Report No. 44

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THE COLLEGE OF AERONAUTICS

CRANFIELD

The evaluation of matrix elements for the analysis of swept-back wing structures by the method of oblique coordinates

- by -

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SUMMARY

This report is an addition to the College of Aeronautics Report No. 31 [1]. The purpose of the report is to enable one to obtain the matrix elements used in the analysis of swept-back wing structures by oblique coordinates in a very rapid manner.

Part I gives the numerical evaluation of the matrix element A_{ij} together with tables and graphs. The parameters chosen are those common to aircraft design.

Part II gives the numerical evaluation of a_{ij} and $|a_{ij}|$ together with tables and graphs.

Part III gives approximations for certain section constants, and the methods of extrapolation and interpolation of the constants from the curves of figures 3 - 14. These simplified expressions give a reasonable degree of accuracy and are satisfactory for preliminary investigations.

YSB

NOTATION

A_S = section area of the stringer.

A_R = section area of the rib boom.

a_S = stringer pitch measured parallel to the ribs.

a_R = rib pitch measured parallel to the stringers.

t = thickness of the top and bottom skins.

$(\pi/2-\alpha)$ = angle of sweepback of the mid-chord point line (see fig. 1).

E = Modulus of Elasticity.

σ = Poissons ratio. Value taken as 0.3 for all calculations.

A_{ij}, a_{ij} $(i = 1,2,3; \; j = 1,2,3)$ = Matrix elements for the determination of the stresses and strains.

Ox, Oy, Oz = axes of reference.

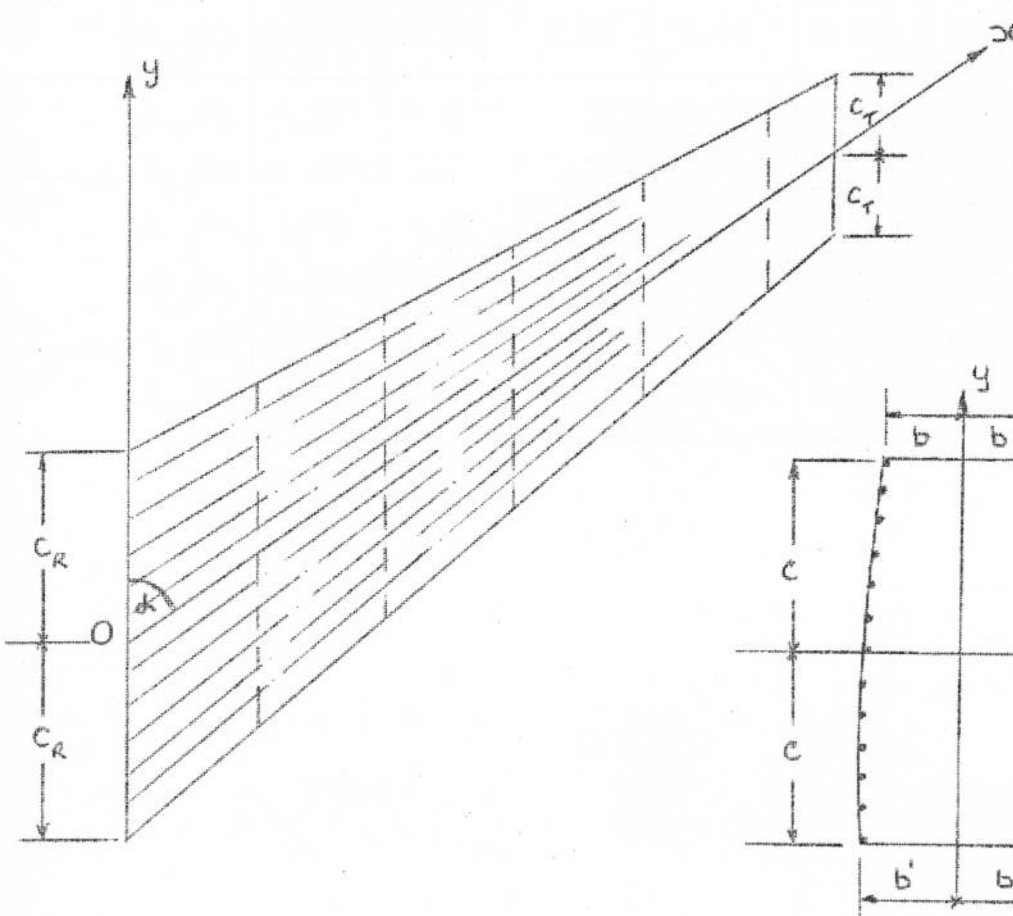


Figure 1

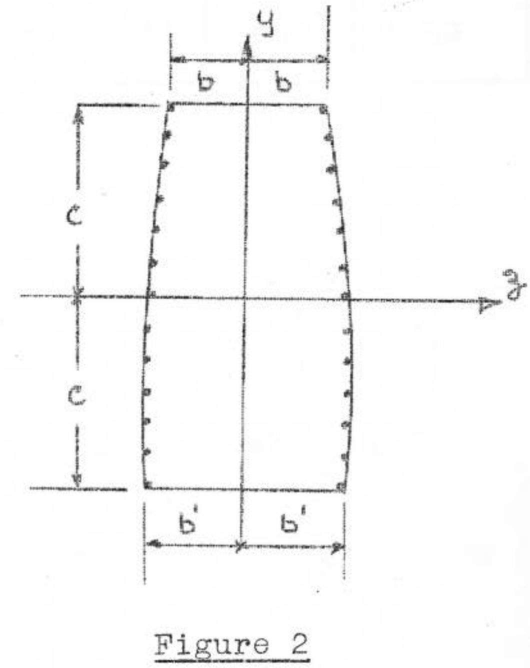


Figure 2

Part I.

Values of the matrix elements A_{ij} for varying values of α^o and $\frac{A_S}{a_S t}$, $\frac{A_R}{a_R t}$.

TABLE I

α^o		30	35	45	55	60	75	90
$\frac{A_S}{a_S t}=.5$	$A_{11}Et$	0.930	0.887	0.815	0.757	0.732	0.679	0.660
	$A_{12}Et$	0.516	0.411	0.229	.0779	.0149	-0.121	-0.168
	$A_{13}Et$	1.50	1.35	1.09	.849	.729	0.367	0
	$A_{22}Et$	1.18	1.14	1.06	0.976	.938	.847	0.812
$\frac{A_R}{a_R t}=.2$	$A_{23}Et$	1.75	1.61	1.35	1.07	0.930	0.477	0
	$A_{33}Et$	3.50	3.31	3.03	2.84	2.77	2.64	2.74
$\frac{A_S}{a_S t}=.5$	$A_{11}Et$	0.858	0.841	0.801	0.755	0.732	0.674	0.652
	$A_{12}Et$	0.350	0.283	0.189	.0560	.0108	-0.0901	-0.126
	$A_{13}Et$	1.25	1.17	1.01	.825	.725	0.384	0
	$A_{22}Et$	0.802	0.783	0.743	.702	.682	.632	0.613
$\frac{A_R}{a_R t}=.6$	$A_{23}Et$	1.19	1.11	0.947	0.771	0.676	0.357	0
	$A_{33}Et$	2.66	2.60	2.52	2.51	2.52	2.57	2.71
$\frac{A_S}{a_S t}=.5$	$A_{11}Et$	0.821	0.817	0.793	0.754	0.732	0.672	0.646
	$A_{12}Et$	0.265	0.215	.124	.0437	.0085	-0.072	-0.102
	$A_{13}Et$	1.12	1.08	.960	.811	.722	0.394	0
	$A_{22}Et$	0.607	0.596	.573	.548	.536	.505	0.492
$\frac{A_R}{a_R t}=1.0$	$A_{23}Et$	0.902	0.846	0.730	0.602	0.531	0.285	0
	$A_{33}Et$	2.23	2.22	2.25	2.33	2.38	2.53	2.69

/Table II

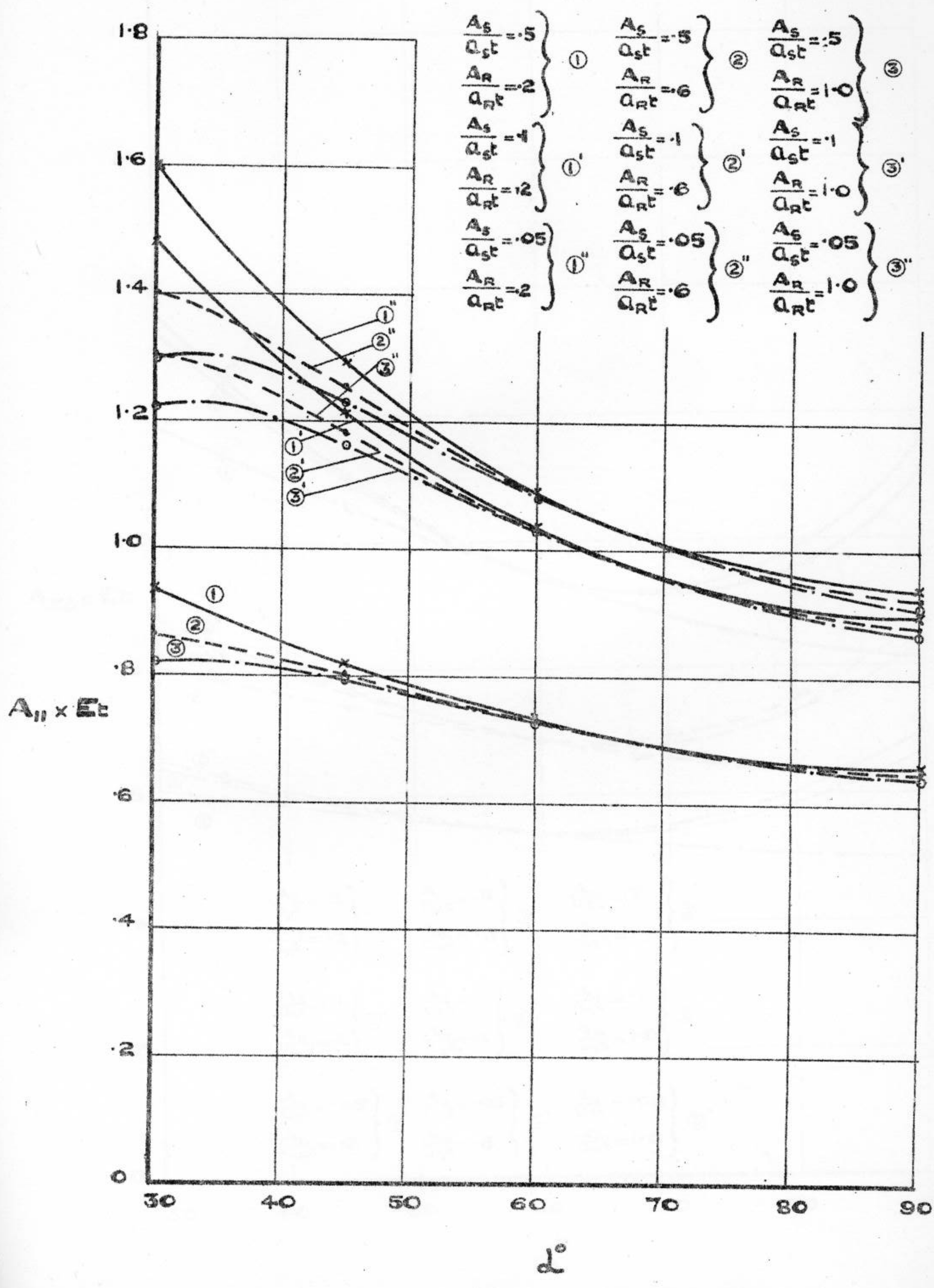
TABLE II

α^{o}		30	35	45	55	60	75	90
	$A_{11}Et$	1.48	1.38	1.21	1.08	1.04	0.932	0.897
$\frac{A_S}{a_S t}=.1$	$A_{12}Et$	0.822	0.638	0.339	0.112	0.0210	-0.166	-0.228
	$A_{13}Et$	2.38	2.10	1.62	1.22	1.03	0.504	0
	$A_{22}Et$	1.35	1.25	1.09	0.980	0.938	.855	1.10
$\frac{A_R}{a_R t}=.2$	$A_{23}Et$	2.25	1.96	1.50	1.11	0.936	0.453	0
	$A_{33}Et$	4.92	4.45	3.74	3.26	3.07	2.71	2.80
	$A_{11}Et$	1.31	1.27	1.18	1.08	1.03	0.923	0.881
$\frac{A_S}{a_S t}=.1$	$A_{12}Et$	0.534	0.426	0.236	0.0803	0.0153	-0.123	-0.171
	$A_{13}Et$	1.90	1.76	1.48	1.18	1.02	0.526	0
	$A_{22}Et$	0.876	0.831	0.759	0.704	0.682	.637	0.829
$\frac{A_R}{a_R t}=.6$	$A_{23}Et$	1.46	1.31	1.04	0.798	0.680	0.338	0
	$A_{33}Et$	3.61	3.42	3.12	2.90	2.82	2.65	2.75
	$A_{11}Et$	1.22	1.21	1.16	1.08	1.03	0.919	0.872
$\frac{A_S}{a_S t}=.1$	$A_{12}Et$	0.395	0.320	0.181	0.0626	0.012	-0.0984	-0.137
	$A_{13}Et$	1.67	1.60	1.41	1.16	1.02	0.539	0
	$A_{22}Et$	0.649	0.624	0.582	0.549	0.536	.508	0.664
$\frac{A_R}{a_R t}=1.0$	$A_{23}Et$	1.08	0.983	0.800	0.622	0.535	0.269	0
	$A_{33}Et$	2.98	2.91	2.79	2.70	2.67	2.62	2.72

TABLE III

α^{o}		30	35	45	55	60	75	90
	$A_{11}Et$	1.60	1.48	1.29	1.15	1.09	0.977	0.939
$\frac{A_S}{a_S t}=.05$	$A_{12}Et$	0.887	0.685	0.361	0.118	0.0222	-0.174	-0.238
	$A_{13}Et$	2.57	2.25	1.73	1.29	1.09	0.528	0
	$A_{22}Et$	1.39	1.27	1.10	0.981	0.938	.856	1.16
$\frac{A_R}{a_R t}=.2$	$A_{23}Et$	2.35	2.03	1.53	1.12	0.937	0.449	0
	$A_{33}Et$	5.23	4.69	3.88	3.33	3.13	2.73	2.80
	$A_{11}Et$	1.40	1.35	1.25	1.14	1.09	0.968	0.922
$\frac{A_S}{a_S t}=.05$	$A_{12}Et$	0.571	0.455	0.251	0.0849	0.0161	-0.129	-0.179
	$A_{13}Et$	2.04	1.88	1.57	1.25	1.08	0.552	0
	$A_{22}Et$	0.892	0.841	0.762	0.704	0.682	.638	0.867
$\frac{A_R}{a_R t}=.6$	$A_{23}Et$	1.512	1.35	1.06	0.803	0.681	0.334	0
	$A_{33}Et$	3.80	3.59	3.24	2.97	2.87	2.67	2.75
	$A_{11}Et$	1.30	1.29	1.23	1.14	1.09	0.963	0.911
$\frac{A_S}{a_S t}=.05$	$A_{12}Et$	0.421	0.340	0.192	0.0662	0.0127	-0.103	-0.143
	$A_{13}Et$	1.78	1.70	1.49	1.23	1.08	0.565	0
	$A_{22}Et$	0.657	0.629	0.584	0.549	0.536	.508	0.694
$\frac{A_R}{a_R t}=1.0$	$A_{23}Et$	1.11	1.01	0.813	0.626	0.535	0.266	0
	$A_{33}Et$	3.13	3.04	2.89	2.77	2.73	2.63	2.72

FIG 3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$\frac{A_S}{a_S t} = \cdot 5$, $\frac{A_R}{a_R t} = \cdot 6$ ②
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$\frac{A_S}{a_S t} = \cdot 05$, $\frac{A_R}{a_R t} = \cdot 6$ ②"
$\frac{A_S}{a_S t} = \cdot 05$, $\frac{A_R}{a_R t} = 1\cdot 0$ ③"
$A_{11} \times Et$
α°
1·8
1·6
1·4
1·2
1·0
·8
·6
·4
·2
0
30
40
50
60
70
80
90

FIG 4

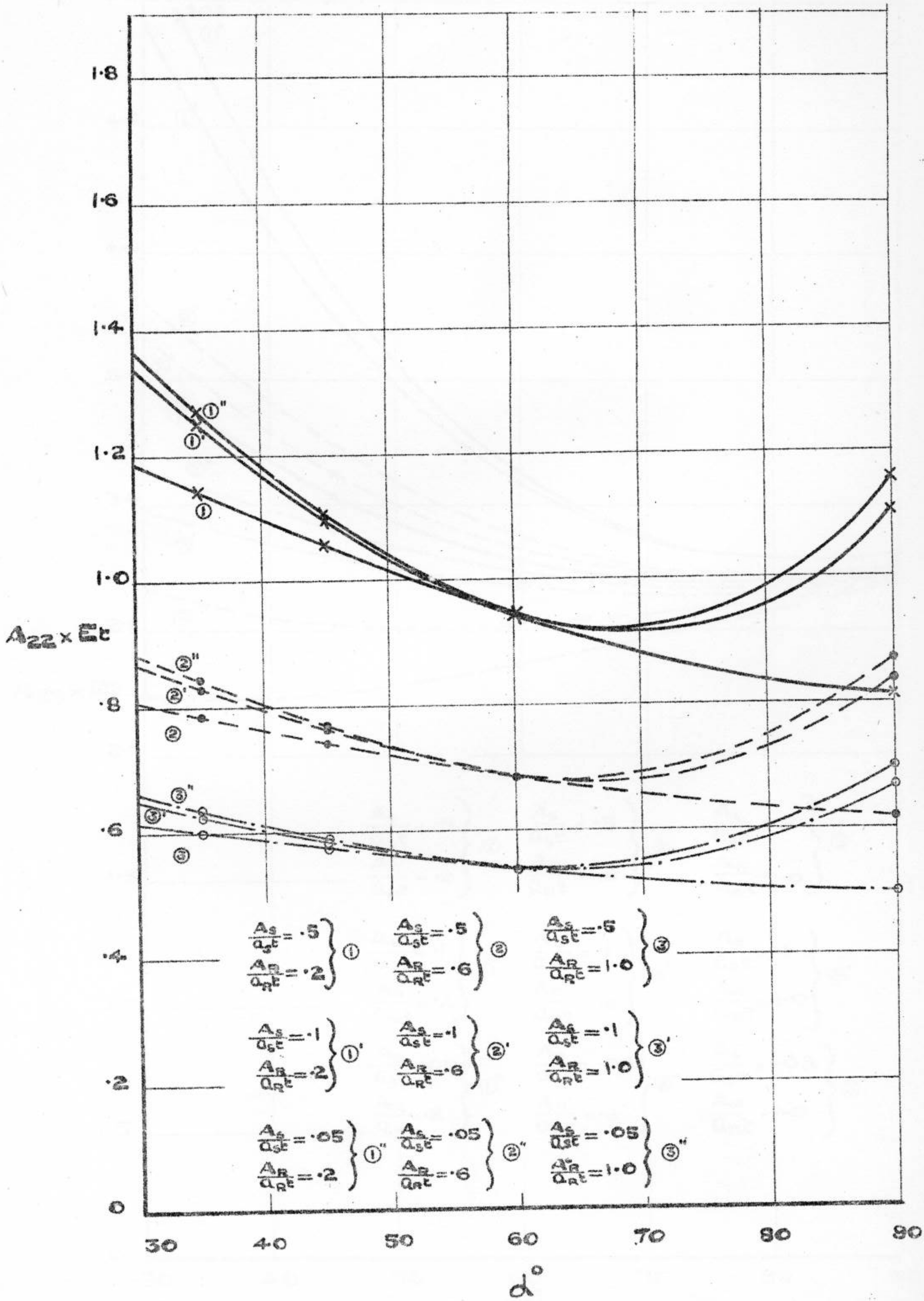

1·8
1·6
1·4
1·2
1·0
·8
·6
·4
·2
0
$A_{22} \times Et$
① ①' ①"
② ②' ②"
③ ③' ③"
$\frac{A_S}{a_S t} = \cdot 5$, $\frac{A_R}{a_R t} = \cdot 2$ ①
$\frac{A_S}{a_S t} = \cdot 5$, $\frac{A_R}{a_R t} = \cdot 6$ ②
$\frac{A_S}{a_S t} = \cdot 5$, $\frac{A_R}{a_R t} = 1 \cdot 0$ ③
$\frac{A_S}{a_S t} = \cdot 1$, $\frac{A_R}{a_R t} = \cdot 2$ ①'
$\frac{A_S}{a_S t} = \cdot 1$, $\frac{A_R}{a_R t} = \cdot 6$ ②'
$\frac{A_S}{a_S t} = \cdot 1$, $\frac{A_R}{a_R t} = 1 \cdot 0$ ③'
$\frac{A_S}{a_S t} = \cdot 05$, $\frac{A_R}{a_R t} = \cdot 2$ ①"
$\frac{A_S}{a_S t} = \cdot 05$, $\frac{A_R}{a_R t} = \cdot 6$ ②"
$\frac{A_S}{a_S t} = \cdot 05$, $\frac{A_R}{a_R t} = 1 \cdot 0$ ③"
30
40
50
60
70
80
90
α°

FIG 5

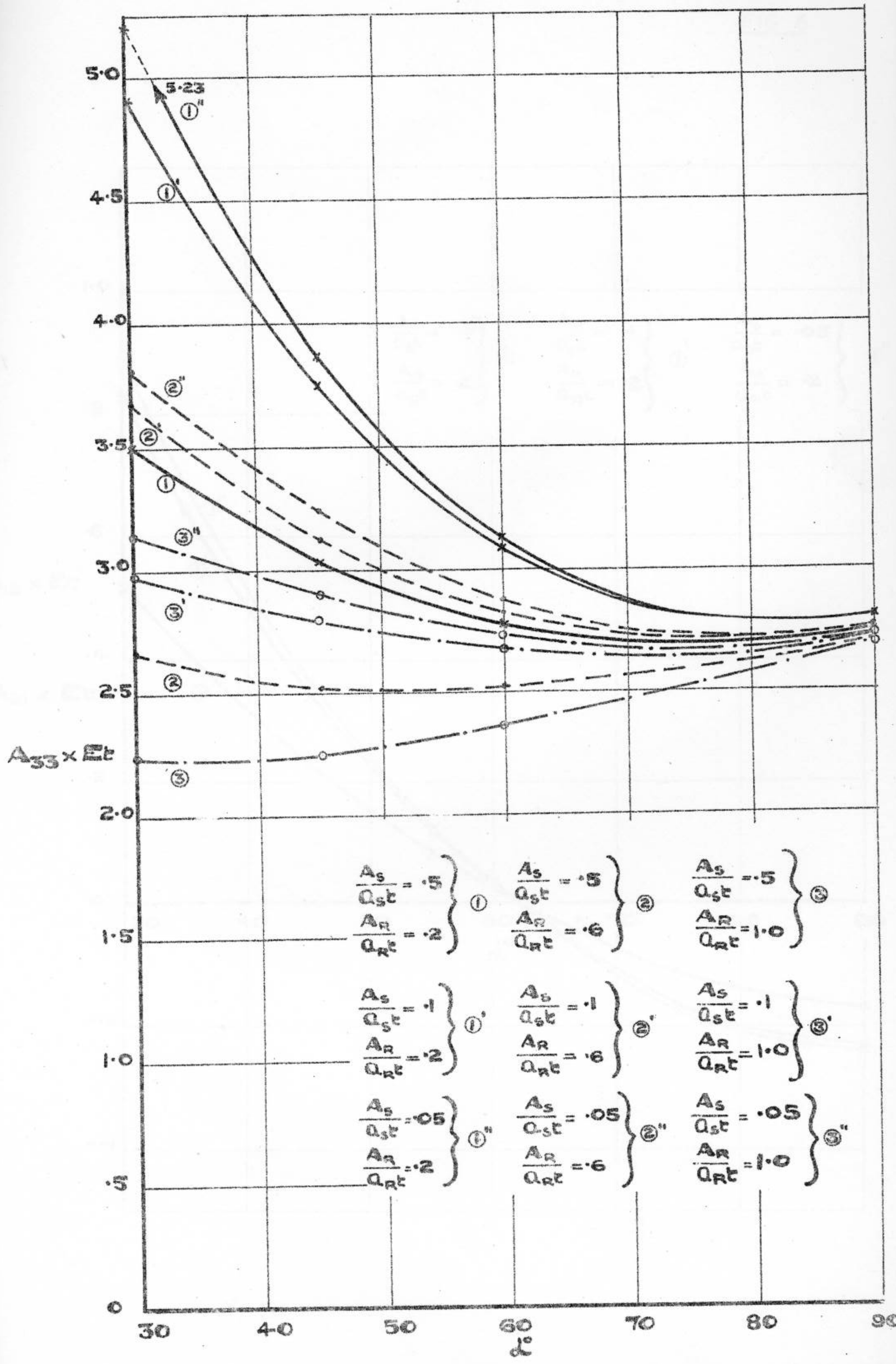

5·0
4·5
4·0
3·5
3·0
2·5
2·0
1·5
1·0
·5
0
5·23
①"
①'
①
②"
②'
②
③"
③'
③
$A_{33} \times Et$
30
40
50
60
70
80
90
$\alpha°$
$\frac{A_S}{a_S t} = \cdot 5$, $\frac{A_R}{a_R t} = \cdot 2$ ①
$\frac{A_S}{a_S t} = \cdot 5$, $\frac{A_R}{a_R t} = \cdot 6$ ②
$\frac{A_S}{a_S t} = \cdot 5$, $\frac{A_R}{a_R t} = 1\cdot 0$ ③
$\frac{A_S}{a_S t} = \cdot 1$, $\frac{A_R}{a_R t} = \cdot 2$ ①'
$\frac{A_S}{a_S t} = \cdot 1$, $\frac{A_R}{a_R t} = \cdot 6$ ②'
$\frac{A_S}{a_S t} = \cdot 1$, $\frac{A_R}{a_R t} = 1\cdot 0$ ③'
$\frac{A_S}{a_S t} = \cdot 05$, $\frac{A_R}{a_R t} = \cdot 2$ ①"
$\frac{A_S}{a_S t} = \cdot 05$, $\frac{A_R}{a_R t} = \cdot 6$ ②"
$\frac{A_S}{a_S t} = \cdot 05$, $\frac{A_R}{a_R t} = 1\cdot 0$ ③"

FIG 6

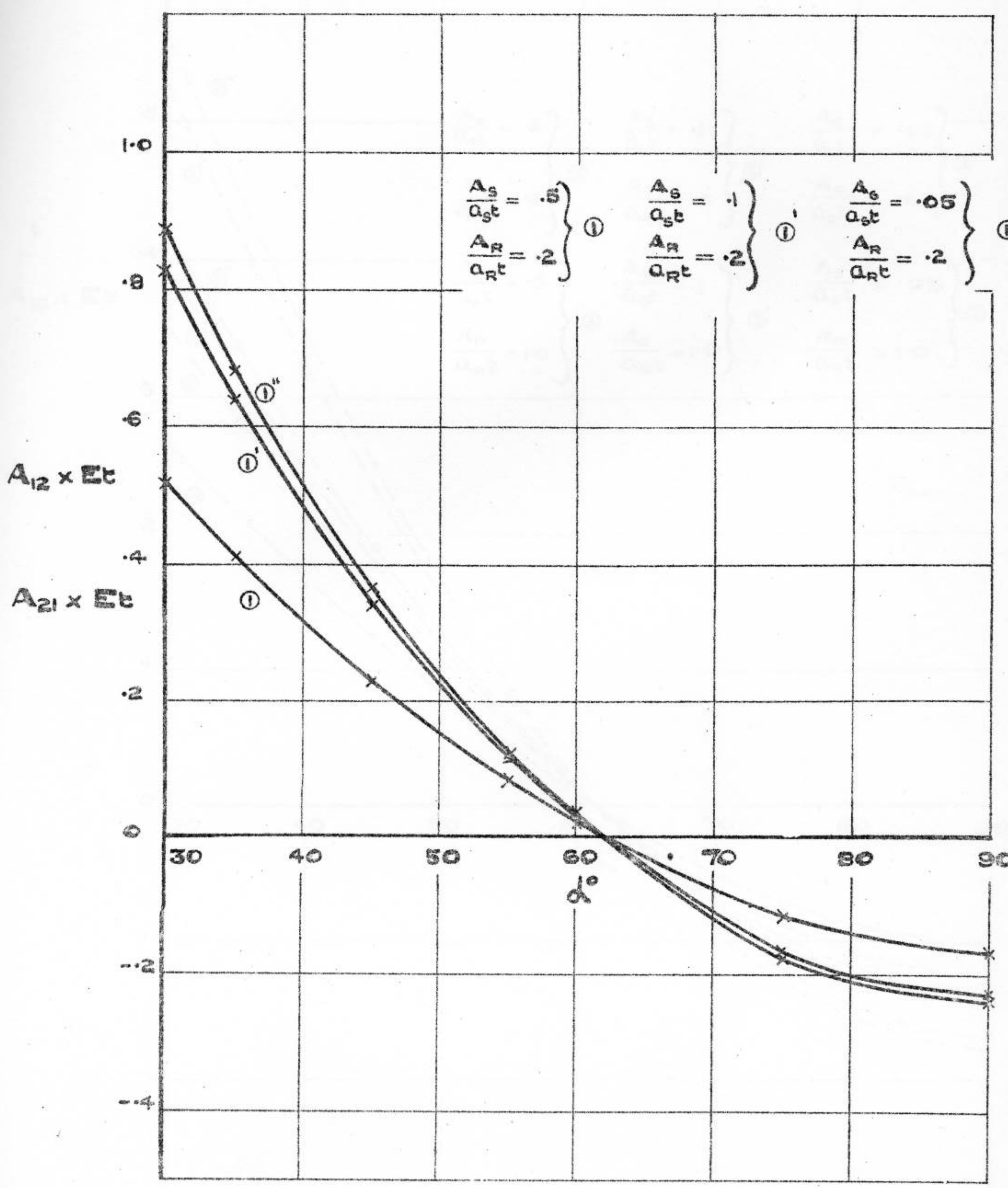
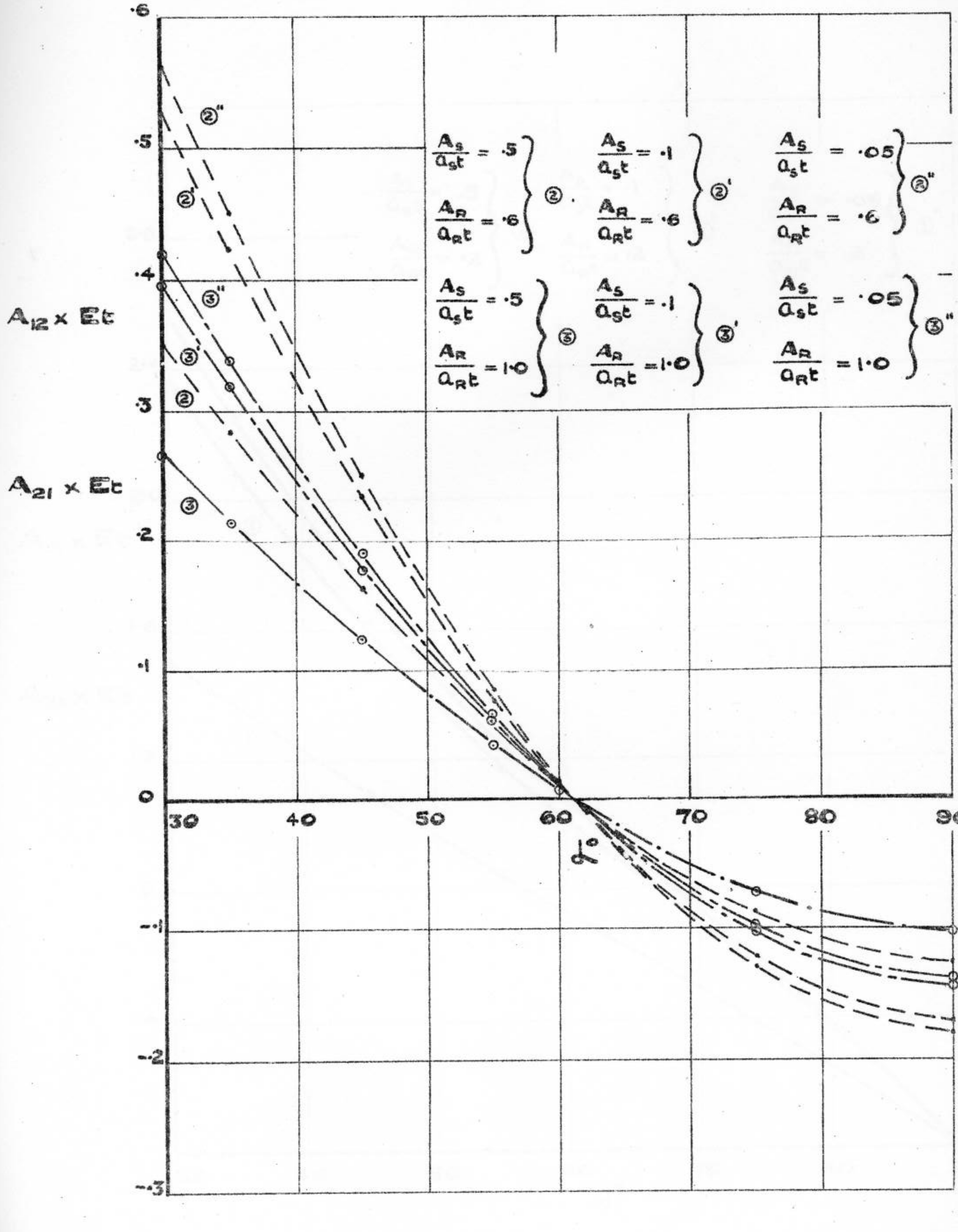

1·0
·8
·6
·4
·2
0
-·2
-·4
30
40
50
60
70
80
90
α°
$A_{12} \times Et$
$A_{21} \times Et$
$\frac{A_S}{a_S t} = \cdot 5$, $\frac{A_R}{a_R t} = \cdot 2$ ①
$\frac{A_S}{a_S t} = \cdot 1$, $\frac{A_R}{a_R t} = \cdot 2$ ①'
$\frac{A_S}{a_S t} = \cdot 05$, $\frac{A_R}{a_R t} = \cdot 2$ ①''
①
①'
①''

FIG 7



·6
·5
·4
$A_{12} \times Et$
·3
$A_{21} \times Et$
·2
·1
0
-·1
-·2
-·3
30
40
50
60
70
80
90
$\alpha°$
$\frac{A_S}{a_S t} = ·5$, $\frac{A_R}{a_R t} = ·6$ ②
$\frac{A_S}{a_S t} = ·1$, $\frac{A_R}{a_R t} = ·6$ ②'
$\frac{A_S}{a_S t} = ·05$, $\frac{A_R}{a_R t} = ·6$ ②"
$\frac{A_S}{a_S t} = ·5$, $\frac{A_R}{a_R t} = 1·0$ ③
$\frac{A_S}{a_S t} = ·1$, $\frac{A_R}{a_R t} = 1·0$ ③'
$\frac{A_S}{a_S t} = ·05$, $\frac{A_R}{a_R t} = 1·0$ ③"
②"
②'
③"
③'
②
③

FIG 8

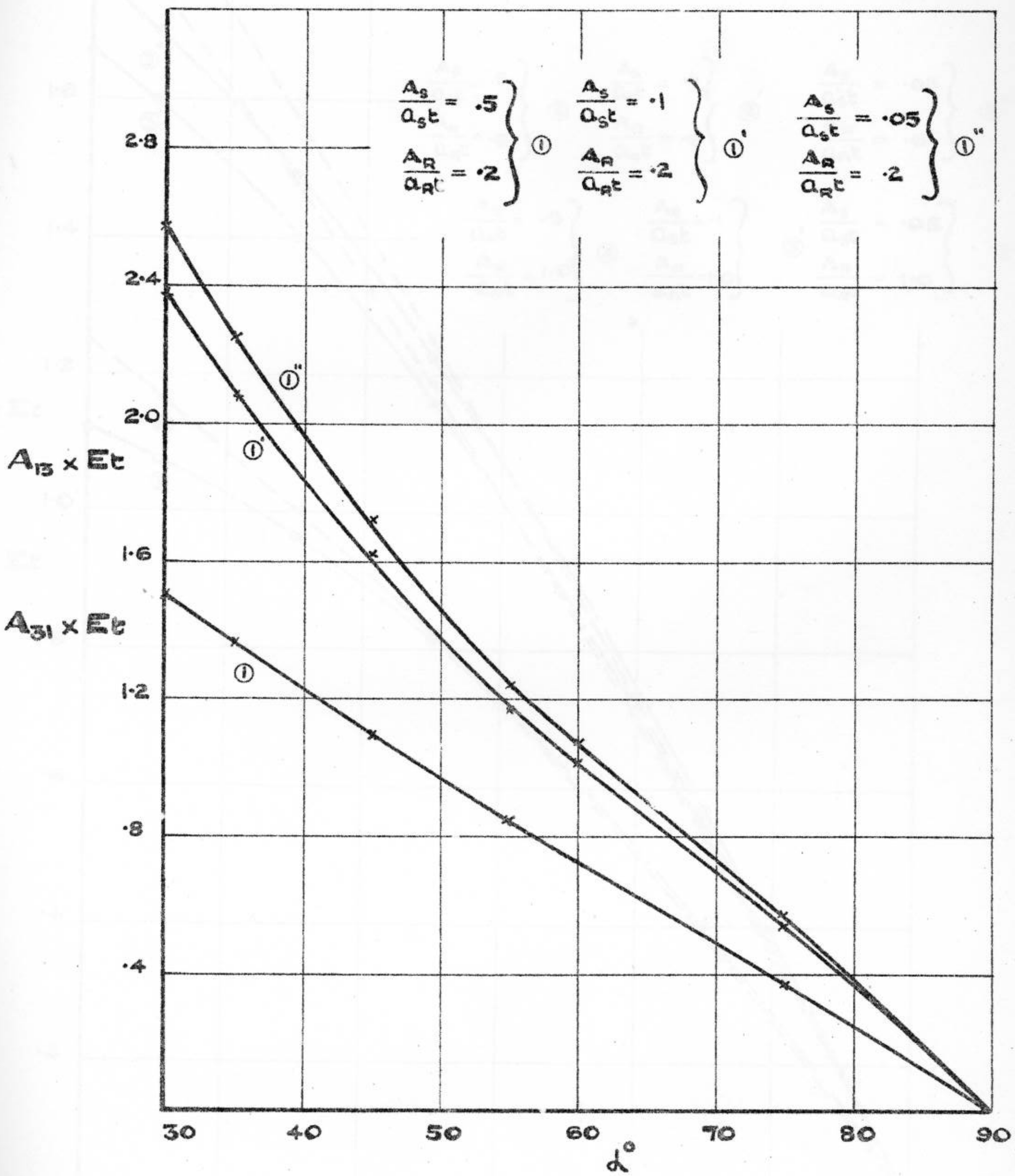

$\frac{A_s}{a_s t} = .5$
$\frac{A_R}{a_R t} = \cdot 2$
①
$\frac{A_s}{a_s t} = \cdot 1$
$\frac{A_R}{a_R t} = \cdot 2$
①'
$\frac{A_s}{a_s t} = \cdot 05$
$\frac{A_R}{a_R t} = \cdot 2$
①"
2·8
2·4
2·0
1·6
1·2
·8
·4
$A_{13} \times Et$
$A_{31} \times Et$
①"
①'
①
30
40
50
60
70
80
90
α°

FIG 9

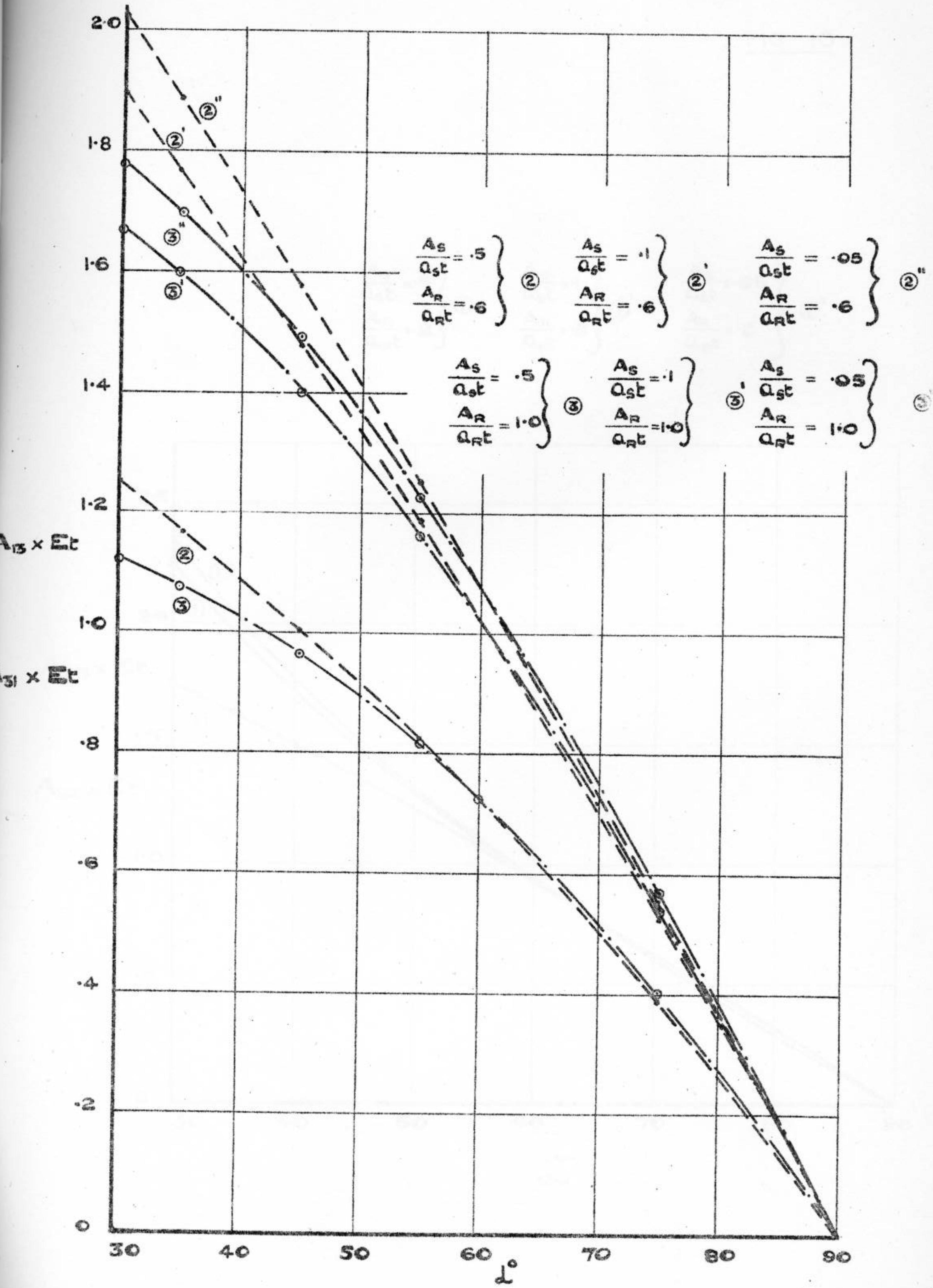

2·0
1·8
1·6
1·4
1·2
1·0
·8
·6
·4
·2
0
30
40
50
60
70
80
90
α°
$A_{13} \times Et$
$A_{31} \times Et$
$\frac{A_S}{a_S t} = \cdot 5$, $\frac{A_R}{a_R t} = \cdot 6$ ②
$\frac{A_S}{a_S t} = \cdot 1$, $\frac{A_R}{a_R t} = \cdot 6$ ②'
$\frac{A_S}{a_S t} = \cdot 05$, $\frac{A_R}{a_R t} = \cdot 6$ ②''
$\frac{A_S}{a_S t} = \cdot 5$, $\frac{A_R}{a_R t} = 1\cdot 0$ ③
$\frac{A_S}{a_S t} = \cdot 1$, $\frac{A_R}{a_R t} = 1\cdot 0$ ③'
$\frac{A_S}{a_S t} = \cdot 05$, $\frac{A_R}{a_R t} = 1\cdot 0$ ③''

FIG 10

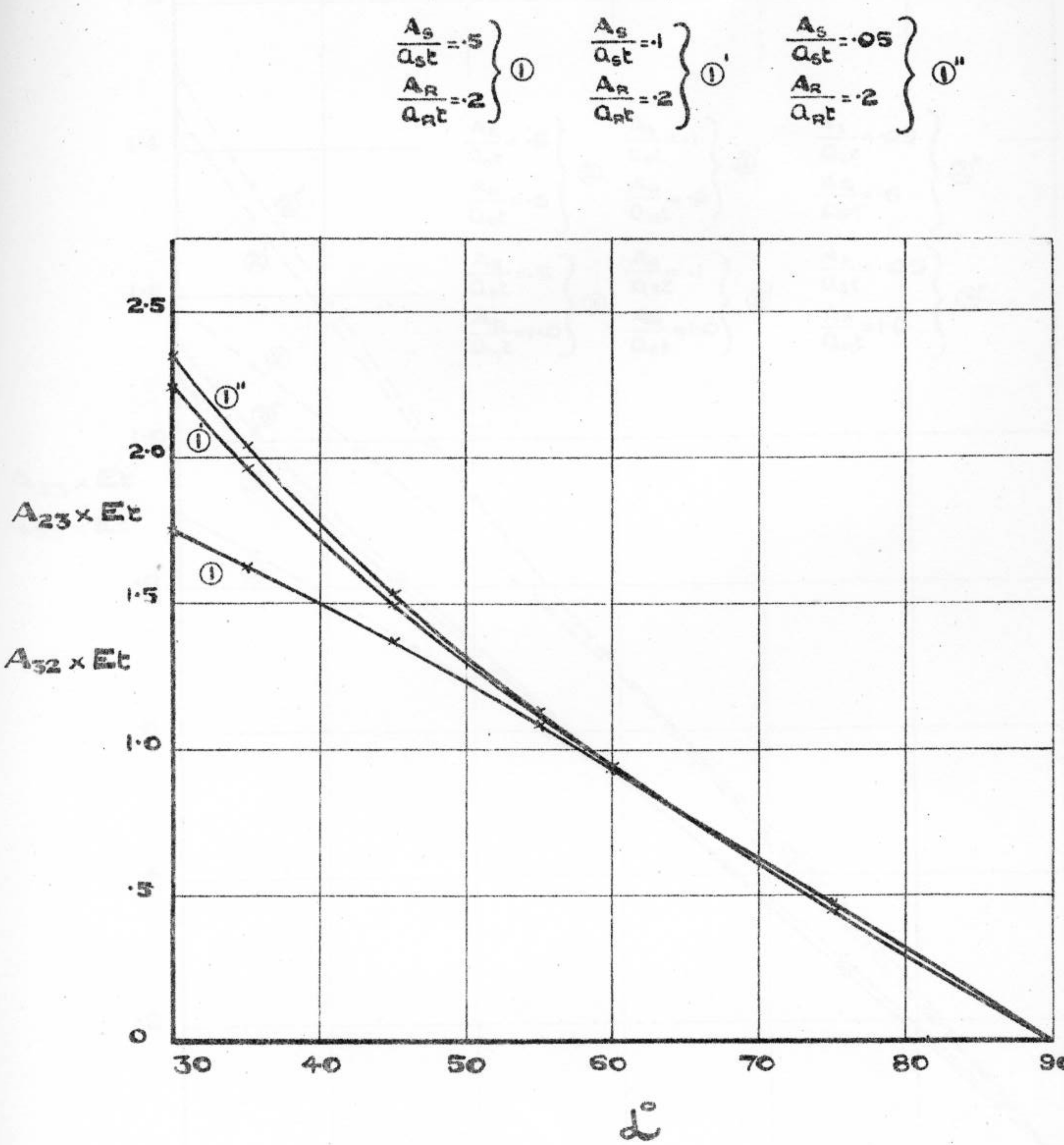
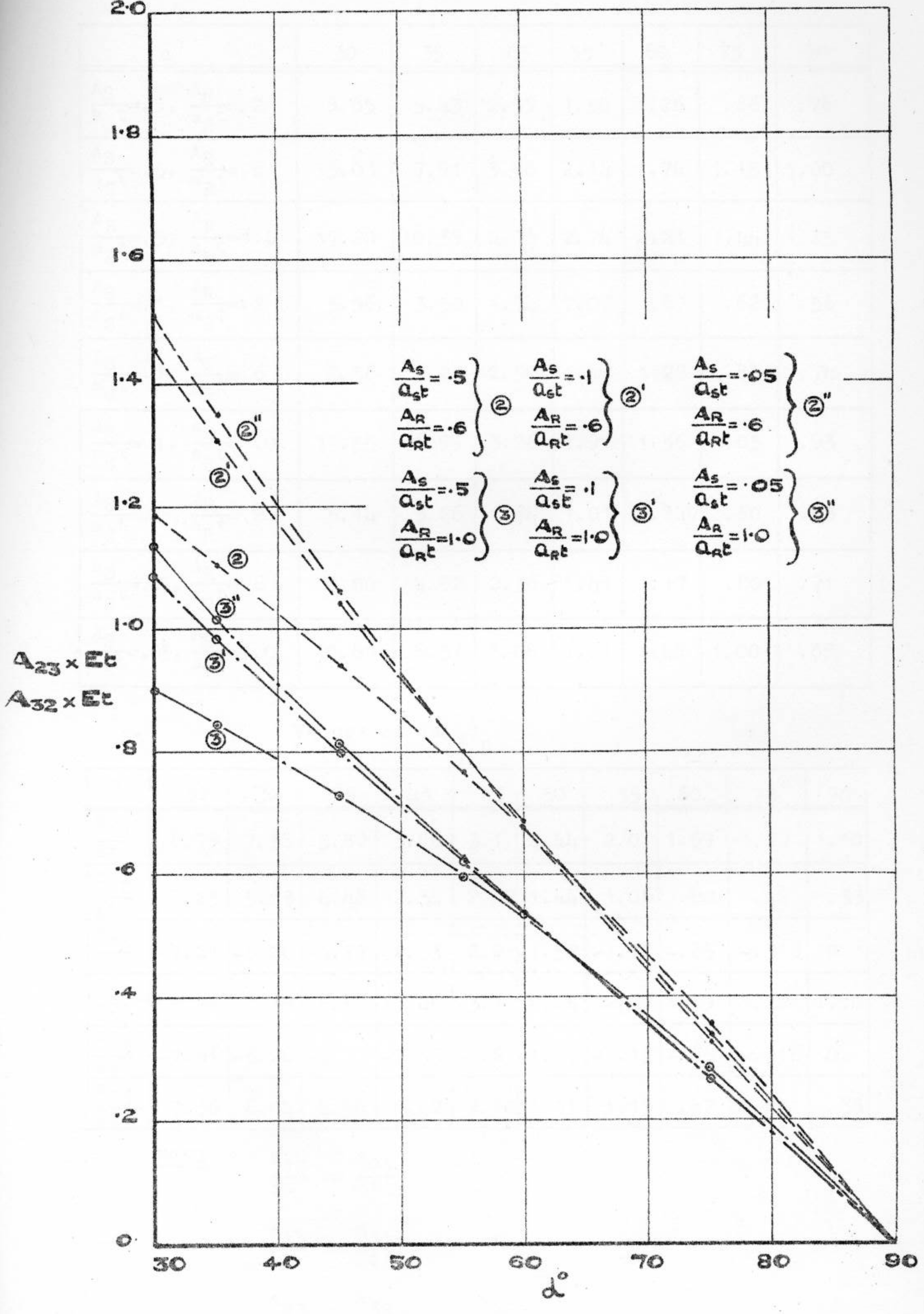

As/ast = ·5, AR/aRt = ·2 } ①
As/ast = ·1, AR/aRt = ·2 } ①'
As/ast = ·05, AR/aRt = ·2 } ①"
2·5
2·0
1·5
1·0
·5
0
A23 × Et
A32 × Et
①"
①'
①
30
40
50
60
70
80
90
∠°

FIG 11


2·0
1·8
1·6
1·4
1·2
1·0
·8
·6
·4
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0
30
40
50
60
70
80
90
$\alpha°$
$A_{23} \times Et$
$A_{32} \times Et$
$\frac{A_S}{a_{St}} = \cdot 5$, $\frac{A_R}{a_{Rt}} = \cdot 6$ ②
$\frac{A_S}{a_{St}} = \cdot 1$, $\frac{A_R}{a_{Rt}} = \cdot 6$ ②'
$\frac{A_S}{a_{St}} = \cdot 05$, $\frac{A_R}{a_{Rt}} = \cdot 6$ ②"
$\frac{A_S}{a_{St}} = \cdot 5$, $\frac{A_R}{a_{Rt}} = 1 \cdot 0$ ③
$\frac{A_S}{a_{St}} = \cdot 1$, $\frac{A_R}{a_{Rt}} = 1 \cdot 0$ ③'
$\frac{A_S}{a_{St}} = \cdot 05$, $\frac{A_R}{a_{Rt}} = 1 \cdot 0$ ③"
②"
②'
②
③"
③'
③

Part II

Values of $\frac{|a_{ij}|}{(Et)^3}$

α^o	30	35	45	55	60	75	90
$\frac{A_S}{a_S t}=.5$, $\frac{A_R}{a_R t}=.2$	8.85	5.43	2.59	1.54	1.26	.86	.76
$\frac{A_S}{a_S t}=.5$, $\frac{A_R}{a_R t}=.6$	13.03	7.91	3.68	2.14	1.74	1.15	1.00
$\frac{A_S}{a_S t}=.5$, $\frac{A_R}{a_R t}=1.0$	17.20	10.39	4.78	2.74	2.21	1.44	1.25
$\frac{A_S}{a_S t}=.1$, $\frac{A_R}{a_R t}=.2$	5.56	3.50	1.74	1.07	.89	.62	.56
$\frac{A_S}{a_S t}=.1$, $\frac{A_R}{a_R t}=.6$	8.56	5.25	2.50	1.49	1.22	.84	.74
$\frac{A_S}{a_S t}=.1$, $\frac{A_R}{a_R t}=1.0$	11.55	6.99	3.26	1.91	1.56	1.05	.93
$\frac{A_S}{a_S t}=.05$, $\frac{A_R}{a_R t}=.2$	5.14	3.26	1.64	1.01	.847	.60	.53
$\frac{A_S}{a_S t}=.05$, $\frac{A_R}{a_R t}=.6$	8.00	4.92	2.36	1.41	1.17	.80	.71
$\frac{A_S}{a_S t}=.05$, $\frac{A_R}{a_R t}=1.0$	10.85	6.57	3.08	1.81	1.48	1.00	.89

Values of $(a_{ij})_p$

α^o	30	32	35	43	45	50	55	60	75	90
$\frac{a_{11}}{Et}$	8.79	7.38	5.82	3.46	3.1	2.44	2.0	1.69	1.22	1.10
$\frac{a_{12}}{Et}$	7.25	5.93	4.48	2.34	2.02	1.44	1.06	.80	.42	.33
$\frac{a_{13}}{Et}$	-7.61	-6.26	-4.77	-2.53	-2.2	-1.57	-1.15	-.85	-.316	0
$\frac{a_{22}}{Et}$	8.79	7.38	5.82	3.46	3.1	2.44	2.0	1.69	1.22	1.10
$\frac{a_{23}}{Et}$	-7.61	-6.26	-4.77	-2.53	-2.2	-1.57	-1.15	-.85	-.316	0
$\frac{a_{33}}{Et}$	7.36	6.04	4.58	2.42	2.10	1.51	1.13	.87	.48	.38

NOTE

$$\frac{a_{12}}{Et} = \frac{a_{21}}{Et}$$

$$\frac{a_{13}}{Et} = \frac{a_{31}}{Et}$$

$$\frac{a_{23}}{Et} = \frac{a_{32}}{Et} .$$

FIG 12

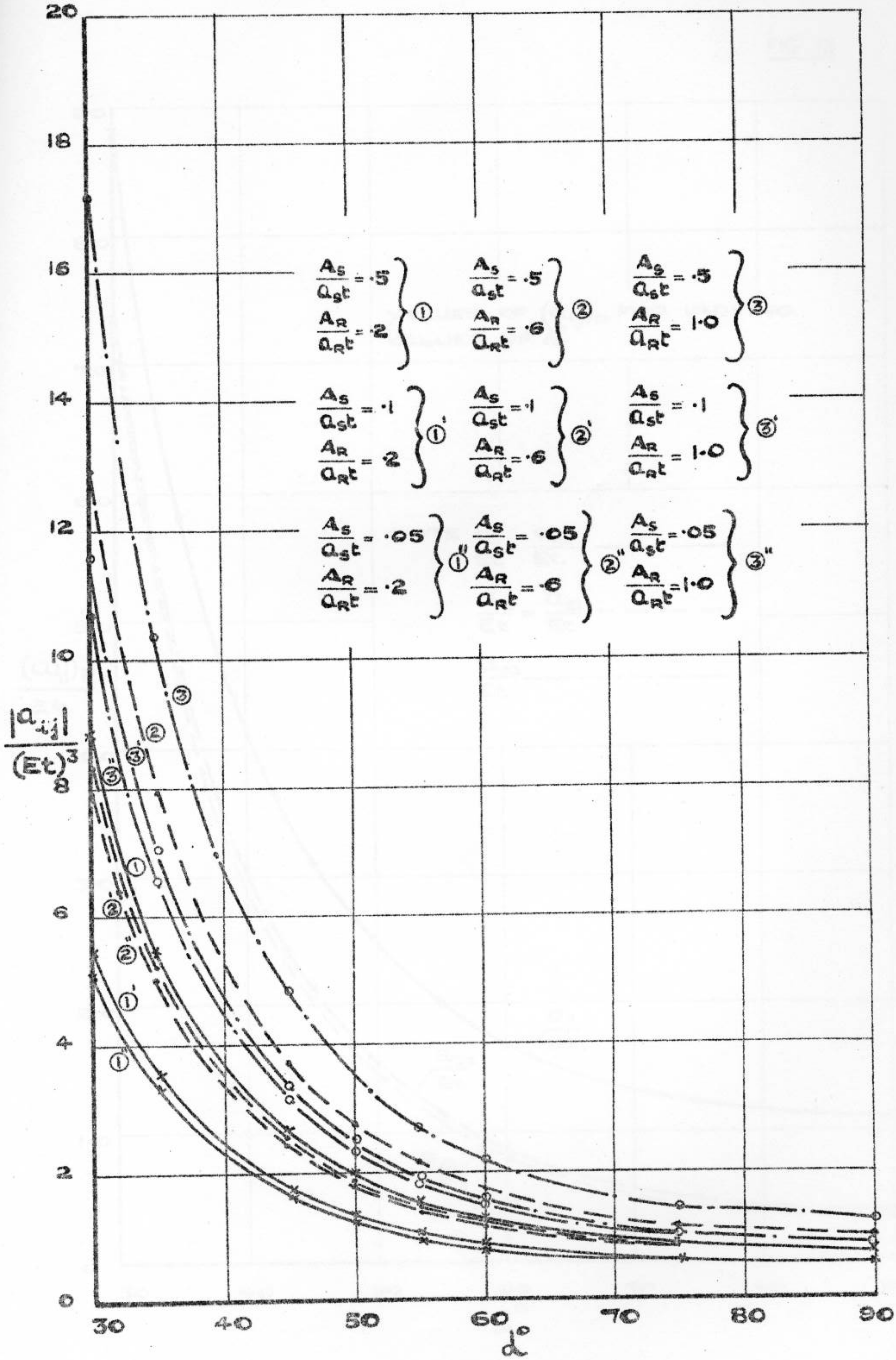

20
18
16
14
12
10
8
6
4
2
0
30
40
50
60
70
80
90
$\alpha°$
$\frac{|a_{ij}|}{(Et)^3}$
$\frac{A_S}{a_S t} = \cdot 5$, $\frac{A_R}{a_R t} = \cdot 2$ ①
$\frac{A_S}{a_S t} = \cdot 5$, $\frac{A_R}{a_R t} = \cdot 6$ ②
$\frac{A_S}{a_S t} = \cdot 5$, $\frac{A_R}{a_R t} = 1\cdot 0$ ③
$\frac{A_S}{a_S t} = \cdot 1$, $\frac{A_R}{a_R t} = \cdot 2$ ①'
$\frac{A_S}{a_S t} = \cdot 1$, $\frac{A_R}{a_R t} = \cdot 6$ ②'
$\frac{A_S}{a_S t} = \cdot 1$, $\frac{A_R}{a_R t} = 1\cdot 0$ ③'
$\frac{A_S}{a_S t} = \cdot 05$, $\frac{A_R}{a_R t} = \cdot 2$ ①''
$\frac{A_S}{a_S t} = \cdot 05$, $\frac{A_R}{a_R t} = \cdot 6$ ②''
$\frac{A_S}{a_S t} = \cdot 05$, $\frac{A_R}{a_R t} = 1\cdot 0$ ③''

FIG 13

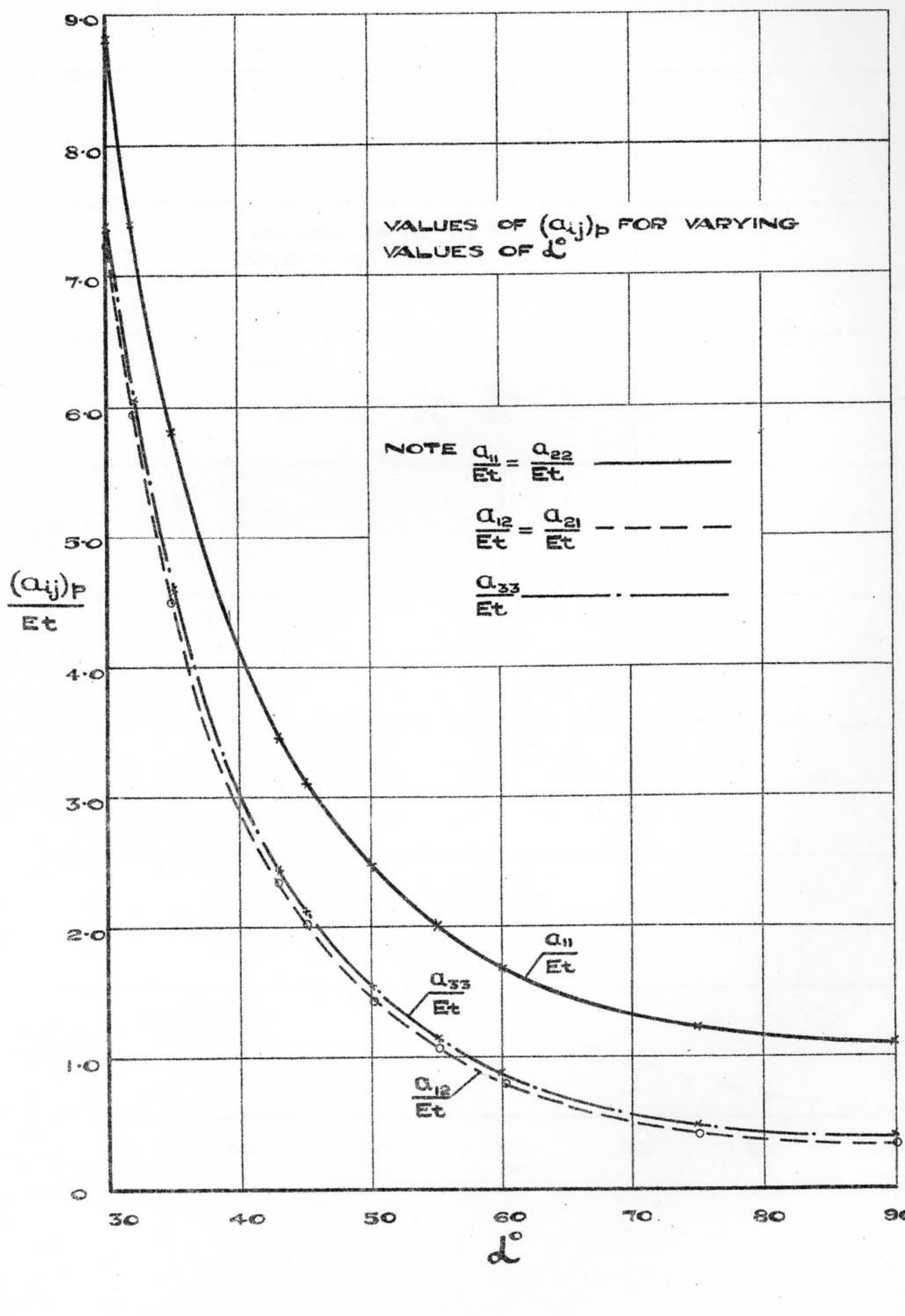
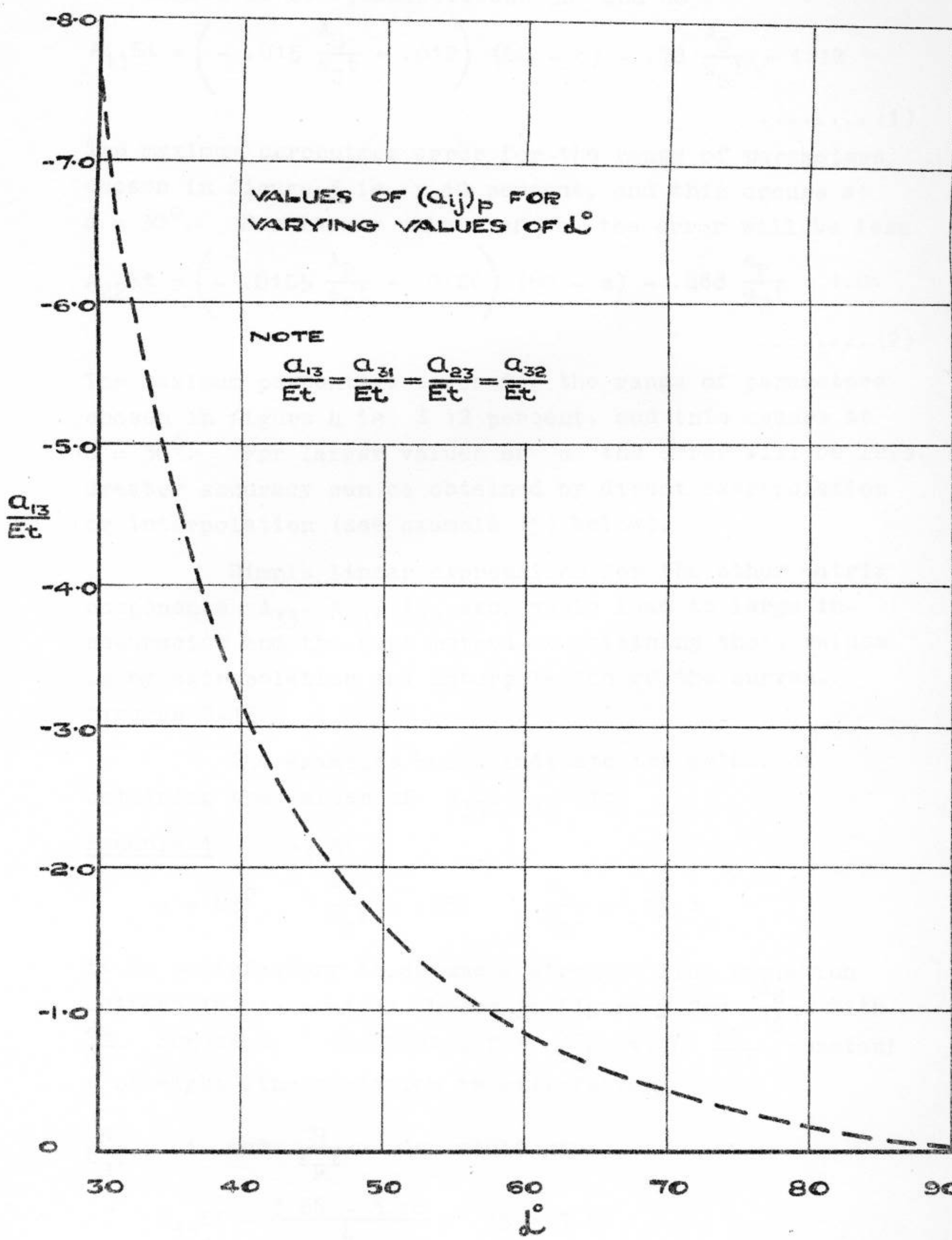

9·0
8·0
7·0
6·0
5·0
4·0
3·0
2·0
1·0
0
30
40
50
60
70
80
90
$\frac{(a_{ij})_P}{Et}$
α°
VALUES OF $(a_{ij})_P$ FOR VARYING VALUES OF α°
NOTE $\frac{a_{11}}{Et} = \frac{a_{22}}{Et}$
$\frac{a_{12}}{Et} = \frac{a_{21}}{Et}$
$\frac{a_{33}}{Et}$
$\frac{a_{11}}{Et}$
$\frac{a_{33}}{Et}$
$\frac{a_{12}}{Et}$

FIG 14


-8·0
-7·0
-6·0
-5·0
-4·0
-3·0
-2·0
-1·0
0
30
40
50
60
70
80
90
$\frac{a_{13}}{Et}$
α°
VALUES OF $(a_{ij})_p$ FOR VARYING VALUES OF α°
NOTE
$\frac{a_{13}}{Et} = \frac{a_{31}}{Et} = \frac{a_{23}}{Et} = \frac{a_{32}}{Et}$

Part III

Formulae giving approximate values of the matrix elements with maximum percentage variation from the actual calculated values.

The following expressions (1 and 2) hold for angles of sweepback between 30° and 60°.

$$A_{11}Et = \left(-\ .015\ \frac{A_S}{a_S t} + .012\right)(60 - \alpha) - .78\ \frac{A_S}{a_S t} + 1.12 \qquad \ldots\ldots\ldots(1)$$

The maximum percentage error for the range of parameters chosen in figure 3 is ± 10 percent, and this occurs at $\alpha = 30°$. For larger values of α the error will be less.

$$A_{22}Et = \left(-\ .0105\ \frac{A_R}{a_R t} + .0126\right)(60 - \alpha) - .488\ \frac{A_R}{a_R t} + 1.01 \qquad \ldots\ldots\ldots(2)$$

The maximum percentage error for the range of parameters chosen in figure 4 is ± 12 percent, and this occurs at $\alpha = 30°$. For larger values of α the error will be less. Greater accuracy can be obtained by direct extrapolation or interpolation (see example (5) below).

Simple linear expressions for the other matrix components A_{33}, A_{13}, A_{32} etc. would lead to large inaccuracies and the best method of obtaining these values is by extrapolation and interpolation of the curves, figures 9-15.

The examples below indicate the method of obtaining the values of A_{33}, A_{13} etc.

Example 1 $\quad A_{33}Et$

$$\alpha = 43° \qquad \frac{A_S}{a_S t} = .288 \qquad \frac{A_R}{a_R t} = .05\ ;$$

it is satisfactory to assume a straight line variation between two parameters chosen in figure 5 for $\frac{A_S}{a_S t}$ with $\frac{A_R}{a_R t}$ constant. Similarly, for $\frac{A_R}{a_R t}$ with $\frac{A_S}{a_S t}$ constant a straight line variation is satisfactory.

$\frac{A_S}{a_S t} = .1$ and $\frac{A_R}{a_R t} = .05$ would give

$$A_{33}Et = \frac{3.88 - 3.18}{.4} \times .15 + 3.88$$

$$= 4.142. \qquad \ldots\ldots\ldots(3)$$

$\frac{A_S}{a_S t} = .5$ and $\frac{A_R}{a_R t} = .05$ would give

$$A_{33}Et = \frac{3.09 - 2.5}{.4} \times .15 + 3.09$$

$$= 3.31 \qquad \ldots\ldots\ldots(4)$$

for the value of $\frac{A_S}{a_S t}$ of .288 by interpolation of the values (3) and (4) above, we have

$$A_{33}Et = 4.142 - \frac{4.142 - 3.31}{.4} \times .188 = 3.76.$$

The actual calculated value of $A_{33}Et$ is 3.81, which shows a percentage error of 1.3 %.

Example 2 $\quad A_{12}Et$

The assumptions are as for Example 1, (see figs. 6 and 7)

$$\alpha = 43^\circ \qquad \frac{A_S}{a_S t} = .288 \qquad \frac{A_R}{a_R t} = .05$$

$\frac{A_S}{a_S t} = .1$ and $\frac{A_R}{a_R t} = .05$ would give

$$A_{12}Et = \frac{.394 - .27}{.4} \times .15 + .394$$

$$= .44 \qquad \ldots\ldots\ldots(5)$$

$\frac{A_S}{a_S t} = .5$ and $\frac{A_R}{a_R t} = .05$ would give

$$A_{12}Et = \frac{.26 - .187}{.4} \times .15 + .26$$

$$= .287 \qquad \ldots\ldots\ldots(6)$$

for the value of $\frac{A_S}{a_S t} = .288$ by interpolation of the values (5) and (6) above, we have

$$A_{12}Et = .44 - \frac{.44 - .287}{.4} \times .188 = .368.$$

The actual calculated value of $A_{12}Et$ is .381, which shows a percentage error of 3.4 %.

Example 3 $\quad A_{13}Et$

The assumptions are as for Example 1, (see figs. 8 and 9)

$$\alpha = 43^\circ \qquad \frac{A_S}{a_S t} = .288 \qquad \frac{A_R}{a_R t} = .05$$

$\frac{A_S}{a_S t} = .1$ and $\frac{A_R}{a_R t} = .05$ would give

$$A_{13}Et = \frac{1.688 - 1.535}{.4} \times .15 + 1.688$$

$$= 1.745 \qquad \ldots\ldots\ldots(7)$$

$\frac{A_S}{a_S t} = .5$ and $\frac{A_R}{a_R t} = .05$ would give

$$A_{13}Et = \frac{1.148 - .95}{.4} \times .15 + 1.148 = 1.222 \qquad \ldots\ldots\ldots(8)$$

for the value of $\frac{A_S}{a_S t} = .288$ by interpolation of the values (7) and (8) above, we have

$$A_{13}Et = 1.222 + \frac{1.745 - 1.222}{.4} \times .188 = 1.467.$$

The actual calculated value of $A_{13}Et$ is 1.472, which shows a percentage error of 0.3 %.

Example 4 $\quad A_{23}Et$

The assumptions are as for Example 1, (see figs.10 and 11)

$$\alpha = 43^\circ \qquad \frac{A_S}{a_S t} = .288 \qquad \frac{A_R}{a_R t} = .05$$

$\frac{A_S}{a_S t} = .1$ and $\frac{A_R}{a_R t} = .05$ would give

$$A_{23}Et = \frac{1.575 - 1.095}{.4} \times .15 + 1.575$$

$$= 1.755 \qquad \qquad ....(9)$$

$\frac{A_S}{a_S t} = .5$ and $\frac{A_R}{a_R t} = .05$ would give

$$A_{23}Et = \frac{1.41 - .975}{.4} \times .15 + 1.41$$

$$= 1.573 \qquad \qquad ....(10)$$

for the values of $\frac{A_S}{a_S t} = .288$ by interpolation of the values (9) and (10) above, we have

$$A_{23}Et = 1.573 + \frac{1.755 - 1.573}{.4} \times .212 = 1.67.$$

The actual calculated value of $A_{23}Et$ is 1.77 which shows a percentage error of 5.6 %.

Example 5

The values of $A_{22}Et$ can be obtained by direct interpolation or extrapolation of the curves of figure 4. For large variations in $\frac{A_S}{a_S t}$ there is very little change in value of $A_{22}Et$ for fixed value of $\frac{A_R}{a_R t}$.

$$\alpha = 43^\circ \qquad \frac{A_S}{a_S t} = .288 \qquad \frac{A_R}{a_R t} = .05.$$

The value can be obtained by extrapolation of the curves between values of $\frac{A_R}{a_R t} = .2$ and .6, respectively.

Taking mean values of the curves

$$\frac{A_R}{a_R t} = .2 \qquad A_{22}Et = 1.10$$

$$\frac{A_R}{a_R t} = .6 \qquad A_{22}Et = .76$$

Therefore for $\frac{A_R}{a_R t} = .05$, $A_{22}Et = 1.10 + \frac{1.10 - .76}{.4} \times .15$
$= 1.23.$

The actual calculated value of $A_{22}Et$ is 1.31, which shows a percentage error of 6 %.

CONCLUSIONS

(1) Variations in the values of A_{11} Et for specified values of $\frac{A_S}{a_S t}$ are very small for large variations in $\frac{A_R}{a_R t}$ throughout the range of α.

This term occurs in the expression for direct stress and shear stress and indicates that the stresses are not critical for quite large variations in the parameters.

(2) Variations in the values of A_{22}/Et for specified values of $\frac{A_R}{a_R t}$ are very small for large variations in $\frac{A_S}{a_S t}$, for values of $30^\circ < \alpha < 75^\circ$. Beyond this the variation is large.

(3) Variations in the values of A_{33} Et for specified values of $\frac{A_S}{a_S t}$ are large for small variations in $\frac{A_R}{a_R t}$.

(4) Variations in the values of A_{12}/Et or A_{21}/Et for specified values of $\frac{A_S}{a_S t}$ are reasonably large for variations in $\frac{A_R}{a_R t}$.

(5) Variations in the values of A_{13}/Et or A_{31}/Et for specified values of $\frac{A_S}{a_S t}$ are reasonably large for variations in $\frac{A_R}{a_R t}$.

(6) Variations in the values of A_{23}/Et or A_{32}/Et for specified values of $\frac{A_R}{a_R t}$ are reasonably small for large variations in $\frac{A_S}{a_S t}$.

---ooOoo---

I am indebted to Dr. Kirkby and the Computing Section of the Aerodynamics Department for the valuable aid given in the computation of the numerical examples.

Reference 1

College of Aeronautics Report No. 31, "On the application of oblique coordinates to problems of plane elasticity and swept-back wings", by W. S. Hemp.