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ADVANCED SCHOOL OF AUTOMOBILE ENGINEERING

Automobile Axle Accelerations

Statistical Analysis of Field Measurements

- by -

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Summary

Accelerations produced at the stub-axles of a vehicle during passage over a road surface are a prominent source of vibrations of the vehicle. This memo describes the instrumentation of a motor car to record these accelerations, and the statistical analysis of records obtained, in terms of Amplitude Distribution, Power Spectral Density Function and Auto-correlation Function. Cross-correlation functions between some of the acceleration signals are included and their salient features discussed, together with the utility of the other statistical descriptions. Suggestions are made for laboratory work to increase the information to be gained from such measurements and the generality with which it can be applied.

Introduction

The non-linear nature of vehicle suspension systems, tyres and body structures results in some difficulty and confusion in the definition of dynamic response characteristics from the results of sinusoidal testing. The investigation here described was carried out to ascertain to what extent the theory of random noise analysis can be applied to vibration problems in road vehicles. Instruments were carried out on a vehicle to measure and record the accelerations of various parts of the vehicle when driven along a road or track. The recorded signals were then analysed in the Laboratory to obtain descriptions of the accelerations in terms of frequency and amplitude. Auto- and cross-correlation functions also were computed, and their significance is discussed.

Vehicle Instrumentation

The vehicle used for all the tests was a Ford Zephyr V6, as described in Appendix 1. Accelerations were measured on the suspension units, close to the wheel hubs. In the first test only the two hubs on the near-side of the car were used, the accelerations of each being measured in 3 mutually perpendicular directions, viz. longitudinally, laterally and vertically. In the second test, accelerometers were fitted to all four hubs to measure vertical accelerations only.

The accelerometers used were of the variable inductance type, and a 3 kc/s. carrier/amplifier system was used.

The signals were recorded on an Elliott 14-track magnetic tape recorder, frequency-modulation being used to enable d.c. and low-frequency signals to be reproduced.

Data Analysis

This was carried out on a Noratom ISAC (Instrument for Statistical Analogue Computation), using samples of signals recorded on a loop of magnetic tape.

Amplitude Domain: Amplitude Distribution Function is computed according to the equation:

$$P(x) = P\ y(t) < x$$

$P(x)$ is the probability that the instantaneous value of $y(t)$ is less than (or equal to) the amplitude level x. Integration is performed on the time for which signal value is less than x, over the sample time, or loop length. The function is plotted automatically as the level of x is increased incrementally between each integration.

Frequency Domain: The Power Spectral Density or P.S.D. Function is computed according to:

$$Y(f) \simeq \frac{1}{BT} \int_{0}^{T} x_B^{2}(t)dt$$

where x_B is the signal passed by a band-pass filter of bandwidth B Hz centered on frequency f, and integration is performed over the sample duration T secs. In practice the centre frequency f is swept at a constant rate, and R-C averaging is used instead of pure integration. For the computation frequency range of 0-200 Hz, two bandwidths are available, 6 Hz from an L-C filter which can be narrowed to 1.5 Hz by the addition of a tuning-fork filter. Analysis of lower frequencies is improved by recording on the tape loop at lower speeds, thus increasing actual sample duration, and computing at the normal speed. The frequency change involved results in a reduction of the effective filter band width relative to the original signal. (e.g. for analysis over the range 0-25 Hz, the increase in sample duration is by a factor of $\frac{200}{25}$ and the filter bandwidths become $\frac{6x25}{200}$ and $\frac{1.5x25}{200}$).

Time Domain: Auto-correlation and cross-correlation Functions are plotted automatically, according to:

$$R_{xx}(\tau) = \frac{1}{T} \int_{0}^{T} x(t)x(t+\tau)dt$$

$$R_{xy}(\tau) = \frac{1}{T} \int_{0}^{T} x(t)y(t+\tau)dt$$

Integration is perforsed over the duration of signal contained by the tape loop. The delay τ is increased incrementally between integrations. It is obtained by recording on two magnetic head stacks and then moving one to a different position on playback. The time delay is the product of tape speed and head stack displacements.

Uncertainty of Computed Results: Uncertainty always exists in the computed results of statistical analysis. The basic uncertainty is caused by using a finite sample to describe a signal, or set of signals, hypothesized to have fixed statistical parameters for all time, i.e. infinite duration. The confidence placed in results can be further reduced by shortcomings of the computing techniques used, e.g. use of R-C averaging and swept frequency analysis in estimation of P.S.D. Functions.

Confidence Bands: are calculated which, for signals with normal or Gaussian distribution, define the range within which 68% of the computed p.s.d. function must lie, i.e. for a computed p.s.d. function, 68% of the actual function represented lies within the defined band on either side of the computed function. For this 68% confidence band, computed function lies between:

$$\pm \epsilon = \pm\sqrt{\frac{1}{BT}}$$

of actual function. B = bandwidth of band-pass filter in Hz, T = integrating time, at best T = length of sample.

Considering analysis range of 0-200 Hz, B = 1.5 Hz and T = 5 sec.

$$\therefore \qquad \epsilon = \sqrt{\frac{1}{1.5 \times 5}} \simeq 0.36$$

If the sweep rate is no more than one filter bandwidth per length of sample (i.e. per passage of loop) then it has negligible effect on the uncertainty. The additional uncertainty due to RC averaging can also be neglected when the RC time constant is no less than the sample duration.

For correlation functions, BT is now the product of the total bandwidth and sample duration, so

$$\epsilon = \sqrt{\frac{1}{200 \times 5}} \simeq 0.032$$

Field (on the road) Measurements

Records were all made under conditions of constant vehicle speed. This simplifies interpretation of the analyses by restricting signals to be nearly stationary.

The conditions were as follows:

First tests (2 wheels only):

Motorway 50 m.p.h.
Unrepaired road (perimeter track) 30 m.p.h.
Unrepaired road 50 m.p.h.

Second tests (4 wheels):

Motorway 50 m.p.h.
Poor country road 50 m.p.h.
Unrepaired surface 30 m.p.h.
Unrepaired surface 50 m.p.h.

Record lengths varied from 30 seconds to 3 or 4 minutes.

Analyses

First Tests

Table 1 shows r.m.s. levels of the outputs of all 6 accelerometers. Table 2 shows the Crest Factors of all the records obtained, defined by:

$$\text{Crest Factor} = \frac{\text{Peak value}}{\text{r.m.s. value}}$$

and in this case the Peak value is approximated by that value exceeded for only 4% of the time (i.e. the 96% confidence band). A Gaussian signal would have a value of 2.0 for such a Crest Factor, and so some numerical value is put to the shape of the amplitude distribution.

Figures 3 to 8 show samples of the computed results in all 3 domains for two accelerometers for the conditions at 50 m.p.h. over the unrepaired surface.

Subsidiary Test: Stationarity and Ergodicity

The results so far were all computed from single samples, and are only representative if the statistical parameters of the particular surfaces do not vary appreciably, either along the length of road used for recording or across it. In the first case samples taken from different points along the record might yield different results (stationarity check). In the second case variation might be obtained for samples from corresponding sections of different records (ergodicity check). A subsidiary test was conducted to check the stationarity and ergodicity, in which three records were made for each set of test conditions. Although statistically inadequate, three samples provide a qualitative indication of the properties in question and results can reasonably be plotted on the same sheet of paper. The results for the front vertical accelerometer at 30 m.p.h. over the unrepaired surface are typical:

From the auto-correlation functions, the range for the mean squared acceleration values varied:

a) along single record, ± 37%
b) across ensemble of records, ± 14%

Figures 9 to 14 show computed functions for each case of P.S.D., Auto-correlation and Amplitude Distribution. Cross-correlation functions (Figs. 15 and 16) show a larger uncertainty. It was decided that for subsequent tests, particularly when cross-correlation was involved, it would be preferable to take averages across a set of at least three records. For the duration of the single sample of a record, non-stationary effects are neglected.

Three records provide a fairly reliable average for the rough surface accelerations, but not for smooth (motorway) surface accelerations. This is thought to stem from the fact that in the latter case the inputs are largely due to rotational imbalance in vehicle components. These cause near periodic accelerations with slowly varying amplitude and phase. Figures 18 and 19 show averaged auto-correlation functions of this type. The results for a single run are far more stable than those for a set of runs. This means that each run is less representative of the complete ensemble of possible runs than in the rough surface case. Therefore averaging must be carried out over a large number of records. Note that the P.S.D. Functions (e.g. Fig. 17) appear more stable than the correlation functions, due to the different method it forms for presenting the same information. The cross-correlation functions, for which phase variations must be considered, have been omitted since no meaning can be assigned to the average of three results as obtained.

Second Tests

The results of the second tests fall into two groups:

1) P.S.D. Functions and r.m.s. levels for vertical accelerations of all 4 hubs for the set of different road surfaces and speeds: Table 3 and figures 20 to 23 (not averaged).

2) Correlation of vertical accelerations for all 4 hubs at two speeds over unrepaired surface only (averages of 3 records): Tables 4 and 5 and figures 24 to 27.

Discussion of Results

The results of the above analyses serve as classifications of the accelerations signals recorded. On the stub-axles of the automobile used in these tests, r.m.s. accelerations of between 0.07g and 0.50g were measured. As the Crest Factors and distribution curves for the first tests show, the amplitude distributions deviate considerably from Gaussian so that the r.m.s. level or the mean squared value is not sufficient description of the accelerations in the amplitude domain. The significance of the form of the distribution has yet to be defined. It is usually assumed that passage of the disturbance through further elements of the mechanical system renders its distribution more Gaussian. This ignores the effect on the local elements, which are distributed in a non-Gaussian fashion, and assumes linearity of the rest of the system.

Further classification of an individual signal involves either the Auto-correlation function or the P.S.D. Function. These functions are a Fourier Transform pair and contain the same information. For simple signals, e.g. Figs. 3 and 4, the auto-correlation function can provide a classification as simple as that of the p.s.d. function (Bendat, Ref. 1). As the description of the signal becomes more difficult, so does interpretation of the auto-correlation function. Two and sometimes three distinct frequencies can be discovered from the auto-correlation function (Figs. 6 and 7) and estimates made of their band width and power content. Beyond this, interpretation of the P.S.D. function is usually more straightforward, although numerical description (e.g. natural frequencies and bandwidths) is no more compact.

Frequencies of dominant vibration modes of the system, such as wheel hop, body pitch and bounce can be observed in the P.S.D. functions.

The significance of non-stationarity of input, as would be obtained during acceleration and braking or from changes of road surface, has yet to be defined. This, and the importance of the amplitude distribution mentioned above, may be ascertained by laboratory testing.

For analysis of smooth road acceleration records, digital methods are required to compute average statistics for sufficiently large ensembles of records.

The second set of tests provides information for laboratory simulation of the vertical accelerations of all 4 wheels of the car as encountered when running over each of several road surfaces. The individual acceleration signals are classified by r.m.s. level and power spectral density functions only, as given in the analysis results.

Since front and rear wheels run over almost the same ground, there is expected to be a large maximum coefficient of cross-correlation between acceleration signals. This should occur for the delay corresponding to the time of passage of the vehicle over a distance equal to its wheel-base at the measured constant speed. This can be clearly seen in Tables 4 and 5 and Fig. 24. It is of interest to determine whether there is any correlation between the road inputs to opposite sides of the vehicle. Assuming any wide undulations are crossed at right angles, there should be some positive correlation at zero delay. If such undulations are crossed at any other angle, the corresponding correlation peak would occur for some finite delay. For the first of these conditions, the cross-correlation between diagonally opposite inputs will exhibit a maximum at the same delay as that for the cross-correlation between two wheels on the same track. This would be of reduced amplitude (Fig. 25).

The cross-correlation analyses are complicated by the presence of transmission paths between hubs existing in the vehicle itself. Laboratory tests using single wheel inputs, and multiple inputs with known correlation, are required to fully define the effective functions and correlation coefficients of these paths. Then enough will be known to extract all the information from cross-correlations of the acceleration signals considered here.

Conclusions

Statistical parameters describing the accelerations of the stub-axles of a motor car have been computed from time-history records. These records were obtained when the vehicle was driven at constant speed over three different types of road. The adequacy of description by mean square value and p.s.d. function only have been considered, and laboratory work suggested to investigate the importance of the assumptions of record stationarity and of Gaussian distribution.

Sufficient information has been obtained to enable simulation of vertical inputs using a multiple-exciter system to be made. Data concerning the required correlation between each pair of the multiple inputs cannot be extracted from the field measurements until some representative tests have been conducted in the laboratory. It then remains necessary to determine the effects of such correlation on parameters of the vehicle's vibratory response, in a like manner to experiments concerning amplitude distribution and stationarity.

Acknowledgements

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References

1. Bendat, J.S. — Principles and Applications of Random Noise Theory, Wiley, 1958.

Appendix I: Instrumentation

Details of Vehicle

Make	Ford
Model	Zephyr V-6
Type	Saloon
Weight	26 cwt.
Wheel-base	9 ft. 7 ins.
Engine	Front
Drive	Rear
Suspension	Independent on all four wheels

Details of Accelerometers

Make	Electro-Mechanisms
Type	AEM
Principle	Variable inductance
Range	± 6g. and ± 9g.

Frequency response - see typical curves - Fig. 2.
Carrier Frequency 3 kc/s
Carrier System S.E. Laboratories
Master Oscillator : Type SE 511
Amplifier-Demodulators : Types SE 423/1, 449/1

Tape Recorder

Elliott Series T.P.4.
Tape width : 1 inch
No. of channels : 14
Speed used : $3^3/_4$ inches/sec.
Recording channels : F.M.11 ; Direct 1
Playback channels : F.M.3 ; Direct 1
Frequency response (± 3 db) : F.M.: d.c. to 1.25 Kc/s.; Direct 50 c/s to 12 Kc/s.

Figures

Table I Hub Accelerations (r.m.s. 'g')

Accelerometer (Nearside)	Motorway 50 m.p.h.	Unrepaired Road 50 mph	Unrepaired Road 30 mph
Front vertical	0.22	0.44	0.37
Front lateral	0.10	0.22	0.19
Front longitudinal	0.07	0.24	0.24
Rear vertical	0.20	0.39	0.32
Rear lateral	0.10	0.23	0.20
Rear longitudinal	0.28	0.52	0.50

Table II Crest Factors (defined in text)

Accelerometer (Nearside)	Motorway 50 m.p.h.	Unrepaired Road 50 mph	Unrepaired Road 30 mph
Front vertical	2.0	1.21	1.33
Front lateral	1.7	1.42	1.50
Front longitudinal	3.6	4.0	1.97
Rear vertical	1.5	1.25	1.70
Rear lateral	1.5	1.26	1.45
Rear longitudinal	2.9	2.0	1.65

Table III

Vertical accelerations (r.m.s. 'g')

Location	o/s front	o/s rear	n/s front	n/s rear
Test condition				
Motorway 50 mph	0.10	0.19	0.15	0.14
Country road 50 mph	0.18	0.24	0.36	0.32
Unrepaired surface 50 mph	0.26	0.26	0.39	0.32
Unrepaired surface 30 mph	0.19	0.21	0.30	0.26

Table IV

Correlation coefficients (max.) 30 m.p.h. Unrepaired surface

Channel A	Channel B	C_c max.	T.m. sec.
o/s front	o/s rear	+ 0.577	254
o/s front	n/s front	+ 0.250	0
o/s front	n/s rear	+ 0.271	245
o/s rear	n/s front	+ 0.188	-254
o/s rear	n/s rear	+ 0.178	0
n/s front	n/s rear	+ 0.523	242

Table V

Correlation coefficients (max.) 50 m.p.h. Unrepaired surface

Channel A	Channel B	C_c max	T.m.sec.
o/s front	o/s rear	+ 0.46	+ 156
o/s front	n/s front	+ 0.18	0
o/s front	n/s rear	+ 0.22	+ 153
o/s rear	n/s front	+ 0.22	- 159
o/s rear	n/s rear	+ 0.24	0
n/s front	n/s rear	+ 0.74	+ 159

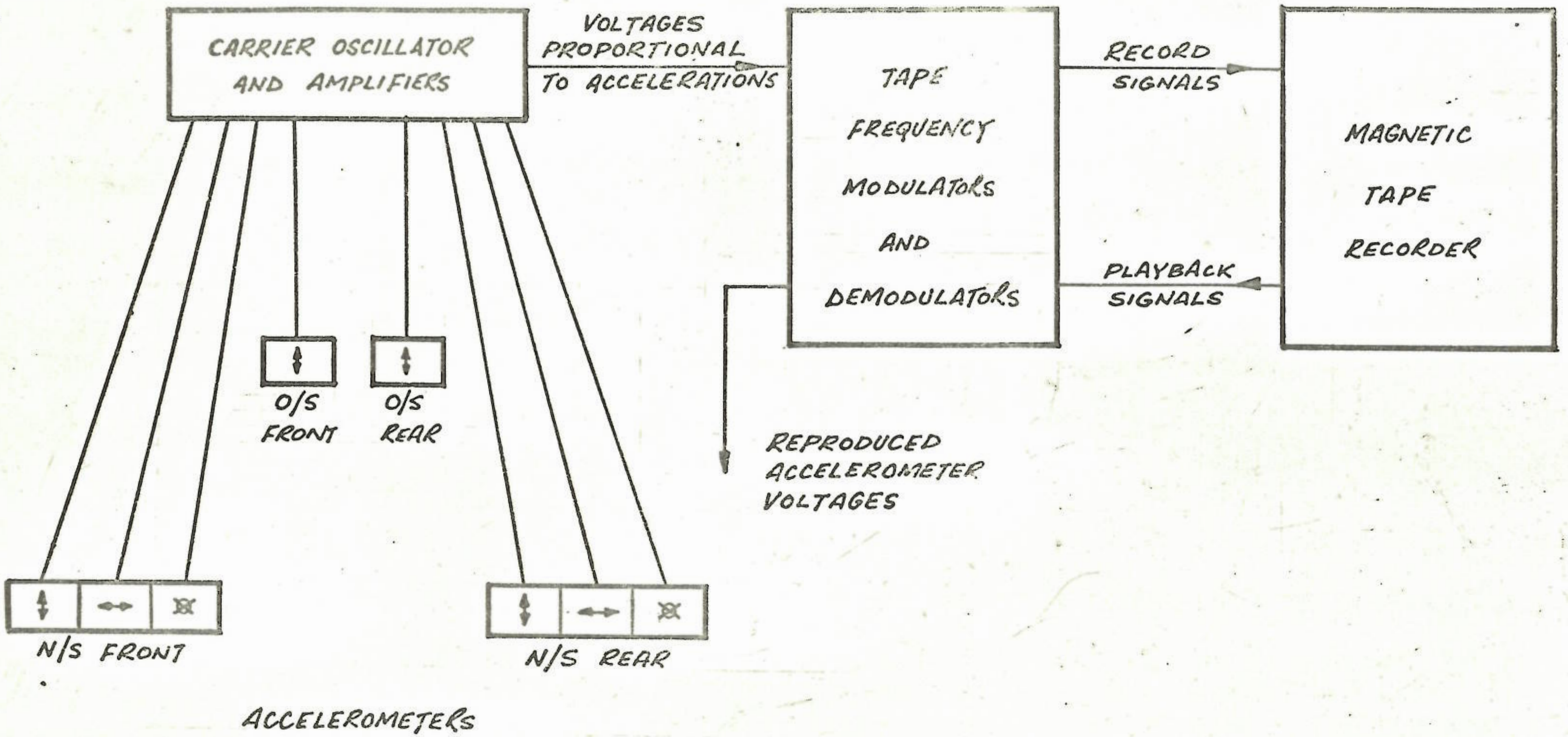

FIG. I BLOCK DIAGRAM OF VEHICLE INSTRUMENTATION SYSTEM.

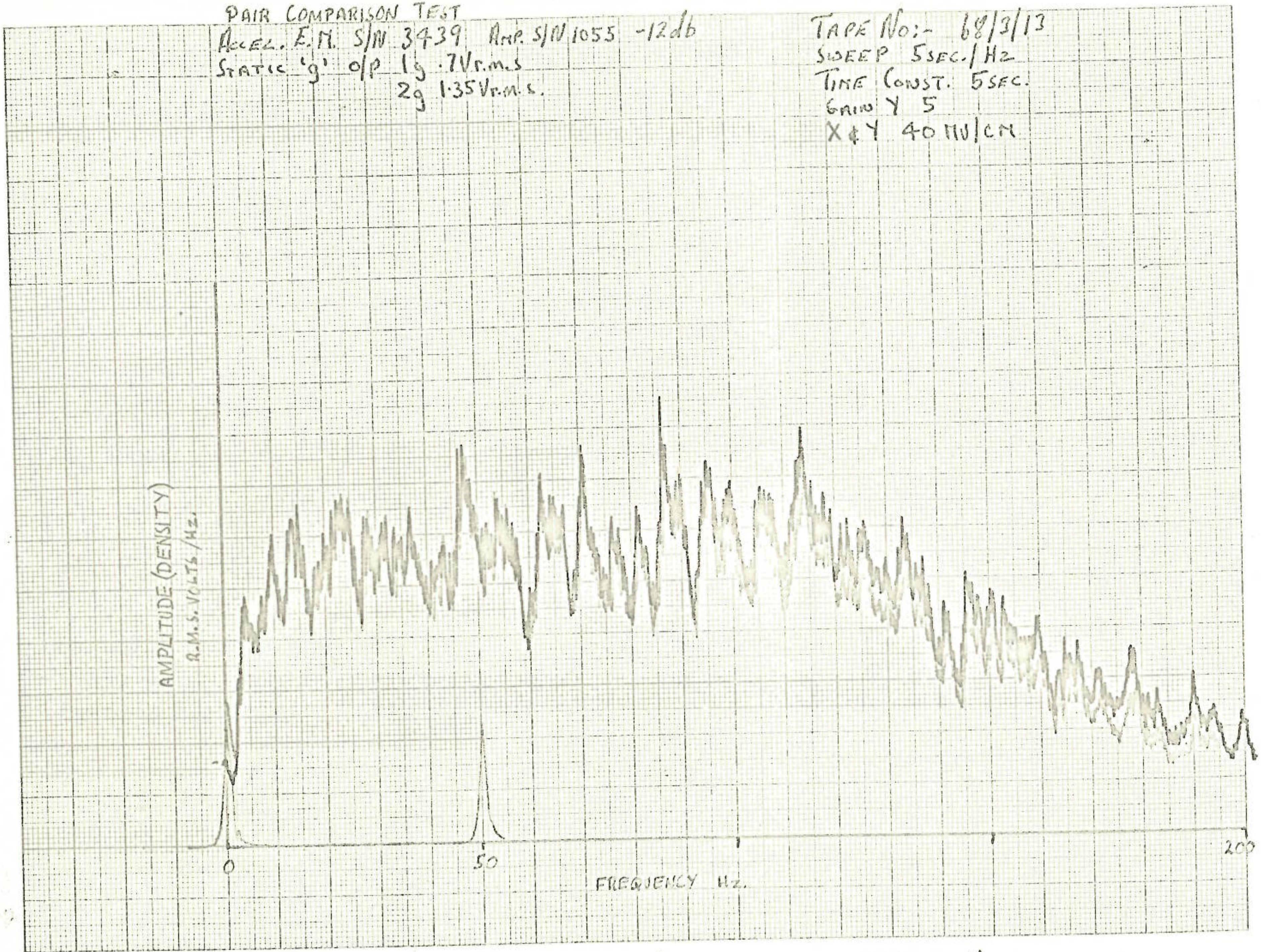

FIG. 2. ACCELEROMETER FREQUENCY RESPONSE (TO BAND-LIMITED WHITE NOISE)

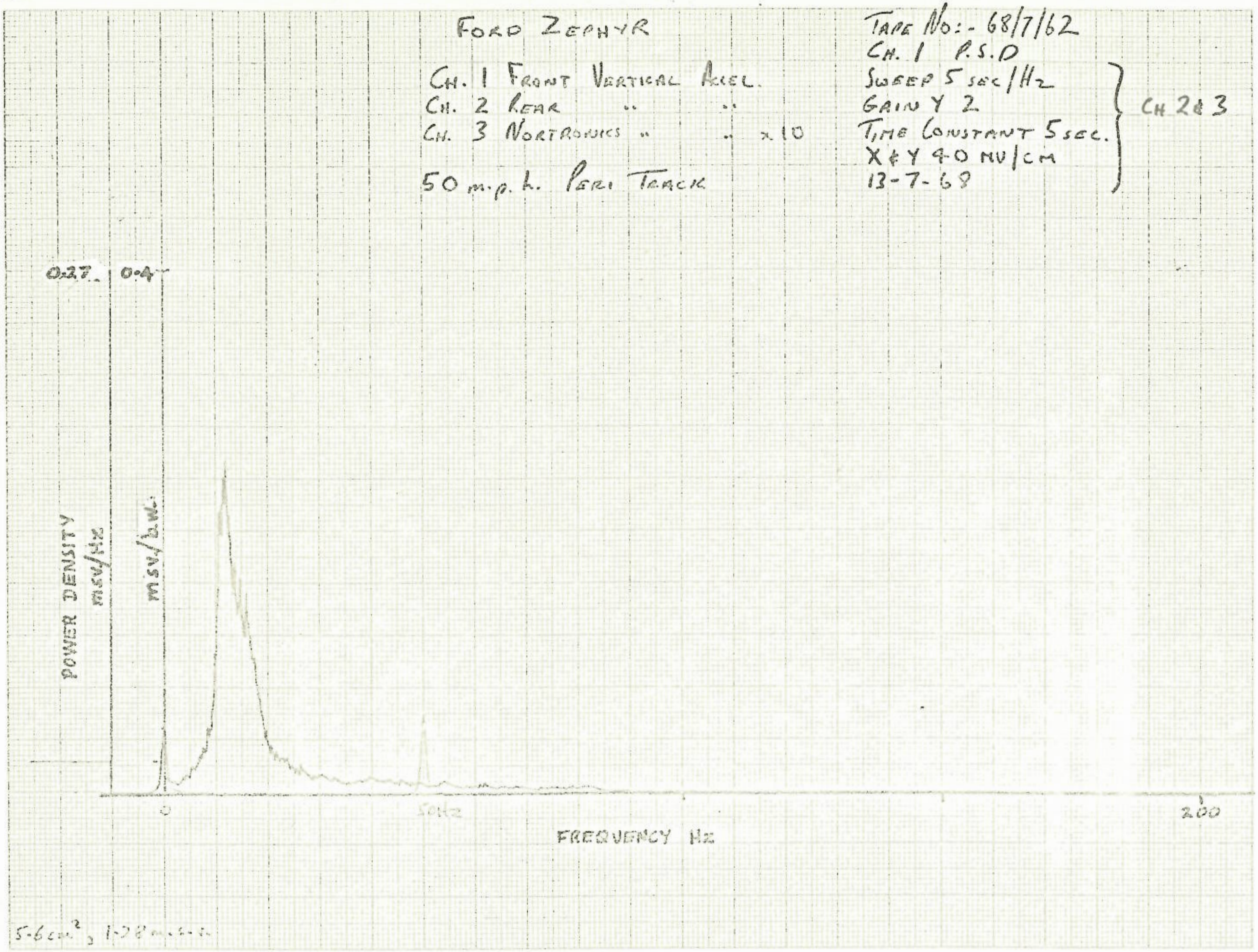

FIG. 3. P.S.D. FUNCTION: VERTICAL ACCELERATION, N/S FRONT HUB, UNREPAIRED SURFACE, 50 M.P.H.

5

68/7/62 CH. 1 GAIN 10

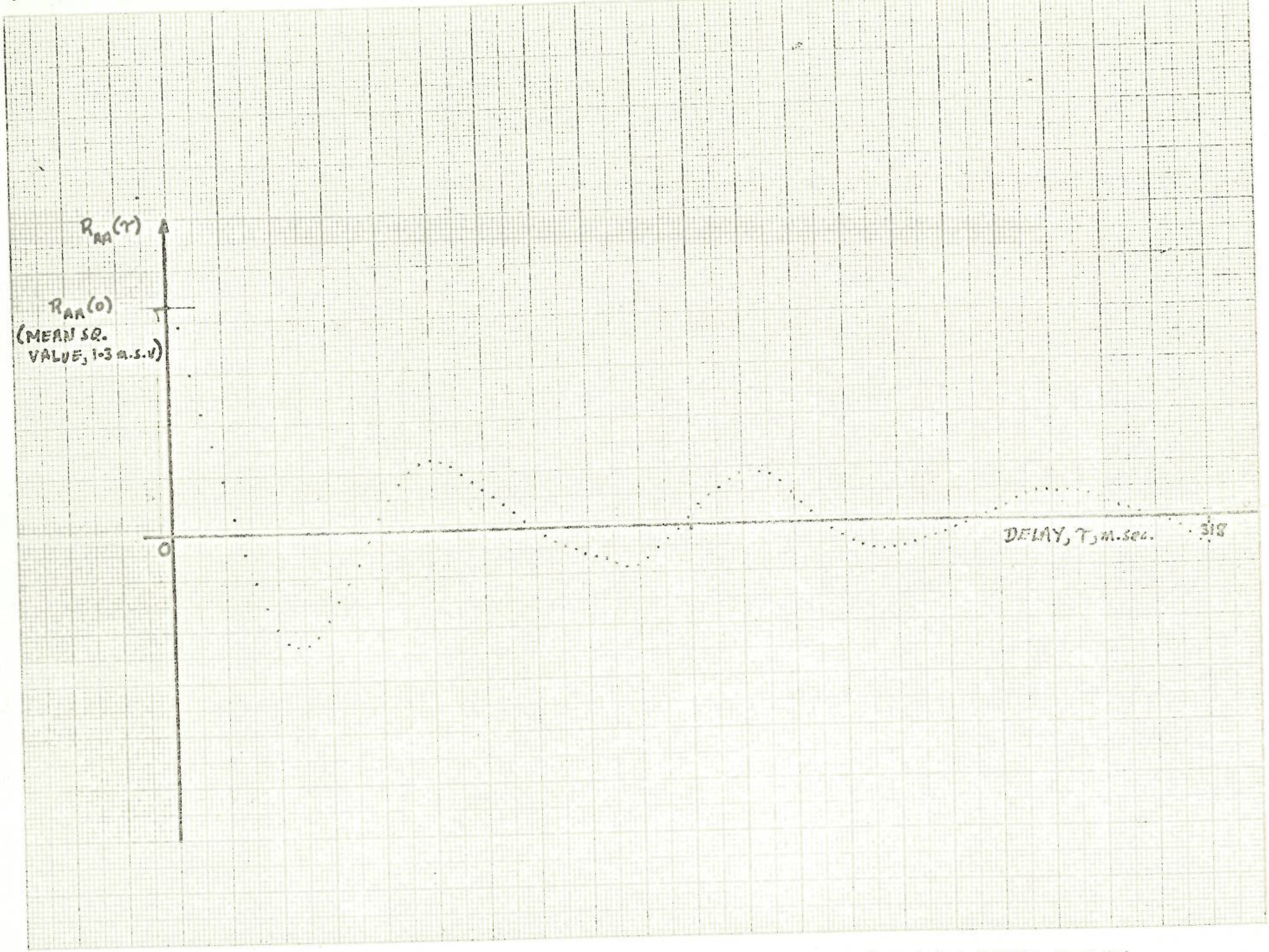

FIG. 4. AUTOCORRELATION FUNCTION: VERT. ACCELN. N/S FRONT HUB, UNREPAIRED SURFACE 50 M.P.H.

68/7/62 CH 1 A.D.F.

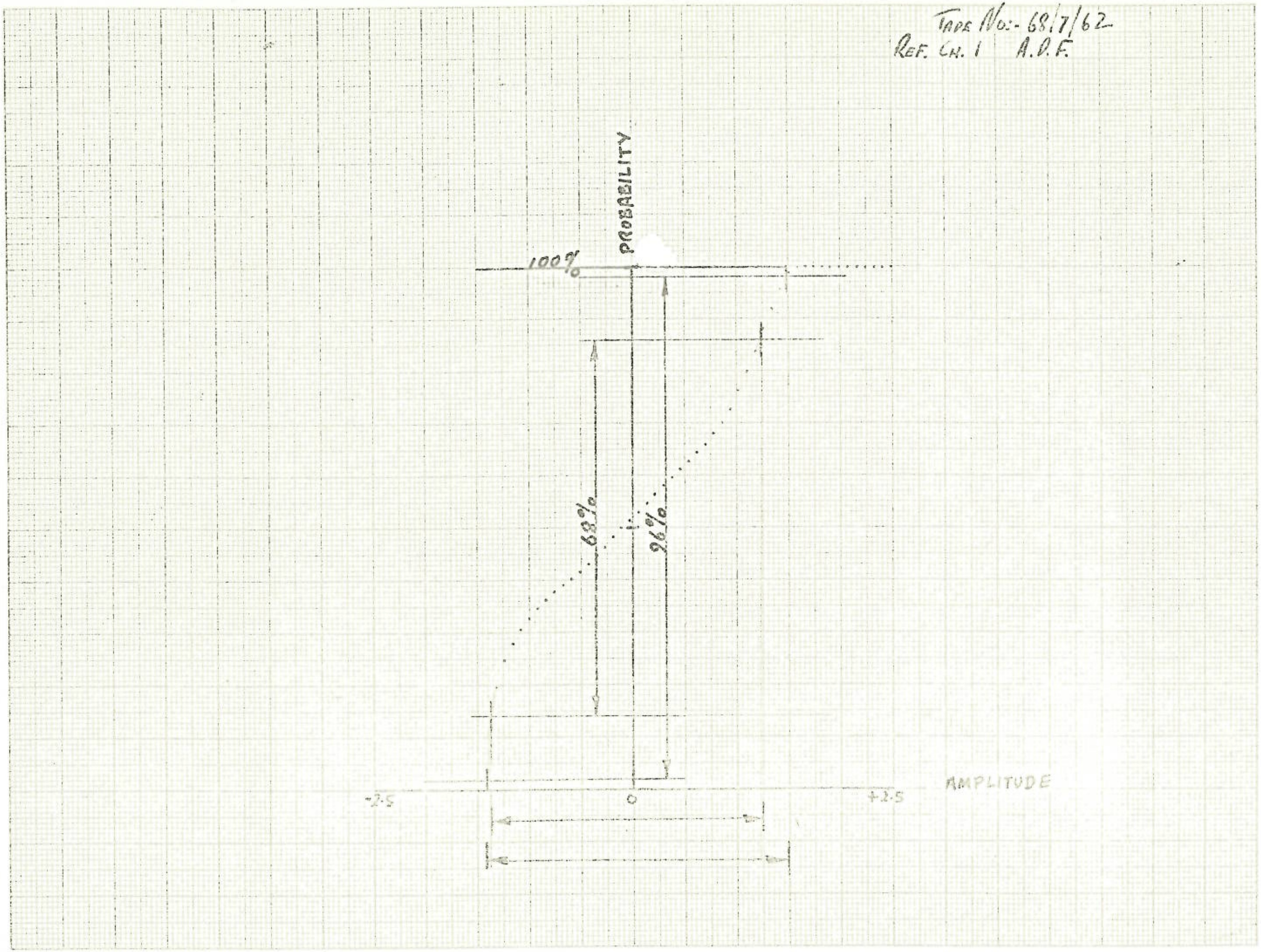

FIG. 5. AMPLITUDE DISTRIBUTION, VERT. ACCELN. N/S FRONT HUB UNREPAIRED SURFACE 50 M.P.H.

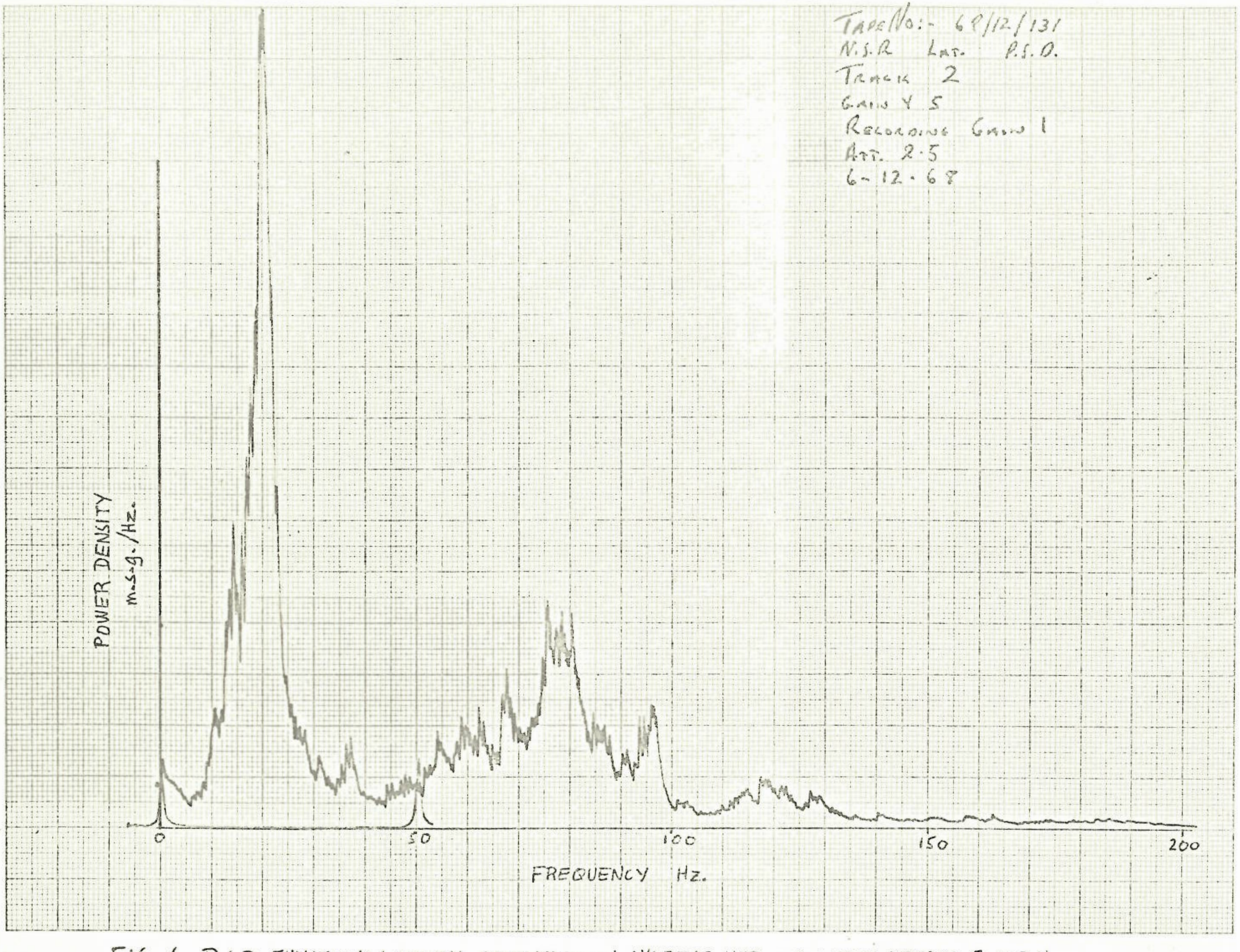

FIG. 6. P.S.D. FUNCTION: LATERAL ACCELERATION, N/S REAR HUB, UNREPAIRED SURFACE, 50 M.P.H.

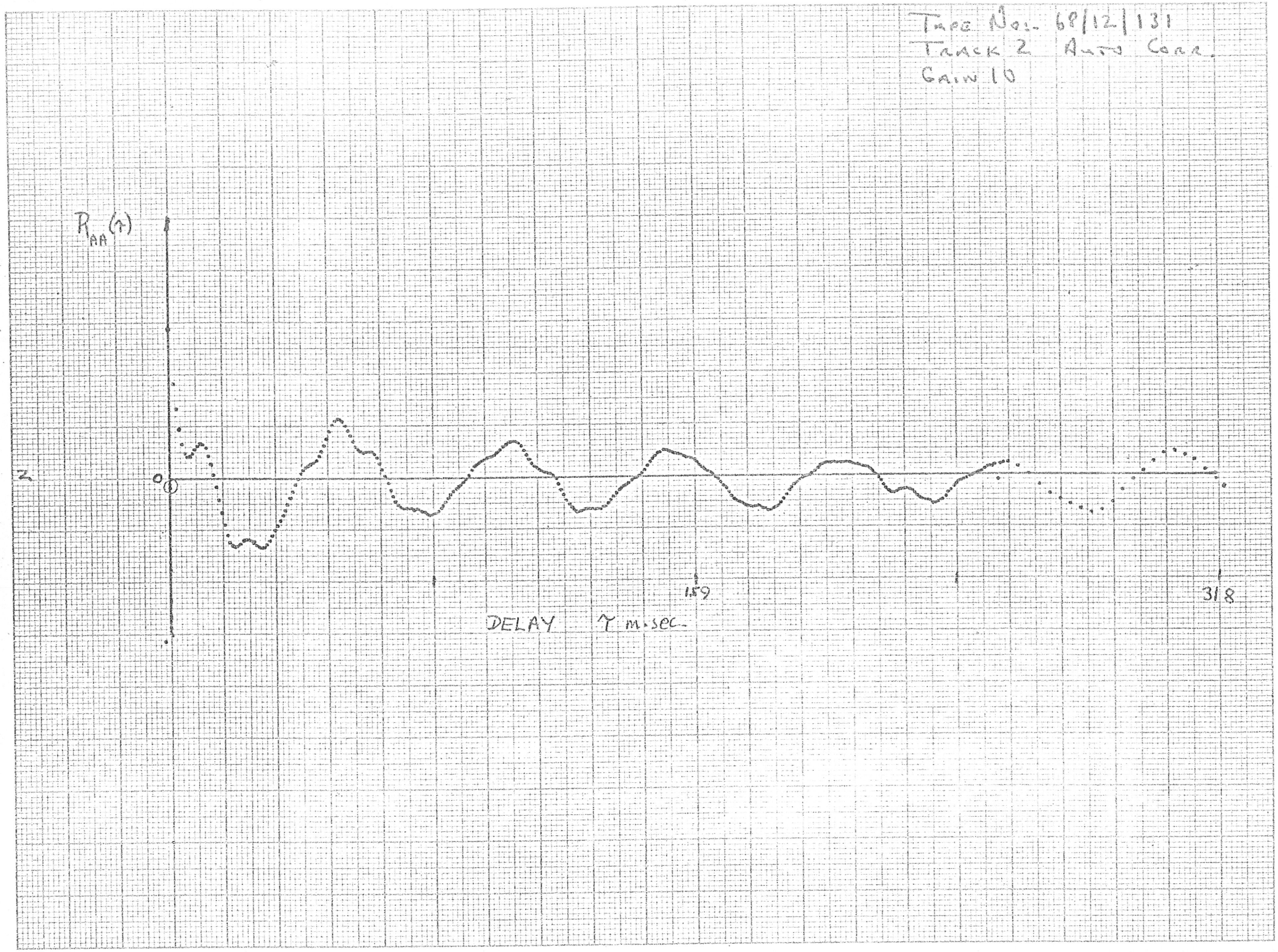

FIG. 7 AUTOCORRELATION FUNCTION: LATERAL ACCELN. N/S REAR HUB, UNREPAIRED SURFACE, 50 M.P.H.

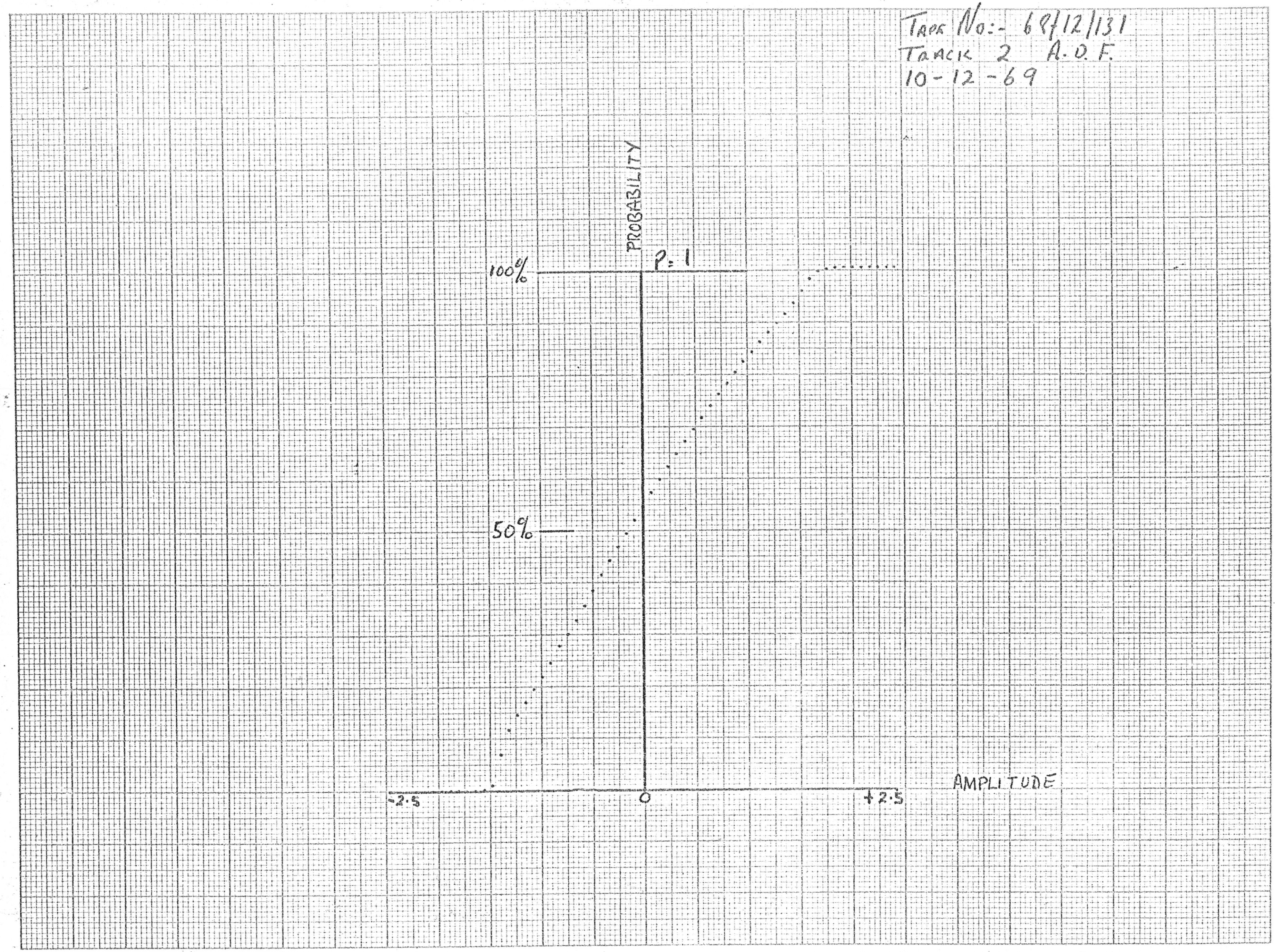

FIG. 8. AMPLITUDE DISTRIBUTION: LAT. ACCELN. N/S REAR HUB, UNREPAIRED SURFACE, 50 M.P.H.

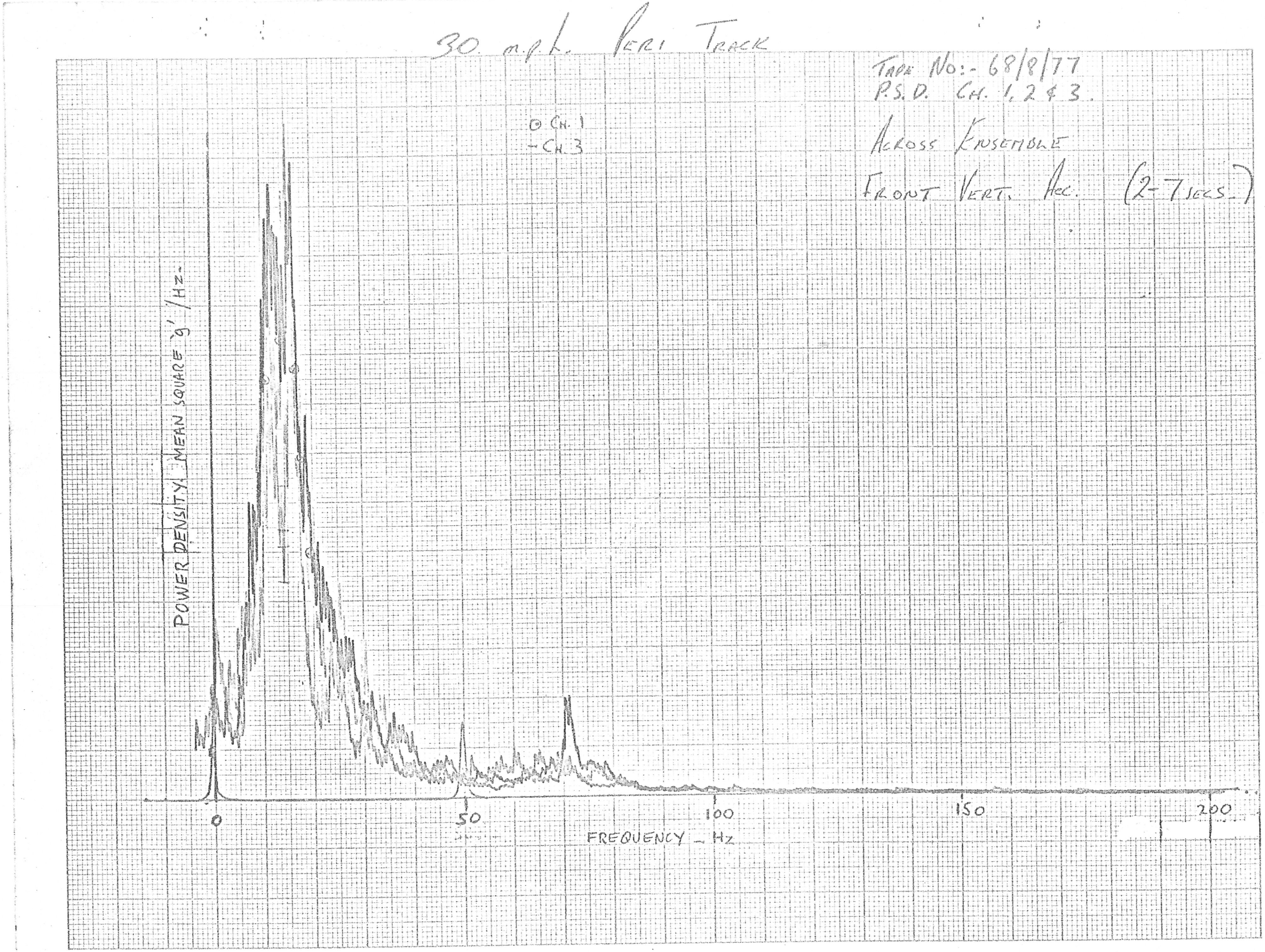

FIG. 9. P.S.D. FUNCTION (ENSEMBLE) VERTICAL ACCELN. N/S FRONT HUB, UNREPAIRED SURFACE, 30 M.P.H.

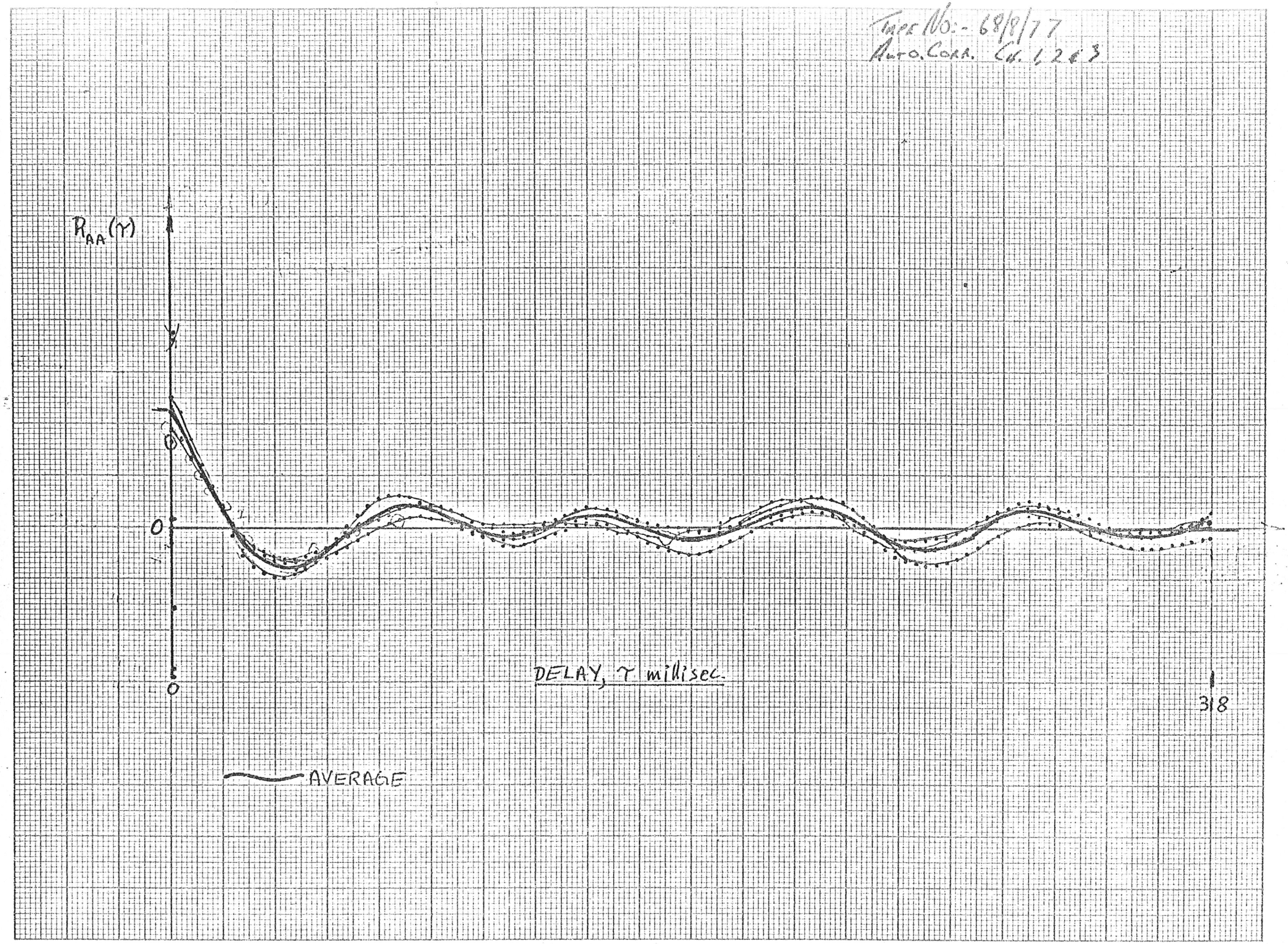

FIG. 10. AUTO CORRELATION FUNCTION (ENSEMBLE), VERT. ACCELN., N/S FRONT HUB, UNREPAIRED SURFACE, 30 M.P.H.

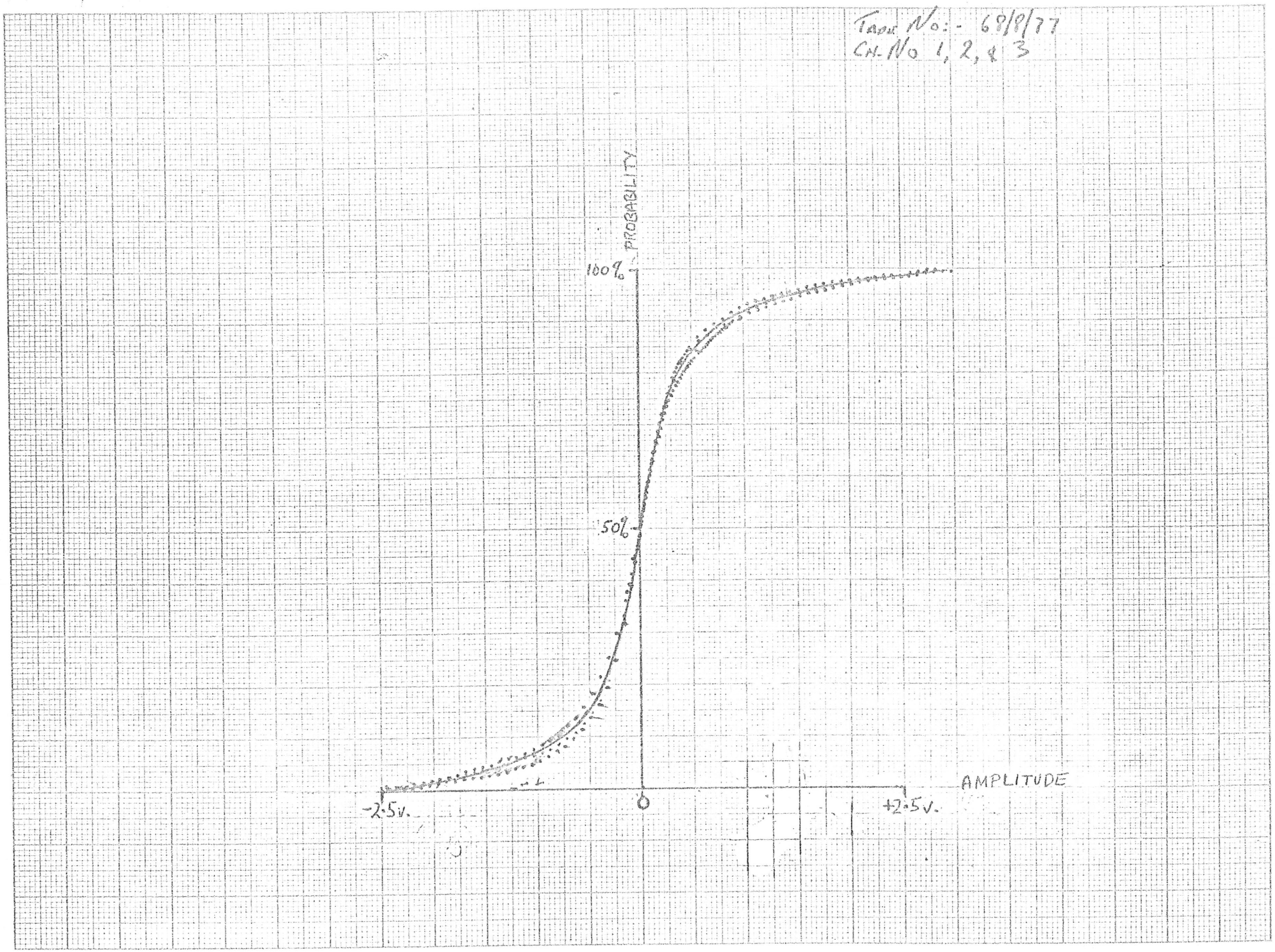

FIG. 11. AMPLITUDE DISTRIBUTION (ENSEMBLE), VERT. ACCELN. N/S FRONT HUB, UNREPAIRED SURFACE, 30 M.P.H.

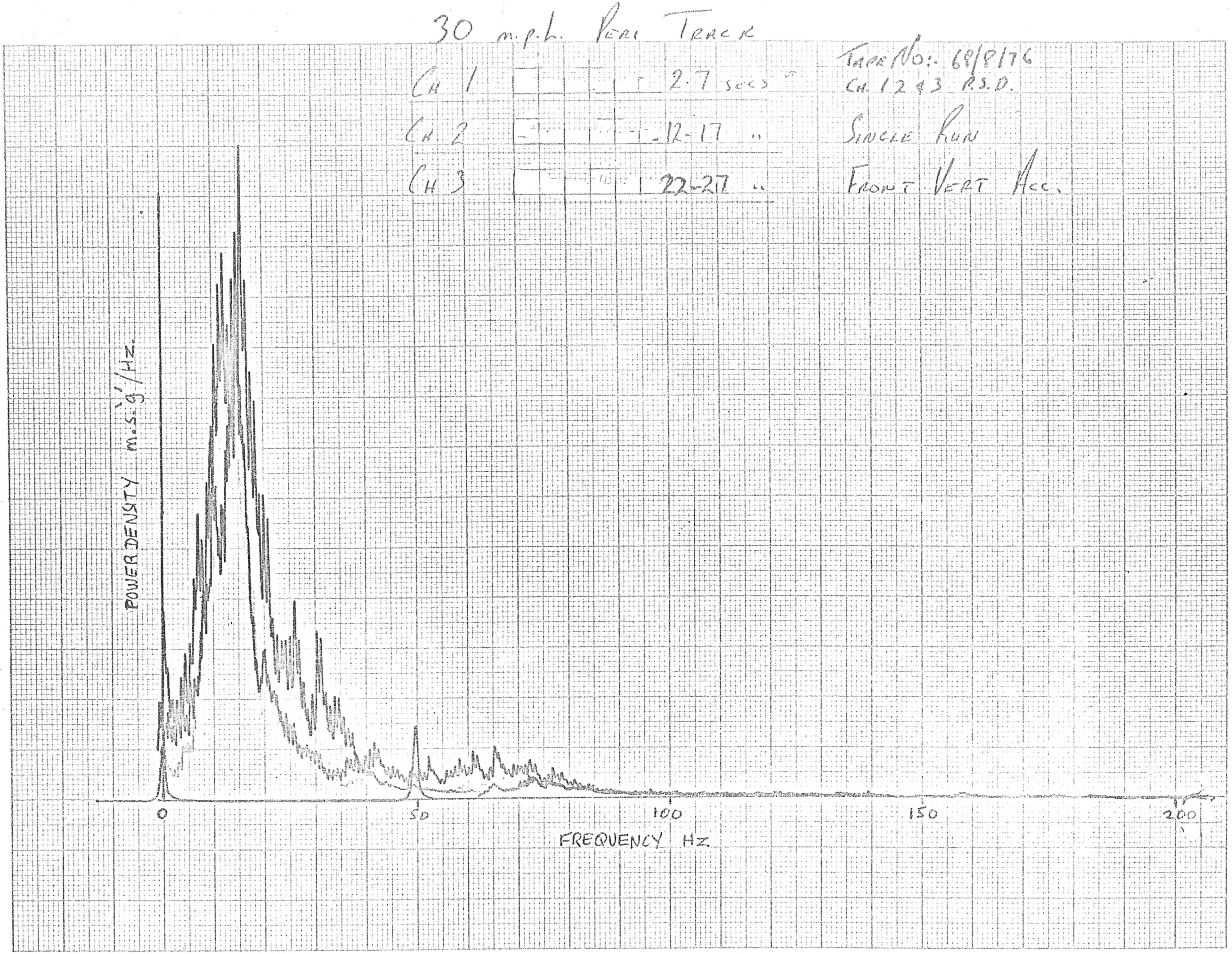

FIG. 12. P.S.D. FUNCTION (SINGLE RECORD) VERTICAL ACCELERATION, N/S FRONT HUB, UNREPAIRED SURFACE, 30 M.P.H.

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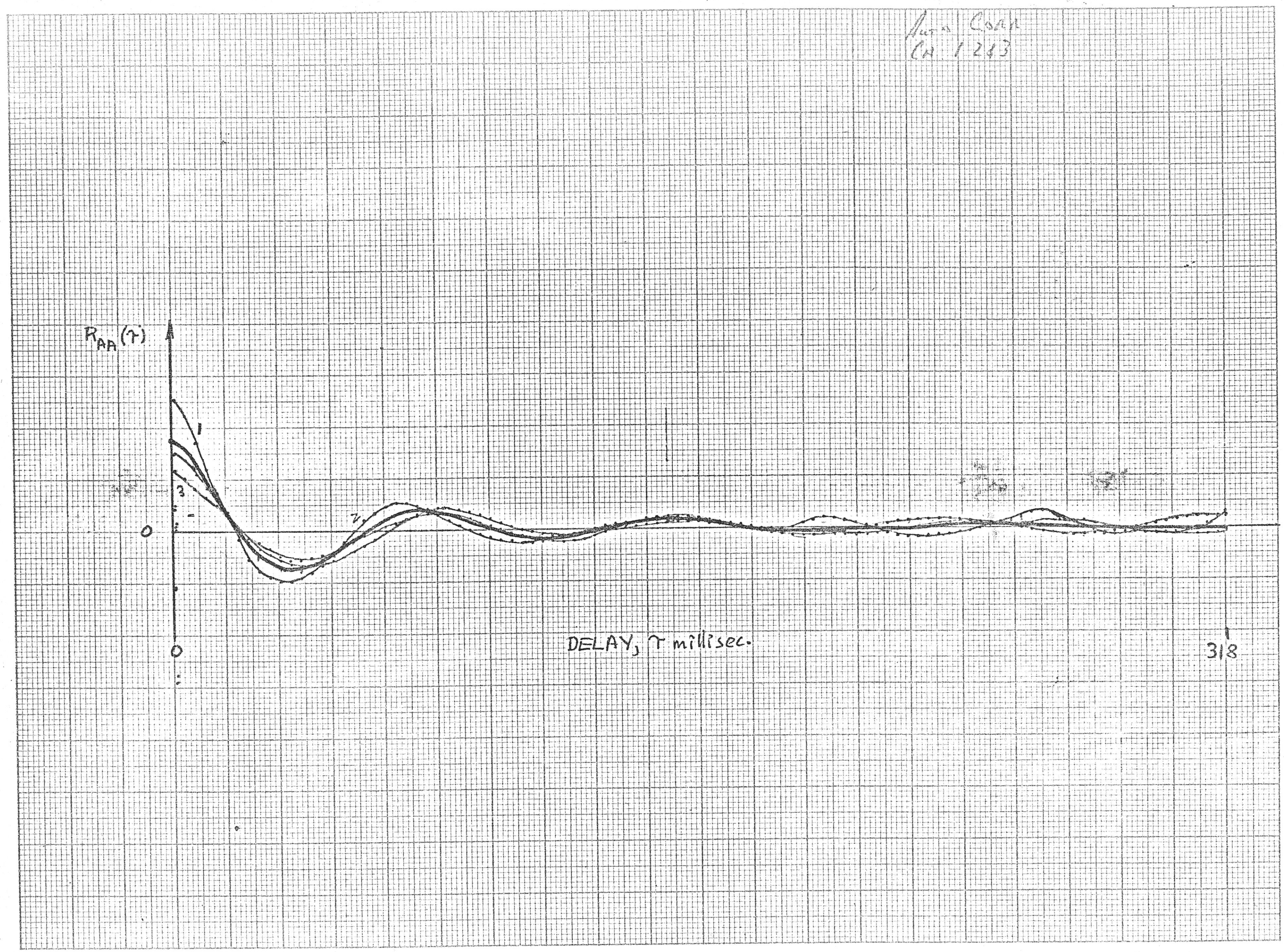

FIG. 13. AUTOCORRELATION FUNCTION (SINGLE RECORD) VERT. ACCELN. N/S FRONT HUB, UNREPAIRED SURFACE, 30 M.P.H.

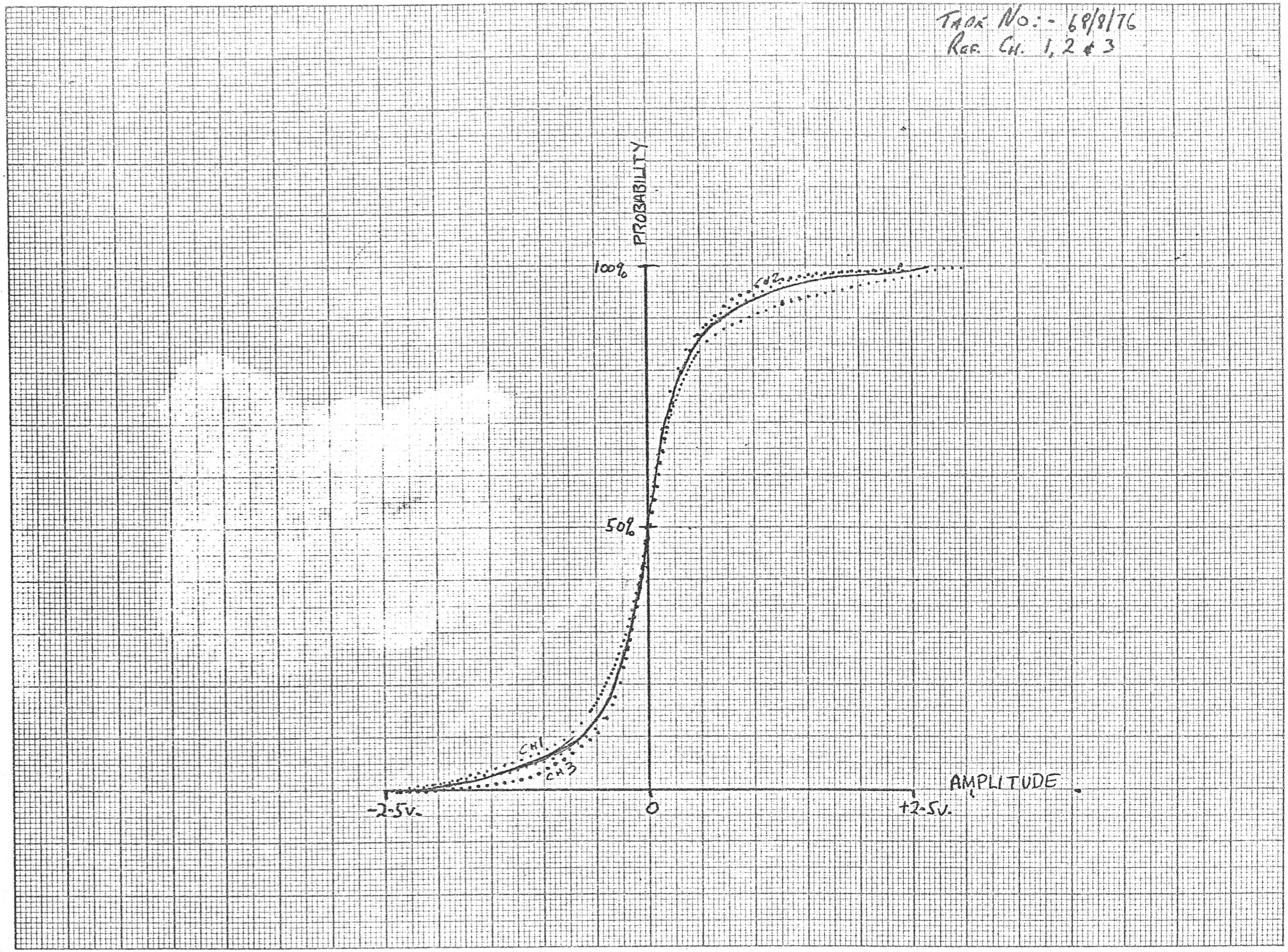

FIG.14. AMPLITUDE DISTRIBUTION (SINGLE RECORD) VERT. ACCELN. N/S FRONT HUB, UNREPAIRED SURFACE, 30 M.P.H.

Across Ensemble (2-7 secs)

30 m.p.h. Peri Track

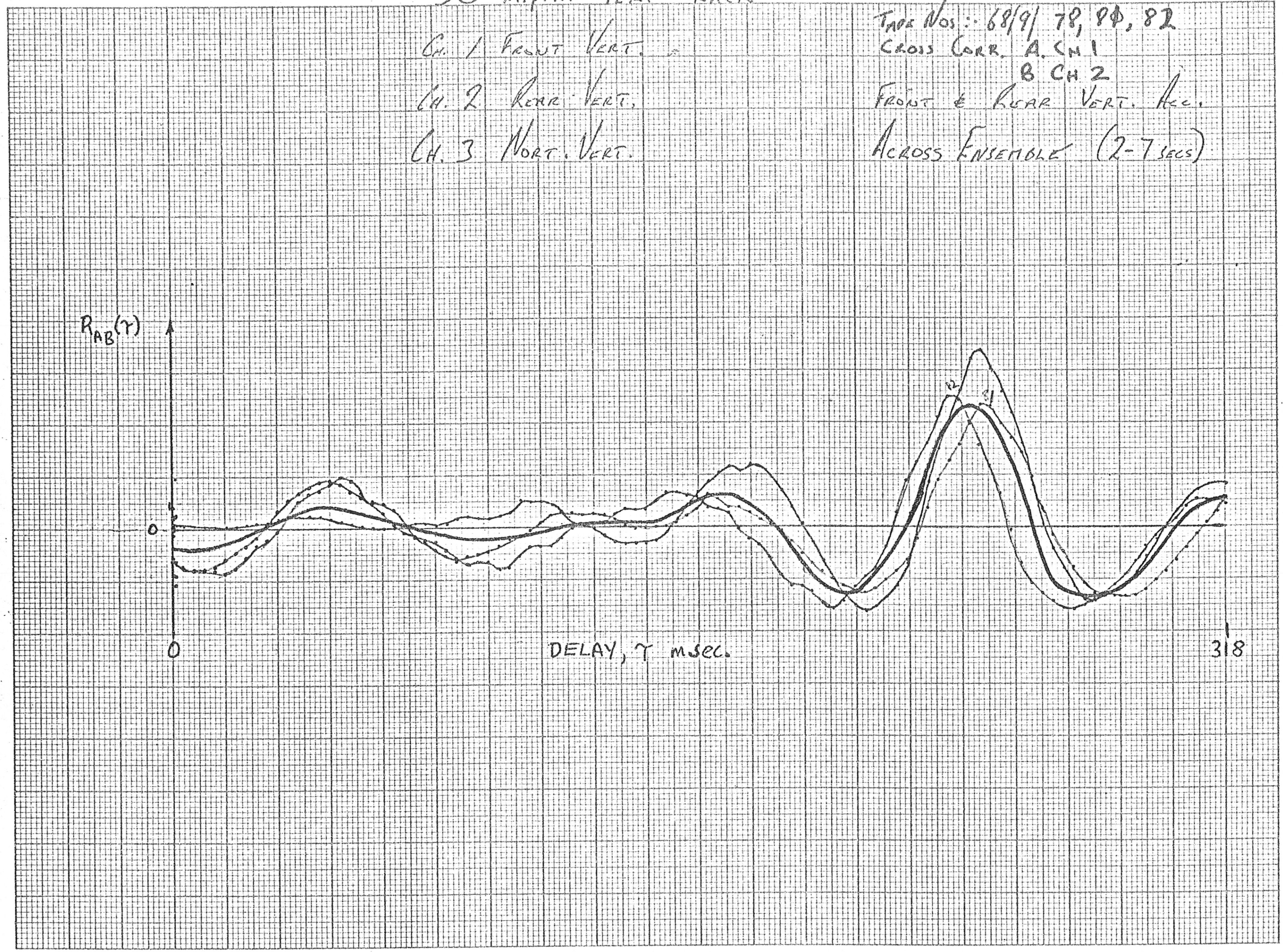

FIG. 15. CROSS-CORRELATION FUNCTION (ENSEMBLE) VERT. ACCELNS. N/S FRONT-REAR HUBS, UNREP. SURFACE 30 M.P.H.

30 m.p.h. Peri Track

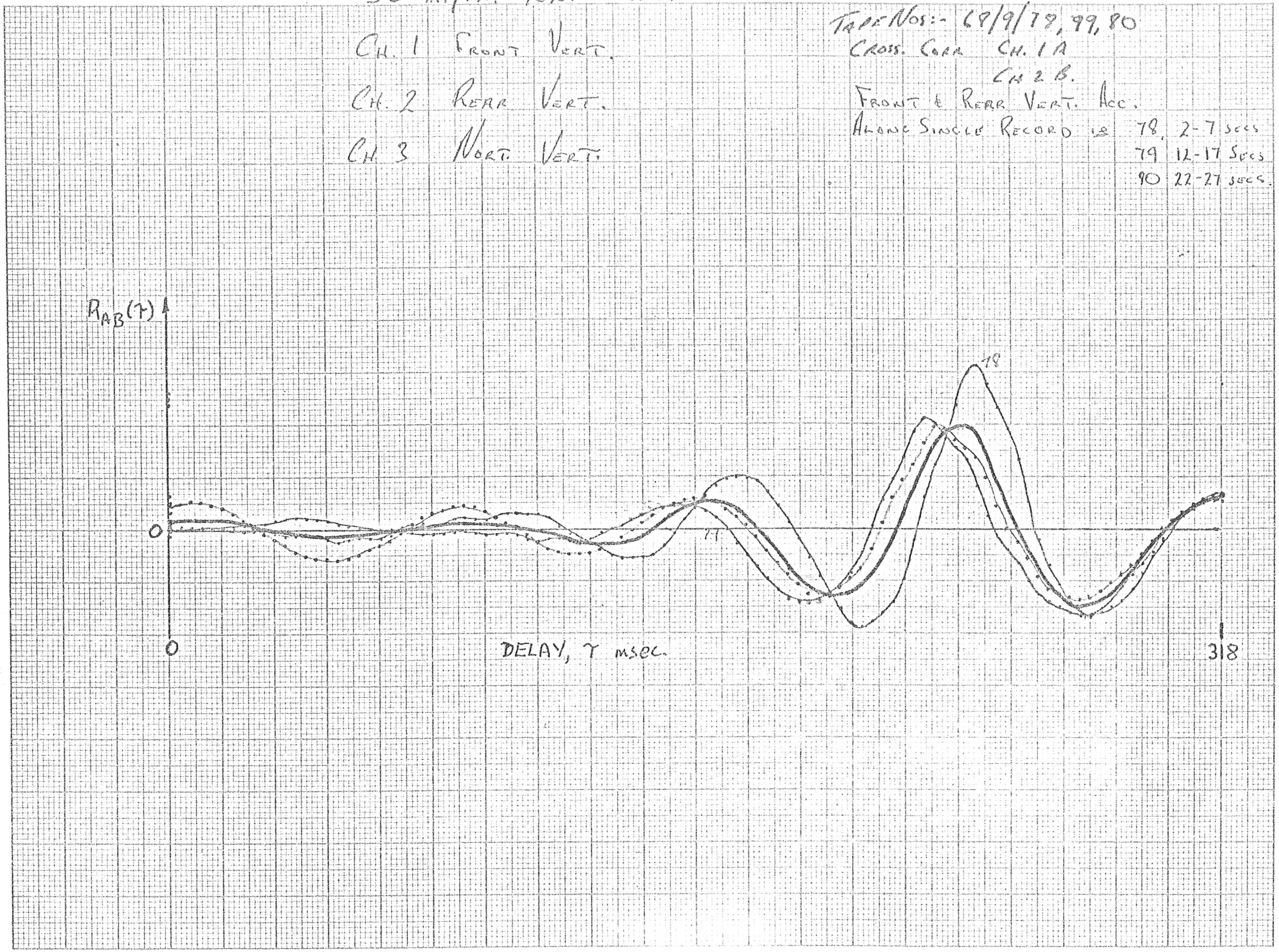

FIG. 16. CROSS-CORRELATION FN. (SINGLE RECORD) VERT. ACCELNS. N/S FRONT-REAR HUBS, UNREP. SURFACE, 30 M.P.H.

50 m.p.h. M.1

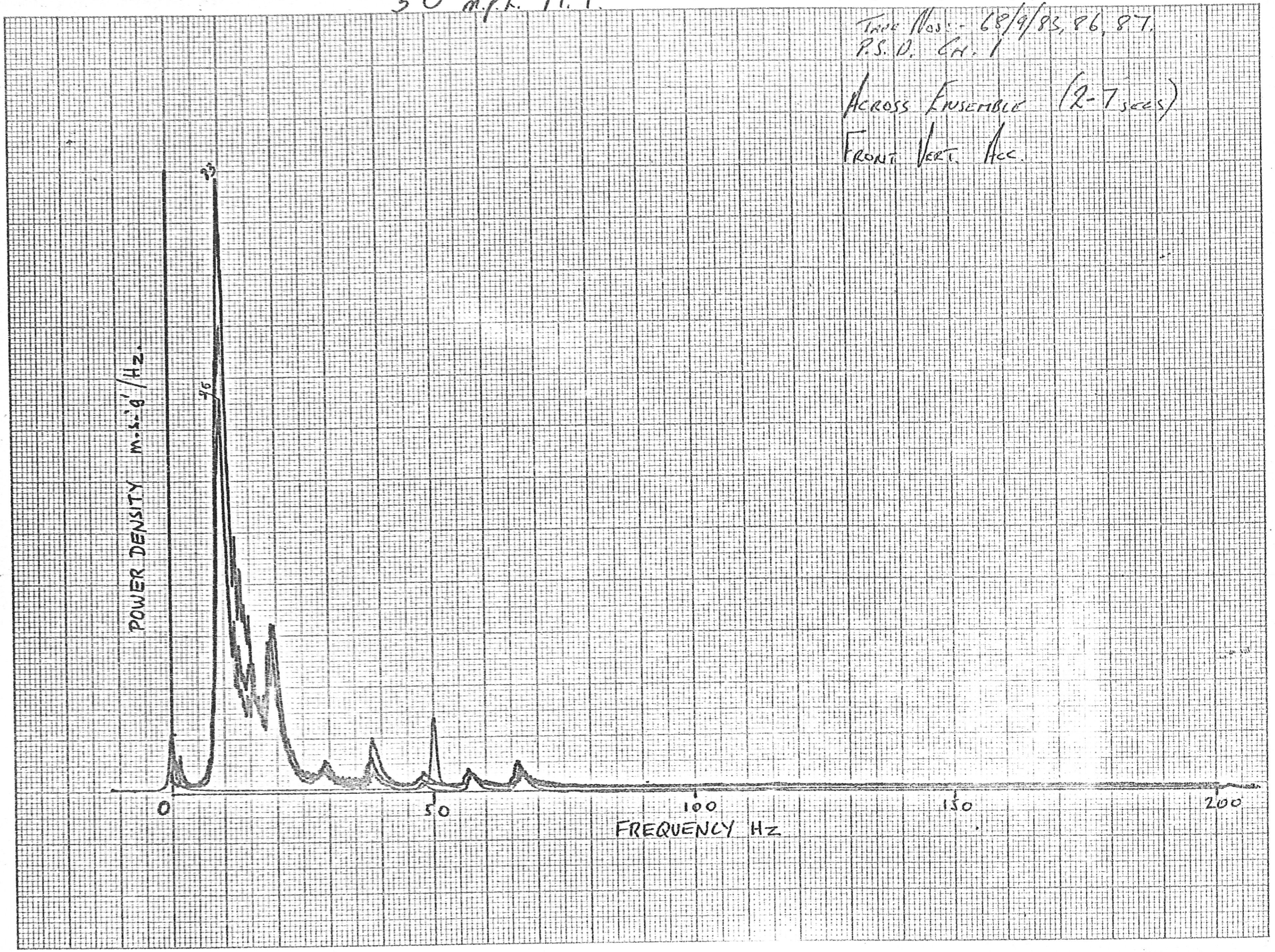

FIG. 17. P.S.D. FUNCTION (ENSEMBLE) N/S FRONT HUB VERT. ACCELN., MOTORWAY 50 M.P.H.

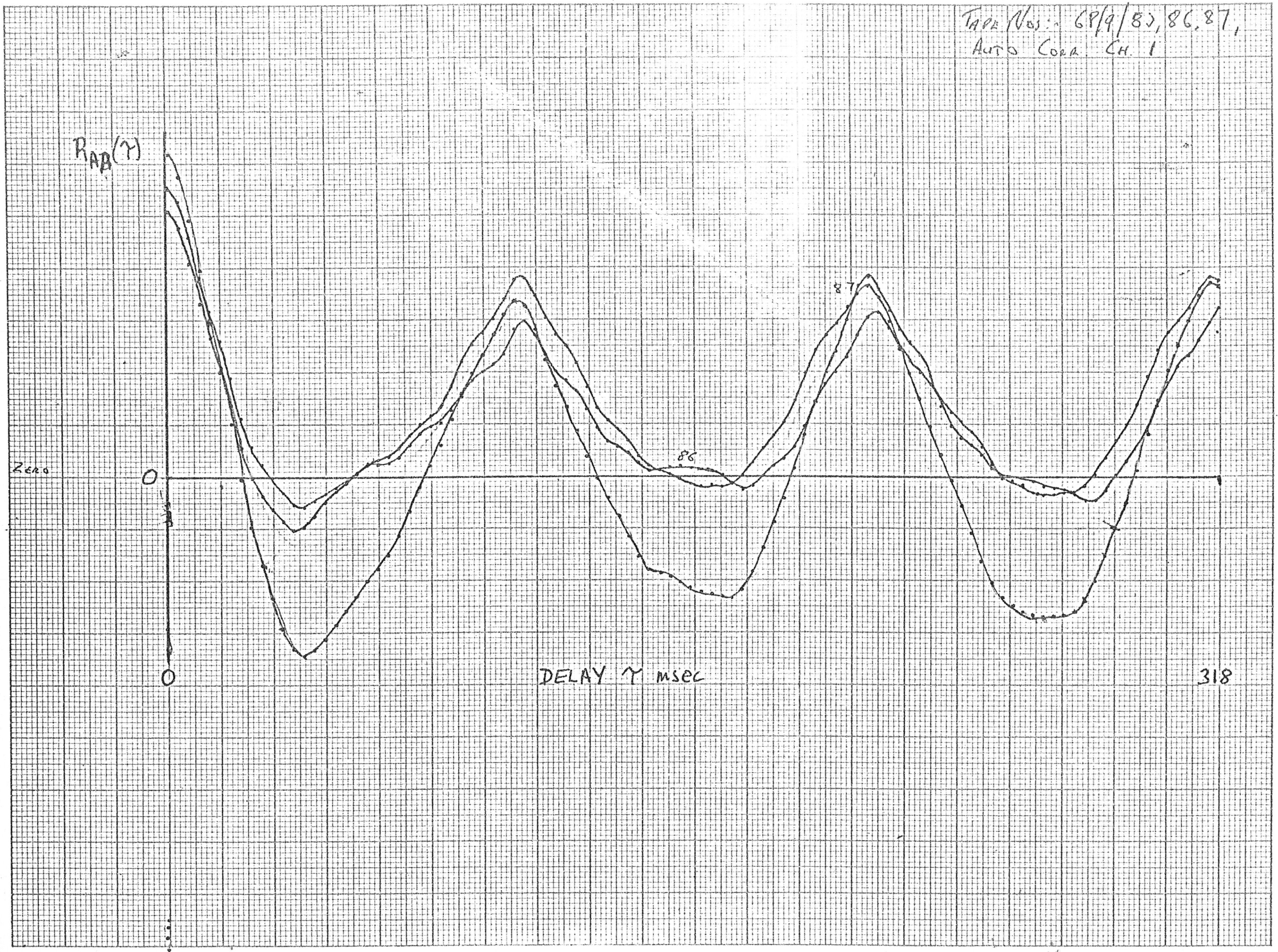

FIG. 18. AUTO CORRELATION FN. (ENSEMBLE) VERT. ACCELN. N/S FRONT HUB, MOTORWAY, 50 M.P.H.

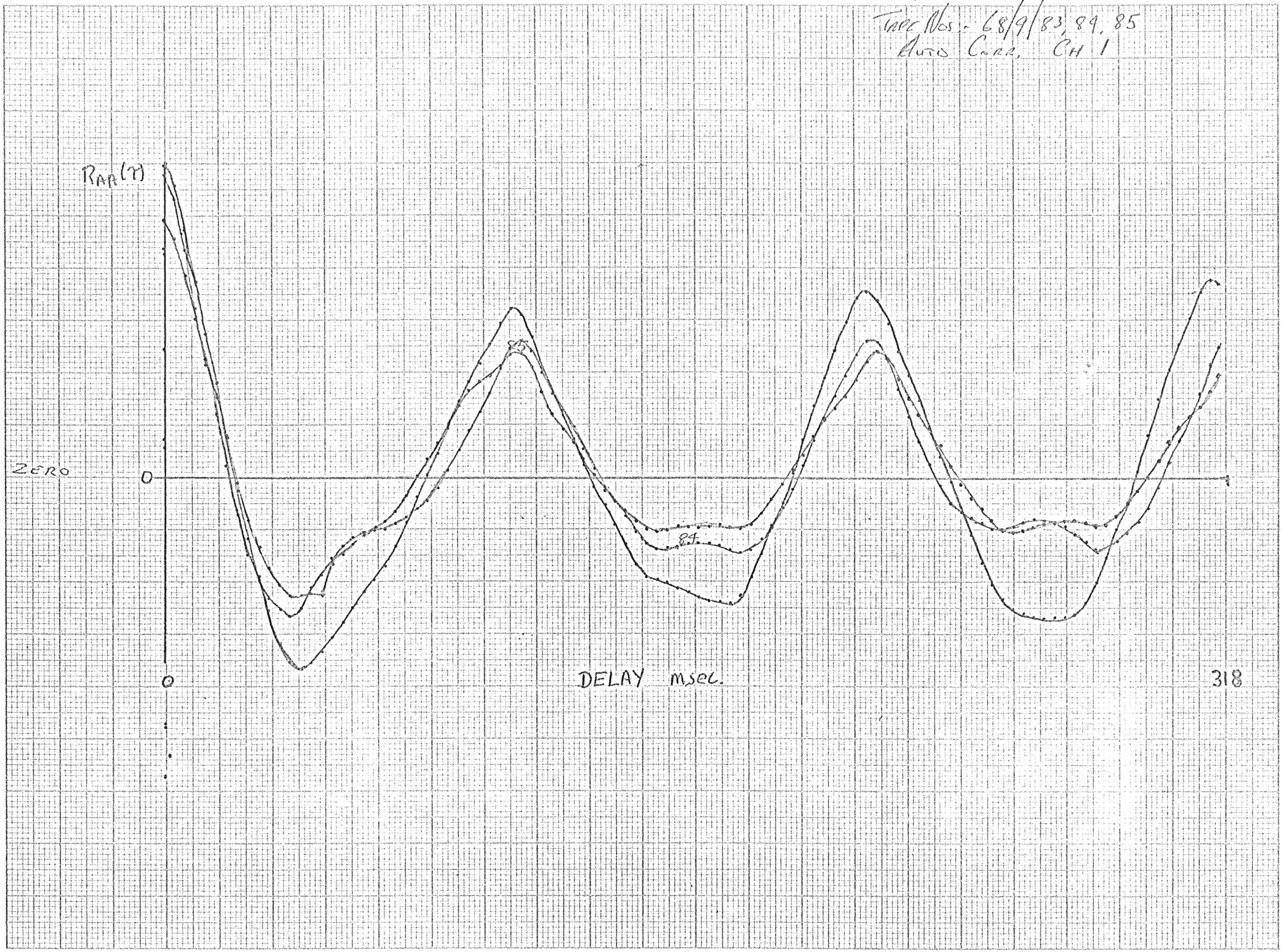

FIG. 19. AUTO-CORRELATION FN. (SINGLE RECORD) VERT. ACCELN. N/S FRONT HUB, MOTORWAY 50 M.P.H.

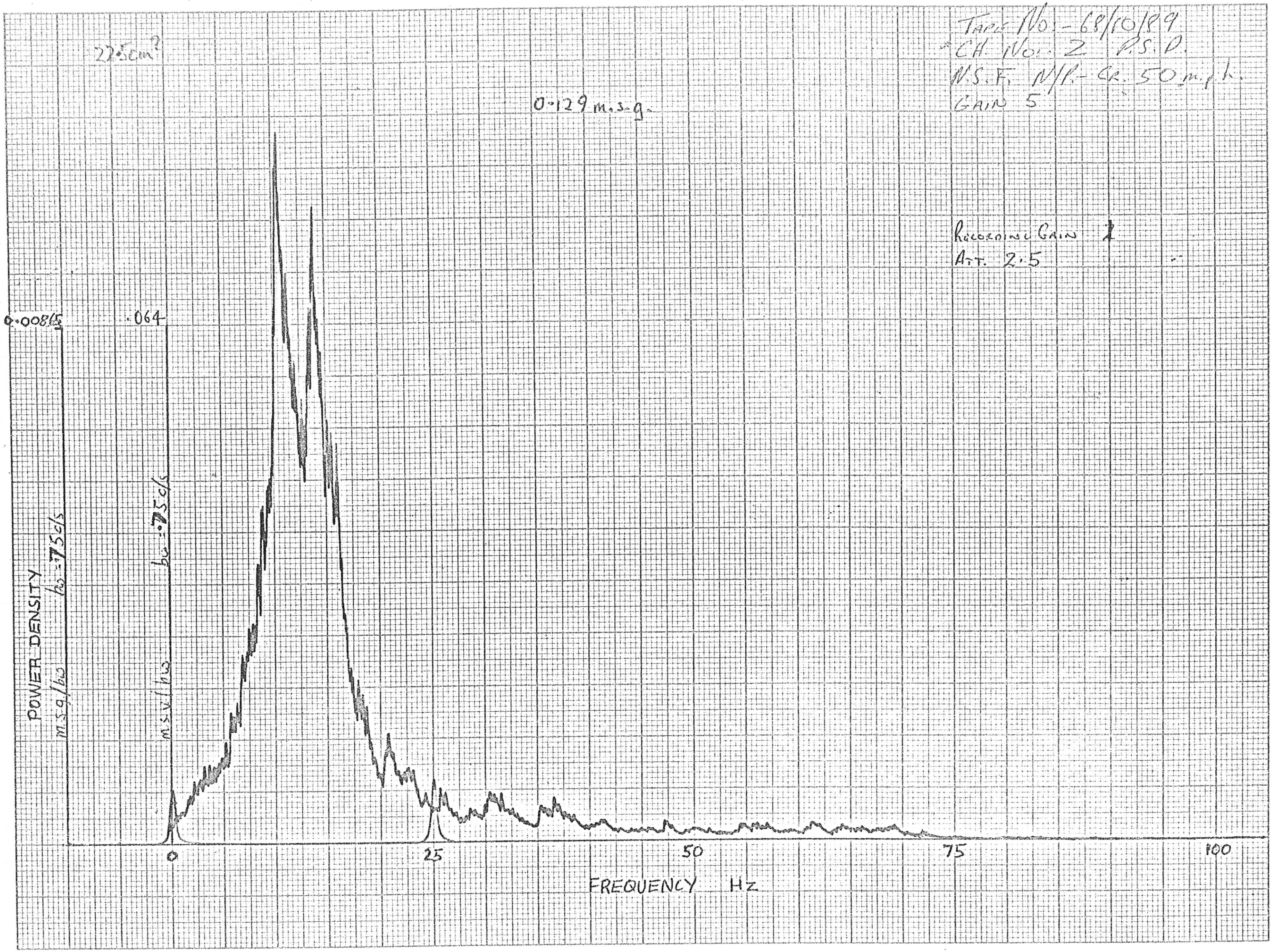

FIG. 20. P.S.D. FUNCTION, VERTICAL ACCELERATION, N/S FRONT HUB, COUNTRY ROAD, 50 M.P.H.

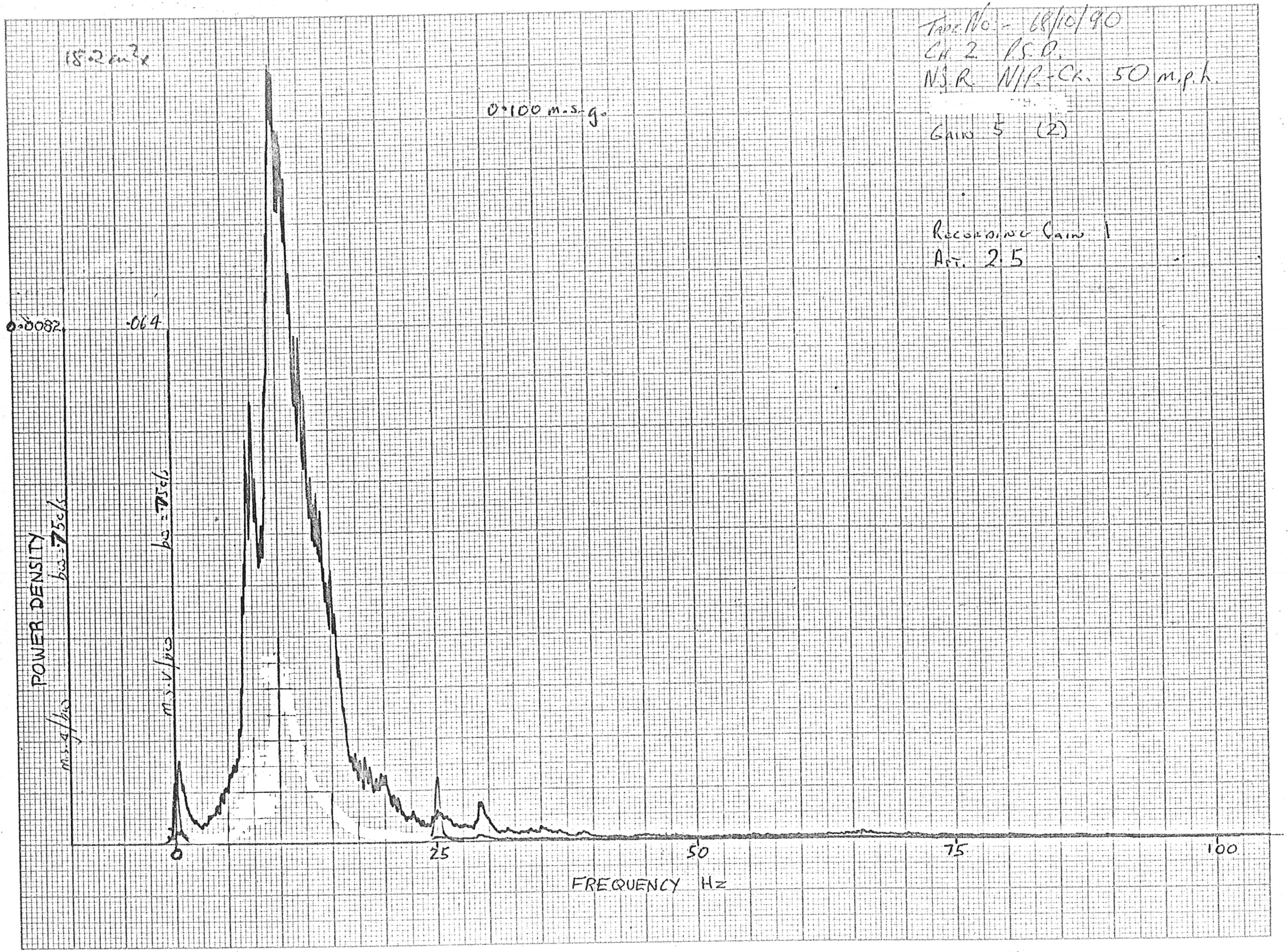

FIG. 21. P.S.D. FUNCTION, VERTICAL ACCELERATION, N/S REAR HUB, COUNTRY ROAD, 50 M.P.H.

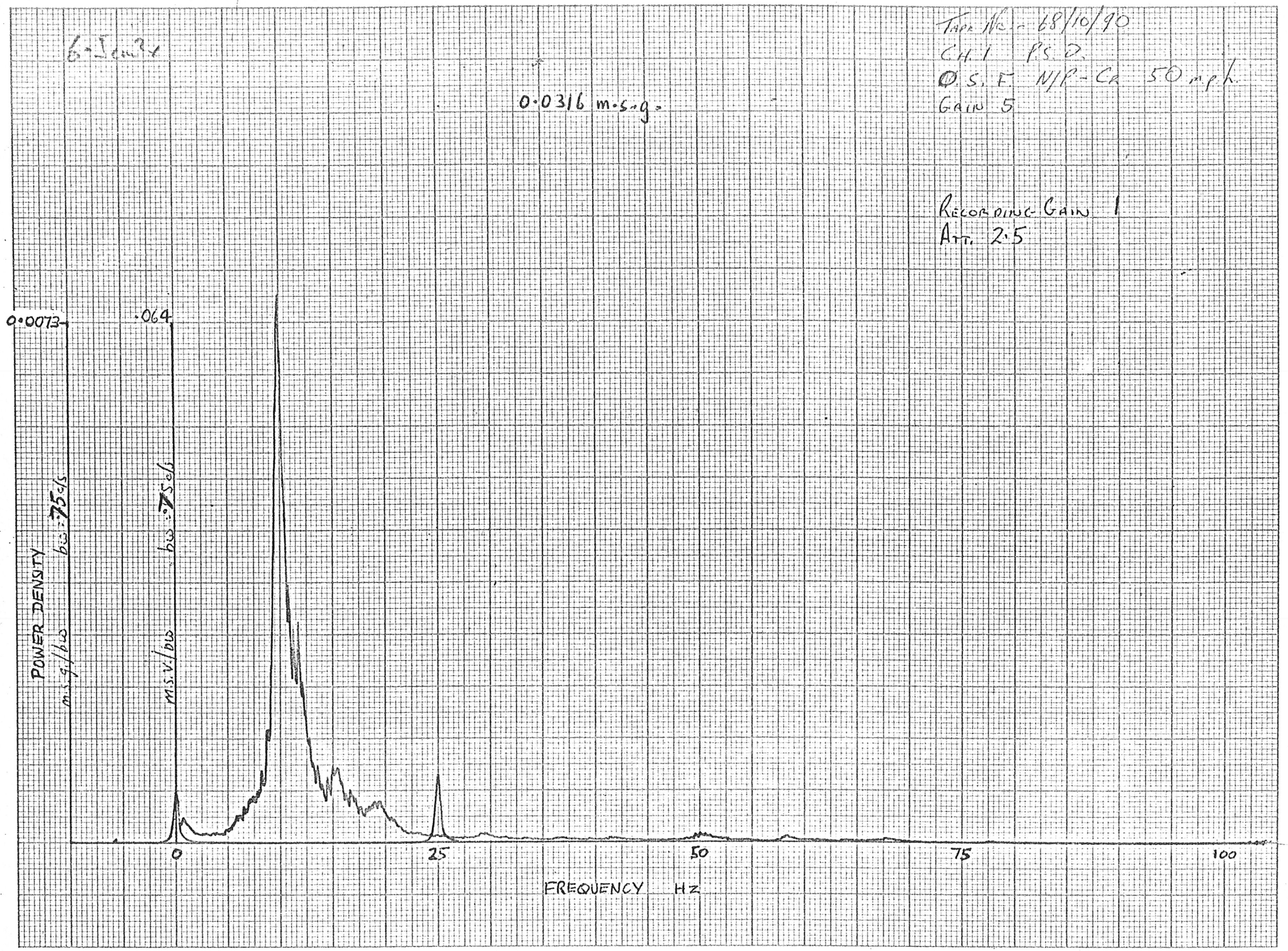

FIG. 22. P.S.D. FUNCTION, VERTICAL ACCELERATION, O/S FRONT HUB, COUNTRY ROAD, 50 M.P.H.

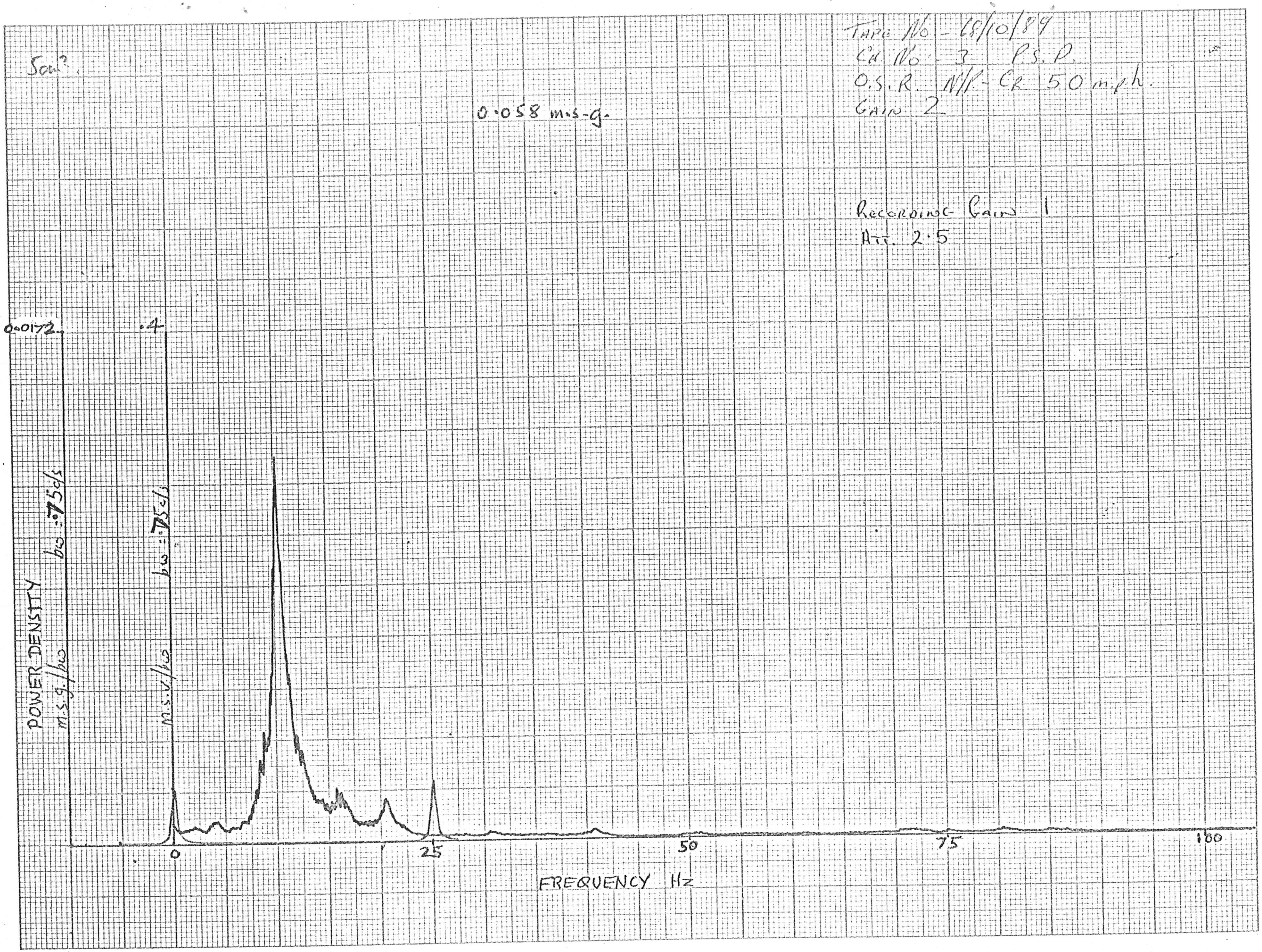

FIG. 23. P.S.D. FUNCTION, VERTICAL ACCELERATION, O/S REAR HUB, COUNTRY ROAD, 50 M.P.H.

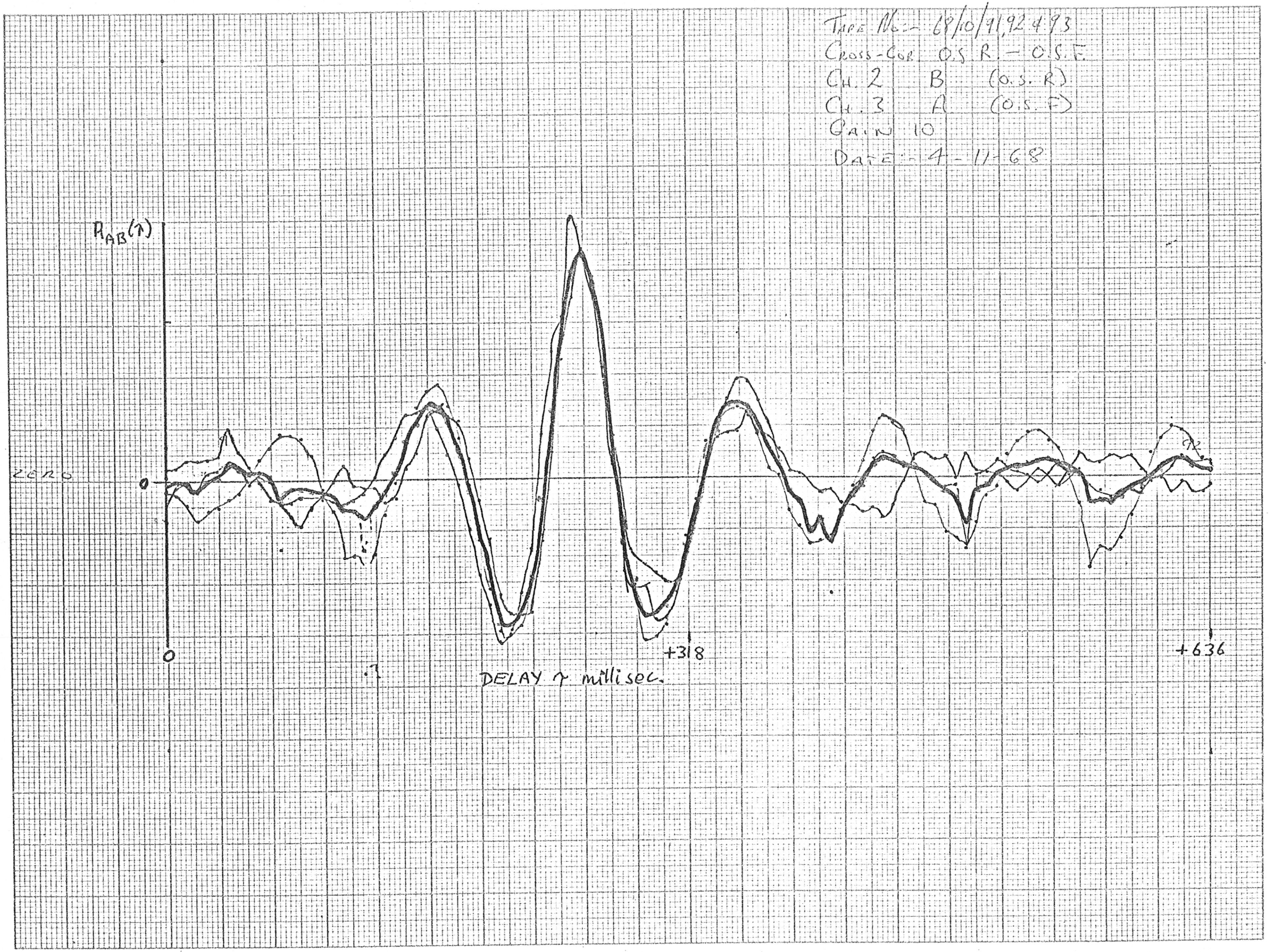

FIG. 24. CROSS CORRELATION FUNCTION, VERT. ACCELNS., O/S FRONT – O/S REAR HUBS, UNREPAIRED ROAD 30 M.P.H.

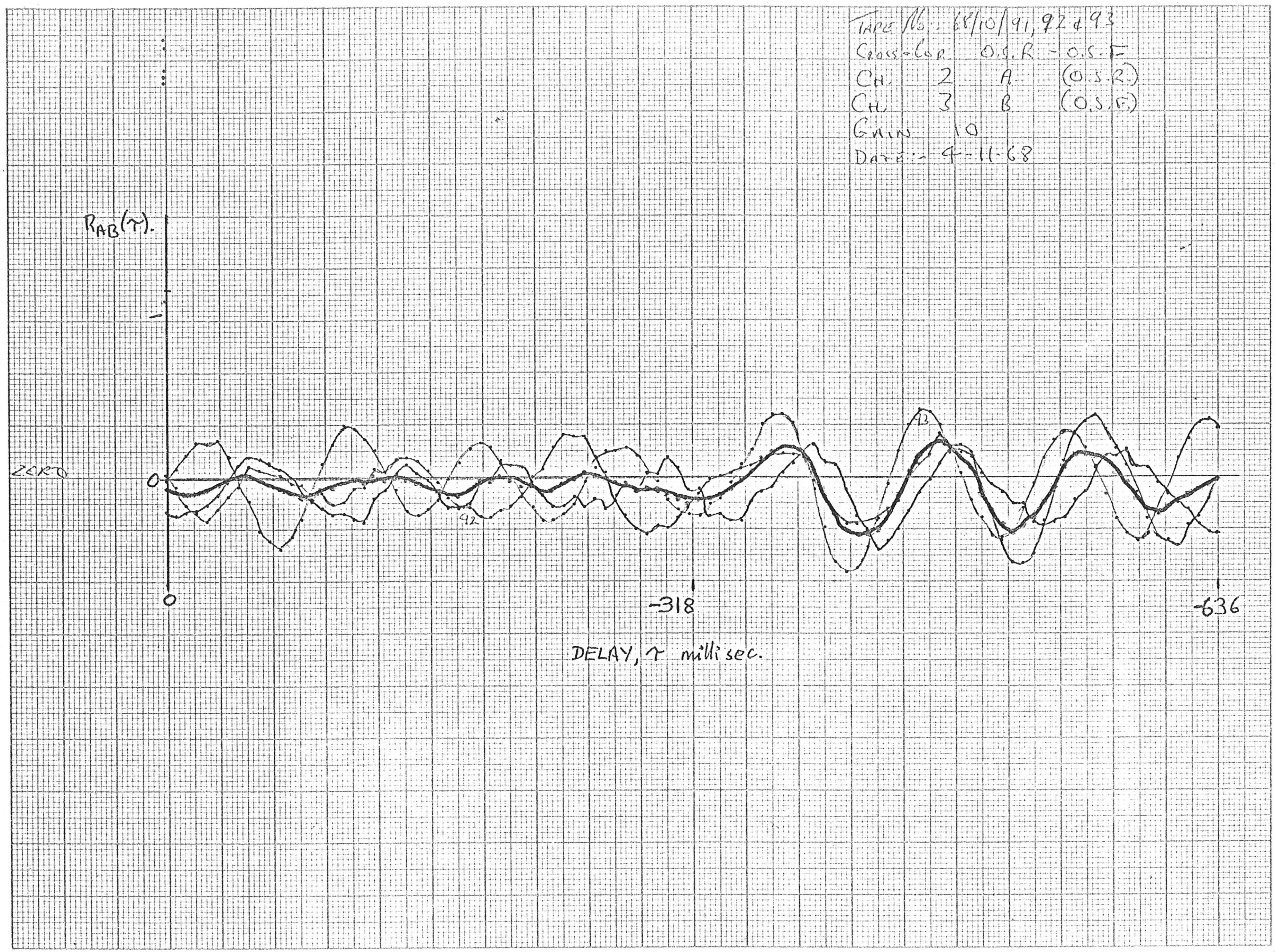

FIG 24(a): CROSS-CORRELATION FUNCTION as FIG 24 BUT τ NEGATIVE

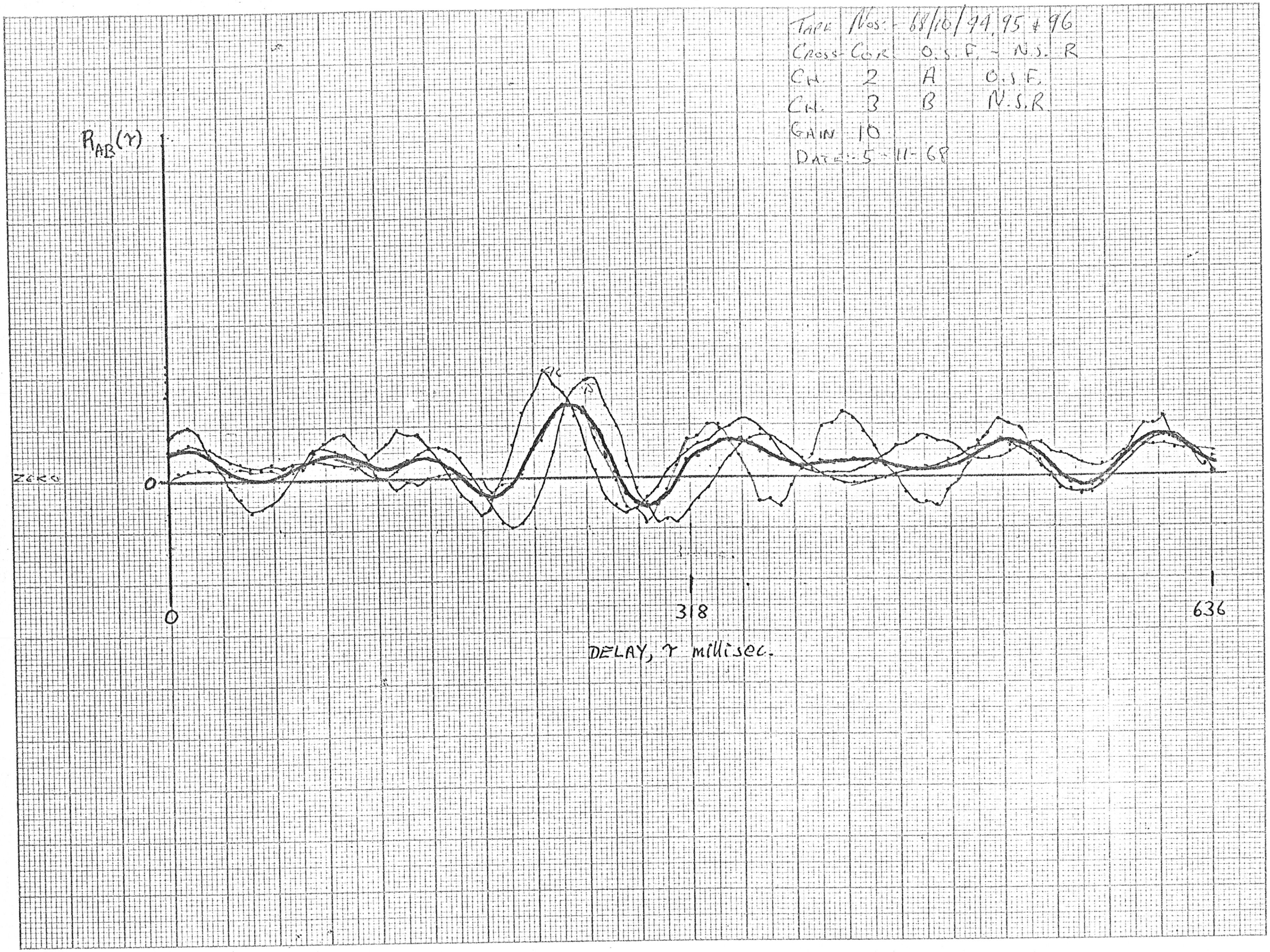

FIG. 25. CROSS CORRELATION FUNCTION, VERT. ACCELNS. O/S FRONT – N/S REAR HUBS, UNREPAIRED ROAD, 30 MPH.

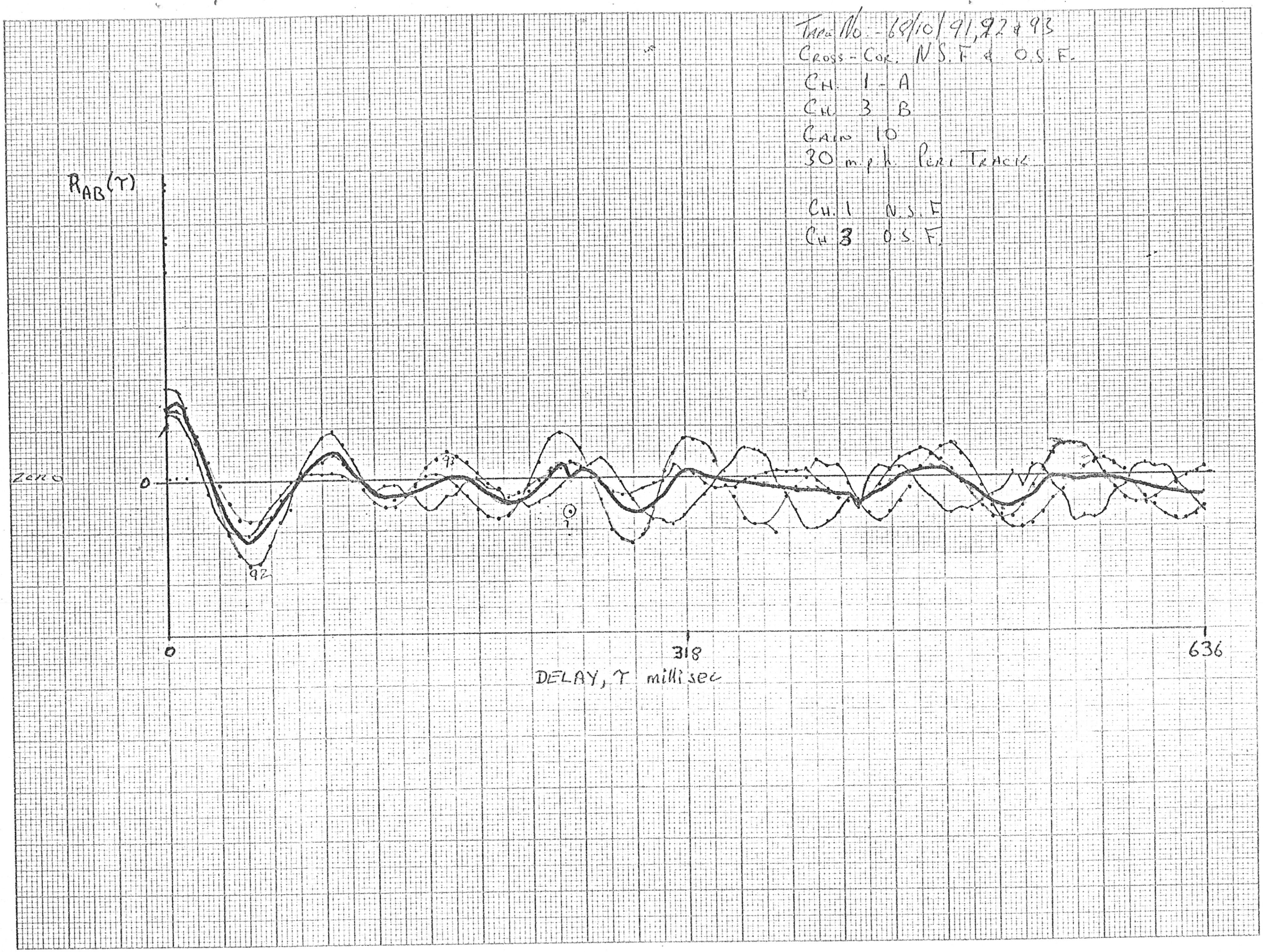

FIG. 26. CROSS CORRELATION FUNCTION, VERT. ACCELNS., N/S FRONT – O/S FRONT HUBS, UNREPAIRED ROAD, 30 M.P.H.

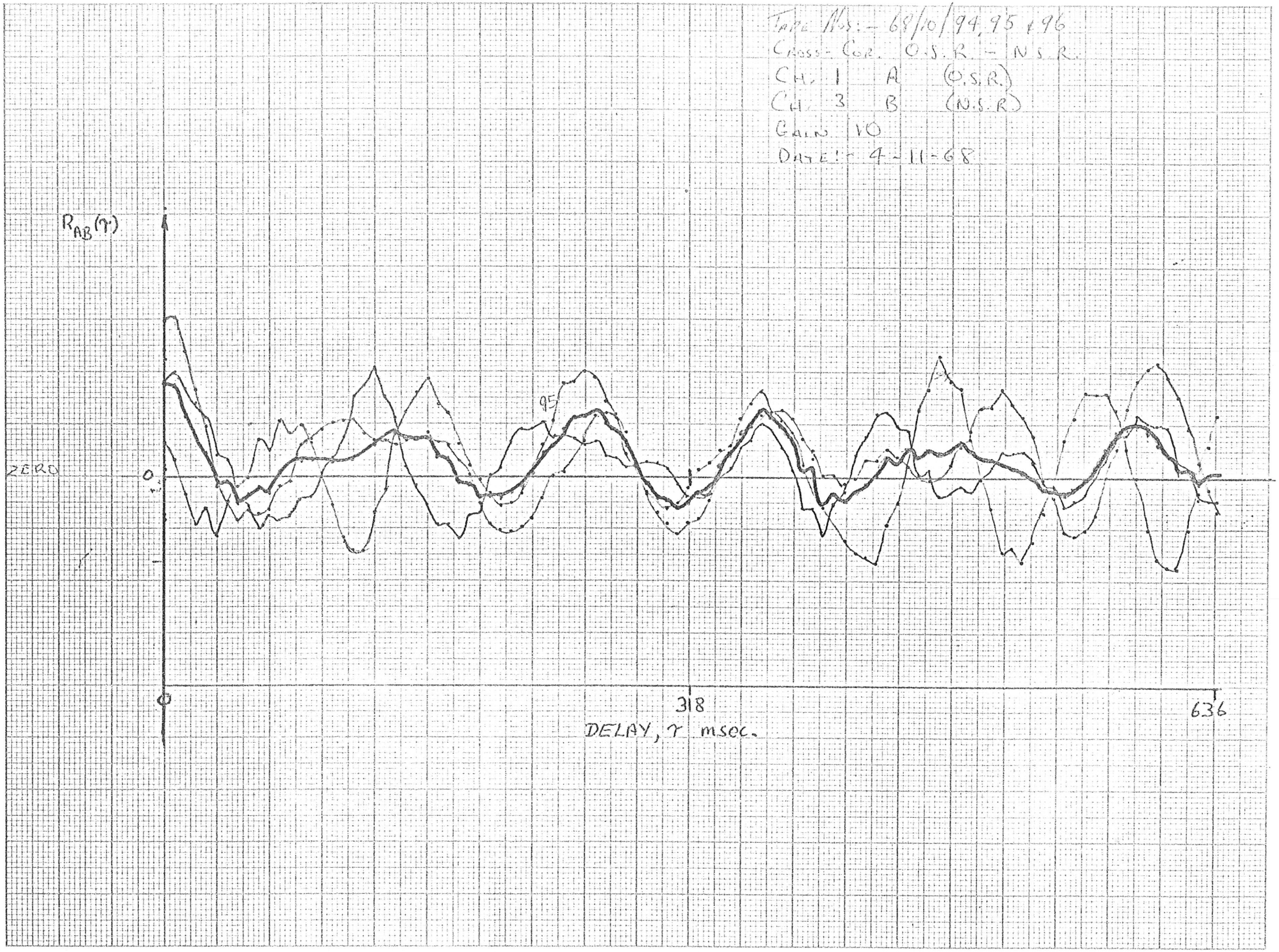

FIG. 27. CROSS CORRELATION FUNCTION, VERTICAL ACCELNS., O/S REAR – N/S REAR HUBS, UNREPAIRED ROAD, 30 MPH.