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Time Domain and Frequency Domain Measurements for Transistor Characterization

- by -

H.W. Loeb

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Summary of a lecture given at a symposium on 'The Properties and Uses of Transistors at High Frequencies' held at Bristol College of Science and Technology, 10th-11th May, 1966.

Two-Port Descriptions

Under small signal conditions, a transistor represents a linear, time-invariant two-port network. Such networks are usually characterized in terms of one of six sets of frequency dependent Open/Closed Termination ('O/C') parameters, each set involving a different choice of dependent and independent variables from among the two-port currents and two-port voltages. Table 1 summarizes these O/C parameter sets and indicates the conditions required for the direct measurement of each parameter.

Open/Closed Termination Parameters, while widely used, are not the only means for small-signal transistor characterization. The set of Finite Termination ('FT') Parameters represents a useful alternative form. These comprise the forward and reverse Insertion Transmissions between specified source and load impedances together with Input and Output Immittances for the same terminations. The formal definitions of the FT parameters and conditions for their direct measurement are set out in Table 2.

Knowledge of any one complete set of parameters at any one frequency permits computation of any other set at that frequency. Transformation tables are given in references 1 and 2. As an illustration of the Conversion formulae Table 3 shows the relation between h and Insertion parameters.

Experimental Implications

These follow directly from the defining statements. To measure O/C parameters, means for providing effective AC short and open circuits at each frequency of interest must be found. This requirement becomes progressively more difficult to fulfil as the frequency increases since the presence of small residual reactances will introduce errors, especially into the phase measurements.

At frequencies above 100 Mc the difficulties involved in O/C measurements can only be overcome by the use of transmission line techniques. Even and odd multiples of $\lambda/4$ line lengths can be used to transform terminating conditions and to translate them to the transistor terminals, but only at the cost of separate line length adjustments for each frequency.

Many of the drawbacks of O/C measurements can be avoided by direct FT parameter determinations. While transmission line techniques must again be used at higher frequencies, the use of the characteristic impedance of the line as the terminating impedances removes the need for line re-adjustment at each frequency and thus makes broad-band measurement systems possible.

The choice of method of measurement will be governed by the amount of information required. Any single parameter, at a particular frequency, may be obtained either by a direct measurement, or indirectly by conversion. In the latter case, in general, one complete set of four parameters is required, involving a series of measurements, followed by some complex number arithmetic. Decisions regarding the most appropriate method of measurement must thus depend upon the total amount of information required.

Time Domain Methods, to be discussed below, are inherently capable of providing a great deal of information from a few measurements. The experimental procedure is largely unaffected by the amount of data required, so that, if only a small amount of data, such as one or two parameters at one frequency, are required, little saving in experimental time results. From a practical point of view, therefore, established Frequency Domain methods and Time Domain methods are complementary.

Examples of Equipment for Frequency Domain Measurements

O/C Parameter Measurement

Equipment	Ref.	Frequency Range	Accuracy
Transformer Ratio	3, 4	15 Kc-5 Mc	≈ 1 - 2%
Arm Bridges (Wayne Kerr 601,801)		5 - 100 Mc	≈ 2% 2-terminal and 3-terminal $f < 50$ Mc; ≈ 10% 3-terminal $f > 50$ Mc
Transmission Line Bridge	5	25 - 100 Mc	≈ 5% 25-100 Mc*

FT Parameter Measurement

Equipment	Ref.	Frequency Range	Accuracy
'Z-g Diagraphs' (Rohde and Schwarz ZDU, ZDP)	6	30-420 Mc	≈ 2-10%* 2-terminal; ≈ 5%*, 3-terminal
		300-2400 Mc	≈ 3-20%*, 2-terminal; ≈ 5%*, 3-terminal
'Loss and Phase Measuring Set' (Bell Telephone Labs.)	7	5-250 Mc	± 0.1 db; ± 0.5°

*Accuracy depends upon type and magnitude of parameter measured.

Time Domain Methods

Instead of characterization in terms of sets of frequency dependent parameters which describe amplitude and phase responses of a transistor to an applied sinusoidal excitation ('Frequency Domain Characterization'), a transistor or other linear time invariant network may be specified by a description of its response if a prescribed pulse is applied to one of its ports. Such 'Time Domain Characterization' may be based, for example, upon the impulse function $\phi(t)$ obtained at the output terminals if a unit impulse $\delta(t)$

$$\left[\text{where} \int_{-\infty}^{\infty} \delta(t)dt = 1,\ \delta(0) = \infty,\ \delta(t) = 0,\ t \neq 0\right]$$

is applied to the input port. More generally, if a time varying input signal $V_i(t)$ is applied to such network, the resulting output will be

$$V_o(t) = \int_0^t V_i(t)\phi(t-\tau)\, d\tau$$

This relation represents the Time Domain equivalent of the Frequency Domain response equation

$$v_o(\omega) = H(\omega)v_i(\omega)$$

where $$v_o(\omega) = \int_{-\infty}^{\infty} V_o(t)e^{-j\omega t}dt$$

$$v_i(\omega) = \int_{-\infty}^{\infty} V_i(t)e^{-j\omega t}dt$$

i.e. $v_i(\omega)$, $V_i(t)$ and $v_o(\omega)$, $V_o(t)$ constitute Fourier transform pairs.

The response $H(\omega)$ can thus be obtained by either of the following procedures:

(1) F D : $H(\omega) = v_o(\omega)/v_i(\omega)$

'Apply signal of known amplitude and frequency to input port, measure amplitude and phase of output signal and form complex ratio. Repeat for each frequency of interest.'

(2) T D :

$$H(\omega) = \frac{\int_{-\infty}^{\infty} V_o(t) e^{-j\omega t} dt}{\int_{-\infty}^{\infty} V_i(t)\ e^{-j\omega t} dt}$$

'Apply pulse $V_i(t)$ of known shape and adequate spectral composition to input port, record output pulse $V_o(t)$.

Compute ratio of Fourier transforms for each frequency of interest.'

In the above description the term 'response' is used to cover any reflected or transmitted signal produced by the input excitation.

Examples of practical methods for computing the required Fourier transforms from empirically obtained time functions are described in references 8 and 9.

The simplest form of T D Measurement is provided by 'Time Domain Reflectometry'. In TDR the type and location of a non-uniformity within a transmission line system is inferred from a study of the waveform reflected by it when a voltage impulse or voltage step is applied to one end of the line. The method is discussed in detail in reference 10.

Description of experimental T D measurement set (Ref. 11)

To determine the full set of insertion parameters, (Z_{in}, $e^{-\psi_{21}}$, $e^{-\psi_{12}}$, Z_{out}) from T D measurements, the transistor or other two-port is embedded in a transmission line system and a short rise time pulse applied. The following time functions are recorded:

(i) Pulse reflected from input terminal (to yield Inport Reflection coefficient and hence Z_{in})

(ii) Pulse transmitted in forward direction (to yield $e^{-\psi_{21}}$)

(iii) Pulse transmitted in reverse direction (to yield $e^{-\psi_{12}}$)

(iv) Pulse reflected from output terminal (to yield Outport reflection coefficient and hence Z_{out}).

(v) Pulse transmitted with transistor replaced by direct link (to yield reference pulse, $V_i(t)$).

The last measurement is important in providing a true reference. Instead of the usual assumption (justifiable at Control Engineering frequencies, but unrealistic at 1,000 Mc) that the input consists of an ideal step, the actual pulse which excites the network is analysed. This procedure eliminates the effect of pulse generator shortcomings and can be used, with suitable system design, to eliminate pulse degradation effects due to the coaxial line system.

The system is shown in schematic form in Fig. 1.

Practical design features are summarised in the following list:-

(1) Use of 50 ohm coaxial airlines throughout, except for time marker system.

(2) Elimination of line losses by symmetry: both reflected and transmitted pulses are measured after traversing identical line lengths.

(3) Location of bias networks, including blocking capacitors, at points in system, such that mismatch reflections occur at non-critical times.

(4) Use of two-channel 50 ohm 'feed-through' type of sampling oscilloscope with pen recorder output to record pulses.

(5) Provision of separate time marker system to permit measurement of pure delay and accurate establishment of phase relationships.

(6) Computation of Fourier Transforms by means of Samulon's method, which provides an exact spectrum if no frequencies in excess of a cut-off frequency equal to 1/(2 x sampling interval) are present.

Preliminary results show good agreement with G - R bridge measurements over range 100 - 750 Mc. Measurements on a passive network indicated that the bias insertion units introduced some error. A straight comparison between F D and T D measurements obtained by direct measurement of amplitude ratios and phase shifts in the coaxial system used for the T D experiments, yielded agreement, within the experimental accuracy, to above 625 Mc in magnitude and 1 Gc in phase.

Conclusions

The feasibility and usefulness of T D measurements as a means for obtaining one-port and two-port parameters has been established. While further assessments of experimental accuracies will have to be made, such methods appear to yield results which are at least as accurate as those obtainable with conventional methods. Future developments can be expected to result in further enhancement of accuracy and reduction of experimental labour.

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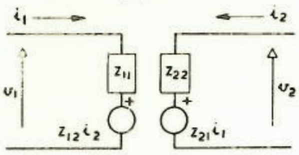
TABLE 1

TWO-PORT PARAMETERS INVOLVING OPEN CLOSED PORT TERMINATIONS

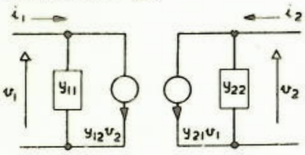
IMPEDANCE (Z)



$$v_1 = z_{11}\, i_1 + z_{12}\, i_2 = z_i i_1 + z_r\, i_2$$
$$v_2 = z_{21}\, i_1 + z_{22}\, i_2 = z_f i_1 + z_o i_2$$

$z_{11} = z_i = \frac{v_1}{i_1}\Big\vert_{i_2=0}$	Inport Impedance, Outport Open
$z_{12} = z_r = \frac{v_1}{i_2}\Big\vert_{i_1=0}$	Reverse Transfer Impedance, Inport Open
$z_{21} = z_f = \frac{v_2}{i_1}\Big\vert_{i_2=0}$	Forward Transfer Impedance Outport Open
$z_{22} = z_o = \frac{v_2}{i_2}\Big\vert_{i_1=0}$	Outport Impedance Inport Open

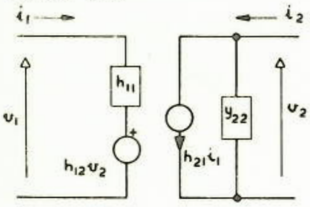
ADMITTANCE (Y)



$$i_1 = y_{11}v_1 + y_{12}v_2 = y_i v_1 + y_r v_2$$
$$i_2 = y_{21}v_1 + y_{22}v_2 = y_f v_1 + y_o v_2$$

$y_{11} = y_i = \frac{i_1}{v_1}\Big\vert_{v_2=0}$	Inport Admittance Outport Closed
$y_{12} = Y_r = \frac{i_1}{v_1}\Big\vert_{v_1=0}$	Reverse Transfer Admittance Inport Closed
$y_{21} = y_f = \frac{i_2}{v_1}\Big\vert_{v_2=0}$	Forward Transfer Admittance Outport Closed
$y_{22} = y_o = \frac{i_2}{v_2}\Big\vert_{v_1=0}$	Outport Admittance Inport Closed

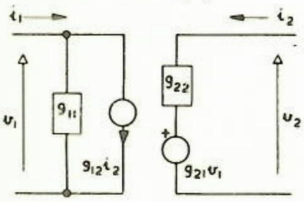
HYBRID (H)



$$v_1 = h_{11}i_1 + h_{12}v_2 = h_i i_1 + h_r v_2$$
$$i_2 = h_{21}i_1 + h_{22}v_2 = h_f i_1 + h_o v_2$$

$h_{11} = h_i = \frac{v_1}{i_1}\Big\vert_{v_2=0}$	Inport Impedance Outport Closed
$h_{12} = h_r = \frac{v_1}{v_2}\Big\vert_{i_1=0}$	Reverse Transfer Voltage Ratio, Inport Open
$h_{21} = h_f = \frac{i_2}{i_1}\Big\vert_{v_2=0}$	Forward Transfer Current Ratio, Outport Closed
$h_{22} = h_o = \frac{i_2}{v_2}\Big\vert_{i_1=0}$	Outport Admittance Inport Open

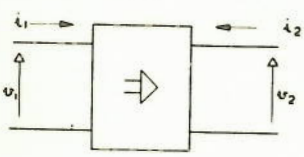
INVERSE HYBRID (G)



$$i_1 = g_{11}v_1 + g_{12}i_2 = g_i v_1 + g_r i_2$$
$$v_2 = g_{21}v_1 + g_{22}i_2 = g_f v_1 + g_o i_2$$

$g_{11} = g_r = \frac{i_1}{i_1}\Big\vert_{i_2=0}$	Inport Admittance Outport Open
$g_{12} = g_r = \frac{i_1}{i_2}\Big\vert_{v_1=0}$	Reverse Transfer Current Ratio. Inport Closed
$g_{21} = g_f = \frac{v_2}{v_1}\Big\vert_{i_2=0}$	Forward Transfer Voltage Ratio Outport Open
$g_{22} = g_o = \frac{v_2}{i_2}\Big\vert_{v_1=0}$	Outport Impedance Inport Closed

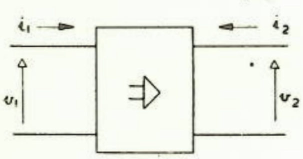
TRANSFER FUNCTION (A)



$$v_1 = a_{11}v_2 - a_{12}i_2$$
$$i_1 = a_{21}v_2 - a_{22}i_2$$

$a_{11} = \frac{v_1}{v_2}\Big\vert_{i_2=0}$	Reverse Transfer Voltage Ratio Outport Open
$-a_{12} = \frac{v_1}{i_2}\Big\vert_{v_2=0}$	Reverse Transfer Impedance Outport Closed
$a_{21} = \frac{i_1}{v_2}\Big\vert_{i_2=0}$	Reverse Transfer Admittance Outport Open
$-a_{22} = \frac{i_1}{i_2}\Big\vert_{v_2=0}$	Reverse Transfer Current Ratio, Outport Closed

TRANSFER FUNCTION (B)



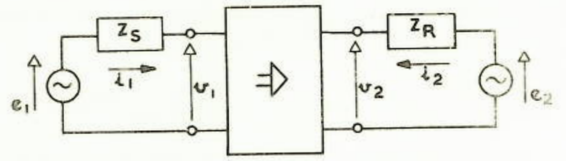
$$v_2 = b_{11}v_1 + b_{12}i_1$$
$$-i_2 = b_{21}v_1 + b_{22}i_1$$

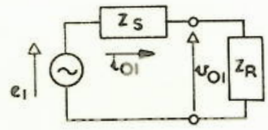
$b_{11} = \frac{v_2}{v_1}\Big\vert_{i_1=0}$	Forward Transfer Voltage Ratio Inport Open
$b_{12} = \frac{v_2}{i_1}\Big\vert_{v_1=0}$	Forward Transfer Impedance Inport Closed
$-b_{21} = \frac{i_2}{v_1}\Big\vert_{i_1=0}$	Forward Transfer Admittance Inport Open
$-b_{22} = \frac{i_2}{i_1}\Big\vert_{v_1=0}$	Forward Transfer Current Ratio Inport Closed

TABLE 2

TWO-PORT PARAMETERS INVOLVING FINITE PORT TERMINATIONS

INSERTION PARAMETERS



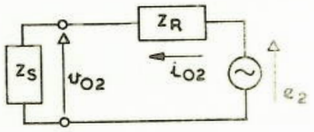


$$i_{01} = \frac{e_1}{Z_S + Z_R}$$

$$v_{01} = \frac{-e_1 Z_R}{Z_S + Z_R}$$

$$i_{02} = \frac{e_2}{Z_S + Z_R}$$

$$v_{02} = \frac{-e_2 Z_S}{Z_S + Z_R}$$



$$Z_{in} = \left.\frac{v_1}{i_1}\right|_{e_2=0} = \frac{1}{Y_{in}}$$ Inport Immittance with Output terminated in Z_R

$$e^{-\psi_{12}} = \left.\frac{-i_1}{i_{02}}\right|_{e_1=0} = \left.\frac{v_1}{v_{02}}\right|_{e_1=0}$$ Reverse Insertion Transmission between Z_s and Z_R

$$e^{-\psi_{21}} = \left.\frac{-i_2}{i_{01}}\right|_{e_2=0} = \left.\frac{v_2}{v_{01}}\right|_{e_2=0}$$ Forward Insertion Transmission between Z_s and Z_R

$$Y_{out} = \left.\frac{i_2}{v_1}\right|_{e_1=0} = \frac{1}{Z_{out}}$$ Outport Immittance with Inport terminated in Z_s

TABLE 3

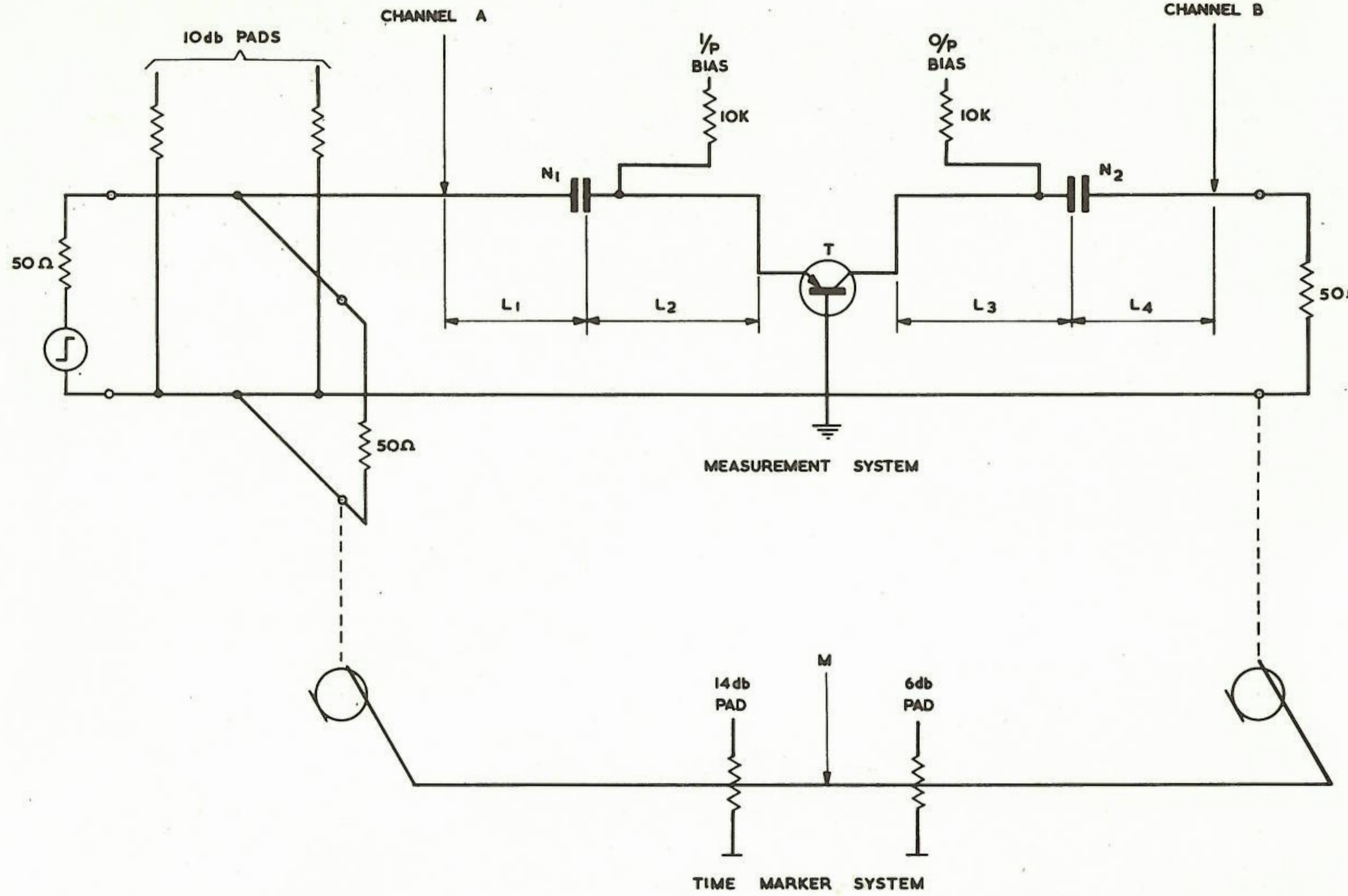
RELATION BETWEEN HYBRID AND INSERTION PARAMETERS

HYBRID	INSERTION [EXACT]	INSERTION [APPROXIMATE, FOR $\Gamma \to 1$, $Z_s = Z_R = 50$]
h_{11}	$\dfrac{Z_{in} + (1-\Gamma)Z_s}{\Gamma}$	$\doteqdot Z_{in}$
h_{12}	$\dfrac{(Y_{out} + Y_R)(Z_{in} + Z_s)e^{-\psi_{12}}}{B\Gamma}$	$= h_{21} \dfrac{e^{-\psi_{12}}}{e^{-\psi_{21}}}$
h_{21}	$-\dfrac{(Y_{out} + Y_R)(Z_{in} + Z_s)e^{-\psi_{21}}}{B\Gamma}$	$\doteqdot -\dfrac{(Y_{out} + 1/50)(Z_{in} + 50)e^{-\psi_{21}}}{2}$
h_{22}	$\dfrac{Y_{out} + (1 - \Gamma)Y_R}{\Gamma}$	$\doteqdot Y_{out} + \dfrac{h_{21}\, e^{-\psi_{12}}}{100}$

$$B = \frac{Z_s + Z_R}{Z_R}$$

$$\Gamma = 1 + \frac{(Y_{out} + Y_R)(Z_{in} + Z_s)e^{-\psi_{21}}\, e^{-\psi_{12}}}{B^2}$$

For complete conversion tables see References marked *.



EXPERIMENTAL SCHEME FOR TIME DOMAIN MEASUREMENT OF TRANSISTOR PARAMETERS