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Note on the Efficiency of Adiabatic Shock

- by -

A. W. Morley, Ph. D.

of the Department of Aircraft Propulsion

SUMMARY

This note records some calculations of the efficiency of adiabatic shock in air ($\gamma = 1.40$) ; where the efficiency is defined as the ratio of the work used to compress the air in the shock wave, from the inlet static to the outlet total head pressure, to the work required if the compression were isentropic. This is the efficiency usually of interest when considering intakes of propulsive units for supersonic flight.

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Introduction

It was desired to calculate the isentropic efficiency of the ordinary adiabatic shock wave as a compressive process from inlet static to outlet total head pressure over a wide range of possible conditions.

For this purpose, the parameters defining the wave, found most suitable, were the wave angle (μ) to the incident flow, and the wedge angle (δ) through which the gas is deflected.

The well-known equations for conservation of energy, equilibrium of forces, continuity of flow and constant tangential velocity applied to adiabatic shock may be written:

$$T_1 \left(1 + \frac{\gamma - 1}{2} M_1^2\right) = T_2 \left(1 + \frac{\gamma - 1}{2} M_2^2\right) \quad \ldots\ldots\ldots \quad (1)$$

$$P_1 \left(1 + \gamma M_1^2 \sin^2 \mu\right) = P_2 \left(1 + \gamma M_2^2 \sin^2 \overline{\mu - \delta}\right) \quad \ldots\ldots\ldots \quad (2)$$

$$\frac{P_1 M_1}{\sqrt{T_1}} = \frac{P_2 M_2}{\sqrt{T_2}} \; \frac{\sin \overline{\mu - \delta}}{\sin \mu} \quad \ldots\ldots\ldots \quad (3)$$

$$M_1 \sqrt{T_1} \cos \mu = M_2 \sqrt{T_2} \cos \overline{\mu - \delta} \quad \ldots\ldots\ldots \quad (4)$$

From these equations, expressions for M_1 , M_2 and P_2/P_1 were obtained in terms of the basic quantities μ and δ as follows:

$$M_1^2 = \frac{2 \cos \overline{\mu - \delta}}{A \sin \mu} \quad \ldots\ldots\ldots \quad (5)$$

$$M_2^2 = \frac{2 \cos \mu}{B \sin \overline{\mu - \delta}} \quad \ldots\ldots\ldots \quad (6)$$

and

$$\frac{P_2}{P_1} = \frac{B}{A} \quad \ldots\ldots\ldots \quad (7)$$

where

$$A = \sin \overline{2\mu - \delta} - \gamma \sin \delta \quad \ldots\ldots\ldots \quad (8)$$

$$B = \sin \overline{2\mu - \delta} + \gamma \sin \delta \quad \ldots\ldots\ldots \quad (9)$$

With these relationships the required efficiency is calculated quite easily.

Efficiency of Compression

The work required per lb. to compress the gas from P_1 to P_{2t} is proportional to $(T_{2t} - T_1)$ whereas ideally it would be proportional to $(T_{2t}' - T_1)$. Here T_{2t}' is the hypothetical downstream temperature corresponding to isentropic compression from T_1 over the same pressure ratio P_{2t}/P_1. Then we define the efficiency as:

$$\eta = \frac{T_{2t}' - T_1}{T_{2t} - T_1} \qquad \text{........ (10)}$$

or

$$\eta = \frac{(P_{2t}/P_1)^{\frac{\gamma - 1}{\gamma}} - 1}{T_{2t}/T_1 - 1} \qquad \text{........ (11)}$$

Now

$$\frac{P_{2t}}{P_1} = \frac{P_{2t}}{P_2} \cdot \frac{P_2}{P_1} = \left(\frac{T_{2t}}{T_2}\right)^{\frac{\gamma}{\gamma - 1}} \cdot \frac{P_2}{P_1}$$

$$= \left(1 + \frac{\gamma - 1}{2} M_2^2\right)^{\frac{\gamma}{\gamma - 1}} \cdot \frac{P_2}{P_1}$$

and, since $T_{2t} = T_{1t}$,

$$\frac{T_{2t}}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2$$

Substitution for P_{2t}/P_1 and T_{2t}/T_1 in equation (11) gives for η the expression:

$$\eta = \frac{\left\{1 + \frac{\gamma - 1}{2} M_2^2\right\} \left\{\frac{P_2}{P_1}\right\}^{\frac{\gamma - 1}{\gamma}} - 1}{\frac{\gamma - 1}{2} M_1^2} \qquad \text{........ (12)}$$

This last equation may be rewritten, so as to express efficiency in terms of wave angle and wedge angle only, by substituting for M_1^2, M_2^2 and P_2/P_1 from equations (5), (6) and (7). We obtain, as a working formula, the expression:

$$\eta = \frac{A \sin \mu}{\gamma - 1 \cos \overline{\mu - \delta}} \left\{ \left(\frac{B}{A}\right)^{\frac{\gamma - 1}{\gamma}} - 1 \right\} \div \frac{A}{B}^{\frac{1}{\gamma}} \frac{\sin 2\mu}{\sin 2\overline{\mu - \delta}} \qquad \text{.(13)}$$

where A and B are as given in (8) and (9). Tables of the function $x^{\gamma - 1/\gamma}$ were already available.

/ Results ....

Results

The values of efficiency were worked out for 5° steps in δ from 0 to 30° and 5° steps in μ from 25° to 85° for atmospheric air ($\gamma = 1.40$). These are given, with the corresponding values of M_1 , M_2 and P_2/P_1 in Table I and curves, Fig. 1, 2, 3 and 4.

From these curves the points of maximum efficiency may be located. This efficiency is plotted in Fig. 5 and compared with that obtainable with normal shock* at the same entry Mach number.

It is seen that with oblique shock the efficiency is nearly 100% until M_1 reaches about 1.8 when an appreciable decrease occurs with increase in M_1. Though the pressure ratio P_2/P_1 is lower the same pressure ratio can be obtained at a higher efficiency with oblique than with "normal" shock.

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FIGURES ATTACHED

Fig. 1. Relationship between η , μ and δ .

Fig. 2. Relationship between M_1, μ and δ .

Fig. 3. Relationship between M_1, M, μ and δ .

Fig. 4. Relationship between P_2/P_1, μ and δ .

Fig. 5. Relationship between η_{max} and M_1 .

TABLE ATTACHED

Table 1. Oblique Shock Data.

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* With "normal" shock:

$$M_2^2 = \frac{2 + \gamma - 1 \; M_1^2}{2\gamma M_1^2 - \gamma + 1}$$

which may be substituted directly in equation (12) to obtain the efficiency at the corresponding M_1.

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FIG. 1.

ORDINARY SHOCK

RELATIONSHIP BETWEEN ISENTROPIC EFFICIENCY OF COMPRESSION (FROM UP-STREAM STATIC TO DOWN-STREAM TOTAL HEAD) AND WAVE AND WEDGE ANGLES DRAWN FOR $\gamma = 1.4$

$$\eta = \frac{A}{\gamma-1}\,\frac{\sin\mu}{\cos(\mu-\delta)}\left\{\left(\frac{B}{A}\right)^{\frac{\gamma-1}{\gamma}}-1\right\}+\left(\frac{A}{B}\right)^{\frac{1}{\gamma}}\frac{\sin 2\mu}{\sin 2(\mu-\delta)}$$

WHERE $A = \sin(2\mu-\delta) - \gamma\sin\delta$

$B = \sin(2\mu-\delta) + \gamma\sin\delta$

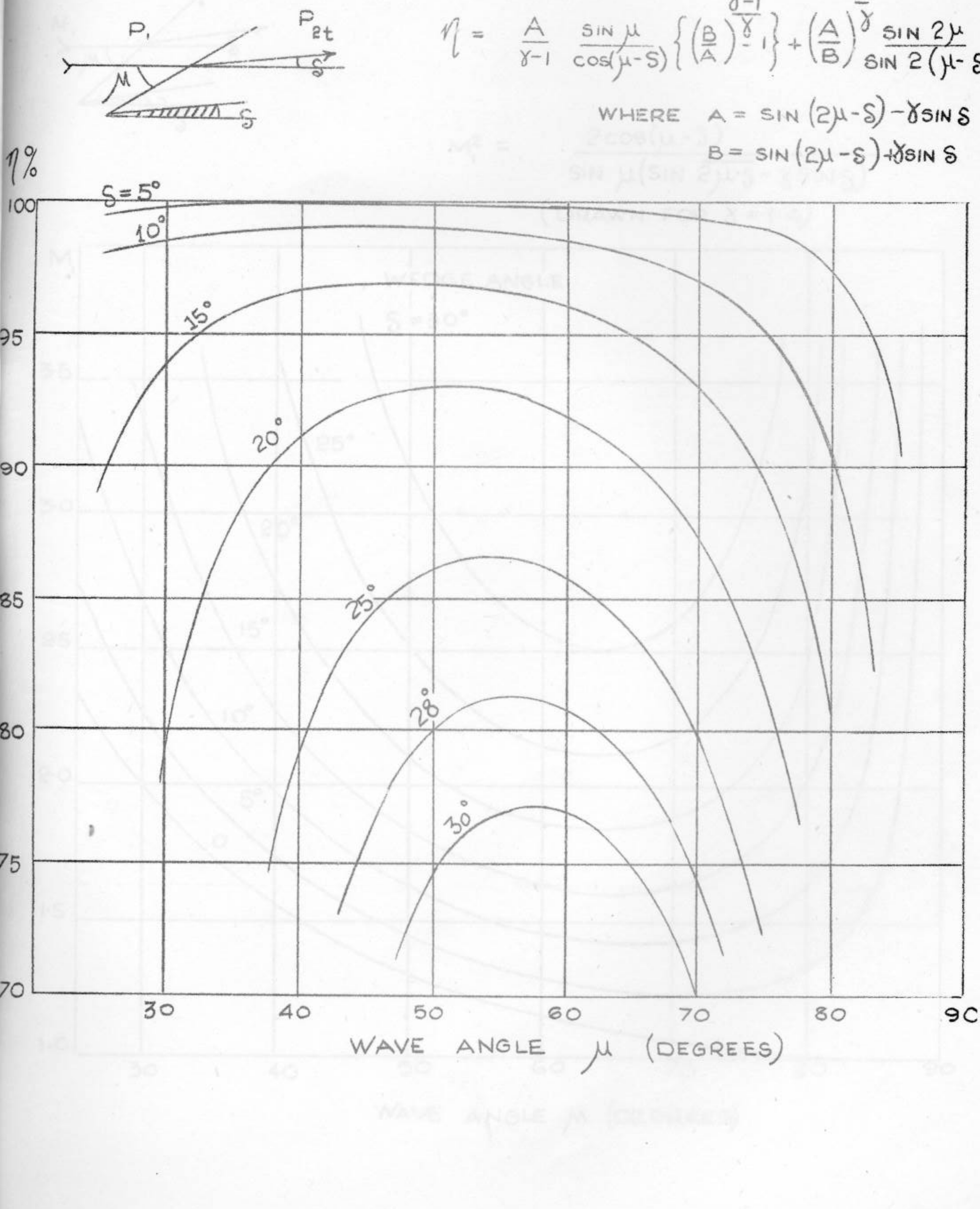

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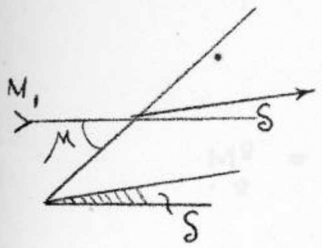
FIG. 2

ORDINARY SHOCK

RELATIONSHIP BETWEEN ENTRY MACH NUMBER WAVE ANGLE AND WEDGE ANGLE



$$M_1^2 = \frac{2\cos(\mu - \delta)}{\sin\mu(\sin \overline{2\mu - \delta} - \gamma \sin\delta)}$$

(DRAWN FOR $\gamma = 1{\cdot}4$)

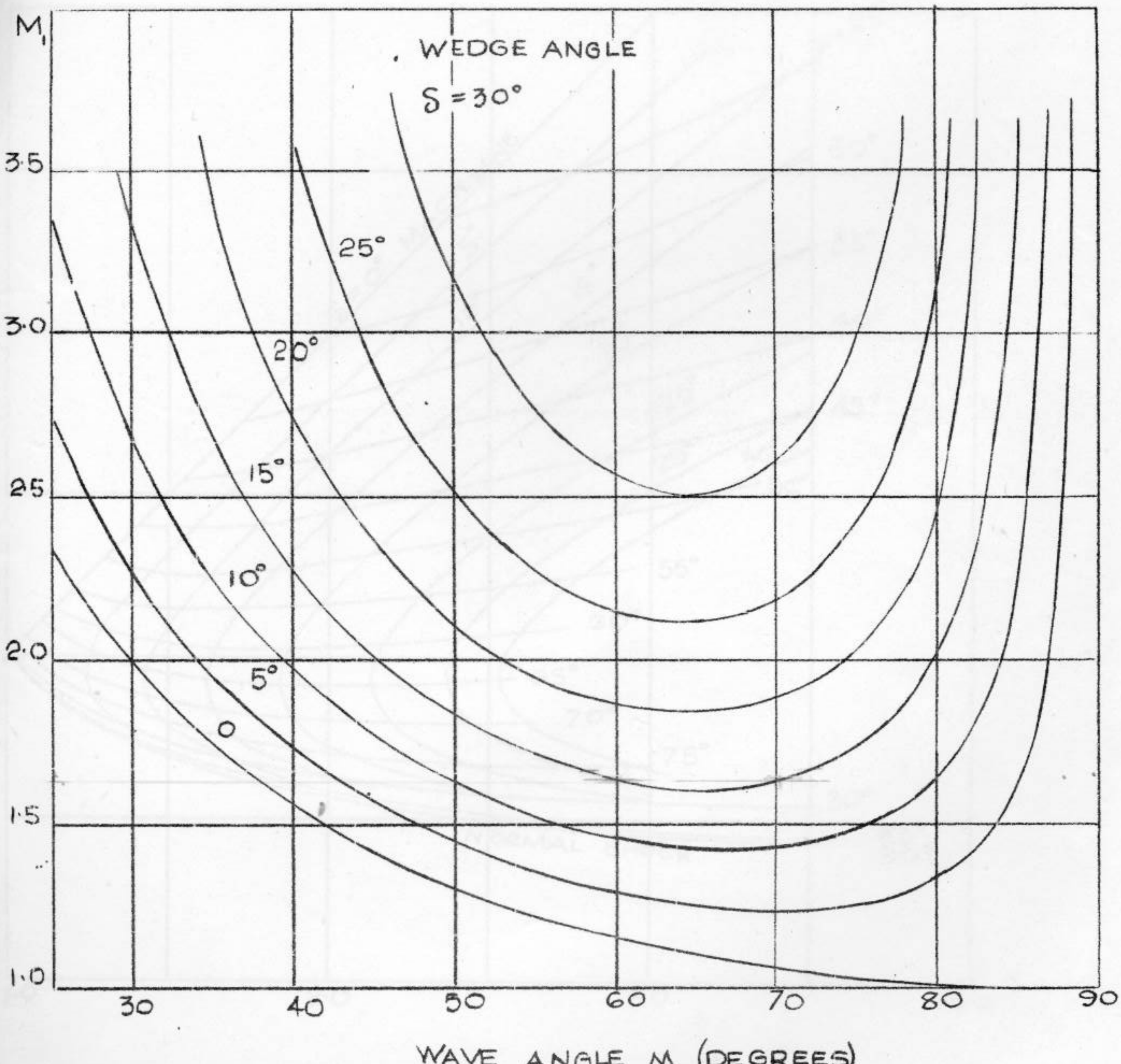


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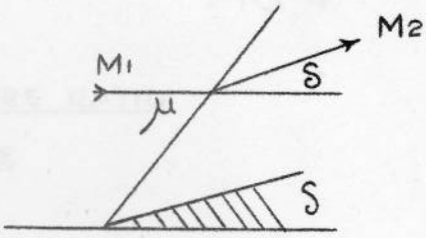
FIG. 3

RELATIONSHIP BETWEEN ENTRY MACH. NUMBER AND EXIT MACH. NUMBER FOR OBLIQUE SHOCK WAVE IN TERMS OF WAVE ANGLE μ AND WEDGE ANGLE δ.

(DRAWN FOR $\gamma = 1{\cdot}4$)

$$M_1^2 = \frac{2\cos\overline{\mu-\delta}\,\operatorname{cosec}\mu}{\sin 2\mu-\delta - \gamma\sin\delta}$$

$$M_2^2 = \frac{2\operatorname{cosec}\overline{\mu-\delta}\,\cos\mu}{\sin\overline{2\mu-\delta} + \gamma\sin\delta}$$



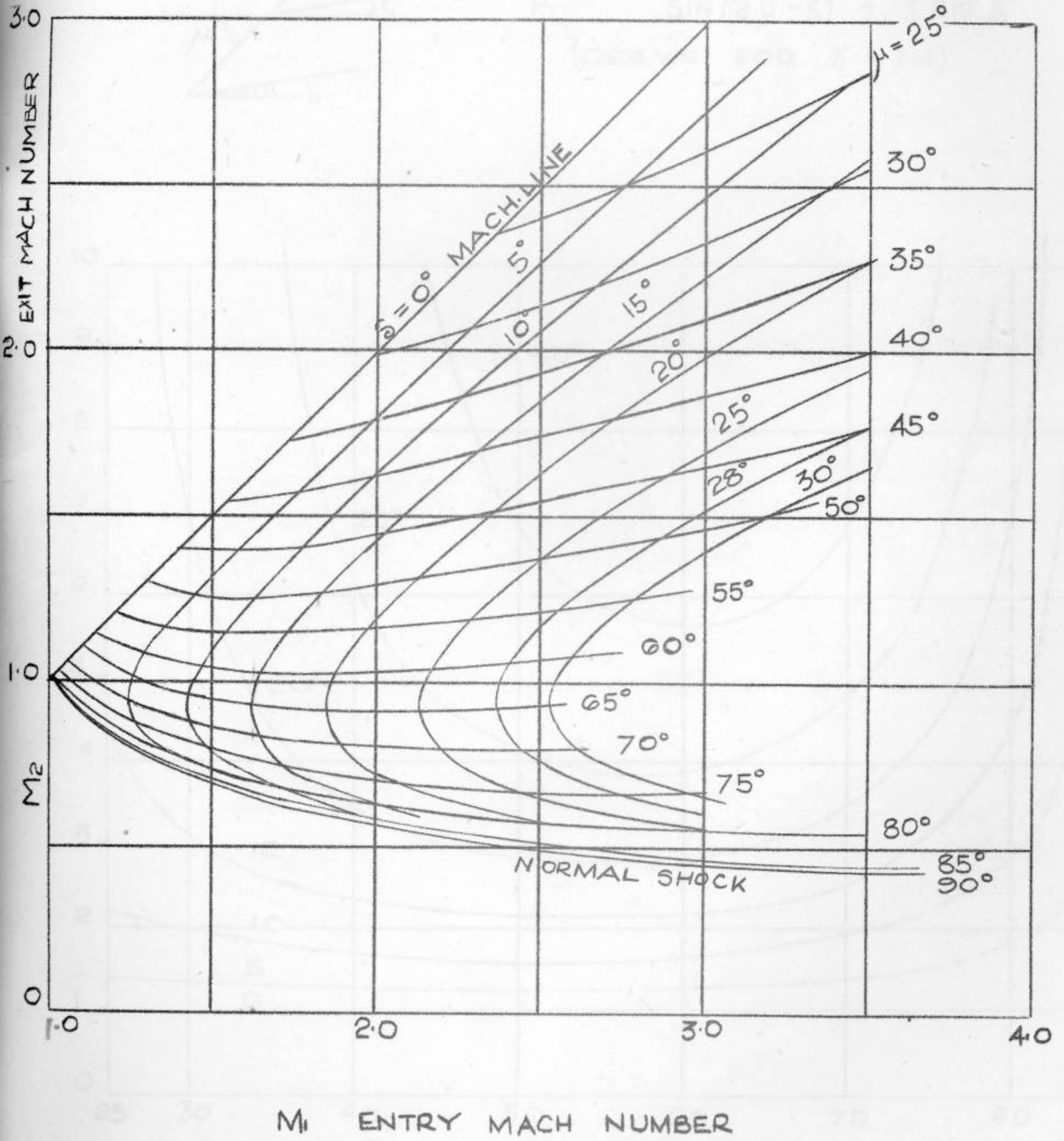


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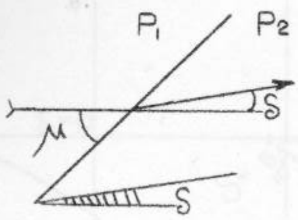
FIG. 4.

ORDINARY SHOCK

RELATIONSHIP BETWEEN PRESSURE RATIO WAVE ANGLE AND WEDGE ANGLE



$$\frac{P_2}{P_1} = \frac{\sin(2\mu-\delta) + \gamma \sin\delta}{\sin(2\mu-\delta) - \gamma \sin\delta}$$

(DRAWN FOR $\gamma = 1.4$)

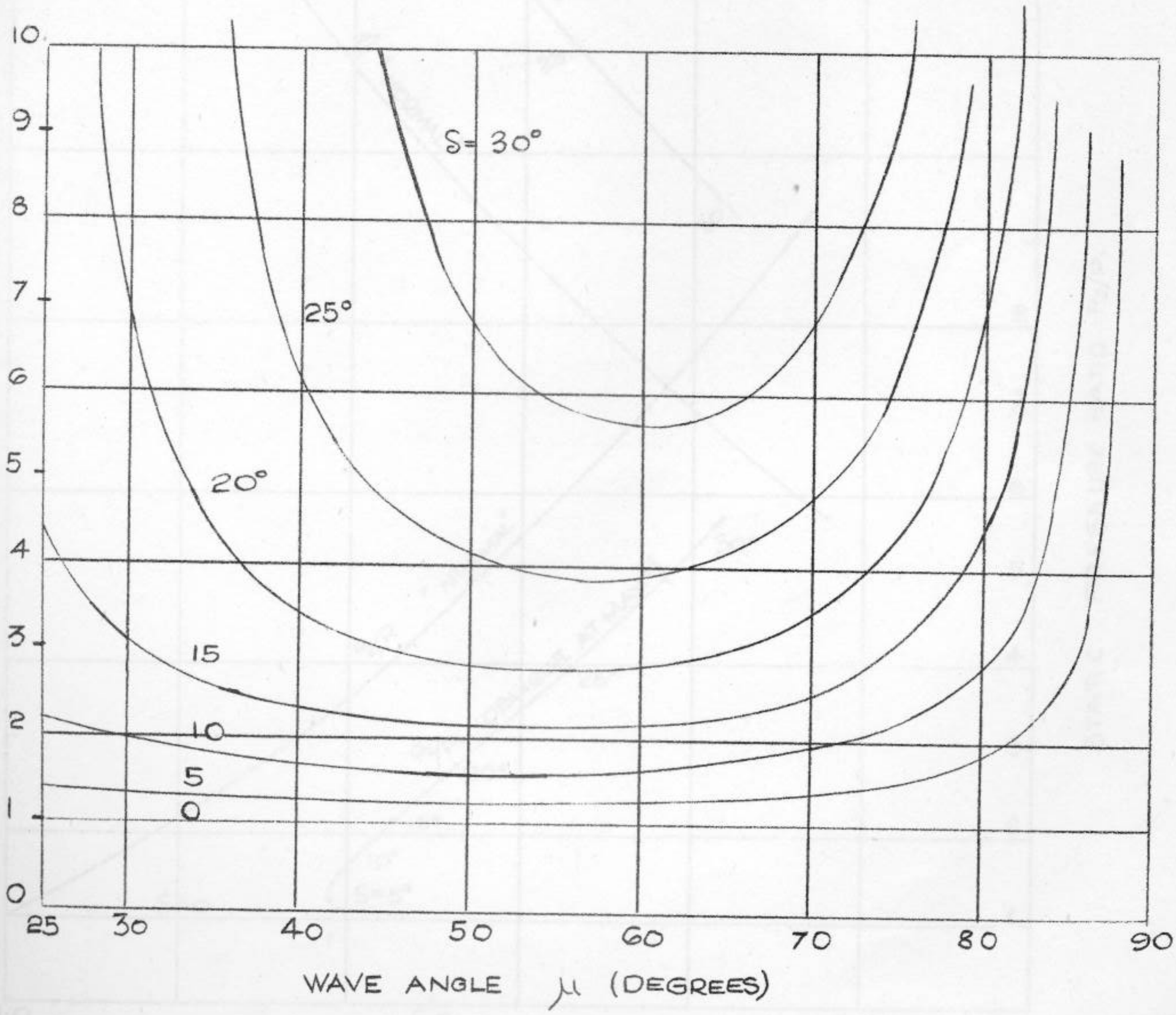
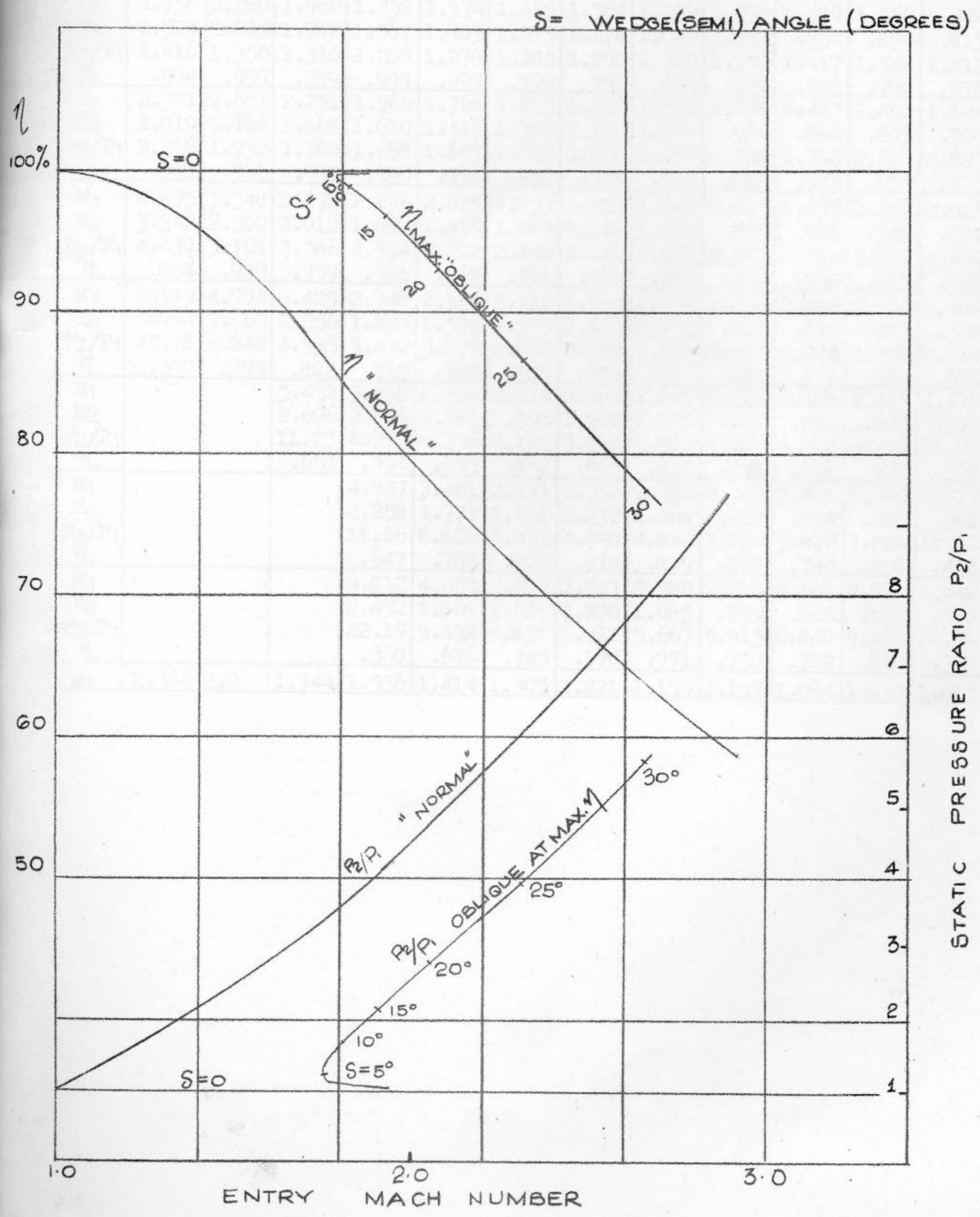


FIG. 5

OBLIQUE SHOCK.

MAXIMUM ISENTROPIC EFFICIENCY
AND COMPARISON WITH NORMAL SHOCK
(FOR $\gamma = 1\cdot4$)

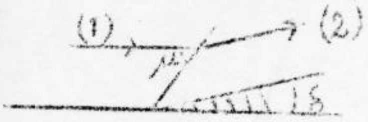


OBLIQUE SHOCK DATA

For adiabatic flow and γ = 1.4

μ = wave angle

δ = wedge (semi) angle



η = isentropic efficiency of compression from P_1 to P_{2t}

	μ	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°
δ = 5°	M_1	2.756	2.280	1.962	1.738	1.574	1.453	1.364	1.299	1.258	1.240	1.252	1.322	1.596
	M_2	2.528	2.085	1.782	1.567	1.403	1.275	1.174	1.090	1.018	.954	.890	.813	.682
	P_2/P_1	1.416	1.350	1.310	1.288	1.278	1.280	1.290	1.312	1.350	1.417	1.540	1.811	2.783
	η	.994	.999	.999	.999	.999	.999	.998	.998	.997	.995	.990	.976	.913
δ = 10°	M_1	3.381	2.681	2.252	1.966	1.768	1.625	1.526	1.460	1.426	1.427	1.480	1.644	2.293
	M_2	2.810	2.166	1.868	1.610	1.417	1.302	1.150	1.050	.964	.884	.803	.705	.555
	P_2/P_1	2.216	1.932	1.780	1.698	1.657	1.643	1.656	1.698	1.782	1.932	2.217	2.893	5.923
	η	.981	.986	.989	.990	.991	.990	.988	.985	.981	.972	.953	.905	.727
δ = 15°	M_1	4.695	3.348	2.678	2.276	2.015	1.837	1.718	1.644	1.615	1.635	1.732	2.016	3.373
	M_2	3.342	2.500	2.012	1.690	1.459	1.285	1.146	1.032	.933	.841	.748	.640	.486
	P_2/P_1	4.430	3.101	2.586	2.334	2.202	2.142	2.142	2.201	2.332	2.586	3.101	4.430	13.02
	η	.894	.939	.959	.966	.969	.968	.966	.960	.950	.930	.893	.806	.497
δ = 20°	M_1	14.83	4.902	3.425	2.748	2.358	2.112	1.960	1.870	1.840	1.882	2.034	2.489	6.329
	M_2	4.64	3.665	2.256	1.825	1.536	1.325	1.163	1.031	.918	.815	.713	.598	.443
	P_2/P_1	45.78	6.840	4.335	3.472	3.079	2.892	2.837	2.892	3.077	3.472	4.332	6.840	46.17
	η	.330	.793	.882	.913	.926	.929	.928	.919	.902	.871	.816	.691	.258
δ = 25°	M_1			5.452	3.635	2.906	2.514	2.286	2.162	2.124	2.186	2.417	3.175	
	M_2			2.696	2.048	1.661	1.398	1.202	1.048	.919	.804	.692	.571	
	P_2/P_1			11.25	6.195	4.756	4.160	3.924	3.922	4.160	4.762	6.194	11.25	
	η			.658	.794	.843	.862	.866	.858	.838	.798	.721	.558	
δ = 28°	M_1				4.827	3.461	2.871	2.556	2.392	2.343	2.421	2.720	3.817	
	M_2				2.258	1.771	1.461	1.239	1.068	.927	.804	.686	.561	
	P_2/P_1				11.06	6.826	5.475	4.948	4.842	5.094	5.870	7.886	16.32	
	η				.647	.758	.800	.814	.809	.788	.744	.658	.466	
δ = 30°	M_1				6.812	4.059	3.199	2.787	2.582	2.518	2.608	2.970	4.448	
	M_2				2.452	1.869	1.519	1.270	1.085	.935	.806	.684	.556	
	P_2/P_1				22.19	9.432	6.838	5.912	5.667	5.913	6.840	9.428	22.20	
	η				.360	.676	.745	.770	.771	.750	.702	.610	.398	
δ = 0°	M_1	2.366	2.0	1.744	1.556	1.414	1.305	1.221	1.155	1.103	1.064	1.035	1.015	1.004