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Note on the Limits to the Local Mach Number on an Aerofoil in Subsonic Flow

-by-

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SUMMARY

It has been noted in some experiments that the local Mach number just ahead of a shock wave on an aerofoil in subsonic flow is limited, values of the limit of the order of 1.4 are usually quoted. This note presents two lines of thought indicating how such a limit may arise. The first starts with the observation that the pressure after the shock will not be higher than the main stream pressure. Fig.1 shows the calculated relation between local Mach number ahead of the shock (M_{S1}), shock inclination (ξ), mainstream Mach number (M_1) and pressure coefficient just aft of the shock, (C_p). It is noted that, for given M_1, C_p and ξ, two shocks are possible in general, a strong one for which $M_{S1} > 1.48$, and a weak one for which $M_{S1} < 1.48$; and it is argued that the latter is the more likely. The second approach is based on the fact that a relation between stream deflection (δ) and Mach number for the flow in the limited supersonic regions on a number of aerofoils has been derived from some experimental data. Further analysis of experimental data is required before this relation can be accepted as general. If it is accepted, however, then it indicates that the Mach numbers increase above unity for a given deflection is about one-third of that given by simple wave theory (Fig.2). An analysis of the possible deflections on aerofoils of various thicknesses (Fig.3) then indicates that deflections corresponding to local Mach numbers of the order of 1.5 or higher are unlikely except at incidences of the order of 5° or more, and may then be more likely for thick wings than for thin wings. Flow breakaway will make the attainment of such high local Mach numbers less likely.

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1. Introduction

It has been noted in some experiments that ahead of a shock wave on a wing section in subsonic flow the ratio of static to reservoir pressure, and therefore the local Mach number, tends to some limiting value with increase of main stream Mach number, and then remains roughly constant whilst the shock wave moves back towards the trailing edge. At this stage, therefore, the magnitude of the minimum static pressure coefficient ahead of the shock tends to decrease with further increase of the main stream Mach number. If this result is accepted as generally true, it can be of considerable assistance in developing a qualitative explanation of the aerodynamic characteristics of a section in the transonic range, when account is taken of the relative growth and movement of the shock waves on the upper and lower surfaces. It is obviously desirable, therefore, to investigate more fully, both by further experiment and analysis, the validity of the above conclusion that the local Mach number is limited in the way described. The following preliminary discussion is by no means a proof but is offered as a stimulant to those who are thinking about this problem.

2. The limit on local Mach number determined by the pressure after the shock

The shock wave may be regarded as the means whereby part of the compression takes place from the maximum suction in the supersonic region to the free stream pressure far downstream, when completely isentropic compression is impossible. It may be noted from such experimental data as is available (see, for example, Ref.1) that the compression across the shock is never such as to overshoot the free stream pressure; in other words the pressure just behind the shock is always less than the pressure in the free stream. It can be argued that in any natural process the energy wasted is likely to be a minimum, and it is to be expected that something less rather than something more than the total compression required will occur in the relatively wasteful process of a shock wave.*

Writing suffix 1 to denote main stream conditions, suffix S1 to denote conditions just ahead of the shock and suffix S2 to denote conditions aft of the shock, we have therefore

$$p_{S2} < p_1. \qquad \ldots\ldots(1)$$

The pressure coefficient C_p just aft of the shock is given by

$$C_p = \frac{p_{S2} - p_1}{\frac{1}{2}\rho_1 U_1^2} = \frac{2}{\gamma M_1^2}\left(\frac{p_{S2}}{p_1} - 1\right)$$

and hence

$$\frac{p_{S2}}{p_1} = 1 + \frac{\gamma M_1^2}{2} \cdot C_p$$

$$= 1 + 0.7\, M_1^2\, C_p, \qquad \ldots\ldots(2)$$

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* This fact is not essential to the argument that follows, but it provides a limit to the range of pressure aft of the shock that need be considered.

where from (1) $C_p < 0$. The value of γ is taken as 1.4.

The Mach number and pressure just ahead of the shock are related to the corresponding quantities in the main stream by the equation

$$\frac{p_{S1}}{p_1} = \left(\frac{5 + M_1^2}{5 + M_{S1}^2}\right)^{7/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Across the shock we have

$$\frac{p_{S2}}{p_{S1}} = \frac{7 M_{S1}^2 \sin^2 \xi - 1}{6}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

where ξ is the angle between the shock and the incident stream

Hence, from (2), (3) and (4)

$$1 + 0.7\, M_1^2 .\, C_p = \left(\frac{5 + M_1^2}{5 + M_{S1}^2}\right)^{7/2} \left(\frac{7 M_{S1}^2 \sin^2 \xi - 1}{6}\right) \ldots(5)$$

Writing $H\left(M_1, C_p\right) = \left(5 + M_1^2\right) \Big/ \left(1 + 0.7\, M_1^2\, C_p\right)^{2/7}$

and $G\left(M_{S1}^2, \xi\right) = \left(5 + M_{S1}^2\right) \Big/ \left(\frac{7 M_{S1}^2 \sin^2 \xi - 1}{6}\right)^{2/7}$,

then it follows from (5) that given the values of M_1 and C_p, the values M_{S1} and ξ must satisfy

$$H\left(M_1, C_p\right) = G\left(M_{S1}^2, \xi\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

The functions $H\,(M_1, C_p)$ and $G\left(M_{S1}^2, \xi\right)$ are plotted in Fig.1. The former is plotted against M_1 for values of C_p of 0, -0.1, -0.2, and -0.4, the latter is plotted against M_{S1}^2 for values of ξ of 60°, 70°, 80° and 90°. It will be noted that for any particular value of $H\,(M_1, C_p)$ and a chosen ξ there may be two values, one value or no value of M_{S1}^2 that satisfies (6). The single value corresponds to the turning point at the base of the appropriate curve of the function G. The interesting feature here is that all these turning points occur at approximately the same value of M_{S1}^2, namely, $M_{S1}^2 = 2.2$, or $M_{S1} = 1.48$ (more accurately the turning value corresponds to $M_{S1}^2 = 2 + \frac{1}{5 \sin^2 \xi}$.

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For example, taking $M_1 = 0.8$, and $C_p = -0.2$, then $H(M_1, C_p) = 5.81$, and the possible combinations of M_{S1}^2 and ξ range from $M_{S1}^2 = 1.24$, $\xi = 90^\circ$, through $M_{S1}^2 = 2.26$, $\xi = 70^\circ$, and then with increasing ξ to $M_{S1}^2 = 3.86$, $\xi = 90^\circ$. For all such possible combinations for given values of M_1 and C_p the entropy change across the shock increases with increase of M_{S1}. This follows from the fact that the entropy change increases with increase of the Mach number component normal to the shock, i.e., with $M_{S1} \sin \xi$, and for a given value of G, we have

$$M_{S1}^2 \sin^2 \xi = \frac{1}{7} \left\{ 1 + 6 \left[\frac{5 + M_{S1}^2}{G} \right]^{7/2} \right\}.$$

Hence, for constant G, $M_{S1} \sin \xi$ increases with M_{S1}.

We cannot specify the value of ξ near the surface in any particular case, that will presumably depend on the boundary conditions and in particular on the interaction of the boundary layer and shock wave. If we accept, therefore, that the value of ξ is determined by the boundary conditions, and the value of C_p is determined by the shape of the wing aft of the shock and the behaviour of the boundary layer, there will be in general two possible shocks, a weaker shock corresponding to $M_{S1}^2 < 2.2$ and a stronger shock corresponding to $M_{S1}^2 > 2.2$. Again, we may argue that in such a case (as in the case of a wedge in supersonic flow) the weaker shock is most likely to occur in practice, and hence the local Mach number ahead of the shock is unlikely to exceed 1.48.

In this discussion no account has been taken of the possibility of the shock being bifurcated. In the usual theory of the bifurcated wave it is assumed that the pressure drop across the bifurcation is the same as that across the main wave. If this were true then the salient points of the above discussion would still apply. If, however, appreciable changes in pressure and Mach number occur between the point of bifurcation and the surface on either side of the shock, then the discussion does not apply.

3. Relation between local Mach number and slope of surface

The following presents a different line of attack on the problem. In a simple wave motion in supersonic flow (i.e., one in which only one family of straight characteristics is present) there is a unique relation between the difference in the Mach numbers at two points on a streamline and the difference in the slopes of the streamline at those points. Starting with a datum of the tangent to the streamline where a local Mach number of unity is first attained, the relation between the angle of the stream deflection (δ) relative to this datum and the corresponding local Mach number (M) is shown in Fig.2. On the surface of a wing section at a Mach number greater than the critical we may expect that in the limited supersonic region the deflection for a given Mach number will be greater than that of a simple wave motion. This is because the backwardly inclined expansion waves from the surface are reflected from the sonic boundary back to the surface as forwardly inclined compression waves which help to increase the stream deflection whilst tending to reduce the Mach number. In Fig.2 are plotted some results due to Göthert (taken from Fig.6 of Ref.2) of stream deflections against Mach numbers in the region of 0.85. It will be noted that these results are reasonably well described by the mean curve drawn through them. Until a great deal more data have been analysed it would be premature to conclude that in general the relation between δ and M is unique and given by some such curve. If, however, for the sake of argument we accept this hypothesis then it appears from Fig.2 that the Mach number increase above unity attained for a given deflection on an aerofoil is of the order of one-third of that given by the simple wave theory. Thus, to attain a Mach number of 1.4 a deflection of the order of 34° is required and a Mach number of 1.5 would require a deflection of about 40°. But the possible deflection is limited by the shape of the aerofoil and the point on it where the supersonic flow begins. From a study of the available data it appears that this point moves very little with increase of Mach number once the shock is well defined and is in general surprisingly little forward of the maximum suction point in incompressible flow. Fig.3 summarises some data derived from Ref.1 and shows, for symmetrical wings of various thicknesses, the angle Θ between the slope of the surface at the beginning of the upper surface supersonic region relative to the chord line plotted against incidence. In each case the value of Θ applies to a range of main stream Mach number from a value a little higher than that at which a shock wave first appears to the highest tested. The latter depends on the wing thickness and incidence, for the 6% thick wing at zero incidence it is 0.88, for the 18% thick wing at zero incidence it is 0.85 and at an incidence of 5.7° it is 0.54. At incidences higher than that at which the full curves terminate in Fig.3 it is impossible to specify Θ from the data given in Ref.1 except to note that the supersonic region begins very close to the leading edge, but in some cases there is evidence of a backward movement of the sonic boundary with increase of Mach number. The value of half the trailing edge angle is also noted on Fig.3. The angle $(\Theta + \tau/2)$ is the maximum deflection possible, and, if the above deductions from Fig.2 are accepted, it appears that over the range of thickness and incidence tested, only on the thicker wings at the largest incidences is the Mach number ahead of the shock likely to reach a value of 1.5 or higher. Nothing can be said about the thinner wings outside the incidence ranges covered by the full curves of Fig.3. The possible Mach number will be reduced if the boundary layer thickens appreciably or separates in the region of the shock.

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As a corollary to this discussion we may note that if the aerofoil surfaces are flat towards the trailing edge, as is usually the case, then once a shock wave has reached a point on the surface where little further deflection is possible, little further increase in local Mach number is to be expected with increase in mainstream Mach number. Presumably, however, large movements of the shock are possible in such cases with relatively little increase in the main stream Mach number and this may lead to large changes in pitching and lift characteristics and even oscillations. This suggests that the relations between the shape of a wing over its rear and the variation of its characteristics in the transonic range as well as the stability of the flow are worth investigating.

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REFERENCES

No.	Author	Title, etc.
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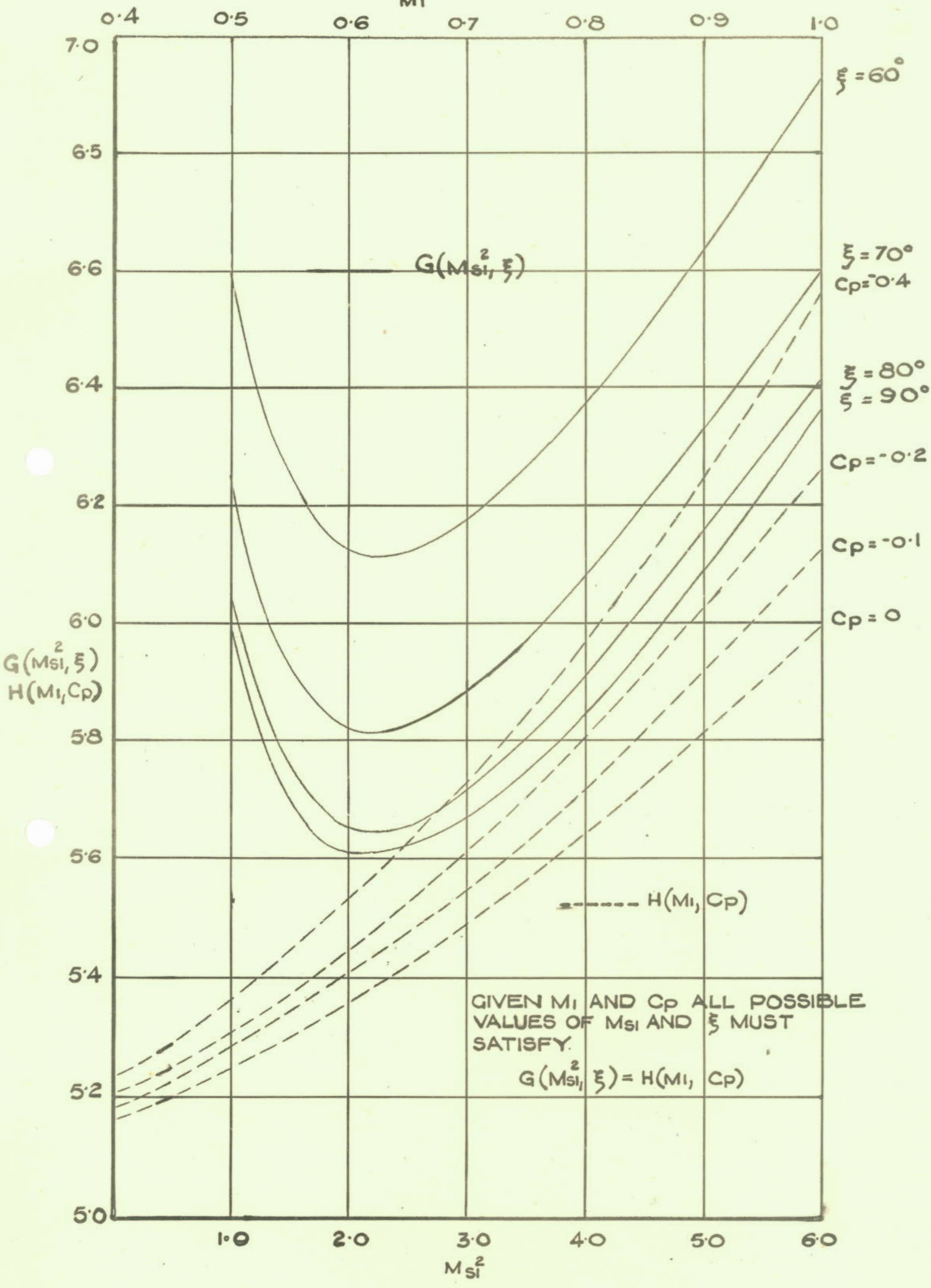
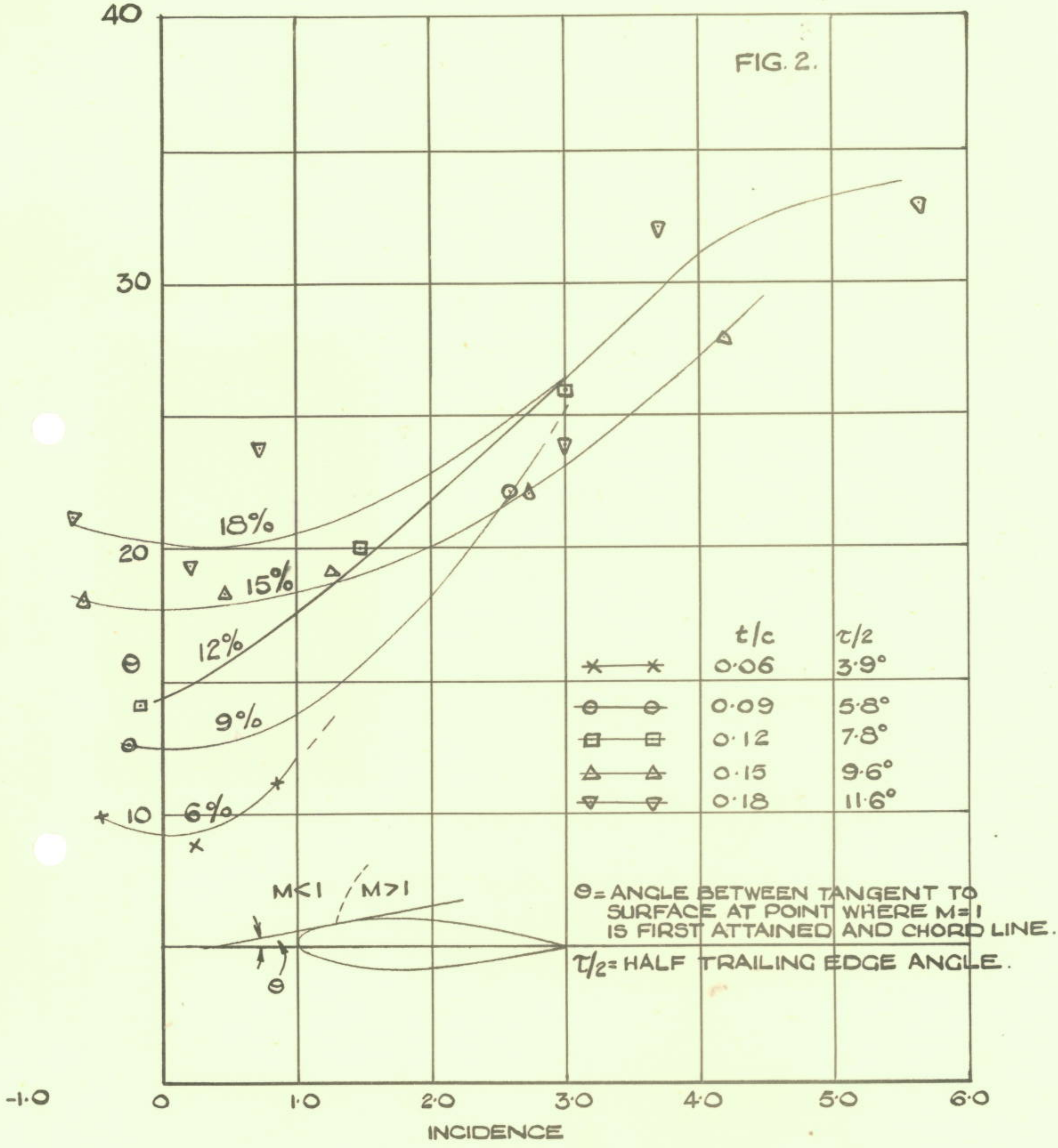
2	B. Göthert	High speed tests on symmetrical profiles of various thickness ratio in the D.V.L. High Speed Wind Tunnel, and comparison with measurements in other wind tunnels. Reports and Translation No. 394, (GDC10/287T).

3. see addendum

FIG. 1.

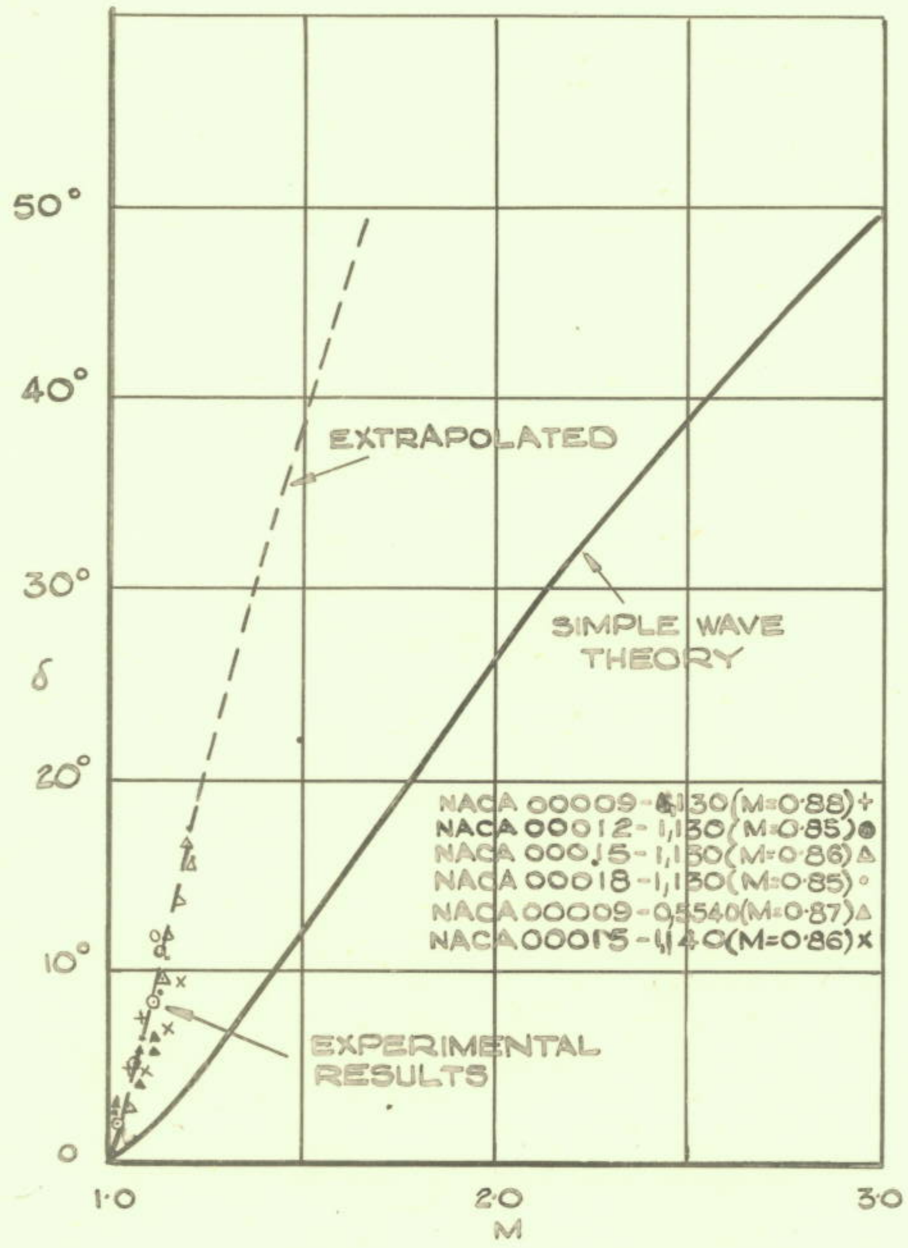


THE FUNCTIONS $G(M_{S1}, \xi) = H(M_1, C_p)$



RELATION BETWEEN θ AND α FOR WINGS OF VARIOUS THICKNESS OF NACA 00 – FAMILY. DATA DERIVED FROM REF. 1.

FIG. 3.



RELATION BETWEEN STREAM DEFLECTION AND LOCAL MACH NUMBER. COMPARISON BETWEEN SIMPLE WAVE THEORY AND EXPERIMENTAL RESULTS FOR ISOLATED AEROFOILS.