# Statistical Parameters in Planning <br> Aero-Engine Production <br> - by - 

A. H Atkinson and J. T. Harris

## ERRATA

Page 16 Ref. 41 should read: DeJong,J.R. The effect of increasing skill on cycle time and its consequences for batch production time standards. Raadgevand Bureau Berenschot, Amsterdam/ Hengelo.

Ref. 42 should read: Manson, N. Practice curves. Memoires Societe des Ingenieurs Civils de France, Jan. - Feb., 1955.

Page 18 (e) Last column for "Ex. last 1" read Ex. 1st 1
(h) "-b" column for "0.437" read 0.427

Fig. 3 Insert in bottom left hand corner "Average production per month".

# THE COLLEGE OF AERONAUTICS 

## CRANFIELD

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## SUMMARY

This report considers the estimation of statistical parameters and their application to production planning in the aero-engine industry. A similar pattern of behaviour to that already recognised as existing in the airframe industry is found to operate, though there are indications of quantitative differences.

The build-up time to the planned peak rate of production in a particular situation is found to be about eighteen months but with variation between firms. The reduction in operator performance time, which occurs with repetition during the build-up period and afterwards, is discovered to be present in machining as well as assembly, but to a lesser extent. The logarithmic function generally descriptive of such a tendency is found to fit the actual man-hour content values rather than the cumulative average ones. The relationship between the logistic of output and the logarithmic function is established and made use of to estimate labour requirements from the commencement of production onwards.

In addition to the above consideration of production variables, examples are given of the use of engine performance ratings to estimate costs. Finally, because of the importance of planning to productivity, a typical production programme has been included.

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## 1. INTRODUCTION

The establishment of empirical relationships, the quantitiative estimation of parameters and the useful application of the results have an established history in air-frame manufacture. Little, however, of a similar nature has been attempted in respect of the production of the power unit of an aircraft. This seems to be due to the fact that airframe production is a younger technology.

However, recent developments in the aircraft industry suggest that empirical planning parameters could be beneficially employed in aero-engine production. Aircraft are now much more costly than formerly with the power plant claiming a greater proportion of that cost and assembly itself playing a more important part in power plant manufacture. Furthermore, the emergence of more competitive conditions makes it essential to pay more attention to estimating and planning methods.

It was with the intention of establishing planning parameters that the research forming the basis of this report was undertaken. Unfortunately, the statistical information obtained was not sufficient to treat the subject as rigorously as was desired; it prevented the consideration of many factors that could influence the situations examined and it made the use of more powerful statistical techniques impossible. Nevertheless, the results are considered valuable. Behaviour in the industry is revealed to be similar to that in the airframe industry, though there are indications of quantitative differences.

Originality in the report is confined mainly to Sections 2-4. Section 5 was included to indicate other types of relationships that could be employed, the form of their behaviour and the data required to estimate the parameters in them. Significant differences were found in the results from different firms and, since sound production planning makes a difference to a firm's performance, Section 6 was added, mainly to give an example of a typical production programme.

An interesting feature of the research is the light it throws on the importance placed on statistical information by different countries. American industry has long since been recognised to be more statistically conscious than British industry. However, it is revealing to observe that the Germans had reduced the collection of detailed statistical information, its analysis and application to a routine procedure using standardised documents, for the purpose of furthering the war effort.

The report is based on a thesis submitted by A. H. Atkinson in partial fulfilment of the requirements for a Diploma of the College of Aeronautics. We should like to convey our thanks for help received, particularly from the Ministry of Aviation, where Mr. S. Bentall has always been of very valuable assistance. Security reasons have prevented a more detailed description of the data employed but the statistical methods used enabled the data to be presented in a modified form.

## 2. PRODUCTION BUILD-UP RATES IN THE BRITISH AERO-ENGINE INDUSTRY

This section considers the behaviour of the monthly rate of output of an aeroengine in relation to the time from the output of the first of a series.
(a) Statistical information

Data were obtained in respect of nine turbo-jet engines built in Britain for military purposes, at seven different factories, in the early 1950's. As indicated in Table 1, five of the engines were of the same type.

In view of the large amount of variation in the original data, each series was smoothed by a 5 -month moving average. In addition, the output rates were adjusted to yield an attained peak production rate of 100 in each case. The data, so modified, are plotted in Figs. 1, (a) - (j).
(b) The logistic

The symmetrical logistic

$$
\begin{equation*}
Y=k /\left(1+10^{a+b X}\right) \tag{1}
\end{equation*}
$$

or, in another form,

$$
\begin{equation*}
\log _{10}\left(\frac{k-Y}{Y}\right)=a+b X \tag{2}
\end{equation*}
$$

where $Y$ is the output rate of month $X(X=1$ being the earliest month of any output) and $\mathrm{a}, \mathrm{b}$ and k are constants to be determined, was found to be descriptive of the general trend of each series, though in three of them it was confined to a section of the original data only.

The decision to fit the logistic to part only of the original data in certain of the series was justified by the behaviour of that series itself. It reflects a change of programme which is liable to occur in the industry. The fitted logistics are inserted in each of the figures.
(c) The meaning of the constants
(i) The constant $k$ is an asymptote of the logistic and, since $b$ is negative in these studies, it becomes the upper asymptote. It is the value that $Y$ approaches as $X$ tends to $\infty$. In a similar way, the value $Y=0$ is the lower asymptote, which is approached as X tends to $-\infty$.
(ii) The constant a locates the logistic on the time scale and it will accordingly be positive and finite. It determines the value of the logistic at $X=0$. As such, it will not reflect reality as no output occurs in month 0 . This is, however, of small consequence.
(iii) The constant $b$ describes the rate of approach to peak production and the smaller it is in absolute value, the longer the build-up period. In view of this inverse relationship and its negative sign, Stanley ${ }^{(7)}$ proposed the use of B where

$$
\begin{equation*}
B=-1 / b \tag{3}
\end{equation*}
$$

the build-up period is then directly related to $B$ and the steeper the logistic in its central part, the smaller the value of $B$.
(d) The fitted values of the constants

The values of the constants obtained by fitting the logistic to the several series are shown in Table 1. In addition, those of B and M - the time to achieve the peak output rate - are included.
(i) The values of M vary generally between 15 and 21 with a well defined mode at 18 months.
(ii) The values of $k$ vary generally between 101 and 107 with a mean of 104 .
(iii) The values of the constant a vary between 1.24 and 2.01 but without a defined average.
(iv) The values of -b vary between 0.091 and 0.212 , again without a well defined average.
(e) Factors affecting the results

The sample cannot be considered sufficient in size to investigate the many factors that could be expected to influence the results. Certain tentative conclusions can be drawn, based on the results and on additional information made available with the original data. These are that :-
(i) Neither the scale of output nor the size of the unit manufactured affected the results significantly.
(ii) The variation in results between factories was greater than that within factories.

Section 4 will reveal that the logistic can be influenced by other factors, namely

> the reduction coefficient, the rate of labour build-up, the cycle time of production and the average working week.

These will not only in general affect $b$ but also the constant $a$, since the amount of work-in-progress will itself be influenced by them.

Finally, there will be, in addition to the usual unexplained variance, the effects of subjective errors of fitting.
(f) Use of the logistic for planning
(i) The values $k=104, \mathrm{a}=1.40,-\mathrm{b}=1.55$ yield a value of $\mathrm{Y}=100$ at $\mathrm{X}=18$ and can be considered as representing average behaviour.
(ii) In view of (e) (ii) of this Section, quantitative values for the parameters
derived from a factory's own historical data would be subject to smaller standard errors than those obtained from the average behaviour of a number of firms and they would be accordingly more satisfactory for its own planning.
(iii) The earlier an estimate is made, generally the less reliable will be the value obtained. However, once deliveries have begun, an increasing amount of data will become available for comparison with the estimated standard and for purposes of extrapolation. To this end, a set of curves with a range of values for the parameters could be drawn on transparent material, placed over observed data and the placing adjusted to reflect the trend in that data.

## (g) Fitting the logistic curve - the method of three selected points

The equation for the logistic curve is usually quoted as

$$
\begin{equation*}
\mathrm{Y}=\mathrm{k} /\left(1+\mathrm{e}^{\mathrm{a}+\mathrm{bX}}\right) \tag{4}
\end{equation*}
$$

but since, in practical computation, it is much more convenient to take logarithms to the base 10 , as tables of these are usually readily available, the curve fitted to the data will be

$$
\begin{equation*}
Y=k /\left(1+10^{a+b X}\right) \tag{5}
\end{equation*}
$$

Select three points, not necessarily included in the data, through which the curve is to pass. Denote these by $\left(X_{1}, Y_{1}\right),\left(X_{2}, Y_{2}\right)$ and $\left(X_{3}, Y_{3}\right)$ where $X_{3}>X_{2}>X_{1}$. A restriction placed on these points is that they must be equally spaced on the $X$ co-ordinate, $\mathrm{i} .\left(\mathrm{X}_{3}-\mathrm{X}_{2}\right)=\left(\mathrm{X}_{2}-\mathrm{X}_{1}\right)=\mathrm{n}$.

Then the parameters of the curve are calculated from the following expressions.

$$
\begin{align*}
& k=\frac{2 Y_{1} Y_{2} Y_{3}-Y_{2}^{2}\left(Y_{1}+Y_{3}\right)}{Y_{1} Y_{3}-Y_{2}^{2}}  \tag{6}\\
& b=\frac{1}{n} \log _{10} \frac{Y_{1}\left(k-Y_{2}\right)}{Y_{2}\left(k-Y_{1}\right)}  \tag{7}\\
& a=\log _{10}\left(\frac{k-Y_{1}}{Y_{1}}\right)-b X_{1} \tag{8}
\end{align*}
$$

Sufficient points can be calculated using the expression

$$
\begin{equation*}
\log _{10}\left(\frac{k-Y}{Y}\right)=a+b X \tag{9}
\end{equation*}
$$

to enable the curve to be fitted
(h) The growth index B (Ref. 7)

Equation (4) can be expressed in the form

$$
\begin{equation*}
\log \left(\frac{k-Y}{Y}\right)=a+b X \tag{10}
\end{equation*}
$$

Let $\left(X_{1}, Y_{1}\right)$ and $\left(X_{2}, Y_{2}\right)$ be two points on the logistic. Substituting the values in (10) and subtracting, we have

$$
\begin{align*}
& b\left(X_{2}-X_{1}\right)=\log \left(\frac{k-Y_{2}}{Y_{2}}\right)-\log \left(\frac{k-Y_{1}}{Y_{1}}\right)  \tag{11}\\
& \left(X_{2}-X_{1}\right)=\frac{1}{b} \log \frac{Y_{1}\left(k-Y_{2}\right)}{Y_{2}\left(k-Y_{1}\right)} \tag{12}
\end{align*}
$$

Hence, if $Y_{1}$ and $Y_{2}$ are fixed quantities, $\left(X_{2}-X_{1}\right)$ is inversely proportional to - $b_{j}$ i.e. the time taken to increase from $Y_{1}$ to $Y_{2}$ is proportional to

$$
-1 / b=B
$$

## 3. TIME REDUCTION IN AERO-ENGINE MANUFACTURE

This section considers the behaviour of the manufacturing man-hour content of an engine in relation to the cumulative number produced.

## A. BRITISH PRODUCTION

(a) Statistical information

Data were obtained in respect of two different turbowjet engines manufactured under differing circumstances. Information was provided in respect of the details of manufacture as indicated in Table 2.

Actual man-hours are not recorded but only an index of them. The value to which each series tended, after a considerable number of components had been produced, was chosen as a base and given a value of unity in each case. Certain of the series are proportional to the actual man-hour content of the given item (unit) whilst others are proportional to the average man-hour content for production, up to and including the given item (cumulative average). The data, as such, are plotted in Figs. $2(\mathrm{a})-(\mathrm{q})$.
(b) The time reduction curve

The mathematical function generally descriptive of this is given by

$$
\begin{equation*}
Y=a X^{b} \tag{13}
\end{equation*}
$$

where $Y$ is the man-hours for the $X$ th unit and $a$ and $b$ are constants to be determined. Where $Y$ is the unit value, it is denoted by $Y_{u}$ and where it is the cumulative average, it is denoted by $Y_{c}$. The function can be put into the form

$$
\begin{equation*}
\log Y=\log a+b \log X \tag{14}
\end{equation*}
$$

so that by plotting the observed values of Y against those of X on a $\log -\log$ scale, it can be readily determined to what extent the function will prove a satisfactory fit. The fitted functions, excluding certain values in some cases, are inserted in each of the Figs. 2 (a)-(q).

## (c) The meaning of the constants

(i) The constant $a$ is the value of $Y$ at $X=1$, reflecting in this study the manhour content of the first of a series as a multiple of that of a high cumulative value.
(ii) The constant $b$ describes the slope of the line on $\log -\log$ scale. It will be negative and the greater its absolute value, the greater the rate of time reduction. Directly from it we have the coefficient of time reduction, given by

$$
\begin{equation*}
\mathrm{L}=\left(100 / 2^{-\mathrm{b}}\right) \% \tag{15}
\end{equation*}
$$

It represents the value of $Y$ for a given $X$ as a percentage of that of $X / 2$.
If the function fits either form of the series satisfactorily, then it will fit the value obtained by transforming that series into the other form, provided certain initial values are excluded. The value of $L$ will remain significantly unaffected by this procedure.
(d) The fitted values of the constants
(i) The values of the constant a vary widely where the cumulative average is employed and between 2.3 and 3.2 , without a well defined mode, where unit values are considered.
(ii) The average value of -b , for machining, is approximately 0.185 and, for assembly, approximately 0.250 . The corresponding coefficients of time reduction are approximately $88 \%$ and $84 \%$ respectively.
(e) Discussion of results

The sample itself is too small for other than tentative conclusions.
(i) Time reduction is experienced in machining, as well as assembly, but to a lesser extent.
(ii) Differences between firms are likely to be significant. Engine B, in particular, was produced under war-time circumstances.
(iii) The logarithmic function describes more satisfactorily the trend of unit data values than that of cumulative average data. The greater scatter of unit data around the fitted trend is due to the smoothing effect of averaging.
(iv) A discussion of the values of the constant a serves little purpose in view of the subjective element in the choice of the index base. However, it indicates that a reduction to about one-third of the man-hour content of the first of a series is at least possible.

## B. GERMAN PRODUCTION

(a) Statistical information

A number of war-time standard statistical documents was obtained, showing detailed breakdown in terms of cost, weight, time and man-hours for raw material, components, processes and performance, as they applied. One of these documents, representing an overall summary for a number of types as indicated, is shown in Fig. 3. A semi-logarithmic scale has been employed and $85 \%$ time reduction curves superimposed.

## (b) Discussion of results

(i) Close accord with an $85 \%$ law is observed generally. The agreement between this and engine $B$ of the British data indicates that this law applies, very generally, under war-time circumstances.
(ii) The Germans recognised the importance of detailed statistical information for historical and planning purposes. The more data that are available, the more reliable the parameters essential for estimating future requirements.

## C. GENERAL

(a) The employment of the coefficient of time reduction in planning
(i) The unit function, rather than the cumulative function, should be employed.
(ii) Different coefficients of time reduction apply to machining, as against assembly and for differing circumstances.
(iii) Pre-planning requires that an estimate of the constant $a$, in man-hours, be obtained by consideration outside the scope of this section. When the first of the series is known, a rough estimate of the constant a is obtained. When further data become available, curve fitting can be employed and comparison with an estimated standard and extrapolation undertaken. For this purpose, various coefficients can be plotted on transparent material. On $\log -\log$ scale, curves having the same coefficient of time reduction will be parallel.
(b) The least squares method of fitting

$$
\begin{equation*}
\mathrm{b}=\frac{\Sigma w v}{\Sigma \mathrm{v}^{2}} \tag{16}
\end{equation*}
$$

$a=\bar{W}-b \bar{V}$
where:

$$
\begin{align*}
\Sigma W V & =\Sigma W V-\bar{W} \Sigma V  \tag{18}\\
\Sigma V^{2} & =\Sigma V^{2}-\bar{V} \Sigma V  \tag{19}\\
\bar{W} & =\frac{\Sigma W}{n}, \bar{V}=\frac{\Sigma V}{n}  \tag{20}\\
W & =\log Y, V=\log X  \tag{21}\\
n & =\text { number of pairs of readings }
\end{align*}
$$

EXAMPLE - Figure 2(b)

| X | Y | W | V | WV | $\mathrm{V}^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 3.64 | 0.5611 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 3.42 | 0.5345 | 0.3010 | 0.1609 | 0.0906 |
| 5 | 3.05 | 0.4842 | 0.6990 | 0.3385 | 0.4886 |
| . | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ |
| $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ |
| $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ |
| $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ |
| 100 | 1.33 | 0.1239 | 2.0000 | 0.2478 | 4.0000 |
| 115 | 1.15 | 0.0607 | 2.0607 | 0.1251 | 4.2465 |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

$$
\begin{aligned}
\overline{\mathrm{W}} & =\frac{3.9835}{14}=0.2845 \\
\overline{\mathrm{~V}} & =\frac{17.8594}{14}=1.2757 \\
\Sigma \mathrm{wV} & =3.8015-(0.2845)(17.8594) \\
& =-1.2795 \\
& =27.9630-(1.2757)(17.8594) \\
\Sigma \mathrm{v}^{2} & =5.3744 \\
& =-\frac{1.2795}{5.3744}=-0.2381 \\
\mathrm{~b} & =0.2845-(-0.2381)(1.2757) \\
\log \mathrm{a} & =0.5882 \\
\mathrm{a} & =3.875
\end{aligned}
$$

## 4. THE TIME CYCLE OF PRODUCTION AND LABOUR BUILD-UP

This section considers the relationship between the logistic of output, the time reduction coefficient, the cycle time and the rate of build-up of man-power.

## (a) Statistical information

Data were obtained of cycle times in respect of one turbo-jet engine. These are not sufficient to establish empirical parameters but they will serve to illustrate the use that can be made of such material. The data are recorded in Fig. 4, where it will be observed that detail for processes have also been made available.
(b) Time cycle reduction

A reduction in the time cycle of manufacture can be expected, following the reduction in man-hour content. However, it cannot be expected to be so strong and our example is, in fact, one with a reduction coefficient of approximately $90.2 \%$. There is, too, variation between processes.

## (c) Time cycle chart

This can be obtained by the following procedure.
(i) Derive a delivery schedule by cumulating the values of the relevant logistic.
(ii) Derive the time cycle for the first of the series by the use of parameters or by production control methods.
(iii) Apply the relevant time cycle reduction coefficient to the above to obtain the starting schedule for the process.

The chart will readily yield the number of partly processed units at any one time. Thus Fig. 4 indicates that, eight months after the start of production, 37 engines have commenced and 33 have completed the first engine build-up process, making five engines in the partly processed stage at that time.
(d) Man power requirements chart

The cycle time reduction chart can be used in conjunction with the time reduction coefficient and an estimate of the man-hour content of the first of a series to establish the manpower requirements at any one time during production.
(i) Obtain the average unit man-hour content of those in process at any one time by means of the reduction coefficient and an estimate of the man-hour content of the first of the series.
(ii) Obtain the average cycle time for the stage considered.

Proceeding with the example above we have

| No. of engines at first engine build | $=$ | 5 |
| :--- | :--- | :---: |
| Average cycle time in hours | $=$ | 72 |
| Average man hours per engine | $=$ | 430 |
| Therefore, number of operators required |  | $=\frac{5 \times 430}{72}=30$ |
| at this stage |  |  |

By this means, manpower requirements can be obtained for the total period of production as shown in Fig. 5.

## 5. PERFORMANCE AND COSTS

This section considers the improvement in performance with time and its relationship with costs, both monetary and physical.

## A. AMERICAN STUDIES

(a) Source, purpose, procedure and definitions

The empirical relationships set out below are taken directly from Ref. 33. They are based on the experience of turbo-jet manufacture before 1950. The purpose of the studies was to relate costs to military effectiveness. The procedure adopted involved, firstly, obtaining the time trend of performance factors and then relating costs to deviations from the trend. Certain common terminology, generally employed in the studies, is

$$
\begin{aligned}
\mathrm{N} & =\text { the number of observations } \\
\mathrm{r} & =\text { correlation coefficient } \\
\text { S.E. } & =\text { the standard error of forecast }
\end{aligned}
$$

Other terms employed are definedwith the relationships.
(b) The relationships
(i) Trend of engine thrust with time

$$
T=9584(1.127)^{t}
$$

| T | $=$ thrust in lbs. military rating |
| :--- | :--- |
| t | $=$ time in units of six months, to design initiation |
| Range of $\quad \mathrm{T}$ | $=1560$ to 16400 |
| Range of $\quad t$ | $=1943$ to 1950 |
|  | N | r,$\quad \mathrm{r}=0.945$

(ii) Trend cf specific fuel consumption with time

$$
\begin{aligned}
& \text { S.F.C. }=0.9531(0.9897)^{-t} \\
& \text { S.F.C. = Specific fuel consumption in lbs/hr/lb. thrust }
\end{aligned}
$$

Range of
Range of $t=1943$ to 1950
(iii) Development costs and performance

$$
C_{D}=3301.0 \quad(\Delta T)^{0.220}(\Delta \text { S.F.C. })^{0.0356}
$$

$C_{D}=$ development costs ( $\$ 000^{\prime} s$ ) at 1948 prices. The costs are those incurred from the initiation of design, after the award of an experimental contract, to the first bench test of an experimental engine.
$\Delta T=$ the percentage increase over the thrust trend
$\Delta$ S.F.C. $=$ the percentage decrease from the S.F.C. trend
Range of $\Delta T=0$ to 92
Range of $\Delta$ S.F.C. $=0$ to 11.1

$$
\mathrm{N}=8, \quad \text { S.E. }=1.43
$$

(iv) Improvement development costs and performance

$$
\mathrm{C}_{\mathrm{DM}}=394.1 \quad\left(\Delta \mathrm{~T}_{\mathrm{m}}\right)^{0.164} \quad\left(\Delta \mathrm{~S} . \mathrm{F} . \mathrm{C} \cdot{ }_{\mathrm{M}}\right)^{0.141}
$$

$\mathrm{C}_{\mathrm{DM}} \quad=$ development costs for improvement of basic engine ( $\$ 000$ 's) at 1948 prices.
$\Delta \mathrm{T}_{\mathrm{m}} \quad=$ the percentage increase in thrust over previous model of series. Where $\Delta \mathrm{T}_{\mathrm{M}} \leqslant 1, \quad \mathrm{~T}_{\mathrm{M}}=1$ is employed.
$\Delta$ S.F.C. ${ }_{M}=$ the percentage decrease in the S.F.C. from previous model Where S.F.C. ${ }_{M} \leqslant 1, \quad$ S.F.C. ${ }_{M}=1$ is employed.
Range of $\Delta \mathrm{T}_{\mathrm{M}}=7.7$ to 27.2
Range of $\Delta$ S.F.C. ${ }_{M}=0$ to 9.3

$$
\mathrm{N}=8, \quad \text { S.E. }=1.40
$$

(v) Development time and performance

$$
T_{D}=10.6(\Delta T)^{0.118}
$$

$T_{D}=$ peace-time months between design initiation and first bench test.

Range of $\Delta T=0$ to 92

$$
\mathrm{N}=8, \text { S.E. }=1.16
$$

(vi) Production cost and performance

$$
C_{p}=8.069 \quad T^{0.233} \quad(\Delta \text { S.F.C. })^{0.0357} \quad\left(\frac{M}{500}\right)^{-0.152}
$$

$C_{p}=$ cumulative average cost per engine for $M$ engines ( $\$ 000^{\prime} s$ ) at 1948 prices.
$\mathrm{M}=$ cumulative number of engines produced
Range of $T=1560$ to 16400
Range of $\Delta$ S.F.C. $=0$ to 11.1

$$
\mathrm{N}=10, \quad \text { S.E. }=1.18
$$

(vii) Production time and performance

No significant relationship found to exist.
Production time $(P)=$ elapsed time from the date of the first test to peak production.

```
Range of P}=5\mathrm{ to }3
Mean of P = 17.6
    N}=1
```

(c) Discussion of results

The relationship between costs and performance, as defined, are revealed to be logarithmic, i.e. a given proportionate increase in a given determining variable is associated with a given percentage change in the level of costs. The effect of the thrust, in this respect, is greater than that of the specific fuel consumption. However, the range of the latter is much smaller than that of the former.

## B. GERMAN STUDIES

(a) Source, purpose and definitions

The empirical relationships in graphical form are shown in Figs. 6 and 7 and have been taken directly from the original German documents. They were used by the Germans for production planning purposes.
(b) The relationships
(i) Direct production manhours and engine dry weight

The reduction factor of direct production manhours per engine, in relation to engine dry weight, was $87 \%$.
(ii) Direct production manhours and engine horse power

The reduction factor of direct production manhours per engine, in relation to engine nominal H.P. rating, was $85 \%$.
(c) Discussion of results

The relationships established are revealed to hold over more than 20,000 engines. However, the reduction factor, including engine dry weight, is higher than that including H. P. rating, the latter being the same as the manhour reduction factor. The reason for the difference arises from the tendency of an engine to increase in weight during a production programme because of modifications or the use of heavier materials to overcome shortages.

It is observed that the value of direct manhours per kilogramme (pound) at 20,000 engines was in the range $2.2-3.2(48-70)$ and that the value of direct manhours per H.P. at the same stage was between 1.7 and 2.1 for radial engines and between 1.25 and 1.9 for in-line engines.

## C. BRITISH STUDIES

Lack of data enabled only a superficial examination of British production costs.
Six samples of 1954 data showed values between 1.6 and 2.8 of direct manhours per lb. of engine dry weight for piston engines and between 2.7 and 4.2 direct manhours per B.H.P. for the three radia! angines of the samples. Compared with the German values, the former are very much reduced and the latter higher. The reduction is due to improved technology and the increase due to greater complexity.

## 6. PRODUCTION PLANNING AND OPTIMUM DECISIONS

This section gives a typical production programme and considers the implications of timing and scale of operation.
(a) Production planning

Sound production planning enables a realistic delivery date to be established and early corrective action and adjustments made. A typical example, treating the first production model of a turbo-jet engine, is shown in Figs. 8-11.

Fig. 8. Items susceptible to delays, those requiring special equipment and those with long cycle times, are given attention before the release of the design to production. The specification of materials and sub-contracting policy needs to be established at an early stage.

Fig. 9. When the design has been released to production, more information will be available to enable parts to be treated in detail.

Fig. 10. Each long dated item, or a group of similar items, is given separate treatment. Details of special equipment are included and its history recorded by filling in the relevant triangles.

Fig. 11. This records the time span for sub-assemblies and subsequent stages and the dates at which the relevant rigs and the fixtures are required.
(b) Time and scale

An aero-engine passes through many distinct phases in its journey from the initial idea to quantity production. Two important problems arise in this time span. It is necessary to decide, firstly, the extent to which the phases should overlap and, secondly, the intensity or scale at which each phase is to operate.

The more a phase overlaps a subsequent one, the less complete or finalised the information available for decisions in respect of that subsequent stage and thus the greater the probability of wastage through incorrect decisions. The economies of a reduced overall time span have thus to be set against the wastage.

There are also economies and diseconomies of scale. A greater rate of output at a lower unit cost can generally be achieved by more capitalistic methods of production. The decision as to which method of production to adopt will depend on expectations in respect of the market. However, a larger output may mean a greater
loss, so that a careful appraisal of the market is necessary to decide the optimum scale of production.

Evaluation is not a straightforward procedure. Many of the values made use of will be based on expectations. Intangibles will need to be considered. Finally, firms operate in differing circumstances and what is best suited to one need not necessarily be the optimum solution for another.

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| Figure 1. | Engine | Factory | k | a | -b | B | M <br> (months) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (a) | 1 | A | 104 | 1.66 | 0.167 | 5.99 | 16 |
| (b) | 1 | B | 113 | 1.51 | 0.120 | 8.31 | 21 |
| (c) | 1 | C | 107 | 1.97 | 0.207 | 4.83 | 15 |
| (d) | 1 | D | 105 | 1.44 | 0.148 | 6.76 | 18 |
| (e) | 1 | E | 101 | 1.51 | 0.163 | 6.12 | 20 |
| (f) | 2 | F | 107 | 1.37 | 0.142 | 7.05 | 18 |
| (g) | 3 | D | 103 | 1.24 | 0.106 | 9.42 | 18 |
| (h) | 4 | G | 101 | 2.01 | 0.091 | 10.95 | 33 |
| (j) | 5 | A | 101 | 1.72 | 0.212 | 4.71 | 18 |

TABLE 1. VALUES OF THE CONSTANTS OF THE FITTED
LOGISTICS AND THE TIME TO ACHIEVE PEAK
OUTPUT RATES

| $\begin{gathered} \text { Fig. } \\ 2 \end{gathered}$ | Operation | Engine | Sample size | Man-hour content | a | -b | L |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (a) | Total direct | B | 14 | Cum.av. | 3.57 | 0.196 | 87.3 | Ex.1st 2 |
| (b) | Total machining | B | 14 | Unit | 3.87 | 0.238 | 84.7 |  |
| (c) | Motor component machining | A | 12 | Unit | 3.08 | 0.155 | 89.8 |  |
| (d) | Total fitting | B | 14 | Unit | 2.27 | 0.162 | 89.4 |  |
| (e) | Total assembly | B | 13 | Cum.av. | 4.11 | 0.227 | 85.4 | Ex.last 1 |
| (f) | Assembly \& subassembly | A | 7 | Unit | 4.04 | 0.264 | 83.3 | Ex.last 3 |
| (g) | Final erection | B | 13 | Cum.av. | 5.69 | 0.241 | 84.6 | Ex.1st 2 |
| (h) | Total testing | B | 13 | Cum.av. | 16.72 | 0.437 | 74.4 | Ex.1st 2 |
| (j) | Compressor casing machining | B | 14 | Unit | 2.85 | 0.199 | 87.1 |  |
| (k) | Rotor sub-assembly machining | B | 14 | Unit | 2.35 | 0.172 | 88.7 |  |
| (1) | Rotor sub-assembly machining | A | 12 | Unit | 2.60 | 0.133 | 90.3 |  |
| (m) | Turbine subassembly machining | B | 7 | Cum. av. | 5.30 | 0.278 | 82.5 | Ex.1st 7 |
| ( n ) | Turbine blades machining | A | 10 | Unit | 3.15 | 0.138 | 90.9 |  |
| (p) | Gearbox machining | A | 13 | Unit | 3.52 | 0.190 | 87.6 |  |
| (q) | Gearbox machining | B | 13 | Cum.av. | 8.87 | 0.400 | 75.7 | Ex.1st 2 |

TABLE 2. THE VALUES OF THE CONSTANTS FOR TLME



FIG.I. RELATIONSHIP BETWEN ENGINES DELIVERED PER MONTH (AS A PERCENTAGE OF PEAK RATE) AND THE MONTH OF OUTPUT.




FIG.I. RELATIONSHIP BETWEEN ENGINES DELIVERED PER MONTH (AS A PERCENTAGE OF PEAK RATE) AND THE MONTH OF OUTPUT.



FIG.I. RELATIONSHIP BETWEEN ENGINES DELIVERED PER MONTH (AS A PERCENTAGE OF PEAK RATE) AND THE MONTH OF OUTPUT.


FIG. 2. CUMULATIVE AVERAGE OR UNIT MANHOURS IN RELATION TO NUMBERS PRODUCED


FIG. 2. CUMULATIVE AVERAGE OR UNIT MANHOURS IN RELATION TO NUMBERS PRODUCED


FIG. 2. CUMULATIVE AVERAGE OR UNIT MANHOURS IN REILATION TO NUMBERS PRODUCED


FIG. 2. CUMULATIVE AVERAGE OR UNIT MANHOURS IN RELATION TO NUMBERS PRODUCED

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FIG. 3. GERMAN WAR-TIME STANDARD DOCUMENT TYPE 2. OVERALL SUMMARY OF THE PRODUCTION

MANHOURS PER ENGINE FOR AERO-ENGINE MANUFACTURING PROJECTS


FIG. 4. TYPICAL TIME CYCLE CHART FOR THE ASSEMBLY AND TESTING OF TURBO-JET ENGINES


FIG. 5. TYPICAL MANPOWER FORCAST CHART FOR TURBO-JET ENGINE PRODUCTION


FIG. 6. GERMAN WAR-TIME DOCUMENT. SUMMARY OF DIRECT MANHOURS PER UNIT OF ENGINE DRY WEIGHT FOR PISTON AERO-ENGINE MANUFACTURING PROJECTS


FIG. 7. GERMAN WAR-TIME DOCUMENT SUMMARY OF DIRECT MANHOURS PER B. H. P. FOR ALL AERO-ENGINE MANUFACTURING PROJECTS


FIG. 9. PRODUCTION PLAN CHART FOR THE MANUFACTURE OF COMPONENTS FOR THE MANUFACTURE OF THE FIRST PRODUCTION MODEL OF A TURBOJET ENGINE MODEL

## LONG-DATED MATERIAL ITEMS

ALL OTHER ITEMS, EXCEPT BLADES, ARE WITHIN A 6 MONTH DELIVERY PERIOD


FIG. 8. TYPICAL OVERALL TIMING PLAN FOR THE MANUFACTURE OF THE FIRST PRODUCTION MODEL OF A TURBO-JET ENGINE PROJECT


FIG. 10(a) PRODUCTION PLAN CHART FOR THE MANUFACTURE OF LONGDATED ITEMS FOR THE FIRST PRODUCTION MODEL OF A TURBO-JET ENGINE PROJECT


FIG. 10 (b) PRODUCTION PLAN CHART FOR THE MANUFACTURE OF A GROUP OF SIMILAR LONG-DATED ITEMS FOR THE FIRST PRODUCTION MODEL OF A TURBO-JET ENGINE PROJECT

## KEY TO SUB-ASSEMBLY STAGES



FIG. 11. TYPICAL CYCLE RELATIONSHIPS FOR ASSEMBLING AND TESTING
A TURBO-JET ENGINE (FIRST PRODUCTION MODEL)

