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The Analysis of Reinforced Circular and Elliptical Cutouts under Various Loading Conditions

- by -

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SUMMARY

The effect of reinforced cutouts in a plane sheet under various loading conditions is considered, and a number of experimental results are given for circular and elliptical cutouts with a uniform plate reinforcement, subjected to various systems of biaxial tension and pure shear.

These experiments were conducted using a plane loading frame, and the results are compared with the theoretical plane stress solution. For the circular cutout the effect of neglecting the bending stiffness of the reinforcement is considered.

Some additional experiments were carried out on a 60 in. diameter pressurised cylinder containing an elliptical hole reinforced according to Mansfield's neutral hole theory. The strains in the sheet in the region of the neutral hole are compared with the corresponding strains in the uncut sheet.

The experimental results obtained generally show a good agreement with the theory.

CONTENTS

NOTATION

ϕ	Airy stress function
r, θ	polar co-ordinates
σ_r	radial stress
σ_θ	tangential stress
$\tau_{r\theta}$	shear stress
ϵ_r	radial strain
ϵ_θ	tangential strain
a	inner radius of reinforcement for circular cutout
b	outer radius of reinforcement for circular cutout
m	$= a/b$
t	thickness of sheet
n	ratio of reinforcement thickness to sheet thickness
f_t	tensile stress at infinity
f_s	shear stress at infinity
$A_1, A_2, C_2, c_1, d_1, B_1, F_1$	constants in solution for reinforced circular cutout
x, y	rectangular co-ordinates
p, q, c	arbitrary constants
f	tensile stress
A	cross-sectional area of reinforcement
T	tension in reinforcement
R	$= \frac{a_o + b_o}{2}$ for elliptical cutout, or radius of circular cutout
a_o	semi-major axis of ellipse
b_o	semi-minor axis of ellipse

Notation (Continued)

Q	critical stress combination
β	angle in plane of ellipse
ν	Poisson's ratio
λ	stress concentration factor

1. Introduction

An exact solution for a circular cutout having a plate reinforcement of uniform cross-section in an infinite plane sheet has been given by Gurney[1] who made use of the real stress function. For other shapes of cutout the method of conformal transformation may be employed, by making use of the complex stress function, developed by Muskhelishvili[6]. In a previous paper[4] this method has been applied to the solution of unreinforced cutouts of various shapes, and a number of experimental results have been obtained which show good agreement with the theory.

Wittrick[2,7] has extended the method to the solution of reinforced elliptical cutouts and cutouts of other shapes for which the transformation function can be expressed as a simple polynomial function. The assumption is made that the cutout has a compact reinforcing member concentrated at its edge, and the bending stiffness of the reinforcement is neglected.

Mansfield[3] has extended the use of the real stress function to investigate the problem of the neutral hole, that is, a hole of such a shape and distribution of reinforcement that the stresses in the sheet are unaffected by the presence of the hole. Again the assumption is made that the reinforcement has negligible bending stiffness.

In nearly all of the theoretical work, it is assumed that the problem can be reduced to that of a cutout in an infinite flat plate. When the cutout is in a flat panel it may be assumed that the effect of the boundary of the panel is negligible provided that the ratio of cutout dimension to panel dimension is not greater than $\frac{1}{4}$. The plane stress solution may be applied to problems of cutouts in curved shells provided that the dimensions of the cutout are small compared with the radius of curvature of the shell. The results may therefore be applied for example to cutouts for windows or doors in an aircraft pressure cabin, or to problems of cutouts in nuclear reactors.

However, there is little experimental evidence recorded to support the theoretical solutions for reinforced cutouts, although both Manfield[3] and Richards[8] have carried out some experimental work using plastic material.

In this paper, some experimental results[5] are given for circular and elliptical cutouts having a symmetrical plate reinforcement of uniform cross-section. The tests were carried out in a plane loading frame, using aluminium alloy panels, which enabled various systems of biaxial tension and pure shear to be applied. The experimental results are compared with the infinite flat plate theory, and for the reinforced circular cutout a comparison is made with both the theoretical solutions of Gurney and Wittrick in order to investigate the effect of the bending stiffness of the reinforcement on the stresses in the sheet.

Some additional tests were carried out on a 60 in. diameter pressurised cylinder containing an elliptical cutout which was reinforced in accordance with Mansfield's netral hole theory. The strains in the sheet in the region of the neutral hole are compared with the strains in the uncut sheet.

2. Experimental Work

Some preliminary experimental work was carried out on reinforced circular and elliptical cutouts in a pressurised cylinder. This had previously been used for tests on unreinforced cutouts, and is identical to that described in Ref. 4.

The cylinder was 60 in. diameter and 72 in. long, constructed from 18 s.w.g. (0.048 in.) L72 aluminium alloy sheet. The cylinder was pressurised using air to give a nominal hoop stress of the order of 10,000 lb/in^2, and was mounted on a trolley at one end to permit free longitudinal expansion under pressure to take place. The air pressure was contained by a plate, rolled to the contour of the cylinder, and shaped to fit accurately into the cutout. This plate was supported externally so that the transverse pressure loading would not be reacted at the boundary of the cutout.

Flat cast iron bulkheads were used for the ends of the cylinder, and these were designed to withstand the pressure load and in addition to act as suitable loading points for combined pressure/torsion and pressure/axial tension tests which it had been originally intended to carry out.

A number of tests were carried out on reinforced circular and elliptical cutouts in the pressure cylinder, having maximum dimensions not greater than about 8 in. However, because of the considerable experimental difficulties arising, it was decided to use the preliminary results obtained on the pressure cylinder to confirm the use of flat plate theory, and to carry out the subsequent work (with the exception of the neutral hole tests) using a plane loading frame (Figs. 11 and 12). This enabled the use of stress ratios other than the 2:1 biaxial pressure stresses, and in addition the effect of combined shear and direct loading could be examined if required.

The plane loading frame had also previously been used for tests on unreinforced cutouts, and is fully described in Ref. 4. The panels used in the plane loading frame were 28 in. square, and made from 16 s.w.g. (0.064 in.) L72 aluminium alloy sheet. It was found that provided the maximum dimension of the cutout in the panel did not exceed about 5 in. the effect of the panel boundary would be negligible, and in this way reasonable comparison should be obtained with the infinite flat plate theory.

The panel was loaded through heavy edge members by means of two 3000 lb. turnbuckles, used in conjunction with calibrated dynamometers. The maximum direct stress in the panel was 1000 lb/in^2, but the shear stress was restricted by panel buckling to 700 lb/in^2.

Tests were carried out, using the plane loading frame, on a circular cutout and a $\sqrt{2:1}$ elliptical cutout, having a constant section reinforcement. In the case of the circular cutout, the reinforcement was chosen so that the weight of the reinforcement added was equal to the weight of the sheet removed by the cutout. For the elliptical cutout, the cross-sectional area of reinforcement added corresponded to the minimum area of reinforcement (i.e. at the minor axis) for a $\sqrt{2:1}$ elliptical neutral hole of the same dimensions.

The cutouts were reinforced by doubler plates of uniform width symmetrically placed each side of the sheet, and attached to the sheet by Araldite 901 cold setting adhesive. Under applied loading this proved to be an adequate joint.

The tangential strains in the sheet were measured around the cutout at the junction of the reinforcement, under various systems of biaxial tension and pure shear. The experimental results are compared with the theoretical plane stress solutions in Figs. 1 - 4. In addition the tangential strains in the sheet were measured along radial lines at certain points around the cutout, to show the reduction in tangential strain away from the junction of the reinforcement.

Tinsley type 6H electrical resistance strain gauges were used in conjunction with a Savage and Parsons recorder. To obtain strain measurements as close as possible to the junction of the reinforcement, the backing paper of the strain gauge was trimmed, and it was found that measurements could effectively be made at a distance of 0.15 in. from the junction. The dimensions of the circular and elliptical cutouts and the strain gauge positions are shown in Figs. 13 and 14 respectively.

Some further experimental work was carried out on a neutral hole, using the 60 in. diameter cylinder which was pressurised by air to a nominal stress of 10,000 lb/in^2. A $\sqrt{2}$:1 elliptical cutout was made in the cylinder, and this was reinforced by plates placed symmetrically each side of the sheet, and attached to the sheet by 6 B.A. bolts placed at 1.2 in. pitch. The width of the reinforcing plate was varied so that the distribution of reinforcement around the cutout was in accordance with Mansfield's neutral hole theory.

Measurements of radial and tangential strains were made on radial lines at certain points around the cutout, and these strains were compared with the strains previously measured at corresponding points on the sheet before the cutout was made, and compared also with the theoretical strains. The dimensions of the neutral hole and the strain gauge positions are shown in Fig. 15. Tinsley type 6H strain gauges were again used, with the Savage and Parsons recorder.

3. Theoretical Considerations

3.1. Reinforced Circular Cutouts (Fig. 16)

The solution to the problem of a circular cutout with a uniform plate reinforcement in an infinite plane sheet under various loading conditions has been given by Gurney[1]. It is assumed that the reinforcement is symmetrically placed each side of the sheet, and the effect of the abrupt change in thickness at the junction of the reinforcement and sheet has been neglected. Apart from this, if it is assumed that a linear stress-strain law exists for the material, then the solution given is exact.

Gurney uses the real stress function ϕ of the plane theory of elasticity satisfying, in the absence of body forces, the biharmonic equation

$$\nabla^4 \phi = 0 .$$

A general form of solution is obtained, and this is used to satisfy the boundary conditions at the inner and outer edges of the reinforcement and at infinity.

The solution gives the stress distribution in the sheet and in the reinforcement when the cutout is subjected to either equal biaxial tensions or pure shear, and by super-position of these two cases any system of biaxial tension and shear may be obtained.

Gurney has assumed a value for Poisson's ratio $\nu = \frac{1}{4}$ and uses this value in the numerical work. The solution may be obtained in a more general form for any value of ν, as follows:

(a) Solution for equal biaxial tension f_t.

The stresses in the plate are given by

$$\sigma_r = f_t \left(\frac{A_1}{r^2} + 1 \right) ,$$

$$\sigma_\theta = f_t \left(- \frac{A_1}{r^2} + 1 \right) ,$$

and

$$\tau_{r\theta} = 0 ,$$

where

$$A_1 = \frac{(1-\nu)(1-m^2)n - (1+\nu)m^2 - (1-\nu)}{(1+\nu)(1-m^2)n + (1+\nu)m^2 + (1-\nu)} \cdot b^2 ,$$

and a is the radius of the cutout (inner radius of reinforcement), b is the outer radius of the reinforcement, n is the ratio of total thickness of reinforcement to thickness of sheet, and m is defined by

$$m = \frac{a}{b} .$$

The stresses in the reinforcing ring are given by

$$\sigma_r = f_t \left(\frac{A_2}{r^2} + 2 C_2 \right) ,$$

$$\sigma_\theta = f_t \left(- \frac{A_2}{r^2} + 2 C_2 \right) ,$$

and

$$\tau_{r\theta} = 0 ,$$

where

$$A_2 = \frac{-2 m^2}{(1+\nu)(1-m^2)n + (1+\nu)m^2 + (1-\nu)} \cdot b^2 ,$$

and

$$C_2 = \frac{1}{(1+\nu)(1-m^2)n + (1+\nu)m^2 + (1-\nu)}$$

The values of the constants A_1, A_2 and C_2 have been calculated in Table 1 for Poisson's ratio $\nu = \frac{1}{3}$ for a range of values of m and n.

(b) Solution for pure shear f_s.

The stresses in the plate are given by

$$\sigma_r = f_s \left(1 - 4d_1 \cdot \frac{b^2}{r^2} - 6c_1 \cdot \frac{b^4}{r^4}\right) \sin 2\theta \quad ,$$

$$\sigma_\theta = f_s \left(-1 + 6c_1 \frac{b^4}{r^4}\right) \sin 2\theta \quad ,$$

$$\tau_{r\theta} = f_s \left(1 + 2d_1 \frac{b^2}{r^2} + 6c_1 \frac{b^4}{r^4}\right) \cos 2\theta \quad ,$$

and the stresses in the reinforcement are

$$\sigma_r = f_s \left[B_1 \left(-2 + 8m^2 \frac{b^2}{r^2} - 6m^4 \frac{b^4}{r^4}\right) + F_1 \left(12m^4 \frac{b^2}{r^2} - 12m^6 \frac{b^4}{r^4}\right)\right] \sin 2\theta \ ,$$

$$\sigma_\theta = f_s \left[B_1 \left(2 + 6m^4 \frac{b^4}{r^4}\right) + F_1 \left(\frac{12r^2}{b^2} + 12m^6 \frac{b^4}{r^4}\right) \right] \sin 2\theta \ ,$$

$$\tau_{r\theta} = f_s \left[B_1 \left(-2 - 4m^2 \frac{b^2}{r^2} + 6m^4 \frac{b^4}{r^4}\right) + F_1 \left(-6 \frac{r^2}{b^2} - 6m^4 \frac{b^2}{r^2} + 12m^6 \frac{b^4}{r^4}\right)\right] \cos 2\theta$$

The constants in the above equations may be obtained by the solution of the four simultaneous equations:

$$1 - 4d_1 - 6c_1 = B_1 n(-2 + 8m^2 - 6m^4) + F_1 n(12m^4 - 12m^6)$$

$$1 + 2d_1 + 6c_1 = B_1 n(-2 - 4m^2 + 6m^4) + F_1 n(-6 - 6m^4 + 12m^6)$$

$$(1 + \nu) + 4d_1 + 2(1 + \nu)c_1 = B_1 \left[-2(1 + \nu) - 8m^2 + 2(1 + \nu)m^4\right] + F_1 \left[-4\nu - 12m^4 + 4(1 + \nu)m^6\right]$$

$$(1 + \nu) + 2(1 - \nu)d_1 - 2(1 + \nu)c_1 = B_1 \left[-2(1 + \nu) - 4(1 - \nu)m^2 - 2(1 + \nu)m^4\right] + F_1 \left[-(6 + 2\nu) - 6(1 - \nu)m^4 - 4(1 + \nu)m^6\right].$$

The values of the constants c_1, d_1, B_1 and F_1 have been calculated in Table 2 for Poisson's ratio $\nu = \frac{1}{3}$ for a range of values of m and n.

3.2. Neutral Hole Theory (Fig. 17)

Mansfield[3] has made use of the real stress function to investigate the problem of the neutral hole. It has been shown that in a plane sheet under any particular loading system, certain reinforced holes may be made which do not alter the stress distribution in the sheet. Such neutral holes must have exactly the same stiffness as the part of the sheet that has been removed.

For any particular loading system the shape of the neutral hole and the distribution of reinforcement are specified, and the hole will not then be neutral for any other system of loading.

The results may be applied to neutral holes in a curved sheet provided that the dimensions of the hole are small compared with the radius of curvature of the sheet.

Mansfield makes the assumption that the hole has a compact reinforcing member at its edge, so that the bending stiffness of the reinforcement is negligible compared with its tensile stiffness.

The state of stress in the sheet may be completely defined by the stress function ϕ, and it is shown that the conditions of equilibrium between the sheet and the reinforcement may be satisfied by putting

$$\phi = 0 .$$

This equation is then used to determine the shape of the neutral hole.

The area of reinforcement required is obtained by equating the strains in the sheet and in the reinforcement.

The shape of the neutral hole and distribution of reinforcement are given here for two particular systems of loading :

(a) Sheet subjected to equal biaxial tension.

Under this system of loading the stress function may be written in a general form

$$\phi = \frac{f}{2}(x^2 + y^2) + px + qy + c ,$$

where f is the tensile stress in the sheet, and p q and c are arbitrary constants.

Putting $\phi = 0$ gives the equation for the shape of the neutral hole.

The neutral hole is therefore a circle, having a uniform reinforcement of cross-sectional area A given by

$$A = \frac{Rt}{1 - \nu} ,$$

where R is the radius of the hole, and t is the thickness of the sheet.

The tension T in the reinforcement is

$$T = fRt .$$

(b) Sheet subjected to 2 : 1 biaxial tension.

The general form for the stress function may be written

$$\phi = \frac{f}{2}(x^2 + \frac{y^2}{2}) + px + qy + c ,$$

and the equation for the shape of the neutral hole is obtained by putting $\phi = 0$.

The neutral hole is therefore an ellipse with major and minor axes in the ratio $\sqrt{2} : 1$.

The distribution of reinforcement is given by

$$\frac{A}{b_o t} = \frac{\sqrt{2} \cdot \left(1 + \frac{x^2}{b_o^2}\right)^{3/2}}{1 - 2\nu + 3\frac{x^2}{b_o^2}} ,$$

where b_o is the semi-minor axis of the ellipse, and the tension in the reinforcement is

$$T = ft(x^2 + 4y^2)^{\frac{1}{2}} .$$

3.3. Reinforced Elliptical Cutouts

Wittrick[2] has used the complex stress function and the method of conformal transformation to obtain a solution for the stress distribution around a reinforced elliptical cutout in an infinite plane sheet. The assumptions made in the analysis are similar to those made by Mansfield for the neutral hole. The effect of the in-plane bending stiffness of the reinforcement has been neglected by considering the reinforcement as a compact member concentrated at the edge of the cutout. The solution gives the stress distribution in the sheet and the tension in the reinforcement for an elliptical cutout with any distribution of reinforcement under any system of biaxial tension and shear.

A circular cutout with a compact reinforcement may be regarded as a special case of an elliptical cutout and a comparison is made with Gurney's solution for a circular cutout with a uniform plate reinforcement in order to investigate the effect of neglecting the bending stiffness of the reinforcement.

The method of solution may be applied to a reinforced cutout of any shape for which the transformation function can be expressed as a simple polynomial function[7]. The solution may also be used to obtain the stress distribution around an elliptical neutral hole due to loadings other than that for which it is neutral.

For the case of an elliptical cutout with uniform reinforcement, the stresses around the boundary have been computed and are tabulated in Ref. 2 for three different shapes of ellipse (including the circle) under three basic systems of uniform stress at infinity, for a range of valuès of the parameter A/Rt. A value of Poisson's ratio $\nu = 1/3$ has been taken in the computation.

These values have been used in this paper to calculate the tangential strains around the circular and elliptical cutouts, for comparison with the experimental results.

To apply Wittrick's solution for a cutout with a compact reinforcing member to the problem of a plate reinforced cutout (where the reinforcement has finite width and its bending stiffness may not be negligible) it is necessary to make some assumptions concerning the geometry of the cutout. Here the boundary of the cutout is defined as the outer edge of the reinforcement (i.e. junction of sheet and reinforcement) since this is where the strains in the sheet are to be measured experimentally, and the area of reinforcement is taken to be the total cross sectional area of the reinforcing plates and the part of the sheet between them.

3.4. The Stress Concentration Factor λ

The stress concentration factor λ is based on the 'critical stress combination'[9] which is given by

$$Q = \sigma_r^2 + \sigma_\theta^2 - \sigma_r \sigma_\theta + 3\tau_{r\theta}^2 \quad ,$$

and the stress concentration factor is then given by

$$\lambda = \sqrt{\frac{Q}{[Q]_\infty}}$$

4. Discussion of Results

The tangential strains around the reinforced circular and elliptical cutouts, under various systems of biaxial tension and pure shear, have been measured using the plane loading frame and the experimental results are compared with the theoretical values in Figs. 1 - 4. In all the theoretical work a value of Poisson's ratio $\nu = \frac{1}{3}$ has been used, and to modify Gurney's solution for this value of ν the values of the constants in the solution have been re-calculated and are given in Tables 1 and 2. It is found however, that the change in the value of Poisson's ratio has only a very small effect on the stress distribution around the cutout.

For the circular cutout a comparison is made with both the theoretical solutions of Gurney and Wittrick, to investigate the effect of the in-plane bending stiffness of the reinforcement. It is found that in some cases there are large differences in the strains predicted by the two theories, particularly when the cutout is subjected to 3 : 1 biaxial tension and uniform shear. However for the symmetrical stress distribution (equal biaxial tensions) for which no bending deformation of the reinforcement is to be expected, the discrepancy is small, and it may be concluded that this difference for the unsymmetrical stress systems is largely attributable to the effect of the bending stiffness of the reinforcement. It should be noted however that the ratio of reinforcement width to cutout diameter is probably considerably greater for these small experimental cutouts than it would be in many practical applications, in which case the effect of the bending stiffness may be of less significance.

A fair agreement was obtained between the experimental and theoretical results for the circular cutout under biaxial tension. It was found that there was a tendency for the experimental results to follow more closely to Wittrick's solution, which neglects the bending stiffness of the reinforcement, than to Gurney's solution, and no satisfactory explanation can be given for this. The results for the circular cutout under uniform shear show the experimental strains to be somewhat greater than the strains predicted by either of the theoretical solutions. Again, no satisfactory explanation for this discrepancy can be given, but the same tendency had been noticed previously in similar tests on unreinforced cutouts[4].

The experimental results for the elliptical cutout under biaxial tension show good agreement with Wittrick's solution. The experimental values for the tangential strains are generally somewhat lower than the theoretical, and this is probably to be expected since the theoretical solution neglects the bending stiffness of the reinforcement, and also owing to the physical size of the strain gauge, it was necessary to measure the strain at a distance of 0.15 in. away from the edge of the reinforcement. The results for the elliptical cutout under uniform shear show the experimental strains to be higher than the theoretical values, in a similar manner to the results obtained for the circular cutout.

For the circular and elliptical cutouts subjected to biaxial tension, measurements of the tangential strain were also made on a radial line at the point of maximum strain, in order to show the reduction in strain away from the edge of the reinforcement, and these results are shown in Figs. 5 and 6. It can be seen from these graphs that the error introduced by measuring the strains around the cutout at 0.15 in. from the edge of the reinforcement is probably fairly small.

The preliminary experimental work carried out on reinforced cutouts in the pressurised cylinder showed no significant difference between the results for the cylinder and those obtained using the plane loading frame, and it was concluded that for the range of cutouts under examination any effects of shell curvature are small and that the use of the flatplate theory is justified.

A further series of tests was carried out on an elliptical neutral hole in the pressure cylinder, and these results are shown in Fig. 7. A comparison is made between the strains in the sheet around the neutral hole and the corresponding strains measured in the uncut sheet before the hole was made, and these strains are also compared with the theoretical strain in the sheet.

The results obtained show little change in the stress distribution in the sheet to have been caused by the presence of the hole, indicating that the hole is neutral, or nearly so. However, in some cases the experimental results show some scatter, and further experimental work would be necessary to achieve more conclusive results.

For a hole to be neutral under any particular loading system, the shape of the hole and the distribution of reinforcement are specified. Wittrick has shown that by keeping the same shape as the neutral hole but using a uniform reinforcement it is possible to achieve a stress concentration factor only slightly greater than unity with a weight of reinforcement considerably less than that of the neutral hole. The effect of variation of the area of reinforcement on the maximum stress concentration factor is shown in Figs. 8 and 9, for uniformly reinforced circular and elliptical cutouts subjected to 1 : 1 and 2 : 1 biaxial tensions respectively. These values have been calculated using Wittrick's solution, neglecting the bending stiffness of the reinforcement.

To investigate further the effect of the in-plane bending stiffness of the reinforcement, the maximum stress concentration factor has been plotted against the ratio a/b in Fig. 10 for the particular case of a circular cutout having a uniform plate reinforcement, subjected to 2 : 1 biaxial tension. The width of the reinforcement and its thickness have been varied so that the area of reinforcement is maintained constant. The part of the sheet between the reinforcing plates has been included in the total area of reinforcement. A comparison is made between the stress concentrations predicted by the theories of Gurney and Wittrick.

It is found that the stress concentration factor in the sheet obtained from Gurney's solution is lower than that given by Wittrick, neglecting the bending stiffness of the reinforcement, as would probably be expected. However, as the width of the reinforcement is increased, the stress concentration in the reinforcement increases rapidly.

It should be noted here however that since the area of reinforcement is constant, increase in its width implies a reduction in its thickness.

It would appear therefore that a compact reinforcement is the most efficient, and in this case the assumption that the reinforcement has negligible bending stiffness may be justified.

5. Conclusions

A number of experimental results are given for reinforced circular and elliptical cutouts under various loading conditions which generally show good agreement with the theoretical solutions.

It is concluded that for a cutout with a plate reinforcement the effect of the in-plane bending stiffness of the reinforcement may not be negligible, although in many practical cases the assumption may be justified.

The tests on the elliptical neutral hole in the pressure cylinder indicated that the presence of the hole caused little change in the stress distribution in the adjacent sheet.

6. References

1. Gurney, C. — An analysis of the stresses in a flat plate with a reinforced circular hole under edge forces. A.R.C. R & M. 1834, 1938.

2. Wittrick, W.H. — The stresses around reinforced elliptical holes in plane sheet. A.R.L. Report SM.267, 1959.

3. Mansfield, E.H. — Neutral holes in plane sheet. A.R.C. R & M. 2815, 1955.

4. Houghton, D.S., Rothwell, A. — The stresses around some unreinforced cutouts under various loading conditions. College of Aeronautics Report 146, 1961.

5. Arthurs, T.D., Hillel, A. — Unpublished thesis work conducted at the College of Aeronautics.

6. Muskhelishvili, N.I. — Some basic problems of the mathematical theory of elasticity. Noordhoff, Groningen, Holland, 1953.

7. Wittrick, W.H. — The analysis of stress concentrations at reinforced holes in infinite sheets. Aeronautical Quarterly, vol.11, August, 1960.

8. Richards, T.H. — Stress distribution in pressurised cabins: an experimental study by means of Xylonite models. A.R.C. 19,360, Strut.1999, 1957.

9. Timoshenko, S., Goodier, J.N. — Theory of elasticity. McGraw-Hill, New York, 1951.

TABLE 1.

Values of the constant A_1, A_2 and C_2 computed for Poisson's ratio $\nu = 1/3$

m = 0

n	0	1	3	5	10
A_1/b^2	- 1.0000	0	+ .2857	+ .3636	+ .4285
A_2/b^2	0	0	0	0	0
C_2	+ 1.5000	+ .5000	+ .2143	+ .1364	+ .0714

m = .5

n	0	1	3	5	10
A_1/b^2	- 1.0000	- .2500	+ .1250	+ .2500	+ .3636
A_2/b^2	- .5000	- .2500	- .1250	- .0833	- .0455
C_2	+ 1.0000	+ .5000	+ .2500	+ .1667	+ .0909

m = .6

n	0	1	3	5	10
A_1/b^2	- 1.0000	- .3600	+ .0360	+ .1823	+ .3223
A_2/b^2	- .6279	- .3600	- .1942	- .1330	- .0744
C_2	+ .8721	+ .5000	+ .2698	+ .1847	+ .1033

m = .7

n	0	1	3	5	10
A_1/b^2	- 1.0000	- .4900	- .0893	+ .0819	+ .2562
A_2/b^2	- .7424	- .4900	- .2920	- .2076	- .1207
C_2	+ .7576	+ .5000	+ .2976	+ .2119	+ .1232

m = .8

n	0	1	3	5	10
A_1/b^2	- 1.0000	- .6400	- .2703	- .0816	+ .1392
A_2/b^2	- .8421	- .6400	- .4324	- .3265	- .2025
C_2	+ .6579	+ .5000	+ .3378	+ .2501	+ .1582

m = .9

n	0	1	3	5	10
A_1/b^2	- 1.0000	- .8100	- .5452	- .3695	- .1121
A_2/b^2	- .9275	- .8100	- .6463	- .5376	- .3785
C_2	+ .5725	+ .5000	+ .3989	+ .3319	+ .2336

m = 1.0

n	0	1	3	5	10
A_1/b^2	- 1.0000	- 1.0000	- 1.0000	- 1.0000	- 1.0000
A_2/b^2	- 1.0000	- 1.0000	- 1.0000	- 1.0000	- 1.0000
C_2	+ .5000	+ .5000	+ .5000	+ .5000	+ .5000

TABLE 2

Values of the constants B_1, F_1, c_1, d_1 computed for Poisson's ratio $\nu = 1/3$

m = 0

n	0	1	3	5	10
B_1	- 1.5000	- .5000	- .2143	- .1364	- .0714
F_1	0	0	0	0	0
c_1	- .5000	0	+ .1429	+ .1818	+ .2143
d_1	+ 1.0000	0	- .2857	- .3636	- .4286

m = .5

n	0	1	3	5	10
B_1	- .8163	- .5000	- .3294	- .2564	- .1698
F_1	- .1633	0	+ .0471	+ .0513	+ .0437
c_1	- .5000	- .0313	+ .1485	+ .1971	+ .2323
d_1	+ 1.0000	+ .2500	- .0853	- .2019	- .3190

m = .6

n	0	1	3	5	10
B_1	- .6805	- .5000	- .3844	- .3266	- .2453
F_1	- .1745	0	+ .0655	+ .0771	+ .0736
c_1	- .5000	- .0648	+ .1288	+ .1865	+ .2312
d_1	+ 1.0000	+ .3600	+ .0374	- .0838	- .2173

m = .7

n	0	1	3	5	10
B_1	- .5738	- .5000	- .4420	- .4081	- .3504
F_1	- .1639	0	+ .0803	+ .1020	+ .1106
c_1	- .5000	- .1200	+ .0824	+ .1505	+ .2104
d_1	+ 1.0000	+ .4900	+ .1968	+ .0815	- .0519

m = .8

n	0	1	3	5	10
B_1	- .5049	- .5000	- .4906	- .4816	- .4606
F_1	- .1278	0	+ .0843	+ .1150	+ .1402
c_1	- .5000	- .2047	- .0043	+ .0733	+ .1500
d_1	+ 1.0000	+ .6400	+ .3884	+ .2844	+ .1675

m = .9

n	0	1	3	5	10
B_1	- .4816	- .5000	- .5000	- .5258	- .5335
F_1	- .0687	0	+ .0681	+ .1018	+ .1397
c_1	- .5000	- .3280	- .1573	- .0725	+ .0260
d_1	+1.0000	+ .8099	+ .6204	+ .5252	+ .4135

m = 1.0

n	0	1	3	5	10
B_1	- .5000	- .5000	- .5000	- .5000	- .5000
F_1	0	0	0	0	0
c_1	- .5000	- .5000	- .5000	- .5000	- .5000
d_1	+1.0000	+1.0000	+1.0000	+1.0000	+1.0000

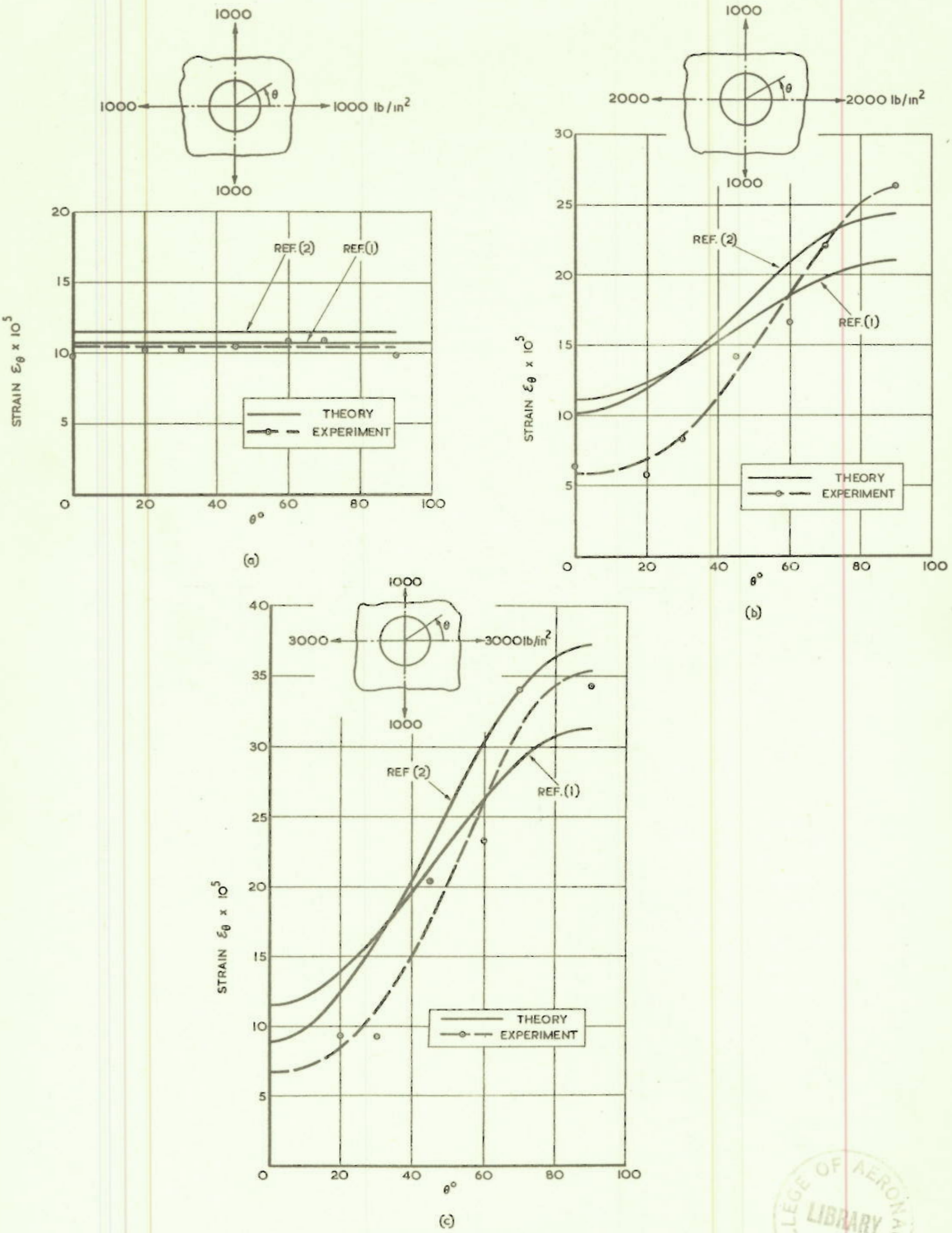


FIG. 1. TANGENTIAL STRAIN AROUND REINFORCED CIRCULAR CUTOUT UNDER BIAXIAL TENSION. A/Rt = 0.55.

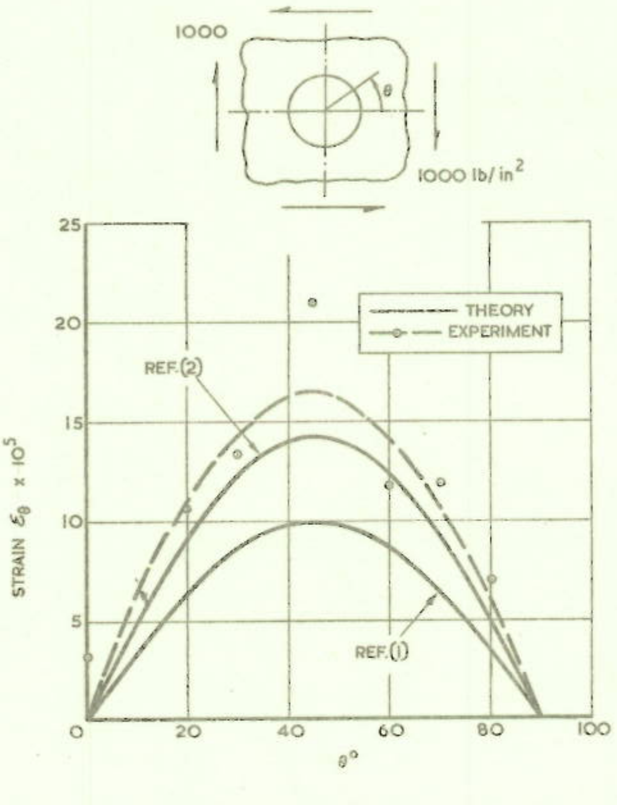


FIG. 2. TANGENTIAL STRAIN AROUND REINFORCED CIRCULAR CUTOUT UNDER UNIFORM SHEAR. A/Rt = 0.55.

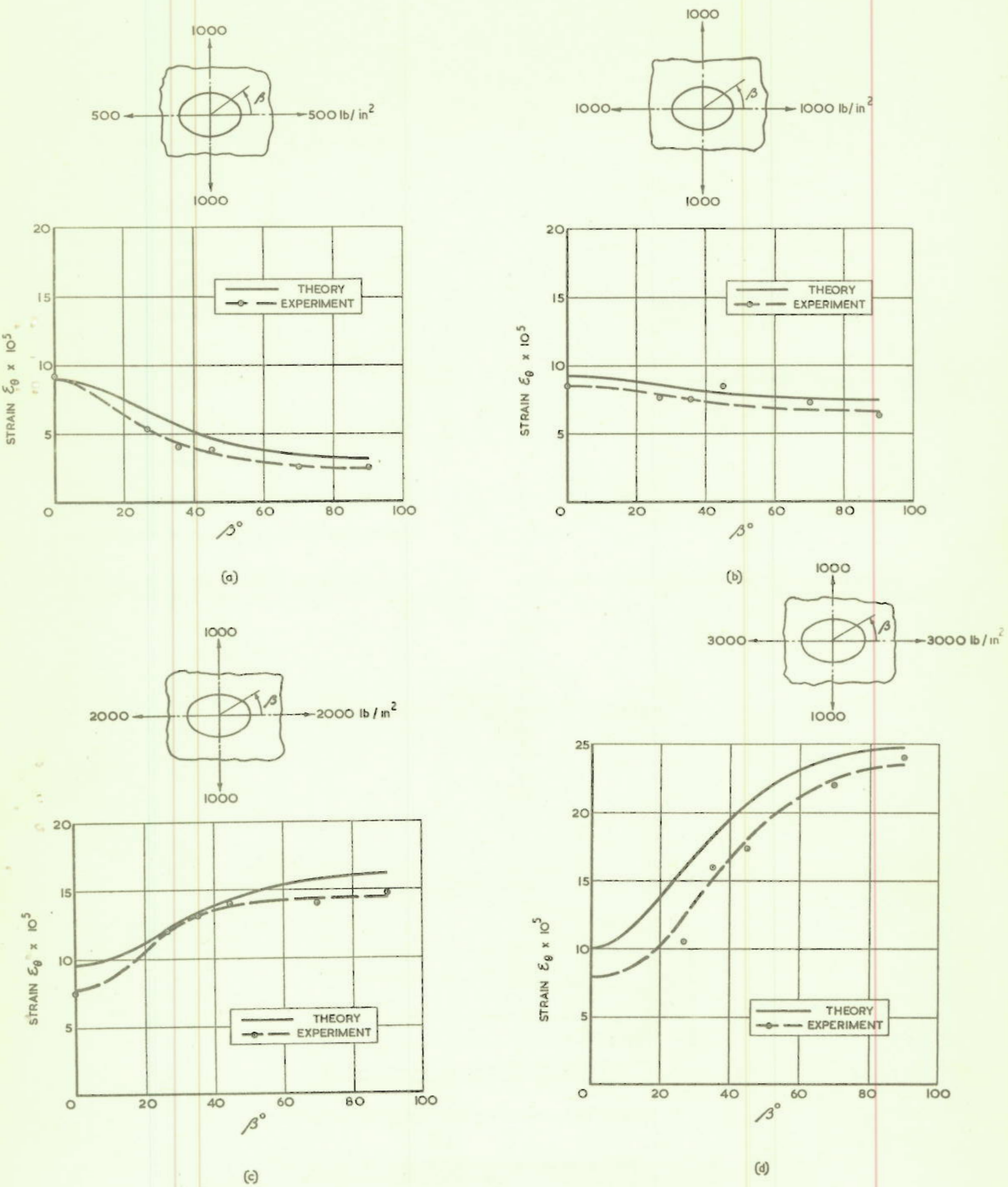


FIG. 3. TANGENTIAL STRAIN AROUND REINFORCED ELLIPTICAL CUTOUT UNDER BIAXIAL TENSION. A/Rt = 1.00.

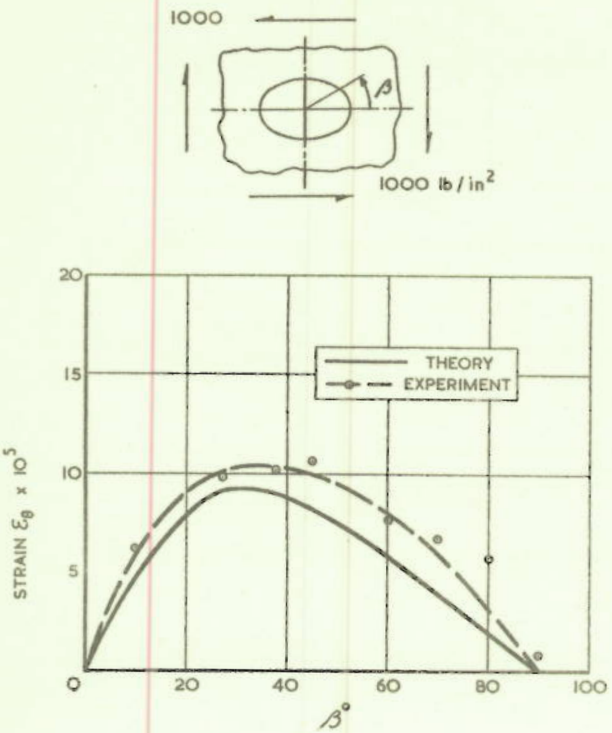


FIG. 4. TANGENTIAL STRAIN AROUND REINFORCED ELLIPTICAL CUTOUT UNDER UNIFORM SHEAR. A/Rt = 1.00.

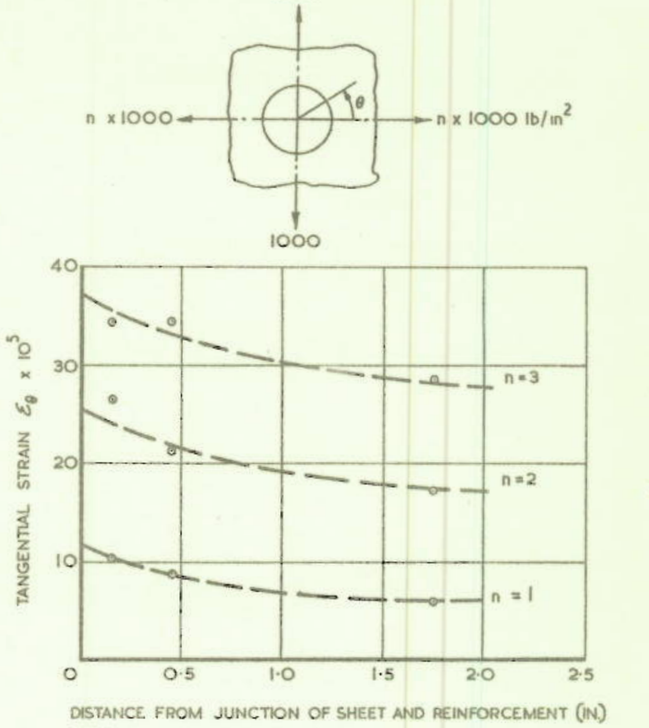


FIG. 5. EXPERIMENTAL TANGENTIAL STRAIN AT $\theta = 90^\circ$ FOR REINFORCED CIRCULAR CUTOUT UNDER BIAXIAL TENSION. A/Rt = 0.55.

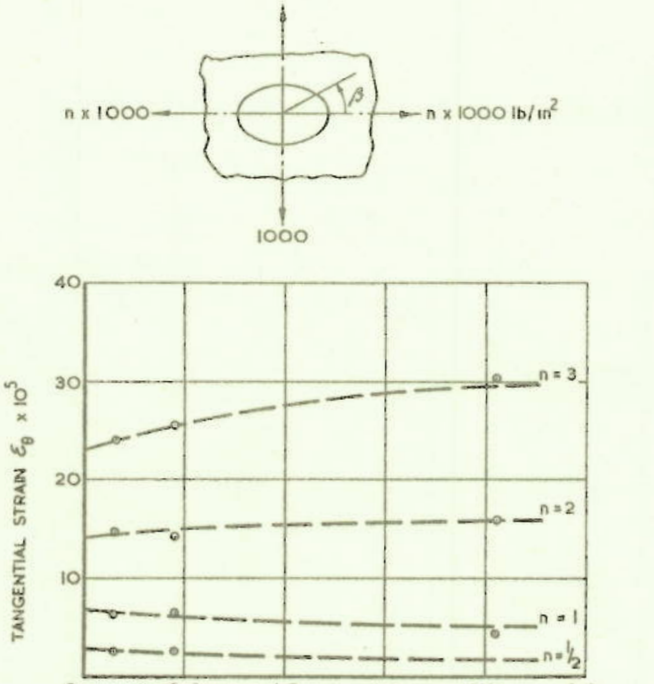


FIG. 6. EXPERIMENTAL TANGENTIAL STRAIN AT $\beta = 90^\circ$ FOR REINFORCED ELLIPTICAL CUTOUT UNDER BAXIAL TENSION. A/Rt = 1.00.

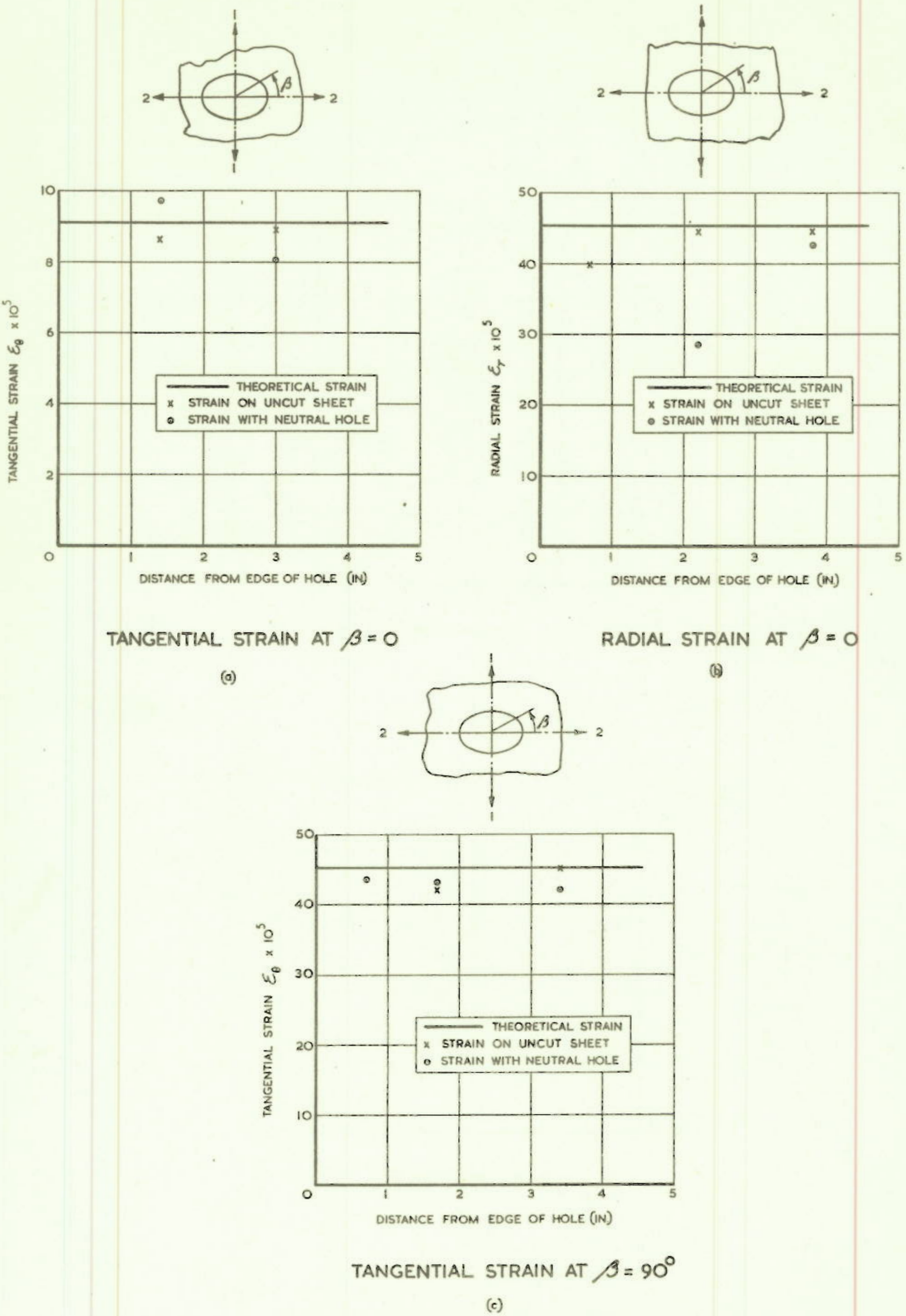
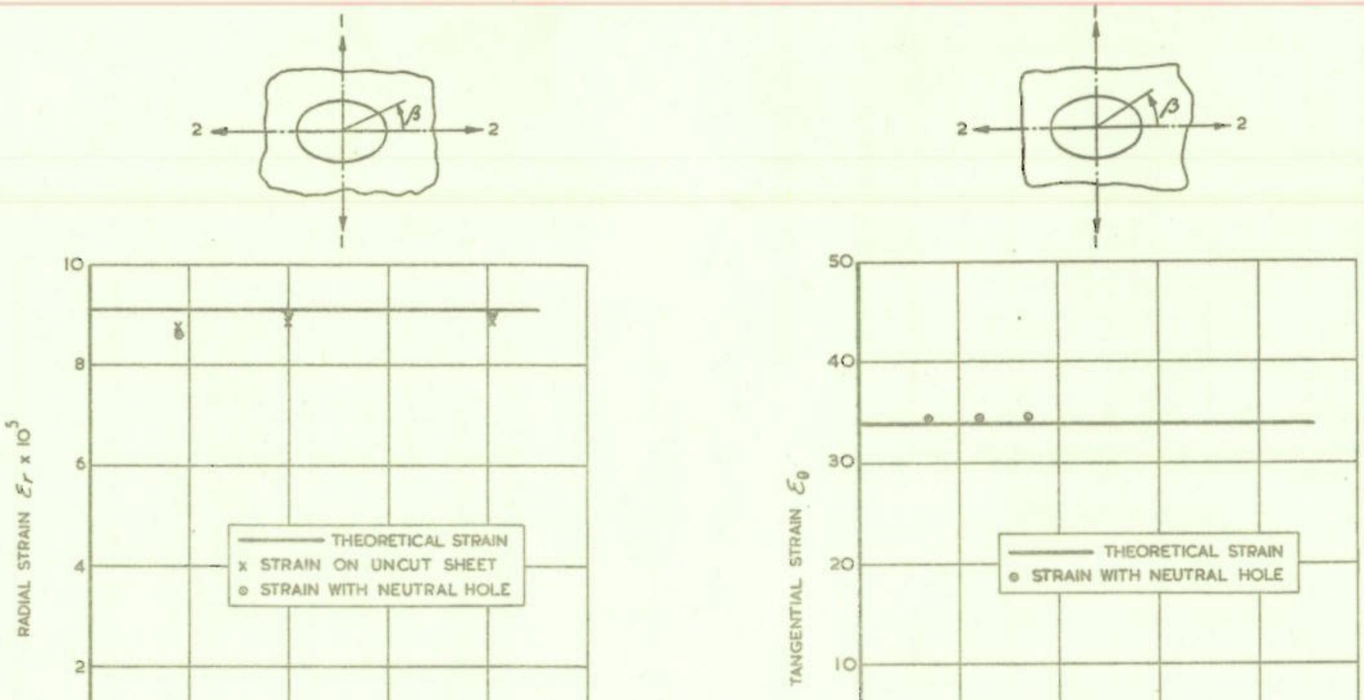


FIG. 7. TESTS ON $\sqrt{2}$:1 ELLIPTICAL NEUTRAL HOLE: DISTRIBUTION OF TANGENTIAL AND RADIAL STRAINS.



2
2
β
10
8
6
4
2
RADIAL STRAIN $\mathcal{E}_r \times 10^3$
THEORETICAL STRAIN
x STRAIN ON UNCUT SHEET
o STRAIN WITH NEUTRAL HOLE
50
40
30
20
10
TANGENTIAL STRAIN $\mathcal{E}_\theta$
THEORETICAL STRAIN
o STRAIN WITH NEUTRAL HOLE

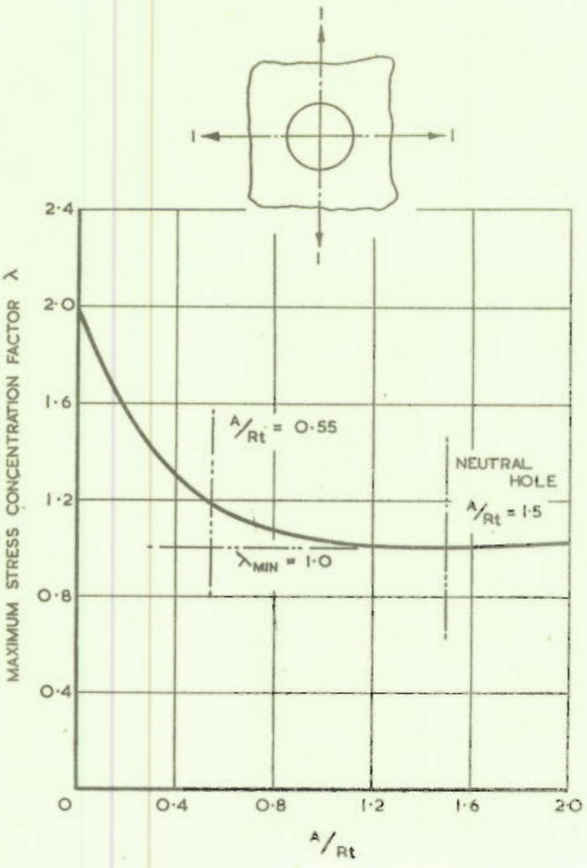


FIG. 8. THEORETICAL STRESS CONCENTRATION FACTOR FOR REINFORCED CIRCULAR CUTOUT UNDER EQUAL BIAXIAL TENSION.

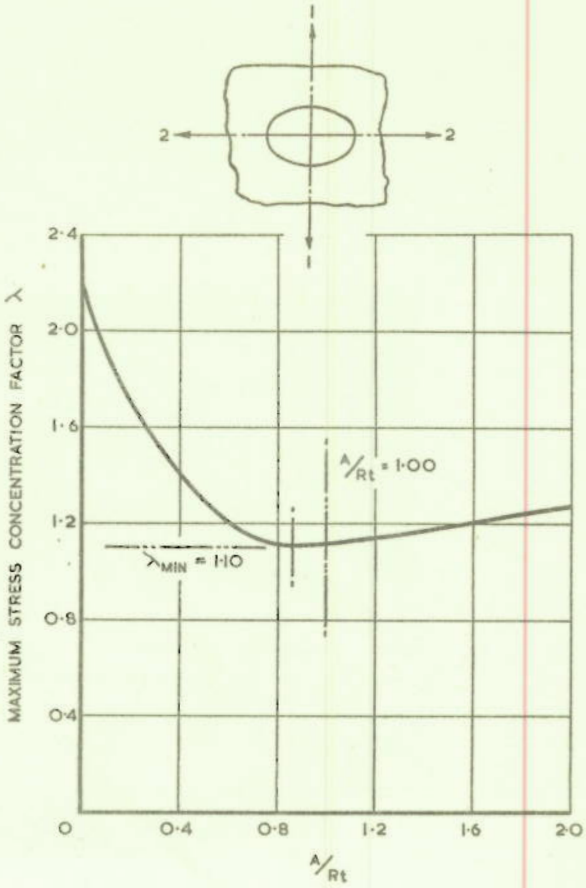


FIG. 9. THEORETICAL STRESS CONCENTRATION FACTOR FOR REINFORCED ELLIPTICAL CUTOUT UNDER 2:1 BIAXIAL TENSION.

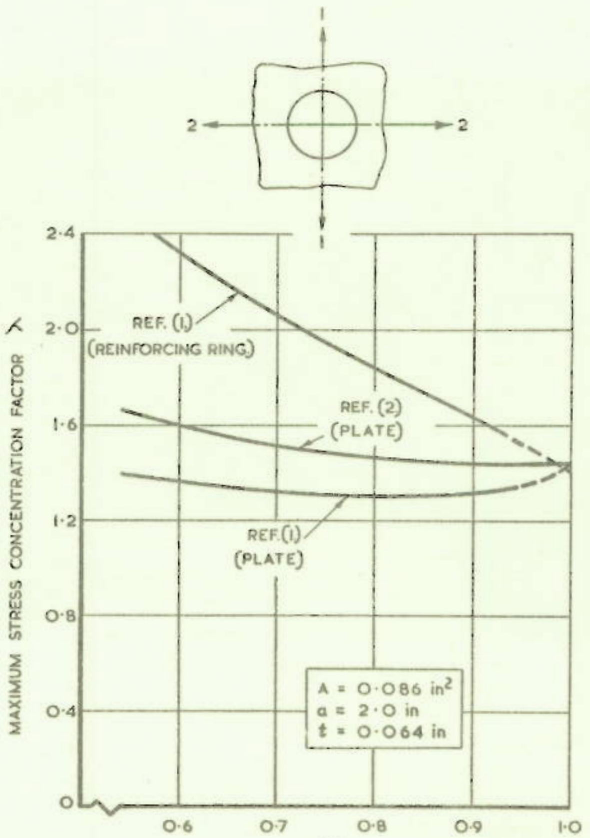
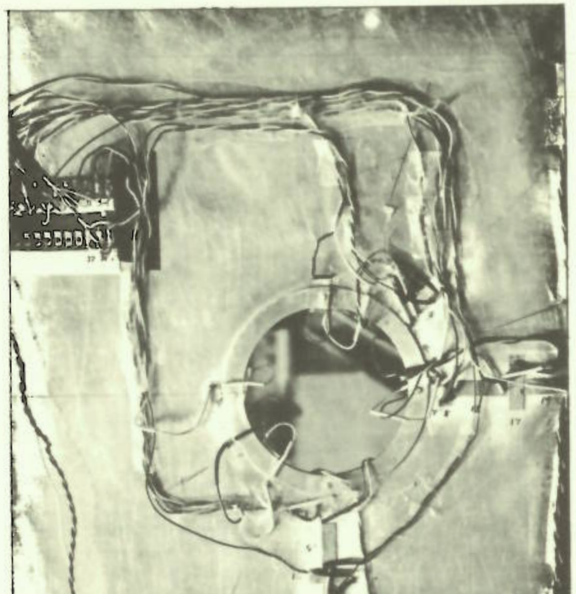
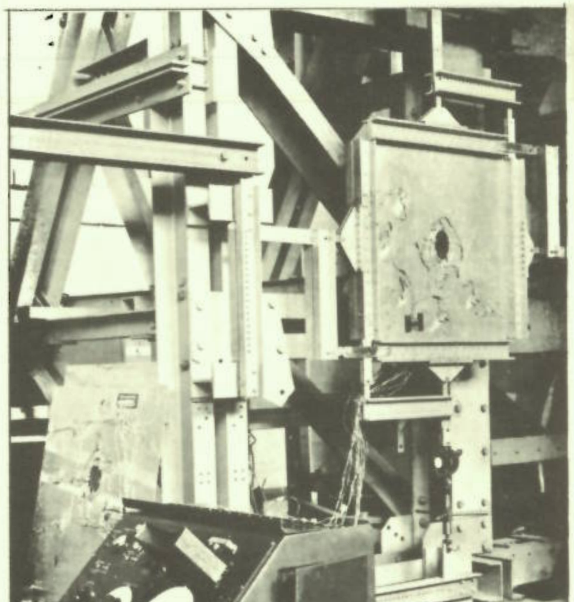
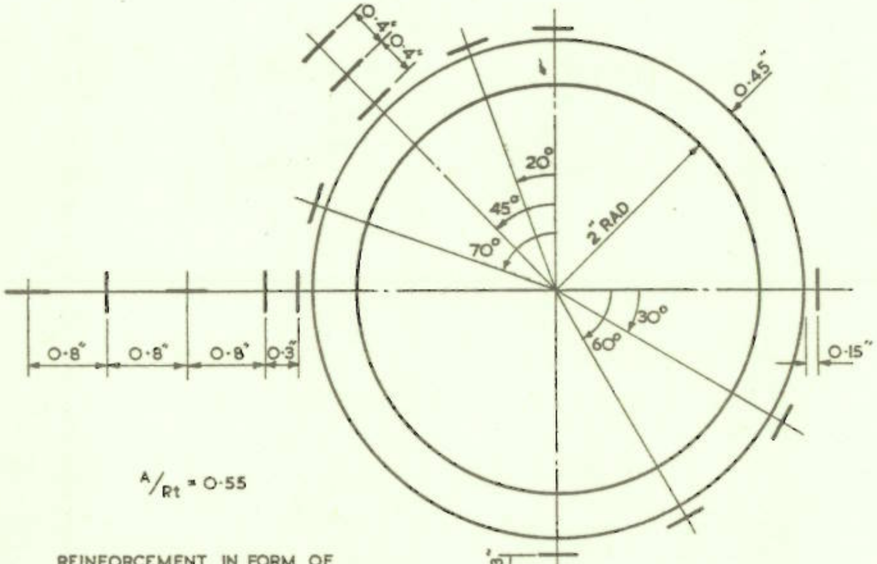


FIG. 10. EFFECT OF WIDTH OF REINFORCEMENT ON MAXIMUM STRESS CONCENTRATION FACTOR FOR CIRCULAR CUTOUT UNDER 2:1 BIAXIAL TENSION.

FIG. 12. GENERAL VIEW OF PLANE LOADING FRAME.

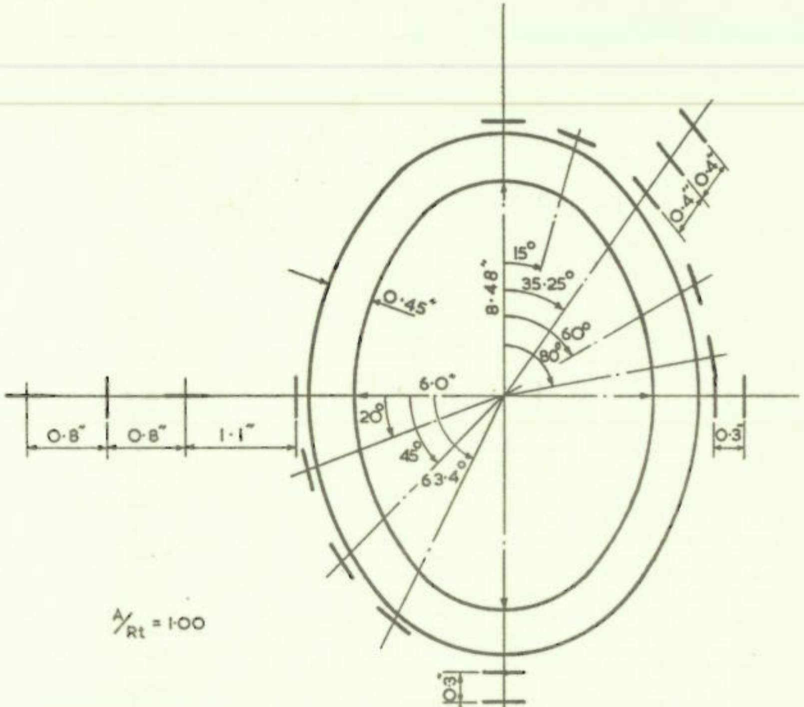






$^A/_{Rt}$ = 0·55

REINFORCEMENT IN FORM OF



$^A/_{Rt}$ = 1·00

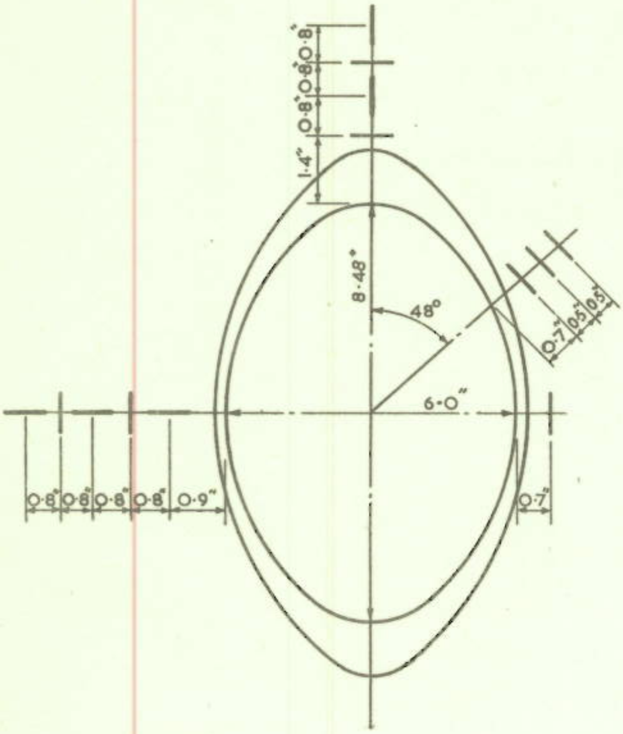


FIG. 15. DIMENSIONS AND STRAIN GAUGE POSITIONS FOR $\sqrt{2}$:1 ELLIPTICAL NEUTRAL HOLE.

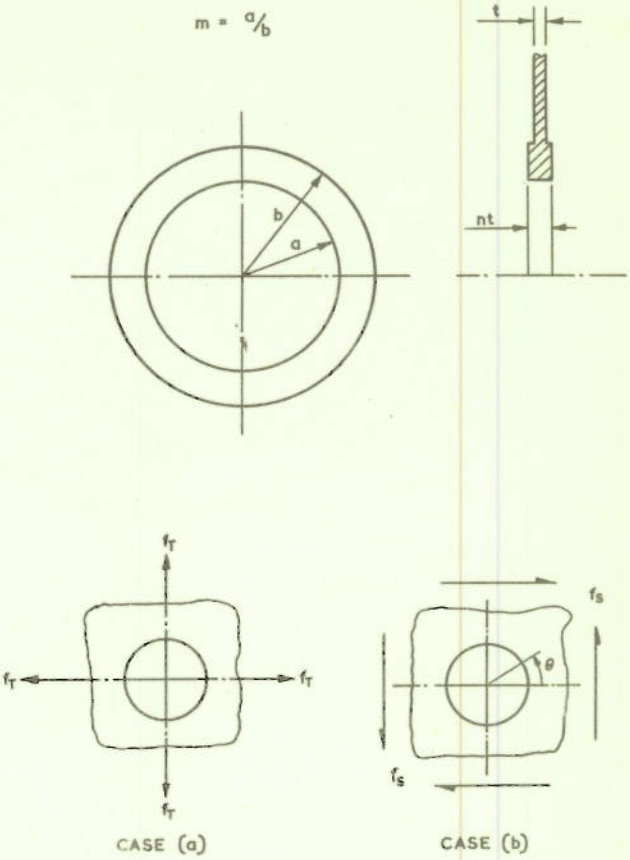


FIG. 16. NOTATION USED FOR REINFORCED CIRCULAR CUTOUT.

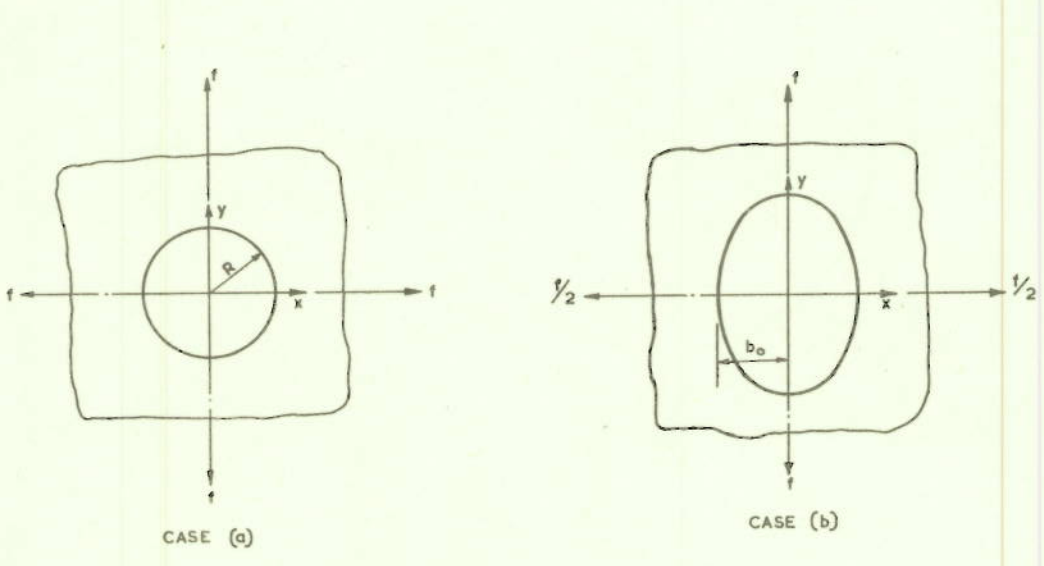


FIG. 17. NOTATION USED FOR NEUTRAL HOLE.