CoA Memo. No. 110

WAREHOUSE MODEL

REPORT NO. 2.

by

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INDEX:

1. INTRODUCTION

1.1. Report No. 1 dealt with the simple warehouse model in a static sense. This was sufficient, the only consideration being optimisation of handling, i.e. the problem posed was spatial.

The problem becomes a dynamic one as soon as stock policies are taken into consideration, that is, as soon as the time dimension enters the problem.

1.2. Section 3 of this report deals with the most usual of stock policies, FIFO. Section 4 discusses the rarer policies of MINDIS, a phrase coined by the author to describe the policy of minimising movement in the warehouse regardless of age of stock, - and LIFO. Section 5 details the mathematics used.

2. SUMMARY OF RESULTS

2.1. The investigation of time dependent warehousing systems has resulted in only a slight alteration of the formulae given in Report No. 1. All the formulae remain the same as long as, wherever the terms K is used, one substitutes $K'=(2-\mu)\bar{K}$ if FIFO applies, and $K''=(2-2\mu)\bar{K}$ if MINDIS applies.

2.2. In the case of LIFO the substitution is somewhat more complicated as one needs to replace the formula $\frac{3}{2}(K-1)$ by

$$\frac{3}{4}\bar{K}^{\frac{1}{3}}\left\{(1-\mu)^{\frac{1}{3}}+(2-\mu)^{\frac{1}{3}}+1/\left[(2-\mu)^{\frac{2}{3}}+(2-\mu)^{\frac{1}{3}}+1\right]\right\}-\frac{3}{2}$$

LIFO, however, is the most rarely used stock policy and is in general only applied, because the warehouse lay-out prevents any other policy.

2.3. The proportion γ is given by the ratio $\bar{\bar{K}}/\bar{K}$, where $\bar{\bar{K}}$ is the averaged minimum stock and $\bar{K}$ has the usual meaning of average stock.

2.4. We should point out that in particular the results described in Report No.1. sections 3.7 - 3.13, still hold.

3. MINIMUM HANDLING DISTANCE UNDER FIFO

3.1. The "Saw Tooth Diagram" (Fig.1 and Fig.5) is familiar to all stock controllers. It depicts the stock level over time. The interval between two adjacent peaks is the "replenishment period." The average stock in any replenishment period is given by the mid-point of the line connecting the peak with the lowest point, i.e. the end point. Thus, if the beginning stock level is K_1 and the stock at the end of the period, that is the stock just before a further replenishment arrives, K_2, the average stock during the period is given by $\frac{1}{2}(K_1 + K_2)$

3.2. The average stock level over time can be estimated by averaging the mid-points of all periods. Similarly one can estimate an average maximum stock, say $\bar{K}^*$ and an average minimum stock $\bar{\bar{K}}$, by averaging all the peaks and lowest points on the diagram respectively.

3.3. Conversely, given $\bar{K}^*$ and $\bar{\bar{K}}$, the average stock level over time, $\bar{K}$, is given by $\frac{1}{2}(\bar{K}^* + \bar{\bar{K}})$.

If we write $\mu = \bar{\bar{K}}/\bar{K}$, then $\bar{K}^* = (2 - \mu)\bar{K}$.

μ and $\bar{K}$ can easily be obtained from stock records.

3.4. The total average movement, in and out, in the average replenishment period is, therefore, given by $M = (1 - \mu)\bar{K}$, and the average volume occupied by this movement is $(2-2\mu)\bar{K}$.

3.5. Consider now what happens in the average period, given all movements are average and given the rule of always filling the nearest empty cell and taking from the nearest full cell, modified by FIFO. This means that the shortest distance rule applies only within groups of equally old stock units. Assume first that $\mu = \frac{2}{3}$ and we start with a full store, that is the stock equals $1\frac{1}{3}\bar{K}$, all equally old and located in the nearest $1\frac{1}{3}\bar{K}$ cells. During the first replenishment period $(2-2\mu)\bar{K} = \frac{2}{3}\bar{K}$ units are issued and according to rule are taken from the nearest location, as the entire stock is of the same age.

At the start of the second period $\frac{2}{3}\bar{K}$ units are received and go into the free locations. The $\frac{2}{3}\bar{K}$ issues, however, are taken from the farther half of the store as they are older. At the start of the third period, the $\frac{2}{3}\bar{K}$ incoming stock units go into the locations vacated by issues during the second period, and issues during the third period are the receipts at the start of the second period.

3.6. Obviously, this process repeats itself continuously and equally obviously, the average distance over which a unit, in or out, must be handled is given by formula (3.1) of Report No. 1 which explicitly becomes

$$\bar{d}_{FIFO} = \frac{3}{2} \left[\sqrt[3]{(2-\mu)\bar{K}} - 1 \right] \qquad (3.1)$$

if the cells are cubical.

The statements about non-cubical cells and handling functions in sections 3.5 and 3.6 of Report No. 1 are valid in this case provided one substitutes $(2-\mu)\bar{K}$ for K in all expressions. The movements in and out are graphically depicted for $1\frac{1}{3}\bar{K} = 26$ in figure 2a.

3.7. Imagine now the same operation with a $\mu = \frac{1}{2}$, again starting from a stock equal to $(2-\mu)\bar{K}$, that is $1\frac{1}{2}\bar{K}$, all equally old. During the first period issues will empty the nearest $\frac{2}{3}$ of the cells, which will be filled at the start of the second period with new receipts. During the second period issues are taken first from the furthest $\frac{1}{3}$ of cells and then from the nearest $\frac{1}{3}$. Receipts at the start of the third period go into the cells vacated during the second period, and issues are taken from the nearest $\frac{2}{3}$ of cells, and so on (See fig.2b).

3.8. Clearly in these conditions the average distance an unit moves is now less than that given in formula (3.1), but not much less. In fact (3.1) gives an upper limit for the average distance, and can be used as a conservative estimate of that average.

3.9. Similar considerations of average movement, in and out, when the ratio μ is greater than $\frac{2}{3}$, illustrated in figs. 2c and 2d, lead us to the same conclusion. Section 5 specifies the exact formulae, but for all practical purposes formula (3.1)is sufficiently accurate.

3.10. In Report No.1, section 4.2. we stated that the imposition of a stock policy such as FIFO may result in a disturbance of the tendency of cubical clustering. Our investigation has shown that this is not so. Imposition of FIFO in effect only expands the space in which movement takes place on average by a factor of $(2-\mu)$. Thus all the formulae given in Report No.1. hold as long as one substitutes the value $(2-\mu)\bar{K}$ everywhere for K, the stock volume.

4. MINIMUM HANDLING DISTANCE UNDER MINDIS AND LIFO

4.1. The discussion in section 3 above leads directly to an appropriate expression for the average distance over which units are moved in the case of MINDIS. Although it may appear that this case has been treated in Report No.1, this is not so as there the static case was considered. In actual fact stock movements have a time dimension, i.e. the saw tooth diagram is a proper representation of stock movement under all policies.

4.2. If no account of age of stock is taken and only the minimum distance rule applies, issues will always be taken from the nearest full cells and receipts stored in the nearest empty cells. Thus, on average movement will take place in a cube of volume $(2-2\mu)\bar{K}$ with origin at the reference point if the cells are cubical and we have

$$d_{MINDIS} = \frac{3}{2}\left[\sqrt[3]{(2-2\mu)\bar{K}} - 1\right] \qquad (4.1)$$

4.3. The statements in section 3.5 and 3.6 of Report

No.1 are also valid in this case, if one substitutes $(2-2/\mu)\bar{K}$ for K in all expressions.

4.4. It is not possible to consider movements under LIFO purely on the basis of average movement and average stocks. If indeed there were no fluctuations from the average, LIFO would be equivalent to MINDIS. One can, however, fairly easily imagine the average picture of a LIFO store.

Starting from a store whose nearest cells are full, a new receipt will occupy the farther locations. During the ensuing period, if issues exceed receipts, all the cells filled at the start of the period will be emptied, and some of the nearest cells as well. The next batch of receipts will occupy these empty cells if it does not exceed the previous issue; if it does some further far cells will be filled. Issues then will be taken first from the cells filled during that period, then from cells filled in the proceeding period and so on. Thus, there will be a tendency for old stock to accumulate in the cells which are about average distance from origin.

4.5. The average maximum stock is given again as $(2-\mu)\bar{K}$. The old stock which moves relatively rarely, will on average be $\mu\bar{K}$, and movement will take place in the space in front and behind this barrier of old stock, which is centred on the mean line of the cube of volume $(2-\mu)\bar{K}$. See fig. 3.

The average distance for cubical cells can thus be approximated by averaging the average distance in a cube of volume $(1-\mu)\bar{K}$ and the average distance of the $(1-\mu)\bar{K}$ farthest cells in a cube of volume $(2-\mu)\bar{K}$.

$$d_{LIFO} = \frac{3}{4}\bar{K}^{\frac{1}{3}}\left\{(1-\mu)^{\frac{1}{3}} + (2-\mu)^{\frac{1}{3}} + 1 \Big/ \left[(2-\mu)^{\frac{2}{3}} + (2-\mu)^{\frac{1}{3}} + 1\right]\right\} - \frac{3}{2} \quad (4.2)$$

Details of derivation of (4.2) are given in section 5. Again the statements of Section 3.5 and 3.6 of Report No. 1 apply given the proper substitution for K derived from (4.2).

5. MATHEMATICAL DERIVATION OF THE FORMULAE

5.1. Given a cube of K cubical cells, and side k, we wish to derive the average distance of the (K-Y) cells farthest away from origin, where the Y cells form a cube, of side y, based on the origin. The volume of the cube, K, is thus divided into four parts: The cube Y, and three rectangular blocks of sides:

k x k x (k-y), y x y x (k-y), and k x y x (k-y), respectively.

5.2. Using the results of Report No.1 we can write the average distance of the cells not in the cube Y, $\bar{d}_y$ as

$$\bar{d}_y = \frac{1}{2}\left\{\frac{k^2(k-y)}{k^3-y^3}\left[k + k + (k-y) - 3\right] + \frac{y^2(k-y)}{k^3-y^3}\left[y + y + (k-y) - 3\right] + \frac{ky(k-y)}{k^3-y^3}\left[k + y + (k-y) - 3\right]\right\} + y \quad (5.1)$$

(5.1) is the properly proportioned average distance of cells in each of the rectangular blocks to the corner in each block nearest to the origin, plus the distance from these corners to the origin. It simplifies to:

$$\bar{d}_{y} = \frac{1}{2}\left\{ \frac{k - y}{k^3 - y^3} \left[k^2(k-y) + y^2(k-y) + ky\,(k-y) \right] + \right.$$

$$+ \frac{1}{k^3 - y^3} \left[2k^3\,(k-y) + 2y^3(k-y) + k\,y\,(k+y)(k-y) \right] -$$

$$\left. - 3 + 2y \right\} =$$

$$= \left\{ \frac{1}{2}\ k+y - 3 + \frac{2k^3 + 2y^3 + k^2y + ky^2}{k^2 + ky + y^2} \right\} =$$

$$= \frac{1}{2}\left\{ 2k + y - 3 + \frac{k^3 + 2y^3}{k^2+ky+y^2} \right\} \qquad (5.1a)$$

$\bar{d}_y \geqslant$ uy for all $0 \leqslant y < k$ since $2k + y-3+\frac{k^3+2y^3}{k^2+ky+y^2} > 3k-3$, as. $y \geqslant 0$.

The condition y = k is meaningless and the condition y = 0 gives the formula for $\bar{d}$ in Report No.1.

5.3. Set $y/k = \beta$ The average distance $\bar{d}_y$ now becomes $\bar{d}_\beta$

$$\bar{d}_\beta = \frac{1}{2}\left\{ 2k + \beta k - 3 + \frac{k^3 + 2\beta^3k^3}{k^2+\beta k^2+\beta^2k^2} \right\} =$$

$$= \frac{1}{2}\left\{ (2 + \beta)\,k-3 + \frac{1 + 2\beta^3}{1 + \beta + \beta^2}\ k \right\} =$$

$$= \frac{3k - 3}{2} + \frac{3}{2}\ \frac{k\beta^3}{1 + \beta + \beta^2} \qquad (5.2)$$

5.4. In what follows we shall designate by $\bar{d}$ (X) the average distance of all cells in a cube of volume X, and by $\bar{d}_\beta$ (X) the average distance of the $(X-\beta^3X)$ cells farthest away from origin. Considering first the statements in section 3.7 and 3.8 above, a glance at the Figs. 2a, 2b makes it obvious that the average distance under FIFO is, for $0 < \mu \leqslant \frac{2}{3}$

$$\bar{d}_{FIFO} = \frac{1}{2}\left\{ \bar{d}\,(N) + \eta\,\bar{d}\,(N') + (1 - \eta)\,\bar{d}_\beta(\bar{K}^*) \right\} \qquad (5.3)$$

$$\begin{aligned} \text{where } N &= (2-2\mu)\ \bar{K} \\ N' &= (2-3\mu)\ \bar{K} \\ \bar{K}^{*} &= (2-\mu)\ \bar{K} \\ \beta &= \sqrt[3]{\frac{2-2\mu}{2-\mu}} \\ \eta &= \frac{2-3\mu}{2-2\mu} \end{aligned}$$

Substituting the explicit expressions in (5.3) we have

$$d_{FIFO} = \tfrac{3}{4}\left\{\left[(2-2\mu)^{\frac{1}{3}}\bar{k}-1\right] + \frac{2-3\mu}{2-2\mu}\left[(2-3\mu)^{\frac{1}{3}}\bar{k} - 1\right] + \right. \tag{5.3a}$$

$$+ \frac{\mu}{2-2\mu}\left[(2-\mu)^{\frac{1}{3}}\bar{k} - 1\right] + \frac{\mu}{2-2\mu}\bar{k}\ \frac{(2-2\mu)/(2-\mu)}{1+\left(\frac{2-2\mu}{2-\mu}\right)^{\frac{1}{3}} + \left(\frac{2-2\mu}{2-\mu}\right)^{\frac{2}{3}}}$$

5.5. It was stated in Section 3 that (5.3) can be approximated by, say, $\tilde{d}_{FIFO}$, as

$$\tilde{d}_{FIFO} = \frac{3}{2}\ \left[(2-\mu)^{\frac{1}{3}}\bar{k} - 1\right] \tag{5.4}$$

Let R be a ratio

$$R = \frac{d_{FIFO} + 3/2}{\tilde{d}_{FIFO} + 3/2}$$

From (5.3a) and (5.4) we have

$$R = \frac{\frac{3}{4}\left\{\left[(2-2\mu)^{\frac{1}{3}} + \frac{2-3\mu}{2-2\mu}(2-3\mu)^{\frac{1}{3}} + \frac{\mu}{2-2\mu}(2-\mu)^{\frac{1}{3}}(1+B)\right]\right\}}{\frac{3}{2}(2-\mu)^{\frac{1}{3}}} =$$

$$= \tfrac{1}{2}\left[\beta + \eta\left(\frac{2-3\mu}{2-\mu}\right)^{\frac{1}{3}} + (1-\eta)\ \ (1+B)\right] \tag{5.5}$$

$$\text{where } B = \frac{\beta 3}{1+\beta+\beta^2}$$

From (5.5) we have

$$2R = \beta + \eta a + (1-\eta)\ (1 + B) = 1 + \beta + B + \eta(a-1-B) =$$
$$= \beta + 1 + \beta^3/(1+\beta+\beta^2) + \eta\left[a-1-\beta^3/(1+\beta+\beta^2)\right] =$$
$$= \frac{1}{1+\beta+\beta^2}\left[1 + \eta a - \eta + (2 + \eta a - \eta)\,(\beta+\beta^2) + (2-\eta)\beta^3\right]$$

Now $2 - \eta = 2 - \frac{2 - 3\mu}{2 - 2\mu} = \frac{2 - \mu}{2-2\mu} = 1/\beta^3$

$$\therefore\ 2R = \frac{2 + \eta a - \eta(1 + \beta + \beta^2)}{1 + \beta + \beta^2} = 1/\beta^3 + \eta a =$$

$$= \frac{(2-\mu)^{4/3} + (2-3\mu)^{4/3}}{(2-2\mu)\ (2-\mu)^{\frac{1}{3}}} \qquad (5.5a)$$

Obviously, if $\mu = 0$. $R = 1$

If $\mu = \frac{2}{3}$, $R = 1$, by substitution in (5.5) or (5.5a).

The maximum deviation from $R = 1$ occurs at $\mu = \frac{1}{2}$, where it is of the order of 0.94. The maximum overestimate in using $\tilde{d}_{FIFO}$ instead of d_{FIFO} is therefore of the order of 7%. (Fig.4)

5.6. When $\frac{2}{3} < \mu < 1$, we first consider the case for which

$$\frac{2 - \mu}{2-2\mu} = n \quad \text{an integer} > 2$$

This implies that the average volume of movement divides the average maximum stock. Hence $2 - \mu = n\ (2-2\mu) = 2n-2n\mu$

$$(2n-1)\mu = 2n - 2$$

$$\mu = \frac{2n - 2}{2n - 1}$$

In this case we have from Fig. 2c

$$d_{FIFO} = \frac{1}{n} \sum_{i=1}^{n} d_{\beta_i}\ (iN) \qquad (5.6)$$

with $\beta_i = \sqrt[3]{\frac{i-1}{i}}$, $\beta_1 = 0\ \therefore\ d_{\beta_1}\ (N) = d\ (N)$

$$N = (2-2\mu)\ \bar{K} = \frac{2\bar{K}}{2n - 1}$$

Substituting explicit expressions into (5.6) we have

$$d_{FIFO} = \frac{1}{n} \sum_{i=1}^{n} \frac{3}{2} \bar{k} \left(\frac{2i}{2n-1}\right)^{\frac{1}{3}} (1 + B_i) - \frac{3}{2} =$$

$$= \frac{3\sqrt[3]{2}\,\bar{k}}{2n\sqrt[3]{2n-1}} \sum_{i=1}^{n} (1 + B_i)\sqrt[3]{i} - \frac{3}{2} \qquad (5.6a)$$

with $B_i = \dfrac{\beta_i^{\,3}}{1 + \beta_i + \beta_i^{\,2}}$

Now $2 - \gamma = 2n/(2n-1)$, hence

$$d_{FIFO} = \frac{3}{2} \bar{k} \left(\frac{2n}{2n-1}\right)^{\frac{1}{3}} - 1 \qquad (5.7)$$

The ratio R now becomes

$$R = \frac{3\sqrt[3]{2}\,\bar{k}\left[(1 + B_i)\sqrt[3]{i}\,\right]/2n\sqrt[3]{2n-1}}{3\sqrt[3]{2}\,\bar{k}\sqrt[3]{n}\;\;/2\sqrt[3]{2n-1}} =$$

$$= \frac{1}{n\sqrt[3]{n}} \sum_{i=1}^{n} (1 + B_i)\sqrt[3]{i} \qquad (5.8)$$

5.7. We now show that

$$\sum_{i=1}^{n} (1 + B_i)\sqrt[3]{i} = n\sqrt[3]{n} \qquad (5.9)$$

$$B_i = \frac{(i-1)/i}{1 + \left(\frac{i-1}{i}\right)^{\frac{1}{3}} + \left(\frac{i-1}{i}\right)^{\frac{2}{3}}}$$

Let $a_i = \sqrt[3]{i}$, $\quad b_i = \sqrt[3]{i - 1}$

The expression under the Summation sign in (5.9) now becomes

$$\left(\frac{1 + b_i^{\,3}/a_i^{\,3}}{1 + b_i/a_i + b_i^{\,2}/a_i^{\,2}}\right) a_i \qquad (5.10)$$

$$= \frac{a_i^{\,3} + b_i a_i^{\,2} + b_i^{\,2} a_i + b_i^{\,3}}{a_i^{\,2} + b_i a_i + b_i^{\,2}}$$

Multiplying both Numerator and Denominator of (5.10) by (a-b) we have

$$\frac{a_i^4 - b_i^4}{a_i^3 - b_i^3} = \frac{i\sqrt[3]{i} - (i-1)\sqrt[3]{i-1}}{i - (i-1)} =$$

$$= i\sqrt[3]{i} - (i-1)\sqrt[3]{i-1} \qquad (5.10a)$$

(5.9) can thus be written

$$\sum_{i=1}^{n}\left[i\sqrt[3]{i} - (i-1)\sqrt[3]{i-1}\right] = n\sqrt[3]{n} \qquad (5.9a)$$

Substituting the first n natural numbers in (5.9a) it is clear that (5.9a) holds for all positive integers n. Hence $R = 1$ for all cases for which $(2-\mu)/(2-2\mu) = n$

5.8. In the case where the ratio $(2-\mu)/(2-2\mu) = n + x/m$, x/m a proper fraction, the integer $n' = mn + x$ can be substituted for n in (5.6a) to estimate the average distance. Hence again the ratio $R = 1$, and $\tilde{d}_{FIFO} = d_{FIFO}$. It is obvious that in cases where the ratio $(2-\mu)/(2-2\mu)$ is neither n nor n' the ratio R will differ from 1. It would require a complicated analysis to estimate this difference, but it is clear that, considering the underlying process, the deviation of R from 1 can be only very small.

5.9. From para. 4.5. the average distance under LIFO is obtained as follows

$$d_{LIFO} = \frac{1}{2}\left\{d\left(\left[1-\mu\right]\bar{K} + d_{\frac{1}{2-\mu}}\left(\left[2-\mu\right]\bar{K}\right)\right\}\right.$$

$$= \frac{1}{2}\left\{\frac{3}{2}\left[\bar{k}\,(1-\mu)^{\frac{1}{3}} - 1\right] + \frac{3}{2}\left[\bar{k}\,(2-\mu)^{\frac{1}{3}} - 1\right] + \right.$$

$$\left. + \frac{3}{2}\,\bar{k}\,(2-\mu)^{\frac{1}{3}}\,B\right\} =$$

$$= \frac{3}{4}\left\{\bar{K}\left[(1-\mu)^{\frac{1}{3}} + (2-\mu)^{\frac{1}{3}}(1+B)\right] - 2\right\} \qquad (5.11)$$

with B = $\dfrac{1/(2-\mu)}{1+\left(\dfrac{1}{2-\mu}\right)^{\frac{1}{3}}+\left(\dfrac{1}{2-\mu}\right)^{\frac{2}{3}}} = 1/\left[(2-\mu)+(2-\mu)^{\frac{2}{3}}+(2-\mu)^{\frac{1}{3}}\right]$

Substitution and re-arrangement in (5.11) leads to (4.2)

APPENDIX

A COMPUTER SIMULATION

A.1. In order to test the applicability of the formulae given in this report we set up a computer simulation of a simple warehouse, storing one item only.

A.2. We assumed that daily issues have a Poisson Distribution with mean, 10 units. Supplies of 50 units come in once a week (of 5 days), with a probability of arrival on the 1st day of 0.5, on the 2nd day of 0.3 and on the 3rd day of 0.2.

A.3. The warehouse consisted of 5 floors of cubical cells, with 5 x 6 cells on each floor. Fig. 5 shows a plan of the warehouse with distances from reference point marked in each cell.

A.4. The "Saw-Tooth Diagram" of the Stock movement over 50 weeks is given in Fig. 6, and the distribution of stock in hand in Table 1.

A.5. The computer programme, using MONTECODE simulated the daily working, on figures obtained from the arrival and issue distributions and calculated the distances over which each item moved under FIFO, MINDIS, and LIFO, for 50 five day weeks and produced the distributions of distances given in Tables 2a,b,c. Table 3 gives a comparison between average distances calculated by computer and by the formulae of this report.

A.6. As can be seen, the formulae give nearly the same values as the computer. We should have liked to carry out further simulations, under different conditions of demand and supply, but pressure of time prevented this. We intend, however to apply further tests, if possible on actual data, which we now may be able to obtain.

TABLE 1 - Distribution of Stock in Hand.

Stock Value	Frequency	% Frequency	Cum.Frequency
0 - 7	1	0.4	0.4
8 - 15	0	0	0.4
16 - 23	2	0.8	1.2
24 - 31	3	1.2	2.4
32 - 39	7	2.8	5.2
40 - 47	15	6.0	11.2
48 - 55	21	8.4	19.6
56 - 63	32	12.8	32.4
64 - 71	35	14.0	46.4
72 - 79	38	15.2	61.6
80 - 87	34	13.6	75.2
88 - 95	28	11.2	86.4
96 - 103	19	7.6	94.0
104 - 111	12	4.8	98.8
$\geq$ 112	3	1.2	100.0

$\bar{x} = 72.560$

$\sigma^2 = 404.00$

$\sigma = 20.1$

$\sqrt[3]{\bar{x}} = 4.1709$

$\sigma/\bar{x} = 27.7$

TABLE 2 - Distribution of distances of movement.

FIFO - TABLE 2a.

Distance	Frequency	% Frequency	Cum.Frequency
0	65	1.3	1.3
1	191	3.8	5.1
2	360	7.2	12.3
3	544	10.9	23.2
4	769	15.4	38.6
5	918	18.4	57.0
6	948	19.0	76.0
7	766	15.4	91.4
8	335	6.7	98.1
9	96	1.9	100.0

$\bar{x} = 4.969$

$\sigma^2 = 3.89$

$\sigma = 1.97$

MINDIS - TABLE 2b

Distance	Frequency	% Frequency	Cum.Frequency
0	101	2.0	2.0
1	303	6.1	8.1
2	604	12.1	20.2
3	988	19.8	40.0
4	1395	27.9	67.9
5	1079	21.6	89.5
6	332	6.6	96.1
7	149	3.0	99.1
8	27	0.6	99.7
9	14	0.3	100.0

$\bar{x} = 3.771$

$\sigma^2 = 2.41$

$\sigma = 1.55$

LIFO - TABLE 2c

Distance	Frequency	% Freqency	Cum.Frequency
0	101	2.0	2.8
1	303	6.1	8.1
2	602	12.0	20.1
3	932	18.7	38.8
4	528	10.6	49.4
5	72	1.4	50.8
6	1143	22.9	73.7
7	644	12.9	86.6
8	553	11.1	97.7
9	114	2.3	100.0

$\bar{x} = 4.726$

$\sigma^2 = 5.45$

$\sigma = 2.34$

TABLE 3 - Comparison of Average Distance, Computer - Formula

	Computer Distance	Formula Distance	% Error
FIFO	4.97	5.29	6.4
MINDIS	3.77	3.66	- 2.9
LIFO	4.73	4.89	- 3.4

SAW-TOOTH DIAGRAM
FIG.1

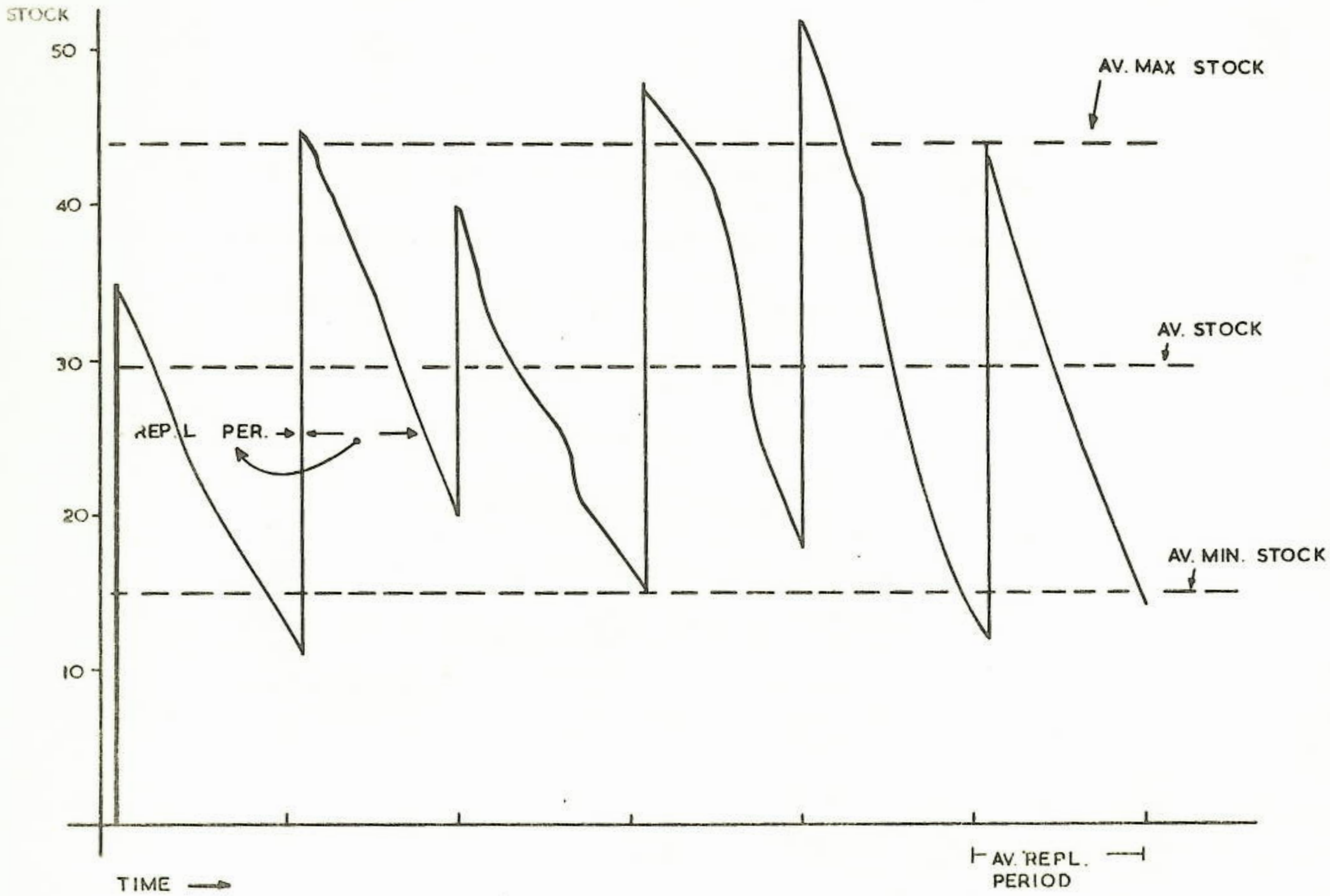


Fig. 2.

MOVEMENT IN AND OUT OF WAREHOUSE
under FIFO

Notes to Figs. 2a, b, c, d.

Each line of the grid shows the complete number of cells available in each period. Cells are identified by number and distance from reference point.

↙ denotes movement out

↗ denotes movement in

A number in the cell denotes the period of receipt of stock that does not move during the period.

Fig. 2a. $\mu = 2/3$ $(2-\mu)\,\bar{K} = 26$

Cell No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
Dist	0	1	1	1	2	2	2	2	2	2	3	3	3	3	3	3	3	4	4	4	4	4	4	5	5	5	6
Period 1	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	0	0	0	0	0	0	0	0	0	0	0	0	0	
2	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	
3	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	
4	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	
5	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	
6	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	
7	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	
8	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	
9	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	
10	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	
11	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	
12	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	

Fig. 2b $\mu = \frac{1}{2}$ $(2 - \mu)\,\bar{K} = 27$

Cell No	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
Dist	0	1	1	1	2	2	2	2	2	2	3	3	3	3	3	3	3	4	4	4	4	4	4	5	5	5	6
Period 1	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	0	0	0	0	0	0	0	0	0
2	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗	↗	↗	↗	↗	↗	↗	↗	↗	↙	↙	↙	↙	↙	↙	↙	↙	↙
3	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↗	↗	↗	↗	↗	↗	↗	↗	↙
4	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗	↗	↗	↗	↗	↗	↗	↗	↗	↙	↙	↙	↙	↙	↙	↙	↙	↙
5	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↗	↗	↗	↗	↗	↗	↗	↗	↗
6	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗	↗	↗	↗	↗	↗	↗	↗	↗	↙	↙	↙	↙	↙	↙	↙	↙	↙
7	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↗	↗	↗	↗	↗	↗	↗	↗	↗
8	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗	↗	↗	↗	↗	↗	↗	↗	↗	↙	↙	↙	↙	↙	↙	↙	↙	↙
9	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↗	↗	↗	↗	↗	↗	↗	↗	↗
10	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗	↗	↗	↗	↗	↗	↗	↗	↗	↙	↙	↙	↙	↙	↙	↙	↙	↙
11	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↗	↗	↗	↗	↗	↗	↗	↗	↗
12	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗ ↙	↗	↗	↗	↗	↗	↗	↗	↗	↗	↙	↙	↙	↙	↙	↙	↙	↙	↙

Fig. 2c $\mu = 4/5$ $(2-\mu)\bar{K} = 27$

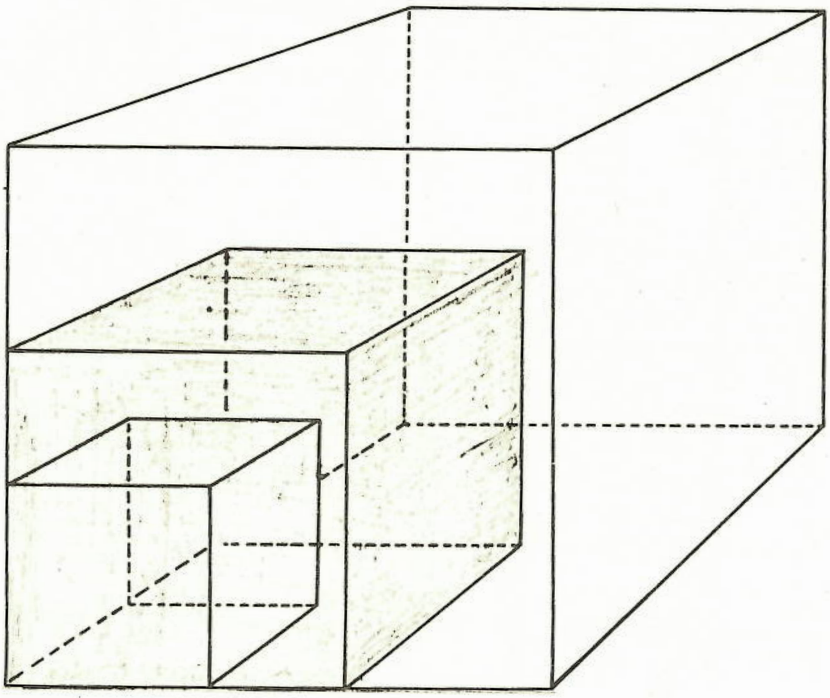
Cell No		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
Dist.		0	1	1	1	2	2	2	2	2	2	3	3	3	3	3	3	3	4	4	4	4	4	4	5	5	5	6
Period	1	↙	↙	↙	↙	↙	↙	↙	↙	↙	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	2	↗	↗	↗	↗	↗	↗	↗	↗	↗	↙	↙	↙	↙	↙	↙	↙	↙	↙	0	0	0	0	0	0	0	0	0
	3	2	2	2	2	2	2	2	2	2	↗	↗	↗	↗	↗	↗	↗	↗	↗	↙	↙	↙	↙	↙	↙	↙	↙	↙
	4	↙	↙	↙	↙	↙	↙	↙	↙	↙	3	3	3	3	3	3	3	3	3	↗	↗	↗	↗	↗	↗	↗	↗	↗
	5	↗	↗	↗	↗	↗	↗	↗	↗	↗	↙	↙	↙	↙	↙	↙	↙	↙	↙	4	4	4	4	4	4	4	4	4
	6	5	5	5	5	5	5	5	5	5	↗	↗	↗	↗	↗	↗	↗	↗	↗	↙	↙	↙	↙	↙	↙	↙	↙	↙
	7	↙	↙	↙	↙	↙	↙	↙	↙	↙	6	6	6	6	6	6	6	6	6	↗	↗	↗	↗	↗	↗	↗	↗	↗
	8	↗	↗	↗	↗	↗	↗	↗	↗	↗	↙	↙	↙	↙	↙	↙	↙	↙	↙	7	7	7	7	7	7	7	7	7
	9	8	8	8	8	8	8	8	8	8	↗	↗	↗	↗	↗	↗	↗	↗	↗	↙	↙	↙	↙	↙	↙	↙	↙	↙
	10	↙	↙	↙	↙	↙	↙	↙	↙	↙	9	9	9	9	9	9	9	9	9	↗	↗	↗	↗	↗	↗	↗	↗	↗
	11	↗	↗	↗	↗	↗	↗	↗	↗	↗	↙	↙	↙	↙	↙	↙	↙	↙	↙	10	10	10	10	10	10	10	10	10
	12	11	11	11	11	11	11	11	11	11	↗	↗	↗	↗	↗	↗	↗	↗	↗	↙	↙	↙	↙	↙	↙	↙	↙	↙

Fig. 2d $\mu = 5/7$ $(2-\mu)\bar{K} = 27$

Cell No.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
Dist		0	1	1	1	2	2	2	2	2	2	3	3	3	3	3	3	3	4	4	4	4	4	4	5	5	5	6
Period	1	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	2	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	0	0	0
	3	↙	↙	↙	↙	↙	↙	↙	↙	↙	2	2	2	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↙	↙	↙
	4	↗	↗	↗	↗	↗	↗	↗	↗	↗	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	3	3	3	↗	↗	↗
	5	↙	↙	↙	↙	↙	↙	↙	↙	↙	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↙	↙	↙	4	4	4
	6	↗	↗	↗	↗	↗	↗	↗	↗	↗	↙	↙	↙	↙	↙	↙	↙	↙	↙	5	5	5	↗	↗	↗	↙	↙	↙
	7	↙	↙	↙	↙	↙	↙	↙	↙	↙	↗	↗	↗	↗	↗	↗	↗	↗	↗	↙	↙	↙	6	6	6	↗	↗	↗
	8	↗	↗	↗	↗	↗	↗	↗	↗	↗	↙	↙	↙	↙	↙	↙	↙	↙	↙	↗	↗	↗	↙	↙	↙	7	7	7
	9	↙	↙	↙	↙	↙	↙	↙	↙	↙	↗	↗	↗	↗	↗	↗	↗	↗	↗	8	8	8	↗	↗	↗	↙	↙	↙
	10	↗	↗	↗	↗	↗	↗	↗	↗	↗	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	9	9	9	↗	↗	↗
	11	↙	↙	↙	↙	↙	↙	↙	↙	↙	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↗	↙	↙	↙	10	10	10
	12	↗	↗	↗	↗	↗	↗	↗	↗	↗	↙	↙	↙	↙	↙	↙	↙	↙	↙	11	11	11	↗	↗	↗	↙	↙	↙

SHADED VOLUME REPRESENTS CARRIER, IN WHICH ON AVERAGE NO MOVEMENT OCCURS UNDER LIFO

FIG. 3



GRAPH OF R AGAINST μ $(0 < p < 2/3)$

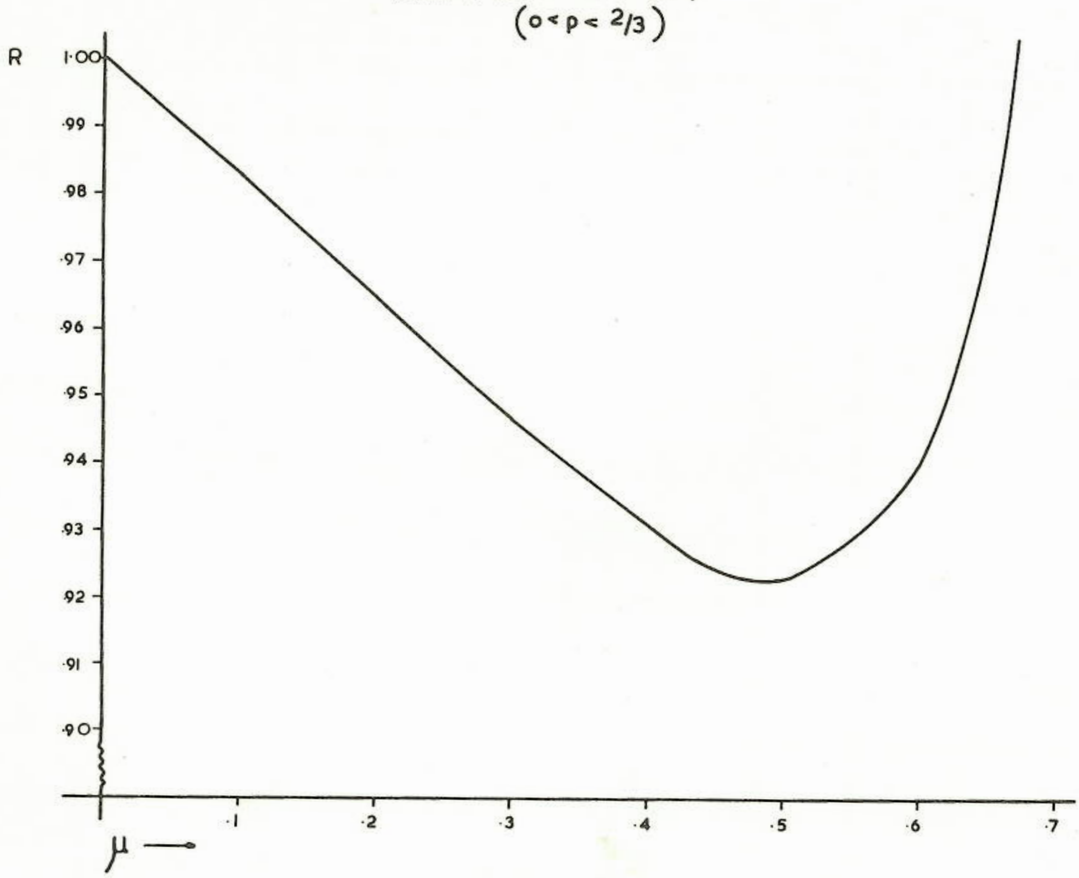


FIG.4

Lay-out of Warehouse for Simulation (With cell distances) Fig.5.

Ground Floor					1st Floor					2nd Floor				
5	6	7	8	9	6	7	8	9	10	7	8	9	10	11
4	5	6	7	8	5	6	7	8	9	6	7	8	9	10
3	4	5	6	7	4	5	6	7	8	5	6	7	8	9
2	3	4	5	6	3	4	5	6	7	4	5	6	7	8
1	2	3	4	5	2	3	4	5	6	3	4	5	6	7
0	1	2	3	4	1	2	3	4	5	2	3	4	5	6

Reference Point

3rd Floor					4th Floor				
8	9	10	11	12	9	10	11	12	13
7	8	9	10	11	8	9	10	11	12
6	7	8	9	10	7	8	9	10	11
5	6	7	8	9	6	7	8	9	10
4	5	6	7	8	5	6	7	8	9
3	4	5	6	7	4	5	6	7	8

Fig.6.

SIMULATION - STOCK