The report is confined to the first two categories with regard to the practical manination of the problem.

2. SUILIARY

The object of the investigation was to examine the existing sorting procedures for railway parcels in order to evaluate existing methods both manual and machine-assisted for efficiency of sorting. To postulate a standard of sorting efficiency it was necessary to lock more closely at the theoretical basis of the sorting process, and to develop a theory of sorting. This has been done by the application of Information Theory, using the concept of Entropy as a measure of the state of disorder of a given mix of parcels.

The relationship between the entropy index and the number of bays or divisions into which the parcels are required to be sorted will enable the amount of sorting per parcel to be found. This in turn will given an indication of the most efficient method to be used.

The theoretical sorting method can be applied to both manual and mechanical sorting, and an indication can be given as to which method is likely to be more efficient in a given situation, and thus a cost estimate can be carried out.

Some comparisons have been made between the cost of sorting parcels both manually and by different mechanical methods, making certain cost assumptions, and relating this to the volume of parcels to be sorted.

3. THEORETICAL SORTING

The speed with which parcels can be sorted is dependent on the number of categories into which the items must be sorted and the number of sorts, necessary to separate them to destinations. With letter post, there is little actual transporting of the letter at the sorting point, most of the sorting time is dependent on how quickly the sorter can recognise the destination. With parcel-sorting, the opposite is true, the greater the length of time is spent in carrying the parcel from the sorting point to the sorting destination.

Sorting speed, though, is not the only factor for optimal sorting. The increased sorting speed, due to the smaller number of sorts, is off-set by the number of times a parcel must be sorted and the increased handling time between sorts. The total sorting efficiency, therefore, is dependent on the speed of sorting and the average number of sorts.

If there are 100 destinations to be sorted to, and an equal number of parcels for each, then two sorts each into ten bays can be used. At the first sort, the parcels are placed into ten bays, each having ten destinations allotted The parcels in each bay are then sorted into a to it. further ten bays, one bay per destination. As the volume for each destination has been assumed to be equal, then sorting to 100 destinations has only required twice the number of sorts that would be needed for ten destinations. Likewise. sorting for 1,000 destinations will only require three sorts of ten bays each. As the effective work done is the same for each sort, then it can be said that sorting for 100 destinations is twice the effective work of sorting for ten destinations. This means that the effective work done in sorting can be related to the logarithm of the number of bays. This logarithm can be called the Index of Effectiveness and allows for the effect of the varying number of handlings occurring with different numbers of bays.

Thus if the sorting speeds are multiplied by this Index, the sorting efficiency can be determined. For parcels sorted into less than ten bays, the effective sorting speed will be reduced, as the logarithms used are to the base of 10. This allows for the increased number of handlings. If the number of bays is greater than ten, then the effective sorting speed will be increased, due to the lesser number of handlings required. If these effective sorting speeds are now compared, the most efficient number of bays can be determined.

The following table has been prepared using standard time data on parcel sorting provided by BR. The time elements referred to in the table and the text following are the analysis of the task of sorting parcels as under:

Element No.	Operation
1	kemove package from 'arrival' vehicle.
2	Identify destination code number.
3	Hove package to appropriate trolley.
4	Stow in trolley
5	Operator returns to pick-up point.

The above is dependent on parcels being sorted into Dritish Railways Universal Trolleys (DRUTEC) a wheeled cage-type container approx. 5' x $3\frac{1}{2}$ ' in area.

No of sorts,I	Area required (5'x3½')	Time per parcel sec.s	Parcels/ hr	Total time Parcel allow- ing 3 secs for elements 1,2 & 4		ive sorting speed : log F
40	2,400ft ²	13.7	263	16.7	421	334
30	1,600	12.0	300	15.0	443	352
20	1,200	10.25	350	13.25	452	338
15	603	9.1	395	12.1	463	350
12	600	9.1	395	12.1	430	322
10	600	0.3	450	11.0	450	327
8	400	6.85	525	9.85	475	330
6	350	6.25	575	9.25	440	300
4	250	4.0.	904	7.0	540	310
					E1.183 only	(all elements)

If we assume that the parcel permits free movement and is between 10 and 75 lbs in weight, then, a first analysis of the basic time per parcel for elements 3 and 5 shows that a sort of four is the most effective, allowing even, for the increased number of sorts. However, the data provided was only for a comparison of manual and automated methods, and did not take into account the time taken for the other elements - unloading and loading the parcels and reading the destination address or code number. When making a normal comparison of methods, these times would not be required, as they are the same for each method. Although the time for elements 1, 2 and 4 is short, it is reasonably constant, and when added to the time for carrying the parcel and returning it, it reduces the overall effective sorting speed, particularly for the sorts with a smaller number of bays.

For the calculations of parcels per hour sorted, the assumption was made that the sorter was working, nonstop, for a full hour, But the sorter does not spend all of his time sorting, he has to move railway cage type trolleys (PRUTES) etc. around, removing full ones and collecting empty ones; he has to assist in the loading of trains specially designed to accept BRUTES, etc. He may also have to wait for 'arrival' vehicles and must also be given a 'personal' allowance. This will probably reduce his actual sorting time to about $\frac{2}{3}$ hour for each hour worked.

From the corrected effective sorting speeds, it will be seen that there is little difference in sorting speeds between 0 and 40 bays, the sorts into 15 and 30 bays, giving the highest speeds; however the sort into 30 bays requires twice the area of that of 15, and four times that of 8 sorts. It has been assumed above, that the time taken to read and identify a parcel's address is constant, for all numbers of bays. But, although no figures for parcels are available, letter sorting has shown that the time taken for recognising the destination and its appropriate bay, particularly when there are a large number of possible destinations, will be longer when there are a larger number of bays. This time will be difficult to measure, as each sorter's time will vary with experience and method of remembering. It will have the effect of reducing the sorting speeds for higher numbers of bays.

4. INFORMATION TLEORY

All control systems function by transmitting signals which contain information about the controller's requirements. This information is then interpreted and any appropriate changes take place. This information can then be put into a memory store, as in the human brain or an electronic computer. Thus, information can be transmitted, translated and stored. It is an essential content of any control system - Information Theory answers quartatively how much information a system contains.

Information Theory was originally developed in the field of Communications Engineering - upon which the media of radio, telegraph etc., are technologically based - by Shannon in his "Mathematical Theory of Communication" in 1948. This theory was used to deal with the question of noise in a radio channel and how to organise a message at the transmission point in order to minimise the effect of this noise. It took into account prior information, of a statistical nature, on the form of the message - thus becoming the application of probability theory to communications problems.

Because of the depth and generality of Information Theory, it can be applied to all forms of communication. As well as communication engineering, it also contributes to the basic concepts of cryptography and language translation, and its use in information retrieval, perception and learning, and other behavioural sciences is being explored.

The fundamental problem of communication is reproducing at one point a message selected at another. The messages may have meaning; i.e. they refer to, or are correlated, according to some system with physical or conceptual meanings. However, this is irrelevant to the engineering problem. The significant aspect is that the actual message is one selected from a set of possible messages. The system must operate for all these possible selections not just the one which will actually be chosen, since this is unknown beforehand.

Shannon, then, is concerned with the average amount of 'uncertainty' within the set or ensemble of messages. He called this uncertainty "Entropy", because the formula he derived for it -"will be recognised as that of entropy as defined in certain formulations of statistical mechanics". It is also the term used in physics to measure the excitation or disorder of molecules in a gas. Shannon's work does not only hold for radio messages, but for any application where the uncertainty of picking one member from a collection is required.

A formula can be derived relating the size of an ensemble to the uncertainty (or entropy) associated with it. If the ensemble has only one member, then there is no uncertainty; we know the solution before we start. If there are two members, then one question is needed to identify between them. In a binary system, it can be shown that each time the ensemble doubles, the number of questions to be asked, thus the entropy, is increased by one. Thus the entropy, E, can be connected to the size of the ensemble by

$$I = -log_n$$

when all of the solutions are equiprobable.

However, where the solutions are not equiprobable, i.e. where there is bias towards certain members, this value of E does not apply, as the entropy is reduced. Therefore some account must be taken of the weighting, or probability, of each member.

Suppose there is a set of possible selctions, whose probabilities of occurrence are $p_1, p_2 \dots p_n$. The probabilities are known, but nothing else concerning which selection will be made is known. If there is a measure of 'choice' involved in the selection of the event, say $E(p_1, p_2 \dots p_n)$, it is reasonable to require that it satisfies the following conditions:-

- 1/ E should be continuous in p;.
- 2/ If all p_i are equal, $p_i = \frac{1}{n}$, then the value of E increases as the size of the set increases.
- 3/ The unit of uncertainty is additive, i.e. if a choice is broken down into successive choices, the original E should be the weighted sum of the individual values of E. For example, the amount of information on two punched cards is the sum of the amount of information on each.

Shannon offered as his second theorem, that the only formula for E satisfying these three conditions is:-

$$E = -K \sum_{i=1}^{n} p_i \log p_i.$$

Where K is merely a positive constant, determining the units of uncertainty.

For a given set, I is a maximum when all the possible outcomes are equiprobable. The minimum value of E is 0, which occurs when there is only one possible outcome. Since the result is then known, uncertainty vanishes.

A collection of unsorted parcels will be in a state of disorder and thus will have a value of uncertainty connected with the possible destinations of any of the parcels. This value will be proportional to the amount of sorting that is required on the parcels, to get them to their proper destinations. Therefore, as this uncertainty exists, the Theory of Information can be applied to Parcel Sorting to determine the amount of sorting required on a collection of parcels.

The mathematical derivation of the entropy value for a mass of assorted parcels is given in Appendix I, and shows that the number of sorts is equal to $\frac{E}{\log F}$, where F is the number of bays into which

the parcels are to be sorted.

5. OPTIMAL SORTING

2

It was shown in the section on Information Theory, that the entropy of a collection of parcels shows the amount of sorting work to be done, and that the reduction in entropy after one sort into a set number of bays is equal to the log, of that number i.e. F. It was stated above that the sorting 'effectiveness'was also related to log F. The reduction of entropy or disorder, verifies this relationship. It was also shown that the entropy divided by the logarithm of the number of bays was equal to the number of sorting passes required.

If an equal number of parcels are allocated to the bays available, then the number of destinations per bay will vary widely. However, since destinations are not equi-probable, this may result in the number of sorts being reduced. Wide discrepancies in the number of destinations per bay may make it more difficult for sorters to remember, and it would be better to have the number of destinations per bay as powers of a common figure, (e.g. 1, 10, 100).

Thus, although the sorting effectiveness requires equal numbers of parcels per bay, in practice this figure can only be used as a Buide when allocating destinations to bays. This means that the reduction in entropy will be less than log F in the majority of cases.

A Practical Example

Data supplied by British Rail for a typical depot was used as a practical example for the above theoretical method.

The total number of parcels handled was 26,813. Of these 10,742 were collected for forwarding, 14,023 were received for transhipment, and 2,042 were received for delivery. There were 176 possible destinations and the number of parcels per destination varied between 2051 and 1.

The entropy per parcel was calculated for the whole collection and this figure came to 1.81. For an equal number of parcels per destination the entropy would have been log 176 (or 2.245).

The method used was to sort to 57 destinations, the traffic being re-sorted to the 176 at the various transhipment points.

Using the theoretical method of optimal sorting, the parcels could be sorted with an average number of 1.8 handlings per parcel into 10 bays. Alternatively, if 15 bays were used, the parcels could be sorted with an average of 1.54 handlings per parcel.

For the moment, we shall assume that the collection is fully sorted at the depot, thus no account will need to be paid to any geographical destinations. If 15 sorting bays are used, the average number of parcels per bay will be 1,000. Of the whole

collection the parcels received for delivery were 2,042, therefore one of the bays can be used solely for these parcels. For the remainder, 2 destinations each receive more than 1000 parcels, thus, these destinations can be allotted individual bays, removing the need for further sorting. The next two destinations, however, account for a further 2500 parcels which will mean that if one bay is allotted to them, they will have to be sorted into two at a later stage. It would be better if they were allotted a bay each. This will leave 172 destinations for 10 bays. Sorting could be arranged as 17 destinations per bay, but this will result in a large variation of parcels in each bay. If 9 bays are allotted, 15 destinations each, there will be a remainder of 37 destinations with a total of 352 parcels, assuming that the groupings are made in order of quantity. The number of parcels per bay for the remainder will vary between 7,000 and 400, to be sorted into 15 each at the secondary sort, although the quantities per bay at this sort will be This range of parcels is rather high, approximately equal. when, for optimum sorting, there should be an equal number of parcels per bay.

If a lower number of bays is used, say 12, the sorting speed will be slightly less efficient, but the quantities per bay, required for optimum sorting will be more closely approached. The parcels could be hundled as shown:-

CAB.1. Bay at	(B.R. Zone Codes)	No. of	parcels s	sorted at
1st Sort	No. of destinations	1	2	3
A	1	2851		
35	1	2217		
C	1	1400		
D	1	1112		
Е	12 from 862-321	6159	6159	
F	12 from 311-200	3089	3089	
G	12 from 200-134	1845	1845	
11	12 from 127-95	1298	1298	
J	12 from 95-80	1058	1058	
ĸ	12 from 20-66	858	258	
L	100 from 64-1	2887	2887	2887
И	Delivery	2042		

This results in a total of 44,849 parcel handlings, for 24,771 parcels, or an average of 1.83 handlings per parcel. The entropy for this collection of parcels was 1.81, therefore with equal quanities of parcels per bay, the theoretical number of parcel handlings would have been 1.74, thus the result of unequal quantities of parcels in the bays has increased the number of handling by .07 per parcel, or an entra 7% handling overall.

This assumed, however, that all of the parcels were sorted at the one depot and then sent to their final destinations in a fully sorted condition. This cannot be the case, in the majority of depots, due to lack of space and facilities, therefore close attention must be paid to the geographical region of the destination. The parcels would be partly sorted and then sent 'rough' sorted to another zone centre for final sorting in that area.

This will have particular effect on the destinations with low volumes of parcels, as the largest parcel volumes are sorted out at the first pass, and the allotment of destinations in the middle ranges can be re-arranged to suit regions quite easily. Although this will mean that destinations of approximately equal volumes will no longer be combined, the new grouping may improve the variance of the number of parcels per bay after the primary sort, to give a closer figure to the average.

For the 100 destinations of small volume traffic, it should be possible to collect 10 groups of 10 destinations each, within the same railway region. If necessary, the 10 destinations per bay need not be strictly acherred to. For example, if there are 12 destinations that can be forwarded together, and another group of 2, then this will be better for forwarding than sorting out 2 groups of 10. These groups can then be combined with the traffic for a main zone centre, and forwarded there for final sorting. In addition, any small destination that may have traffic forwarded to the sorting depot in question, as a zone centre, must be sorted out individually, in any event.

The following table (Tab.2) is an amended version of the above one giving the code numbers for B.R. zone centres and showing how destinations could be combined geographically by regions.

Table 3 shows a breakdown of destinations for bay L on the second sort. (This shows numbers of parcels only, the destination codes being omitted.) Tab. 2.

Sc.R

$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	ay at st soz	ct Re	egion	le cei		cue	centz	nd parce e	13 per	20116	
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E E.R. $171/862;156/753; 153/693;161/575;165/528$ 151/442;152/402;164/373;155/347;100/301; 149/235;109/152 F E.R. $103/135;139/125;102/82; 140/70; 118/62;$ 115/62; 104/62; H.E. $173/295;181/105;197/206;174/80;163/74;177/67;$ G S.R. $221/321;200/293;225/273;202/200;212/200;226/170$ 210/134;204/120;227/95;223/70;220/63 H U.R. $306/255;312/207;300/144;316/110;301/105;303/95;$ 323/92;330/70; 314/70;310/66; G c.R. $66/138; 81/127; 84/140$ J L.M.R. $23/399; 56/293;24/193;18/141;49/109;52/96;$ 17/95;13/90;19/85;34/85; 9/75; 35/70 K L.H.R. $6/465;72/311;57/220;27/164;55/134;54/123;$ 30/98;33/07;60/66;7/90;3/92 L The remainder; on the second sort they could be sorted to the following groups (number of parcels only):- Fable 3. I II III IV V VI VII VIII IX X <u>X.R. M.E. M.E. W.R M.R S.R LMR LMR Sc.R</u> 35 16 43 6 35 40 5 10 20 50 $46 30 20 40 15 20 35 41 50 4010 10 23 25 30 55 50 25 80 50 10 5 30 60 1 100 35 33 540 5 20 10 30 40 35 17 42 90$	С	E.	R.	16	6/140	3					
$\frac{151/442; 152/402; 164/373; 155/347; 100/301; 149/235; 109/152}{149/235; 109/152}$ F E.R. 103/135; 139/125; 100/82; 140/70; 113/62; 115/62; 104/62; IV.E. 173/295; 161/105; 197/206; 174/80; 163/74; 177/67; G S.R. 221/321; 200/253; 222/273; 202/200; 212/200; 226/170 210/134; 204/120; 227/95; 223/70; 220/63 II U.R. 306/255; 312/207; 300/144; 316/110; 301/105; 303/95; 323/92; 330/70; 314/70; 310/66; Gc.R. 66/138; 61/127; 84/140 J L.M.R. 23/399; 56/293; 24/193; 13/141; 49/109; 52/96; 17/95; 13/90; 15/25; 312/207; 27/164; 55/134; 54/123; 30/98; 33/07; 60/66; 7/90; 3/92 I L.M.R. 6/465; 72/311; 57/220; 27/164; 55/134; 54/123; 30/98; 33/07; 60/66; 7/90; 3/92 I The remainder; on the second sort they could be sorted to the following groups (number of parcels only):- Table 3. I II III IV V VI VII VIII IX X K.R. M.E. M.R. S.R. LMR. S.R. K.R. M.R. S.G. S.	D	E.	R.	15	4/111	2					
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$\begin{array}{c ccccccccccccccccccccccccccccccccccc$				15	1/442	;152/4	02;164/	373;155/	347;10	0/301;	
$\frac{119/62; 104/62;}{1.73/295; 181/105; 197/206; 174/80; 183/74; 177/67;}{G S.R. 221/321; 200/293; 225/273; 202/200; 212/200; 226/170 210/134; 204/120; 227/95; 223/70; 220/63} \\ H W.R. 306/255; 312/207; 300/144; 316/110; 301/105; 303/95; 323/92; 330/70; 314/70; 310/66; \\ Sc.R. 66/138; 81/127; 84/140 \\ J L.M.R. 23/399; 56/293; 24/193; 13/141; 49/109; 52/96; 17/95; 13/90; 15/65; 34/85; 9/75; 35/70 \\ K L.M.R. 6/405; 72/311; 57/220; 27/164; 55/134; 54/123; 30/96; 33/07; 60/66; 77/90; 3/92 \\ L The remainder; on the second sort they could be sorted to the following groups (number of parcels only):- \\ \hline Table 3. \\ \hline I II III IV V VI VII VIII IX X \\ \hline N.R. N.E. N.E. N.R. S.R LMR LMR LMR Sc.R \\ 35 16 49 6 35 40 5 10 20 50 \\ 46 30 20 40 15 20 35 41 50 40 \\ 10 10 23 25 30 55 50 25 60 5 \\ 5 10 5 30 60 1 10 35 33 5 \\ 40 5 20 10 30 40 35 17 42 90 \\ \hline$				14	9/235	;109/1.	52				
H.E. $173/295;181/105;197/206;174/80;183/74;177/67;$ G S.R. $221/321;200/293;225/273;202/200;212/200;226/170$ $210/134;204/120;227/95;223/70;220/63$ H W.R. $306/255;312/207;300/144;516/110;301/105;303/95;$ $323/92;330/70;$ $314/70;310/66;$ Sc.R. $66/138;$ $81/127;$ $84/140$ J L.M.R. $23/399;$ $56/293;24/193;13/141;49/109;52/96;$ $17/95;13/90;19/85;34/85;$ $9/75;$ $35/70$ K L.M.R. $23/399;$ $56/293;24/193;13/141;49/109;52/96;$ $17/95;13/90;19/85;34/85;$ $9/75;$ $35/70$ K L.M.R. $6/405;72/311;57/220;27/164;55/134;54/123;$ $30/98;33/67;60/66;7/90;3/92$ L L The remainder; on the second sort they could be sorted to the following groups (number of parcels only):- Table 3. I II III IV V VI VII VII X M.E. N.E. W.R. S.R. LMR LMR Sc.R. M.E. N.E. W.R. S.R. LMR LMR Sc.R. M.E. N.E. W.R. S.R. LMR LMR Sc.R. </td <td>F</td> <td>E,</td> <td>R.</td> <td>10</td> <td>3/135</td> <td>;139/1</td> <td>25;108/</td> <td>82 ; 140</td> <td>/70 ;</td> <td>118/62;</td> <td></td>	F	E,	R.	10	3/135	;139/1	25;108/	82 ; 140	/70 ;	118/62;	
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If each 'bay' used takes the form of 1 DRUTE, then there will still be a need to sort further for some of the regions, where destinations in that region may be far apart. However, if a 'bay' consists of a number of BRUTES, placed together, traffic to be forwarded to different zone centres in the same region can be placed in different DRUTES, allowing the parcels to be forwarded via the most convenient zone centre.

Hore than one BRUTE will be required at each bay, at the same time, but, on the other hand, there will be less need to replace full BRUTES with empty ones.

Each sorting depot will have different quantities of parcels for each other depot. Likewise, day to day quantities will vary at each depot, but over a period of time, a representative number of parcels per depot per day, can be obtained and this figure can be used for determining the allocation of sorting bays. Attention should be paid to these figures, which should be anended to keep up with altering trends.

It has been assumed throughout this section that the parcels received for sorting have been in a completely mixed collection. However, this is not strictly accurate, parcels received for transhipment at the larger depots will already have been passed through a primary sort elsewhere. This means that the uncertainty, or entropy of a part of the collection has already been reduced elsewhere, as the possible destinations for this group has been greatly reduced.

This will tend to cancel out the effect which the unequal numbers of parcels in each bay will have on the reduction in total entropy after one pass.

Manual Sorting

A great many parcels carried by British Rail are sorted by hand. A gaug of men, sorting a collection of parcels, can either work individually, each man collecting, sorting and loading on his own, or two or more can work as a team, one selecting the parcel and reading the destination, the other taking it to the receiving vehicle. There are three basic types of sort a parcel can receive - it can be the first sort of traffic collected at the depot to the area of its destination; it can be a parcel received from a depot for delivery; or it can be a 'transhipment'parcel, which has been forwarded from another area, to be sent on to a third depot.

When the parcels are collected, they pass through a primary sort, either directly to its destination, or more commonly, to a destination area. These parcels are then loaded into appropriate railway wagons and forwarded. At the destination depot, the wagons are unloaded and the parcels resorted, either to final cartage posts, or if transhipment traffic, to the final destination zone centre.

The parcels are often rehandled several times in the course of transit, by a large number of men. Many of these men simply cancel out the work done by earlier men, mainly because sorting and loading involves the use of barrows, which cause unnecessary loading and unloading. Parcels offloaded from barrows accumulate on platform while awaiting the arrival of a train. As it takes 40-60 minutes to fully load a van with parcels, trains tend to leave lightly-loaded, to clear platform space and keep to time-In addition, segregation of parcels for different tables. destinations within a single van is difficult and long delays sometimes occur at intermediate stations, for unloading and loading This means that some vans are detached from trains parcels. with less parcels than would normally justify this action.

To ease this problem, a form of unit load has been developed. At first, single pallets were tried, but these were not successful, as the dimensions of the pallets were not compatible with those of the vans, and, with the end-forces subjected to vans when shunted, unless the packages were strapped to the pallets, or the wagon was fully-loaded, damage could occur to the goods. Box pallets would have been able to deal with the problem, but they were heavy, immobile and, theoretically, wasted a lot of space in the rail van, compared with filling the van to capacity with parcels. However, it was found after investigation that on average, parcel vans were underfilled.

There were some routes where the vans were filled to capacity, but adequate segregation of the parcels was impossible, consequently unnecessary handling resulted.

This resulted in a wheeled, box container being developed. This was known as the British Railways Universal Trolley Equipment (B.R.U.T.E.). It has two fixed wheels towards the rear and two swivelling castor wheels at the front. These wheels are offset, to provide an inherent amount of "toe-in", which helps prevent 'snaking', when being towed, a well known characteristic of the old type platform barrow trains.

Another aid to good tracking was found by having the drawbar at the rear of the trolley, instead of the front, and having it spring-loaded, so that it extended when the trolleys were traversing a curve and pulled back afterwards. This results in very accurate tracking of a train of BRUTEs, each one following exactly in the path of the tractor. Another important purpose of the spring draw-bars, is during the initial tractive effort, enabling the traccor to take up the load one trolley at a time; this can result in the use of a smaller tractor.

To avoid the rehandling occurring during transit, the parcels should be loaded into LAUTES at the earliest opportunity, and laft there to the final destination. The trolleys can be loaded onto parcel trains with the maximum speed and minimum labour. 18 BRUTES can be loaded onto a CUV van in five minutes, compared with the full hour to load it with individual parcels.

The BRUTH's are best used for total movement of a group of parcels; however, they can be used instead of the old-type barrows for storage and transport around depots. Where intermediate stations do not have the facilities for handling BRUTEs, the parcels can be initially sorted into BRUTEs, loaded onto trains directly from BRUTEs into vans and unloaded manually at the other end. Alternatively, when quantity for one destination is consistently sufficient to fill a van, then BRUTEs need not be used, and the van can be detached at the receiving station. This is particularly useful for terminal depots. At present a number of routes are solely for BRUTE trains. A train, consisting of ten GUV vans, each capable of holding 18 BRUTEs runs a set route, depositing and collecting BRUTEs from stations on the route. A good example of this is the East Anglia BRUTE train, from Peterborough East to Liverpool Street. The train travels around East Anglia twice a day, each van having a set number of BRUTEs for one or more of the fifteen stations. The train can be loaded at the originating depot some time before departure, in a slack period, allowing the staff to deal with other traffic, as the need requires.

At the moment, unfortunately, there is still a lot of traffic not handled in BRUTES. However, with the increasing use of BRUTES, handling of parcels will be reduced, resulting in a speedier service, with less chance of damage and confidence in parcels arriving at the correct destinations. The following section deals with parcel-sorting from a Work Study view-point, comparing the work involved in manual sorting to different sorts, and also with mechanical sorting. As the time to 'rough-load' a parcels van varies considerably, all of the sorts will be into BRUTES.

As the theoretical sorting developed is not restricted to BRUTEs, or even to British Rail parcels, the parcels will be sorted into what will be termed 'bays'; these could conceivably be BRUTEs, rail vans or parcel bags.

Mechanical Sorting of Parcels

In addition to manual sorting, there are a number of sorting machines available. Although these machines differ in design and method of operation, they are all of the same basic concept. The parcels pass, on a conveyor of one form or another, a sorting point at which an operator records the destination of the parcel in the memory system of the machine. The parcel then moves along the conveyor, until this destination is reached, whence the parcel is removed from the conveyor. As the actual sorting process is still carried out by a human operator, this method is more accurately described as an automatic conveying of sorted parcels to their receiving bays.

Sorting machines provide a faster and more efficient method of sorting, with, when a sufficient number of parcels is sorted, a reduction in labour costs. Efficiency is increased because the operator remains at the sorting point, reading the destination code and keying this into the memory. After this all of the sorting is carried out automatically, with little chance of a wrong sort. As the operator does not have to move away from the coding point, the speed of sorting is increased, all of the carrying of the parcel through this sort being done on conveyors, the speed of which can be controlled. If the sorting operator has a loading assistant, who places the parcels with the code number showing in the correct position, and who can also call out the code number, then peak periods of traffic can be sorted rapidly and correctly.

The machine delivers sorted parcels to one of a number of sorting bays, where the parcels are accumulated, awaiting formation into unit loads, such as BRUTES. To sort completely by mechanical means, however, will incur very high cost, due to the large number of discharge points and long length of conveyors required. Of all the possible destinations B.R. parcels are liable to be forwarded to, 75% receive less than 100 parcels per day each. This means that a mechanical/manual combined system could be better used. The primary sort would use the sorting machine, followed by a manual secondary sort. This would result in a system similar to the theoretical manual system. The destinations with a large number of parcels, would have separate discharge points, whilst the remainder are sorted from the discharge bays into BRUTEs, for their respective destinations, grouped around the discharge bays.

Alternatively, traffic for other transhipment depots could be stowed in a number of BRUTES, without thorough sorting. This would allow traffic for destinations within the same transhipment depot, and with low quantities, to be combined for transporting conveniently.

For a large depot, handling a high throughput of traffic, another method would be to have a 'rough' preliminary sort before passing traffic to the sorting machine for a main sort for the destinations within this sort.

As the sorting is carried out at high speed, the sorting can be better planned to fully utilise the facilities and labour of the depot. Staff will be able to undertake other work on the same shift. Provided the demand is sufficiently great, received traffic for delivery could be sorted on the machine, into final cartgae posts, thus enabling more time to be spent on making out delivery notes and better loading of the delivery vehicles, than if the traffic had to be sorted manually as well. Labour would still be required to load the parcels from the discharge chutes into parcels vans or unit loads, even if no further sorting was needed. However, these stowing operators will have a choice of package shape and size from the accumulation of parcels, to assist them in the loading of the BRUTE, etc to its fullest capacity. The distance that the parcels are carried, also, will be reduced to a inimum, as will the choice of destinations, although there will still have to be a certain amount of package recognition by the operators.

The discharge chutes will act as temporary reservoirs, provided they are large enough, so that once a chute has been cleared, the operator can move on to another, reducing to a minimum his waiting time. This will also mean that other operators can be moved to a certain set of chutes, to prepare all of the traffic to meet particular trains, thus ensuring the prompt dispatch of traffic and a decrease in vaiting-time and 'under-loading' of through parcel trains.

With mechanical sorting, a higher numbers of sorts can be carried out in the primary sort than would be used in a manual sort. Other work has indicated that 40 discharge points is the maximum an operator can key to efficiently and although this number of discharges reduces the number of secondary sorts, it means a fair increase in the capital cost of the equipment. It will also require a larger area for the sorting equipment. Unlike manual sorting, there is little difference in sorting rates for up to 40 sorts, as all carrying of the parcel is done automatically, the Thus number of parcels passing the operator being constant. the number of discharge chutes chosen will be dependent on cost of equipment area available and amount of primary sorting required.

Types of Sorting Machine

There are several types of sorting machine, all based on the use of a conveyor of one form or another.

a) Tilting Tray

The Bagshawe "Tilt-Tray" sorter consists of a series of closely spaced trays, mounted between axially disposed pivots, on carriages which travel around a fixed track. Each tray is held in a horizontal position on a cam plate, until the destination point is reached, when an external roller is raised causing the tray to tilt.

Parcels are placed on the trays at an induction point, where the destination for each tray is selected on a key-board. This destination is fed into a logic memory unit, following the tray, round the memory as the tray moves along the track, until the selected destination is reached, at which point the discharge arm is raised, tilting the tray, so that the parcel slides off. Discharge can be to either side of the track and, if necessary, 'double tray' keying can be added, so that two trays carrying a long parcel can be tilted simultaneously.

The destinations should be spaced at a minimum of $1\frac{1}{2}$ tray lengths, the number of discharge stations being limited only by the length of the sorter, however, the cost of the control equipment may rise sharply with increasing length.

The control equipment used includes an analogue memory unit, constructed from static state switching elements in the form of a shift register.

Photo-electric cells or other devices can be placed across the mouth of each discharge chute to warn of any full chutes and to prevent any more parcels being placed in it. This will mean that an extra chute must be supplied at the end of the run to collect any such parcels, which will then have to be resorted, or alternatively a re-circulating format be used, although this will mean that should the chutes remain full, one or more of the trays will be used as moving storage. Therefore, close attention must be paid to the loading of the chutes, and either ample storage space or rapid removal of the parcels should be available.

The Tilt-Tray system can be used as a re-circulating loop or as a simple over-and-under circuit. If the latter system is used, there can be no re-circulation of any parcels, but the space required is quite narrow for area where there is some restriction in width. With the loop method several different patterns can be used. With only one loading point and discharge to one side of the conveyor only, the width of the sorter will be doubled, but the length halved. If the width between the two paths is increased, discharges can be made to both sides. Alternatively, one sorting machine can be used for two operations, if two loading points are included, one each end of the loop, then one side can deal with the sorting of outgoing parcels, whilst the other handles incoming parcels, sorting them to the separate cartage posts. However, the sorting operator on the latter position would have to be very experienced and have a lot of local knowledge, until address codes become standard throughout the country. No matter which of the above systems were used, a small proportion of the traffic would have to be handled nanually due to excess length.

The sorting machine operates at speeds up to 250 ft/min, which, assuming four foot trays, means handling approximately 5,750 parcels per hour under maximum sorting conditions. The speed can be adjusted to give a steady flow of work.

Tilt-tray installations at work in parcel and freight depots in the USA have been reported as bringing about a saving in operating costs equal to the capital outlay, over a period of 3-4 years. This, however, means making full utilisation of the equipment.

b) Tilting Slat

The Sortrac III tilting-slat sorting conveyor is similar to the Tilt-Tray in operation. Instead of trays, this conveyor consists of a series of C" wide mild steel slats, mounted on centre pivots and moved by a chain-conveyor. The slats are held in a horizontal position by stabilising rollers which run in a central track. The slats can be tilted in either direction by switching the stabilising rollers to a subsidiary track. The track can only be supplied with the return strand under the sorting strand of the conveyor.

The conveyor is scanned by a photo-electric beam at the induction point, so that all of the slats under a particular parcel are tilted at the discharge point. Decause the slats tilt as they approach the discharge point, the change in direction is gradual, allowing a virtually shock-free discharge from the conveyor.

The control system and coding operation are practically identical to that of the Tilt Tray system. But, whereas the length of parcels on the trays is limited by the length of the trays themselves, the parcels sorted on Sortrac III can be up to 6' long and 3' square.

Unlike the Tilt Tray, this system is not capable of negotiating corners and is therefore limited to a single flow direction. It is also slower with a maximum conveyor speed of 200 ft/min. Discharges can be arranged at a minimum pitch of 2 ft on each side of the conveyor, but it is seldom possible to make use of this, particularly with larger parcels.

c) Diverters

This method of automated sorting is different in operation to the above two. It consists of a number of arms which sweep across a flat conveyor, diverting parcels off into discharge chutes. To aid diversion, a vertically mounted belt conveyor is mounted around the diverting arm, moving in the direction of the discharge.

Diverters can operate on any flat conveyor, band or slat, provided that it is sufficiently tough to withstand the sideways movements of the parcels. The diverters can be either single or double acting, i.e. they sweep to one or both sides of the conveyor. However, they have limitations in use; the packages must be capable of absorbing the impact of hitting the diverting arm and must also be of sufficient height to prevent passing or sticking under the arm (approx. $\frac{1}{2}$ " gap.)

It is also slover in operation than the other methods, as the arm has to swing across before the parcel reaches the discharge chute, and swing back before the next parcel arrives. If the parcels are too close together, there is a risk of striking and trapping a parcel preceding the parcel for that destination point. There is also a restriction on throughput caused by the maximum lengths of parcels that can be handled by the arms. The maximum speed of the conveyor is about 150 ft/min.

d) Tilted Band

This system is based on a conventional belt or band conveyor, which is tilted to an angle of about 30° after passing the

coding point. Along the lower edge of the conveyor is a shallow wall with a number of gates included in it. The parcel destination is coded in the normal way, via a keyboard, into the memory unit, the signal generated following the parcel along the conveyor until the appropriate gate is reached, when the latter opens, depositing the parcel into a discharge chute. This method is used by the G.P.O. for mechanical sorting of their parcel post.

It is not suitable for B.R. parcels, however, as the parcels handled by them are larger and heavier. The sidewall must be fairly low, to avoid excess friction. This means that large or irregular parcels are liable to tip over the wall, with risk of damage, as well as the necessity for re-handling. The gates should also be at least the length of the larger parcels and preferably 1½ times larger.

Thus for parcels up to 3' in length, the gates should be at least 42' long. Alternatively two flaps could be opened simultaneously, but with increasing cost in control and operating mechanism.

However, as this method is not suitable for B.R. Parcels, it will not be considered further.

Costs for Automatic Sorting Machines (for equivalent capacity)

Tilting Tray

Basic cost Per foct run Induction stations Destinations

Tilting Slat

Drive Tension Slat Track + Aproi and supports Dischargers Hemory system Supply and erect

Diverters

Basic conveyor,	Belt	14 per ft	
	Slat	20 per ft	
Hemory system		40 per ft.	
Single Diverters		225 eac.) + installation 350 eac.)	
Double Diverters		350 eac.) This carraction	

£

6100

1150

650

130 ea

45 per foot

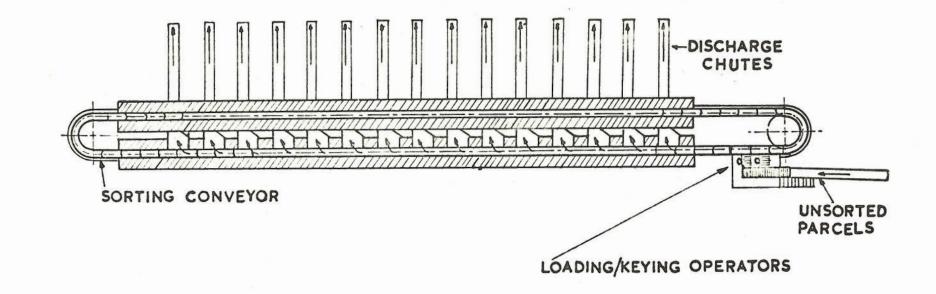
100 per ft.

40-50 per foot of conveyor

36

1300 ea.

380 ea.



A TYPICAL MECHANICAL SORTING LAYOUT

23a -

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7. AMALYSIS OF RESULTS

In this section the application of Information Theory to Parcel Sorting will be dealt with in more detail. Also, comparisons will be made between sorting methods mentioned in earlier sections.

In the section on Information Theory, it was found that the entropy of a collection of unsorted parcels could be found from the formula:-

$$E = -\frac{D}{1} \frac{n_{i}}{N} \log\left(\frac{n_{i}}{N}\right) \qquad (i=1,2...,D)$$

This figure for entropy, in units dependent on the base of the logarithms used, indicated the amount of sorting that was required on the collection, to get them into a fully sorted condition. This means that an increase in entropy is accompanied by an increase in the amount of sorting required.

If, for the sake of analysis, we assume that the number of possible destinations, D, is a constant, then the entropy must vary with N, the total number of parcels, or n_i , the number of parcels per destination. If we also assume that n_i , remains a constant proportion, regardless of N, then an increase in N will produce an increase in n_i , in proportion. Thus $\frac{n_i}{N}$ will remain constant, i.e. there will ne change in entropy. Therefore the entropy of a collection is not dependent on the number in that collection, if the proportion of parcels per destination remains constant.

The entropy must therefore be dependent on n_i . The formula used to determine the entropy is such that E is a maximum and equal to log D, when all of the probabilities $\frac{n_i}{W}$ are equal, i.e. all n_i are equal. As the variation between these probabilities increases, the value of E will decrease, for a given number of destinations and constant total number, until all the $\frac{n_i}{W}$ but one are zero, at which point E = 0. Therefore, if a sorting destinationshas a number of depots which receive an equal quantity of parcels, the entropy, and hence the amount of sorting required, is greater than that of a dustination which sends a large number to one or two depots and smaller numbers to the others.

The reduction in entropy after one sort is also found. If the number of parcels per bay is the same, this reduction will be equal to the logarithm of the number of bays. If the number per bay is equal, then the reduction in entropy will be reduced. However, provided that reasonable care is taken in balancing the numbers, the error will not be too great In the practical example of manual sorting, the reduction in entropy on dividing equally to the 12 bays should have been log 12 = 1.00. For the first sorting pattern designed, i.e. total sorting, the reduction of entropy after the first sort was calculated as 1.01, an error of 6.5%.

If the number of destinations is increased, then E is also increased, as, when all probabilities are equal and there are D possible destinations, E = log D. Therefore to reduce the entropy of a collection of parcels at a depot, the number of destinations to be sorted to should be reduced. This, in fact, is what is intended for B.R. and G.P.O. traffic in the near future, when certain large depots will become area centres for freight and parcels. Each depot will handle an increased amount of transhipment traffic, but with mechanised sorting, and standard unit load transport, i.e. DRUTE trolleys This should be the most efficient way of dealing with long distance parcels. Smaller depots should be mainly concerned with local traffic; traffic for other areas being sent to the area sorting depot, possibly after a simple primary sort.

Comparison of methods

The Work Study investigation showed that the theoretical sorting speed for nanual sorting into 12-15 sorts, is appreximately 400 parcels per hour. An allowance of 10% should be made for normal working rate, which reduces the speed to 360 parcels per hour. A further 20% for a Rest Allowance will bring this down to 208 parcels per hour. A deduction will also have to be made for different sizes of parcels, handwriting on labels, coding checks etc., if this is assumed to be 10% then the final speed for sorting would be 260 parcels per hour, per man. However, these figures were obtained on the assumption that the staff were working full-time on parcel-sorting alone. But they also have to load trains, change BRUTE trolleys, and alter sorting layouts for different types of traffic. This will probably reduce the overall sorting speed to about 200 parcels per hour.

In addition to this, a labour allowance must be made for sickness, holidays etc. and also to cope with peak periods that may occur, requiring an extra 22½% labour force.

The approximate labour cost of one man is approx. £1000 p.c.; thus a introduction of the quantity of parcels to be handled per day, will allow the canount of labour required to be calculated, and therefore the labour costs of sorting to be found. The quantity of parcels handled per day will be the number of parcels received for sorting per day, multiplied by the entropy over the log of the number of bays, assuming optimum sorting is used, and all of the parcels are fully sorted. Otherwise the number of handlings should be less than this, and obtained from tables similar to those in the section on theoretical handling.

To compare mechanical with manual systems, the cost of the mechanical system, with its associated labour costs, should be calculated.

The maintenance, running and depreciation of the mechanical equipment is assumed to be 15% of the total capital cost, per year, the accounting life of the equipment being taken as 10 years. If a cost comparison is made between manual handling for a certain number of parcels, and mechanical handling for the same number, then the difference between the two costs will be the savings expected by use of mechanical equipment.

If a cash flow is then drawn up, showing the initial outlay, against the expected returns, the expected return c_n investment can be found.

The costs obtained for the mechanical equipment are only an approximate figure, drawn up as a tentative guide to allow comparisons to be made. As the mechanical equipment will be used solely for a primary sort, labour will be required to sort the parcels into secondary sorts, the number of sorters required depending upon the number of parcels per day and the amount of sorting required. Additional labour will also be needed for loading the machine and for the primary sorting operation.

Of the three types of mechanical aquipment considered:-

a) The Diverter system is the cheapest; however it has a number of disadvantages, it is the slowest in speed and there are also limitations of size, weight and strength of the package. Therefore, it will not be considered further.

b) The Tilting Slat system has the highest capital cost; however, it is versatile, as the slats can support parcels of varying lengths, whereas tray length is naturally limited. It is restricted though, by the fact that it cannot turn corners, thus layout is restricted to a single run of the conveyor.

c) The Tilt-Tray system appears to have a slightly lower capital cost and has the ability to travel round corners, thus different layouts can be considered as circumstances demand. Extra labour will be required, however, to handle long parcels that cannot be carried on the trays.

For comparison it will be assumed that the entropy of the collection is 1.8 and that there is three-shift working. The conveyors will be of 100 ft. operating length and have one induction station and 15 destinations, each destination being a secondary sort into 10 DRUTES.

Knowing the number of men required allows the labour cost per annum to be calculated for manual sorting. The costs for the mechanical equipment are:-

a) Tilt Tray

Basic Cost	6100
per ft. run	7200
Induction stations	1300
Destinations	5700
Total Cost	20,300

. Annual cost for running and maintenance = £1,013.

be manually sorted from the discharge chute. If, however, a discharge chute was provided for each one or two cartage posts, the loading operation could be carried out more efficiently, although the keying operator would have to have some local knowledge, to know which traffic was assigned to which cartage post. These additional destinations would mean that one of them could be taken over, if a breakdown occurred on one of the other chutes.

If the costs for varying numbers of parcels per day are obtained and drawn up on a cost/volume chart, the relative costs can be compared, and an idea of the comparative costs for a specific parcels volume can be indicated.

Manual Sorting

Parcels, day	/ /hr	Parcel handling/ hr Entropy = 1.8	Men re por shift	quired per day	+ 22½% labour allowance	Cost p.a. at £1000
10,000	416	750	3.75	11.25	14	14,000
15,000	625	1125	5.62	16.9	21	21,000
20,000	832	1500	7.50	22.5	28	28,000
25,000	1040	1875	9.37	23	35	35,000
30,000	1250	2250	11.25	33.0	42	42,000
35,000	1460	2630	13.15	34.5	49	49,000
40,000	1666	3000	15.0	45.0	56	56,000
45,000	1070	3400	17.0	51.0	63	63,000
50,000	2080	3750	18.7	56.1	70	70,000

Automatic Sorting, Labour

Parcels/ day	/hr	llen required for sorting	$= 22\frac{1}{2}\%$	+ 2 for keying operation	Men per day	Cost p.a. at £1000
10,000	416	2.08	3	5	15	15,000
15,000	625	3.13	4	6	18	18,000
20,000	832	4.16	5	7	21	21,000
25,000	1040	5.2	6.4	3.4	25	25,000
30,000	1250	6.25	7.7	9.7	29	29,000
35,000	1460	7.3	9.0	11	33	33,000
40,000	1666	8.33	10.2	12.2	37	37,000
45,000	1870	9.35	11.5	13.5	41	41,000
50,000	2080	10.4	13.0	15	45	45,000

In addition to the labour cost, the following costs will have to be added:-

Tilt-Tray - £1,013

Tilting Slat - £1,680

Costing Exercise on Tilt-Tray Sorting Machine

This exercise uses the tentative costs provided by B.R.B. for a Tilt Tray sorting machine, to give an idea of the expected returns on investment that would be achieved with this equipment. For the purpose of the exercise the following data has been assumed:-

- a) 25,000 parcels per day are handled;
- b) the machine has a working length of 100 ft.
 with one induction point and fifteen discharge points, giving a capital cost of £20,300;
- c) the expected life of the machine is five years, with three shift working; also, an anticipated scrap value of £3,000 is expected

The cost for running, maintenance and depreciation was given as 15% of the capital cost. However, the D.C.F. technique used avoids the need to consider depreciation as a separate factor, a D.C.F. rate of return of 10% implies that the project will earn 10% on the funds invested in the equipment as well as repay original sums invested over life of equipment. The accountancy life of this type of equipment is taken as 10 years, with straight line depreciation, and no residual value, the depreciation, therefore is 10% per annum. Thus, the running and maintenance of the equipment, the actual cash payments are 5% p.a.

As the equipment does not qualify for an Investment Grant, there will be an initial allowance of 30% of the capital cost plus an Annual Allowance of 25% of the Residual Value, for tax purposes, charges at a rate of 421% Corporation Tax.

There may be a Residual Value after the five years, if the equipment is sold for more than this, a balancing charge will have to be made on the difference. If it is sold for less a balancing allowance will be made. It is assumed that the tax is paid one year later than the profit is made.

Year	0	1	2	3	4	5	6
(a) Cash Outlay	(20,300)			-	-	763	-
(b) Cash Returns (Profit over manual system)	-	10,000	10,000	10,000	10,000	10,000	-
(c) Running costs	-	(1,000)	(1,000)	(1,000)	(1,000)	(1,000)	-
(d) Met cash in (b-c)	-	9,000	9,000	9,000	5,000	9,000	-
(e) Capital Allowance		11,500	2,200	1,550	1,260	945	-
(f) Residual Value (Capital value allow- ance)	-	8,300	6,600	5,050	3,790	2,845	sold for 3,000
(g) Taxable returns (d-e)	-	(2,500)	6,200	7,450	7,740	8,055(Balance charge)
(h) Corp.Tax @ 42½% (1 year delay)	-	-	-	(2,890)	(3,160)	(3,290)	(3,490
(i) After tax cash return (d-h)	-	9,000	9,000	6,110	5,840	5,710	(3,490)
(j) 20% factor	-	.833	.694	.579	.482	.402	.335
(k) Net present value (jxi)		7,500	6,250	3,540	2,820	2,290	(1170)

(Figures in brackets are negative quantities)

This results in an N.P.V. cash return of £21,230 for a capital outlay of £20,300, with a return on investment of 20% plus an additional profit of just under £1,000. This is for 25,000 parcels per day, with a higher rate of parcels, the profit over the manual system would be higher, resulting in a higher expected rate of return.

8. CONCLUSIONS

The first conclusion that can be arrived at is that Information Theory can be applied to parcel sorting. One of the basic concepts of Information Theory is that of Entropy, or the uncertainty of receiving the required message from a collection of possible messages. A collection of unsorted parcels for a number of destinations, has an uncertainty connected with selecting the parcel for a specific destination. Thus, as these uncertainties can be regarded as comparable, the concepts of one can be applied to the other, within limitations, dependent upon the similarity of the two. The concept of entropy, therefore, can be applied to parcel sorting. This will give the amount of uncertainty in the collection a finite value, thus enabling comparisons to be made between various collections, and also giving an indication of the amount of sorting required to sort the parcels.

If an optimum sorting method is used, by getting equal quantities of parcels into the optimum number of bays, then the relationship between the entropy and the number of bays will enable the amount of sorting per parcel to be found. The optimum number of bays is arrived at by comparing sorting speeds into different numbers of bays, an allowance being made for the 'effectiveness' or reduced number of sorting passes needed, of the larger number of bays. If these quantities are used as a guide for a more practical system, and this system follows the theoretical system as closely as is conveniently possible, then the theoretical amount of sorting need, the latter being slightly larger,

The above is applicable when the parcels are fully sorted at one depot. If, however, they are only partly sorted, the logarithm of the number of bays indicates the reduction in entropy for each sort. Thus knowing the original entropy, the entropy of the semi-sorted collection can be found.

It was shown in the test that as the total number of parcels was increased, the entropy altered only if the proportion of the parcels to each destination altered. This means that the entropy is not related to the total number of parcels, but to the number of destinations and the number of parcels to each destination. It was seen that as the number of destinations increased, then so did the entropy. Likewise, as the probabilities of parcels per destination became more equal the entropy increased, until a maximum was reached when all of the destinations were equi-probable. This is intuitively the most uncertain position, as all the parcels have to pass through all of the sorts. The entropy will be zero when the probabilities for all destinations but one are zero, as no sorting need be carried out, no uncertainty existing.

Because the entropy is dependent upon the number of destinations and the proportion of the traffic sent to each, then to reduce it, and thus the sorting required, either one or both of these factors should be reduced. If traffic at a depot is passed through a primary sort, the traffic for other areas or regions then being forwarded directly to the areas, whereas traffic for the area in which the depot is situated is passed through a secondary sort, this will reduce the number of destinations sorted to, and hence the entropy for the parcel collection of that depot. This will necessitate large depots in each area receiving more traffic than it would normally, from a number of different areas. But with efficient sorting and concentration on the area covered by the depot, the handling should be improved. Care must be taken in selecting these area centre depots, to ensure they are of sufficient size to handle the traffic and placed in an accessible position, geographically.

This will mean that alternative coding to that at present in use should be provided. At the moment each zone centre depot has its unique code number, depots in the same region having similar code numbers. At present, though, these codes are not always referred to, parcels being referred to by destination address rather than by code number, also many amendments take place to the zone centres for certain towns as stations and lines are closed, and these amendments are often not kept up to date.

With the theoretical sorting, the distribution of parcels to each bay will vary from depot to depot, thus to get even quantities, a complex number of codes would have to be remembered. Thus new codes should be drawn up for each depot, to enable parcel sorters to sort at highest speeds, without having to refer to an address, or remember a complex collection of numbers. The old codes should still be kept, as once a parcel has left the originating depot or area, this new code will be meaningless; thus the two codes should be of different forms.

The theoretical sorting analysis shows that when all factors are taken into consideration, there is not much difference in the effective sorting speeds. These factors include the speed of sorting one sort into various bays, and the effect that the extra re-handling required has men parcels are sorted into a small number lowever, there are peaks in effective sorting of bays. speed at 15 and 30 bays. The layout for 30 bays requires twice the layout area of 15 bays, and there will also be difficulties in equating quantities of parcels and grouping destinations. This means that 15 bays would produce the most effective sorting, when speed is the most important consideration, but, as is pointed out, difficulties would arise in obtaining optimal sorting, after the first sort, and a slower speed, with slightly more sorting per parcel, would give a sorting pattern closer to the optimum. It should be emphasised though, that this only occurs due to the number of destinations, 176. With a different number of destinations, either less or more, then other patterns, probably using 15 sorts should be used.

When the sorting pattern is drawn up, close attention must be paid to the destination areas. Only at the largest depots can traffic be fully sorted, therefore for the remainder a proportion of the traffic will be sent only 'part-sorted'. Time can be wasted in transport and rehandling if parcels are sent to the wrong area zone centre.

To keep segregation of parcels on route, they should be kept in unit loads, the British Railways Universal Trolley Equipment (BRUTE) being ideal for this. BRUTEs are steadily being brought into use, but faster in some regions than others. This means that inter-region traffic has to take into account the facilities of the different regions. It will be a waste of time and effort if traffic is forwarded from one region to another in DRUTEs, if the receiving region cannot handle them. The impression gained from sorting staff using DRUTE trolleys at present, was a very favourable one. They are very much easier to handle around depots than the old-type flat barrows, and a parcels van that would require an hour to load 'rough' takes about five minutes to load with 10 PRUTEs. This means that BRUTEs for different destinations can be loaded into one van, without fear of 'un-sorting' the parcels in transit. Men a destination is reached, the BRUTEs required for that station are removed from the van, without having to detach the van, reducing waiting time to a minimum. This leads to the conclusion that as much traffic as possible be forwarded in BRUTEs. This is being done by D.R. to a limited extent with their BRUTE trains. Hine vans of a train of ten are filled with BRITEs for destinations on a set route, possibly circular, the other van being filled with awkward parcels that will not conveniently fit in BRUTEs (e.g. rolled carpets.

Therefore, if optimum manual sorting is used, as completely as possible, straight into BRUTES, an efficient parcel sorting system should be developed. Once in a particular BRUTE, the parcels should remain in it for as long as possible.

Sorting machines are available to assist in sorting, provided that sufficient quantities of parcels are sorted per day. This speeds up the process by eliminating the labour required to transport parcels between the first and second sorts, this also allows a reduction in labour costs. The costs for low quantities per day are higher than for manual sorting, but as the quantity increases the savings in labour increase. The example in the analysis has shown that for a total of 25,000 parcels per day, with the costs provided, there will be a return on investment of approximately 20% for a life of five years. The sorting time-table can be planned more efficiently, sorting being carried out in quieter periods, thus alloving the sorting staff to concentrate on the rapid loading of parcel trains, when required.

The theoretical sorting method can also be applied to mechanical sorting. As the secondary sort is a manual one, from the discharge chutes, there are different limitations that will apply to this method. On the primary sort, the different speeds of sorting to differing numbers of bays will be almost negligible, as the conveyor runs at constant speed, the operator remaining stationary. An increased number of primary sorts will increase the cost of the equipment and also the area required, but it will decrease the amount of sorting required by the secondary sort. Some parcels will still need to be

combined for transport to other areas, due to the low quantities per destination per day.

In manual sorting, time has also to be spent sorting out parcels for delivery to their cartage posts. With mechanical sorting, extra discharge points could be supplied for this type of traffic, allowing the delivery staff to spend more time on loading the vehicles efficiently, and in the making out of delivery notes.

The number of manual staff at the discharge chutes should be a function of the number of parcels handled, rather than of the number of bays, the bays acting as temporary Each operator having a set number of storage reservoirs. bays to lock after, the operators being concentrated on certain bays when the need arises, e.g. for loading trains This would mean that an increased number of bays etc. would have to be dealt with, with a larger primary sort, although the total quantity of parcels will be the same, therefore some account must be paid to this when determining the labour required, as there will be a limit on the bays one man can look after.

If a fairly low number of bays is used, and one becomes unusable, there being no alternative bay, problems will Additional bays should be supplied for this and arise. any other contingencies that may arise (e.g. full chute, illegible address).

In conclusion, therefore mechanical sorting systems should be used for dealing with quantities of parcels over a set minimum level, determined by comparing the cost of running the system compared with a purely manual method. This cost figure will vary with the entropy of the collection; the higher the entropy, the more chance there will be of exploying sorting machines. A chart is provided comparing (Fig.2) mechanical with manual sorting, showing an approximate break-aven point, and also the point for an expected return on investment of 20% using a D.C.F. emthod. It must be pointed out, though, that the costs supplied were only tentative and therefore a more accurate investigation of costs, machine life, etc., must be carried out before an accurate break-even point is obtained. It is hoped however, that the figures supplied will provide a reasonable guide to expected costs and returns.

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APPENDIX I

The probability, P_i , of a parcel for the ith destination D being selected from the collection will ne n_i , where n_i is the number $\frac{1}{11}$

of parcels for the ith destination and N is the total number of parcels in the collection.

The entropy of the collection, per parcel, will be :-

$$E = -\sum_{i=1}^{D} p_i \log p_i; p_i = \frac{n_i}{M}$$
$$= -\sum \frac{n_i}{M} \log \frac{n_i}{M}$$
$$= -\sum \frac{n_i}{M} \log n_i - \log M$$
$$= \sum \frac{n_i}{M} \log N - \sum \frac{n_i}{M} \log n_i$$

but H is constant and $n_i = H$

The total entropy of an unsorted group of il parcels will be :-

$$1\mathbb{I} = \mathbb{I} \log \mathbb{I} - \Sigma \operatorname{n}_{i} \log \operatorname{n}_{i}$$

If the parcels are corted into F bays, with n parcels in each bay bay then, entropy per bay = n; $\log n_j - \sum_{i=1}^{D_j} n_i \log n_i$ where $D_j = \text{dest}^n$ per bay.

. Total entropy after 1st sort, NE' = $\sum_{j=1}^{F} n_j \log n_j - \sum_{i=1}^{D} n_i \log n_i$ where E' is new level of entropy and summation is over all destinations. The reduction in total entropy = NE - NE' = (N log N - Σn_i log n_i) - (Σn_i log n_j - Σn_i log n_i) $= \mathbb{N} \log \mathbb{N} - \Sigma n_i \log n_i$ $\mathbf{N} = \sum_{i=1}^{F} \mathbf{n}_{i}$ $F = \Sigma n_{i} \log N - \Sigma n_{i} \log n_{i}$ $= \Sigma (n_i \log N - n_j \log n_j)$ $= \sum_{i=1}^{n} \log \frac{1}{n_{i}}$. Reduction in entropy per parcel = E - E' = $\frac{1}{H} \sum_{i=1}^{H} n_{i} \log \frac{H}{n_{i}}$ for any number, n, of parcels per bay. If we assume that the parcels are divided equally into each bay, then, with F bays, $n_{i} = \frac{N}{F}$ $E - E' = \frac{1}{H} \sum_{i=1}^{F} n_{i} \log \frac{H}{n_{i}}$ but n; $\log \frac{H}{n}$; is constant and equals $\frac{H}{F} \log F$

$$E - E' = \frac{1}{1!} \sum_{i=1}^{i} \frac{F}{F} \log F$$

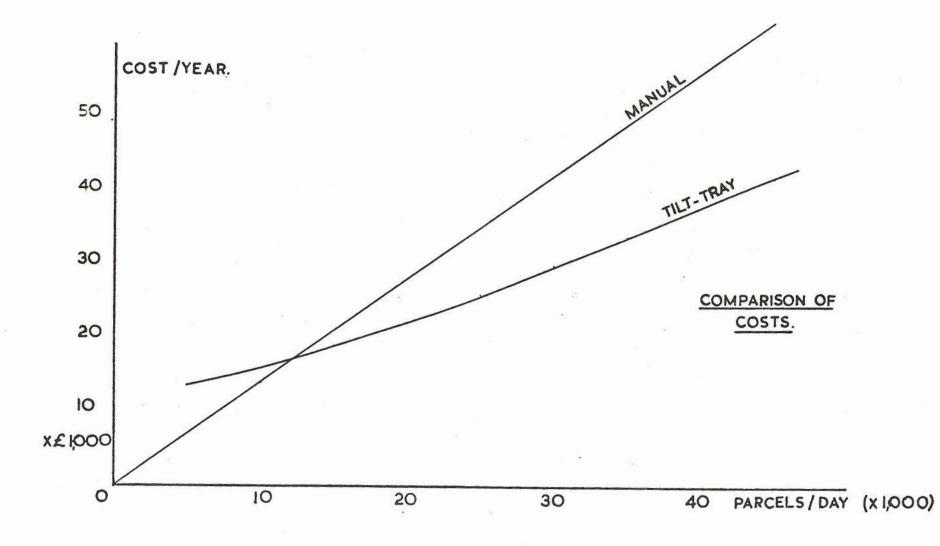


FIG. 2

For equal quantities of parcels per bay, after one sort the entropy will be reduced by log F. (the number of bays).

The units of entropy are dependent on the base of the logarithms. If logs to the base of 2 are used, then the units will be binary digits, or "bits", if to the base 10, then, in decimal digits. As $\log_2 10 = 3.32$, 1 decimal digit = 3.32 bits.

Assume that the quantity of parcels in each bay is the same:-

Change in entropy, $E - E' = \log F$.

This is the reduction of entropy in one pass,

. . if L' = 0, then no further sorting is required.

if E' > 0, then further sorting is required,

and $E = \log F + E'$

$$\frac{E}{\text{or log } F} = 1 + \frac{E'}{\log F}$$

After a further sort,

 $E' - E'' = \log F , assuming same n° of bays.$ if E'' \$ 0, no further sorting is required if E'' > 0, further sorting is required. $\frac{E'}{\log F} = 1 + \frac{E''}{\log F}$ or $\frac{F}{\log F} = 2 + \frac{E''}{\log F}$

Thus the amount of further sorting required is determined by E" and log F.

If
$$E'' = \log F$$
, i.e. $\log F = 1$,

one further sort is needed,

and, E'' - E'' = 0 or a fully sorted collection.

 $\frac{E}{\log F} = 3$, the number of sorts carried out.

if $\frac{\mathbb{E}^n}{\log F} > 1$, then further sorting will be required

if $\frac{E''}{\log F} < 1$, then only a proportion will be needed to be sorted. If, for $\frac{E''}{\log F} < 1$, log F is reduced, so that $\frac{E''}{(\log F)} = 1$, then E''' = 0, and no further sorting is required. To obtain $\frac{E''}{(\log F)} = 1$, log F must be multiplied by $\frac{E''}{\log F} = 1$. $\frac{E''}{\log F}$ is the proportion of parcels to be re-sorted. Alternatively, each parcel must be sorted an extra $\frac{E''}{\log F}$ times.

This results in $\frac{\text{Entropy}}{\log F}$ = number of sorts required.

When the units of entropy and the base of the log are the same, and optimum sorting is used.

For example:-

1) Say, entropy of 2.0 and F = 10 bays 1st sort $E - E' = \log F$ 2 - E' = 1 L' = 12nd sort $E' - E'' = \log F$ 1 - E'' = 1 E'' = 0. fully sorted in exactly two sorts. 2) entropy of 2.0 and F = 25 bays 1st sort $E - E' = \log F$ 2 - E' = 1.4 E' = 0.6 $\frac{E'}{\log 25} = .43$, . only 43% of the parcels need to be sorted a second time $\frac{E}{\log F} = 1 + \frac{E'}{\log F} = 1.43$, the n° of sorts per parcel required. $\left\{ \text{of.} \quad \frac{E}{\log F} = \frac{2.0}{1.4} = \frac{1.43}{4} \right\}$