

3 8006 10058 6745

REPORT NO. 112

JANUARY, 1957

THE COLLEGE OF AERONAUTICS  
CRANFIELD

On the Choice of the Working Fluid for Intermittent  
Supersonic Wind Tunnels

-by-

G.A. Bird, B.Sc.,\*

and

G.M. Lilley, M.Sc., D.I.C.

SUMMARY

The general advantages and disadvantages of using a gas other than air as the working fluid in a wind tunnel are discussed. It is shown that, in certain cases, an intermittent tunnel may be more suitable than a continuous tunnel for use with these gases.

A number of gases and gas mixtures (in particular, air, Freon 12, helium and Freon 12 - Argon) are compared on the basis of their Reynolds number against Mach number characteristics when expanded from given stagnation conditions and the limitations, which are imposed on the maximum Mach number by the condensation of the gas, are calculated. The size and complexity of the required pressure vessels and associated equipment is discussed for the different gases and some conclusions are reached as to the more suitable gases for various operating ranges.

MEP

\* Whilst carrying out these investigations, Mr. Bird was on leave of absence from the Aeronautical Research Laboratories, Melbourne.

List of Contents

	<u>Page</u>
1. Introduction	3
2. Reynolds number against Mach number characteristics of different gases	4
3. Condensation effects	5
4. Effect of specific heat ratio	6
5. Tunnel layout and ancillary equipment	7
6. Typical gases for use in wind tunnels	9
7. Conclusions	10
References	11
Appendix I. The variation of Reynolds number with Mach number	12
II. The variation of tunnel pressure ratio with Mach number and ratio of the specific heats	14
III. The variation of vacuum tank capacity with Mach number and ratio of the specific heats	15

Figures 1 to 7.

## 1. Introduction

A working fluid other than air may be considered for use in a supersonic intermittent wind tunnel for two reasons;

either (i) to increase the Reynolds number for a given tunnel size,

or (ii) to avoid condensation of the working fluid in the case of hypersonic tunnels.

The advantages to be gained by using these other gases must be compared with the disadvantages - of which the main ones are;

(i) The heavy gases which are most suitable for increasing the Reynolds number have, in general, a specific heat ratio ( $\gamma$ ) which is different from that of air. In addition, the use of these gases may introduce error due to differences in relaxation times and in their degree of departure from either a perfect gas or air.

(ii) The design of the tunnel components will be complicated by the use of a gas other than air and the general layout of the tunnel will probably be made more elaborate.

(iii) In most cases, the heavy gases have higher boiling points than air so that the maximum Mach number before condensation will be less than for air at the same stagnation temperature.

(iv) Many of these gases are expensive to purchase and are difficult to obtain in large quantities.

For most experiments conducted at low supersonic Mach numbers, the attainment of the highest possible Reynolds number will be the dominant consideration. However, at high Mach numbers, it becomes more important to have the stagnation temperature as near as possible to that which would exist in actual flight at these speeds in the atmosphere. Also, in order to simulate conditions approximately at very high altitudes, it may be necessary to conduct tests at an extremely low Reynolds number. In the sections which follow consideration will be given to wind tunnels having these ranges of Mach number, Reynolds number and stagnation temperature, although it is not envisaged that the complete range of conditions will be obtained in any one tunnel.

## 2. Reynolds number against Mach number characteristics of different gases

Figure 1 shows the Reynolds number per inch for a stagnation pressure of one atmosphere as a function of Mach number for a number of gases at various stagnation temperatures. The gradient of these curves at any Mach number depends critically on the specific heat ratio of the gas (see Appendix I). In the case of Freon ( $\gamma = 1.125$ ) the Reynolds number falls more sharply with increasing Mach number than in the case of air ( $\gamma = 1.40$ ) and with helium ( $\gamma = 1.66$ ) the reverse applies.

At a Mach number of one, the Reynolds number obtained when using Freon as the working fluid is more than twice that obtained with air when it is expanded from the same stagnation conditions. However this margin is reduced as the Mach number increases and vanishes at about  $M = 2.7$ . Therefore it will only be of advantage to use Freon in a tunnel running at Mach numbers less than about two.

The Reynolds number with helium as the working fluid is well below that with air until a Mach number of about five is reached. For hypersonic tunnels it is almost essential (from the Reynolds number standpoint) to use a gas with a high value of  $\gamma$  as the working section to stagnation density ratio ( $\rho/\rho_{st}$ ) is much higher at high Mach numbers for these gases. It can be seen that considerable advantage is to be gained by refrigerating the high pressure storage vessels to reduce the stagnation temperature. This can only be done, of course, when using a gas with an extremely low boiling point such as helium. If the stagnation temperature of helium is reduced to  $108^\circ\text{K}$  (which is still well above the temperature of liquid air at atmospheric pressure), the Reynolds number with helium is above that for air even at transonic Mach numbers. Under similar conditions of pressure and temperature hydrogen has approximately a Reynolds number half that for air. In view of its relatively high vapour pressure characteristic it could be refrigerated over a wide range of stagnation pressures to give at least the Reynolds number attained with air at the same Mach number.

The considerations above do not apply to those experiments in which the heat transfer at high Mach numbers is important. In such experiments, the stagnation temperature must be far greater than atmospheric and it is not possible to avoid the consequent drop in Reynolds number. At very high Mach numbers the required stagnation temperatures are so high that it is impossible to simulate them in a conventional wind tunnel. If however the effects of gas dissociation and ionization at high temperature on the heat transfer

rate and general flow over a body are ignored the effects of the elevated stagnation temperatures at very high Mach numbers can be simulated in a wind tunnel by the employment of cooled models & moderate stagnation temperatures. Although this test method has the advantage that relatively high Reynolds numbers can still be attained at high Mach numbers, it suffers from the disadvantage that gases different from air must be employed to avoid condensation of the working medium.

### 3. Condensation Effects

The maximum Mach number to which any particular gas may be used is limited by the condensation of the working fluid when the temperature of the expanding gas falls below its boiling point. Figures 2, 3 and 4 show the saturation conditions for air, Freon 12 and Helium respectively. Condensation effects will not occur in helium at Mach numbers less than about twenty except at very low stagnation temperatures.

These saturation limits are marked on Figure 1 for a stagnation pressure of one atmosphere and it can be seen that, in general, by the time a gas reaches its saturation limit it has already been superseded by another on considerations of Reynolds number. These limits do not mark the end of usefulness of the gas, as a certain amount of supersaturation will take place before sufficient condensation occurs to appreciably affect the flow. Experimental work (e.g. ref. 1) indicates that, in the case of air, this extension is of the order of one or two Mach numbers but, since the effects of condensation are rather selective to the phenomenon being investigated, it is difficult to predict the range of usefulness of any gas beyond its saturation limit.

To avoid condensation at high Mach numbers in wind tunnels using air, heaters are included in the circuit to increase the stagnation temperature. It can be seen from Figure 1 that this involves a considerable penalty in terms of Reynolds number.

One possible objection to the use of Freon 12 in intermittent tunnels is that the stagnation pressure will be limited by condensation under stagnation conditions. However the curves in Figure 3 show that only a moderate amount of heating is required to produce a substantial rise in this limit.

It is assumed in Figures 2, 3 and 4 that the gases behave as perfect gases. To check this assumption, the

calculations were repeated for air as an imperfect gas using the methods of Reference 2. The modified curves are plotted as dotted lines in Fig. 2 and it can be seen that in this case, the imperfect gas effects are very small. Increasing pressure tends to increase  $\gamma$  and increasing temperature tends to reduce it and, in this example, the two effects tend to cancel each other. Where the change in  $\gamma$  is sufficient to affect the temperature and pressure ratios the changes in these also tend to cancel one another as far as their effect on the Mach number for saturation is concerned.

#### 4. Effect of Specific Heat Ratio

Figure 5 shows the ratio of the values of a number of flow functions in helium and Freon 12 to their corresponding values in air. It is seen that the change in  $\gamma$  has a large effect on the one-dimensional isentropic flow functions, and that this effect increases with Mach number. However its effects on pressure coefficients, pressure ratios across shock waves, shock wave angles and similar variables are comparatively small and are almost independent of Mach number. This indicates that the corrections which would have to be applied to quantitative tests may not be so large as to preclude the use of gases other than air. Tests of a comparative and qualitative nature in such gases would certainly be possible. The use of Freon 12 for testing in the transonic range has already been proved to be feasible (ref. 6).

It has been suggested (Ref. 3) that the advantages of using a heavy gas can be retained while preserving the correct value of  $\gamma$  if the heavy polyatomic gas (such as Freon 12), having a low  $\gamma$ , is mixed in the correct ratio with a heavy monatomic gas (such as argon, krypton or xenon) which has a high value of  $\gamma$ . However, apart from xenon which is unobtainable in the required quantities, the advantages in terms of Reynolds number are smaller than in the case of pure Freon 12 and the condensation of Freon in the mixture is reached at a lower Mach number than with pure Freon<sup>\*</sup>. The curve for a mixture of argon and Freon 12 is shown in Fig. 1.

At hypersonic Mach numbers in flight with dissociation present the specific heat ratio first falls below 1.4 and then exceeds it. It has been suggested (Ref. 3) that the

---

\* At the same Mach number the static temperature will be lower in the gas mixture due to the change in  $\gamma$  from 1.13 for Freon 12 to 1.4 for the mixture.

variation of  $\gamma$  with temperature for an undissociated polyatomic gas - monatomic gas mixture may, under wind-tunnel conditions, represent at least some of the conditions imposed by the fall in  $\gamma$  under flight conditions. However, since it is not possible to represent the changes in viscosity and thermal conductivity, as a result of dissociation, by this method such tests would have little bearing on the problems of skin friction and heat transfer.

### 5. Tunnel Layout and Ancillary Equipment

The tunnel components will necessarily be more elaborate when a gas other than air is used as the working fluid although an intermittent tunnel will suffer less in this respect than a continuous tunnel. In the case of a continuous tunnel, the complexity of the design will be greatly increased by the necessity of providing ancillary compressors, vacuum pumps, filters and storage tanks for the gas. The sealing problems will also be difficult, particularly where the drive shaft enters the tunnel shell, and it will probably be necessary to provide means of isolating and purging the test section for gaining access to the model. On the other hand, there need not be any basic complication of the circuit of an intermittent tunnel as the required compressors, storage vessels, isolating valves etc. will normally be included in the circuit.

An important factor concerning the design of compressors and vacuum pumps for intermittent tunnels at high Mach numbers is the tunnel pressure ratio. As shown in Appendix 2 and Fig. 6\* this ratio increases rapidly with Mach number, above  $M = 2$ , being greatest for the smaller values of  $\gamma$ . The decrease in pressure ratio with helium compared with air is very significant at high Mach numbers.

The tunnel layout should preferably be such that the gas is not lost after each run and, if it is an intermittent tunnel, it will probably be of the combined blow-down and suction type. One of the major tunnel items is then the vacuum tank and Appendix 3 and Fig. 7\* shows the required

---

\* The results given in Fig. 6 are for a supersonic diffuser having a normal shock wave at working section Mach number followed by zero pressure recovery to zero velocity. In practice it should be possible to improve on this efficiency by the use of multi-shock diffusers thereby causing a large reduction in the required tunnel pressure ratio.

\* The curves in Fig. 7 are based on a normal shock supersonic diffuser followed by zero pressure recovery to zero velocity. If diffusers having greater efficiencies are used in practice the vacuum tank capacity will correspondingly be reduced.

capacity of this tank for the various gases as a function of Mach number. It is assumed that after each run the pressure in the tank is twenty times its initial value. The tank capacity is then independent of the stagnation pressure.

The required tank capacity for Freon 12 is less than half that for air over the Mach number range in which it might be used. Also as Freon would be used only at relatively low Mach numbers, for which the required pressure ratio is small, it would be possible to design a tunnel having atmospheric outlet pressure and to collect the gas in a flexible storage bag from which it would be pumped to the high pressure reservoir.

In the case of helium, the required tank capacity is over three times that required for air. To overcome the difficulty of providing a very large vacuum tank, one tunnel (Ref. 4) uses instead an ejector system to reduce the back pressure and provide the necessary pressure ratio. With the system, however, the charge of helium is wasted during each run. It can be seen from Fig. 7 that the refrigeration of helium is advantageous from the point of view of reducing the required tank capacity as well as increasing the Reynolds number.

One difficulty with hypersonic tunnels is the construction of the nozzle section because of the very large area ratio which is required at high Mach numbers. In this respect helium offers great advantages over air as the required area ratios are in the ratio of 5.5:1 at  $M = 8$  and 17.5:1 at  $M = 15$ .

In the case of the high pressure storage tank it is desirable from size considerations to have the storage pressure as high as possible compared with the stagnation pressure. Reynolds number considerations make it desirable to keep the stagnation pressure as high as possible. If the Reynolds number in the working section is to be kept constant during a run, the gas must be heated or passed through a heat regenerator before it enters the working section. The necessary amount of heating is reduced if the storage capacity is made larger than the minimum.\* The best compromise between these factors can only be determined for each particular case.

---

\* Where very high storage pressures are required for the operation of hypersonic wind tunnels the temperature change in throttling must be added to the temperature drop due to isentropic expansion of gas in the storage vessel in order to determine the capacity of the heat regenerator. A small gain is obtained when hydrogen or helium are used, compared with air, as the working medium, for the Joule-Thomson effect in these cases at ordinary temperatures causes heating instead of cooling.



## 6. Typical Gases for Use in Wind Tunnels

Besides those gases which have been mentioned, a number of others may be considered for possible use in wind tunnels.

The discussion on heavy polyatomic gases was limited to Freon 12 which is the one normally considered to be most suitable as it is easily available, non-toxic and non-reactive. Polyatomic gases as a whole are reviewed in Refs. 3 and 5 and the fluorochemicals were found to be the most useful group. Many new fluorocarbons are still being synthesized and some of these would give larger increases in Reynolds number than Freon 12 ( $\text{C Cl}_2 \text{ F}_2$ ) over the transonic range and, if available, could be used with advantage. For instance,  $\text{C}_4 \text{ F}_{10}$  has a molecular weight almost twice that of Freon 12, although its boiling point is slightly higher and its specific heat ratio lower.

Air is, of course, the most suitable of the diatomic gases. Hydrogen, although dangerous to handle, could be used to higher Mach numbers before condensation and the Reynolds number would decrease by only a factor of two.

The heavy monatomic gases (argon, krypton and xenon) could be considered for use only at very low Mach numbers as they have high boiling points. Neon is only slightly lighter than air and its boiling point is much lower, but, as it has a  $\gamma$  of 1.66, its temperature drop with Mach number is greater than for air and its use is therefore very limited.

When polyatomic gases or gas mixtures are used under wind-tunnel conditions their relaxation effects, due to the time lag in exciting the vibrational modes of the molecules, are in general different from those for air under flight conditions. The conclusion drawn by Chapman (Ref. 3) is that these effects are of more importance in changing boundary layer flow than in altering pressure distribution and are of greatest importance at low Reynolds numbers. It is also suggested (Ref. 3) that by the use of polyatomic gas mixtures in suitable proportions relaxation effects in flight through air at hypersonic speeds could be simulated in a wind tunnel.

When tests at very high Mach numbers are required to simulate conditions in flight through air it becomes necessary to use very high stagnation temperatures in order to represent correctly the effects of air dissociation. The temperatures involved are such that the use of conventional wind tunnels must be excluded. Tests can be performed in a shock-tube type wind tunnel where the time duration of the flow is small

and that of pseudo-steady flow conditions is even smaller. Apart from the inconvenience of the very short running time truly similar flow conditions are not obtained with this latter arrangement because the high pressure, high temperature gas partially dissociates before the test section is reached. (It is unlikely that the nozzle length will be sufficiently long for complete recombination to take place). This means that the physical state of the gas ahead of the model will be different from that in flight, even if it were possible for the temperature and pressure to be correctly represented.

It is possible that some of the aerodynamic effects connected with air dissociation at high temperatures can be represented by dissociating a suitable polyatomic gas at a much lower stagnation temperature. Such gases could be employed in a conventional wind tunnel having a running time of reasonable length. Tests performed in this way, especially for some research purposes, would possess some advantages over the shock-tube test method even though all the effects of air dissociation would not be completely represented.

## 7. Conclusions

Some of the main observations which may be made on the choice of the working fluid for tunnels operating in the various speed ranges are.-

(i) The use of Freon 12 confers large advantages in terms of Reynolds number to tunnels which operate in the transonic and low supersonic speed ranges. In this speed range, condensation difficulties are not likely to be encountered and the errors introduced by the change in specific heat ratio should not be appreciable.

(ii) The use of a mixture of heavy polyatomic and monatomic gases in proportions to give the same specific heat ratio as air confers smaller gains in Reynolds number and is limited to relatively low Mach numbers because of condensation.

(iii) For a tunnel which has an operating range around  $M = 2.5$  to  $M = 5$  and which has normal stagnation temperatures, air gives the highest Reynolds number. To avoid condensation difficulties above  $M = 5$  the stagnation temperature of the air must be increased and the Reynolds number then falls sharply. This will be a disadvantage for many wind tunnel tests although the thermal conditions in actual flight will be better simulated at these higher stagnation temperatures.

(iv) Even with pre-heating, the use of air is impracticable above a Mach number of about 9. For higher Mach numbers it is necessary to use helium, despite the large vacuum tank capacity which is required if the gas is to be re-used after each run. Above a Mach number of 5, helium gives a higher Reynolds number than air at the same stagnation temperature. Still higher Reynolds numbers can be obtained by reducing the stagnation temperature, although by such means, simulation of actual flight conditions is not possible.

### References

- | <u>No.</u> | <u>Author</u>                         | <u>Title, etc.</u>   |
|------------|---------------------------------------|--|
| 1.         | H.C. Stever and<br>K.C. Rathbun       | Theoretical and experimental investigation of condensation of air in hypersonic wind tunnels. N.A.C.A. T.N. 2559, Nov. 1951. |
| 2.         | A.G. Eggers, Jr.                      | One-dimensional flows of an imperfect diatomic gas. N.A.C.A. T.N. 1861, April 1949.  |
| 3.         | D.R. Chapman                          | Some possibilities of using gas mixtures other than air in aerodynamic research. N.A.C.A. T.N. 3226. August 1954.            |
| 4.         | S.M. Bogdonoff and<br>A.G. Hammit     | The Princeton helium hypersonic tunnel and preliminary results above $M = 11$ . Princeton Univ. Rep. No. 260, June 1954.     |
| 5.         | R. Smelt                              | Power economy in high speed wind tunnels by choice of working fluid and temperature. R.A.E. Rep. No. Aero.2081, Aug. 1945.   |
| 6.         | von Doenhoff A.E. and<br>Braslow A.L. | Studies of the use of Freon 12 as a wind tunnel testing medium. N.A.C.A. T.N. 3000 (1953).                                   |
| 7.         | J. Lukasiewicz                        | Development of large intermittent wind tunnels. J. Roy. Aero. Soc. April (1955).   |

APPENDIX I

Variation of Reynolds Number with Mach Number

Notation

$R_e$	Reynolds number
$M$	Mach number
$v$	flow velocity
$l$	typical length
$\mu$	coefficient of viscosity
$a$	speed of sound
$\gamma$	specific heat ratio
$T$	temperature
$p$	pressure
$\rho$	density
$m$	molecular weight
$R$	universal gas constant
$st$	denotes stagnation conditions

$$\begin{aligned}\text{Now } R_e &= \frac{\rho v l}{\mu} \\ &= \rho a \frac{M l}{\mu} \text{ as } M = v/a \\ &= \frac{\gamma p M l}{\mu a} \text{ as } a^2 = \frac{\gamma p}{\rho} \\ &= \frac{p M l}{\mu} \sqrt{\frac{\gamma}{RT}} \text{ as } a^2 = \frac{\gamma RT}{m}\end{aligned}$$

But in an isentropic expansion

$$\frac{p_{st}}{p} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}} \text{ and } \frac{T_{st}}{T} = \left(1 + \frac{\gamma-1}{2} M^2\right)$$

Therefore

$$\frac{R_e}{l p_{st}} = \frac{1}{\mu_{st}} \sqrt{\frac{m \gamma}{RT_{st}}} \frac{M}{\left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{\gamma+1}{2(\gamma-1)} - \omega}}$$

making use of the empirical relation

$$\frac{\mu}{\mu_{st}} = \left(\frac{T}{T_{st}}\right)^{\omega}$$

The gradient of  $\ln (R_e/l p_{st})$  with Mach number is

$$\frac{d \ln (R_e/l p_{st})}{dM} = \frac{1}{M} - \frac{M \left[ \left(\frac{\gamma+1}{2}\right) - \omega(\gamma-1) \right]}{\left(1 + \frac{\gamma-1}{2} M^2\right)}$$

It is shown in Fig. 1 that the variation of  $\gamma$  has a dominant effect on the Reynolds number - Mach number relation at high Mach number. For  $M = 4$  this expression changes from a value of about  $-0.015$  for  $\gamma = 1.66$  to  $-1.69$  for  $\gamma = 1.125$ .

APPENDIX II

The variation of tunnel pressure ratio with Mach number and the ratio of the specific heats

If the gas expands isentropically from stagnation conditions ( $p_{st}$ ) to the working section pressure ( $p$ ) then

$$\frac{p_{st}}{p} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}} \dots\dots\dots(1)$$

If the working section is followed by a diffuser in which the gas velocity is reduced to zero then the final pressure ( $p_f$ ) to working section pressure ratio can be written

$$\frac{p_f}{p} = \left(1 + \frac{\gamma-1}{2} M^2 \eta_{\sigma}\right)^{\frac{\gamma}{\gamma-1}} \dots\dots\dots(2)$$

where  $\eta_{\sigma}$  is the overall diffuser efficiency. If the pressure and Mach number at the end of the supersonic diffuser are found then equation (2) with the appropriate subsonic values of  $p$  and  $M$ , and  $\eta_{\sigma}$  replaced by  $\eta_{\sigma(sub)}$ , can be used to find the pressure recovery in the subsonic diffuser. The simplest case is that for a normal shock at the end of the working section followed by no pressure recovery in the subsonic diffuser. For this case  $\eta_{\sigma(sub)} = 0$  and

$$\eta_{\sigma} = \frac{\left[\left(\frac{2\gamma M^2 - \gamma + 1}{\gamma + 1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}{\frac{\gamma-1}{2} M^2} \dots\dots\dots(3)$$

From equations (1), (2) and (3) the stagnation to final pressure ratio for the case  $\eta_{\sigma(sub)} = 0$  is

$$\frac{p_{st}}{p_f} = \frac{\left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}}}{\left(\frac{2\gamma M^2 - \gamma + 1}{\gamma + 1}\right)^{\frac{\gamma-1}{\gamma}}} \dots\dots\dots(4)$$

Calculated values are shown in Fig. 6 for Helium ( $\gamma = 1.67$ ), Air ( $\gamma = 1.4$ ) and Freon 12 ( $\gamma = 1.125$ ). Also shown is the mean experimental curve for air tunnels given by Lukasiwicz (Ref. 7).

APPENDIX III

The variation of vacuum tank capacity with Mach number  
and the ratio of the specific heats

If the stagnation pressure and temperature are maintained constant throughout the running time ( $\Delta t$ ) of the tunnel then the rate of mass flow ( $m$ ) through the tunnel is constant. If  $V$  is the vacuum tank capacity then the increase in density of the gas inside the tank during the time  $\Delta t$  is

$$\rho_f - \rho_i = \frac{m}{V} \Delta t \quad \dots\dots\dots(1)$$

where  $m = \frac{\gamma p_{st}}{a_{st}} A \frac{M}{\left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{1+\gamma}{2(\gamma-1)}}} \dots\dots\dots(2)$

and  $p_{st}$  and  $a_{st}$  are the stagnation pressure and speed of sound respectively,  $A$  is the working section area, and  $M$  is the working section Mach number. After some rearrangement we obtain, on the assumption that the initial temperature in the tank is equal to the upstream stagnation temperature,

$$\frac{V}{A \cdot \Delta t} = a_{st} \cdot \frac{p_{st}}{p_f} \cdot \frac{1}{\left(\frac{T_i}{T_f} - \frac{p_i}{p_f}\right)} \cdot \frac{M}{\left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}} \dots\dots\dots(3)$$

If we put  $T_i = T_f$  and assume that  $p_{st}/p_f$  is given by equation (4) (Appendix II) then

$$\frac{V}{A \cdot \Delta t} = \frac{a_{st}}{\left(1 - \frac{p_i}{p_f}\right)} F(M) \quad \dots\dots\dots(4)$$

where  $F(M) = \frac{(\gamma+1)M \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{1}{2}}}{2\gamma M^2 - \gamma + 1}$

and 
$$\lim_{M \rightarrow \infty} \frac{L}{A_o t} = \frac{\gamma+1}{2\gamma} \sqrt{\frac{\gamma-1}{2}} \frac{a_{st}}{\left(1 - \frac{p_i}{p_f}\right)} \dots\dots\dots(5)$$

These relations show that the vacuum tank capacity is independent of the tunnel stagnation pressure and proportional to the square root of the tunnel stagnation temperature.

The vacuum tank capacity, based on equations (4) and (5) and with  $p_f/p_i = 20$ , for a wide range of Mach numbers and different gases is shown in Fig. 7. Also shown is an experimental curve for Mach numbers up to 5 in air based on the mean experimental pressure ratios given by Lukasiewicz (Ref. 7).

Since the vacuum tank capacity is proportional to the tunnel pressure ratio  $p_{st}/p_f$  a reduction in pressure ratio from that given in Fig. 6 will cause a corresponding reduction in the required vacuum tank capacity.



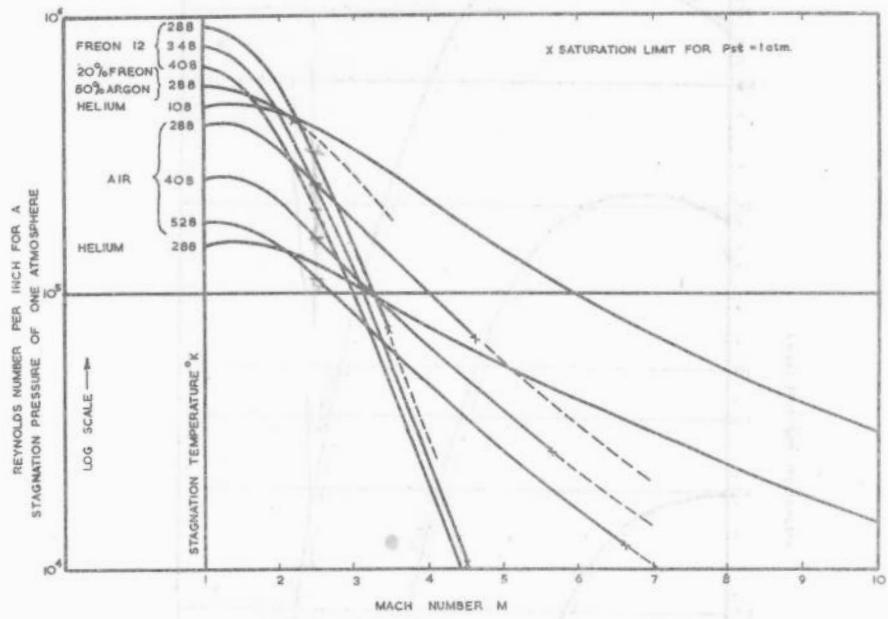


FIG. 1 MACH NUMBER AGAINST REYNOLDS NUMBER CHARACTERISTICS FOR A NUMBER OF GASES.

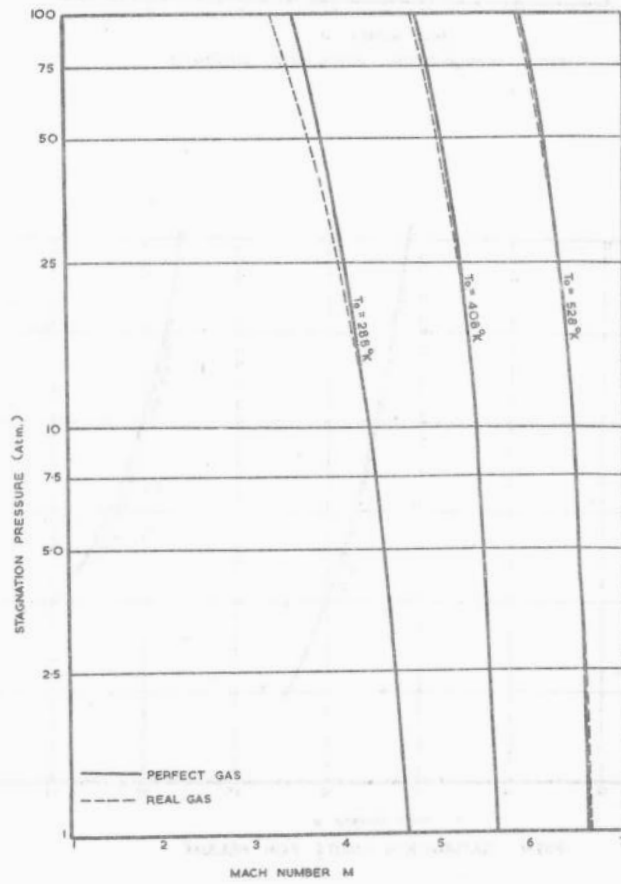


FIG. 2 SATURATION LIMITS FOR AIR

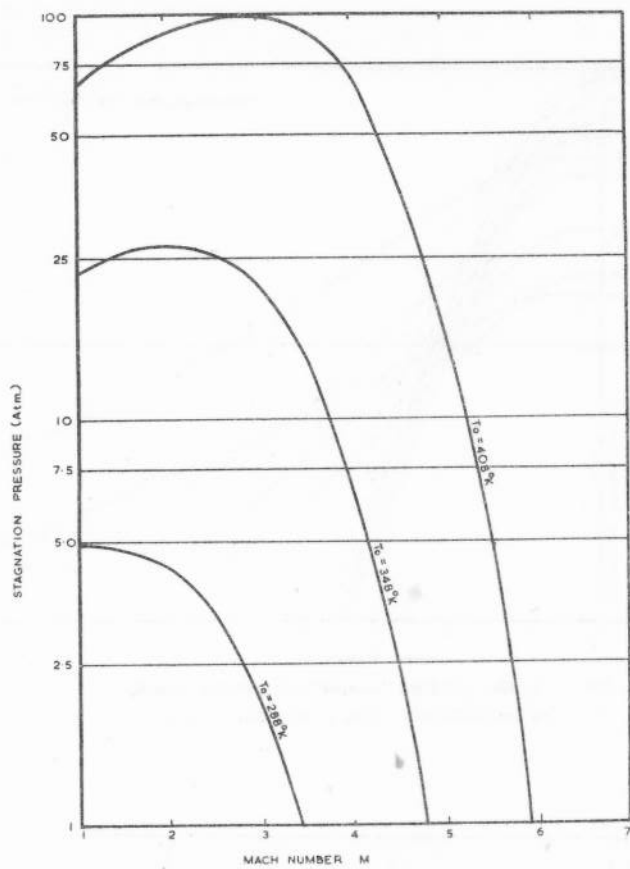


FIG. 3 SATURATION LIMITS FOR FREON 12

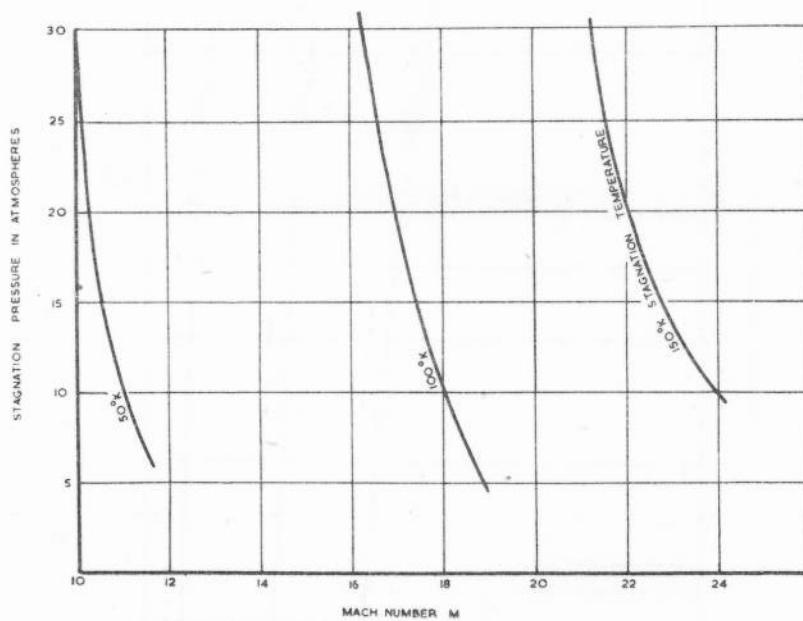


FIG. 4. SATURATION LIMITS FOR HELIUM

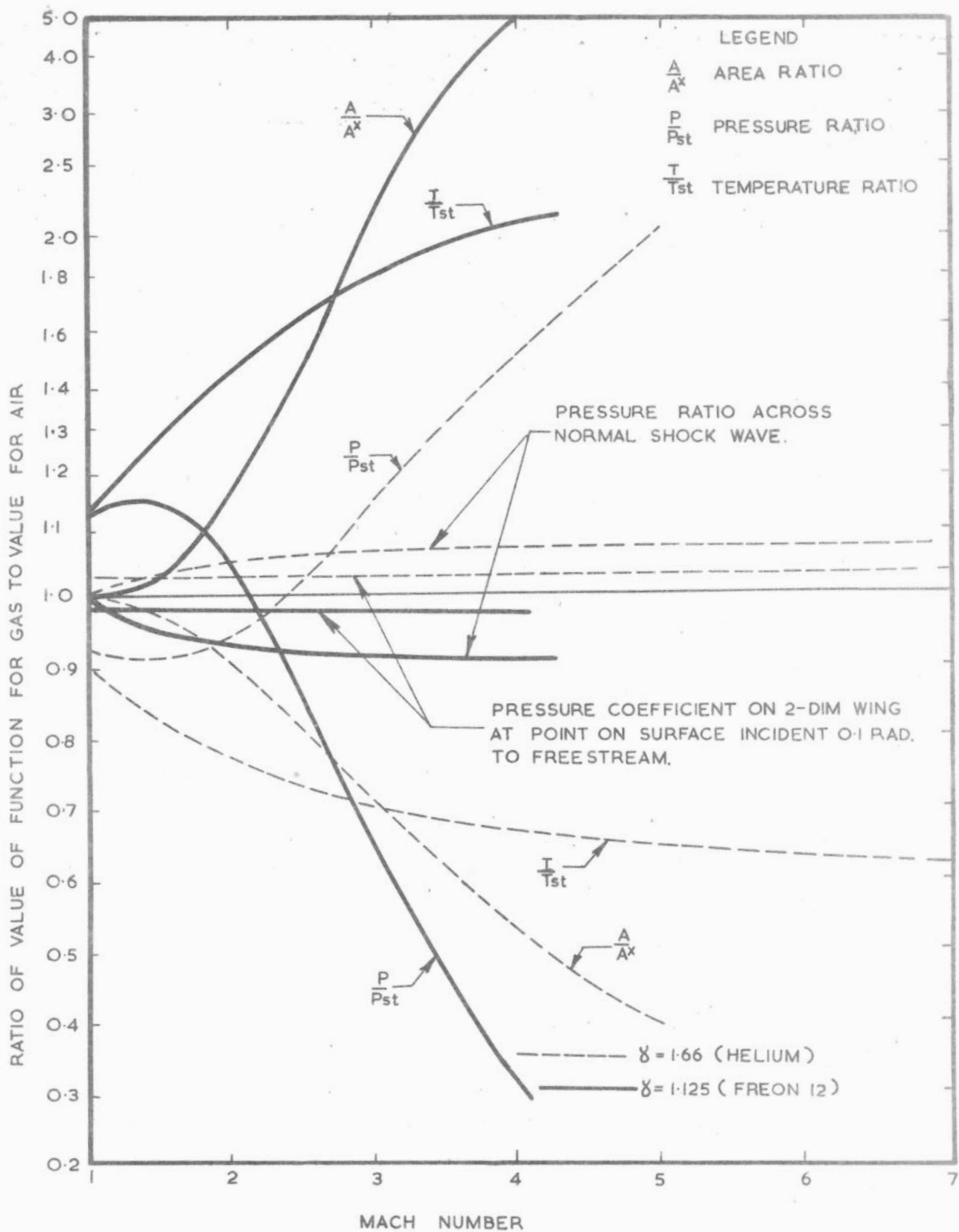


FIG. 5 EFFECT OF  $\gamma$  ON TYPICAL FLOW FUNCTIONS

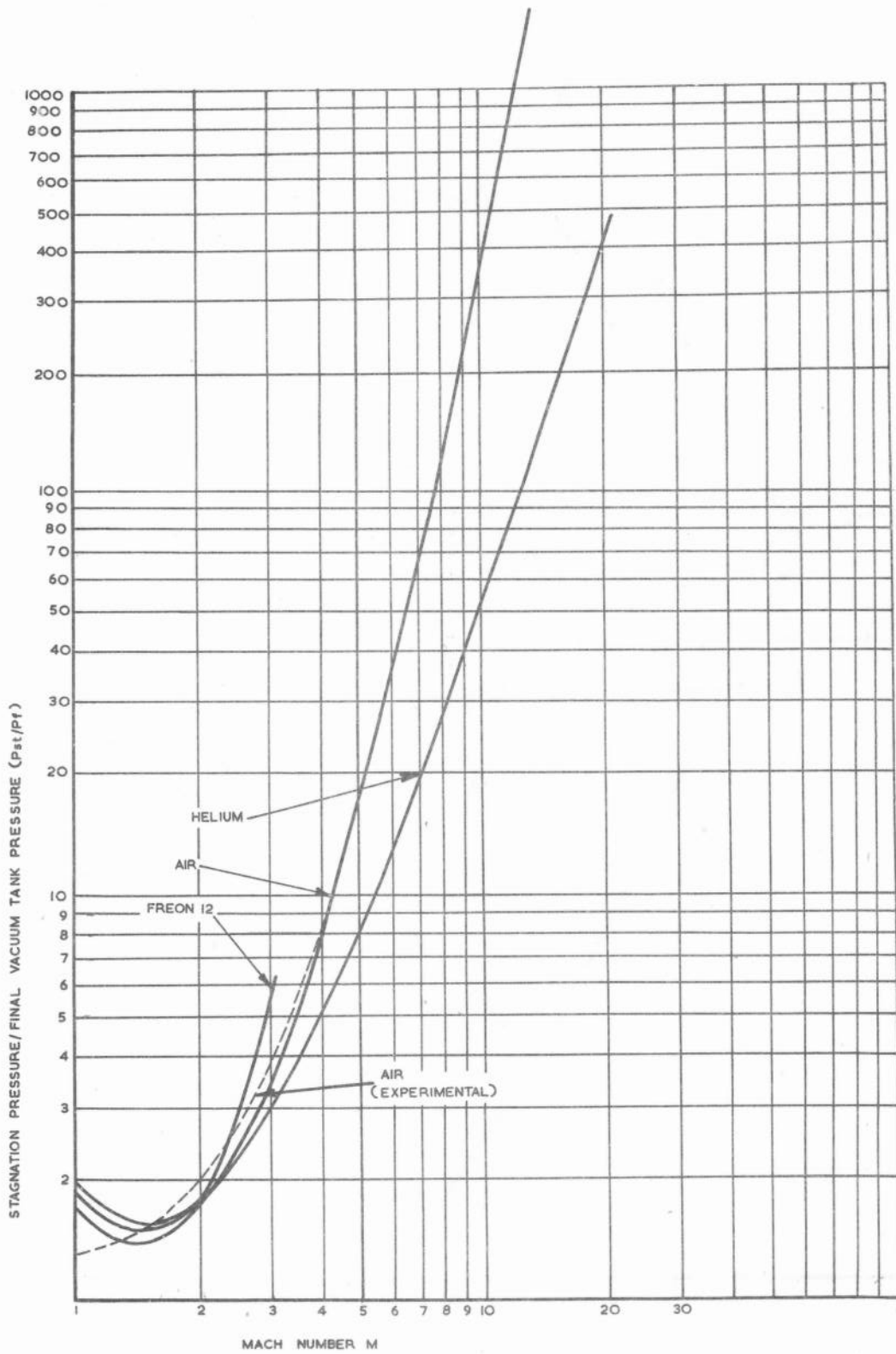


FIG. 6 THEORETICAL TUNNEL PRESSURE RATIO

$(\gamma_{\sigma}(\text{SUB}) = 0)$

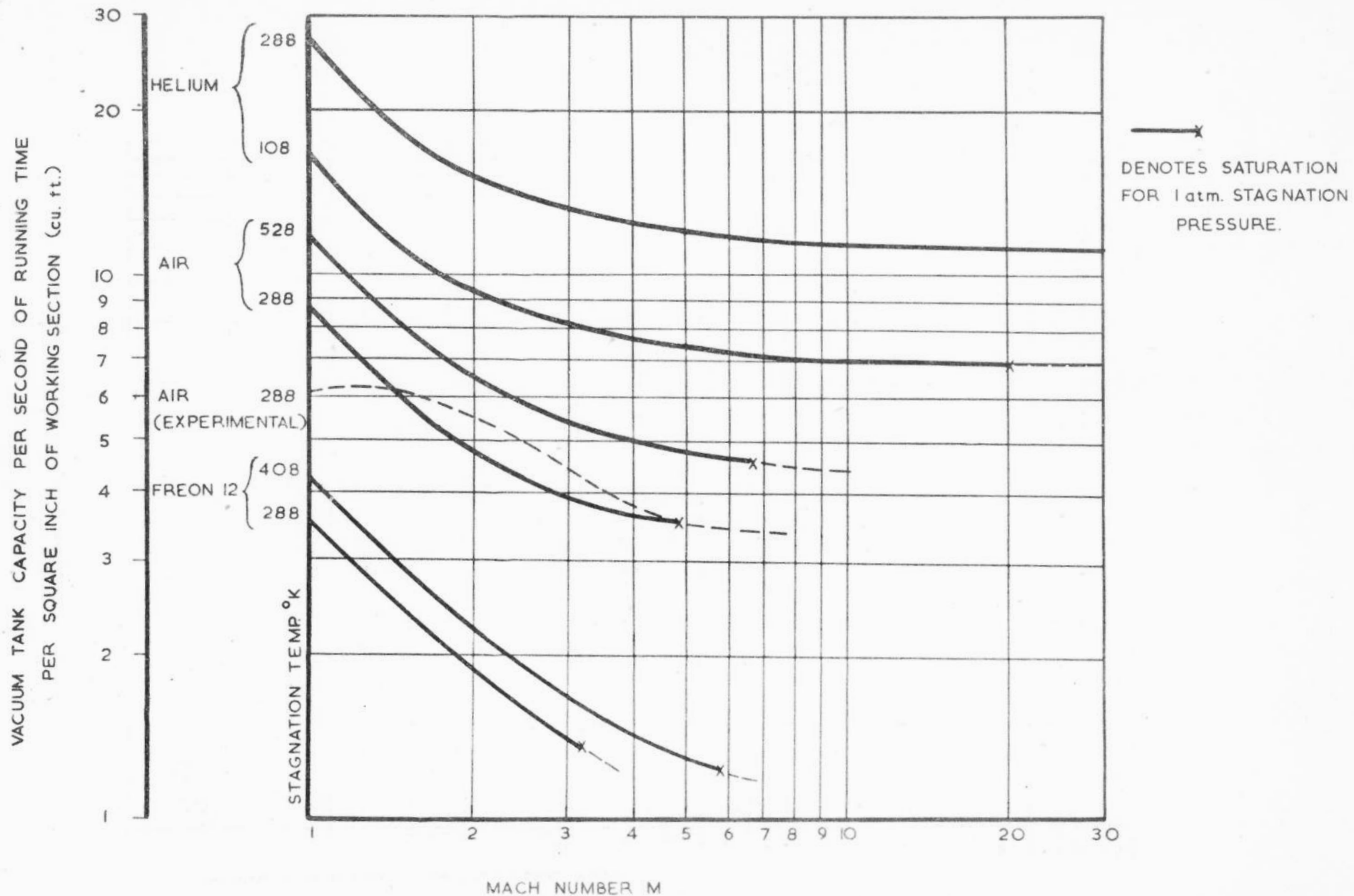


FIG. 7 REQUIRED CAPACITY OF VACUUM TANK  
 BASED ON THEORETICAL PRESSURE RATIO WITH  $(\gamma_{\sigma(SUB)}=0)$