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# Aerodynamic Characteristics of a Hypersonic Parachute

- by -

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## SUMMARY

Newtonian theory, both in the form of the Modified-Newtonian and the Newton-Busemann pressure laws, is used to find the shape, cloth area and drag of the axisymmetric canopy of a hypersonic parachute, whose only load-carrying fibres are longitudinal ones. As an example, an estimate is made of the size of canopy needed to give a drag of 20,000 lb. in flight at a Mach number of 10 at 100,000 feet altitude.

# CONTENTS

## List of Symbols

| | |
|---|---|
| $A$ | total area of cloth in canopy |
| $C_D$ | drag coefficient of canopy |
| $D$ | drag of canopy |
| $M_\infty$ | free-stream Mach number |
| $p$ | pressure |
| $p_o$ | stagnation pressure behind a normal shock |
| $q_\infty$ | free-stream dynamic pressure |
| $(r, z)$ | cylindrical polar co-ordinates |
| $r$ | radial co-ordinate |
| $s$ | arc length along canopy generator, measured from the leading edge |
| $T$ | longitudinal tension in cloth per unit radian |
| $z$ | longitudinal co-ordinate |
| $\theta$ | local angle of canopy generator |

## Subscripts

| | |
|---|---|
| $L$ | leading edge |
| $T$ | trailing edge |
| $\infty$ | free stream conditions |

## 1. Introduction

The canopy of the parachute we consider here is a surface of revolution and is impermeable. Its shape, cloth area and drag are calculated using Newtonian theory, on the assumption that the load in the canopy is carried by longitudinal fibres only. Results derived from the Modified-Newtonian pressure law (the "uncorrected" parachute) and from the Newton-Busemann pressure law (the "corrected" parachute) are compared. Two possible canopy shapes, one for a drogue parachute, the other for an umbrella-shaped parachute, are examined.

One example is given. The size of the canopy, which offers 20,000 lb. drag in flight at $M_\infty = 10$ at 100,000 ft. is calculated.

## 2. Equilibrium of parachute element

We assume that the parachute is a surface of revolution and examine two possible canopy shapes, one for a drogue parachute, the other for an umbrella-shaped parachute. The drogue canopy is formed by revolving the generator shown in Fig. 1 about the z axis. The generator of the umbrella canopy is shown in Fig. 2.

The hypersonic parachute is in a Newtonian flow, so that a thin shock layer coincides with its forward-facing surface. To allow the flow in the shock layer to stream away, the umbrella-shaped canopy must be vented at the crown. The vent area need only be infinitesimal for the shock layer approaching it is infinitesimally thin because of the infinitely large density ratio across the shock. The shock layer is free to leave the trailing edge of the drogue canopy. The leading edge may be considered vented or blanked off. Here, for algebraic simplicity, we take the drogue generator to have a zero radius at the leading edge.

Now consider the canopy element formed by revolving an arc of length ds about the z axis; Figs. 1 and 2. The pressure p on this element of area $2\pi r ds$ is normal to the forward-facing surface and depends on the local slope. For very high Mach numbers ($M_\infty \to \infty$) the pressure on the rearward-facing surface is zero. The load on the canopy is in equilibrium with the tension in the canopy cloth. Further we assume that the load is carried by the longitudinal fibres only. A circumferential fibre will, of course, not be in tension if its length equals the optimum length given by the radial co-ordinate we calculate below. There is no friction between the shock layer and the parachute so that the longitudinal tension is constant. Thus for equilibrium of the canopy element we require that

$$2\pi p\, r \cos\theta\, ds = d(2\pi T \sin\theta) = 2\pi T \cos\theta\, d\theta$$



where $\theta$ is the local slope,
p the pressure difference across the element,
and T the longitudinal tension in the cloth per unit radian.

Simplifying we have

$$p\, r\, ds = T\, d\theta \qquad (1)$$

Substituting for p in terms of the local slope we can find from this equation the shape of the canopy generator. When p is given by the Modified Newtonian pressure law, $p = p_o \sin^2\theta$, the resulting shape is labelled in the following the "uncorrected parachute". The "corrected parachute" follows from the Newton-Busemann pressure law,

$$p = p_o\left(\sin^2\theta + \frac{\sin\theta}{r}\ \frac{d\theta}{dr}\int_{\theta_L}^{\theta} r'\cos\theta'\ dr'\right)$$

In this result we correct the ordinary Newtonian sine-squared law for the centrifugal pressure difference across the shock layer due to the curvature of the streamlines on the concave forward-facing surface of the canopy.

## 3. Shape of canopy generator

### 3.1. Drogue parachute

The arc length $s$ is measured from the leading edge. From Fig. 1, $ds = dr/\sin\theta$. Substituting this is equation (1) we have

$$\frac{p\,r}{T}\ dr = \sin\theta\ d\theta \tag{2}$$

#### 3.1.1. Uncorrected parachute

Substituting for $p$ in (2) from the Modified Newtonian pressure law gives

$$\frac{p_o r}{T}\ dr = \frac{d\theta}{\sin\theta} \tag{3}$$

from which we find

$$\frac{1}{\sqrt{2}}\sqrt{\frac{p_o}{T}}\ r = \sqrt{\ln\left(\frac{\tan\frac{1}{2}\theta}{\tan\frac{1}{2}\theta_L}\right)} \tag{4}$$

Now

$$dz = dr\cot\theta \tag{5}$$

so that

$$\sqrt{2}\sqrt{\frac{p_o}{T}}\ dz = \frac{\cos\theta\ d\theta}{\sin^2\theta\sqrt{\ln\left(\frac{\tan\frac{1}{2}\theta}{\tan\frac{1}{2}\theta_L}\right)}} \tag{6}$$

using (3) and (4).

Integrating (6) yields

$$\sqrt{2}\sqrt{\frac{p_o}{T}}\ z = \cot\tfrac{1}{2}\theta_L\ \mathrm{erf}\left(\frac{1}{\sqrt{2}}\sqrt{\frac{p_o}{T}}\ r\right) - \tan\tfrac{1}{2}\theta_L\ \mathrm{erfi}\left(\frac{1}{\sqrt{2}}\sqrt{\frac{p_o}{T}}\ r\right), \tag{7}$$

where

$$\text{erf } x = \int_0^x e^{-u^2} \, du \,, \tag{8}$$

and

$$\text{erfi } x = \int_0^x e^{u^2} \, du \,. \tag{9}$$

Equation (7) defines the shape of the generator of the uncorrected drogue parachute, and this is plotted in Fig. 3.

### 3.1.2. Corrected parachute

Putting the Newton-Busemann pressure law in equation 2, we obtain

$$p_o \left( \sin^2 \theta + \frac{\sin \theta}{r} \frac{d\theta}{dr} \int_{\theta_L}^{\theta} r' \cos \theta' \, dr' \right) r \, dr = T \sin \theta \, d\theta$$

or

$$r \sin \theta \frac{dr}{d\theta} + \int_{\theta_L}^{\theta} r' \cos \theta' \frac{dr'}{d\theta} \, d\theta' = \frac{T}{p_o} \,. \tag{10}$$

Differentiating (10) yields

$$\frac{d}{d\theta} \left( r \frac{dr}{d\theta} \right) + 2 \cot \theta \left( r \frac{dr}{d\theta} \right) = 0 \tag{11}$$

Integrating (11) gives

$$r \frac{dr}{d\theta} = \frac{k}{\sin^2 \theta} \,, \quad \text{where k is a constant.} \tag{12}$$

Now from (10) we have, when $\theta = \theta_L$ ,

$$\left( r \frac{dr}{d\theta} \right)_{\theta = \theta_L} = \frac{T}{p_o \sin \theta_L} \tag{13}$$

so that

$$k = \frac{T}{p_o} \sin \theta_L \,.$$

Thus

$$r \frac{dr}{d\theta} = \frac{T}{p_o} \frac{\sin \theta_L}{\sin^2 \theta} \tag{14}$$

and

$$\frac{1}{2} r^2 = \frac{T}{p_o} \sin \theta_L \left( \cot \theta_L - \cot \theta \right) \tag{15}$$

Hence

$$\frac{1}{\sqrt{2}}\sqrt{\frac{p_o}{T}}\ r = \left(\cos\theta_L - \cos\theta\ \frac{\sin\theta_L}{\sin\theta}\right)^{\frac{1}{2}} \tag{16}$$

Again $dz = dr \cot\theta$

$$= \frac{T\sin\theta_L}{p_o}\ \frac{\cot\theta\, d\theta}{r\sin^2\theta}\ , \text{ using (14).}$$

$$\sqrt{2}\sqrt{\frac{p_o}{T}}\ dz = \frac{\sin\theta_L\ \operatorname{cosec}^2\theta\ \cot\theta\ d\theta}{(\cos\theta_L - \sin\theta_L\cot\theta)^{\frac{1}{2}}}$$

This integrates to

$$\sqrt{\frac{p_o}{T}}\ z = \frac{1}{3}\sqrt{\frac{p_o}{T}}\ r\ (\cot\theta + 2\cot\theta_L) \tag{17}$$

Finally eliminating $\cot\theta$ from equation (17) by using (15),

$$\sqrt{\frac{p_o}{T}}\ z = \frac{1}{3}\sqrt{\frac{p_o}{T}}\ \frac{r}{\sin\theta_L}\left(3\cos\theta_L - \frac{1}{2}\ \frac{p_o r^2}{T}\right) \tag{18}$$

The shape of the generator of the corrected drogue parachute is a cubic, and is shown in Fig. 3.

### 3.2. Umbrella parachute

As the arc length $s$ is measured from the leading edge it follows from Fig. 2, $ds = - dr/\sin\theta$. Substituting this in equation (1) we have

$$\frac{pr}{T}\,dr = -\sin\theta\, d\theta\ . \tag{19}$$

#### 3.2.1. Uncorrected parachute

Integrating (19), with the pressure given by the Modified Newtonian law, yields

$$\frac{1}{\sqrt{2}}\sqrt{\frac{p_o}{T}}\ r = \sqrt{\ln\left(\frac{\tan\frac{1}{2}\theta_T}{\tan\frac{1}{2}\theta}\right)} \tag{20}$$

The $z$ co-ordinate follows as in section 3.1 from $dz = dr \cot\theta$ and is given by

$$\sqrt{2}\sqrt{\frac{p_o}{T}}\ z = \cot\tfrac{1}{2}\theta_T\ \operatorname{erfi}\left(\frac{1}{\sqrt{2}}\sqrt{\frac{p_o}{T}}\ r\right) - \tan\tfrac{1}{2}\theta_T\ \operatorname{erf}\left(\frac{1}{\sqrt{2}}\sqrt{\frac{p_o}{T}}\ r\right) \quad \ldots\ldots \tag{21}$$

where erf x and erfi x are defined in equations (8) and (9).

The generator of the uncorrected umbrella parachute, described by equation (21), is shown in Fig. 4.

### 3.2.2. Corrected parachute

Substituting the Newton-Busemann pressure law in (19) yields

$$r \sin\theta \frac{dr}{d\theta} + \int_{\theta_L}^{\theta} r' \cos\theta' \frac{dr'}{d\theta'} d\theta' = -\frac{T}{p_o} \tag{22}$$

The method for reducing this has already been given in section 3.1.2. Thus we find

$$r \frac{dr}{d\theta} = -\frac{T}{p_o} \frac{\sin\theta_L}{\sin^2\theta} \tag{23}$$

so that

$$\frac{1}{2} r^2 = \frac{T}{p_o} \sin\theta_L (\cot\theta - \cot\theta_T) \tag{24}$$

$$\sqrt{\frac{p_o}{T}}\, r = (2\sin\theta_L)^{\frac{1}{2}} (\cot\theta - \cot\theta_T)^{\frac{1}{2}} \tag{25}$$

Substituting from (23) for dr in $dz = dr \cot\theta$ and using (25) gives finally for the z co-ordinate

$$\sqrt{\frac{p_o}{T}}\, z = \frac{1}{3}\sqrt{\frac{p_o}{T}}\, r (\cot\theta + 2\cot\theta_T) \tag{26}$$

Using (24) allows us to eliminate $\cot\theta$ and find

$$\sqrt{\frac{p_o}{T}}\, z = \frac{1}{3}\sqrt{\frac{p_o}{T}}\, r \left( \frac{1}{2} \frac{p_o}{T} \frac{r^2}{\sin\theta_L} + 3\cot\theta_T \right) \tag{27}$$

The shape of the generator of the corrected umbrella parachute is the cubic shown in Fig. 4.

### 4. Canopy cloth area

The cloth area A of the canopy is

$$A = \int_{\theta_L}^{\theta_T} 2\pi r \, ds = 2\pi T \int_{\theta_L}^{\theta_T} \frac{d\theta}{p}, \text{ using equation (1).} \tag{28}$$

Clearly, since pressure p depends on the local slope, the cloth area will depend only on the initial and final angles $\theta_L$ and $\theta_T$, and not on whether the parachute is drogue or umbrella-shaped.

### 4.1. Uncorrected parachute

$$p = p_o \sin^2\theta$$

$$\frac{p_o A}{2\pi T} = \int_{\theta_L}^{\theta_T} \frac{d\theta}{\sin^2\theta} = \cot\theta_L - \cot\theta_T . \tag{29}$$

4.2. Corrected parachute

With equation (14) or (23) and the appropriate expression for ds it is easy to show that for both the drogue and umbrella parachutes

$$r\,ds = \frac{T\sin\theta_L}{p_o}\ \frac{d\theta}{\sin^3\theta} \tag{30}$$

so that

$$\frac{p_o A}{2\pi T} = \sin\theta_L \int_{\theta_L}^{\theta_T} \frac{d\theta}{\sin^3\theta}$$

$$= \frac{1}{2}\left[\cot\theta_L - \cot\theta_T\,\frac{\sin\theta_L}{\sin\theta_T} + \sin\theta_L\ \ln\left(\frac{\cot\frac{1}{2}\theta_L}{\cot\frac{1}{2}\theta_T}\right)\right] \tag{31}$$

The non-dimensional cloth areas, $p_o A/T$, given by (29) and (31), are compared in Fig. 5, for $\theta_T = 90^o$.

5. Canopy drag

The longitudinal tension per unit radian is constant so that the drag follows most easily from resolving the attaching forces. The same result can be found by integrating the pressures on the canopy. Thus the drag D of the canopy is

$$D = 2\pi T(\cos\theta_L - \cos\theta_T) \tag{32}$$

The corresponding canopy drag coefficient, which refers only to the wave drag for the frictional drag is not evaluated here, is defined by

$$C_D = \frac{D}{q_\infty A}$$

$$= \frac{p_o}{q_\infty}\ \frac{(\cos\theta_L - \cos\theta_T)}{(p_o A/2\pi T)} \tag{33}$$

The drag coefficients of the uncorrected and corrected parachutes are compared in Fig. 5, for $\theta_T = 90^o$.

6. Results

The calculated shape of the generator of the drogue canopy is shown in Fig. 3 for $\theta_L = 10^o$. The drogue canopy is produced by revolving this about the z axis, and can be seen to be practically conical along most of its length. Clearly the corrected parachute requires a smaller area of cloth than the uncorrected one if both work at the same tension. This is confirmed in Fig. 5 which shows that this difference in cloth area rapidly falls as $\theta_L$ becomes large.

Fig. 4 shows the shape of the generator of the umbrella canopy, and compares the corrected and uncorrected parachutes for $\theta_L = 4^o$. Again the large difference in cloth area is clear. The effect of changing the leading edge angle from $\theta_L = 4^o$ to $\theta_L = 10^o$ can be seen by comparing the two corrected canopies.

Fig. 5 yields the expected result that, for a given cloth area, the drag increases as the surface becomes more normal and flat to the stream. However the figure also shows that at the same time the tension becomes very large, and, of course, in the limit as $\theta_L \to 90^o$, $T \to \infty$. The variation in T is got most simply from the result that, when $\theta_T = 90^o$, the canopy drag equals $2\pi$ T cos $\theta_L$.

To achieve a given drag from the corrected and uncorrected canopies, working at the same tension, we must make $\theta_L$ the same for both. In this case the cloth area required for the corrected canopy is much smaller than for the uncorrected canopy. See Fig. 5.

For a given drag from a given area of cloth, $C_D$ is fixed. Here the advantage of the corrected parachute is that the tension in the cloth is much smaller, though the difference is less marked when $q_\infty C_D/p_o$ is less than 0.3 or near 1.0. Again see Fig. 5.

To give some estimate of the size of the quantities involved we consider a canopy which offers a drag of 20,000 lb. We must keep $\theta_L$ small to keep T small. It is convenient to choose $\theta_L = 9^o$ where, see Fig. 5, $q_\infty C_D/p_o = 0.3$ and $p_o A/T = 20$ for the corrected parachute. Now for $M_\infty = \infty$, $p_o/q_\infty = 1.839$ and it is close to this limiting value for $M_\infty > 4$. Using these values we find that for flight at $M_\infty = 10$ at 100,000 ft. the cloth area required to produce a 20,000 lb. drag is approximately A = 20 sq. ft. Thus $\sqrt{p_o/T} = 1$ and we can read the size of the canopy directly from Figs. 3 and 4, using the values for $\theta_L = 10^o$. Both the drogue and umbrella parachutes have a maximum radius of 1.4 ft. The axial length of the drogue parachute is 5.3 ft., and that of the umbrella parachute 2.7 ft. The tension around the maximum circumference would be of the order of 2,000 lb/ft.

## 7. Conclusions

We have compared the uncorrected parachute, derived from the Modified-Newtonian pressure law, with the corrected parachute, derived from the Newton-Busemann pressure law, which corrects for the centrifugal pressure drop across the shock layer. The corrected parachute offers a better performance than the uncorrected one. If both are working at the same tension the corrected parachute requires less cloth for a given drag, particularly at small leading edge angles $\theta_L$. $\theta_L$ will be small so that the tension in the cloth is kept as small as possible for a given drag.

Two canopy shapes, for drogue and umbrella-shaped parachutes, have been derived. A particular example has been included to give an estimate of the size of canopy required.

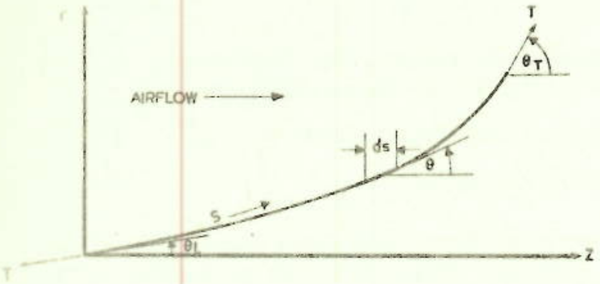


FIG. 1. COORDINATE SYSTEM.

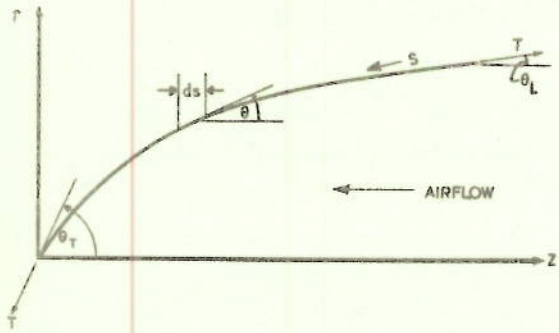


FIG. 2. COORDINATE SYSTEM.

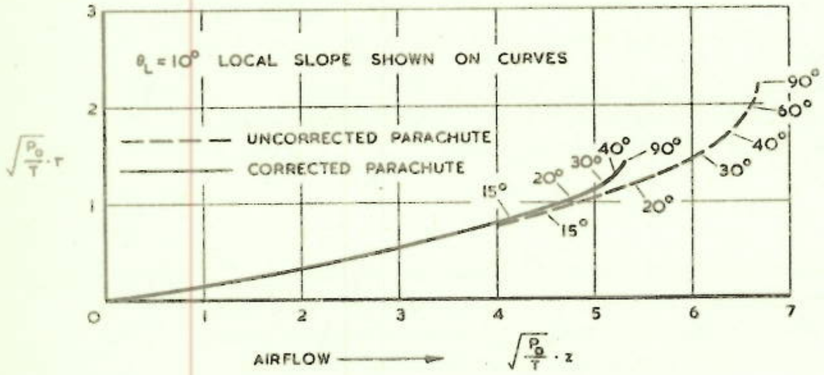


FIG. 3. DROGUE-CANOPY GENERATOR

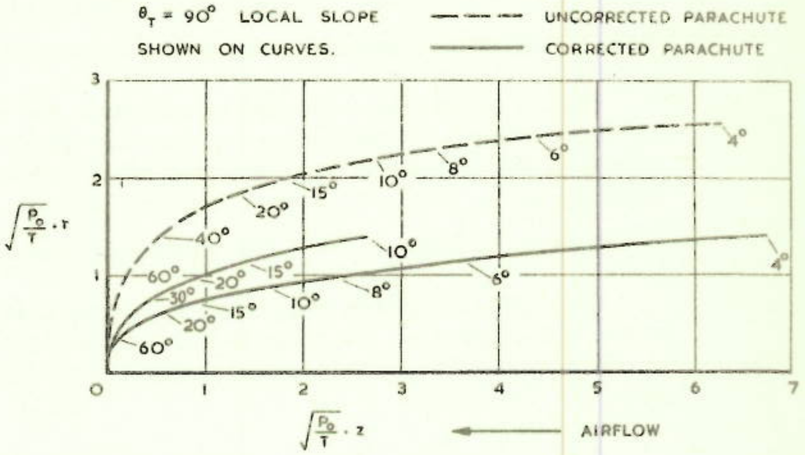


FIG. 4. UMBRELLA-CANOPY GENERATOR

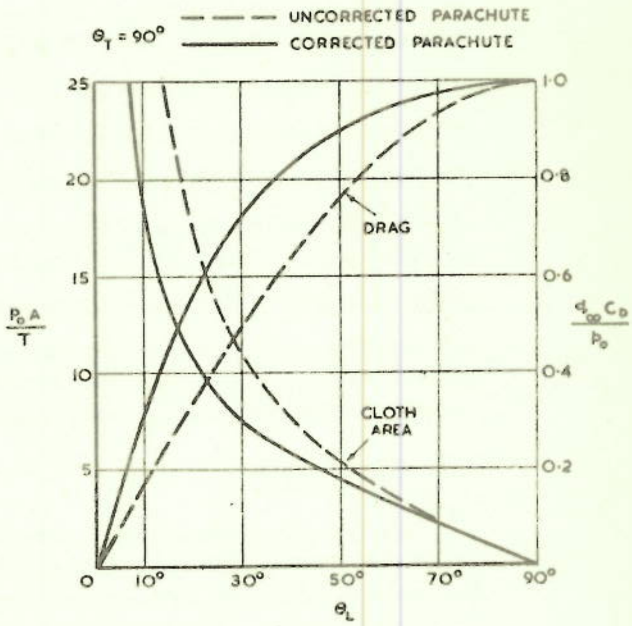


FIG. 5. CLOTH AREA AND DRAG OF CANOPY