

THE COLLEGE OF AERONAUTICS
CRANFIELD

A Theoretical and Experimental Investigation of Tail Unit
Flutter on The M.S. 760 'Paris' *

- by -

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Summary

After an incident which, it was suspected, was due to symmetric elevator flutter, had occurred on the Morane-Saulnier 760 operated by the College of Aeronautics, Cranfield, a theoretical and experimental investigation of the aircraft's flutter characteristics was undertaken.

The theoretical investigation consisted of binary and ternary symmetric flutter calculations with and without the control circuit included. These showed the aircraft to be liable to flutter for mass distributions similar to that which existed at the time of the incident.

The experimental work consisted of flight flutter tests using control jerk excitation with both film and magnetic tape recording. These showed that the aircraft as supplied by the makers had a critical speed for symmetric elevator flutter of 380 knots, but that this could be lowered to 240 knots by the installation of a stick force indicator combined with unfavourable distributions of fuel load and fuselage mass distribution. The tests also showed the tail unit mode excited by rudder kicks to be safe, but as doubt exists as to whether this mode is the most critical antisymmetric one, further work is needed on this aspect. A 75 c.p.s. rudder buzz was encountered that was not caused by compressibility effects.

As a result of this investigation the elevator mass balance was increased and the aircraft proved to be free from elevator flutter up to at least 400 knots.

A general conclusion reached in this investigation was that static balancing of control surfaces should include the effect of components of the control circuit attached to them if those components contribute to the inertia couplings induced by vibration in other elastic modes.

* This report is based on a thesis submitted by Mr. Mitchell in partial fulfillment of the requirements for the Diploma of the College of Aeronautics. The investigation was supervised by Mr. D.J. Johns, who prepared this report.

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1. Introduction

The College of Aeronautics took delivery of a Morane-Saulnier M.S. 760 'Paris', registration G-APRU, in December 1958. This is a straight wing all metal aircraft powered by two Turbomeca 'Marbore' gas turbines giving 883 lb. thrust each. The aircraft (Figures 1 and 2) has an all up weight of 7650 lb. It seats four crew in a pressurised cabin ahead of the leading edge of the wing. It has an altitude limitation of 25,000 feet and an indicated airspeed limit of 350 knots, this latter being said to be set by wing/aileron flutter at 410 knots.

In February 1960 when a stick force transducer was fitted to the top of the control column an oscillation of the tail structure and elevators was noticed. On February 4th a flight to observe this was made, when it was recorded as a vibration of the elevators whose amplitude increased with increasing airspeed from 200 knots up to the maximum the pilot considered safe, 320 knots. It could be greatly reduced, if not entirely stopped, by holding the control column firmly.

The manufacturers, when consulted, stated that they had experienced this vibration but that it only occurred when a force transducer was fitted to the control column.

It was felt advisable to investigate the effect further, particularly as the aircraft's tailplane is mounted at the top of the fin. In this country in the last ten years at least five aircraft have been built and flown with this tailplane position. Of these one lost both elevators due to symmetric flutter (Reference 1) and another lost the complete tailplane due to anti-symmetric flutter (Reference 2).

The advent of rear mounted jet engines for civil aircraft has meant that several more aircraft now in the project or construction stage are to be fitted with high tailplanes. It is thus of interest to investigate the flutter of the M.S. 760 tail unit with the aims of firstly establishing the safety, or otherwise, of the particular aircraft; and secondly, of attempting to learn more about the flutter of high tailplanes that might be applicable to future aircraft.

This report covers the investigation of the tail unit flutter of the M.S. 760. The investigation is both theoretical and practical. The former consisted mainly of making standard flutter calculations to find the effect of mass balance and control circuit on the flutter characteristics of the aircraft. The latter consisted of flight tests to measure the aircraft's damping for structural vibrations at various speeds. From these the flutter characteristics can be estimated.

The two parts of the investigation must be considered together. In this report although the details are separated the results are brought together whenever this provides a clearer picture of the behaviour of the aircraft.

2. Ground Resonance Tests

2.1 The theoretical analysis of the M.S. 760 tail unit flutter uses normal modes

measured in ground resonance tests.

Three sets of ground resonance tests were made during this investigation with, in general, the aircraft excited at the tailplane, using Goodman 390A electro-mechanical exciters (Figure 3). These sets of tests were each aimed at establishing different aspects of the aircraft's vibration characteristics and the results of each set of tests were plotted in such a way as to highlight the aspect under consideration. In all tests the control column was held in the neutral position fore and aft by an elastic cord. Figure 4 shows the location of measurement points for all the tests in which the mode shapes were mapped using a hand-held Lan Elec seismic accelerometer.

Three different sets of exciter positions were used in all. For most of the tests two exciters were attached to the tailplane rear spar, one at each of the outboard elevator hinge fittings, using brackets which bolted on to the underside of the tailplane below the fittings. The exciters were connected in series and could be driven either in phase or in anti-phase to give symmetric or antisymmetric oscillations respectively.

To find the effect of exciter position on the results one exciter was fitted to apply a lateral horizontal force to the fuselage below the fin, instead of the two exciters on the tailplane. This could only produce very small vibration amplitudes but the modes excited agreed well both with regard to mode shape and frequency with those excited on the tailplane tips. In addition, in the third series of tests the elevators were excited directly at their trailing edge in order to determine the circuit frequency. Measurements were then only taken for symmetric excitation but a scan through the frequency range using antisymmetric excitation showed that nothing unusual was happening.

The first set of tests were aimed at obtaining a general picture of the aircraft's normal mode shapes and frequencies to guide the positioning of transducers and the design of the instrumentation system as a whole for the flight tests. The aircraft had two normal control columns fitted, had full main fuel, no fuel in the wing tip tanks, and carried no cockpit ballast to represent the crew. It was supported on its undercarriage with the tyre pressures reduced to about half their normal values for this and for both the other ground resonance tests.

The normal mode frequencies from the tests are given in Table 1.

The second set of tests was aimed specifically at measuring symmetric modes for use in the flutter calculations. The only modes measured were the wing fundamental bending, fuselage vertical bending, elevator rotation and tailplane first overtone bending. These modes were measured with the aircraft in each of the following conditions:

- (i) Main tank full, tip tanks empty, no nose ballast.
- (ii) Main tank full, tip tanks empty, 152 lb. nose ballast.
- (iii) Main tank empty, tip tanks empty, no nose ballast.
- (iv) Main tank empty, tip tanks empty, 152 lb. nose ballast.

In each case the aircraft carried 705 lb. ballast in the cockpit to represent

four crew with parachutes. The undercarriage was lowered and excitation was at the tailplane rear spar tips. One normal control column and the stick force transducer on a stub stick were fitted.

Table 2 gives the frequencies and, where they can be measured, the dampings of the symmetric modes excited during the second set of tests and Figure 5 shows the amplitude/frequency response at the tailplane rear spar tip in condition (i).

As related elsewhere in this report it was decided during the investigation to increase the elevator mass balance. This had been tried rather crudely during the second resonance test by attaching clamps to the elevator horns. This showed that increasing the mass balance from 5.5 lb. to 7.5 lb. stopped elevator rotation being excited by fuselage vertical bending.

Once the elevator mass balance had been modified to permit it to be increased for flight it was necessary to repeat the ground resonance tests to decide what value of mass balance was likely to be best, and to confirm that no new inertial coupling occurred with the larger mass balance fitted.

For this test the aircraft was in the same condition as for case (ii) of the second set of tests. Elevator mass balance values of 5.5 lb. (makers), 6.5 lb. and 7.5 lb. were tested.

2.2 Discussion on the Ground Resonance Tests

The behaviour of the aircraft when excited symmetrically at frequencies between 15 and 20 c.p.s. is very complicated. There appear to be three modes in this frequency range, all containing some fuselage bending and some elevator rotation in varying proportions and with various relative phase. It is difficult, and perhaps not very meaningful, to determine which of these is due to which component of the aircraft. As the elevator phase is 90° relative to the fuselage for the 18.3 c.p.s. mode this is the circuit resonance frequency. The 19.2 c.p.s. mode is termed elevator rotation (ii) for want of a better name.

In the second set of tests only two of these modes could be excited. This difference must be due either to the different control circuit inertia or the cockpit ballast causing two of the previous modes to merge into one. Both the modes that could be excited are sensitive to the fuselage mass distribution.

During the third set of tests it was observed that increasing the elevator mass balance to 6.5 lb. reduced the elevator rotation caused by fuselage bending, and a further increase to 7.5 lb. prevented it completely. 7.5 lb. is the mass necessary to statically balance the elevators and the control run down the fin.

With 5.5 lb. mass balance elevator rotation could always be excited by exciters attached to the tailplane, for any value of power used. With 6.5 lb. and low excitation power a structural mode was excited at 15.6 c.p.s. which changed to an elevator rotation mode when the power was increased or the elevator struck a sharp blow. With 7.5 lb. mass balance elevator rotation could not be excited from the tailplane. This indicates that 7.5 lb. mass balance gives effective dynamic balance of the elevators.

No new couplings were found in the frequency range 10-50 c.p.s. when the elevator mass balance was increased.

Throughout the tests, regardless of mass balance, symmetric excitation at about 18 c.p.s. caused antisymmetric motion of the rudder accompanied by out of phase motion of the rudder pedals. Under antisymmetric excitation the most easily excited mode is fin torsion at 13.0 c.p.s. This gives a very violent vertical as well as fore and aft motion of the tailplane tips.

The structural damping is obtained from the response spectra. Reference 3 shows that the sharper the resonance peak, the lower is the damping for that mode. The simplest way to evaluate the damping is to measure the width of the peak at a height of $1/\sqrt{2}$ that of the top of the peak. If the resonance is at a frequency of ω rad./sec. and the peak width is $\delta\omega$ rad./sec., then the damping is

$$\gamma = \frac{1 \delta\omega}{2 \omega}$$

This expression is developed in reference 3, pages 72-79.

Although there is no reason why the structural damping should be the same for all resonance modes, on the M.S. 760 it is reasonably constant at 4.8 per cent, except when the main fuel tank is empty, when it is rather lower.

3. Flight Tests

3.1 Flight Test Programme

The flight tests divided into three phases. The first established the flutter characteristics of the basic aircraft with empty wing tip fuel tanks. It was initially (and incorrectly) assumed that variation of the contents of the main fuel tank would not affect these characteristics. The second phase investigated the effect of variation of the contents of the main and wing tip fuel tanks, the control column inertia and the aircraft centre of gravity on the flutter behaviour.

This second phase became more complicated as it progressed and it was found that more aircraft parameters were significant. It was not completed rigorously but sufficient flights were made to understand how the various parameters affected the aircraft.

Both the first and second phases of flight tests used film recording. Before the third phase started the instrumentation system was modified to allow the use of both magnetic tape and film recording. A more important modification was to the aircraft itself. The flutter calculations had indicated that an increase of elevator mass balance would solve the flutter problem on this aircraft. After consultation with the makers the mass balance was increased from 5.5 lb. to 7.5 lb.

The third phase of flight testing was, therefore, intended primarily to establish that the aircraft was free from flutter with the increased elevator mass balance. In addition it was also an opportunity to gain experience of the use of magnetic tape recording, and to compare this technique with that of recording on film.

3.2. Excitation of the M.S. 760

The best method of excitation appears to be to use a sinusoidal exciter fitted with a brake. The required mode can be selected by choice of frequency, the amplitude response measured, then the brake is applied and the decay of the oscillations used to measure the damping. This method was originally considered for the M.S. 760, but rejected because of lack of electric power in the aircraft. An electrically driven inertial exciter such as that used by the R.A.E. Farnborough for the flutter testing of a Meteor (Reference 4) required a large power surplus to avoid frequency hunting near resonance. For the M.S. 760 at least 1 H.P., preferably 2 H.P., would have been needed. The aircraft can supply a maximum of 60 amps at 28 volts, giving the equivalent to 2.2 H.P. This ruled out an electrically driven inertial exciter.

Bonkers were ruled out on the grounds of safety, modification time and expense. An inertial exciter on the control column using about $\frac{1}{4}$ H.P. was seriously considered. As it was known that the control column affected the flutter characteristics of the M.S. 760 it was decided to use stick jerks initially to investigate the safety of fitting a heavy exciter to the control column. These tests showed that while it would be safe, it would so change the characteristics of the control system that the results would not be applicable to the aircraft in its normal configuration.

Use of a solenoid to excite the control system sinusoidally was considered (Reference 5). This reduced the electrical power required at the expense of a complex control system. The design and modification time involved made this system impracticable.

This left stick jerks as the only possible method of excitation; as these proved very effective during the initial flights it was decided to continue using them throughout the investigation.

3.3 Aircraft Instrumentation

The aircraft was fitted with ten \pm 9g Lan Elec seismic accelerometers in the rear fuselage and tail unit. Five of these were sensitive to vertical (or normal) acceleration, five to lateral. R.A.E. torsional velocity transducers (Reference 6) were fitted to each elevator and the rudder. Figure 6 shows the positions and reference numbers of the accelerometers and velocity transducers in the aircraft. Figures 7 and 8 show typical installations. The signals from these were integrated once or twice as necessary to give displacement. This was done using fully transistorised equipment designed and built by the Department of Flight Instrumentation Section.

For the first two phases of flight testing the displacement signals were recorded on a Hussenot A 13 film recorder. To avoid overcrowding the film a switching system was used to select six signals at a time. Five switch channels were used. Channels 1 and 2 carried the vertical accelerometers and both elevator transducers and were used to record symmetric oscillations. Channels 3, 4 and 5 were used to record antisymmetric oscillations.

When recording a number of signals on film it is necessary to choose

between two forms of presentation. These are, firstly, spacing the mean points of the signals across the film and using small signal amplitude. This gives greater clarity at the expense of accuracy. The alternative is to bunch all the traces at the centre of the film and use the largest possible amplitude. The authors chose this latter alternative. It certainly is more difficult to identify the signals, particularly as it has not been possible to use trace identification breaks, but the gain of a factor of four on amplitude has helped the analysis considerably once a trace has been identified.

For the final phase of flight testing a magnetic tape recorder was installed in addition to the Hussenot A 13. This was to gain experience of the technique of recording on magnetic tape, and to enable the transient response of the aircraft to a control jerk to be analysed electrically.

3.4 Flight Test Techniques

For the initial tests the aircraft was flown with a normal control column on the pilot's (port) side and a stub stick on the starboard side. The wing tip fuel tanks were empty. To measure the damping in symmetric and anti-symmetric modes at a given speed the aircraft was trimmed to that speed. The pilot struck the control column a sharp forward blow with his fist and then left the column free to vibrate. The instrument channel was then switched from 1 to 2 and the stick jerk repeated. Next the rudder pedal was kicked and a recording made on each of the three antisymmetric channels. This completed the measurements at that airspeed.

Measurements were made at between 8,000 and 12,000 feet. This altitude range was chosen as being small enough to have no effect on flutter speed. The mean altitude of 10,000 feet was decided on as being sufficient to make it possible to escape from the aircraft if a structural failure occurred.

This initial phase was completed in five flights.

These flights showed that two oscillation modes could be excited by control jerks. These are a symmetric fuselage vertical bending/elevator rotation mode at 18.3 cycles per second with a critical speed of 380 knots equivalent air speed, and an antisymmetric fin bending/torsion mode at 9.5 - 10.5 c.p.s. Over the speed range tested this latter mode has no critical speed for large amplitude oscillations but has non-linear damping that decreases with decreasing amplitude to zero for an amplitude at the tailplane tip of order 0.03 inches. Thus the aircraft flutters very gently in this mode throughout its flying life. These results are given in greater detail and discussed in later sections.

This stage of the investigation, corresponding to the initial flutter clearance of a prototype, was completed in about one fifth the flying time normally needed to clear a new aircraft. One reason for this was the reliability of the instruments, but a more important one was that the aircraft had previously been flown to 350 knots and so was believed to be safe up to that speed. Also the flutter incident of February 1960 had shown that if elevator flutter was encountered it was mild at speeds above the critical.

On a prototype being flight flutter tested it is prudent to take records at speeds up to a predetermined one on any one flight. After the flight the

records are analysed and it is decided how much this maximum speed can be raised for the next flight. Increases are usually about 10 knots. According to Bisplinghoff (Reference 7, page 554) an aircraft wing can be destroyed in two to three cycles of flutter 5 knots above its critical speed.

Because the M.S. 760 was believed to be safe, and the flutter was of a control surface rather than the primary structure, this test procedure was not rigorously followed. The first two flights were exploratory. On the third records were taken up to 300 knots. The fourth increased maximum speed to 330 knots and indicated a flutter speed of at least 360 knots, taking the most pessimistic set of points. The fifth flight concentrated on speeds between 320 knots and the design diving speed of 350 knots.

3.5 The Effect of Aircraft Configuration on the Flutter speed

Once the flutter characteristics of the basic aircraft were known the investigation was concentrated on the effect of two aircraft parameters on the flutter speed of the symmetric mode. These were the presence or absence of fuel in the wing tip tanks, and the moment of inertia and mass moment of the control column. It was believed that the former would affect the aerodynamic damping by altering the amount of wing flexing that occurred, while it was known that adding a mass, in this case a force transducer, to the control column made the aircraft more liable to flutter. Whether this effect was due to stick moment of inertia (mr^2) or the stick mass moment (mr) only was not known. As the control column slopes back in the trimmed position it appeared possible for pitching of the aircraft to cause an inertial moment tending to rotate the elevators to act on the control column.

A series of stick jerks with the wing tip fuel tanks full and then empty showed that tip fuel has a negligible effect on the flutter speed of the aircraft when the transducer is not fitted to the stick.

To investigate the effect of masses on the control column on the symmetric flutter speed a small rig was built. This was a 'T' shaped arm which would fit onto the stub stick in the aircraft. The stub stick is a control column about 8 inches shorter than the normal one fitted to the aircraft. The upper end is designed to carry the force transducer or other experimental equipment that might be required. The vertical leg of the 'T' fitted onto the stub stick with the cross piece horizontal fore and aft when the controls were central (Figure 9). Weights were made from mild steel cylinders with central holes so that they could slide on the cross piece. These could be fitted either forward or aft of the stick. They were held in position by a pin that passed through a hole in the weight and the cross piece. There were four holes in the cross piece, two forward and two aft of the stick.

Early tests with a 2 lb. weight on the arm showed very little variation of aircraft damping between the weight in the extreme forward and extreme aft positions. It was therefore concluded that the effect of the stick centre of mass varying from ahead of to behind the column pivot was unimportant. The same series of tests showed that the damping decreased considerably when the arm alone was fitted to the stick. It increased slightly (relative to that with the arm only) when a 1 lb. mass was fitted. These results

were almost independent of the position of the masses fore and aft, and indicated that the control column moment of inertia had a significant effect on the flutter characteristics of the aircraft.

Further flights with the stick force transducer (weight 2 lb. 14 oz.) fitted in an attempt to repeat the original flutter incident failed to do so. On three occasions the aircraft was flown to 350 knots with no trace of vibration. From stick jerks the flutter speed still appeared to be 380 knots, but the damping at subcritical speeds was reduced. On these flights the effect of changing the aircraft's centre of gravity was investigated qualitatively (owing to an instrument failure no records were obtained), and this appeared to be insignificant. This was later found to be incorrect. The authors were thus forced to the conclusion that something had changed in the aircraft and that it was no longer flutter prone below 380 knots.

Shortly after this the aircraft with the transducer fitted fluttered at 260 knots. This occurred on a routine student flight, when the fuel in the main (fuselage) tank was almost exhausted. The incident was repeated and recorded the next day, again with very little fuel in the main tank. The fuel load in this tank had originally been assumed not to affect the flutter characteristics of the aircraft as the tank has only a small displacement in the fuselage vertical bending mode. These results showed that this assumption was not valid.

It may be felt that it was rash to deliberately fly the aircraft into a flutter regime. The authors do not believe it was as there was ample evidence that flutter with the transducer fitted would not be catastrophic. From the incidents and the flight of February 1960 it was known that the flutter was mild, probably amplitude limited and could be stopped almost entirely by the pilot holding the control column. It was of the utmost value to get a measurement of the flutter mode shape and frequency to confirm that this mode was the one excited by stick jerks.

This flight was made with 99 lb. of ballast fitted in the nose of the aircraft. On the next flight, this was not carried but the aircraft was excited at intervals as fuel was burnt off. No large amplitude flutter occurred, indicating that the amount of nose ballast carried is a significant parameter. The flight also showed that the variation of damping with main fuel tank contents is complex.

The main tank capacity is 930 litres. When it contains 600 litres the damping is markedly reduced compared to the case of a full tank. With 400 litres the damping is much the same, but further reducing the contents to 200 litres causes the damping to increase again. These results are given in greater detail in a later section.

This rather complicated behaviour is supported by further records from the same flight. After excitation the structural vibrations decayed to a certain amplitude and then continued at this amplitude until the control column was held again. This amplitude depends both on speed and main tank contents, points of large amplitude corresponding to points of low damping and vice versa.

This behaviour with the transducer in and without nose ballast is so complex that it cannot be expected that accurate predictions of the behaviour of the aircraft in other configurations can be made from it. Similarly, the flutter calculations are not sufficiently detailed to predict it. To clear the aircraft it would be necessary to fly at all possible combinations of fuel state, ballast and stick inertia.

Further tests to discover the interaction of these parameters, and particularly to measure the variation of flutter characteristics with fuel state of the basic aircraft without the transducer fitted, were left until after the aircraft had been resonance tested and the magnetic tape recorder fitted.

3.6. Flight Tests with Increased Elevator Mass Balance

The test techniques used for the third phase of flight testing were identical to those used previously, except for the inclusion of the magnetic tape recorder in the instrumentation system. The recorder was fitted in the starboard rear passenger seat with 88 lb. of ballast under the seat. This was the maximum that could be fitted and, together with the recorder, was the equivalent of a passenger weighing 134 lb. and a 24 lb. parachute. The difference between this and the standard 165 lb. passenger was not thought to be significant.

For the first three flights of this phase the aircraft was flown with the stick force transducer on the port side and a normal control column on starboard. 99 lb. of ballast was fitted to the nose of the aircraft. This is the condition in which flutter previously occurred at about 250 knots.

The tests were conducted fairly cautiously. The first flight was to 250 knots, the second to 300 knots and the third to 375 knots. The makers permission had been obtained to exceed the normal Certificate of Airworthiness speed limit of 350 knots. In addition, the aircraft was flown under experimental category B conditions because of the change of mass balance.

These tests showed that the damping of the elevator mode was now almost constant at 2.5 per cent critical throughout the speed range of the aircraft.

The final flight was made to check that the increase of mass balance had also improved the aircraft with normal control columns. Once more the aircraft was flown to 375 knots. This flight showed that the aircraft with 7.5 lb. of elevator mass balance but otherwise as supplied by the makers was now free from flutter up to at least 400 knots.

4. Analysis of Flight Records

4.1 Analysis of film records

If the aircraft can be considered a single degree of freedom system when vibrating, as it is if it vibrates in one normal mode, then the decay of the transient response to an impulse will be exponential. Since the aircraft is a poorly damped structure the decay will be an exponentially decreasing oscillation. Thus, if the logarithm of the amplitude of successive peaks of the oscillation is plotted against the number of the peak after the start of the motion a straight line should result, the slope of which is related to the damping

by

$$\gamma = \frac{1}{2\pi} \log_e A$$

where $\gamma = c/c_c$, A is the amplitude ratio of successive full waves.

The technique of analysing the film record obtained from a control jerk was as follows. When the trace from the transducer to be used had been identified the envelopes of its peaks and troughs were sketched in freehand. This is shown for a stick jerk in Figure 10. The height between the envelope lines was then measured at each peak or trough. The logarithm to base ten of the height was then plotted against the number of the wave, as in Figure 11. The best straight line was then drawn through the curve and the slope of this line used to determine the damping.

In practice the decay is not exponential, but contains beats depending on the purity of the mode forced. In the vast majority of cases, however, a very good approximation to a straight line was obtained.

There were two exceptions to this. The mode forced by rudder kicks showed non-linear damping in that the logarithmic plot started steeply downwards immediately after the kick and then became less and less steep until at some small amplitude the slope was zero. Figure 12 shows an example of this. The damping was evaluated from the slope of the curve at an arbitrary amplitude of 0.316 ins. peak to peak on the film.

The other irregularity was not so easy to evaluate. When the transducer was fitted to the control column and under certain conditions of speed and fuel state, the oscillation resulting from a stick jerk did not decay for about half a second, and then suddenly stopped in a very few cycles. Figure 13 shows a film record of a decay of this type, and Figure 14 shows the corresponding logarithmic amplitude plot. It is clearly not possible to fit a straight line to a curve of this type.

The technique of film analysis used does highlight non-linearities in the aircraft's response. For a good, linear trace the damping could be accurately obtained with a repeatability of ± 10 per cent. Frequency could be measured to 0.1 c.p.s. for a 20 c.p.s. oscillation. For this frequency the phase could be determined to $10^\circ - 20^\circ$, the accuracy depending on the number of modes superimposed on the signals whose phases were being compared.

The highlighting of non-linearities makes the plotting of logarithmic amplitude graphs for each control jerk very well worth while, even though it is a lengthy and tedious process. The line obtained makes it possible to estimate the accuracy of the result and the quantity of unwanted modes present. This is not possible if the damping is obtained directly from the flight results by comparison with standard exponential decay curves.

4.2 Analysis of Magnetic Tape Records

When the aircraft response is recorded on magnetic tape the existence of the signal in an electrical form makes possible an analogue type of analysis. To



separate the various modes the signal from the tape is played through a narrow pass filter which removes all frequencies except the one required. If the filter has a near zero damping it will not distort the signal.

In practice the tape is made into an endless loop and played backwards. This is to avoid setting the filter ringing with the initial high amplitude disturbance and swamping the interesting part of the decaying oscillation. This technique has been tried on the signals obtained from the M.S. 760 with considerable success.

The endless loop of tape carrying the signal to be analysed is played backwards through a standard replay unit. The loop is about four feet long, with at least a foot between the splices and the signal. The signal from the tape is amplified, demodulated and put into the analyser. This is a filter whose damping is kept to zero by the use of an amplifier with variable feed back.

In the equipment used the amplifier is a modified Solartron analogue computer trainer with the filter components in a separate unit. The output can be permanently recorded on paper by a New Electronic Products Ltd. ultra-violet recorder type 1185. The complete set of equipment is shown in Figure 15. From left to right the units are the tape replay deck, demodulator, analyser, oscilloscope and ultra-violet recorder.

The system is first used to determine what frequencies are present on a tape. The signal is repeatedly put through the filter while the filter frequency is changed by small steps through its full range, from 6 - 55 c.p.s. Each time the signal goes through the filter it sets it ringing, the amplitude of oscillation depending on the amplitude of vibration, at the frequency to which the filter is tuned, present in the signal. This procedure provides data from which can be plotted an amplitude/frequency curve similar to that obtained from a ground resonance test.

The filter is then tuned to each of the response peaks in turn and at each peak an ultra-violet recorder trace is taken of the signal from the filter. These are comparatively pure modes and are analysed by plotting logarithm of amplitude curves as before.

As a test of the system the elevator response to a single stick jerk at 150 knots was analysed. The trace from the film recording of the jerk is shown in Figure 16. This contains a 4 c.p.s. component with initial amplitude 3.1 inches and high damping, and a 20 c.p.s. mode whose logarithmic amplitude graph, from the film, is given as Figure 17. The initial amplitude is about 0.4 inches.

It will be seen that the damping of the high frequency component is between 3.4 and 4.1 per cent critical, and that the decay curve shows beats at 4 c.p.s. indicating a mode with frequency of either 24 or 16 c.p.s. is also present.

Figure 18 shows the amplitude/frequency curve obtained from the analysis of the magnetic tape recording of the same stick jerk. Clearly frequencies of 20.0 c.p.s., 16.8 c.p.s., 13.3 c.p.s., 11.5 c.p.s. and less than 6 c.p.s. are present. Figures 19 and 20 show the filtered traces and the logarithmic

amplitude graphs for the 20.0, and 16.8 c.p.s. modes.

In the case of the 20.0 c.p.s. mode the original ratio of wanted to unwanted signal was 1 : 8. After filtering it became 2 : 1, a gain factor of 16. When the trace is analysed the decay is nearer a perfect exponential than the author has ever obtained from a film recording. The decay rates obtained from film and tape are the same, although the range of possible dampings that could be obtained from the film by suitable choice of the best straight line is far greater than could be obtained from the tape recording.

The decay of the 16.8 c.p.s. mode is also a near perfect exponential; this is even more impressive as the mode is quite invisible on the film record. The 13.3 c.p.s. mode had a much less perfect decay, but this is hardly surprising when it is considered that it is a rather weaker mode. The 11.5 c.p.s. mode is so heavily damped that its damping cannot be evaluated numerically.

These modes can all be identified as ones that would be expected to be present. In order of descending frequency they are the control circuit (only one control column fitted), fuselage bending, a low frequency mode predicted by the flutter calculations and observed by Morane-Saulnier but not previously by The College of Aeronautics, and finally at 11.5 c.p.s. tailplane bending. It is possible that the last two modes may be reversed, tailplane bending occurring at 13.3 c.p.s.

5. Theoretical Investigation

The technique of flutter calculations is well established, there being several textbooks (reference 3, 7 and 8) and a very detailed report (reference 9) on the subject. The theoretical investigation of the flutter of the M.S. 760 tail unit applies the technique to the particular aircraft.

From the start an energy solution of the equations of motion was used. The generalised coordinates needed for Lagrange's equations are taken to define measured normal modes for the aircraft and a control rotation. The generalised aerodynamic forces on the aircraft are obtained in two ways, both attempting to correct two-dimensional derivatives to the comparatively low aspect ratio (3.9 for the tailplane) of the M.S. 760 surfaces.

5.1 Symmetric Flutter

For symmetric flutter the modes used were an elevator rotation (the elevator being assumed torsionally stiff), the wing fundamental flexion mode and the fuselage vertical flexion mode. These were measured modes and each included all parts of the aircraft as well as the component after which the mode was named. Thus the fuselage vertical flexion mode included wing and tailplane flexion and torsion, and some elevator rotation.

Two sets of calculations were made. The first used modes measured on the third prototype M.S. 760 by O.N.E.R.A. (Reference 10), and aerodynamic forces obtained from the two-dimensional derivatives by a method developed by the first author. The second calculation used modes measured on the College aircraft and aerodynamic terms recommended by Minhinnick (Reference 11).

It was noticed that although the elevators are statically balanced when out of the aircraft, and normally rest nose down when in the aircraft, they are in fact under balanced. This is due to the control run up the fin which provides an underbalancing moment of 18 lb.in. With the complete circuit connected this is not apparent as the sloping back control columns provide an overbalancing moment that overrides the effect of the control run. However, when the aircraft is oscillating the displacement amplitude is much greater at the back of the fuselage than at the control column. Thus the apparent static overbalance becomes a dynamic underbalance for the elastic modes referred to as wing fundamental bending and fuselage vertical bending. This should be made clear by inspection of Figure 21, which shows a schematic drawing of both the elevator control circuit and main fuel tank disposition relative to the fuselage vertical bending mode.

In the flutter calculations the mass of the control run up the fin is included in the mass distribution for the elevators. This is believed to provide the inertia coupling that causes flutter in this case.

5.2 First Flutter Calculation

The first calculation used a wing bending mode that included some fuselage bending with a frequency of 10.3 c.p.s. (Reference 10, page 50) and a fuselage vertical bending mode at 18.3 c.p.s. that included some wing and tailplane flexure and torsion (Reference 10, page 29). These modes were checked for mass orthogonality and Broadbent's criterion

$$\frac{a_{rs}}{\sqrt{a_{rr} a_{ss}}} < 0.10$$

was evaluated at 0.025. The modes were measured with the wing tip fuel tanks fitted but empty, the main fuel tank full and ballast representing four crew in the cabin.

The procedure for evaluating the aerodynamic derivatives for the M.S.760 tail unit (tailplane aspect ratio 3.9, taper ratio 0.6) was then as follows. The steady spanwise lift distribution for this planform was obtained from Reference 12, as was the steady lift distribution for a rectangular aerofoil of the same aspect ratio. The ratio of the dynamic forces at a given spanwise station on a rectangular wing of aspect ratio 3.9 to the two-dimensional dynamic forces was obtained from reference 13 for a chosen mode shape. The ratio of steady lifts at that station obtained from reference 12 is then used to correct the ratio of dynamic forces for a rectangular wing to that for a tapered one. The ratio of three-dimensional to two-dimensional forces obtained by this method was assumed to apply to all aerodynamic derivatives at the particular station.

When checked against the values of three-dimensional derivatives quoted by Minhinnick (reference 9) those obtained by the method outlined above agreed to within 20 per cent except for the elevator derivatives which were much too small.

A great advantage of three dimensional derivatives is that they are insensitive to changes of reduced frequency. Minhinnick recommends that in general they should be regarded as independent of frequency. In view of the work involved the derivatives were evaluated at $\nu = 0.6$ and then assumed to apply for all frequencies.

Stiffnesses for the elastic modes were evaluated by equating kinetic to potential energy in the modes excited during ground resonance tests. This is valid so long as the mode shapes do not change in flight. This assumption is implicit in the semi-rigid method used throughout this investigation. The stiffness of the control circuit was initially taken as zero.

Once the terms of the equations of motion were known the solution was comparatively simple, using the Ferranti Pegasus digital computer owned by The College of Aeronautics. The three simultaneous equations in the three generalised coordinates are solved by equating the determinant of their coefficients to zero with the speed and frequency as unknowns.

A programme was written that expanded the determinant. This gave real and imaginary equations which were cubic and quadratic in ν^2 respectively, and both contained the speed. These were solved by the same programme by substituting a range of values of speed into the equations and at each speed solving each equation for ν . The results were plotted graphically and points at which the results from the two equations coincided were solutions. A developed version of this programme provides a similar solution with the added facility of being able to solve for damping at points away from the flutter boundary. This is a useful check on the stability of flutter solutions.

This type of calculation, with the circuit stiffness zero, is referred to as the circuit cut case. The results of the calculations are shown in Figure 22 where the flutter speed is plotted against the mass balance in the elevator horn. The predicted flutter frequency is written alongside the theoretical points.

The solution will be seen to consist of two lobes of instability. The smaller represents a high frequency mode which has both upper and lower critical speeds for all values of mass balance between zero and 3.6 lb. The other is a lower frequency mode which has no upper critical speed and extends out to values of mass balance of at least seven pounds.

The agreement with the experimental result for the aircraft with the normal control columns fitted is poor both with regard to speed and frequency.

5.3 Second Flutter Calculation

A further set of calculations were made to attempt to achieve a better agreement with experiment. These calculations were the same as the first in principle, but the modes used were measured on the College aircraft and the values of aerodynamic derivatives were as recommended by Minhinnick (Reference 11).

The modes used were measured for the aircraft with the main fuel tank empty, four crew and 152 lb. nose ballast. Flight tests had indicated that these conditions would give the aircraft a critical equivalent air speed of 640 ft/sec. at 10,000 feet. The transducer was fitted to the control column during the mode measurement.

The structural terms obtained from these modes were all within 15 per cent of those for the first calculation. Broadbent's orthogonality criterion for the wing bending mode with respect to the fuselage bending mode was 0.09.

Minhinnick's technique for obtaining aerodynamic derivatives assumes that the variation of the derivatives with frequency is small. If this is so then the aerodynamic derivatives obtained from aircraft control and stability tests can be used as the derivatives for flutter calculations.

The individual derivatives obtained by this method agree with those of the first calculation to within 20 per cent for the main surface and about 50 per cent for the control surface. However, after integration over the aircraft the final aerodynamic terms in the flutter determinant are very different. The terms involving the elastic modes only are about 50 per cent larger in the second calculation, but those involving the elevators are quite different.

The boundaries for ternary flutter obtained by the second calculation are shown in Figure 23. It will be seen that again the two lobes are present, the high frequency one being rather larger this time. The flutter speed at mass balance values close to that used on the M.S. 760 is much lower, 850 ft/sec. at a mass balance of 5 lb. compared with the experimental value of 640 ft/sec. at 5.5 lb.

In addition to the ternary calculation a binary calculation was also undertaken using another programme. This solved the simpler binary flutter equations directly in terms of speed and frequency. The results of the binary calculation using elevator rotation and fuselage vertical bending as degrees of freedom is shown in Figure 23. It will be noticed that the envelope of the two lobes of the ternary calculation is almost identical to the binary curve on Figure 23, but the frequencies around the binary curve in the 5 lb. mass balance regime are very much nearer those measured in flight. Using elevator rotation and the wing bending mode as degrees of freedom does not predict flutter at any value of mass balance between zero and ten pounds.

Since it was known experimentally that the moment of inertia of the control column had a profound effect on the flutter characteristics of the aircraft it was clearly necessary to include the control circuit in the calculations. This was done using the method developed by Templeton. (Reference 14).

The effect of the control circuit on control surface flutter depends on two parameters. These are the circuit stiffness and the frequency of vibration of the control column if the surface is rigidly clamped. The former was obtained from ground tests carried out by Service Technique Aeronautique and described in reference 15. The latter depends on the moment of inertia of the control column or columns. On the M.S. 760 these moments of inertia were calculated from measurements of the weight and centre of gravity of the control columns. The stick frequencies were then calculated to be 28.2 c.p.s.

for the circuit with two normal control columns or 19.7 c.p.s. for the circuit with one normal column and the transducer on its stub stick.

Figure 24 shows the results of the binary calculation with the circuit included. It will be seen that as the stick inertia increases and the frequency drops the aircraft becomes progressively less flutter prone. There is no indication of an initial lowering of the flutter speed as the stick inertia increases.

This result does predict that when the pilot grasps the control column he will stop the aircraft fluttering, as the mass of his hands increases the moment of inertia of the control column. However, it totally fails to predict the effect of the transducer on the aircraft's flutter characteristics.

6. Results

Figure 23 gives the results of the circuit cut elevator flutter calculations for the aircraft in the condition that flight tests had shown to be the most flutter prone. It will be seen from the binary calculation that the predicted flutter speed at the value of mass balance used in the aircraft is very much higher than that measured in flight, but that only a small change in the position of the flutter boundary would be needed to achieve agreement between theory and practice.

As a check on the calculations, and to enable the effect of structural damping to be determined, the six degree of freedom flutter simulator at the R.A.E., Farnborough was also used to solve the flutter equations. The results are given in Table 3.

Figure 25 gives the digitally calculated damping at sub-critical speeds for a mass balance of 4 lb. This was the largest mass balance at which the high frequency lobe of the ternary solution was encountered. The damping of the low frequency lobe fell to zero at its critical speed even more suddenly than did that of the high frequency lobe.

Figure 26 shows an example of the response of the aircraft to a stick jerk taken at 10,000 feet altitude and 200 knots with the makers original mass balance, one normal control column and one stub stick fitted.

Figure 27 shows the continuous antisymmetric oscillation of the tail unit that occurs in both rough and smooth air. Figure 28 shows that continuous self-excited 75 c.p.s. vibration of the rudder also occurs.

A film record of limited amplitude flutter with a frequency of 17 c.p.s. has been obtained for the M.S. 760 but is not included here because of its length. This film was of interest since no record of limited amplitude flutter has apparently ever been presented in a published document. Figure 29 is a logarithmic amplitude plot of the flutter taken at 6000 ft. altitude, a speed of 300 knots with 300 litres of fuel in the main tank. The stick force transducer, one normal control column and 99 lb. of nose ballast were fitted. Figure 29 shows the limiting amplitude most clearly with the initial damping being -0.5 per cent critical. The inset drawing illustrates the form of the amplitude plot as recorded on film.

Figure 30 shows the variation of limiting amplitude with speed for similar aircraft conditions.

Figure 31 shows the variation with speed of the damping of the 18.3 c.p.s. symmetric oscillation obtained from stick jerks on the basic aircraft. It shows that flutter will occur at 380 knots equivalent air speed. When consulted on this the makers stated that they had encountered elevator flutter at 360 knots on the third prototype M.S.760. This could be stopped by the pilot grasping the stick.

Figure 32 shows that the effect of fuel in the tip tanks is very slight, possibly beneficial. This would agree with the destabilising effect of wing motion predicted by the flutter calculations, but the measured result is not really based on enough measurements to be believed implicitly. It could be due entirely to two inaccurate points at the highest speed.

These results are all based on measurements by transducer 2 at the tailplane tip. Similar results are obtained if the traces from the elevator transducer or the rear fuselage transducer 5 are used. In general the rear fuselage damping is some 20 - 30 per cent greater than that of the tailplane tip, while that of the elevators is about 50 per cent less.

Figures 33 and 34 show the effect of the main fuel tank contents on the damping of symmetric vibrations when the stick force transducer is fitted. The frequency ranges from 17.0 to 17.5 c.p.s. The condition of the aircraft was identical to that in which it fluttered at 240 knots except that no nose ballast was fitted.

Figure 35 shows the effect of varying the vibration frequency by fitting weights to the control column while the aircraft maintained a constant speed of 150 knots. Fuller details of this test are given in Table 4.

Figures 36 to 38 show how the damping of symmetric vibration is changed by adding 2.0 lb. additional mass balance to the elevators. It will be seen that the M.S.760 is free from symmetric elevator flutter up to at least 400 knots with this extra mass balance. Tables 5 to 10 give measured dampings, frequencies and phase angles of symmetric vibrations for various aircraft configurations.

Figure 39 shows the variation with speed of the damping of the 9 - 10 c.p.s. antisymmetric vibration forced by rudder kicks. Figure 40 shows the variation with speed of the amplitude of the continuous antisymmetric vibration of the tail unit, as measured at the tailplane tip by transducer 2. Tables 11 and 12 give details of the results plotted on Figures 39 and 40 respectively.

7. Discussion

7.1 Safety of the Flight Tests

In a flight investigation of this sort safety is of paramount importance. In this particular investigation it was most valuable to know that before the investigation commenced the particular aircraft concerned had repeatedly flown to 350 knots with and without the transducer fitted, and would not flutter

catastrophically in this speed range. This made it possible to fly tests that would have been the height of folly on an unproven prototype.

Flight flutter testing is inevitably dangerous in that the aircraft is flown to its design speed limits and if anything goes wrong at these speeds it is more likely to be serious. Escape from the aircraft would also be more difficult at high speed. On the other hand, flutter testing in itself will not cause flutter. If an aircraft is going to flutter at a certain speed it will flutter at that speed, whether or not it is instrumented and tested to detect the onset of this condition. Thus flight flutter testing can only be beneficial. Dangerous conditions can be detected and avoided, and the safety margins at design conditions can be estimated from test results.

Further steps can be taken to improve the safety of flight flutter tests. If sufficient theoretical results are available, as they should be in the case of a prototype, then the damping in each mode that should occur at a given speed will be known. If the measured damping does not agree with that calculated it reduces any temptation to use the theoretical work to bias extrapolation of the flight results to a higher speed. On the other hand, if flight and calculated results agree with regard to damping, mode shape and frequency at one speed then it should be safe to consider theoretical results for a higher speed when planning the next flight. This could be most useful in the case of an aircraft for which the damping first fell, and then increased again, as speed was increased.

It is most valuable during flight tests to know how speed, altitude or aircraft configuration should be changed to reduce the violence of flutter if it occurs. Usually, reducing speed stops flutter, but this need not be so on an aircraft for which the flutter regime changes rapidly with altitude, Mach Number or fuel load. On the M.S. 760 grasping the control column stops elevator flutter as the calculations predicted, but aircraft could be built such that holding the control column would make a control surface flutter more violent.

Points such as these must be borne in mind during the planning of a flight test. The pilot must always know that he can retreat from a dangerous position to one that has been proved safe. The flight must be planned to reach a speed that is certain to be safe, and this speed must not be exceeded. The pilot should be asked to trim his speed up to the final one selected rather than to fly at around the selected speed while he trims the aircraft, as this underlines the need for caution. In a conservatively planned test the odd five or ten knots involved should not be of great practical significance as the safety margin should be considerably larger than this.

The greatest danger in the M.S. 760 investigation has been one of fatigue. Elevator flutter on this aircraft is amplitude limited and can be stopped by the pilot, so will not be catastrophic. On the other hand, the stresses induced during fluttering may be sufficient to cause failure by fatigue.

With the stick force transducer fitted to the aircraft and with the makers elevator mass balance (5.5 lb.) there is slight continuous vibration at speeds above 300 knots. The stresses induced by this should not cause damage, but in view of the lack of fatigue test data and the life the aircraft is expected to reach it should not be flown in this condition without special authority and a

record of these flights should be kept.

7.2 Symmetric Flutter

The results from the first phase of flight testing showed beyond doubt that the aircraft as supplied by the makers would suffer elevator flutter at 380 knots equivalent air speed. This result was later confirmed by Morane-Saulnier, who had experienced elevator flutter on the third prototype M.S. 760 at 360 knots. The flutter was amplitude limited and could be stopped by the pilot grasping the control column.

The mode that the elevator couples with is fuselage vertical bending at 18.3 c.p.s. Both the frequency and mode shape measured in flight agree with those measured during ground resonance tests.

Phase two of the flight tests showed that the behaviour of the aircraft becomes much more complicated when the frequency of the control circuit is varied. The variation of damping with frequency given in Figure 35 could be due to one of two causes. It could be a result of frequency coincidence of the aircraft as a whole with the natural frequency of the fuselage bending mode, or it could be due to some effect directly causing a change in the damping of the aircraft. One example of this latter is fuel sloshing, an effect that will clearly change the damping of the aircraft as a whole. It is far from obvious whether this change will account for the effects observed.

The known variation of damping of the elevator mode with main fuel tank contents and with nose ballast could be due to either mechanism. In the first case burning off of fuel or changing the ballast load will change the natural frequency of the fuselage bending mode and so change the degree of frequency coincidence. From the flight records the extreme frequency limits for the elevator oscillation with the transducer fitted are 17.0 c.p.s. and 17.5 c.p.s. Thus the frequency does not vary by more than 0.5 c.p.s. through the whole range of speed and main fuel tank contents.

By the second explanation varying the ballast load will vary the mode shape, and hence the amplitude of oscillation at the main tank. Varying the tank contents will vary the sloshing behaviour directly.

The authors feel that the second explanation is correct although it cannot be proved as yet. The evidence that points towards it is given below.

From the flight trials the damping at a given speed decreases as the main tank fuel contents fall from full (900 litres) to 600 litres, remains constant for 600 to 400 litres, and then rises again towards that for the full tank case as the contents fall towards zero.

Now the main tank is basically semi-cylindrical with the axis fore and aft, but it has a projection downwards for about the forward third of its length. Figure 41 shows this. The tank has baffles at about 1 foot intervals so sloshing of the contents as a whole will not occur; anyway, the frequency of the effect is far too high for this to be happening. However, a frequency of 17.5 c.p.s. is about that at which a fourth harmonic standing wave will form on the surface of the fuel between the baffles. Reference 16 shows that this will happen in circular and annular tanks. Energy dissipation in a wave system of this type

will be proportional to the free surface area in the fuel.

Now, because of the shape of the tank, as the contents fall from 900 to 600 litres the free surface area of the fuel increases considerably. From 600 to 400 litres there is little change. By the time the contents are down to 300 litres all the remaining fuel is in the forward tank section and the surface area is much reduced.

The flutter calculations show that as the control column inertia is increased the unstable region on the flutter diagram is steadily reduced. This has been shown in Figure 24. The calculation thus indicates that grasping the stick and increasing the stick inertia by the mass of the pilot's hands will stop the elevator flutter. It gives no suggestion of the initial drop of flutter speed that occurs when the flutter frequency is lowered towards 17.5 c.p.s. One of two conclusions can be drawn from this. Either the calculation is not correct, which does not appear likely in view of its accuracy in predicting the flutter speed and frequency, or some effect is occurring which has not been included in the calculation.

If fuel sloshing is occurring it is as a high, probably fourth, harmonic standing wave between the tank baffles and will cause a positive damping of the aircraft structure (reference 16). Energy is absorbed by, and dissipated in, the fuel motion. This appears to be a stabilising effect, but in fact need not be so. Damping, as well as absorbing energy, will cause phase changes in the complete motion of the aircraft. These changes will alter the rate at which the oscillating aircraft absorbs energy from the airstream. If damping increases this rate by more than the rate of dissipation due to the damping the overall effect will be destabilising.

This effect has been shown by Broadbent in reference 17, where the introduction of up to 20 per cent critical structural damping in a wing flutter problem was found to reduce the flutter speed. However, when the Royal Aircraft Establishment six degree of freedom flutter simulator was used to investigate the effect of structural damping on the M.S. 760 elevator flutter this was found to be very beneficial.

From the flight tests the phase of motion of the elevators relative to that of the tailplane has been observed to vary. At 300 knots with the normal control column fitted the elevators are in phase with the tailplane. Fitting the stick force transducer causes phase changes of 60° to 85° , the exact value depending on the contents of the main fuel tank. Large values of phase angle correspond to points of low aircraft damping.

Thus there can be no doubt of one link in the mechanism by which the transducer or main fuel tank contents affect the elevator flutter of the M.S. 760. The mechanism is one of changing the phase angle between the two significant degrees of freedom of the aircraft to enable it to extract more energy from the airstream.

Two minor pieces of evidence support the case of fuel sloshing. One is the 'plateau' type of decay curve, an example of which is shown in Figure 13 and 14. These look as though motion of something in the aircraft builds up for about half a second after a stick jerk. Then, when the amplitude reaches a critical level, the aircraft changes in some way and becomes very heavily damped. The change does not cause a change of phase angle between the

elevators and the structure. It might be that the standing waves in the fuel tank take half a second to build up to a level at which the crests start to break and foam.

The other evidence against a straightforward frequency coincidence being the explanation is that in ground resonance tests the closest frequency coincidence between elevator and fuselage modes occurred when the aircraft with its main tank empty had no ballast in the nose and the stick force transducer fitted. Yet in flight this condition was less critical than the similar one with 99 lb. of nose ballast.

Thus, although the evidence as to the cause of the variation of the damping of the M.S. 760 with frequency may slightly favour the fuel slosh theory it is far from definite. The effect deserves further investigation as an academic exercise, but it was never the primary cause of the elevator flutter and now that the elevator mass balance has been increased it is no longer a danger to the aircraft.

The calculations seem to justify the use of Minhinnick's rules for obtaining unsteady aerodynamic forces. They also indicate the importance of determining the whole flutter diagram as opposed to making a single calculation for the aircraft as it is to be flown. For the M.S. 760 the binary flutter speed at a mass balance of 5.5 lb. (that fitted to the aircraft by the makers) is about 1300 ft./sec., which would appear to be quite safe. However, the calculation for 5.0 lb. mass balance gives a flutter speed of 850 ft./sec., only 20 per cent above the design diving speed for the two seat aircraft.

On the M.S. 760 changing the elevator mass balance does not change the structural deformation mode appreciably; what does change is the ratio of elevator rotation to tailplane linear displacement in the fuselage bending mode. Since the elevator rotation was included as a separate variable it is free to vary with mass-balance, and the calculation is sufficiently flexible to accommodate this change of mode shape.

One effect that does not occur on the M.S. 760 is loss of mass balance effectiveness, unless the control column is included as mass balance. In general loss of mass balance effectiveness is due either to flexibility of the arm or attachment carrying the mass, or the nearness of the balance mass to the nodal line of the mode in question. Either of these factors can cause a mass to contribute inefficiently, or even in the opposite anti-balance sense, to the dynamic balance of a control surface.

On the M.S. 760 the natural frequency of the elevator horns that carry the mass balance is 76 c.p.s., some four times that of the fuselage bending mode in which flutter occurs. The nearest nodal line in this mode is five feet forward of the balance mass.

In the calculations rigid body pitch and plunge have not been included as separate degrees of freedom. This is because the rigid body pitching frequency of the M.S. 760 is about 6.0 c.p.s. at 350 knots at sea level, about one third of the flutter frequency. It is thus most unlikely that this will affect the elevator flutter.

Comparison of the ternary and binary solutions is most interesting. In reference 18, Jahn and others made similar calculations for the Typhoon. They found a very similar effect of the binary calculation giving a single region, which was in fact composed of two flutter lobes of different frequencies which were revealed by ternary calculations.

The calculations suggest that wing damping has no very significant effect on the elevator flutter of the M.S. 760. This is contrary to general experience, but does agree with the experimental evidence that the damping of the elevator mode on the aircraft before modification is not affected by the presence of fuel in the wing tip tanks. This must indicate that there is very little wing motion during elevator flutter.

The calculation of damping at sub-critical speeds is disappointingly inaccurate. However, it does indicate that the low frequency mode is much more heavily damped than the high frequency one. The approach to flutter in the low frequency mode would be extremely sudden.

The failure of the damping calculation is not so surprising in view of the flight result that the damping of the aircraft is not the same for all its components. In general the fuselage damping was about twice that of the elevators, with the tailplane structure in between. Apart from confusing the damping calculation this effect causes the mode shape to vary with time after a stick jerk.

The low frequency mode has not been observed with certainty in flight during this investigation, although immediately after a stick jerk there is always a heavily damped low frequency component in the response. The frequency of this component is 4 - 6 c.p.s. so it is almost certainly the short period pitching of the aircraft as a rigid body. However, the makers have recorded a 12 - 13 c.p.s. symmetric oscillation during their own flight flutter tests. This occurred when the tip tanks were part full. Thus the low frequency lobe of the ternary calculation does correspond to a mode that can occur in flight.

The calculated effect of the control column on elevator flutter does not agree with the actual behaviour of the aircraft, although it does agree with the findings of reference 18.

The elevator mass balance was increased from 5.5 lb. to 7.5 lb. after consideration of static and dynamic ground tests, and the flutter calculations. The tests showed that the 2 lb. increase should achieve effective static and dynamic balance, and the calculations showed that it gave a 15 per cent mass balance safety margin over the most pessimistic result.

The results of flight tests with the increased mass balance have been given earlier. They show that the elevator and fuselage modes have been uncoupled. The damping is almost independent of air speed.

Unfortunately, the increase of mass balance has lowered the frequency of the aircraft to about 17 c.p.s., well within the low damping frequency region. The aircraft damping is low, about 2 per cent critical with a minimum at one low speed point of 1.2 per cent. However, records show that this is adequate for the safety of the aircraft, as it is large enough to avoid undue excitation of the mode by turbulence. With the transducer fitted the aircraft is slightly better

damped than with the normal control columns.

The aircraft as supplied by the makers was basically flutter prone because of effective static underbalance of the elevators. This underbalance is caused by the failure to include any of the control circuit in the balance calculation. In this Morane-Saulnier's action was that which can be interpreted from the British Civil Airworthiness Requirements (reference 19) and Av.P.970 (reference 20). Ignoring the control run is usually justifiable as it is mainly horizontal and symmetric and so it is unaffected by normal or lateral accelerations. But in an aircraft such as the M.S.760 with a considerable vertical control run close to the control surface this will be subjected to similar acceleration induced forces as the surface itself. It should thus be included with the surface for static balancing purposes.

In fact, even when the additional mass balance fitted for the third phase of flight testing is used the centre of gravity of the control surface on its own is 5 per cent of the elevator chord forward of the hinge line, just within the limits set in references 19 and 20. In this condition the elevators and their control circuit are balanced. But with the increased mass balance in the elevators there is no longer anything to counteract the static overbalance of the control column. Consequently, on the ground the elevators rest on the up stop with the control column fully back. This is a little annoying for the pilot when taxiing, but once the aircraft is airborne it is not noticeable as the airload on the elevators balances the stick.

8.3 Antisymmetric Vibration

The fact that a symmetric flutter mode was not satisfactorily damped on the M.S.760 has meant that it has received considerable attention at the expense of the antisymmetric modes. Consequently comparatively little has been learned about the more interesting problem of the anti-symmetric flutter of T-tails.

The mode forced by rudder kicks shows ample damping that increases with increasing air speed. However, it is unlikely that this mode is the most critical antisymmetric one. If records of rudder kicks are examined, it can be seen that the continuous low amplitude oscillation involves motion of transducer 8 (fin leading edge) but much less motion of transducer 9 (fin rear spar). When the rudder is kicked this state is reversed, but the aircraft gradually returns to the original mode as the rudder kick vibration dies away. The frequency of the continuous vibration is lower than that of the mode forced by rudder kicks, 6.9 c.p.s. compared to 8.9 c.p.s. Both frequencies do appear in the continuous oscillation on some of the records.

It seems certain from this result that there are two antisymmetric modes with similar frequencies and that the one forced by rudder kicks has the higher damping. From experience of the symmetric modes the structure is not excited to continuous vibration in smooth air unless the damping is well below 1 per cent critical. This is much less than the large amplitude damping measured from rudder kicks, and indicates that this mode also must have very low damping for low amplitudes.

If the excitation is constant then the variation with speed of the amplitude of the continuous oscillation indicates a variation of damping, high amplitude

corresponding to low damping and vice versa. Thus the damping in the unknown antisymmetric mode probably drops with increasing speed from about 250 knots onwards. The large amplitude damping in the rudder kick mode increases with increasing speed but the amplitude level at which the damping in this mode becomes very small also increases with increasing speed.

The lower frequency antisymmetric mode does not correspond to one measured during ground resonance tests, either on the College aircraft or those carried out by O.N.E.R.A. The continuous vibration is not of sufficient amplitude to cause fatigue damage. On the other hand, its implications are worrying in that there appears to be a low damped mode whose damping may drop with increasing speed and about which nothing is known. If this mode does have a critical speed, which may not be the case, and if this speed is exceeded then the ensuing flutter will involve only the primary structure, will not be controllable by the pilot and will almost certainly cause a catastrophic structural failure. However, provided the aircraft is not flown beyond the speed limits already proved by the makers there is no danger of this happening.

Another antisymmetric mode that has been observed during flight testing is a 75 c.p.s. single degree of freedom rudder buzz. This appears to occur throughout the aircraft's speed range. It is independent of engine thrust, having been recorded in a glide with both engines throttled back, but the amplitude increases with increasing air speed. The mode involved is the rudder torsion which O.N.E.R.A. measured as having a natural frequency of 100 c.p.s. The rudder fitted to G-APRU is foam filled and so should have a slightly lower natural frequency.

This vibration is certainly the cause of the skin cracking that has occurred on the M.S.760 rudder. It is probably due to direct excitation of the rudder by turbulence from the fin/tailplane junction. Fitting a bullet fairing here might solve the problem.

7.4. Limited Amplitude Flutter

Limited amplitude flutter is a common phenomenon, particularly when the flutter is of a control surface. It must be due to the equations of motion being non-linear, for if they are used in their linear form they predict that once an oscillation has become divergent its amplitude will continue to grow indefinitely. To the authors' knowledge there is no published work on the nature of these non-linearities, nor has the mechanism of amplitude limited flutter ever been explained.

The non-linearities must be either structural or aerodynamic in origin. On the M.S.760 the amplitude to which the symmetric flutter is limited is about ± 0.1 inch at the tailplane tips, or $\pm 0.3^\circ$ elevator rotation, and this amplitude is comparatively unaffected by changes of speed. Deformations of the structure of this order involve stresses very much less than those which will cause plastic deformation, or those that cause non-linear behaviour of rivets and joints, so the structure should be linear to this stress.

On the other hand, the boundary layer over the tailplane will be of the order of tenths of inches thick. If there is assumed to be no pressure gradient over the tailplane, and the boundary layer is turbulent from the leading edge, then reference 21, page 147, gives a semi-empirical formula for the boundary layer

thickness as

$$\frac{\delta}{x} = \frac{.37}{\text{Re}^{1/5}}$$

For flight at 400 feet/sec. this gives thicknesses of 0.235 in. and 0.54 in. at stations one foot and three feet aft of the leading edge respectively.

The turbulent boundary layer does not consist of uniformly distributed turbulence merging into potential flow at the edge of the layer. The inner region of the layer is continually turbulent, but the outer region consists of areas of potential flow interspersed between billows of turbulence that extend out to the edge of the layer.

No theoretical work has apparently been reported on the boundary layer over an oscillating surface. It appears reasonable to assume that if the surface frequency is high enough and the amplitude is small compared to the boundary layer thickness, the outer edge of the layer will remain steady while the surface oscillates, the motion normal to the surface being dissipated in the turbulence of the boundary layer.

If this physical model is correct then both the aerodynamic stiffness and damping of an oscillating aerofoil or control surface will be reduced if the oscillation amplitude is small compared to the boundary layer thickness.

To explain the experimental results on the M.S. 760 an increase of the stiffnesses and damping must raise the flutter speed so that at any speed above the critical for small oscillations there is an amplitude at which larger oscillations are neither convergent nor divergent. By calculation it can be shown that if all the stiffnesses and dampings are changed by the same percentage then the flutter speed does not alter. But if the dampings increase more with increasing amplitude than the stiffnesses then the calculations indicate that limited amplitude flutter can occur, as this change raises the flutter speed. This could also explain why the predicted flutter speed for small oscillations is too high.

This effect could work the other way in that an increase of amplitude could change the aerodynamic parameters to lower the flutter speed and make the flutter that was already occurring more violent. This has happened once, on the Javelin (reference 1). An oscillation of the elevators of this aircraft grew more and more divergent with increasing amplitude until the elevators broke away from the aircraft.

This explanation of limited amplitude flutter is not intended to be rigorous. It is put forward simply as a possible mechanism for an effect for which there is no rigorous explanation to date.

7.5 Instrumentation

The extreme reliability of the instrumentation system has been one major factor contributing to the success of this investigation. This is probably due to the use of printed circuits and transistors throughout the complex electronic components.

Because one structural mode was excited much more than any others by stick jerks, and because the frequencies were very different, it was possible to separate the various frequencies on the film by eye. This was simply a matter of luck. It cannot be assumed that this will be possible in general. The reverse is more likely to be true, particularly as aircraft structures become stiffer and more complex.

This investigation strained the use of film recording to its limit. On certain occasions the records showed that something complicated was happening, without being able to give the information in a form that could be analysed. Examples of this are the non-linear decay after rudder kicks, and the 'plateau' type decay curves that occasionally followed stick jerks.

The initial trials of the magnetic tape recorder showed it to be a very much more powerful instrumentation system. The results of an analysis of one stick jerk are given in Section 4.2. This one stick jerk gives as much information as a complete frequency scan using continuous excitation would. The analyser was able to separate the two obvious elevator frequencies on the tape, and also identify two more that could not be seen by eye.

8. Conclusions

8.1. Conclusions relating specifically to the M.S.760 four seat aircraft with tip tanks

i) The aircraft as supplied by the makers will suffer symmetric elevator flutter at 380 knots equivalent air speed. If a stick force transducer or some other mass is fitted to the control column the critical flutter speed can be reduced to 240 knots. The flutter is amplitude limited and can be stopped by the pilot grasping the control column. Lowering of the critical speed depends on the contents of the main fuel tank and the amount of ballast in the nose of the aircraft.

ii) The flutter described above can be prevented by increasing the total elevator mass balance from 5.5 lb. (2.45 Kg) to 7.5 lb. (3.40 Kg.).

(iii) The damping of symmetric vibrations on the M.S.760 is critically dependent on their frequency, frequencies near 17.5 c.p.s. being poorly damped. The mechanism of this effect needs further investigation.

(iv) The tail unit of the M.S.760 vibrates in the 6-9 c.p.s. antisymmetric modes at all airspeeds. Further work is needed to determine if the vibration is a symptom of some effect that might be dangerous.

(v) The rudder vibrates at 75 c.p.s. throughout the speed range of the aircraft. This vibration is not affected by Mach No. or power setting. It does cause fatigue damage. It is probably excited by aerodynamic buffet from the fin/tail-plane junction. A bullet fairing here might cure this effect.

8.2 General conclusions

(i) Recording the response of the aircraft to stick jerks on film can be satisfactory, but rapidly becomes impossible if more than one mode is excited.

Recording on magnetic tape is very much more satisfactory.

(ii) Static balancing of control surfaces should include the effect of components of the control circuit attached to them if those components are essentially normal to the plane of the control surface and are similarly subjected to the accelerations induced by body motions and/or by couplings with other elastic modes.

9. Acknowledgements

The investigation reported here have benefited from the help of many people to whom due thanks are given. In particular they include the following of the College staff:-

Mr. K.H. Griffin who advised on computer programmes,
Mr. M.A. Perry who developed the flight instrumentation,
and Flt. Lt. A.J. Macdonald who piloted the aircraft.

Also Mrs. Mitchell assisted with the ground resonance tests and with the data reduction in general.

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TABLE 1

Results of the First Ground Resonance Test

a) Symmetric modes.

Wing bending fundamental	9.8 c.p.s.
Tailplane bending fundamental	11.5 c.p.s.
Fuselage vertical bending	16.6 c.p.s.
Elevator rotation (i)	18.3 c.p.s.
Elevator rotation (ii)	19.2 c.p.s.
Tailplane bending overtone	30.2 c.p.s.

b) Antisymmetric modes

Fin bending	9.1 c.p.s.
Fin torsion	12.7 c.p.s.
Fuselage side bending	21.9 c.p.s.
Elevator torsion	28.5 c.p.s.
Tailplane bending	30.7 c.p.s.
Fin overtone bending	45.0 c.p.s.

The aircraft had its main fuel tank full, wing tip tanks empty. It carried no cockpit ballast to represent crew, and was thus not typical of a configuration that could occur in flight. Two normal control columns were fitted.

TABLE 2

Frequencies and dampings of symmetric normal modes for various aircraft configurations

Configuration	i	ii	iii	iv
Wing bending	9.8 4.0	9.8 -	9.8 -	9.8 -
Fuselage bending	15.5 5.0	15.5 4.8	17.3 -	16.6 2.8
Elevator rotation	18.1 4.4	17.8 4.8	17.3 -	17.4 1.5
Tailplane overtone	28.3 -	28.3 4.8	29.3 -	28.5 -

The upper figures are frequency in c.p.s., the lower are damping, per cent critical.

TABLE 3

Comparison of the flutter speeds calculated on digital and analogue machines

Case	Digital, ft./sec.	Analogue, ft./sec.
Binary, mass balance 0 lb.	300	310
Binary, mass balance 0. lb. 2% damping	-	420
Ternary, mass balance 0 lb.	300	310
Ternary, mass balance 5 lb.	850	795

TABLE 4

Variation of aircraft damping at 150 knots with masses fitted to the control column

Mass lb.	Position of mass	Damping per cent	Frequency c. p. s.	Phase ^o Angle
0	-	5.8	19.5	20
arm only	-	1.68	17.5	105
1	1	3.13	16.8	150
1	2	2.92	17.0	125
1	4	4.15	16.4	120
2	4	6.9	15.0	180

The position of the mass can be 1, 2, 3 or 4. These are the numbers of the positions on the arm, counting from the front.

The phase angle is that by which the elevators lag the tailplane structure.

TABLE 5

Flight results for the response of the standard
aircraft with empty tip tanks to stick jerks

Equivalent air speed knots	Altitude feet	Mach No.	Damping % critical	Frequency c. p. s.	Phase °
151	12000	0.28	5.8	19.5	20
202	13000	0.39	3.95	19.0	0
228	13000	0.44	3.7	18.9	0
253	14800	0.50	2.7	18.9	0
276	8500	0.48	3.1	18.5	0
303	8500	0.53	2.6	18.2	0
331	10000	0.61	1.65	18.4	20
352	8000	0.63	0.75	18.1	35

These results are typical examples of those measured in flight. They are not averaged values and do not correspond to the best line through all the flight results.

The phase angle is that by which the elevators lag the tailplane structure.

TABLE 6

Flight results for the response of the standard
aircraft with full tip tanks to stick jerks

Equivalent air speed knots	Altitude feet	Mach No.	Damping % critical	Frequency c. p. s.	Phase °
153	10000	0.29	5.0	18.2	-
202	11000	0.39	4.4	18.3	-
253	11000	0.46	2.9	18.5	-
302	10000	0.55	1.9	18.4	0
352	9000	0.63	1.0	18.4	20

The phase angle is that by which the elevators lag the tailplane structure. Where a phase angle is not given it is because the elevators are vibrating at a different, in this case higher, frequency than the structure. The frequencies given are those of the tailplane structure.

TABLE 7

Damping of symmetric vibrations caused by stick jerks. Tip tanks empty, one normal control column and the stick force transducer fitted.

Air speed knots E. A. S.	Altitude feet	Mach No.	Main tank contents litres	Damping % critical	Frequency c. p. s.	Phase °
202	6500	0.33	800	2.35	17.2	85
254	6300	0.42	785	1.60	17.3	83
303	6000	0.50	770	1.00	17.2	80
202	5700	0.34	600	2.35	17.2	84
252	5200	0.42	580	1.00	17.3	75
298	5000	0.50	550	0.35	17.2	78
350	7000	0.57	520	0.70	17.5	87
202	2000	0.27	200	2.00	17.1	130
265	2300	0.35	190	1.20	17.0	80
303	2000	0.40	180	0.85	17.1	70

TABLE 8

Flight results for the response of the modified aircraft to stick jerks. Elevator mass balance 7.5 lb., two normal control columns fitted, tip tanks empty.

Air speed knots E. A. S.	Altitude feet	Mach No.	Damping % critical	Frequency c. p. s.	Phase °
150	6500	0.24	1.40	17.6	75
202	6500	0.33	1.55	17.3	80
252	6500	0.42	1.50	17.0	95
303	6500	0.49	1.90	17.3	82
350	6000	0.55	1.85	17.1	120

TABLE 9

As above, wing tip tanks full.

Air speed knots E. A. S.	Altitude feet	Mach No.	Damping % critical	Frequency c. p. s.	Phase °
150	6500	0.24	1.75	17.4	90
206	6500	0.33	1.75	17.3	85
260	6500	0.42	1.90	17.1	70
310	6500	0.50	2.10	17.1	90
340	6000	0.55	2.15	17.0	85

The phase angle is that by which the elevators lag the tailplane structure.

TABLE 10

Flight results for the response of the modified aircraft to stick jerks. Elevator mass balance 7.5 lb., one normal control column, the stick force transducer and 99 lb. nose ballast fitted. Tip tanks empty.

Air speed knots E. A. S.	Altitude feet	Mach No.	Damping % critical	Frequency c. p. s.	Phase °
150	15000	0.29	1.75	16.8	120
203	15000	0.39	2.40	16.6	100
252	15000	0.49	2.70	16.3	140
305	14500	0.59	2.40	16.1	180
352	13500	0.68	1.75	15.8	150

The phase angle is that by which the elevators lag the tailplane structure.

TABLE 11

Damping of large amplitude vibrations forced by rudder kicks

Air speed knots E. A. S.	Damping % critical	Frequency c. p. s.
150	4.4	9.1
202	4.8	9.0
252	6.6	9.05
306	7.6	9.1
354	10.2	8.9

TABLE 12

The peak-to-peak amplitude of the tailplane tip due to continuous antisymmetric oscillations

Air Speed knots E. A. S.	Altitude feet	Smooth Air		Rough Air
		Average Amplitude inches	Maximum Amplitude inches	Maximum Amplitude inches
100	0	.045	.090	.150
150	10,000	.020	.030	-
200	12,000	.020	.030	-
200	3,000	-	-	.180
250	15,000	.014	.020	-
260	10,000	-	-	.180
308	10,000	.025	.065	-
350	8,000	.030	.090	-

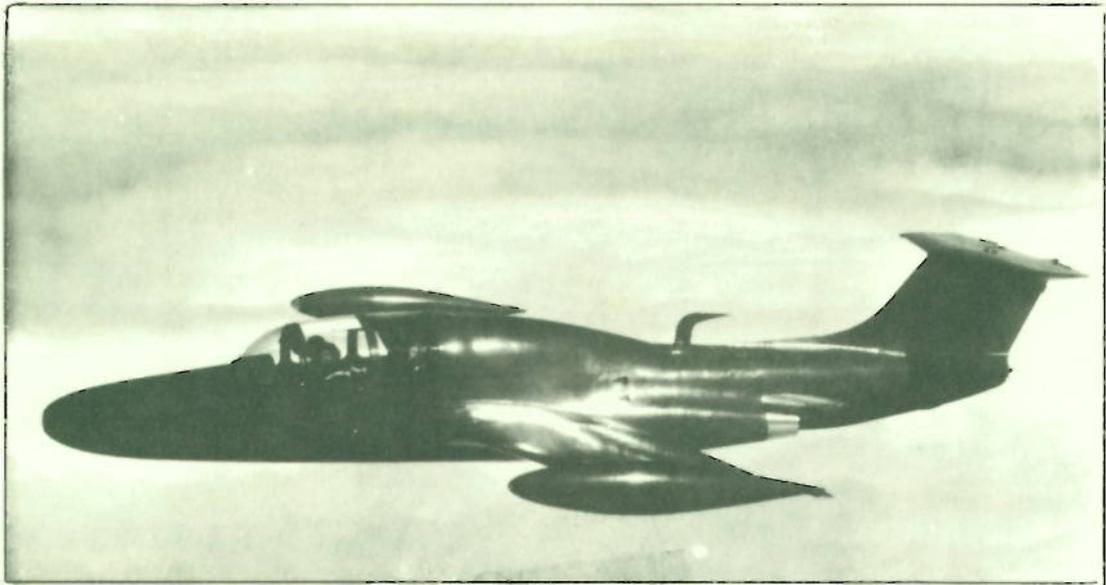


FIG. 1. THE M.S. 760 PARIS

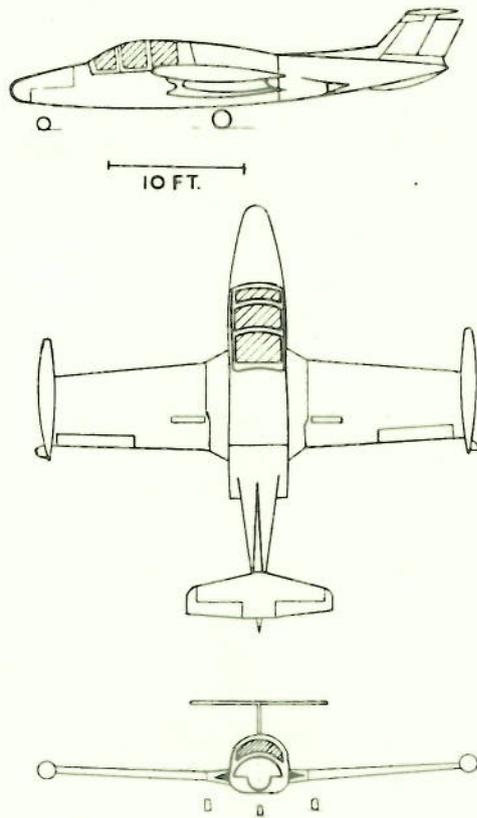


FIG. 2. THE M.S. 760 PARIS

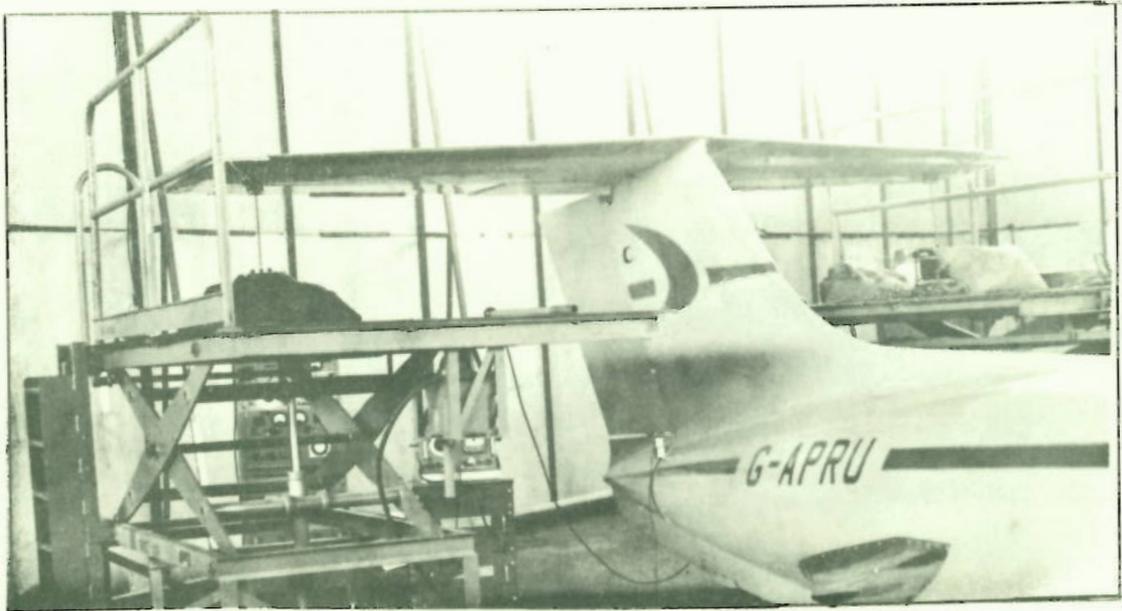


FIG. 3. A GENERAL VIEW OF THE GROUND RESONANCE TEST

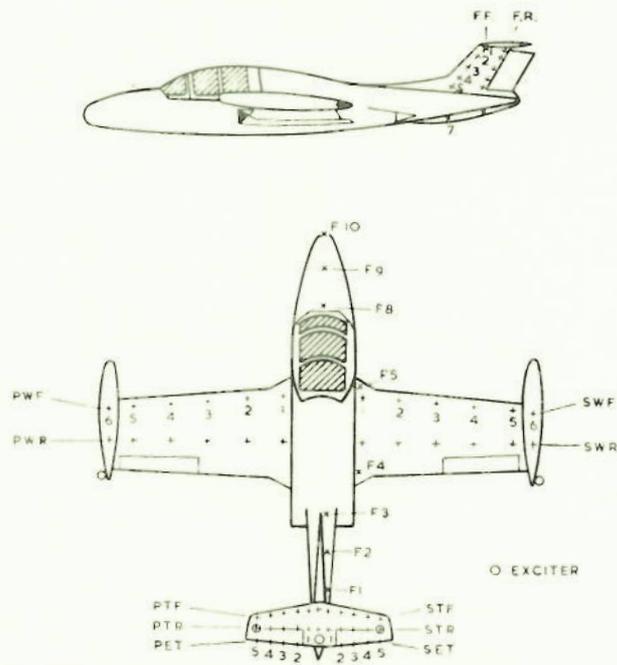


FIG. 4. LOCATION OF MEASUREMENTS POINTS USED DURING GROUND RESONANCE TESTING

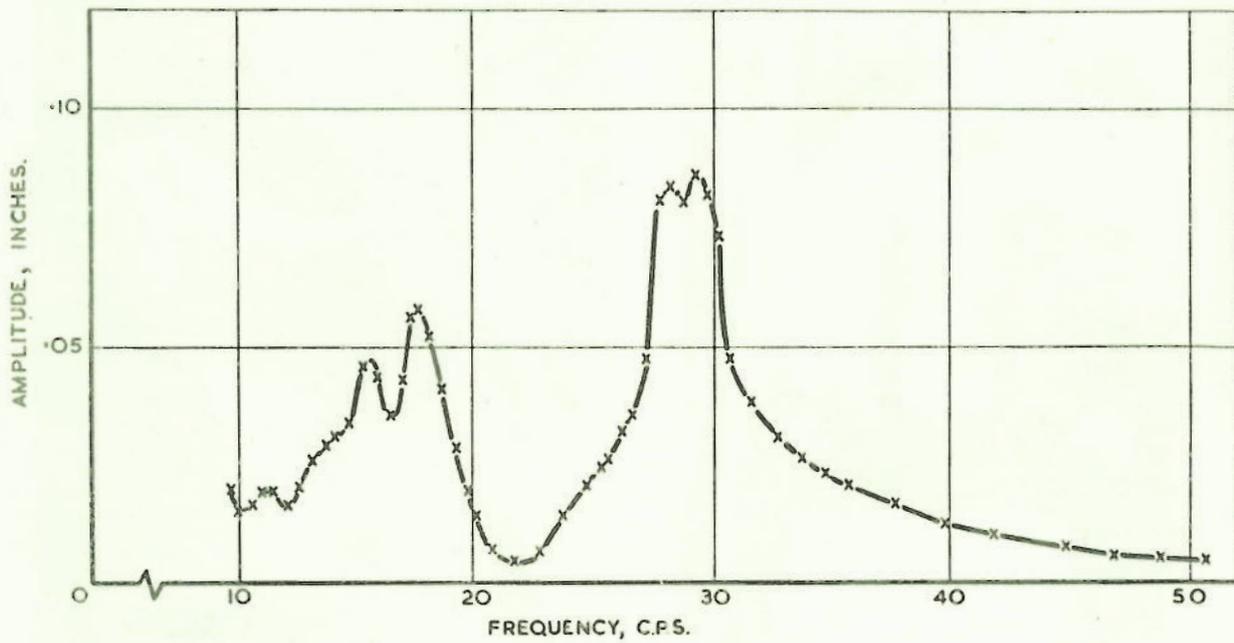


FIG. 5. VIBRATION AMPLITUDE AT THE TAILPLANE REAR SPAR TIP UNDER SYMMETRIC EXCITATION AT CONSTANT POWER

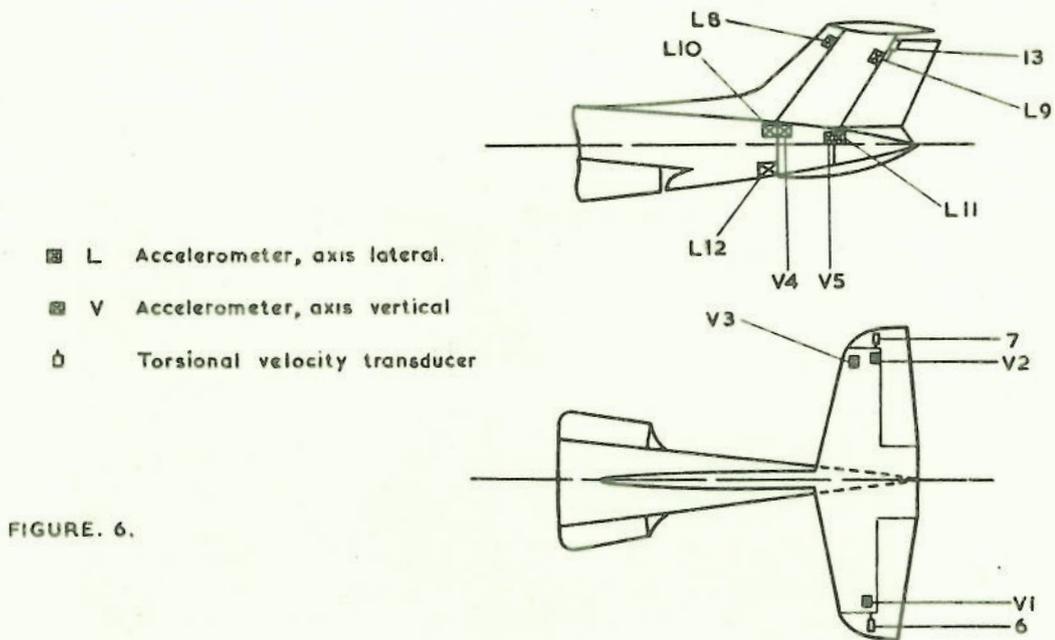


FIGURE. 6.

FIG. 6. POSITION OF TRANSDUCERS IN THE M. S. 760 TAIL UNIT

where they are held by the pin.
 In the aircraft the arm fits into a stub column with the cross-
 piece fore and aft. The weights slide onto the cross-

FIG. 9. THE CONTROL COLUMN ARM AND WEIGHTS.

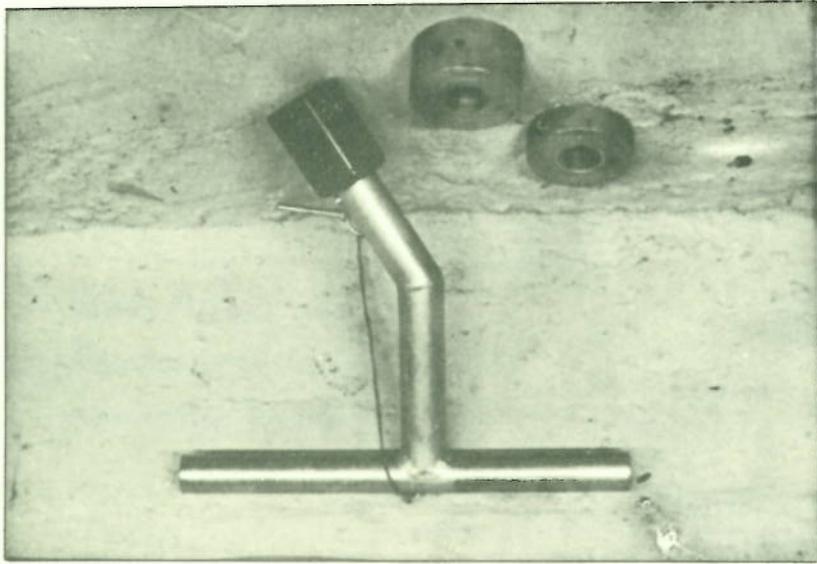


FIG. 8. THE RUDDER TORSIONAL
 VELOCITY TRANSDUCER ON THE
 FIN SPAR. THE SPINDLE IS
 ATTACHED TO THE RUDDER

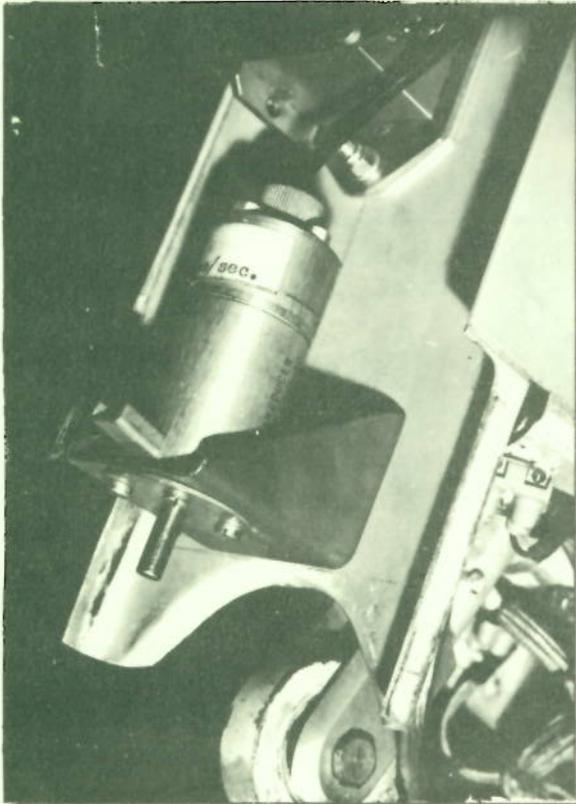


FIG. 7. A VAN ELEC ACCELEROMETER
 FITTED LATERALLY ACROSS THE FIN
 REAR SPAR

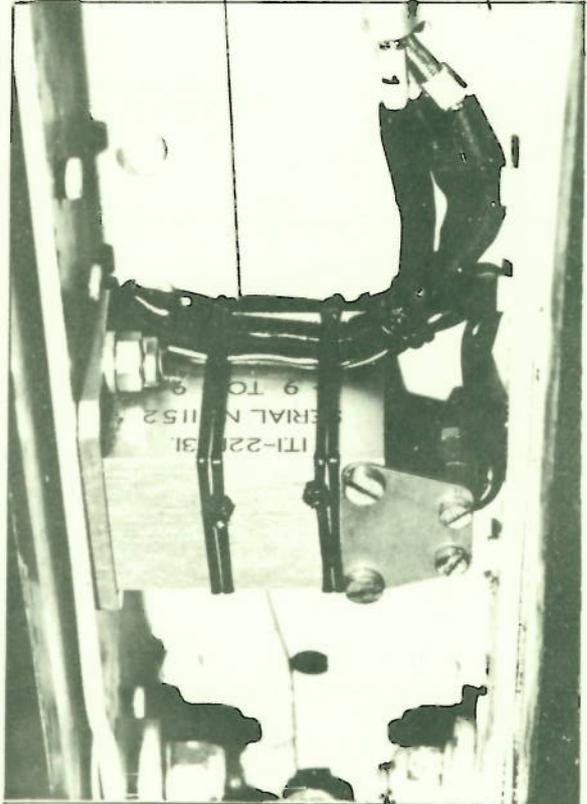




FIG. 10. A STICK JERK WITH THE ENVELOPE OF THE TRACE FROM TRANSDUCER 2 DRAWN IN FREEHAND

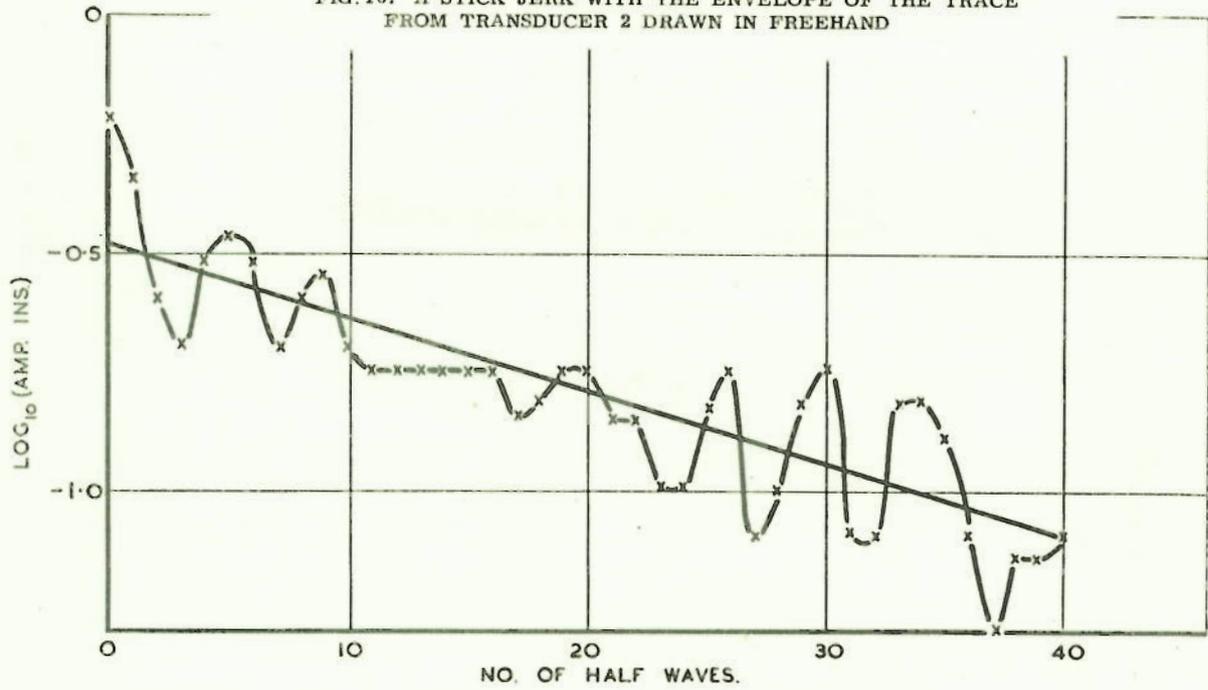


FIG. 11. A LOGARITHMIC AMPLITUDE PLOT FROM FIGURE 10

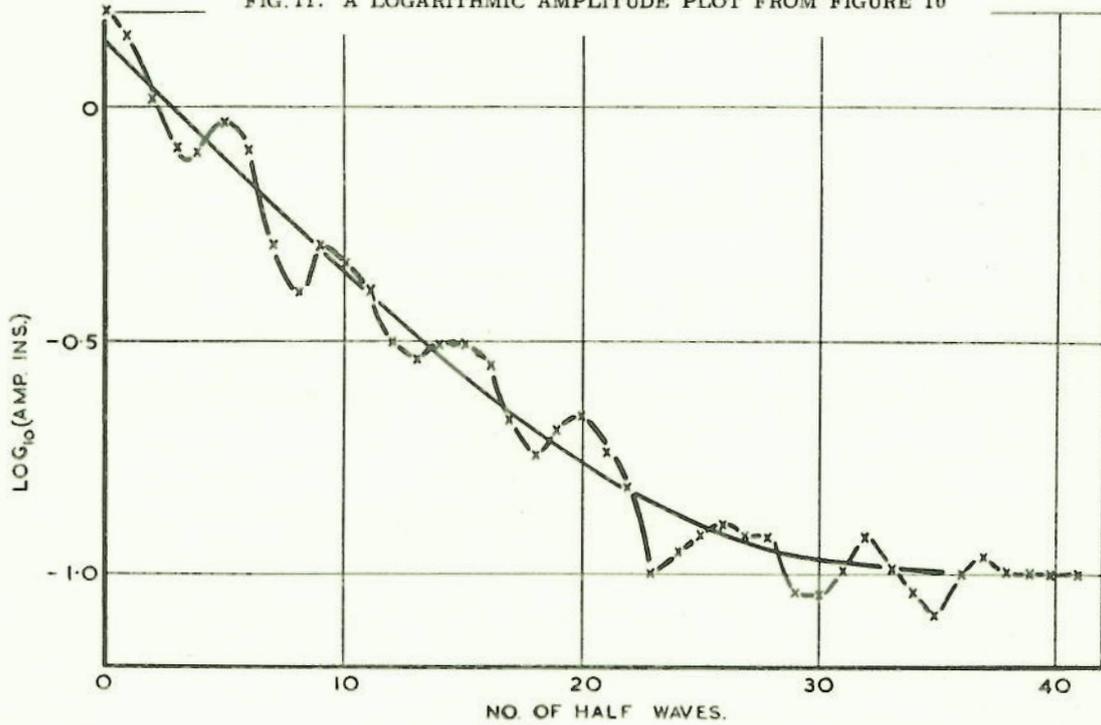


FIG. 12. A LOGARITHMIC AMPLITUDE PLOT OF THE OSCILLATION OF TRANSDUCER 2 FOLLOWING A RUDDER KICK

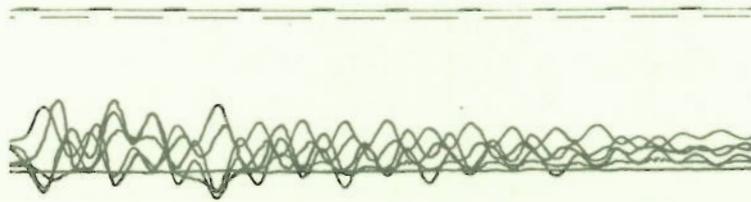


FIG. 13. A STICK JERK SHOWING NON-LINEAR DAMPING

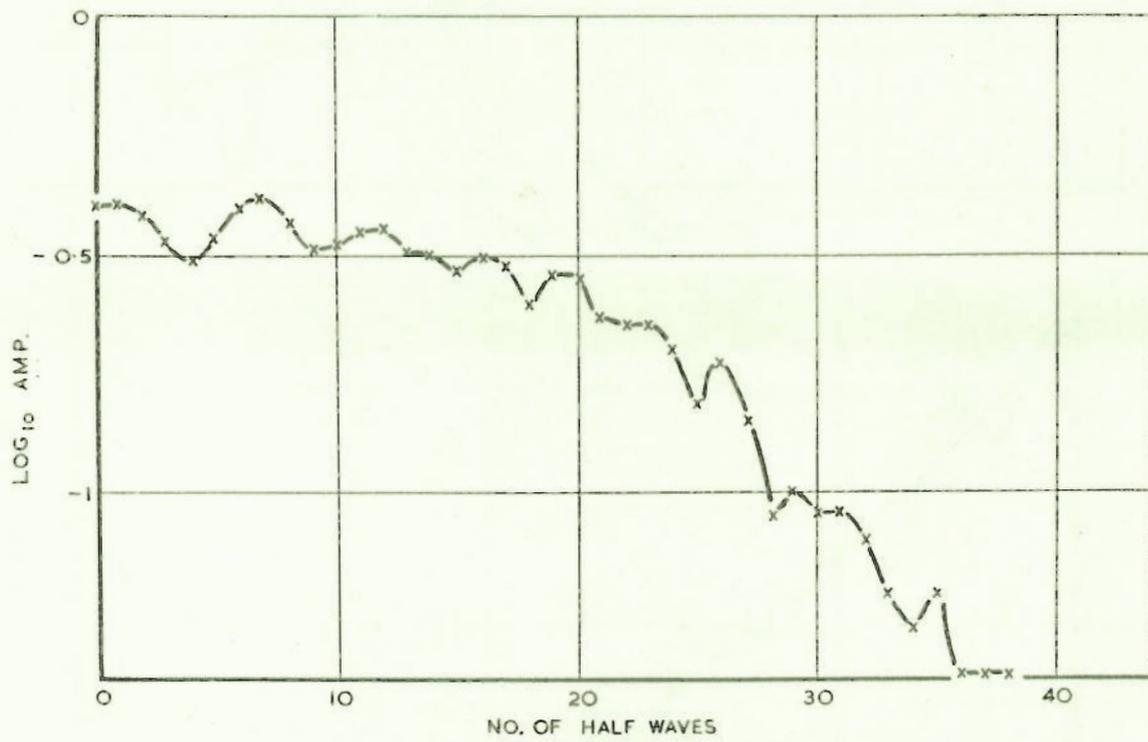


FIG. 14. LOGARITHMIC AMPLITUDE PLOT FROM FIGURE 13

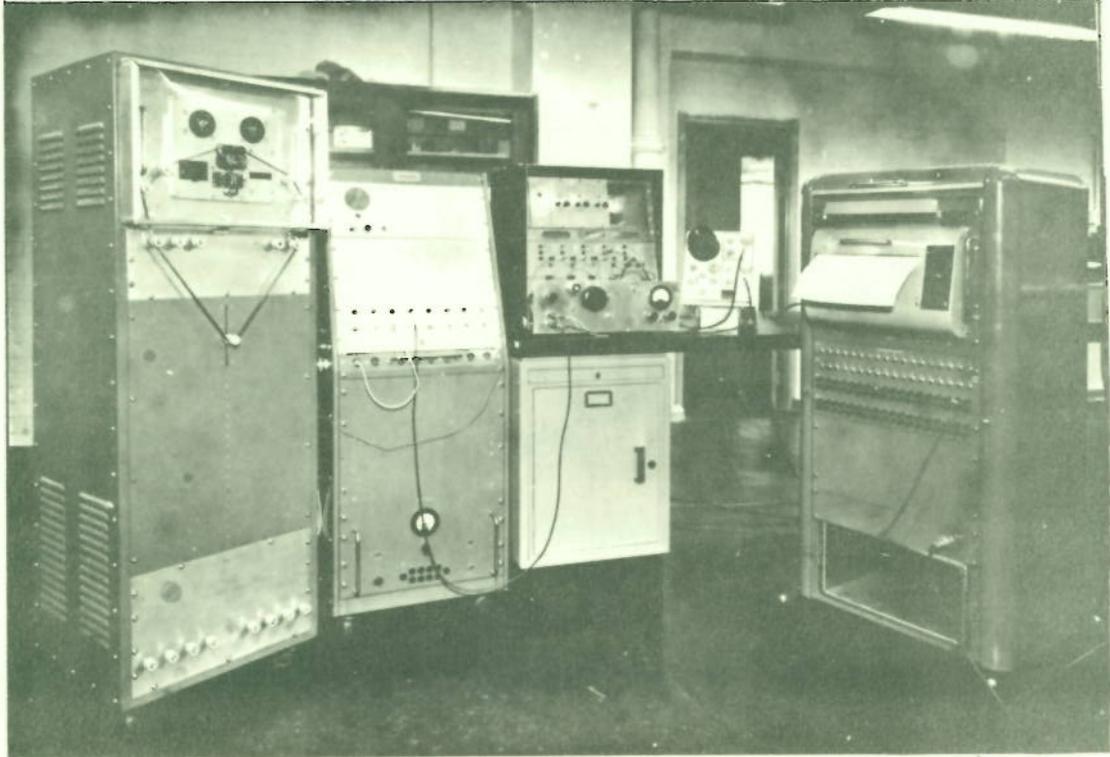


FIG. 15. THE TAPE ANALYSIS EQUIPMENT

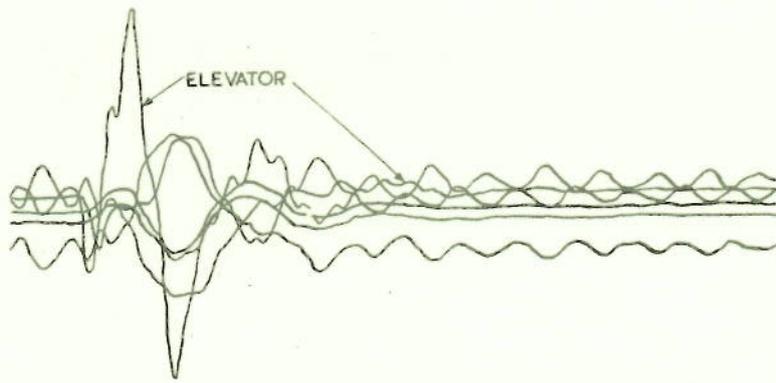


FIG. 16. A FILM RECORD OF A STICK JERK AT 150 KNOTS

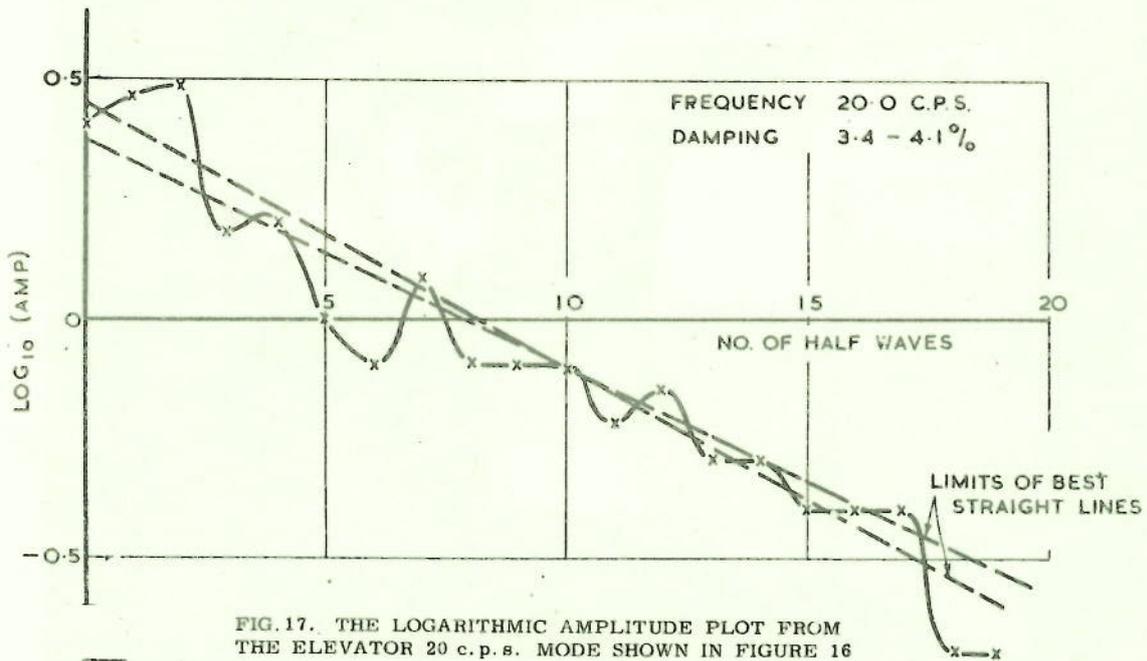


FIG. 17. THE LOGARITHMIC AMPLITUDE PLOT FROM THE ELEVATOR 20 c.p.s. MODE SHOWN IN FIGURE 16

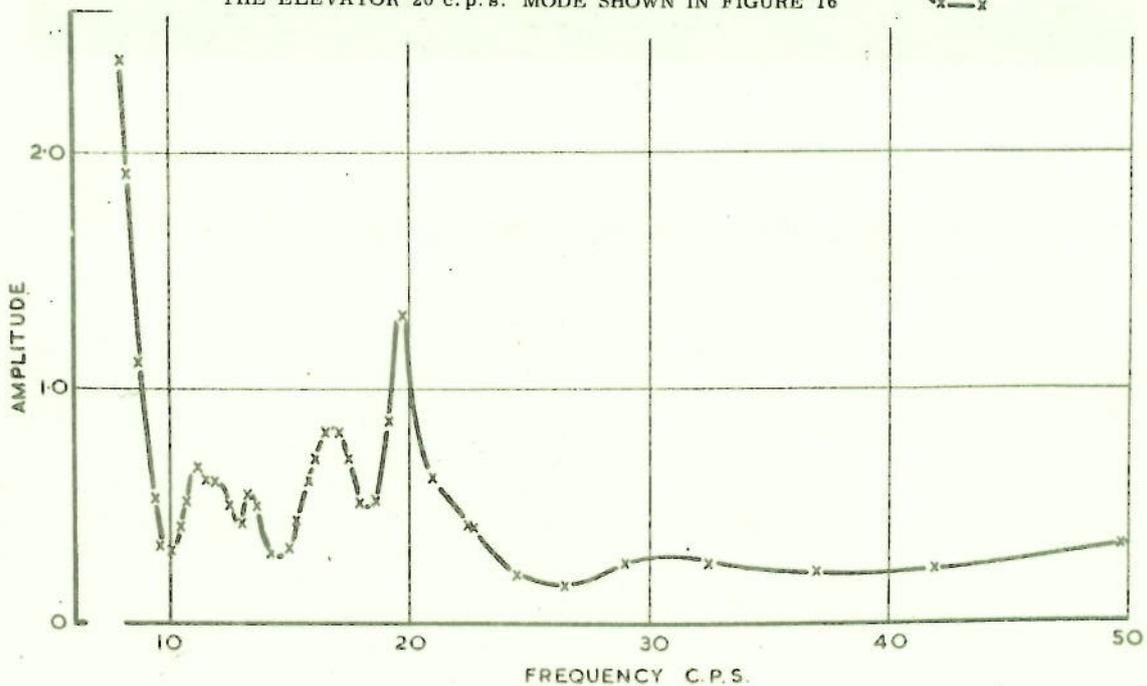


FIG. 18. THE FREQUENCY CONTENT OF THE ELEVATOR RESPONSE TO A STICK JERK, FOUND BY FILTERING THE RECORDING ON MAGNETIC TAPE OF THIS RESPONSE

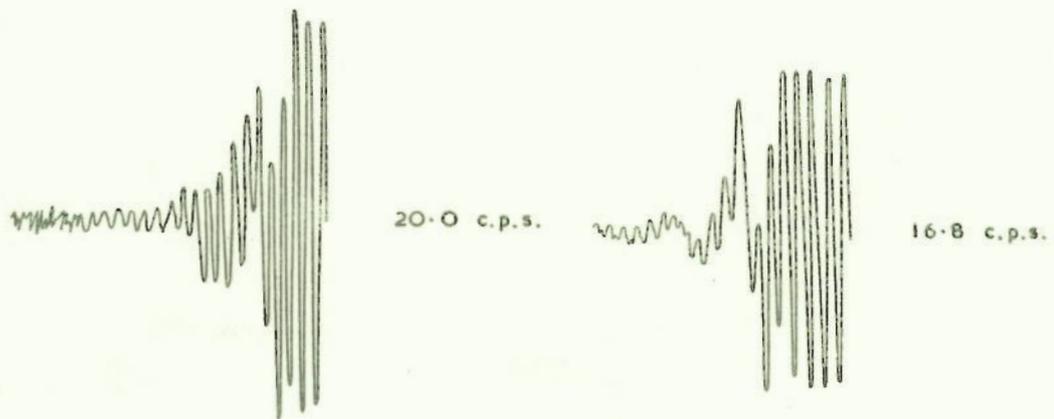


FIG. 19. SIGNALS FROM THE MAGNETIC TAPE RECORDING OF A STICK JERK AT 150 KNOTS AFTER FILTERING

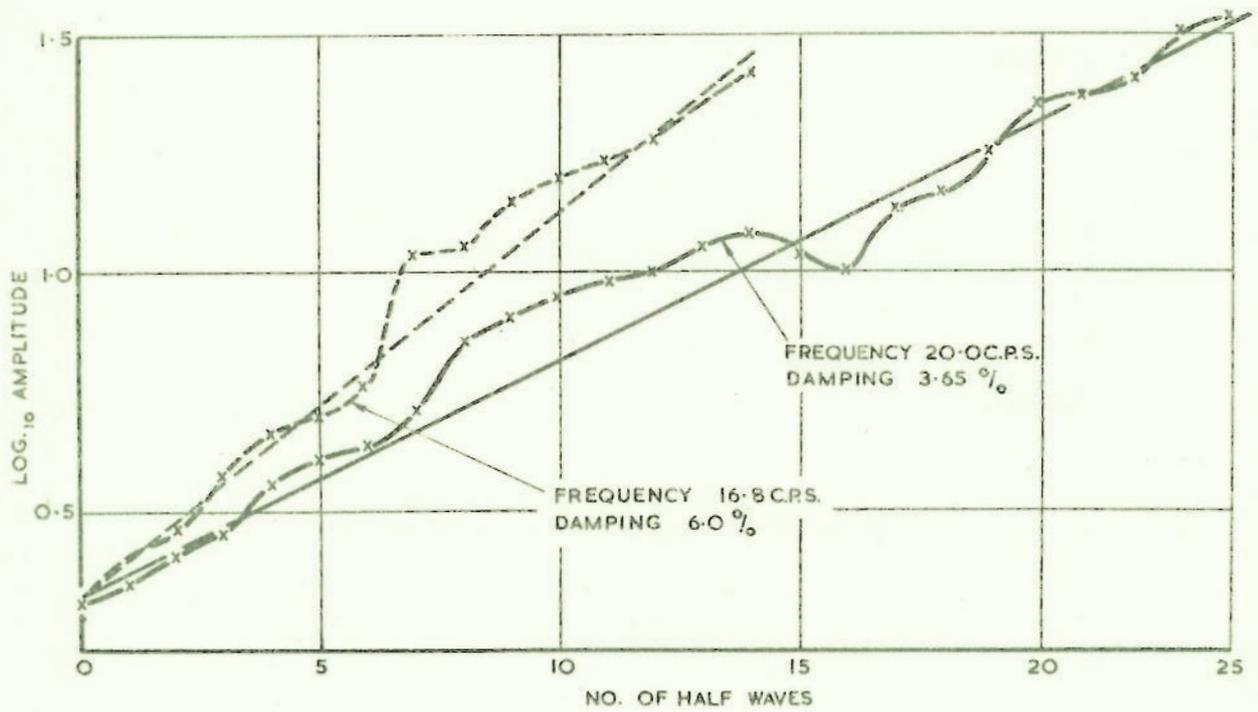


FIG. 20. LOGARITHMIC AMPLITUDE PLOT OF THE 20 c.p.s. AND 16.8 c.p.s. WAVE IN FIGURE 19

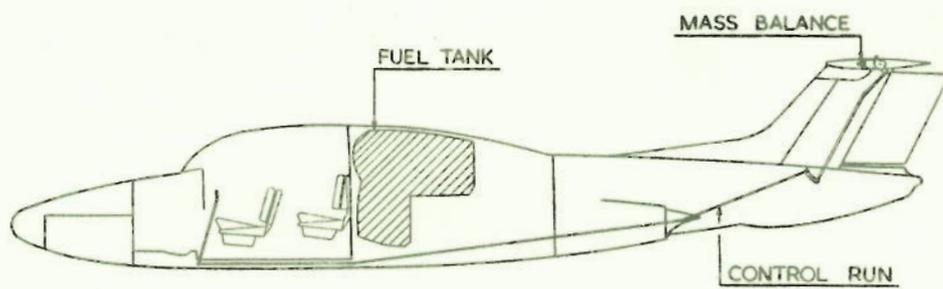


FIG. 21.(a)



FIG. 21.(b)

FIG. 21. SCHEMATIC OF ELEVATOR CONTROL
LINKAGE AND FUNDAMENTAL BODY MODE SHAPE

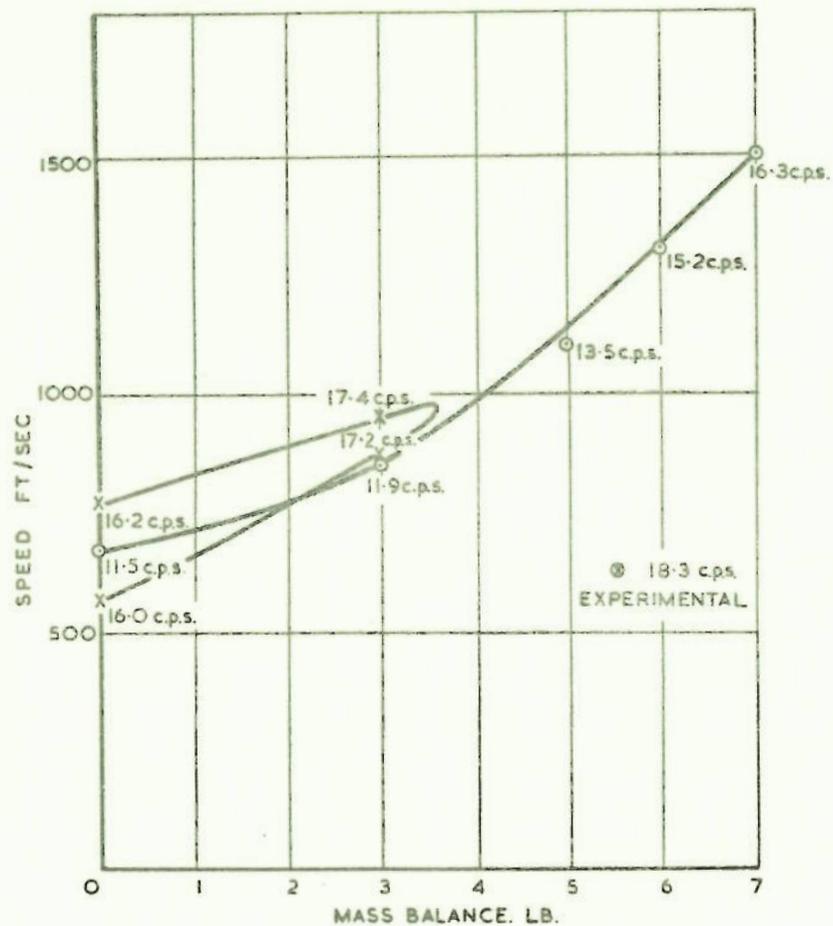


FIG. 22. THEORETICAL FLUTTER BOUNDARY USING O. N. E. R. A. MODES AND THE FIRST SET OF AERODYNAMIC DERIVATIVES.

Circuit cut, sea level, main tank full. Symmetric flutter.

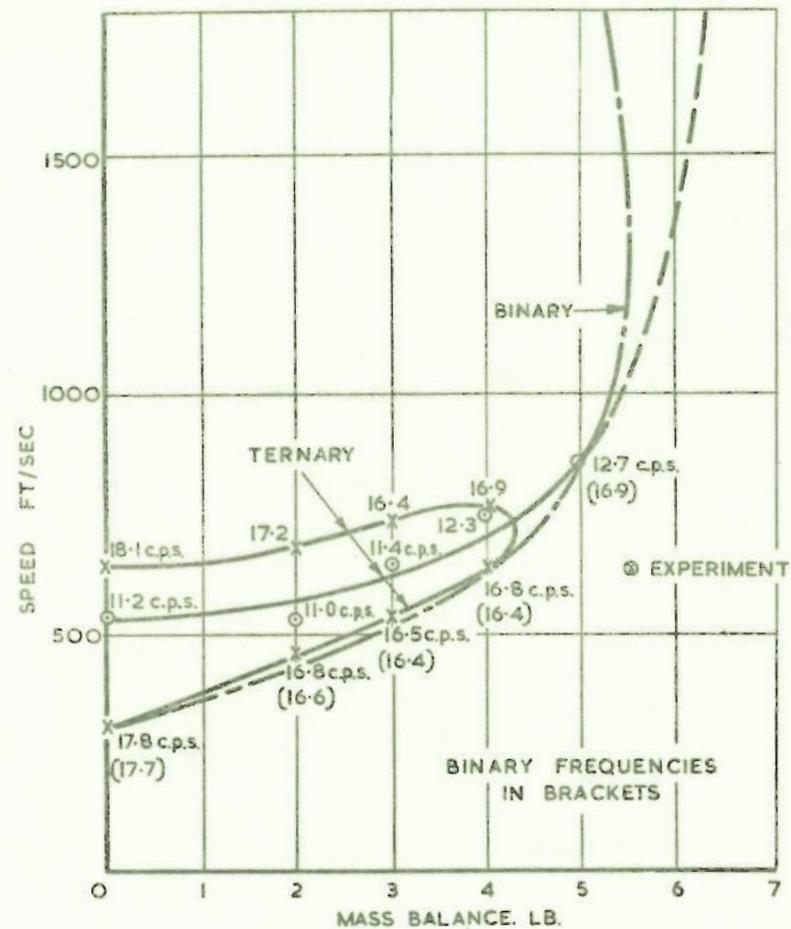


FIG. 23. BOUNDARIES FOR TERNARY AND BINARY FLUTTER.

Minhinnick derivatives, measured modes with the main tank empty and 152 lb. nose ballast fitted. Circuit cut, sea level. Symmetric flutter.

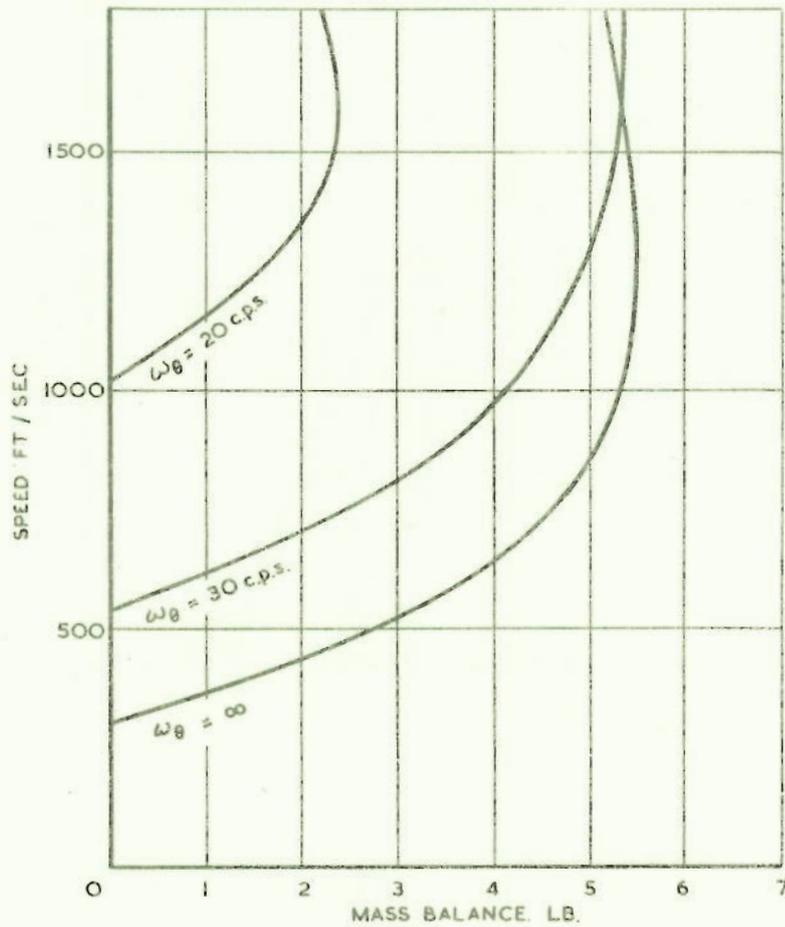


FIG. 24. BINARY FLUTTER BOUNDARIES AS FIGURE 23, BUT WITH THE CONTROL CIRCUIT INCLUDED

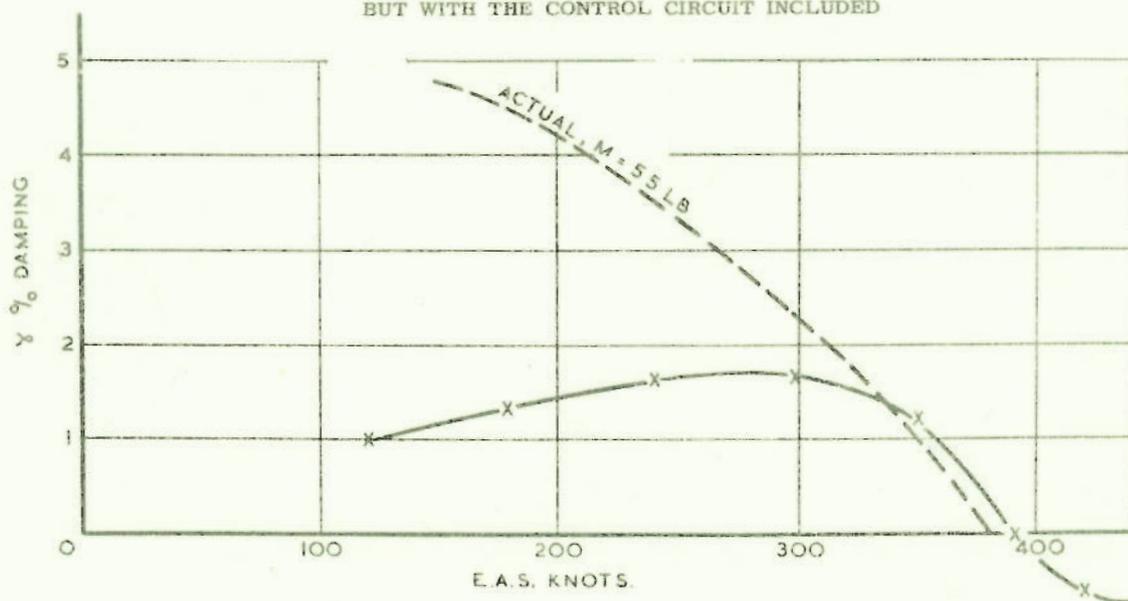


FIG. 25. VARIATION OF DAMPING WITH SPEED CALCULATED FOR 4.0 LB. ELEVATOR MASS BALANCE. TERNARY WITH THE CIRCUIT CUT

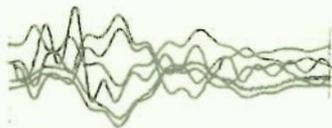
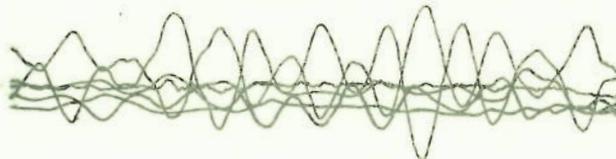


FIG. 26. THE RESPONSE OF THE M.S. 760
TO SYMMETRIC STICK JERKS AT 200 KNOTS



SMOOTH AIR 350 KTS



ROUGH AIR 260 KTS

FIG. 27. CONTINUOUS ANTISYMMETRIC VIBRATION



FIG. 28. 75 c.p.s. RUDDER VIBRATION AT 300 KNOTS

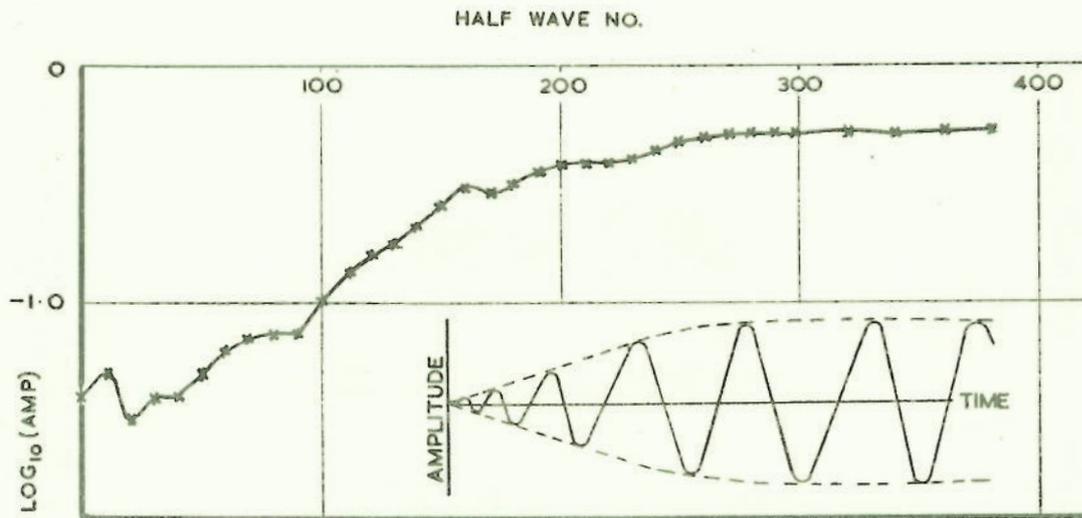


FIG. 29. A LOGARITHMIC AMPLITUDE PLOT FOR LIMITED AMPLITUDE FLUTTER.

Aircraft with stick force transducer and 99 lb. of nose ballast at 300 knots.

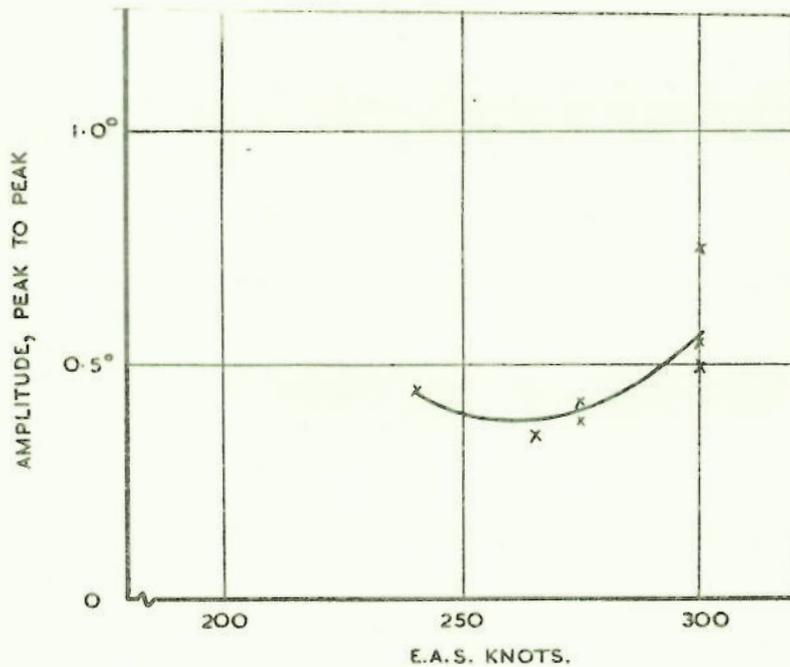


FIG. 30. VARIATION WITH SPEED OF THE LIMITING AMPLITUDE FOR ELEVATOR FLUTTER

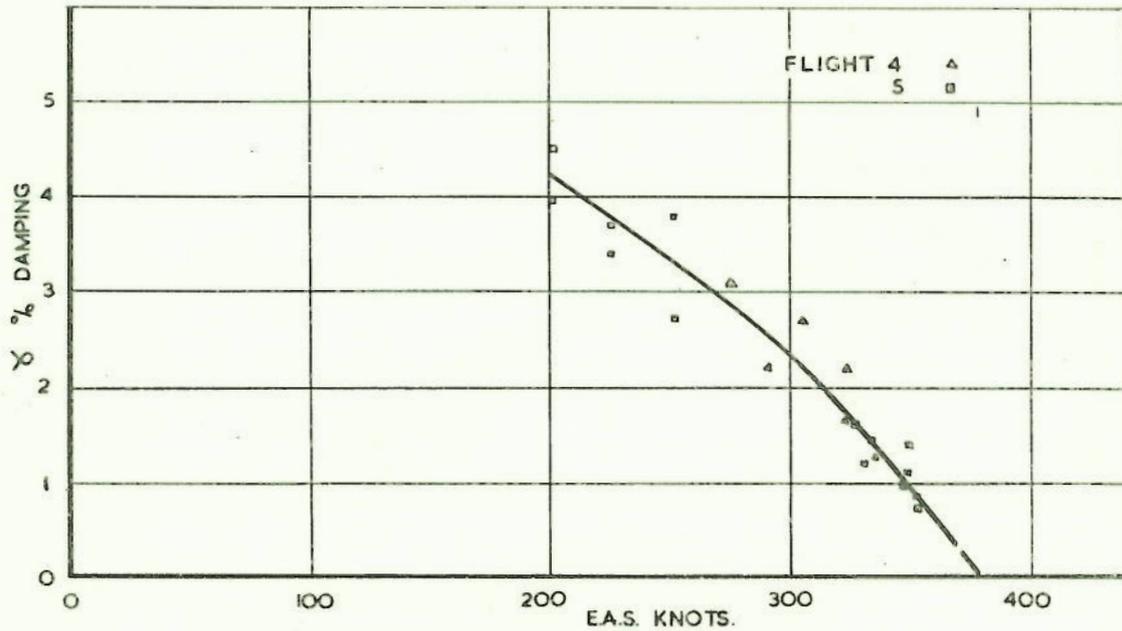


FIG. 31. THE DAMPING OF SYMMETRIC VIBRATION MEASURED AT 10,000 ft. WITH ONE STUB AND ONE NORMAL STICK FITTED.

Displacements measured by transducer 2, tip tanks empty.

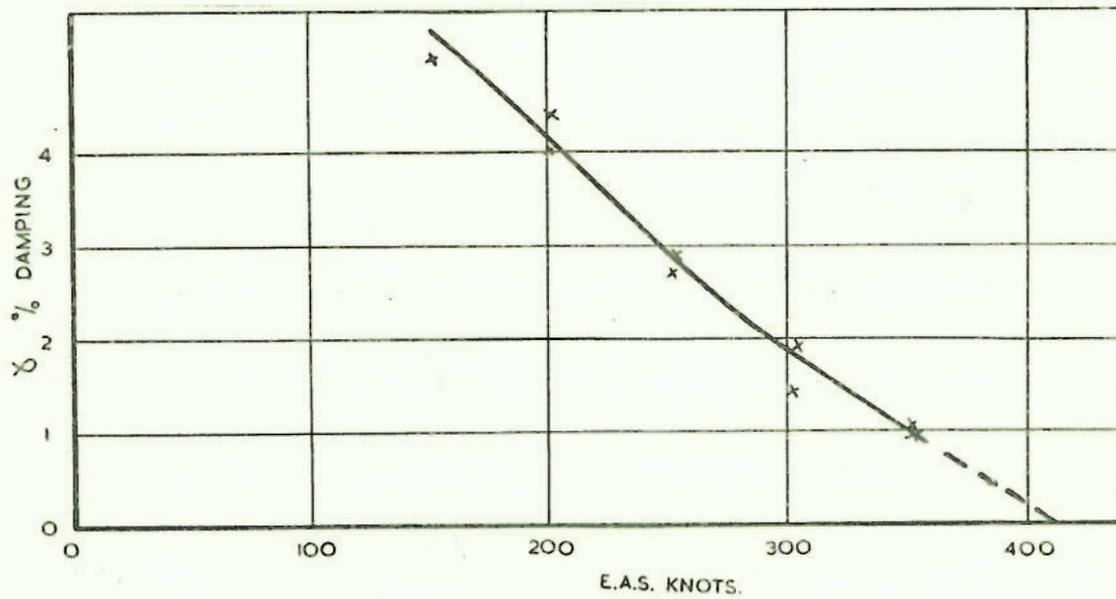


FIG. 32. AS FIGURE 31, BUT WITH TIP TANKS FULL

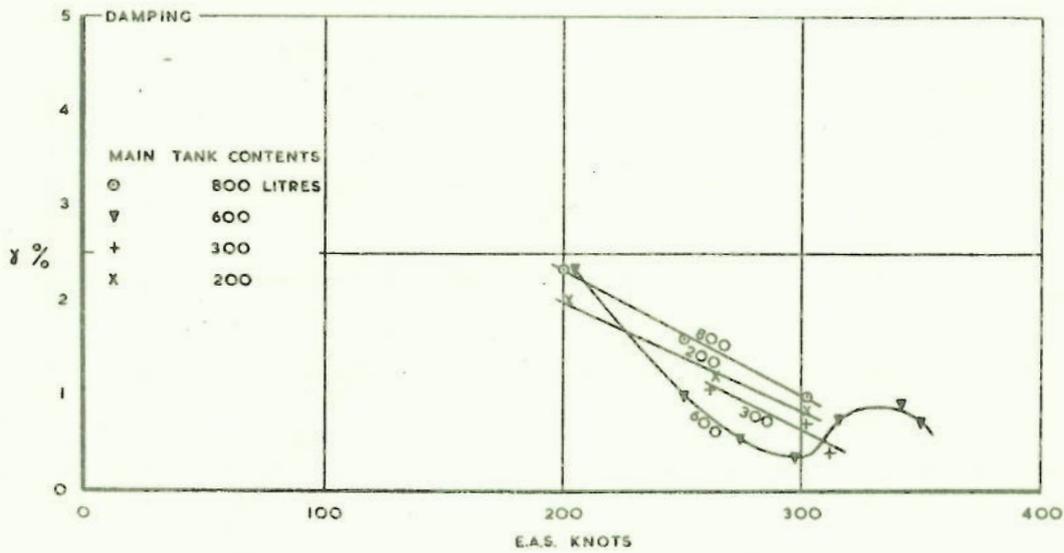


FIG. 33. DAMPING OF SYMMETRIC VIBRATION MEASURED BY TRANSDUCER 2.

Stick force transducer and one normal stick fitted.

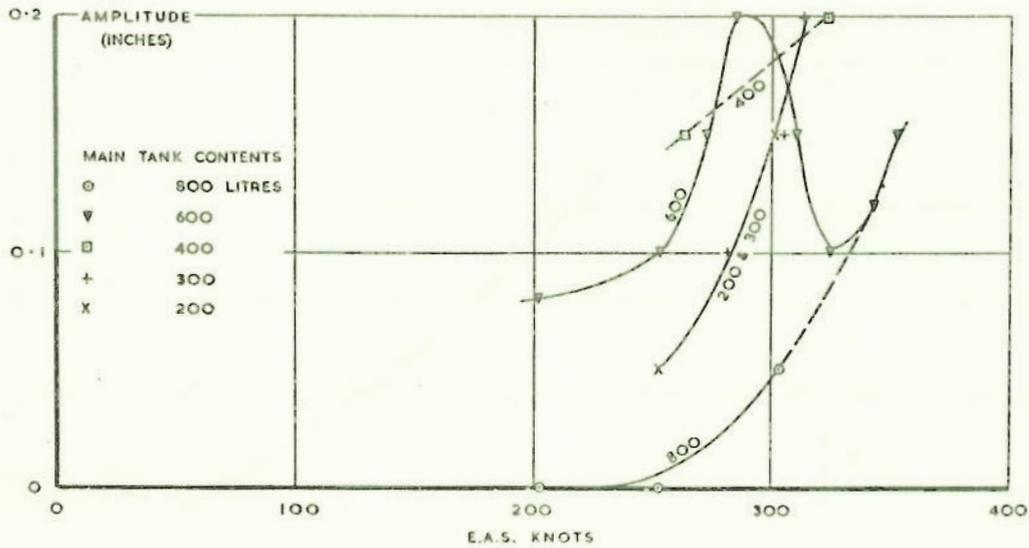


FIG. 34. VARIATION OF THE PEAK TO PEAK AMPLITUDE OF THE FILM TRACE FROM TRANSDUCER 2 WITH SPEED AND FUEL STATE.

Stick force transducer and one normal stick fitted.

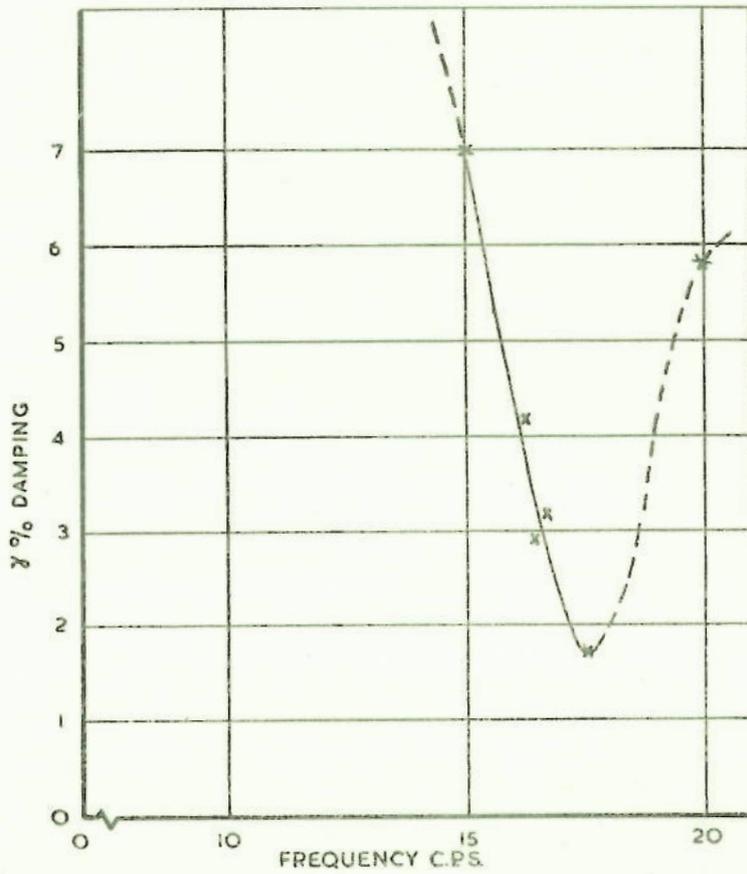


FIG. 35. VARIATION WITH FREQUENCY OF THE DAMPING OF SYMMETRIC VIBRATIONS AT 150 KNOTS

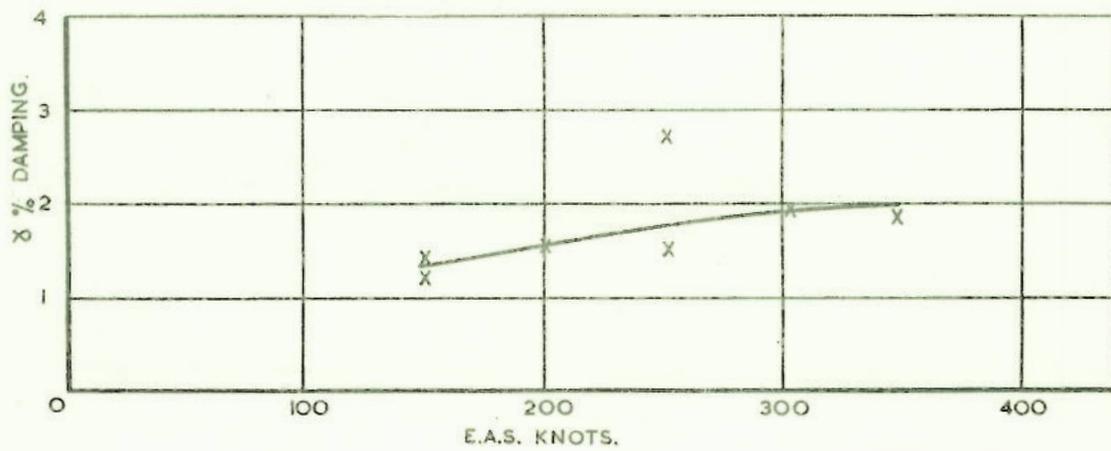


FIG. 36. DAMPING OF SYMMETRIC VIBRATION ON THE MODIFIED AIRCRAFT.

Two normal sticks fitted, tip tanks empty.

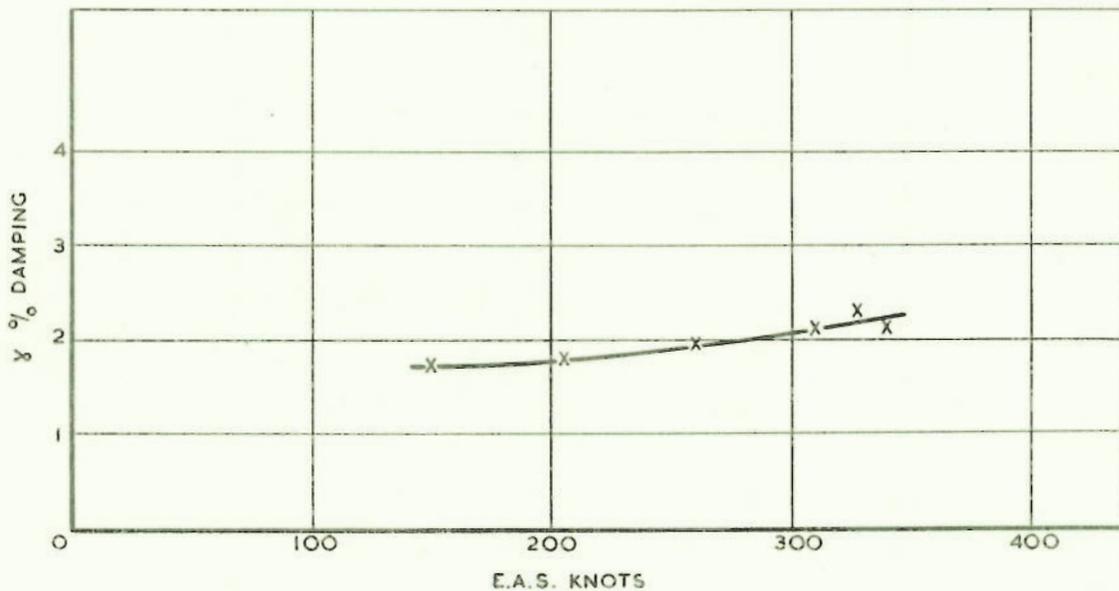


FIG. 37. AS FIGURE 36, WITH TIP TANKS FULL

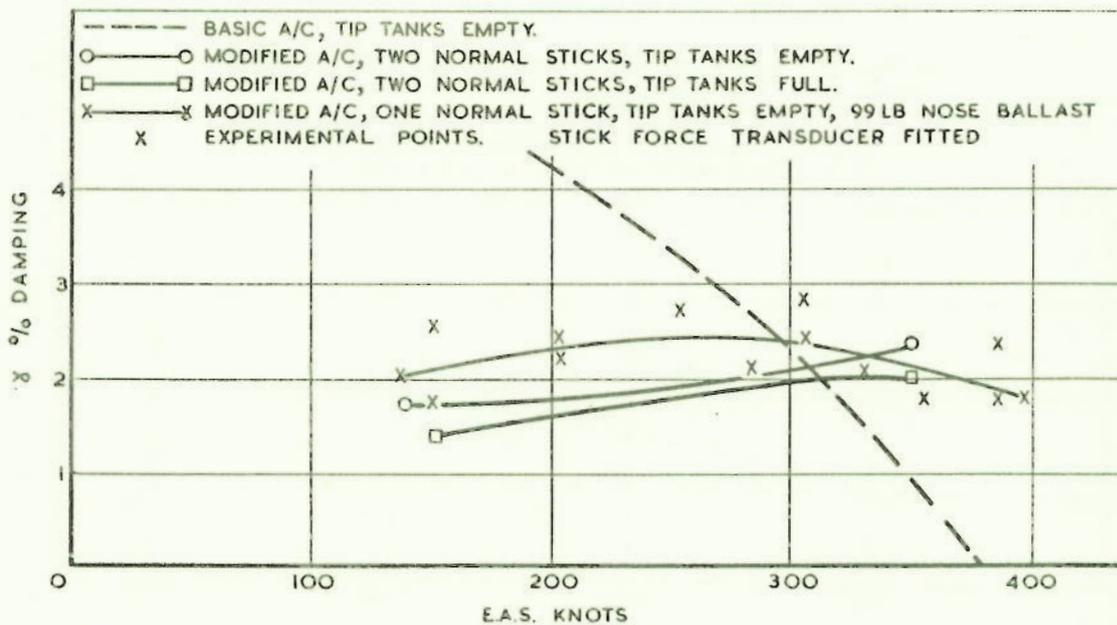


FIG. 38. DAMPING OF SYMMETRIC VIBRATION ON THE MODIFIED AIRCRAFT.

Stick force transducer, one normal stick and 99 lb. nose ballast fitted. Comparison shown with results of Figs. 31, 36 37.

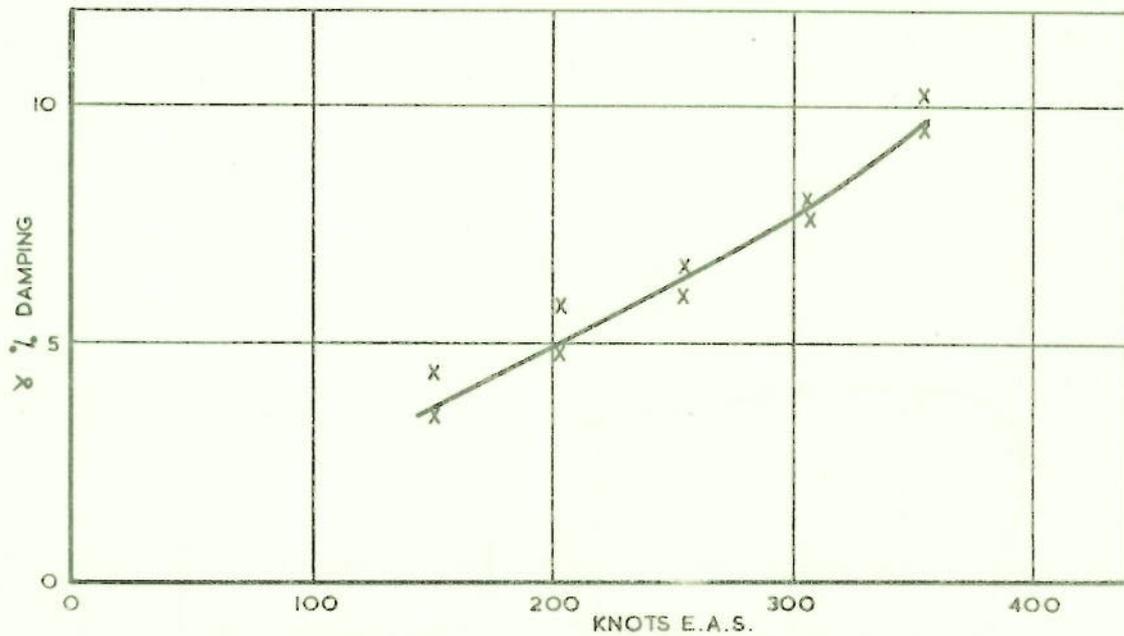


FIG. 39. DAMPING OF LARGE AMPLITUDE ANTISYMMETRIC VIBRATIONS FORCED BY RUDDER KICKS, MEASURED BY TRANSDUCER 2

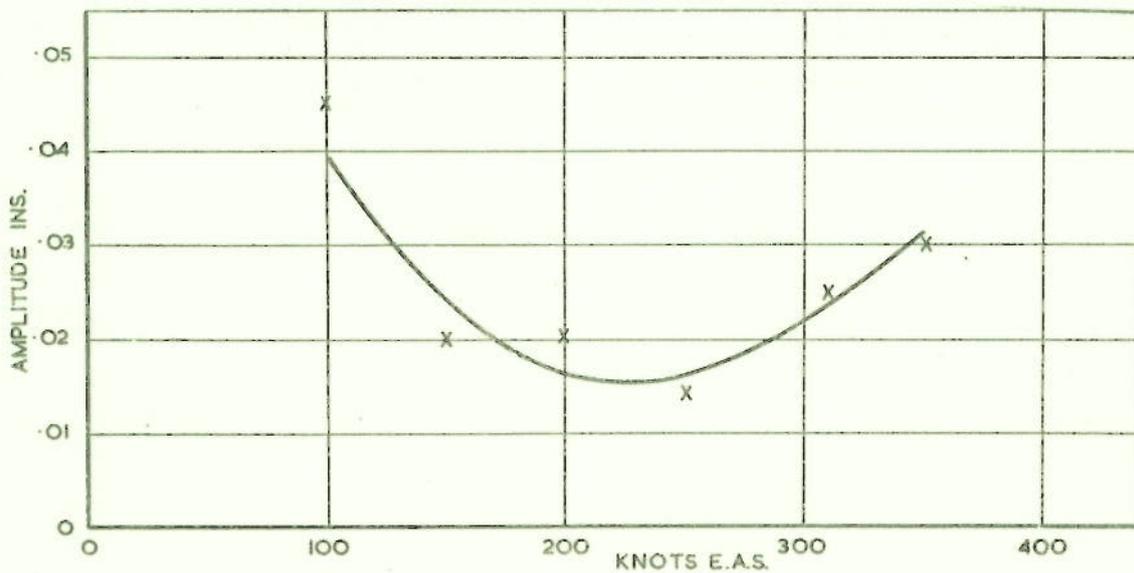


FIG. 40. THE AVERAGE PEAK-TO-PEAK AMPLITUDE OF THE TAILPLANE TIP DUE TO STEADY ANTI-SYMMETRIC VIBRATION IN SMOOTH AIR

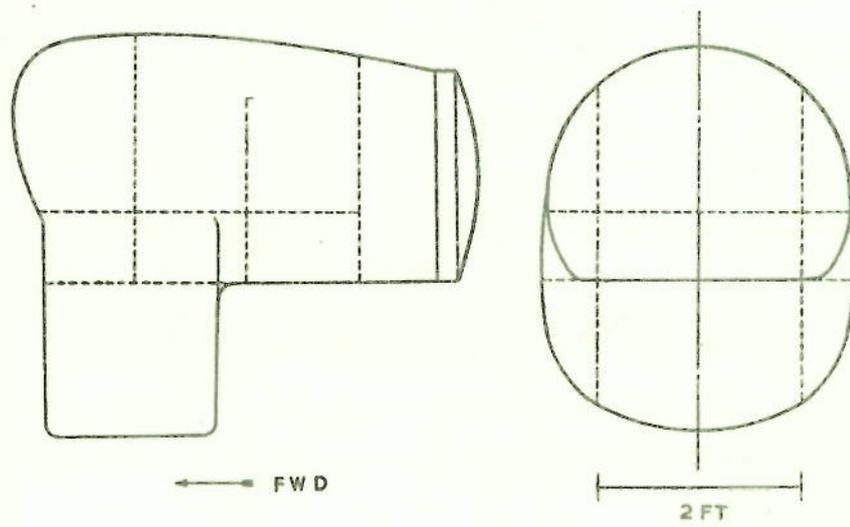


FIG. 41. THE MAIN FUEL TANK