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The creep behaviour of fibre reinforced plastics

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Synopsis

This paper describes the tensile creep behaviour of fibre reinforced plastics under a uniaxial stress system. In particular, unidirectionally and bidirectionally reinforced plastics are considered for the relative ease with which samples of them can be manufactured.

A composite model that has previously been described^{1,2} is used as the basis of theoretical predictions of the variation of the creep compliance with certain material parameters. The parameters whose effects are examined are the volume fraction of the fibres, the distribution of fibre orientations, and the angle between a specified direction and the direction of application of a uniaxial stress system.

Specimen experimental results are included to show the accuracy of these predictions and any shortcomings of the theoretical model are examined in the light of this comparison.

1. Introduction

The stress-strain behaviour of any continuously reinforced composite, where the reinforcement is of a fibrous form, depends on the Young's Modulus and Poisson's Ratio, or their time-dependent equivalents, of the two phases, the distribution of the fibre orientation and the volume fractions of the phases. To calculate this behaviour exactly, it would be necessary to ensure that the equilibrium and compatibility conditions around the individual fibres were satisfied. For most cases this would be an excessively lengthy and difficult process and in consequence a model of the behaviour of fibre reinforced composite materials that gives a reasonably accurate description is required for general use.

Voigt³ and Reuss⁴, in the related field of the mechanical properties of polycrystalline aggregates, proposed that the properties of the composite can be obtained by a volume weighting of the moduli or the compliances respectively of the two phases. It has since been shown by Hill⁵ that these two proposals, based on assumptions of uniform strain or uniform stress throughout the composite, give rise to bounds to the possible behaviour of any composite fabricated from a particular volume ratio of the two phases. Neither of these proposals take any account of the geometry of the phases and so the bounds to the behaviour of the composite are widely separated

Arridge⁶ has proposed a simple model which represents a composite, formed from a mat of straight and continuous fibres embedded in a homogeneous isotropic matrix, by assuming that the two phases are subjected to equal strains. His representation of the fibrous mat assumes all the fibres to be straight, continuous and only capable of transmitting loads in tension as in the 'Ideal' fibrous mat described by Cox⁷. Cox has summed the effects of all his fibres using a distribution function describing the fibre orientations, and Arridge uses the same procedure. This simple model contains no contribution from the flexural or shear stiffness of the fibres as well as ignoring Poisson's Ratio effects and interaction between the phases. The description of composite behaviour obtained from this method contains large errors when considering modes of deformation other than tensile extension parallel to the fibre axis in a unidirectionally reinforced composite.

To eliminate these errors we must consider both phases to be isotropic and homogeneous and also allow for some interaction between the phases. In order to do this Dootson^{1,2} has considered the composite to be built up from simple composite elements of the type shown in Figure 1. They consist of many infinitely long parallel fibres embedded in a cylinder of the matrix material with its axis parallel to the fibre axes. The fibres are placed randomly in this element and the element is assumed to be large enough to be macroscopically homogeneous. This is the same type of element as Hashin and Rosen have used to describe the behaviour of a unidirectionally reinforced composite. The elastic constants of the element, which they have calculated using variational methods, have been used by Dootson as the basis of a general model which covers all planar fibre reinforced composites. The

orientation of fibres in the composite is described by an orientation function in the same way as Cox has done. This orientation function also describes the orientation of the composite elements. These composite elements can be added together assuming either a uniform stress or uniform strain field throughout the composite material. Consequently two limits to the stress-strain behaviour of an composite are obtained rather than an exact solution. Because of the boundary conditions that Hashin and Rosen applied to their element, these limits should only coincide for a unidirectionally reinforced composite.

As we are considering time-dependent matrix materials it is necessary to summate these elements using a Laplace transform technique. The calculations that have been performed to produce the data given in this paper have therefore been computerised.

2. Experimentation

2.1 Materials used

For reasons of ease of manufacture of specimens and of allowing a knowledge of the orientation distribution of the fibres, it has been decided to produce samples of a composite material by a filament winding technique.

The most convenient form in which continuous 'E'-glass fibres are produced commercially is that of rovings. These consist of several parallel strands each composed of 204 individual filaments, bundled together but not twisted. As the main difficulty associated with filament winding is the complete impregnation of the rovings with resin it was decided to use a roving with as few strands as possible. The roving being used is produced by Fibreglass Ltd. and consists of 12 strands.

As the winding process takes about an hour to place one layer of glass onto the mandrel, the resin must be of a type that is slow to gel. Also if the resin is of a low viscosity the air is more easy to remove from the composite. Consequently Araldite MY 753 with HY 951 hardener was chosen as being the most suitable system as it has a gel time of $2\frac{1}{2}$ hours at room temperature and a viscosity of less than 20 poises.

The resin is cured at room temperature for 24 hours and then postcured at 50°C for 30 days to ensure that it has reached a state where the material properties will not alter at the temperature at which the testing will be carried out. As the testing occurs in a temperature and humidity controlled room these remain at a constant level. The temperature is $20.5 \pm 1^{\circ}\text{C}$ and the relative humidity is $50 \pm 5\%$.

2.2 Tensile creep testing machine

The tensile moduli of the materials being tested are likely to lie between 2×10^{10} dynes/cm² and 50×10^{10} dynes/cm² and in order to produce

extensions large enough to be measured accurately, stresses must be applied to the specimen that are larger than can conveniently be applied using a direct loading system. It has consequently been decided to use a lever arm with a ratio of 10:1 to apply the stress to the specimens. The machine has been designed such that the load is applied along the geometric axis of the specimen and that no bending or twisting of the specimen can occur.

The moduli of the specimens to be tested are sufficiently high for an extensometer to be attached directly to the specimen without causing any large deflections. The sensitive element of the extensometer that is in use is a Sogenique linear displacement differential capacitance transducer having a travel of 0.1". This can read the separation of the two ends of the gauge length to 0.00005" and over a 3' gauge length the resulting strain can be measured to within 1% if the minimum strain measured is 0.015%.

2.3 Properties of the component phases

In order to be able to predict how the composite materials will behave, it is necessary to measure the properties of the component phases. As each of the phases is isotropic and homogeneous it is only necessary to measure the tensile compliance and Poisson's ratio, or their time-dependent equivalents, to allow a full characterisation of their stress-strain behaviour.

For the 'E'-glass fibres that have been used as the reinforcing phase, these two properties have been obtained from the manufacturers of the material. These values are:-

$$J_f = 1.365 \times 10^{-12} \text{ cm}^2/\text{dyne}$$

$$\nu_f = 0.20$$

In order to measure these properties for the resin it has been necessary to carry out a series of creep tests on specimens of the unreinforced resin. These tests have been performed at various stress levels in order that the onset of non-linear behaviour in the resin can be found. Figure 2 shows the measured variation of the compliance of the MY 753 resin both with time and stress. At stresses below 6.4×10^7 dynes/cm² it can be seen that the behaviour is linear as the compliance curves below this stress superimpose. Above this stress level however the compliance curves begin to diverge. Consequently, in order to be able to predict the behaviour of composite materials from data obtained in the linear region, we must work in a stress range that corresponds to an initial composite strain of less than 0.2%. This strain corresponds to the upper limit of the linear region of the resin behaviour.

To describe the shape of this compliance curve, a curve fitting technique has been used based on the expression

$$J(t) = S_0 + \sum_{i=1}^n S_i (1 - e^{-t/\delta_i})$$

which uses a discontinuous retardation time spectrum with retardation times, δ_i , chosen at equal intervals of logarithmic time such that they occur at approximately half decade intervals. The S_i are the corresponding values of the spectrum, to be found by a linear regression technique, and the n describes the number of δ_i required to fill the time range over which experimental data is known. The values of S_i calculated to fit the low stress compliance data are plotted in Figure 3. The curve they describe is drawn in Figure 2.

The time-dependent equivalent of the Poisson's ratio, the creep contraction ratio, of this resin has been measured by Darlington using the contraction measuring device that he describes in Paper 1. The 100 second value of the creep contraction ratio that he obtains is 0.388 at stress levels where the resin is behaving in a linear fashion. The time-dependence of the creep contraction ratio for this resin behaves very much in the way that Turner³ has suggested, in that the Bulk modulus of the resin appears to be independent of time. The creep contraction ratio is thus described by

$$\eta(t) = \frac{1}{2} \left(1 - \frac{E(t)}{3K} \right)$$

and hence, knowing the 100 second value, we can calculate its value at any time.

3. Theoretical results

By substituting the material properties of the two isotropic phases into the elemental model of the composite it is now possible to predict the time-dependent stress-strain behaviour of any fibre reinforced composite made from these materials.

The range of composite materials with which the theoretical results will be compared are all fabricated by filament winding techniques. Let us examine the effect of varying the geometric variables within the range that is obtained using these fabrication techniques. These variables are:

(a) Volume fraction of fibres.

The range of v_f obtained using the filament winding machine is between 0.35 and 0.55. This range is likely to cover many filament wound articles made by normal manufacturing processes.

(b) Orientation distribution function.

Using the filament winding technique the fibre distributions obtained using a square plate as a mandrel are unidirectional and bidirectional. The number of fibres in each direction can vary from 0 to 100% of the total.

(c) The angle between the applied stress and a specified direction.

The 0° direction is specified as being parallel to the direction in which there are most fibres. To vary the angle between this and the applied stress it is necessary to cut the tensile creep specimens at different angles to the fibre axis.

The creep compliance also varies with time and as creep tests will be conducted for 100,000 seconds, the creep compliance will be predicted up to this time. The applied stress will not be considered as a variable as the stress will be at a level where the resin behaves linearly and the composite compliance is thus independent of stress.

3.1 Unidirectional reinforcement.

The basic model upon which the predictions are based is the simple unidirectionally reinforced element and so the first predictions to be made will be for the creep compliance of unidirectionally reinforced MY 753. Figure 4 shows the computed variations of the compliance of this system both with angle and time for a volume fraction of fibres of 55%.

The variation of the compliance with angle is as one would expect from physical considerations with the minimum compliance parallel to the fibre direction and the maximum compliance at about 60° to this. The time-dependence varies in a similar fashion with angle as very little creep is predicted parallel to the fibres and most creep occurs at 60° to this direction. From the separation of the isochronous curves it can be seen that the composite is creeping at an increasing rate with respect to logarithmic time in all directions.

The variation of the 100 second compliance of this unidirectional system with the volume fraction of the fibres present is shown in Figure 5. The shapes of the three curves shown are very similar but the reduction of the compliance with increasing volume fraction is greater parallel to the fibre axes than in the direction of the maximum compliance. The compliance in the fibre direction decreases by 20% for an increase in the volume fraction of the fibres from 45% to 55% while in the direction of the maximum compliance this decrease is only 13%.

3.2 Bidirectional reinforcement

A bidirectionally reinforced composite is built up by adding together the simple unidirectional elements. As these can be added assuming either that the stress field or the strain field remain constant, two limits to the compliance within which the real behaviour must lie are obtained. In Figure 6 three bidirectionally reinforced systems are compared with a unidirectional system having the same volume fraction of fibres. The only difference between the cases is the distribution of the fibres which varies from the unidirectional case ($x = 1, y = 0$) to the symmetrically bidirectional case ($x = 0.5, y = 0.5$).

As the number of fibres perpendicular to the 0° direction increase, so the uncertainty about the actual value of the compliance becomes greater and the limits become more widely separated. By the nature of the model these limits coincide at 45° for each of these cases, and the value of the compliance at 45° remains constant for varying x and y . In changing the fibre distribution from unidirectional to symmetrically bidirectional the direction of the maximum compliance has shifted from 60° to 45° while the magnitude of this compliance has decreased by about 10%.

It is interesting to note how the limits predicted by this elemental method are greatly superior to the Voigt and Reuss bounds. In Figure 6, as well as the elemental predictions for the tensile compliance of these bidirectional composites, are included the Voigt and Reuss bounds to a composite composed of these two materials with a volume fraction of glass of 45%. The Voigt bound can be seen to always lie below the predicted compliance while the Reuss bound is always greater. This must, however, be the case as these bounds require no knowledge of the phase geometry.

4. Comparison of experimental and theoretical results

Having seen how the elemental model predicts the creep compliance to vary with variations of several parameters it is now of interest to compare these predictions with experimental results that have been measured.

In Figure 5, which shows the predicted variation of the compliance of a unidirectional composite with volume fraction and angle, are included a set of 100 second compliances measured at various angles to the fibre direction. The average volume fraction of this group of specimens which were cut from one sheet of the composite material, was measured to be 37%. The volume fraction of each specimen was measured by burning the resin from the gauge length of the specimen at a temperature of 680°C for 15 minutes and comparing the weights before and after burning. The experimental points in Figure 5 all appear to lie on a line corresponding to a volume fraction of 41%. This discrepancy of 4% in the volume fractions of the specimens described is typical of the error normally found between the measured and expected volume fractions. This error is thought to be due to the burning off of the layer of finish, which is applied to the surface of the glass fibres during manufacture, while burning the resin from around the fibres. Tests are being conducted to ascertain the exact nature and magnitude of this error. It will be assumed here that an error of this type exists and all further results described here will be corrected for this error.

Having shown that the variation with angle of the compliance is of the same form as predicted for the unidirectional composite, the next step is to examine the variation with time of typical compliances. Figure 7 shows several creep curves of unidirectionally reinforced MY 753 specimens out at various angles to the fibre axes. The circles represent the experimental results and the lines the theoretical predictions of the variation

with time of the compliance for the particular angles and volume fractions associated with the tested specimens. The agreement here is very good and it therefore appears that the elemental model gives an accurate prediction of the variation of the compliance of unidirectionally reinforced composites with time, volume fraction and angle.

The next step is to examine how accurately this elemental model can be extended to bidirectional composites. If the model is good then this can be shown by testing specimens cut at 45° to the fibre axis, for which the upper and lower limits to the compliance coincide. In Figure 8 the creep compliance of a symmetrically bidirectional composite at 45° to the fibre axes is shown. The predicted compliance compares very well with the experimental results for this specimen. Having shown that this check gives good agreement, it is of interest to see the comparison of the creep result at some other angle with the two limits predicted. Figure 8 also shows the experimental results for the creep compliance of a symmetrical bidirectionally reinforced specimen cut at 0° to one fibre axis. These results are compared with the upper and lower limits predicted using the elemental model. The experimental results can be seen to be near to the lower limit of the compliance, indicating that, for this particular case, the assumption of a uniform strain field throughout the composite is better than that of uniform stress.

5. Conclusions

In order to make any general conclusions further experimental data is required. On the basis of the results examined here, however, we can make some relevant remarks about the general accuracy of the elemental method in describing the creep compliance of unidirectional and bidirectional composites.

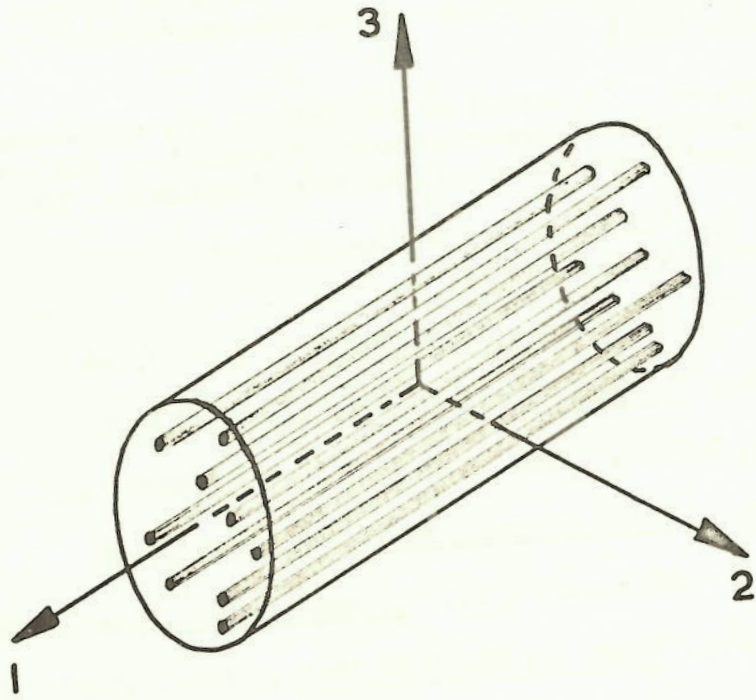
The results for unidirectional composites agree within experimental error with the predictions. This would be expected if the fabricated specimens are of a high standard as the elemental model should be an exact description of this case.

With bidirectional composites, however, we can in general only give an approximate indication of the exact value of the compliance in theory. It is therefore of interest, although without any confirmatory evidence no more can be said, that the bidirectional result at 0° lies near the lower limit that was predicted. If this trend can be shown to continue then the assumption of a constant strain field can be made. This would increase the usefulness of the elemental model considerably as at present the main drawback of this method is the lack of an exact solution for a general fibre distribution. The limits of the existing solution are however more accurate than the simplified fibre model⁶ as has been shown elsewhere.²

6. References

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FIG 1



Composite Element

FIG 2

STRESS 10^7 dynes/cm² CREEP COMPLIANCE OF MY753

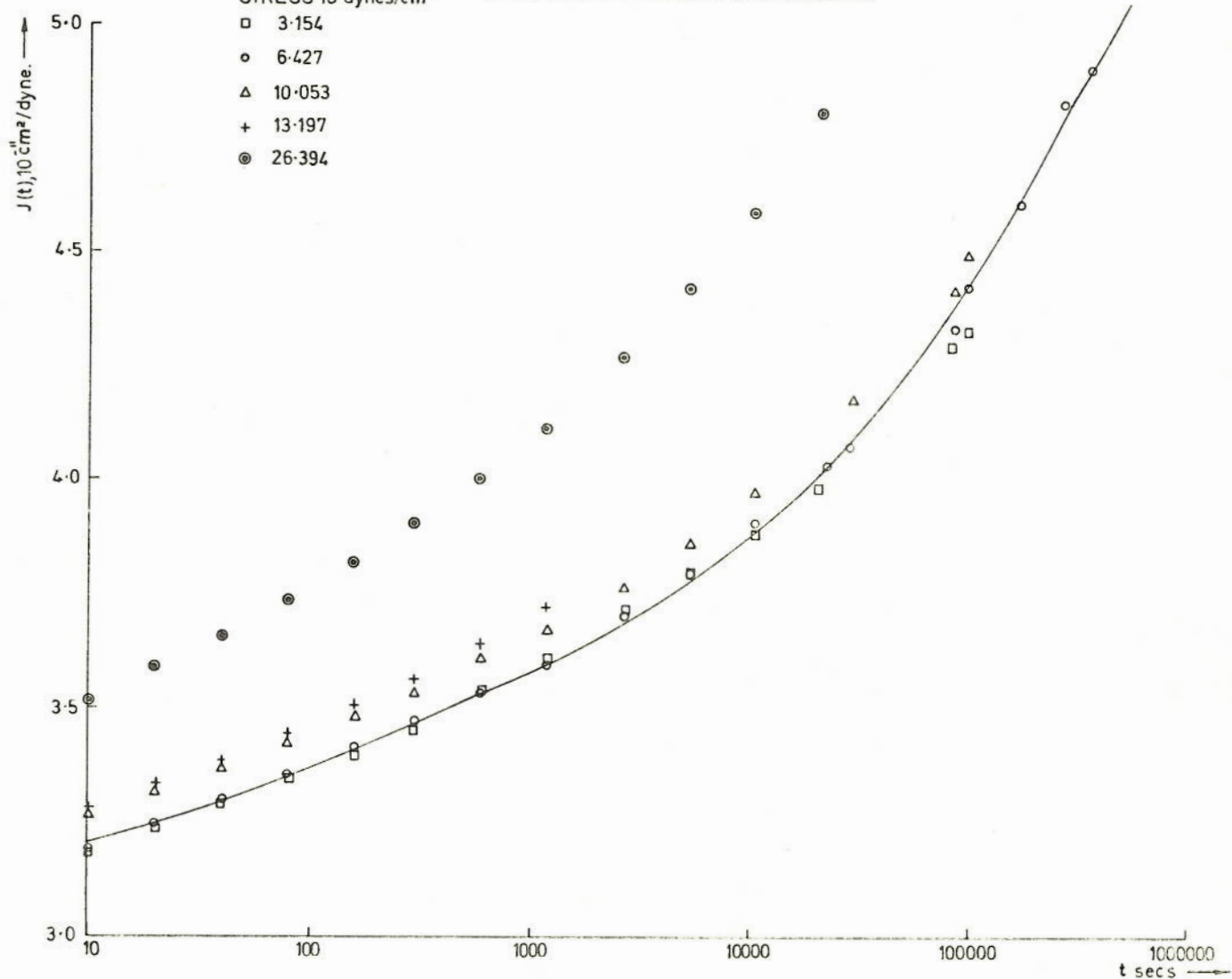
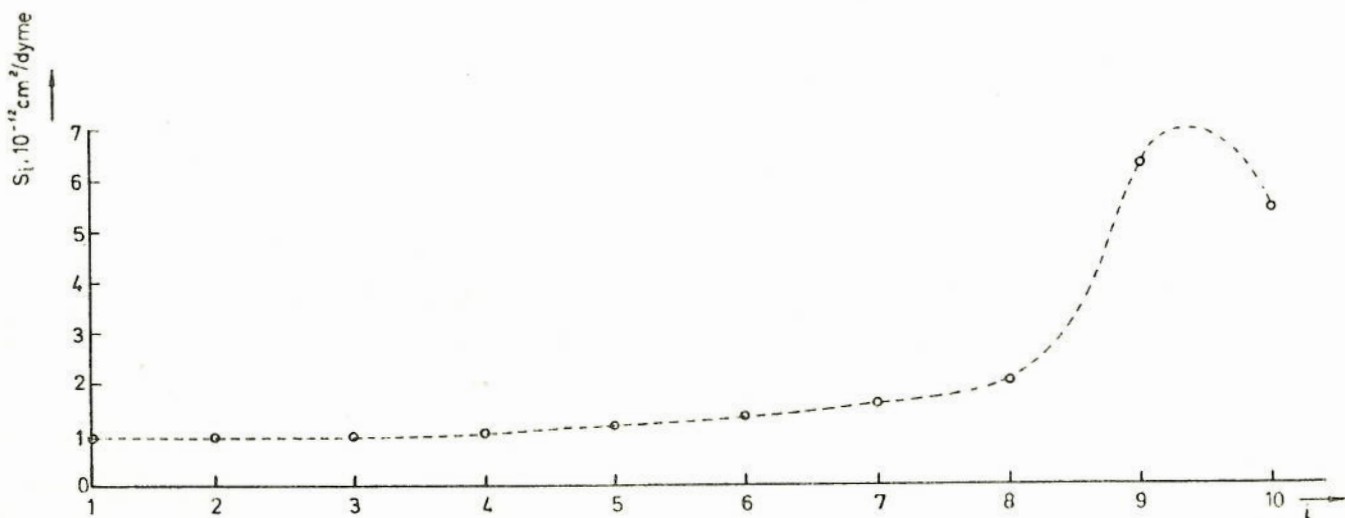


FIG 3

DISCONTINUOUS RETARDATION TIME SPECTRUM FOR MY753

$$S_0 = 3.097 \times 10^{-11} \frac{\text{cm}^2}{\text{dyne}}$$



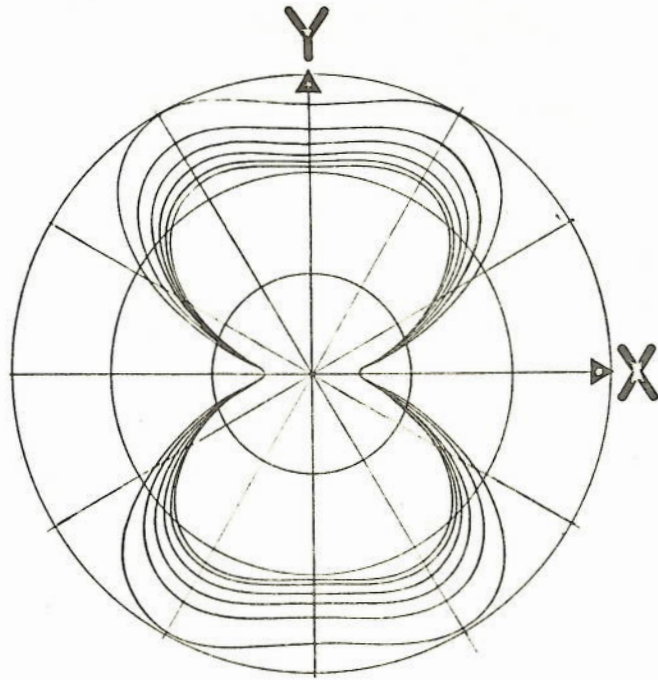


FIG 4. VARIATION OF THE CREEP COMPLIANCE OF UNIDIRECTIONALLY REINFORCED MY753 WITH TIME. $V_f=0.55$
 $t = 0, 10, 100, 1000, 10000, 100000,$

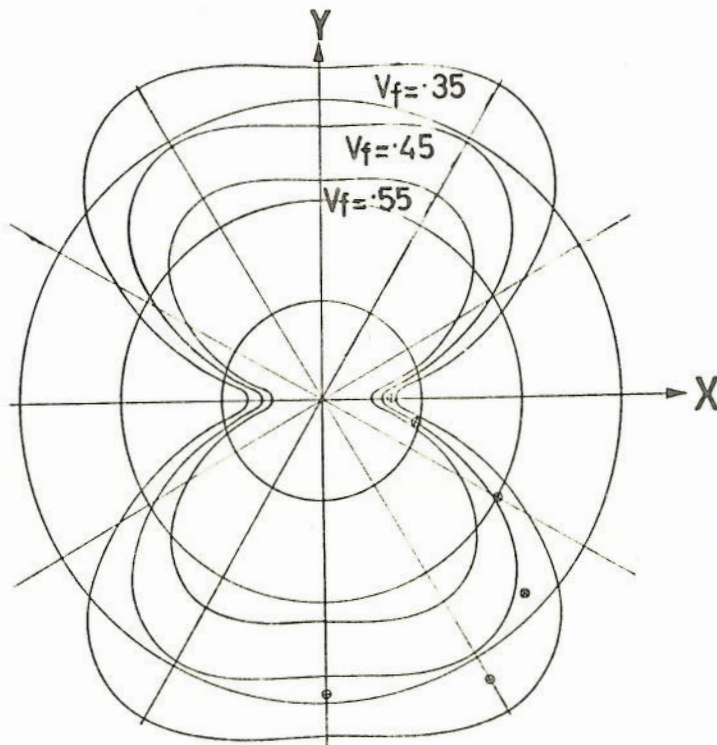


FIG.5 VARIATION OF THE 100 Sec. CREEP COMPLIANCE OF UNIDIRECTIONALLY REINFORCED MY753 WITH THE VOLUME FRACTION OF FIBRES PRESENT.

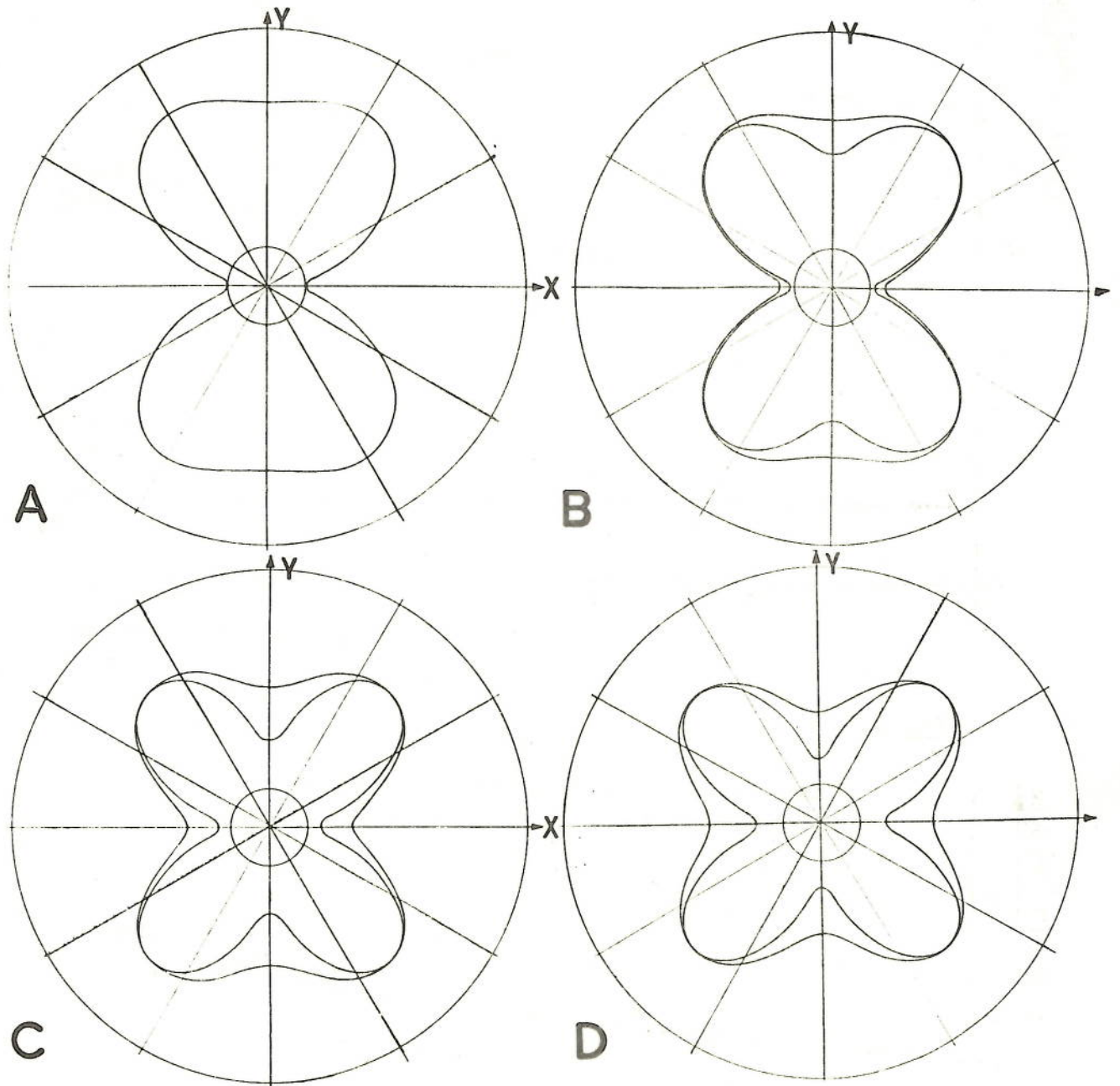


FIG 6. VARIATION OF THE 100 Sec. CREEP COMPLIANCE OF BIDIRECTIONALLY REINFORCED MY753 WITH THE PERCENTAGE OF FIBRES IN BOTH DIRECTIONS. $V_f = 0.45$

a) $x = 100\%$ $y = 0\%$

b) $x = 90\%$ $y = 10\%$

c) $x = 70\%$ $y = 30\%$

d) $x = 50\%$ $y = 50\%$

FIG. 7.

Tensile creep compliance of unidirectionally reinforced MY753.

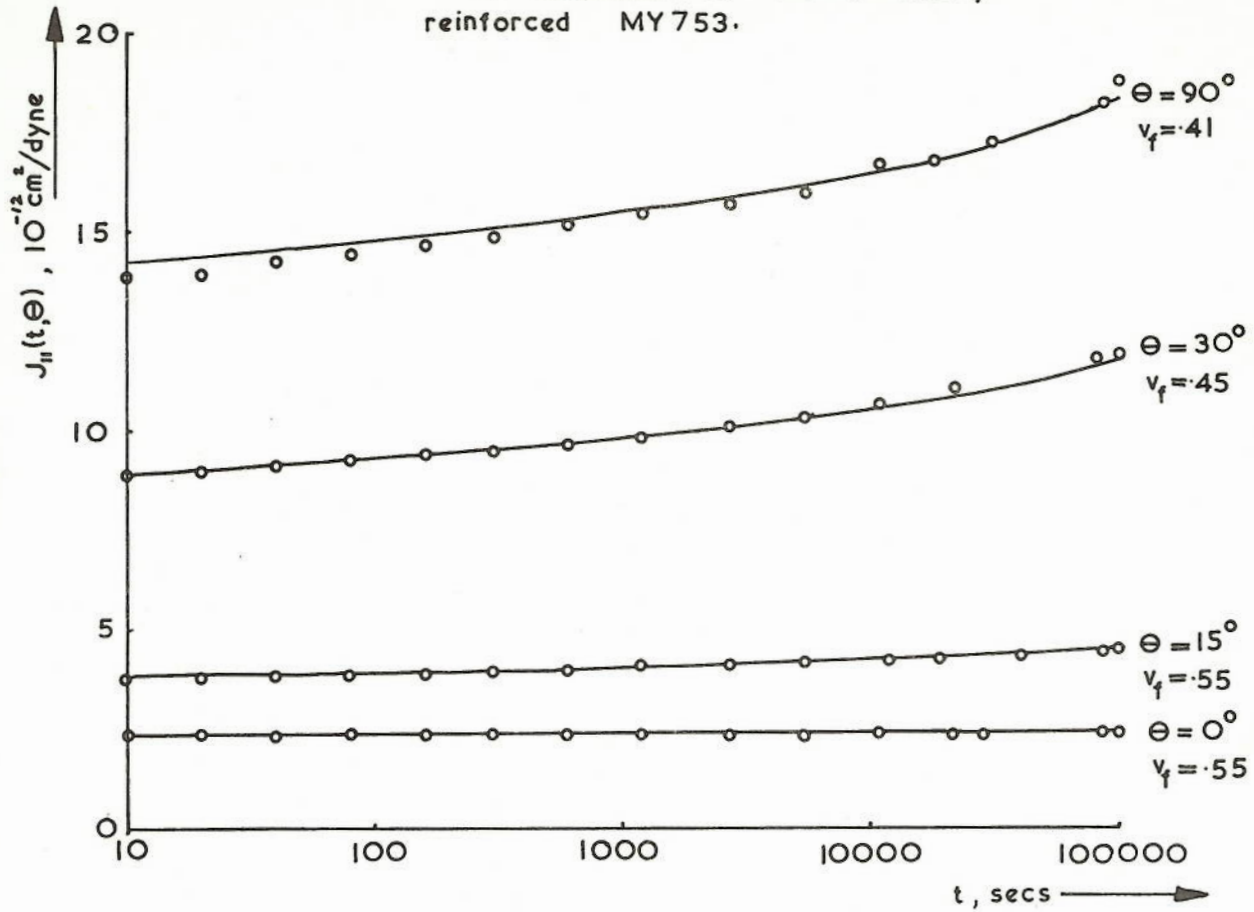


FIG. 8.

Tensile creep compliance of bidirectionally reinforced MY753. $x = y = 0.5$; $v_f = .54$.

