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The Determination in Flight of the Body Drag and the Mean Blade Profile Drag Coefficient of a Helicopter*

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SUMMARY

Slight modifications have been made to the energy equation which enable the results of partial climb tests to be plotted as two straight lines, the slopes of which are measures of the body drag of the helicopter and the mean profile drag coefficient of the rotor blades.

Sufficient data has been analysed to show that the method can be used to obtain an accurate measurement of the body drag.

The values of $(C_Q - \frac{\delta\sigma}{4})$ obtained by the method are of the right order of magnitude, and will give a good indication of the profile drag losses of the rotor if the transmission and tail rotor power can be assessed to an accuracy of one per cent.

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N O T A T I O N (see also Figure 6).

T	Thrust	(lb.)	
Q	Torque	(lb.ft.)	
C_T	Thrust coefficient		$T/\frac{1}{2}\rho(\Omega R)^2\pi R^2$
C_Q	Torque coefficient		$Q/\frac{1}{2}\rho(\Omega R)^2\pi R^3$
D'	Body drag	(lb.)	
D'_{100}	Body drag at 100 ft./sec.	(lb.)	
C'_D	Body drag coefficient		$D'/\frac{1}{2}\rho V'^2\pi R^2$
δ	Mean profile drag coefficient of blade		
R	Radius of rotor	(ft.)	
σ	Rotor solidity		$bc/\pi R$
b	Number of blades		
c	Chord of blades (assumed constant)	(ft.)	
ρ	Density of the air	(slugs/ft.^{3})	
Ω	Angular velocity of rotor	(rads./sec.)	
V_c	Rate of climb	(ft/min)	
V_a	Component of velocity normal to rotor	(ft/sec)	$= V \sin i$
V_t	Component of velocity tangential to rotor	(ft/sec)	$= V \cos i$
V	Velocity of undisturbed flight	(ft/sec)	
v	Induced velocity through the rotor	(ft/sec)	
U_T	Ideal hovering induced velocity	(ft/sec)	$= (T/2\rho\pi R^2)^{\frac{1}{2}}$
u	Total velocity normal to rotor	(ft/sec)	$= V_a + v$
V'	Resultant velocity at rotor disc	(ft/sec)	$(V_t^2 + u^2)^{\frac{1}{2}}$
i	Rotor incidence relative to direction of V. (positive when relative wind is down through the disc)		
χ	Inclination of rotor disc from the horizontal (positive when relative wind is down through disc)		
μ	Tip speed ratio		$V_t/\Omega R$
λ	Inflow ratio		$u/\Omega R$
ν	Resultant velocity ratio		$V'/\Omega R = (\lambda^2+\mu^2)^{\frac{1}{2}}$
ν_1	The value of ν at which V_c is its maximum.		

1. Introduction

It has long been the practice to evaluate the drag constants for a fixed wing aircraft by plotting the results of flight tests as the drag coefficient against the square of the lift coefficient. The relationship is found to be linear, so that

$$C_D = C_{D_Z} + K\, C_L^{\,2}\,.$$

Thus, the constants C_{D_Z} and K can readily be obtained as the intercept and slope of the curve respectively.

At the present time, no attempt appears to have been made to evaluate the drag constants for a helicopter in a similar manner. In this report, use is made of the energy equation for the helicopter, and, by making small changes to the form of the equation, it will be shown that the results of flight tests can be plotted in such a way as to give two straight lines, the slopes of which are measures of the drag constants of the helicopter.

2. Evaluation of the Drag Constants for the Helicopter

2.1 The energy equation

An energy equation has been derived[1] from a consideration that the power supplied to the rotor is used

(a) to overcome the drag of the fuselage,

(b) to overcome the profile drag of the rotor blades,

(c) to provide an induced velocity through the rotor disc,

and (d) to climb.

In a non-dimensional form the energy equation becomes

$$C_Q = C_D'\,.\,\nu^3 + \frac{\sigma\delta}{4}\,(1 + 3\nu^2) + \frac{C_T^{\,2}}{4\nu} + \frac{V_C\,.\,C_T}{\Omega R}\,. \quad \ldots\ldots (2.1)$$

In the derivation of equation (2.1) a component of velocity along the blades has been neglected. Glauert[2] has shown the effect of this velocity component to change the term $(1 + 3\nu^2)$ into a term $(1 + 4.5\nu^2)$. Equation (2.1) thus becomes

$$C_Q = \frac{V_C\,.\,C_T}{\Omega R} + \frac{C_T^{\,2}}{4\nu} + \frac{\sigma\delta}{4}\,(1 + 4.5\nu^2) + C_D'\,\nu^3 \quad \ldots\ldots (2.2)$$

and it is this energy equation which will be used in the subsequent analysis.

The two drag constants in this equation are δ, the mean profile drag coefficient of the rotor blades, and C_D', the drag coefficient of the fuselage.

2.2 Evaluation of the body drag coefficient

Inspection of equation (2.2) reveals that, for a constant value of the torque coefficient C_Q and increasing velocity ν, values for the rate of climb V_c are given which rise to a maximum and then fall with further increase in ν.

The value for ν for the maximum V_c will be designated ν_1. It is found that for values of ν well in excess of ν_1 the variation in the terms

$$\frac{C_T^2}{4\nu} + 4.5\,\frac{\sigma\,\delta}{4}\,\nu^2$$

is negligible compared with the variation in the term $C_D'.\nu^3$. This is to be expected, since at the high forward speed more power is used to translate the fuselage.

Therefore, over the range $\nu >> \nu_1$

$$\frac{V_c\,.\,C_T}{\Omega R} = \text{a constant} - C_D'\,\nu^3 \qquad \text{......(2.3)}$$

from which it is seen that, for a constant thrust coefficient, the rate of climb varies linearly with ν^3. Hence, by plotting V_c against ν^3 the body drag coefficient, C_D', can be evaluated from

$$C_D' = -\,\frac{d\,V_c}{d\,\nu^3}\,.\,\frac{C_T}{\Omega R}\,. \qquad \text{......(2.4)}$$

2.3 Evaluation of the mean profile drag coefficient

If equation (2.2) is multiplied throughout by ν and re-arranged, there results an expression for the rate of climb as follows -

$$\frac{V_c\,.\,C_T}{\Omega R}\,.\,\nu = \left(C_Q - \frac{\delta\sigma}{4}\right)\nu - \frac{C_T^2}{4} - \frac{4.5\,\sigma\,\delta}{4}\,\nu^3 - C_D'\,.\,\nu^4\,. \qquad \text{......(2.5)}$$

In this case, only values of ν well below ν_1 are considered, and over this range the variation in the terms involving ν^3 and ν^4 is negligible in comparison with the term in ν on the left-hand side of the equation.

Thus, for the range $\nu << \nu_1$, the expression for the rate of climb becomes

$$\frac{V_c\,.\,C_T}{\Omega R}\,.\,\nu = \left(C_Q - \frac{\delta\sigma}{4}\right)\nu - \text{a constant} \qquad \text{.....(2.6)}$$

and the value of the mean profile drag coefficient, δ, for the blades can be evaluated from

$$\left(C_Q - \frac{\delta\sigma}{4}\right) = \frac{C_T}{\Omega R}\,.\,\frac{d}{d\nu}\,(V_c\,.\,\nu)\,. \qquad \text{......(2.7)}$$

2.4 The application of the method

The method of analysis described in the preceding paragraph is seen to be readily applicable to the results of flight tests. If measurements of the rate of climb are made at various forward speeds, the results of these partial climb tests can be reduced to curves of V_c against ν^3 (over the range $\nu > \nu_1$) and $V_c . \nu$ against ν (over the range $\nu_1 > \nu$).

In the evaluation of the body and mean profile drag coefficients, it is necessary to assume that the thrust is constant and equal to the weight. It is also necessary to assume that the angle of tilt, χ, of the rotor axis is small, so that the forward speed, as measured by the airspeed indicator, equals the velocity, V_t, tangential to the rotor disc, and that the rate of climb V_c is equal to the velocity V_a, normal to the disc.

It is also necessary to estimate the induced velocity through the disc. A chart has been prepared (Figure 1) which gives the induced velocity v for various values of the velocity normal and parallel to the disc. This chart[10] is based on experimental values obtained by Brotherhood and Stewart[3,4].

3. Results and Discussion

Flight test results were available for the following aircraft.

Sikorsky	S.51	(ref.5)
Hoverfly	Mk.I	(ref.6)
Bristol	171	(ref.7)

The leading particulars of these aircraft are given in Table I.

In each case V_c was plotted against ν^3 and $V_c.\nu$ against ν. The graphs were found to be straight lines, thus supporting the predictions of the energy equation. The only departure from the linear relationship was in the region where the forward speed was close to the forward speed for maximum rate of climb. A specimen reduction is given for the S.51 in Table II and the graphs in Figures 2 and 3.

3.1 The body drag

The results obtained from the analysis are tabulated below -

	$\frac{dV_c}{d\nu^3}$	D'_{100} (lb.)
S.51	-34,800	269
Hoverfly Mk.I	-84,000	250
Bristol 171	-57,100	154

The values for the body drags follow the expected trend, the Bristol 171 being obviously the cleanest of these three aircraft. In the case of the Hoverfly a rough check on the value of D'_{100} is possible, Stewart having made an estimate[6] of the component drags. In this reference the body drag at 100 f.p.s. is quoted as 240 lb., which is seen to be close to the value derived from the flight test results.

3.2 The value of $(C_Q - \frac{\sigma\delta}{4})$

The results obtained from the flight tests are given in the following table -

	$\frac{d}{d\nu}(V_c \nu)$	$C_Q - \frac{\delta\sigma}{4}$
S.51	1860	.000666
Hoverfly Mk.I	1740	.000370
Bristol 171	1130	.000146

With the existing available data it is not possible to make a conclusive independent check of these values. However, Stewart[8] gives information concerning the collective pitch angles; and with the additional aid of Tables[9] of rotor characteristics, an estimate of the quantity $(C_Q - \frac{\sigma\delta}{4})$ is possible. The following table gives the estimated values.

	$C_Q - \frac{\sigma\delta}{4}$
S.51	.00060
Hoverfly Mk.I	.00042

These two estimated values verify the order of the results obtained from the flight measurements.

The usefulness of the parameter $(C_Q - \frac{\delta\sigma}{4})$ is limited, as in itself it does not give an indication of the profile drag of the rotor blades. Separation of the profile drag coefficient, δ, by the evaluation of C_Q requires an accurate assessment of the power expended in overcoming transmission losses and in driving the tail rotor. For the three helicopters considered here, this information regarding wasted power was not readily available. However, if firstly ten per cent and then fifteen per cent of the total engine power is assumed for the power losses, the following values for δ are obtained.

	δ	
	10 per cent Waste power	15 per cent Waste power
S.51	.0134	.0106
Bristol 171	.0184	.0160
Hoverfly Mk.I	.0208	.0181

These values for δ are all of the expected order of magnitude, but the differences with each power loss are such that they are of little value in assessing the profile drag losses of the rotor. Consequently, an assessment of the waste power to an accuracy of one per cent of the total engine power is required before a satisfactory value of δ can be determined.

3.3 <u>The application of the method to the auto-rotative glide</u>

The energy equation (2.2), without alteration, is applicable to the helicopter in an autorotative descent. It must be noted in this case that C_Q is small and negative. It is based on the torque required to overcome the transmission losses and to drive the r tail rotor.

Flight test results for the Bristol 171 have been plotted in Figures 4 and 5. The predictions concerning the linearity of the curves are again verified. Analysis of the results leads to the following results.

D'_{100}	$C_Q - \frac{\delta\sigma}{4}$	δ	
		10 per cent Waste power	15 per cent Waste power
141 lb.	-.00016	.00955	.00795

The value for the body drag gives further support for the method since the ten per cent variation from the value quoted in paragraph 3.1 can easily be accounted to the change in direction of the resultant velocity over the body.

The values for the mean profile drag coefficient for the blade are considerably less than those for powered flight quoted in paragraph 3.2. But at high forward speeds the degree of stalling of the retreating blade is greater in powered flight than in auto-rotation, and this blade stalling would account for the increase in the mean profile drag coefficient of the rotor blades.

4. Conclusions

(a) Sufficient data has been analysed to show that the method can be used to obtain an accurate measurement of the body drag.

(b) The values of $(C_Q - \frac{\delta\sigma}{4})$ obtained by the use of the method are of the right order, and will give a good indication of the profile drag losses of the rotor if the transmission and tail rotor power can be assessed to an accuracy of one per cent.

(c) The value of δ obtained from flight test results by this analysis represents a mean of the values over the range $\nu \ll \nu_1$, and cannot be assigned to any particular forward speed. Further, the value of δ thus obtained will include the effects of blade stalling.

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TABLE I

Leading Particulars of the Aircraft Considered in this Report.

	S.51	Bristol 171	Hoverfly I
Weight lb.	4985	4850	2650
Rotor diameter	48'	47'-5"	38'
Disc loading lb/ft.^{2}	2.756	2.746	2.335
Solidity	0.073	0.050	0.058
Tip speed (under power) ft/sec.	486	669	449
Tip speed (autorotation) " "		640	

TABLE II

Specimen Reduction of Flight Test Results

Sikorsky S.51.

V_c ft/min.	V_i knots	μ	V_t/U_T	V_a/U_T	v/U_T	λ	ν
765	20	.073	1.43	.514	.46	.057	.093
875	25	.091	1.78	.587	.50	.055	.106
960	30	.109	2.14	.645	.43	.055	.122
1030	35	.127	2.48	.691	.39	.055	.138
1070	40	.145	2.84	.719	.34	.054	.155
1090	45	.163	3.19	.732	.30	.053	.171
1075	50	.182	3.57	.722	.28	.051	.189
1045	55	.199	3.90	.702	.26	.049	.204
985	60	.218	4.26	.661	.24	.046	.223
910	65	.236	4.64	.611	.22	.043	.240
815	70	.254	4.99	.547	.20	.038	.256
690	75	.272	5.34	.464	.19	.033	.274
540	80	.291	5.71	.362	.19	.028	.292
370	85	.309	6.06	.249	.19	.022	.310
185	90	.327	6.41	.124	.19	.016	.327

Height = 3,000 ft.

ΩR = 486 ft./sec.

C_T = .0105.

FIG. 1.

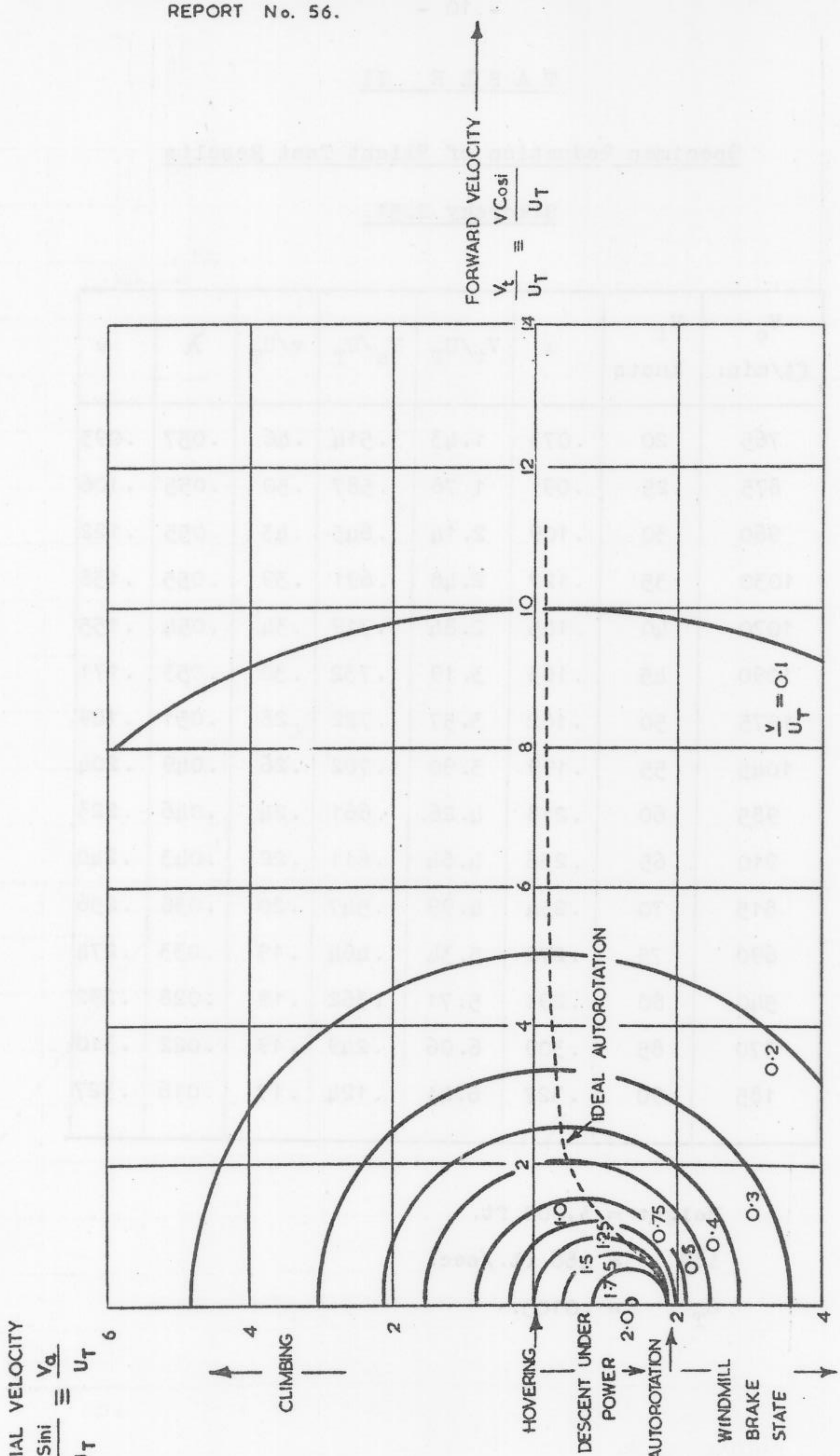
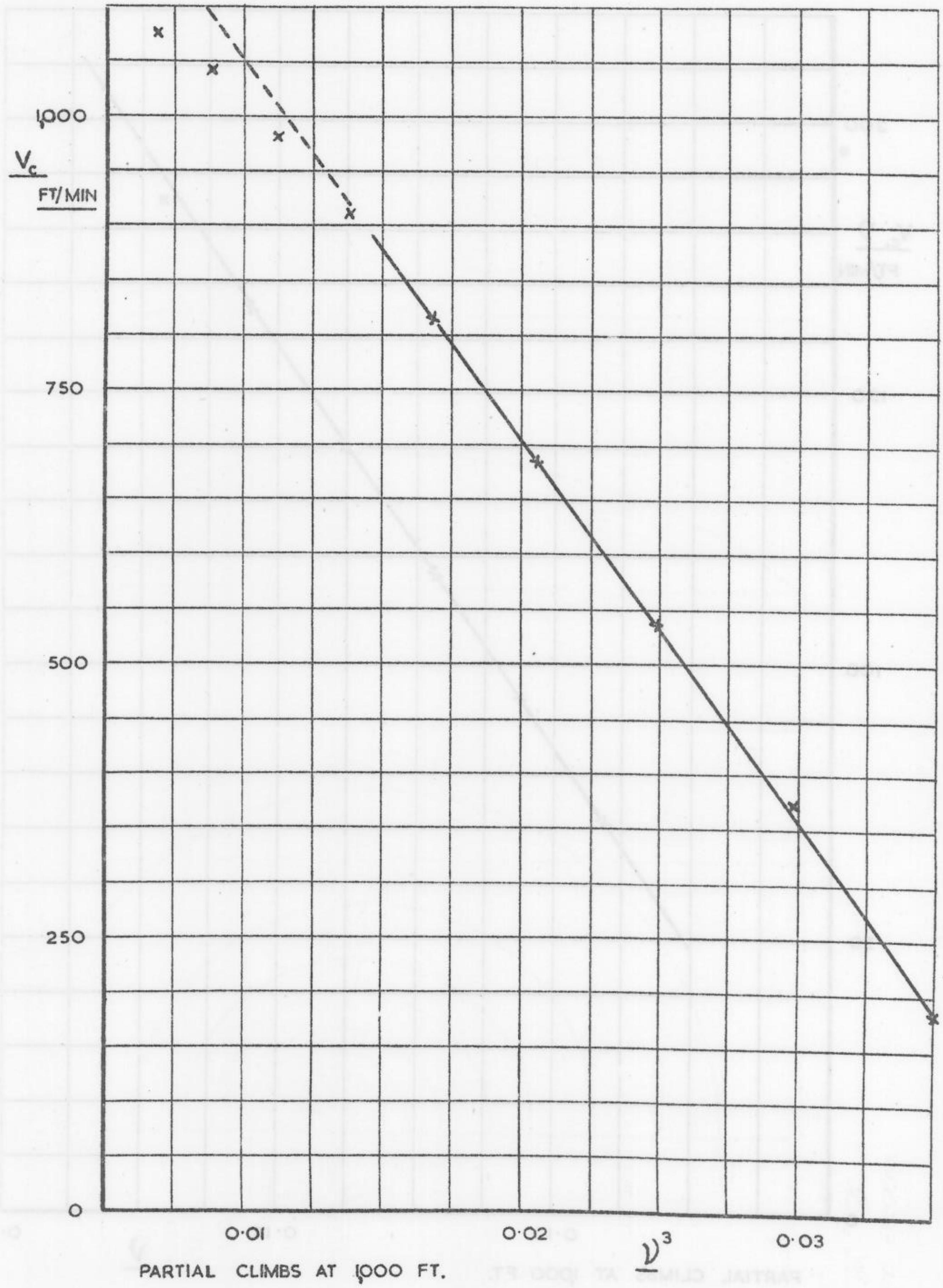


CHART FOR DETERMINATION OF

INDUCED VELOCITY IN FORWARD FLIGHT

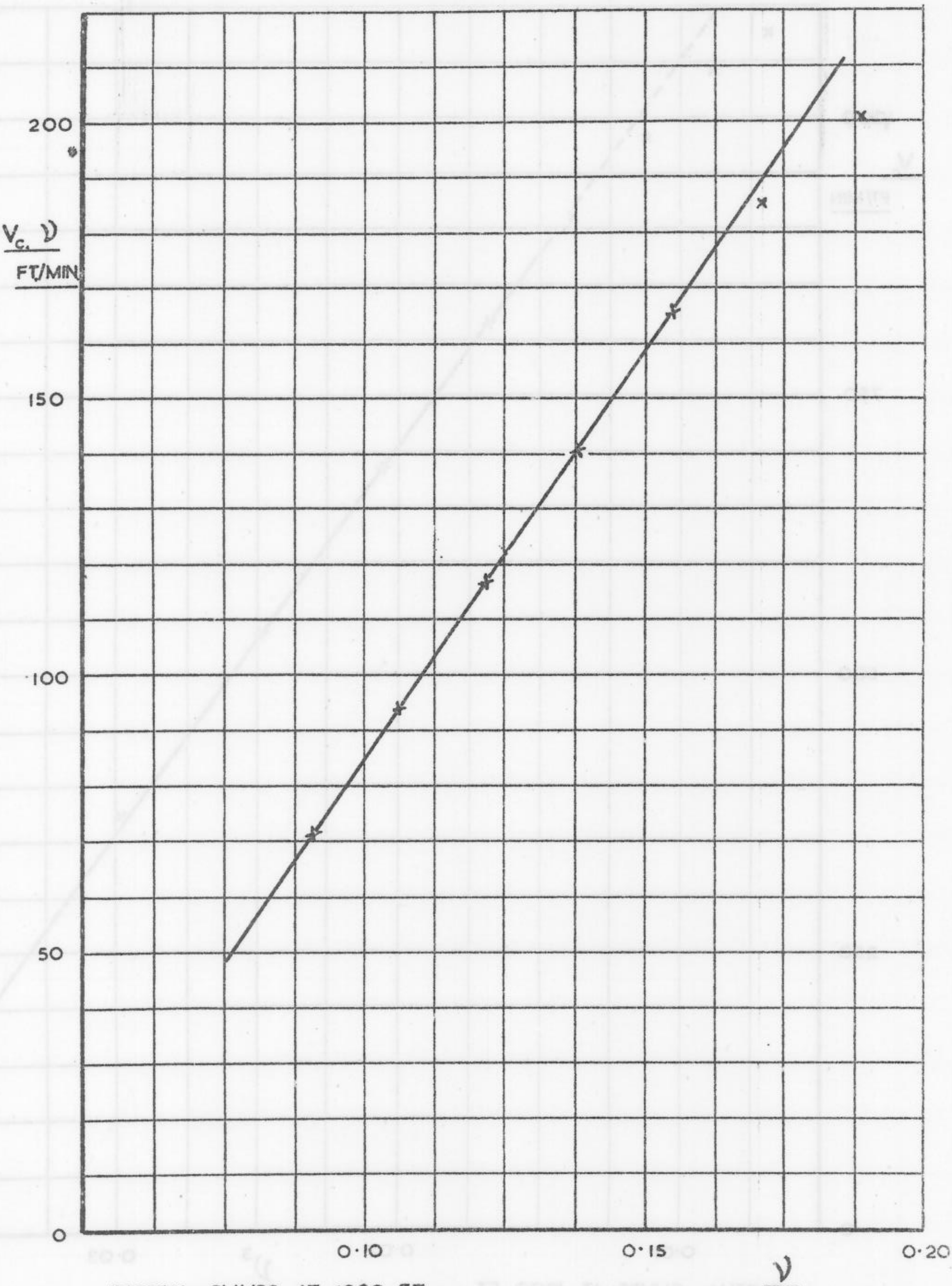
FIG. 2.



$V_c \sim \nu^3$ FOR $\nu > \nu_1$

SIKORSKY S.51.

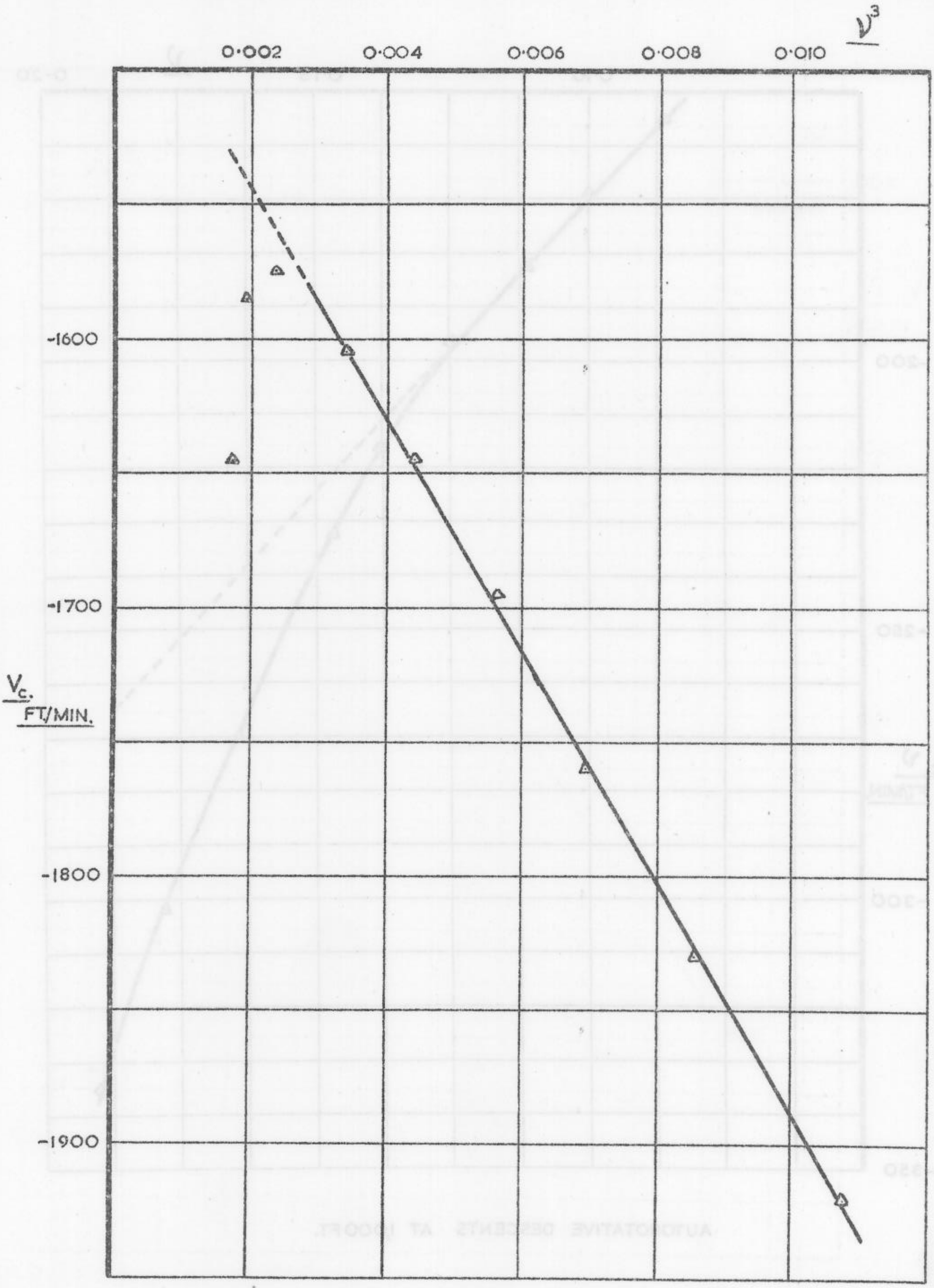
FIG. 3.



PARTIAL CLIMBS AT 1,000 FT.

$V_{c.}\nu \sim \nu$ FOR $\nu_1 > \nu$

SIKORSKY S. 51.

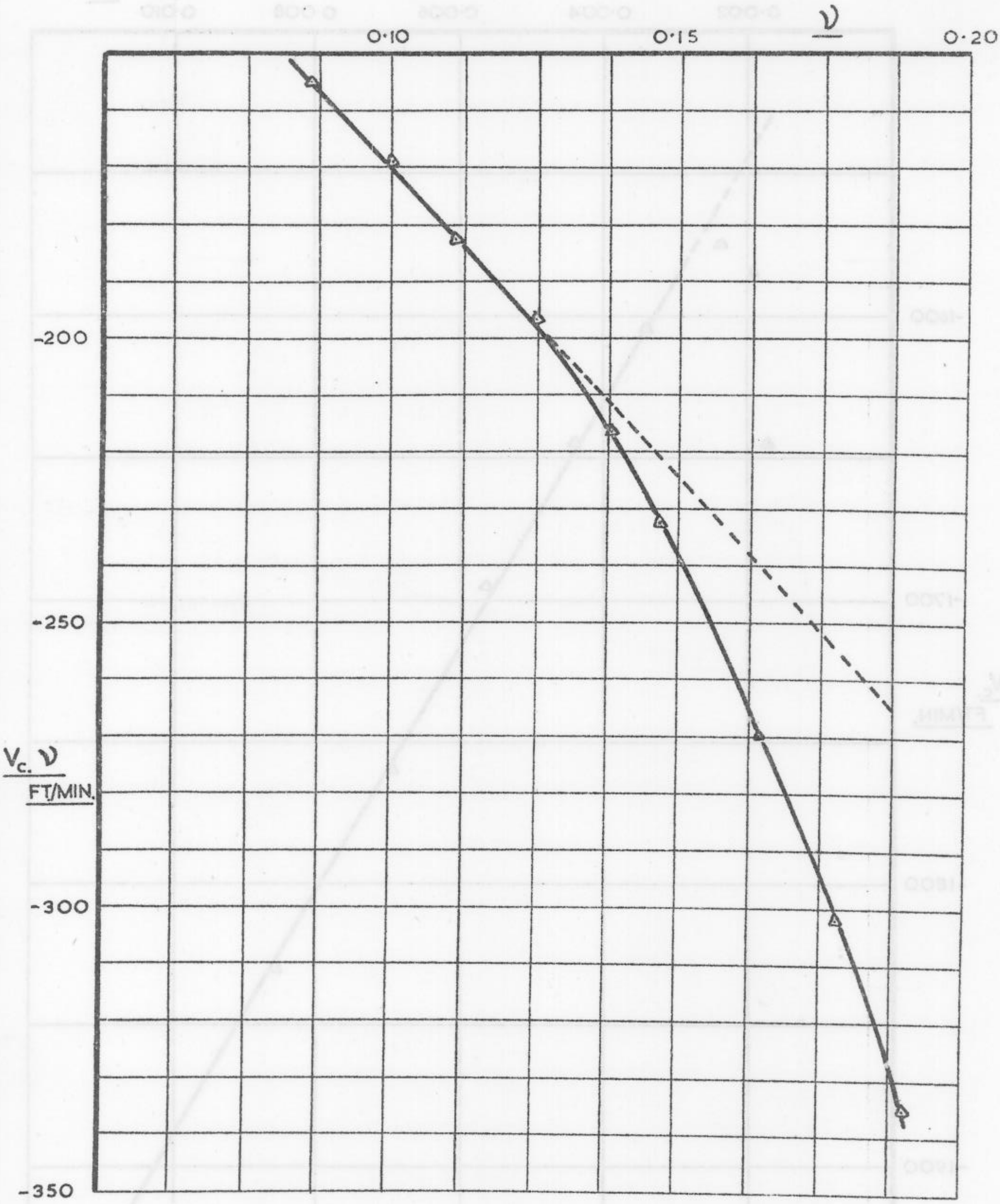


AUTOROTATIVE DESCENTS AT 1000FT.

$V_c \sim \nu^3$ FOR $\nu > \nu_1$

BRISTOL 171

FIG. 5.

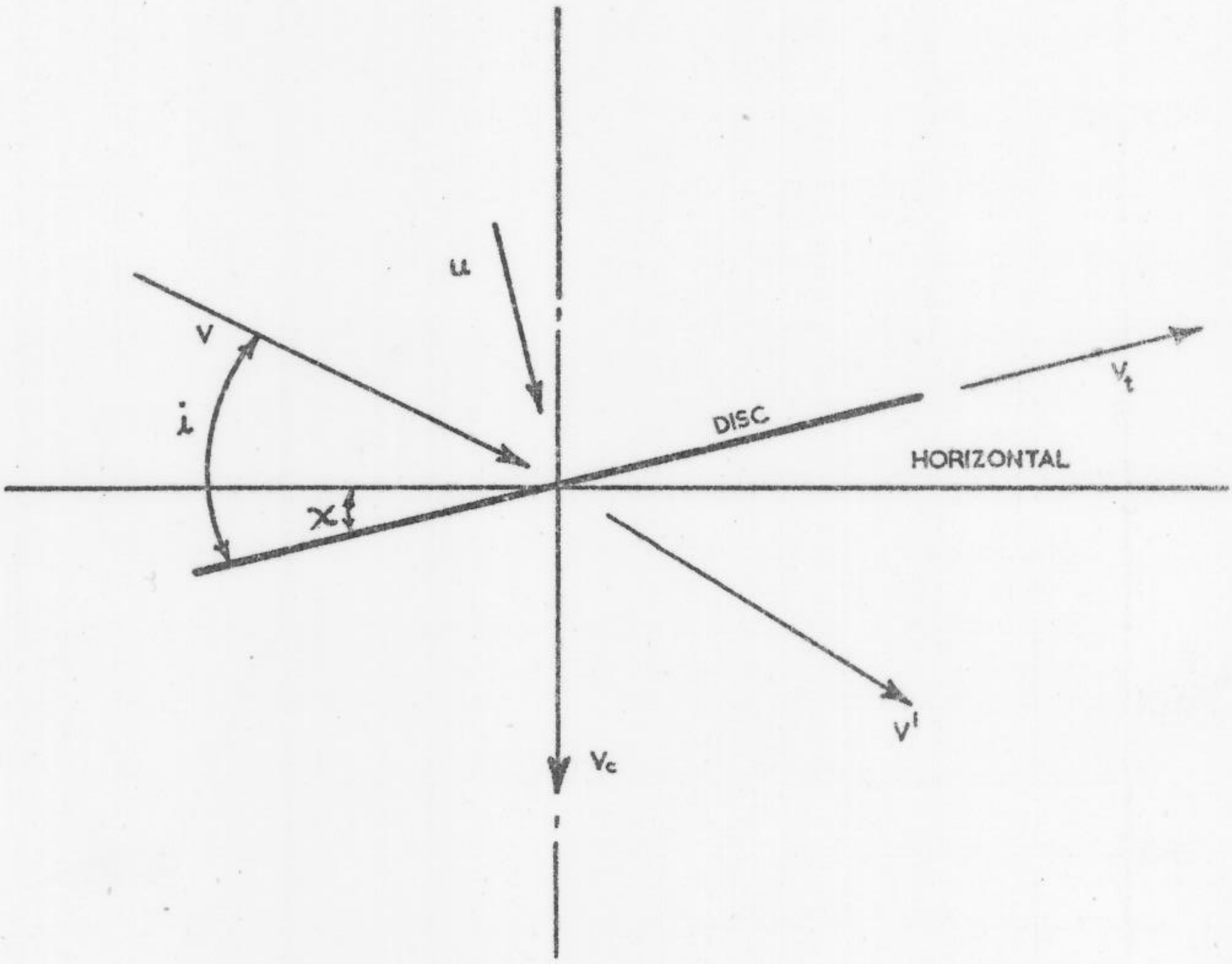


AUTOROTATIVE DESCENTS AT 1,000 FT.

$V_c.\ \nu \sim \nu$ FOR $\nu_1 > \nu$

BRISTOL 171.

FIG. 6.



$u = V_a + \upsilon = V\sin i + \upsilon$

$V_{t_1} = V\cos i$

$V = (V_t^2 + u^2)^{\frac{1}{2}}$

VELOCITIES AND ANGLES AT THE ROTOR DISC.

(SEE ALSO NOTATION)