to marcel sorting.
by
B.a. Deghimid and hom. pmomaton.

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Fine work on parcel sozting taas originally suggestec to the Haterials Lancilng Resemrch Unic by Britiss Rail wich is one of the spousoring oremisations contributine to the Research pund. This wozk hes also formed the subject of a research thests, Which has been used as the basis of this heport. the stuaent, ir. B.R. Defartund is cherefore the main author, in co-oferation witi the staft of the Research Unit.

Quite a high pic,ortion of comercial goods travel frow supplier to customer as a 'parca1'. That is, a single packege secureiy packe, moving fron or in to a destination by various means of transport. Since most parcels orisinating from a single point are destined for different addresses winch nay be fin any part of the country (ve are oniy consicering inlend parcels) a sorting procedure ias to take place. Theoretically, this coule ocerr at any point after despatch, but is usually done at a point central to coliection frow a number of suppliers. the auvent of tuall order busizess which is still expandin, rapicly in this country, has broujht a vascly increased flow of parceis into the various organisations undertaling to deliver hean.

It was appreciated from the beoinning of the work thet this problen was not peculiar to $\%$. or other parcel handine orgenisationc sweh as the Gop.o., but the principles involved shouid be capable of application over a much wider field. There are many cituations wiere a sorting activity takes place in orien to reluce a mass of randomly mizea items into ciscrete caterories, and it is lilely that many of these coulc venefit by beinc exanined by the methoce cuccested in this report.

One of the aurly difficultics wac to decirle what constituted a percel, and certain rules sere propose to arrive at a definition. It vas generally taken to be a paciace of any shape or form which travelled betwecn two poines as a separate entity. It vas nocesaany to place scme restrictions oin size and weight stace articles as lerge and unwielily as a double-bec size whtrese racuiarly tiave? on B.R. as 'parcels traitic', but co not lend themselves very well to tapid handinat, Thus a parcel was generally talen to mean a box or pacinge of wout 2 ft . cube vilici. could readily be handled by one man. This diviston is ioosely observed by L.f., tho ton to treat separately those iteras whict are obviously unvielaly or excessively heavy.

There are three rough divisions into size and "inandleability":-
(i) Postal parcels accepted by the G.P.O. made stringent size ane veight limits.
(ii) Jailway parcels trafiic, which takes ainost any size, both parallel with the G.P.O. range and upwards, and sinilar services rum by ational or private road carriers. The upper size/weight limit tencis to be set by the pricing structuze ratiocz then by ingosed physical 1imitations。
(iii) Lazger and gencraily more irregular packafes which usually travel by rail 'suncries' service or could similarly travei by road haulage. This category also inciucies mulciple packages to ore sddress.
 tio practicut xamination cil the problem.

## 2. SUILIARX

The object of tie investigation was to examine the existing sorting proceuures for railvay parcele in order to evaluate existing methods both manual and nachine-essisted for efficiency of sorting. To postulate a standard of sorting efficiency it was necessary to lock more closely at the theoretical basis of the sorting procese, ard to develop a theory of sortinc. This has been done by the application of Information Theory, using the concept of Eatropy as a measure of the state of disorder of a given mix of parcels.

The relationship between the entropy inciex and the numer of bays or divisions into mich the parcels are required to be sorted will enable the anount of sortinc per parcel to be found. This in turn will given an indication of the most efficient method to be usec.

The theoretical sortinf method can be applied to both manual anc mechanical sortine, and an indication can be given as to which method is likely to be more efficient in a given siouttion, and thus a cost estimate can be carried out.

Some comparisons have been made between the coct of sorting pancels both nanually ani by cifferent mechanical methods, making certain cost assmptions, and relating this to the volume of parcels to be sortaci.

## 3. THODTTICAL SORTHE

The $s_{1}$ eed witi which parcels can be sorted is dependent on the number of categories into thich the itens must be sorted and the numer of sorto, necessary to separate then to destinations. With letter post, there is littie actual transporting of the ietter at the sorting point, wost of the sorting tine is cepancent on how cuickiy the sortus can recognise the destination. Fith parcei-sortine, tice oposite is true, the greater the leneth of tine is spent in cirrying the parcel from the sorting point to the sorting destinetion.

Sorting speed, though, is not the only factor for optimal sorting. the increased sorting speed, cue to the smaller nuiber of sozts, is off-set by the numer of tines a parcel must be sorted and the increased handing time between sorts. The totai sorting efficiency, therefore, is dependent on the speec of sorting and the avezage humber of sorts.

If there are 100 destinations to be sorter to, and an equal namer of parcels for each, then tivo sorts cach into ten bays car be uced. At the first sort, the parcels are placed into ten bays, each having ten destiazticus allotted to it. The parcels in ewein bay are then sorted into a further ten bays, one bay per destination. As the volume for eacl cestination has been assued to be equal, then sortino to 100 destinarions ias only required twice the number of sorts that voul.u be needed for ten destinations. Likevise, sorting for 1,000 destinations will oniy requice three sorts of ten bays each. As the effective wori done is tiee same for each sort, then it can be saiu that scrting for 100 destinations is twice the effective vor: of sorting for ten iostinations. This means that the effective worl done in santing can be releted to the logarithn of the nabar of bayo. This logaritim can be called the Inder of effectiveness ard allows for the cffect of the varying number of handings occurzin rith cifferent numbers of Deys.

Thus if the sorting ofeeds are multipliec by this In ex, the sorting efficiency can be determined. For parcols surteu into less than ten bays, the effective sorting speed will be reiuch, as the logarithms used are to the base of 10. This ailows for the increasei number of handings. If the number of bays is grater than ten, then the offective sorting opeed will be increased, cue to the lesser number of hendings required.

If these effective sorting speecs are now compared, the nost efficient mumber of beys can be deternineci.

The folioning table has beex prepared using standard cime data on parcel sorting provicied by in. The time elaments referred to in the taile and the text following are the analysis of the tasi of sorting parcels as under:

## ziementio.

## Operation

| 1 | Fenove veclage froni 'arrival' vehicle |
| :---: | :---: |
| 2 | İentify destination code nmber. |
| 3 | hove rackage to appropriate trcliey. |
| 4 | Scor in troliey |
| 5 | Operator returas to picli-up point. |

Ta above is depencicnt on parcels being sorted into ritish Railmays Universal Trolleys (mmini) a theeled cage-type cortainer approx. $5^{\circ} \times 3 \frac{1}{2}$ in arce.

| ivo of scrts, F | Arer <br> recuired $\left(5^{1} \times 3^{\frac{1}{2}}\right)$ | Time per parce? sec.s | $\begin{gathered} \text { parcels/ } \\ \text { nr } \end{gathered}$ | ```Total time Parce1 allow- ing 3 secs for elements 1,2 <<4``` | ```Effective sorting speed s y log T``` |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 40 | 2,400 $\mathrm{tt}^{2}$ | 13.7 | 263 | 16.7 | 421 | 334 |
| 30 | I, 600 | 12.0 | 300 | 15.0 | 442 | 352 |
| 20 | 1,200 | 10.25 | 350 | 13.25 | 452 | 338 |
| 15 | coo | 9.1 | 395 | 12.1 | 463 | 350 |
| 12 | 600 | 9.1 | 395 | 12.1 | 430 | 322 |
| 10 | 600 | 0.0 | 450 | 11.0 | 450 | 327 |
| 0 | 400 | 6.35 | 525 | 9.05 | 475 | 330 |
| 6 | 350 | 6.25 | 575 | 9.25 | 146 | 300 |
| 4 | 250 | 4.0 . | 904 | 7.0 | 540 | 310 |
|  |  |  |  |  | L1.1.3 | (a11 |
|  |  |  |  |  | only | elements) |

If re assure that the purcei cimits frec movencnt and is between 10 anc 75 Ibs in weicht, then, a thet analysic of the basic time per parcel for elencnts 3 and 3 shoris that a sort of four is the most effective, allowing ever, for the increased numbr of sorts. However, the data proviued was only for a comparison of manual and autonated methode, an: did not take into account the time taler: for the otho: eluments - whloadng and loading the parceis and reading the destination aderess or code number. Then making a nomal comparison of methods, these times would not be required, as tiley are the sanic for cich methoc.

Although the time for elements 1,2 and 4 is short, it is reasonably constant, and then adied to the time for carrying the parcel and retuming it, it reduces the overall effective screinf speed, particulariy for the sorts with a smallec number of bays.

For tie calcuiations of parcels per hour sorted, the assumption was made that the sorter was vorling, nonstop, for a fuli hour, hut the sorter does not spenc all of his time sorting, he has to move railvay cage type trolleys ( HUTL ) etc. around, zenovinc fuli ones anc collecting enpty ones; he has to ascist in the loading of trains specialiy cesigned to accept 3RUTiSS, etc. ile may aiso have to wait for 'crivival' vehicies and mut also be given a 'personai' aliovance. This vill provabiy reduce his actual sortine tize to about hour for each hour worked.

From tie correcter cffective sorting speeds, it will be seen that ther is littic Gifference in sozting speeis between 0 and 40 bayc, the sorts into 15 mo 30 bays, giving the highest speets; howevor the sort into 00 bays raquires twice the are of that of 15 , and four tines that of 0 sorts. It has been assuned aDove, that the time taken to read and identiffy a parcel's adcross is constant, for all numiters oz bays. But, althouch no figures for parcels are available, ietter sorting has siom that the tine teken fos recogising the destination and its appropriate bay, particulanly when there are a large numer of pocsible destinations, will be longer when there are a larcor muber of bays. This time vill be difficult to meastre, as cach sorten's time vill vary with cxperierce anu mothod of remoberinj. It will heve the effect of reducing the sorting speeds for higher numbers of bays.

All control systems function by transmitting signals wich contain information about the controller's requirements. This information is tien interpreteu and any appropriate changes take place. Snis infonation can then be put into a menory store, as in tho human brain or an eloctronic compter. Thus, information can be transritted, translated and stored. It is an uesential conteitt of any control systen - Infomation Theory answers quartatively how mucir infomation a system contains.

Infomation theory was criginally developed in the rielc of Commications Dngineering - upor wich the media of radio, celegrapi etc., are tecmologically vased - by Shannon in iis "Wathenatical Thecy or Conmuication" in 1948. This theory Tas used to deal witin the question of noise in a radio chanel and how to organise a nossage st the trawsuission pcint in order to minimise the effect of this noise. It tock into accoun prior informetion, of a statistical nature, on the form of the nessage - thus boconing tice application of provability theory to comunicutione problens.

Bocause of the copth an fenerality of Information Theory, it can be applied to all foms of commication. As well as comunication encineeringु, it also contributes to the basic concopts of cryptoçraphy and language translation, and its use in informetion retricvai, porception and learninc, and other behavioural sciences is being explorec.

The funcamental problem of coramication is troroducing at one poirc a messaje scicectec at anotner. The messaces may have meaning; i.e. they refer $: \circ$, or are correlated, accoring to sone systen with piyysical or ccnceptual meanimes. Novever, this is incolevant to the cnoweering problem. The significant aspect is that the actual message is one selected from a set of nosstble messagou. The systen mat operate for all these possinle seloctionshot just the one trisch will actually be chosen, since this is unamom veforenank.

Shamon, then, is concerned vith tise average amount of 'macertainty' within the set or cnscuibe of mossares. he called this uncertaint, "Gntropy", Lecause the frmala he derived for it "vill be recegrisce as that of entropy as defined in certain formiletions of statistical mechanics". It is also the term used in physics to measure the excitation or disorder of wolecuies in a ras.

Shannon's verk does not only tole for racio messages, but for any application wione the wacertanty of piclinc one nenber Erom a collection is required.

A formia can be cerivei relating the size of an cnsente to tive uncertainty (or entropy) associate with it. If the enscmible nas only one mewber, then theite is no uncertainty; ve Finot the volution bufore we start. If there are two rembers, then one question is neecied to identify between then. In a binary systom, it can be shomn that each time the ensemble doubles, the number of quescions to be asked, thus the entropy, is increascd by one. Tius the entropy, $L$, can be connected to the size of tine enscmile by

$$
\mathrm{Z}=-10_{2} n
$$

then ail of the solutions are acuiprobable.

Hovever, where the solutions are not equiprobaile, i.e. where thore is bias tomarcis certain combors, this value of E does not. apply, as tie eintropy is reduced. Therefore some account mast be taker of the vejothing, or probability, of each member.

Suppose there is a set of gossibie selctions, wiose probabilities of cccurrence are $\mathrm{F}_{2}, \mathrm{P}_{2} \ldots \mathrm{n}_{\mathrm{n}}$. The probabilities are known, but nothing else concening wich selection will be mace is kinown. If tiere is a measure of "choice" involved in the selection of the event, say $\mathrm{E}\left(\mathrm{p}_{1}, \mathrm{p}_{2} \cdots{ }^{2}\right.$ ), it is reasonable to require that it satisfies the following conditions:-

1/ I sinould be continuous in $P_{i}$ 。

21 If all $p_{i}$ are cqual, $p_{i}=\frac{1}{n}$, then the value c: E increases as the size of the set increases.

3/ whe unt of uncertainty is additive, i.e. if a choice is broken dom into successive choices, the criginal $Z$ shouic be the weighted sum of the indiviclual values of E. Tor example, the anount of inforration on two punched carls is the sum of the amount of informatica on cach.

Ginanon offered as his secon, theorem, tiat the only formula for E satisfyin these tiree conitiors is:-

$$
\ddot{z}=-\sum_{i=1}^{n} p_{i} \operatorname{lcg} p_{i}
$$

Were it is werely a positive constant, detamining the units of uncertainty.

For a given cet, - is a mainimm wier, all the possible outcomes are equiprobable. The mininum vaiue of it is 0 , wich occurs vinct. tivere is only one possinle outco:c. Since the result is then snom, uncertainty vanishes.

A collection of unscricu parcels will be in a state of disorier and thes vili have a value of uncertainty comected with the possivie destivations of any of the ऐarcels. This value will be proportional to the amount oi sorting that is required on the parcels, to get then to their proper destinations. Finerefore, as this uncertaincy exists, the Theory of Information can be applicd to Parcel Sorting to determine the amount of sorting reruired on a collection cf parcels.

The mathonatical derivation of the entropy value for a mass of asconted parcels is given in hpendiz $I$, and siows that the number of sorts is equal to $\frac{E}{\log F}$, where $T$ is the number of bays into which the parcels are to be sorted.

It was shown in the sectiou on Infomation Theory, that the entropy of a collection of parcels shows the amount of sorting work to be done, and that the reduction in entropy after one sort irto a set number of bays is equal to the log, of that numor i.e. i. it vas stateu ajove that the sorting 'effectiveness' ${ }^{\prime}$ as also related to log F. The reduction of entropy or disorcen, verifies this relationship. It wes also shown that the entropy diviced by the logarithn of the manber of bays vas equal to the number of sorting passes required.

If an equal wuber of parcels are allocates to the beys availcble, then the mumez of destinations per bay vill vary vilely. Fowevor, since destinations are not equi-probable, this may rosult in the number of sorts being reduced. Wide ciscrepancies in the nuber of destinations per bay may make it Hore difficult for sortul:s to romember, and it would be letter to have the number of destinations per bay as powers of a common figure, ( $0.8 .1,10,100$ ).

Thus, aithough the sozting effectivenese requires equal numbers of parcels per bay, in practice this figure can only be used as a Buice then allocacing dentinations to bays. This means that the reriction in entropy will be less than $\log \mathrm{F}$ in the majority of cascs.

## A Practical Exarple

Data supplied by iritish Rail for a typical depot vas used as a practical example for the above theoretical method.

The total nuber of parcels hancied was 26,813. of these 10,740 were collected for formardine, $1.4,02 j$ were received for transhipmeni, and 2,042 vere receivec for delivery. there vere 176 possible ciestinations and the nuber of parcels per destination variec Detween 2051 and 1.

The ertropy ner parcel tas calculated for the whole collection and tinis figure came to 1.81. For an equ. 1 nubler of parcels per destination the entropy would have been 1000176 (or 2.245).

The methoc used vas to soct to 57 cestinations, the traffic being re-surted to the 170 at the various transhipment poinis.

Ucing the theoreticat method of optinal sorting, the parcels couid be sortec with an average number of 1.5 handings per parcel into 10 bays. Alternatively, if 15 bays were used, the parcels coule be sorted with an averace of 1.54 handlings per parcel.

For the zoment, we shali assume that the collection is fully sorted at the depot, thus no account will need to be paid to any gecyraplical destinations. If 15 sorting bays are used, the average nowber of parcels per bay will be 1,800 . Of the whole collection the parects received for delivery vere 2,042 , therefore one of the bays can be used solely for these pareels. For the remainder, 2 destinations each receive more than 1000 parcels, thus, these cestinations can be allotted individual bays, renoving the need for: further sorting. The next two destinations, hovever, account for a further 2500 parcels which viil mean that if one bay is allotted oo then, they will have to le sorted into two at 2 later stace. It vould ie better if they vere allotted a bay eacil. This will leave 172 cestinations for lo bays. Sorting could be arranged as 17 destinations $<x$ bay, but this will reoult in a large variation of percels in each bay. If 9 bays are allotted, 15 destinations each, ti.ere will be a remainder of 37 destinations with a total of 332 parcels, assumiug that the grounings are maria in order of quantity. The number of parcels per bay for the remainder will vary between 7,000 and 400 , to be sorted into 15 each at the secondary sort, altho.g the quantitics per bay at this sort will be approrimately equal. This range of parcels is rather high, when, for optinu sorting, there should be an equal nuber ci parcels per bay.

If a lower number of bayc is uced, say 12, the sorting speed vill be sifgitty less efficiont, but he quentities per bay, veģiran for optiaum sonting wh be zore ciosely approached. The parcols could be nunciled as shom:-

| $\begin{aligned} & 3 \times 1.10 \\ & \text { 1oy } 28 \\ & \text { Sort } \end{aligned}$ | (B.a. Zone Coces) Ho. of (estinations | No. of parcels sorted at |  |  |
| :---: | :---: | :---: | :---: | :---: |
| A | 1 | 2831 |  |  |
| \% | 1 | 2217 |  |  |
| C | 1 | 1400 |  |  |
| D | 1 | 1112 |  |  |
| E | 12 from 862-321 | 6159 | 6159 |  |
| F | 12 fron 311-200 | 3039 | 3089 |  |
| G | 12 Erom 200-134 | 1845 | 1845 |  |
| 12 | 12 from 127-55 | 1290 | 1238 |  |
| $\pm$ | 12 from 55-80 | 1058 | 1058 |  |
| 2 | 12 from $00-66$ | 850 | C53 | 2007 |
| L | 100 from 64-1 | 2827 | 2007 |  |
| 15 | Delivery | 2042 | - |  |

This results in a total of 44,849 parcel handings, for 24,771 parcels, or an averace of $1 . \hat{3} 3$ handlings per parcel. The entropy 50 this collection of parcels was 1.61 , therefore rith equal quanities of parcels per bay, the theoretical number of parcel handings vould have been 1.74 , thus the result of tinequal quantities of parcels in the bays has increased the number of handing by .07 per parcel, or an extra $7 \%$ handing overall.

This assumed, nowever, thet all of the parcels were sortec at the one dept and then sent to their fincl destinations in a Eully sorted conuition. 'his cannot be the case, in the majority of lepots, ciue to lack of space and facilities, therefore c?ose attention nust be paid to the geograpinical region of the destintion. The parcels would be partly sorted and then sent "rough' sorted to another zone centre for final sorting in that area.

This will have perticular effect on the destinations with low volunes cif parcels, as the largest parcel volumes are sorted out at the first pass, and the allotnent of destinations in the middle rances can be re-arwange to suit regions ruite easily. Although this vill mean that destinations of approximately equal volumes will no longer be coubined, the nev groupinf tay improve the variance of ti:e number of parcels per bay after the primary sort, to give a closer figure to the averace.

For tie 103 destinations of small volume trafic, it should be pecsible to collect 10 groups of 10 destinations each, with'n the sarte railway regicn. If necessary, the 10 destimacions per bay need not be strictly acherred to. For example, if there are 12 destinations that can be formarced togettins, and another group of 0 , then this will be better fox fontharding than sorting out 2 grours of 10 . These groups can then be combined with the traffic for a main zone centre, and fonvarued there for final sortinf. In adcition, any small destination chat may have traffic corwarded to the sorting cepot in question, as a zone cencre, mest be sortec out fncivicuajly, in any event.

The Eollowing tailie (Tab.2) is an anenced version of the above one giving the code numbes for D. F. zore centres and shoving how destinations could be comined recorapizically by regions.

Table 3 shows a brealicomn of destinations for bay $L$ on tie secoid sort. (This shovs mumers of parcels only, the cestination codes being owitted.)

Tav．2．

| Bay at 1st sort | Zone centre code wnber and parcels per zone aezion cent：e |  |
| :---: | :---: | :---: |
|  |  |  |
| A | E． P ． | 150／2851 |
| E | L． C 。 | 162／2217 |
| C | E． C 。 | 166／5403 |
| D | E。瓦。 | 154／1112 |
| E | E．${ }^{\text {P．}}$ | 171／862； $156 / 753$ ；153／693； $161 / 575 ; 165 / 523$ |
|  |  | 151／442； $152 / 402 ; 164 / 373 ; 155 / 347$ ；100／301； |
|  |  | 149／235； $309 / 152$ |
| T | E．R． | 103／135； $139 / 125 ; 100 / 82$ ；140／70 ；113／52； |
|  |  | 115／62；104／62； |
|  | IT． 2. | 173／205；101／105；197／206； $174 / 80 ; 100 / 74 ; 177 / 67$ ； |
| 6 | S．R． | $221 / 321 ; 200 / 293 ; 225 / 273 ; 202 / 200 ; 212 / 200 ; 226 / 170 ;$ |
|  |  | 210／134；204／120；227／95；223／70；220／63 |
| E | \％．E． | $\begin{aligned} & 306 / 255 ; 312 / 207 ; 300 / 144 ; 316 / 110 ; 301 / 105 ; 303 / 95 ; \\ & 323 / 92 ; 330 / 70 ; 314 / 70 ; 310 / 66 ; \end{aligned}$ |
|  | Sc．T． | 66／138；81／127；34／140 |
| J | I．M．R． | 23／399； $56 / 293 ; 24 / 193 ; 13 / 141 ; 49 / 109 ; 52 / 96$ ； |
|  |  | 17／95； $13 / 90 ; 15 / 85 ; 34 / 85 ; 9 / 75 ; 35 / 70$ |
| 2． | L．IV．R． | $6 / 405 ; 72 / 311 ; 57 / 220 ; 27 / 164 ; 55 / 134 ; 54 / 123 ;$ |
|  |  | 30／98；33／57；60i心6；7／90；3／92 |
| L | The remaincer；on the second sort they coulc ic sorted to the following eroups（number of parcels only）：－ |  |
|  |  |  |

Table 3.

| I | II | III | IV | V | VI | VII | VIII | IK | \％ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8． P ． | 13．E． | W．E． | W．1． | W．2． | S． 12 | LRP | LMR | LMR | Sc． P |
| 35 | 16 | 45 | 6 | 35 | 40 | 5 | 10 | 20 | 50 |
| 46 | 30 | 20 | 40 | 15 | 20 | 25 | 41 | 50 | 40 |
| 10 | 10 | 2.3 | 25 | 30 | 55 | 50 | 25 | 80 | 5 |
| 8 | 10 | 5 | 30 | 60 | 1 | 10 | 35 | 33 | 5 |
| 40 | 5 | 20 | 10 | 30 | 40 | 35 | 17 | 42 | 20 |
| 30 | 4.5 | 5 | 5 | 40 | 55 | 30 | 13 | 55 | 5 |
| 48 | Sc．il | 5 | 15 | 15 | 17 | 45 | 40 | 25 | 10 |
| 32 | 35 | 45 | 1 | 20 |  | 5 | 20 | 43 | 30 |
| 11 | 45 | 45 | 3 |  |  | 24 | 17 | 46 | 90 |
| 22 | 5 | 30 |  |  |  | 25 | 60 | 46 | 5 |
| 42 | 35 | 50 |  |  |  | 50 |  |  |  |
| 17 | 30 |  |  |  |  |  |  |  |  |

If each＇bay＇used takes the form of 1 DRUTL，then there will still bo a nocd to sort further for some of the regions， vhere destinations in that region may be far apart．Gowever， if a＂bay＂consists of a nuriber of ERUNES，placed togetier， traffic to be forvarded to different zone centres in the same region can be placed in different RRUTRS，allowing the parcels to be forwarded via the most convenient zone centre．

Hore than one BRUTE will be required at each bay, at the sane time, but, on the other hand, there will be less need to "eplace full BRUTLS vith cmpty ones.

Wach sorting cot will have ifferent quantities of parcels for each other depot. Lilevise, diay to day quanticies will vary at each depot, but over a period of time, a representative number of parcels per copot per day, can be obtained and this Figure can be used for cetermining the allocation of sorting Days. Attention should be paid to these figures, which should be antended to keep up vitia aitering trends.

It has been assuned throughout this section that the parcels receivec for sorting have been in a completely mixed collection. However, this is not strictly accurate,parcels received for transhipment at the lazger depots will already havo been passed through a prinary sort elsevhere. This means that the uncertainty, or encropy of a part of the collection has aready been reduced elsewhere, as the possible destinaticns for this group has been greatly reduced.

This vill tend to cancel out the effect which the unequal nubers of parcels in each bay vill have on the reduction in total entropy after one pass.

## Manual Sorting

A great many parcels carried by hritish hail are sorted by hand. A gang of mien, sorting a coilection of parcels, can either work individually, eacin men collecting, sorting and ?oading on his own, or two or more can work as a tean, one selecting the parcel and recaing the destinetion, the otcer taking it to the receiving vehicle. There are three basic types of sort a parcel can receive - it can be the first sort of traffic collected at the ciepot to the area of its destination; it can be a parcel received from a depot for delivery; o: it can be artranchipmentrparcel, which has been forvarded from another area, to be sent on to a third depot.

When the parcels are collected, they pass through a primary sort, either directly to its destination, or more commenly, to a ciestination area. These parcels are thea loaded into appropriate railway wazons and forvarded. At the destination depot, the wagons are unloaded and the parcels resorted, either to final cartage posts, or if caanshipmenc traffic, to the final cestination zone centre.

The parecls ate often rehandled several times in the course of trensit, by a large number of men. Nany of these men simply cancel out the worl: done by eaviar men, mainly because sorting and Loadnis invoives the use of barous, which cause unnecessary loading and unjoading. Parcels offloaded from barrows accumulate on platform while awaitins the arrival of a train. As it takes $40-50$ minutes to fully load a van with parcels, trains tend to leave lightiy-loaded, to clean flatforn space and keep to timetables. In addition, seroxazation of pazcels for different destinations within a ciagle van is difficult and long delays sometimes occur at intemediate stations, for unioeding and loading parcels. This zeans that soue vans are detached from trains wit: less parcels than vould aormally justify this action.

To case this problen, a form of unit load has heen developed. At first, singie palleto vere tried, but these vore not successiti, as the dimensions of the pallets wera not compatible with those of the vans, and, vith the end-forces subjected to vans when shunted, uniess the packages were strapped to the pallets, or the vagon was fully-loaded, damage could occur to the goocs. Boz pallets vould have been able to deal vith tile problem, but they were heavy, imobile and, theoretically,
wasted a lot of space in tie rail van, compared with filling the van to caracity with parceis. ovever, it was found aster investigation that on averase, parcel vans vere underfilleci.

Tiere were some rovtes where the vans were filied to capacity, but acequate segregation of the parcels vas impossible, consequentiy unnecessary handling resulted.

This resulted in a wheeled, box container being developed. This was m now as the iritish Railvays Universal Troliey Equipment (B.R.U.T.E.). It hes two fixed wheels towards the rear and wo swivelling castor wheels at the front. These wheels are offset, to provicie an inierent anount of "toe-in", which helps prevent 'snaling', when being towed, a well known charactoristic of the cld type placform barrov trains.

Another aid to good tracining was found by having the draw bar at the rear of the trolley, instead of the front, and having it sprine-ioaded, so that it extended when the trolleys were traversing a curve and pulled back afterwards. This resuits in vefy accurate tracking of a train of BRUTEs, each one following exactly in the path of the tractor. Another impoitant purpose of the spring draw-bars, is during the initial tractive effort, enabling the traccor to take up the load one trolley at a time; this can result in the use of a smaller tractor.

To avoic the rehaniling occurring during transit, the parcels should je loaded into widutes at the carliest opportumity, and laft there to the final destination. The trolleys can be Ioaded onto parcel trains with the maximum speed and minimum labour. $\quad 10$ bRUTi's can be loaded onto a CUV van in five minutes, compared with the full hour to load it with indivicual parcels.

The DRUTH are best used for total movenert of a group of parcels; hovever, they can be used instead of the oid-type barrows for storage and transport around depots. There intemediate stations do not have the facilities for hanciling DRUTEs, ihe parcels can be initially sorted into BNUTEs, loaded onto trains directly from inUTEs into vans and unloaced manually at the other end. Alternatively, when quantity for one cestination is consistentiy sufficient to fill a van, then Brjurs need not be used, and the van can be detached at the receiving station. This is particularly useful for terminal depots.

Ax present a number of routes are solcly for BRUTL trains. A train, consisting of tea CUV vans, each capable of holding 18 3RUTFs runs a set route, depositing and collecting BRUTEs from stations on the route. A good example of this is the East Anglia BRUTE train, from Peterborough East to Liverpool Street. The train travels around Lest Angiia twice a day, each van laving a set number of BBUTEs for one or more of the fifteen stations. The train can be loaded at the originating depot some tine jefore departure, in a slack period, allowing the staff to deal with cther traffic, as the need requires.

At the moment, unfortunately, there is still a lot of traffic not hancled in BRUTEs. However, with the increasing use of DRUTEs, hanciling of parceis will be reduced, resulting iñ a speedier service, with less chance of damage and confidence in parcels arriving at the correct destinations. The following section deals with parcel-sorting from a Work Study view-point, comparing the work involved in manual sorting to cifferent sorts, and also with mechanical sorting. As the time to 'rouch-ioad' a parcels van varies consicicrably, all of the sorts wiil be into BRUTES.

As the theoretical sorting developed is not restricted to MRUTEs, or even to British Rail parcels, the parcels vill be sorted into what will be termed 'bays'; these could conceivably be LRUTEs, rail vans or farcol baç.

Sechanical Sorting of Parcels

In addition to manual sorting, there are a number of sorting machines available. Although these machines differ in design anc methoci of operation, they are all of the same basic concept. The parcels pass, on a corveyor of one form or anotner, a sorting point at which ar operator records the westination of tae parcel in the memory system of the machine. The parcel. then moves along the conveyor, until this destinetion is reached, whence the tarcel is removed from the conveyor. As the actual sorting process is still camied out by a iman operator, this method is more accurately described as an autonatic conveying of sorted parcels to their receiving bays.

Sorting machines provide a faster and nore efficient method of sorting, with, when a sufficient number of parcels is sorted, a reciuction in labous costs. Efficiency is increased because the operator remains at the sorting point, reading the destination code and keying this into the memory. After
this all of the sorting is carried out automatically, with little chance of a vrong sort. As the operator does not have to move away from the coding point, the speed of sorting is increased, all or the carrying of the parcel through this sort being done on conveyors, the speed of which cait be controlied. If the sorting operator has a loading assistant, who places the parcels with the code number showing in the correct position, and who can also call out the code number, then peak periods of traffic can be soried rapiclly and correctiy.

The machine delivers sorted parcels to one of a number of sorting bays, where the parcels are accumiated, awaiting formation into unit loais, such as BRUTLS. To sort completely by mechanical means, hovever, will incur very high cost, due to the large number of discharge points and long length of convejors required. Of all the possible destinations B.R. parcels are liable to be forvarded to, $75 \%$ receive less than 100 parcels per day each. This means that a mechanical/manual comined system could be better used. The primary sort would use the sorting machine, followed by a manual secondary sort. This would resuit in a system similar to the theoreitical manual system. The destinations with a larce number of parcels, would have separate discharge points, whilst the remainder are sorted from the discharge beys into BRUTEs, for their respective destinations, grouped arouri the discharge bays.

Alternatively, traffic for other transhipment depots could be stowed in a number of BRUTDS, without thorough sorting. This would allow traffic for destinations within the same transhipment depot, and with low quantities, to be combined for tramsporting convenientiy.

For a lazge depot, handing a high throughput of traffic, another method would be to have a 'rough' preliminary sort before passing traffic to the sorting wachine for a main sort for the destinations witnin this sort.

As the sorting is carried out at hifh speed, the sorting can be bettcr planned to fully utilise the facilities and labour of the depot. Staff will be able to undertake other work on the same shift. Providea the demand is cufficiently great, received traffic for dolivery could be sorted on the mechine, into final cartgac posts, thus enabling more time to be spent on making out delivery notes and better loading of the delivery vehicles, than if the trafsic had to be sorted manualiy as well.

Labour would still be required to load the parce1s from the discharge chutes into parcels vans or unit loads, even if no further sortiny was needec. However, these stowing operators wili have a choice of package shape and size from the accumulation of parcels, to assist them in the loading of the JRUT, , etc to its fullest capacity. The distance that the parcels are carried, also, will be reduced to a inimun, as will the choice of destinations, although there will still nave to be a certain amount of package recognition by the operators.

The discharge chutes will act as temporary reservoirs, proviled they are large enough, so that once a chute has been cleared, the operator can move on to another, reducing to a miniman his waiting tiae. This will also wean that other operators can be moved to a certain set of chutes, to prepare all of the traffic to meet particular trains, thus ensuring the prompt dispatch of traffic and a decrease in vaiting-time and 'under-loading' of through parcel trains.

With mechanicai sorting, a higher numbers of sorts can be carried out in the priwary sort than would be used in a manual sort. Other wort has indicated that 40 discharge points i.s the maximum an operator can key to efficiently and aithough this numer of discharges reduces the number of seconary sorts, it means a fair increase in the capital cost of the equipment. It will also require a larger area for the surting equipment. Unlike manual sorting, there is lietice difference in sorting rates for up to 40 sorts, as all carrying of the parcel is done autonatically, the number of parcels passing the operator being constant. Thus the number of discharge chutes chosen will be dependent on cost of equipmeat area availeble and anount of primary sorting requized.

## Tyjes of Sorting ifachine

There arn several types of sortine machine, all based on the use of a conveyor of one form or another.
a) Tilting Tray

The Dagshave "Tilt-Tray" sorter consists of a series of closely spaced trays, mounted between axialiy disposed pivots, on
carriages which travel arounc a fixed track. Each tray is held in a horizontal position on a cam plate, until the destination point is reached, when an external rolier is raised causing the tiay to tilt.

Parcels are placed on the trays at an induction point, Where the destination for each tray is selected on a key-boand. Fhis destination is fed into a logic memory unit, following the tray, rouric the memory as the tray moves along the track, until the selected destination is reachec, at wich point the discharge arm is raised, tilting the tray, so that the parcel slides off. Discharge can be to either side of the track and, if necessary, 'double tray' keying can be added, so that two trays carrying a long parcel can be tilted simultaneousiy.

The destinations should be spaced at a minimun of $1 \frac{1}{2}$ tray lengths, the numer of discharge stations being limited only by the lengtl: of the sorter, however, the cost of the control ecuipnent may rise sharply with increasing length.

The control aquipment used inciudes an analogue memory unit, constructed from static state switching elements in tie form of a sinift register.

Photo-electric cells or other devices can be placed across the mouth of each discharge chute to warn of any sull chutes and to prevent any more parcels being placed in it. This will mean that an extra chute must be supplied at the end of the run to collect any such parcels, wiich will then nave to be resorted, or alternaitively a revizculating fomat ve used, although this will mean that should the chutes remain funl, one or more of the trays will be used as moving storage. Therefore, close attention muct be paid to the loading of che chutes, anc either anple storage space or rapid removal of the parcels should be availabic.

The Tilt-iray system can le used as a re-cisculating loop or as a sinpic over-and-under circuit. If the latter syoten is used, there can be no re-circulation of any parcels, but the seace required is quite narrow for area. where there is some restriction in vidth. With the loop metnod several cifferent gnttems can be used. lith only one loading point and discharge to one side of the conveyor only, the wicth of the sorter will be doubled, but the lengtin halved. If the wicth between the two
paths is increasel, disciarges can be made to both sides. Alternatively, one sorting nachine can be used for two operations, if tho loading points are included, one each enc of the loop, then one si.we can deal with the sorting of outcoing parcels, whilst the other handles inconing parcels, sortinf then to the separate cartage posts. However, the sorting operator on the latter position would have to be very experienced and have a lot o. local knovledge, until aduress cories becone standard throughout the courtry. iNo matter which of the ainove systems were used, a small proportion of the traffic would have to be handled nanually due to excess length.

The sorting macine operates at speeds up to $250 \mathrm{ft} / \mathrm{min}$, which, assuing four foot trays, means handling approximately 3,750 parceis per hour under maximum sorting conditions. The speed can ie adjusted to eive a steady flow of work.

Tilt-tray installations at work in parcel and freifht depots in the USA have been reported as bringing about a saving in operating costs equal to the capital outlay, over a period of $3-4$ years. Tris, however, meens maning iull utilisation of the equimmat.

## b) Mineing Slat

The Sortrac ITI tiltingslat sorting conveyor is similar to the Tilt-Tray in operation. Instead of trays, this conveyor consists of a serics of $\mathcal{S}^{\prime \prime \prime}$ wide mild steel slats, mounted on centre pivots and moved by a chain-conveyor. the slats are held in a horizontal position by stabilising rollers which run in a central track. The slats can be tilted in either cirection by switching the stabilising rollers to a subsidiary tracli. The track can only be supplied witin the return stand under the sorting strand of tiee conveyor.

The conveyor is scemed by a ploto-electric bean at the incuc*ion point, so thet all of the slats under a particular parcol are tilced at the discharge point. Decause the sjats tilt as they apuroach the discharge point, the change in dinection is jradual, allowing a virtually shoci-iree discharge from tiee conveyor.

The control systen and coding operation are practically identical to chat of the Tilt Tray systen. But, whereas
the lengtl of parcels on the trays is limited by the length of the teays themselves, the parcels sorted on Sortrac III can be up to of long and $3^{\prime}$ square.

Unike tie Tilt Tray, this system is not capable of negotiating comers and is therefore limited to a single flow direction. It is also slover with a maximum conveyor opeed of $200 \mathrm{ft} / \mathrm{min}$. Discharges can be arranged at a mininum pitch of 2 ft on each side of the conveyor, but it is seldon possible to make use of this, particularly with larger parcels.
c) Diverters

This method of automated sorting is different in operation to the above two. It consists of a number of arns which sweep across a flat conveyou, diverting parcels off into discharge chutes. To aid diversion, a vertically mounted belt conveyor is mountec around the diverting arm, moving in the direction of the discharge.

Diverters can operate on any flat conveyor, band or slat, provided that it is sufficiontly tough to withstand the sicievays movenents of the parcels. The diverters can be either single or double acting, i.e. they sweep to one or both siues of the conveyor. Hovever, they have limitations in use; the packages must ve capable of absorbinco the finpace of hitting the diverting arm and must also be of sufficient height to prevent passing or sticking uncer the am (aprox. ${ }^{\prime \prime}$ gap.)

It is also slover in operation than the other methods, as the atm has to swing across before the parcel reaches the discharge chute, and swing back before the next parcel arrives. If the parcels are too close together, there is a risk of striking anc trapping a parcel praceuing the parcel for that destination point. There is aiso a restriction on throughput caused by the naximun lencths of parcels that can echaded by the arms. The maximua speci of the conveyor is about $150 \mathrm{ft} / \mathrm{min}$.
c) Tilteu Banc

His system is based on a conventional belt or band conveyor, which is tilted to an ancle of about $30^{\circ}$ after passing the
coding point. Along the lover edre of the conveyor is a sinallow wall with a number of gates included in it. The parcel destination is coced in the norwal way, via a keyboard, into the mewory unit, the signal generated following the parcei along the conveyor uncil the appropriate gate is reached, thien the latter opens, depositing the parcel into a discharge chute. This method is used by the G.P.O. for nechanical sorting of their parcel post.

It is not suitable for B.R. parcels, however, as the parcels handled by then are larger and heavier. The sidewall rust be fairly low, to avoid cxcess friction. This means that large or irreçular parcels are liable to tip over the wall, with risk of camage, as well as the necessity for re-handing. The gates should also be at least the length of the larger parceis and preferabiy $1 \frac{1}{2}$ times larger.

Thus for parcels up to $3^{\prime}$ in length, the gates should ise at least $4 \frac{1}{2}$ ' lorg. Alternatively tyo flaps could be opened simultanecusiy, but with increasing cost in control and operating mechanism.

Hovever, as tisis method is not suitable for B.R. Parcels, it will not be considered further.

Costs for Autoratic Sorting iachines (for equivalent capacity)

Filting Tray

| Basic cost | 6100 |
| :--- | ---: |
| Per foot run | 36 |
| Induction stations | 1300 ea. |
| Destinations | 380 ea. |

Tilting siat
Drive 1150
Tension 650
Slat Tzack + Aproi $\quad 45$ per foot
and supports
Dischargens
ifemory system
Supply and erect
£
6100
1300 ea.
380 єа.

130 ea 100 per ft.

40-50 per foot of conveyor

Diverters

| Basic conveyor, | Lelt | 14 per ft |
| :--- | :--- | :--- |
|  | Slat | 20 per ft |
| Iemory systerl |  | 40 per ft. |
| Sinsle Diverters |  | 225 eac.) |
| Double Diverters |  | 350 eac.) |



A TYPICAL MECHANICAL SORTING LAYOUT
7. ADALYCIS CT RESULSS

In this section the applicetion of Infommation Fracry to Qarcel Sorting will be dealt vith in more detail. Also, comparisons will be mace between sorting methods mentioned in earlifor sections.

In the section on lifomation hecry, it was found that the entropy of a coliection of unsorted parcels could be found Erom the lomula:-

$$
\mathbb{E}=-\frac{L}{L} \frac{n_{2}}{i} \log \left(\frac{n i}{i}\right)
$$

$$
(i=1,2 \ldots . \ldots)
$$

This figure for entropy, in units cepencent on the base of the Jogarithne used, indicated the amount of sorting that vas required on the collection, to get them into a fully soreon exuftion, This means that an increase in entropy is accompanied by an increase in the araont of sorting requizec.

If, for the sake of analysis, fe assume that the number of poosible jestinations, $D$, is a constant, then the entropy muct vary with it, the cotal number of parcels, or $n_{i}$, the nuber of parcels per destination. If te also assume that $n_{i}$, renains a constant roportion, recardless of it, then an increase in in will produce an increase in $r_{i}$, in proportion. Thac $\frac{n}{i v} w i=1$ remain constant, i.e. there will ne change in entropy. Therefore the entropy of a coliection is not dependont on the bubier in thet collection, if the Fropozcion of nareels per Gestination remains constaut.

Tue eatropy must thoncfore be dependent on $\mathrm{n}_{2}$. Tine formia user to detezminc the entrory is such that $E$ is a m ximura and equal to $\log x$, when all of the probabilities ${ }^{n}$ i are equal, i.e. all $r_{i}$ are equal. As the variation between these probabilities thereases, the value of E will decrease, for a given nuber of cicotinations and constant total number, until all the $\frac{n,}{n}$ but one are zero, at which point $\mathrm{E}=0$.

Therefore, if a sorting destinationchas a numer of dopots which receive an equal guanticy of parcels, the entropy, and hence the anount of sorting required, is greuter than that of a duetinetion wisich sends a lorge number to one or two depots anc smailer numbers to the others.

The recuctica in entropy after one sort is also found. If the number of parceis per bay is the same, this reduction will be equal to the logarithe of the number of bays. In the number per bay is equat, then the reduction in atroy vili bu recuces. İovever, proviced that reasonable care is taken in ualanciab the numbers, the error wili not be too great In the practical example of wnual sortim, the reduction in entropy on dividing equally to the 12 bays should have been log $12=1.00$. Fcr the first sorting pettern designeu, i.e. cotal sorting, the zeduction of entropy after the first sort vas calculateu as 1.01 , an error of $6.5 \%$.

If the number of coscinations is increased, then $\mathbb{E}$ is also increased, as, when all provailities are equal and there are D possible destimetions, $\mathrm{z}=\log \mathrm{D}$. Therefore to reduce the entropy of a collection of parcels at a depot, the number of destirations to be sorted to should be reduced. This, in fact, is what is intended for D.R. and G.P.O. trafic in the acar future, when cortain large depote will becone area centres for freight and parcels. Zach depot will handle on incraased anount of transhipnent troffic, but with mechanised soreing , anc ctandard unit load transport, i.e. BRUTE trolleys Fhis should be the most efficient way of caling with long distance parcels. Smalier dopots should be mainly concerned with local traffic; traffic for other areas being sent to the arca sorting dopot, possibly after a simple primary sort.

Comparison of methods.

The work Study investigation showci that the theoreticai sorting speed for namal corting into 12-15 sozts, is approaniztely 400 parceia por hour. in allorance of $10 \%$ shoula be made for romal wonking rate, which reduces the speed to 360 parcels per hour. A furthe: $20 \%$ for a Rest Allonance will breng ti is dow to 208 patcels per hour. A deduction mill also have to be nede for different sizes of parcele, handuriting on labels, coding checks etc., if this is answed to be $10 \%$ then the final speed for sorting would be 260 parcels pei hour, per man. However, these figures were obtained on the assumption that the staff wore working fuil-time on parcel-sortirg alone. Dut they
also heve to load traias, change BRUTE trolleys, and alter sorting iayouts for different types of traffic. Tais vill probably roduce the overall sorting speed to about 200 parcels per incur.

In addition to tlis, a labour allowance thest be made for sichess, holidays ecc. and also to cope with peak periods that nay occur, reçuiring an extra $22 \frac{1}{2} \%$ labour force.

The approsimate iabour cost of on war io cproze E1000 p.a.; thus a
 Gatount of lebour requirud to be colcriated, and therefore the labour costs of sorcing to be found. The quantity of parcels handied per ciay vill be the number of parcels received for sorting per day, multiplied by the entropy over the lof of the number of bays, assuning optinun sorting is used, a.d all of the parceis are fully sorted. Otherwise the number of handings should be less than this, and obtained from tables similar to chose in the section on theoretical handing.

To compare mechanical with manual systems, the cost of the mechonicai system, with its associated labour costs, should be calculated.

The maintenance, runaing and deprociation of the mechanical equiprent is asswed to be $15 \%$ of the total capital cost, per year, the accounting life of the equiprant being taken as 10 years. Ir a cost comparison is made between marval handing for a certain number efparcels, and mechanical handing for the sane number, then the difference between the tro costs will be the savings expected by use of mechamical equipment.

If a cash flow is then drama up, shorine the initial outiay, abainst the expected recurns, the expected return on investnent can be Sound.

The coots outained for the mechanical equipment are only on approzinate figure, dram $u$ as a tentative guice to allor comparicons to be made. As the mechanical equipnent will be used solely for a printary soxt, labour vill be requized to sort the farcels into secondary sorts, the numer of sorters requirec iependine upon the number of parceis per day and the amount of sorting required.

Adiitionai lobour vill also be needed for loading the machine and for the prinary sorting operation.

Of the three types of nechanical equiphent cosiciered:-
a) The Diverter system is the chapost; however it has a number of disacvantages, it is the slovest in speed and there ace also linitetions of size, veifht and strengen of the pachage. inerefore, it vill no 'se considered further.
b) Whe Tilting Elat sycten has the highest capital cost; however, it is varscitic, as the slats can support parce?s of varying lengths, whereas tray length is caturally limiteci. It is restrictec though, by the fact thet it cannot turi corncers, thus layout is restricted to a single run of the conveyor.
c) The Tile-tray syetem appears to have a slightly lower cafital cost and has the ailility to travel round corners, thus different layouts can io considared as circumstances cemand. Intra labour will be required, however, to hande long parcels that cannot be carried on the trays.

Fo: comparison it will be assumed that the entropy of tie collection is 1.0 and that there is three-shift working. The converons will de of 100 ft. operating length and have one inuuction station anc 15 cestinations, cach destination being a secnciary sort into 10 mevins.

Anowing the nomber of men zecuired allows the labour cost per annun to be calculated for manual sorting. The costs for the tuechanical equipnent are:-
a) Poic Pray

| Basic Cost | 6100 |
| :--- | ---: |
| pez ft. Tun | 7200 |
| Incuction stacions | 1300 |
| Destinations | $\underline{5700}$ |
| Total Cost | 20,300 |

- Amual cost for running and maintenance $=£ 1,013$.
be nanually sorted from the disclarge chute. If, however, a discharge chute tues provided for each one or two cartage posts, the loading oparation could be carried out more efficiently, alhough the keying cperator would have to have some local movledje, to know which traffic was assioned to which cartage post. These additional destinations would mean that one of them could be taken over, if a breakdown occurred on one of the other chutes.

If the costs for varying numbers of parceis per day are obtained and dram up on a cost/volune chart, the relative costs can be corfared, and an idea of the comparative costs for a specific parcels volume can be indicated.

Mancal Sortins

| $\begin{aligned} & \text { Parcels/ } \\ & \text { day } \end{aligned}$ | /hr | ```Parcel hancling/ hr Zntropy = 1.0``` | iien re por shift | quired <br> per <br> day | $\begin{aligned} + & 22 \frac{1}{2} \% \\ & \text { labour } \\ & \text { allowance } \end{aligned}$ | Cost p.a. at E 1000 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10,000 | 410 | 750 | 3.75 | 11.25 | 14 | 14,000 |
| 15,000 | 625 | 1125 | 5.62 | 16.9 | 21 | 21,000 |
| 20,000 | 832 | 1500 | 7.50 | 22.5 | 28 | 28,000 |
| 25,000 | 1040 | 1875 | 9.37 | 23 | 35 | 35,000 |
| 30,000 | 1250 | 2250 | 11.25 | 33.0 | 42 | 42,000 |
| 35,000 | 1.460 | 2630 | 13.15 | 34.5 | 49 | 49,000 |
| 40,000 | 1006 | 3000 | 15.0 | 45.0 | 56 | 56,000 |
| 45,000 | 1070 | 3400 | 17.0 | 51.0 | 63 | 63,000 |
| 50,000 | 2000 | 3750 | 16.7 | 56.1 | 70 | 70,000 |

Avomatic Soucing, Labour


| 10,000 | 410 | 2.00 | 3 | 5 | 15 | 15,000 |
| :--- | ---: | :--- | :---: | :---: | :---: | :---: |
| 15,000 | 625 | 3.13 | 4 | 6 | 10 | 16,000 |
| 20,000 | 032 | 4.10 | 5 | 7 | 21 | 21,000 |
| 25,000 | 1040 | 5.2 | 6.4 | 3.4 | 25 | 25,000 |
| 30,000 | 1250 | 6.25 | 7.7 | 9.7 | 29 | 29,000 |
| 35,000 | 1460 | 7.3 | 9.0 | 11 | 33 | 33,000 |
| 40,000 | 1665 | 6.33 | 10.2 | 12.2 | 37 | 37,000 |
| 45,000 | 1670 | 9.35 | 11.5 | 13.5 | 41 | 41,000 |
| 50,000 | 2080 | 10.4 | 13.0 | 15 | 45 | 45,000 |

In audition to the labour cost, the following costs will have to be aducd:-

$$
\begin{array}{r}
\text { Tilt-May }-£ 1,013 \\
\text { Tilting Siat }-£ 1,680
\end{array}
$$

Costing Enercise on Tilt-Tray Sorting Hachine

Whis exercise uses the tencative costs provided by B.R.B. for a Tilt Tray sorting machine, to give an iclea of the expected retums on investment that would be achieved with this ecquipment. For the purpose of the exercise the following data has been assumed:-
a) 23,000 parcels per day are kanded;
b) the machine has a working length of 100 ft . witin one induction point anci fifteen discharce points, giving a copical cost of $\{20,300$;
c) the expected life of the machine is five years, with three shift. working; also, an anticipatcd scrap value of 23,000 is expected

The cost for ruming, maintenance and depreciation was given as $15 \%$ of the empital cost. lowever, the D.C.F. techinique used avoics the need to consider cepreciation as a seperate Eactor, a D.C.F. rate of return of $10 \%$ implies that the project will earn 10\% on the Sunds investea in the equipment as well as repey original suns invested over life of equipment. The account mey life of this type of equipment is taken as 10 years, with straight line dopreciation, and no resicual value, the cepreciation, therefore is $10 \%$ per annum. Thus, the ruming and maintenance of the equipment, the actual cash payments are $5 \%$ p.a.

As the equipment does not qualify $f 0 x$ an investmont Grant, there will be an initial allowance of $30 \%$ of the capital cost plus an Anwal Allowance of $25 \%$ of the Resicual value, for tax purposes, charges at a race of $42 \%$ Corporation Tax.

There may be a Residual Value after the five years, if the equipment is sola for more than this, a balancing charge will have to be made on the difference. If it is sold for less a balaucing allowance will be made. It is assuried that the tais is paid one year later than the profit is made.

| Year | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (a) Cash Outlay | $(20,300)$ | - | - | - | - | - | - |
| (b) Cash Returas (Profit over menual systen) | - | 10,000 | 10,000 | 10,000 | 10,000 | 10,000 | - |
| (c) Rumiling costs | - | $(1,000)$ | $(1,000)$ | ( 1,000 ) | $(1,000)$ | ( 1,000 ) |  |
| (c) Hect cash in (b-c) | - | 9,000 | 9,000 | 9,000 | 5,000 | 9,000 | - |
| (e) Capital Allowance | - | 11,500 | 2,2.0 | 1,550 | 1,260 | 345 | - |
| (E) Rosicual Value (Capital value allow- ance) | - | 8,300 | 6,600 | 5,050 | 3,790 | 2,845 | sold <br> for <br> 3,000 |
| (c) Tamable returns ( $\mathrm{d}-\mathrm{e}$ ) | - | (2,500) | 6,000 | 7,450 | 7,760 | $8,055($ | Balance charge) |
| (h) Comp.Tax @ $42 \frac{1}{2} \%$ <br> (1 year cielay) | - | - | - | (2,000) | $(3,100)$ | (3,290) | (3,490 |
| (i) AEter taz casi return ( $\mathrm{d}-\mathrm{h}$ ) | - | 9,000 | 9,000 | 6,110 | 5,840 | 5,710 | (3,4:0) |
| (j) 20\% factor | - | . 833 | . 694 | . 579 | . 482 | . 402 | . 335 |
| (k) iet prenent value (jzi) | - | 7,500 | 6,250 | 3,540 | 2,820 | 2,290 | (1170) |

(Figures in brackets are negative quantities)

This recults in an ll.P.V. cash return of 221,230 for a capital outlay u! 220,300 , with a zetum on investinent of $20 \%$ pius an awitional profit of just uncier $\mathrm{E} 1,000$. This ic for 25,000 parcels per day, with a hicher rate of parcels, the profit over the manual system would be nigher, resulting in a higher expected rate of retum.

The first conclusion tinat can be arrived at is that mbonation Theory ean be applicd to parcel sorting. Dne of the vasic concepts of Information heory is that of Entrory, or the uncertainty of receiving the required nessage from a collection of possible messages. A collection of msortec parcels for a number of ciestinations, has an uncertainty connected with selecting the percel for a specific destimation. Thus, as these uncertainties cen be regarded as comarable, the concepts of one can be applied to the other, Within innitations, dependent upon rae similarity of the two. The ecncept of entropy, rierefore, can be applied to parcel sorting. This will Give the awount of uncercainty in the coliection a finite value, thus emabling comparisons to be made becween various coliections, and also giving an indication of the azount of sorting required to sort tie parce?s.

If an optinua sorting wechod is used, by getting equal quantities of porcels iato the optimum muber of bays, then the relationsing between the entropy and the number of bays vill enabie the anount of sortins per parcel to be found. The optimum muber of bays is arrived at by comparing soztivg spceds into different numbers oz bays, an allowance beint nacie Eor the 'effectiveness' on recluced nuwber of sorting passes needed, of the larger number of bays. If these quantities are used as a guide for a more practical systen, and this system foliovs the theoretical system as closely as is conveniently possible, then the theoretical anount of sorting required is a good indication of the actual sorting need, the lateer being slighty laygen,

The above is applicaile when the parcels are fuliy sorted at one depot. If, hovever, they are onily fartly sortei, the logarithn of the number of beys indiates the reduction in entropy for each sort. Thus inewing the original entropy, tio entiopy or the semi-sorted collection can be Ëcurc.

It was shown in the test that as the total number of forcels was increasce, the entropy aitered only if the rroportion of the perceis to each destination altered. This means that the entropy is not related to the total number of parcels, but to the numer of destinations and
the number of parcels to eacin destination. It vas seen tlat as the number of cestinations increased, then so diu the entropy. Lilevise, as the probabilities of parcels per destination becane more equal the entropy increase, until a maximu was reached when all of the cestinations vere equi-probaile. This is intuitively the rost uncertain rosition, as all the parcels have to pass through all of the sorts. The entropy will be zero wien the probabilities for all ciestinations but one are zero, as no sortinfo need be carried out, no uncertainty caisting.

Because the entropy is dependent upon the number of destinations and the proportion of the traffic sent to each, then to reduce $i t$, and thus the sorting required, either one or bota of these factors should be reauced. If traffic at a ciepot ic passed through a prinaiy sort, the trafific for other areas or regions then being forwarced directiy to the areas, whereas traffic for the area in thich the lopot is situated is passed through a secomany sort, this will reduce tise number of destinations sorted to, and hence the entropy for the parcel collection of that depot. finis will necessitate large depots in each area receiving more trafic than it would nomally, from a numer of cifferert areas. Tut vith efficient sorting arici concentration on the area covered by the cepot, the handling siould be improves. Care must be taken in selecting these area centre depots, to onsure they are of sufficient size to hancle the trafific and placed in an accessible position, geograpincally.

This will mean that alternative couing to that at present in use should ve providec. At the monent each zone centre depot has its unique code nwaber, depots in the saide region having similar code numbers. At present, ticugt, these codes are not alvays referred to, parcels beinf referred to by destination address rather than by coce number, aisc nany anencments take place to the zone contros for certain toms as stations and lines are closed, anc these amendenes are often not kept up to date.

With the theoretical sorting, the distribution of parcels to each bay vill vary Eon cepot to depot, thus to get even quantities, a complex nunber of codes would have to be renembered. Thus nev codes should be drawn up for each Cenot, to enable parcel sorters to sort at highest speeds, vitiout having to reifer to an aduress, or remember a colylea collection of nubers. The old codes should still
be kept, as once a parcel has left the originating depot or area, tiis nuv cole wili ve neanincless; thus the two codes should be of different forms.

The theoretical sorting awiysis shows that when ail factors are taken into consideration, there is not nuch difference in the cffective sorving speeds. These factors include the spech of sorting one sort into various bays, anc the effect that the extra re-handiac required has mon parceis are sorced into a smoll number of bays. However, there axe peaks in effective sorting speed at 15 ance 30 thys. The layout for 30 bays rervires twice the layout area of 15 bays, ane there vill also be dieficultics in equating quantities of jarceis anu growping destinations. This means that 15 bays vould prowuce the nost cffective sorting, when apeed is the most important consideration, but, as is pointed out, di三!icuities would arise in obtaining optimal sorting g after the first sort, and a slower speed, with slightly more sorting per parcei, would give a sorting pattern closer to hie oftirum. It shouid be emhasised though, that tinis oniy occurs lue to the rumber of eestinations, 176. Jith a different number of destinations, either less or nore, then other patterns, probably using 15 sorts shoule De uset.

Then the sorting pattern is draw up, close attention must be paid to the destination areas. Only at the largest depots can traffic be fully sorted, therefore for the remaincer a propertion of the traffic will be sent only 'part-sortec'. T'ine can be wasted in transport and reiunding if parcels are sent to the wrong area zone centre.

To keep segregation of parcels on route, tiey should be kopt in unit loads, the suitish Railyays universal Trolley Equipment (aRURE) Eang iceal for this. URUPEs are steadily being brought into use, but faster in sone reşions than others. finis incens thet finter-region traffic has to tako into account the facilities of the different regions. It wili ie a waste of time and esfort is traftic js fonvarced from one region to another in BRUTLs, if tie receiving resion canot hanule then. The imprescion gained from sortinc staff using wimit trolleys at present, vas a very favourable one. They are very much easier to hancile arowa depote than the old-type flat baraots, and a parceis van that voula require an hour to load 'rough' takes about five minutes to loal with 10 IRUTEs. this
nems tiat BNUTES EOE different destinations can be lowe into one van, without fear of 'un-sorting' the parcals in transit. Mon a iestination is reached, the DiUTLS recuirea for that station are removed from the von, vithout having to detaci the van, reducing vaiting tine to a nininum. This leads to the conclusion that as much traffic as possiole be formarded in DRUTSS. This is being cone by J.i. to a limited extent with their BROTL trains. iline vans of a train of ten are filied Wen Dhris for destinetions on a set route, possibly circuiar, the cther van being filied with arymard parceis that will not convenienty fit in BKUTEs (e.g. rolled carpets.

Therefore, if oftimum inual soring is usor, as completely as possible, straigint into zidums, an efficient parcel sortius system should be coveloped. Once in a pariicular Whut, the parcels could remain in it for as long as posciole.

Sorting Haclines are avaikavle to assist in sorting, provided that sufficient gunti气jas of parcels are sorted pe:. cay. Nnis speeds un the process by eliminating the inbour require. to transport parcels between the Eirst and seconc sozts, this aiso allows a recuction in labour costs. The costs for low quantities per day are higher than for menual sorting, but as the quantity increases the savinus in labour increase, The eanmple in the analysis hes swom that for a total of 25,000 parcel.s per day, with the costs proviciec, there will be a retum on investnent of approwinately 20\% For a life of five years. The sorting time-iable can be planned more efficiorty, soxting being carried out in quicter poriods, tinus allouinc the sorting staff to concentrete on the rapil loaing of parcel trains, when roguireci.

The thooretical sorting mothot can also be aplited to mecinnical sorting. is tie seconcary sort is a manual one, tron the disciarge chuted, thece are different ininitations tivet vill apuly to this method. On the prinary cort, the afferent speecis of sorting to differing nubers of bays vill ie amost ne ligible, as the convoyor rurs at coustant speou, the operator remaining stationary. An increasca number of prinary corts rill increase the cose of the equipment and also the area required, but it wili decreasc the amome of sorting required by the secondary sort. Some parcels vill still need to be
conbined Efor transport to other areas, due to the low quantities per cestination per day.

In manual sorting, tive lias also to be spent sortind out parcels for delivery to their cartage posts. Vïth mechancal sorting, extra aischarge points could be supriied for tinis type of traffic, ailowing the delivery staff to spen more time on loading the vehicles eificiently, and in the maling out of delivery notes.

The nunder of manal stafs at the uischarge chutes should be a function of the nuriver of parcels handled, rather than of the numbe of bays, tie ivays activg as terporary stcrage rescrvoirs. Tach operator having a set muber of beys to lock atter, the operators being concentrated on ceatain bays wen the need axises, e.g. for loadine trains etc. This vould mean that an increased munber of bays wouid iave to be deait tith, with a larger primary sort, although tie total cuantity of parcels will be the same, therefore sone account must be paid to this vhen detemining the labour raquirer, as there will be a limit on the bays one man can look after.

If a fairly low number of bays is usec, and one becomes wasable, there being no altemative bay, problems vill arise. Acditional bays shoulc be suppliec for this anc any other contingencies that may arise (e.g. Full chute, illogibie akrose).

In conclusion, therefore mechanical sorting systems should bo used for doalinj witi cuantitics of parceis over a set minimun level, detomined by comparing the cost of ruming the ojstem comparec with a purely manual methoc. Whis wost ficure vill waty with the entropy of the collection: the higher the entropy, the more chance there will be of arioying sortinc machines. A chart is provided comparine (Fis. 2 ) necianical with moneal sorting, sucving on arporimate break-aven point, an alco che point for an expected return on investnent of $20 \%$ using a D.O.F. emthod. It must be pointed wut, though, that the costs supplied were only centative ani tuerefore a more accurate investigation of coots, machine life, etc., must be carried out before an accurate breek-even point is obtained. It is hoped hovever, that the gures surplied vill provide a reasonable fuice so expected costs and returns.
P. MPLMAD
E. EDEGMOS.
$\therefore$ Ghwomir

P。PiLELTPS
Z.S. SCamparz
C.E. SLALKON

Cos. CHERES (W)

SALES DROCHUTUS
'Sechanical Landling'February,1964
'Information Transuission' Dublisheci Chapman and Mail (1964)
'A Simple Introdection to Capital
Expenditure Decisions' pub.Pitman(10C7)
'Mechenical Handling' Maxch 1966
'Principles of Codiņ̧, Filtering and Information Theory' pub. Spartan Press jaltir:ore (1963)
'The Ilathematical Theory of Communication' Urbana: University of Illinois press (1949)

(To be pubiishod shortly)

Sortrac Ltd. Eristol
Bacshame : Co. J.td., Dunstable.

## APPGITIK I

The probabizity, $r_{i}$, of a parcel for the ith destination $D$ being selected from the collection will ne $\frac{r_{i}}{}$, where $n_{i}$ is the number
of parcels for the ith destination and is in total number of parcels izi tia collection.

The entropy of the collection, per parce, will be:-

$$
\begin{aligned}
2 & =-\sum_{i=1}^{D} p_{i} \log P_{i} ; p_{i}=\frac{n_{i}}{i i} \\
& =-\sum \frac{n_{i}}{N} \log \frac{n_{i}}{N} \\
& =-\sum \frac{n_{i}}{N}\left\{\log n_{i}-\log i f\right\} \\
& =\sum \frac{n_{i}}{n} \log n-\sum \sum_{i}^{n_{i}} \log n_{i}
\end{aligned}
$$

but is is constant and $n_{i}=$ is

$$
\begin{aligned}
\therefore E & =\frac{1}{i n} \log N \cdot \sum n_{i}-\frac{1}{i} \sum n_{i} \log n_{i} \\
& =\log \left[-\frac{1}{i n} \sum n_{i} \log _{10}{ }^{1} i\right.
\end{aligned}
$$

The cotel meropy on an uncorted group of if parcels will be:-

$$
i 1=i \log n-\sum n_{i} 10_{0}^{0} n_{i}
$$

If the parcels are corted into $F$ baye, with $n_{j}$ parcels in eaci bay tay $\begin{aligned} & \text { then, entropy fer bay }= \\ & n_{j} \\ & \log _{5} n_{j}-\sum_{j} \quad n_{i} \log n_{i}\end{aligned}$
where $D_{j}=$ cicst ${ }^{n}$ per bay.

- Total entropy after $i^{\text {et }}$ sort, $\operatorname{ME}^{\prime}=\sum_{j=1}^{T} n_{j} \log n_{j}-\sum_{i}^{D} n_{i} \log n_{i}$
where in' is new level of entropy and sumation is over all destinations.

Trie recuction in total entropy $=$ ive - 淢'

$$
\begin{aligned}
& =\left(N \log \pi-\sum_{i}^{D} n_{i} \log n_{i}\right)-\left(\sum n_{j} \log n_{j}-\sum_{i}^{D} n_{i} \operatorname{Iog} n_{i}\right) \\
& =I \operatorname{Iog} N-\frac{N}{\sum} n_{j} \log n_{j} \\
& i i_{i}=\stackrel{F}{\Sigma n_{j}}
\end{aligned}
$$

$$
\begin{aligned}
& =\frac{F}{\Sigma}\left(n_{j} \log 1 T-n_{j} \log n_{j}\right) \\
& =\sum \mathrm{n}_{j} \log \frac{2}{\mathrm{r}_{j}}
\end{aligned}
$$

- . Neduction in entropy per parcel $=E^{\prime}-E^{\prime}=\frac{1}{N} \sum n_{j} \log \frac{i^{T}}{n_{j}}$,
for any number, $n_{j}$, of parcels per bay.

If we assume that the parcel.s are divided equally into each bay, then, with $I$ bays,

$$
\begin{gathered}
n_{j}=\frac{N}{E} \\
E-H^{\prime}=\frac{1}{i j} \sum_{j} n_{j} \log \frac{N}{n_{j}}
\end{gathered}
$$

but $r_{j} \log \frac{1 i}{n_{j}}$ is constant and equals $\frac{\pi}{F} \log$

- $L-E^{\prime}=\frac{I}{i} \quad \frac{F}{\Sigma} \frac{H}{T} \log \vec{i}$

$$
=\log 10^{\mathrm{F}}
$$



For equal quantities of parcels per bay, after one sort the entropy vill be reduced by log $F$. (the number of bays).

The wits of eatropy are depencent on the base of the logarithms. If logs to the base of 2 are useci, then the units will be binary digits, or "bits", if to the base 10 , then, in decimal digits. As lof̃ $_{2} 10=3.32,2$ decinai igit $=3.32$ bits.

Ascume that the quantity of parcels in each bay is the same:Ghange in entropy, $\mathrm{E}-\mathrm{L}^{\prime}=10 \mathrm{~F}$.

This is the rectuction of eitropy in one pass, - Dif $^{\prime \prime}=0$, tion no further sorving is nequired. if $I^{\prime}{ }^{\prime}>$, then furtier sorting is required, and $E=\log E \div E^{\prime}$ $\frac{E}{\operatorname{or} \log T}=I+\frac{E^{2}}{\log T}$

After a further sort,

$$
E^{\prime}-E^{\prime \prime}=\log F \quad, \text { assunins same } n_{0}^{0} \text { of bays. }
$$

is $\mathbb{E}^{\prime \prime} \leqslant 0$, no Eurther sorting is required if $i^{\prime \prime}>0$, Ewher souting is required.

- $\frac{E^{8}}{\log F}=1+\frac{2^{\prime \prime}}{20_{3} F}$
or $\quad \frac{\mathrm{N}}{\log \pi}=2+\frac{2^{n}}{\log T}$

Thus the amount of further sortine reguired is detemined by $\mathrm{E}^{2}$ and log F .

one further sort is needed,
and, $\mathrm{E}^{\prime \prime}-\mathrm{E}^{\prime \prime}=0$ or a fully sorted coljection.
$\therefore \frac{\mathbb{E}}{10 \mathrm{E}} \mathrm{F}=3$, the nubiber of sorts carried out.
if $\frac{\mathrm{B}^{n}}{\log \mathrm{~F}}>1$, then Eurther sortine will be required
if $\frac{\mathbb{L}^{\prime \prime}}{10 马 Y}<1$, cinen only a proportion will be neeued to be sorted.
IE, for $\frac{\mathbb{E}^{\prime \prime}}{\log F}<1$, $\log \mathrm{F}$ is reduced, so that $\frac{E^{\prime \prime}}{(\log F)^{\prime}}=1$,
theriz $Z^{\prime \prime \prime}=0$, and no further sorting is required. To obtain
$\frac{\mathbb{E}^{\prime \prime}}{(\log \mathbb{P})^{\prime}}=1, \log F$ must be mitiplied by $\frac{E^{\prime \prime}}{10 G^{\prime}} \cdot \cdots \frac{Z^{\prime \prime}}{10 g F}$
is the proportion of parcels to be re-sorted. Alternatively, each parcel muse be sorted an extra $\frac{E^{\prime \prime}}{\operatorname{leg} F}$ times.

This results in $\frac{\text { Entropy }}{100 \mathrm{~F}}=$ number of sorts required.

Then the units of entropy and the base of the log are the same, and ontinun sorting is used.

For exampe:-

$$
\begin{aligned}
& \text { 1) Say, entropy of } 2.0 \text { and } F=10 \text { bays } \\
& \text { lst soit E - E' = log } \mathrm{F} \\
& \begin{aligned}
2-E^{\prime} & =1 \\
E^{\prime} & =I
\end{aligned} \\
& \text { 2ack sort } \quad \mathrm{H}^{\prime}-\mathrm{E}^{\prime \prime}=1 \mathrm{os} \mathrm{~F} \\
& \begin{aligned}
1-\mathrm{E}^{\prime \prime} & =1 \\
\mathrm{E}^{\prime \prime} & =0
\end{aligned} \\
& \text {. fully sorted in exactly two sorts. } \\
& \text { 2) entropy of } 2.0 \text { and } F=25 \text { bays } \\
& \text { lse sore } \mathrm{E}-\mathbb{E}^{9}=\log \mathrm{T} \\
& \begin{aligned}
2-\mathrm{E}^{\prime} & =1.4 \\
\mathrm{E}^{\prime} & =0.6
\end{aligned} \\
& \frac{\mathbb{L}^{8}}{\log 25}=.43, \quad \therefore \text { only } 43 \% \text { of the parcels } \quad \text { need to be sorted a seconc time } \\
& \frac{E}{10 Z E}=I+\frac{U^{\prime}}{10 G T}=1.43, \begin{array}{l}
\text { the } n \text {. of sorts per parcel } \\
\text { required. }
\end{array} \\
& \left\{\text { or. } \frac{E}{\log \pi}=\frac{2.0}{1.4}=1.43\right\}
\end{aligned}
$$

