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Wing Body Interference at Supersonic Speeds

-by-

S. Kirkby, B.Sc., Ph.D., A.F.R.Ae.S., and
A. Robinson, M.Sc., A.F.R.Ae.S.

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- SUMMARY -

The increment in lift due to wing -
body interference at supersonic speeds is
calculated approximately for an untapered
wing, without sweepback.

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1.1 Introduction

To estimate the increment in wing lift due to the presence of a body, it has been assumed that the body is represented by a cone travelling, vertex foremost, with its axis at incidence ψ. The velocity field due to the cone may then be considered to be generated by an equivalent doublet distribution, along the axis of the cone, for which the induced velocity is determinable. In particular, the upwash velocity is evaluated along the mid-chord line of the wing, to which Ackeret's theory is applied for the estimation of the increment in wing lift due to body interference.

Some calculations are given to show that the use of Ackeret's theory is, for all practical purposes, permissible.

While it is believed that the calculations contained in this report represent an acceptable approximation, further investigation of the subject is desirable.

1.2 Notation

ρ - air density

V - free stream velocity

M - Mach number

μ - Mach angle

ϵ - semi-vertical angle of cone

β - $\sqrt{M^2 - 1}$

γ - $(\cot \epsilon)/\beta$

b - wing span, tip to tip

c - wing chord

c_1 - distance of wing leading edge aft of vertex of cone.

c_2 - distance of wing trailing edge aft of vertex of cone.

c' - distance of mid-chord line aft of vertex of cone.

λ - $\beta b/2c'$

ΔL - lift increment due to wing-body interference

ΔC_L - increment in lift coefficient, based on gross wing area, due to wing-body interference

$$= \Delta L/\tfrac{1}{2}\rho V^2 bc$$

$\Delta C_L'$ - increment in lift coefficient, based on net wing area (or area of wing overhang) within the Mach cone of the body, due to wing-body interference

$$= \Delta L/\tfrac{1}{2}\rho V^2 (b - 2c' \tan \epsilon)c \qquad /x....$$

1.2 <u>Notation</u> (contd.)

x − chordwise co-ordinate (measured from vertex of cone against the direction of flow)

y − spanwise co-ordinate (measured from centre line of cone and positive to starboard)

z − normal co-ordinate (measured from centre line of cone perpendicular to xy-plane and positive downwards)

α − wing incidence (in radians)

ψ − incidence of cone centre line (in radians)

ϕ_1 − induced velocity potential due to wing

ϕ_2 − induced velocity potential due to body

w_c − downwash velocity induced by body

Δp − pressure increment induced by body

r, θ − cylindrical co-ordinates

2. Analysis

It has been shown by Tsien (ref.1, eq.4) that for a cone travelling at supersonic speed the induced velocity potential may be expressed in the form

$$\phi_2 = -\beta \cos \theta \int_{\cosh^{-1}(-x/\beta r)}^{0} f(-x - \beta r . \cosh u) \cosh u \, du,$$

where $\beta = \sqrt{M^2 - 1}$ and (x, r, θ) are cylindrical co-ordinates referred to the vertex and axis of the cone (fig.1).

According to Tsien the appropriate function is given by

$$f(-x - \beta r . \cosh u) = K(-x - \beta r . \cosh u),$$

where K is a constant. Hence, integrating, as in ref.1, eq. 4a,

$$\phi_2 = K\beta \cos \theta \left\{ -\frac{x}{2} \sqrt{\left(\frac{x}{\beta r}\right)^2 - 1} - \frac{\beta r}{2} \cosh^{-1}\left(-\frac{x}{\beta r}\right) \right\}.$$

The conventional rectangular cartesian co-ordinates for an aircraft (figs.1, 2) are related to the above cylindrical co-ordinates by the equations

$$y = r . \sin \theta, \quad z = r . \cos \theta .$$

/Hence......

Hence the downwash velocity, w_c, due to the above doublet distribution, at a point in the x-y plane ($z = 0$) is

$$w_c = \left(\frac{\partial \varphi_2}{r \partial \theta}\right) \theta = \pi/2, \; r = y$$

$$= -\frac{K\beta^2}{2}\left\{-\frac{x}{\beta y}\sqrt{\left(\frac{x}{\beta y}\right)^2 - 1} - \cosh^{-1}\left(-\frac{x}{\beta y}\right)\right\} \quad \ldots\ldots(1)$$

Since this downwash velocity, in the x-y plane, due to the cone is constant along a line x/y = constant, the chordwise average of downwash velocity is approximately equal to its value at mid-chord. It will now be assumed that the lift distribution along a specified chord can be estimated by Ackeret's formula, independently of conditions elsewhere along the span ('strip theory method'). In addition it will be assumed that the incidence is in fact constant along any given chord, and is given by its actual value at mid-chord where

$$x = -\tfrac{1}{2}(c_1 + c_2) = -c' .$$

Thus the increment in wing lift ΔL resulting from the induced upwash, $-w_c$, of the body will, at supersonic speed, be given by

$$\frac{d(\Delta L)}{dy} = \frac{2\rho Vc}{\beta}(-w_c), \quad \ldots\ldots\ldots\ldots\ldots(2)$$

where c is the wing chord. The validity of this approximation is discussed in the appendix.

Since the interference of the body will only exist in its 'after-cone', which is of semi-vertical angle

$$\mu = \sin^{-1}(1/M) = \cot^{-1}\beta$$

and has its vertex and, approximately, its axis coinciding with those of the cone, it is necessary to consider two cases when evaluating the increment in wing lift: namely (i) when the wing extends beyond the Mach cone of the body and (ii) when the wing tips lie within this Mach cone. Then, for a rectangular wing of span b and body cone of semi-vertical angle ϵ, the lift increment is

$$\Delta L = \frac{4\rho Vc}{\beta}\int_{c'\tan\epsilon}^{c'/\beta}(-w_c)dy \quad \ldots\ldots\ldots(3a)$$

when the wing extends beyond the Mach cone of the body, and

$$/\Delta L \ldots\ldots$$

$$\Delta L = \frac{4 \rho V c}{\beta} \int_{c' \tan \epsilon}^{b/2} (- w_c) \, dy \quad \ldots \ldots \ldots (3b)$$

when the wing tips lie within the Mach cone of the body.

Indefinite integration of equations (3a) and (3b) along the lifting line at $x = - c'$ yields, after substituting from equation (1) and writing $u = \beta y / c' < 1$,

$$\frac{4 \rho V c}{\beta} \int (- w_c) \, dy$$

$$= 2 \rho V K c c' \int \left\{ \frac{1}{u} \sqrt{\frac{1}{u^2} - 1} - \cosh^{-1} \left(\frac{1}{u} \right) \right\} du$$

$$= - 2 \rho V K c c' \left\{ \frac{\sqrt{1 - u^2}}{u} + u \cosh^{-1} \left(\frac{1}{u} \right) + 2 \sin^{-1} u \right\}$$

$$+ \text{constant.} \quad \ldots \ldots \ldots (4)$$

Now the value of K is given (ref.1, eq.5b) by the boundary condition at the surface of the cone where the velocity component along the negative z-direction is $V \sin \psi$ so that

$$K = \frac{2 V \sin \psi}{\beta^2 \left\{ \zeta \sqrt{\zeta^2 - 1} + \cosh^{-1} \zeta \right\}}, \quad \ldots \ldots \ldots (5)$$

where $\zeta = (\cot \epsilon) / \beta$. $\quad \ldots \ldots \ldots (6)$

Define $\beta b / 2 c' = \lambda$. $\quad \ldots \ldots \ldots (7)$

After substituting from equations (4) – (7) in equations (3a) and (3b) it is found, on rearrangement, that the increment ΔC_L in lift coefficient based on gross wing area $(= bc)$ may be written

$$\Delta C_L = \frac{\Delta L}{\frac{1}{2} \rho V^2 bc}$$

$$= \frac{\sin \psi}{\beta} \cdot \frac{4}{\lambda} \left\{ \frac{1}{\zeta} + \frac{2 \sin^{-1}(1/\zeta) - \pi}{\zeta \sqrt{\zeta^2 - 1} + \cosh^{-1} \zeta} \right\} .. (8a)$$

for a wing extending beyond the Mach cone of the body,

/or...

or

$$\Delta c_L = \frac{\sin\psi}{\beta}\,\frac{4}{\lambda}\left\{\frac{1}{\zeta} + \frac{2\sin^{-1}(1/\zeta) - \sqrt{(1/\lambda)^2 - 1} - \lambda\cosh^{-1}(1/\lambda) - 2\sin^{-1}\lambda}{\zeta\sqrt{\zeta^2 - 1} + \cosh^{-1}\zeta}\right\} \dots\dots\dots (8\text{v})$$

when the wing tips lie within the Mach cone of the body.

The gross wing area employed in evaluating Δc_L includes the wing centre section, passing through the body, which does not contribute to the lift and upon which there is no body interference. It may therefore be preferable in interpreting the results to base the increment $\Delta c'_L$ in lift coefficient on the net wing area within the Mach cone of the body, i.e., area of wing overhang within the Mach cone of the body, $(= bc - 2c'\tan\epsilon.c)$; and this method has been adopted in presenting the results (figs. 3 - 4). The increment in lift coefficient $\Delta c'_L$ is in general

$$\Delta c'_L = \frac{\Delta L}{\frac{1}{2}\rho V^2 (b - 2c'\tan\epsilon)c} = \frac{\Delta c_L}{1 - 1/\lambda\zeta}.$$

Therefore

$$\Delta c'_L = \frac{\sin\psi}{\beta}\cdot\frac{4}{\lambda - 1/\zeta}\left\{\frac{1}{\zeta} + \frac{2\sin^{-1}(1/\zeta) - \pi}{\zeta\sqrt{\zeta^2 - 1} + \cosh^{-1}\zeta}\right\} \dots (9a)$$

for a wing extending beyond the Mach cone of the body, and

$$\Delta c'_L = \frac{\sin\psi}{\beta}\cdot\frac{4}{\lambda - 1/\zeta}\left\{\frac{1}{\zeta} + \frac{2\sin^{-1}(1/\zeta) - \sqrt{(1/\lambda)^2 - 1} - \lambda\cosh^{-1}(1/\lambda) - 2\sin^{-1}\lambda}{\zeta\sqrt{\zeta^2 - 1} + \cosh^{-1}\zeta}\right\} \dots (9b)$$

when the wing tips lie within the Mach cone of the body. The previous result, equation (9a), may of course be deduced from this last result by substituting $\lambda = 1$ within the curly brackets in equation (9b).

There is in each of the above cases an increment in wing drag associated with the increment in wing lift due to body interference which is of amount

$$\Delta c_D = \Delta c_L \tan\alpha \doteq \Delta c_L \cdot \alpha \dots\dots$$

where α = wing incidence (in radians).

/3..........

3. Limiting Value of $\Delta C_L'$ for Wings of Small Overhang

When the wing overhang, $b/2 - c'\tan \epsilon$, becomes small, tending to zero, it is necessary to calculate the limiting value of $\Delta C_L'$. In this extreme case, the downwash velocity, w_c, due to the body is constant along the wing overhang and equal to its value at the surface of the cone where

$$x = -c', \quad y = c'\tan \epsilon, \quad z = 0$$

and thus, from equations (1) and (5),

$$w_c \longrightarrow -\frac{K\beta^2}{2}\left\{ \mathcal{Y}\sqrt{\mathcal{Y}^2-1} - \cosh^{-1}\mathcal{Y} \right\}$$

$$= V\sin\left\{ \frac{\mathcal{Y}\sqrt{\mathcal{Y}^2-1} - \cosh^{-1}\mathcal{Y}}{\mathcal{Y}\sqrt{\mathcal{Y}^2-1} + \cosh^{-1}\mathcal{Y}} \right\}.$$

Under these conditions $b/2 - c'\tan\epsilon \longrightarrow 0$ so that, from equation (3b),

$$\Delta L \longrightarrow \frac{4\rho Vc}{\beta}(-w_c)\left(\frac{b}{2} - c'\tan\epsilon\right)$$

from which it follows that

$$\Delta C_L' \longrightarrow \frac{\sin\psi}{\beta} \cdot 4 \cdot \frac{\mathcal{Y}\sqrt{\mathcal{Y}^2-1} - \cosh^{-1}\mathcal{Y}}{\mathcal{Y}\sqrt{\mathcal{Y}^2-1} + \cosh^{-1}\mathcal{Y}}.$$

This formula gives the maximum value of $\Delta C_L' \cdot \beta/\sin\psi$ for any given $\mathcal{Y}$ ($= 1/\lambda$, under limiting conditions) and is plotted as the upper boundary in figure 3.

Further $\Delta C_L' \longrightarrow 4\sin\psi/\beta$ asymptotically as $\mathcal{Y} \longrightarrow \infty$. This is the absolute maximum increment in lift coefficient $\Delta C_L'$ for any given wing-body combination with a slender body at incidence ψ and Mach number $M = \sqrt{1+\beta^2}$.

4. Results

The interference experienced by a wing attached to a slender body is due to the upwash generated by the body in its 'after-cone' causing the wing to operate at an increased effective incidence. The resulting increment in lift is proportional to $\sin\psi$ whilst being a function of the Mach number, the cone angle, the position and span of the wing as shown in equations (8a) and (8b).

In general, for a given wing position, c', and span, $2b$, the increment in lift coefficient, $\Delta C_L'$, decreases with the angle of the cone, 2ϵ, (figs. 3 - 4) although this apparently does not apply to relatively thick bodies ($\mathcal{Y} > 3$) with wings almost spanning the Mach cone of the body (i.e., for which $\lambda \doteqdot 1$). Neglecting this region, $1 < \mathcal{Y} < 3$ and $0.7 < \lambda < 1$, in which the

/approximations...

approximations are doubtful, it is seen (fig.3) that there is least interference, with $\Delta C_L^!$. β/sin ψ ≒ 0.3,

for a slender body with a wing at least spanning the Mach cone of the body. On the other hand, maximum interference, with $\Delta C_L^!$. β/sin ψ = 4, is to be

expected for a slender body with extremely small overhang.

If these results are to be used for bodies of shape other than conical, but approximately so, cot ϵ should be taken to be the ratio of the mean radius of a characteristic section of the body to its distance aft of the nose of the body. The mean radius may, for example, be taken to be $\sqrt{A/\pi}$, where A is the area of the cross-section considered.

- REFERENCES -

No.	Author	Title, etc.
1.	H-S. Tsien	Supersonic Flow over an Inclined Body of Revolution. Journal of Aeronautical Sciences, Vol. 5, 1938.
2.	A.E. Puckett	Supersonic Wave Drag of Thin Airfoils. Journal of Aeronautical Sciences, Vol.13, 1946.
3.	A. Robinson	The Wave Drag of Diamond-Shaped Aerofoils at Zero Incidence. R.A.E. Tech. Note No. Aero.1784, 1946.

APPENDIX

Comparison of the Present Method with Exact (Linearised) Theory

It will now be shown that for linear spanwise distribution of upwash velocity the lift increment calculated by the method adopted in this report is identical with that obtained by exact (linearised) theory. There is limited agreement between the two methods for velocity distributions approximating to that given by equation (1).

In general the induced velocity potential ϕ_1 at the point $P = (x, y)$ on the wing is

$$\phi_1 = - \iint_A \frac{\sigma\, dx_0\, dy_0}{\sqrt{(x - x_0)^2 - \beta^2 \left[(y - y_0)^2 + z^2\right]}}$$

where the source strength per unit area, $\sigma = w_c / \pi$, is evaluated from the downwash velocity w_c and integration extends over the area A (fig.5) of the 'fore-cone' of the point P (refs. 2 and 3).

1). Linear spanwise velocity distribution

If it is assumed that

$$w_c = - py_0 - q$$

then

$$\phi_1 = \frac{1}{\pi} \int_{-c_1}^{x} dx_0 \int_{\eta_1}^{\eta_2} \frac{(py_0 + q)\, dy_0}{\sqrt{(x - x_0)^2 - \beta^2 \left[(y - y_0)^2 + z^2\right]}}$$

where $\eta_1 = y + (x - x_0)/\beta$

and $\eta_2 = y - (x - x_0)/\beta$

so that

$$\phi_1 = \frac{1}{\pi} \int_{-c_1}^{x} dx_0 \left[\frac{p}{\beta^2} \sqrt{(x - x_0)^2 - \beta^2 (y - y_0)^2} - \frac{py_0 + q}{\beta} \sin^{-1} \frac{\beta (y - y_0)}{x - x_0} \right]_{\eta_1}^{\eta_2}$$

$$= - \frac{py + q}{\beta} \int_{-c_1}^{x} dx_0$$

$$= - (py + q)(x + c_1)/\beta.$$

Now by the linearised Bernoulli equation the pressure
increment at the point (x, y) is

$$\Delta p = \rho V \frac{\partial \phi_1}{\partial x} = - \frac{\rho V(py + q)}{\beta}.$$

The resulting lift increment over a rectangular area
bounded by $x = -c_1$, $x = -c_2$, $y = y_1$, and $y = y_2$ is then

$$\Delta L = 2 \int_{-c_1}^{-c_2} \int_{y_1}^{y_2} \Delta p . \, dx . \, dy$$

$$= - \frac{2\rho V}{\beta} \int_{-c_1}^{-c_2} dx \int_{y_1}^{y_2} (py + q) dy$$

$$= \frac{2\rho Vc}{\beta} (y_2 - y_1) \left\{ \frac{p(y_1 + y_2)}{2} + q \right\}$$

since $c_2 - c_1 = c = $ wing chord.

Alternatively the lift increment evaluated by
Ackeret's theory yields an identical result with the present
velocity distribution since putting, as in equation (2),

$$\frac{d(\Delta L)}{dy} = \frac{2\rho Vc}{\beta} (-w_c)$$

and integrating over the lifting line equivalent to the above
rectangle gives in the present case

$$\Delta L = \frac{2\rho Vc}{\beta} \int_{y_1}^{y_2} (py + q) dy$$

$$= \frac{2\rho Vc}{\beta} (y_2 - y_1) \left\{ \frac{p(y_1 + y_2)}{2} + q \right\}.$$

Another way of deriving this results is
as follows.

Since the flow of a compressible fluid past a
body of revolution produces small perturbations for which
the induced velocity potential ϕ satisfies the linearised
equation of motion

$$\beta^2 \frac{\partial^2 \phi}{\partial x^2} - \frac{\partial^2 \phi}{\partial y^2} - \frac{\partial^2 \phi}{\partial z^2} = 0, \quad \ldots\ldots\ldots (10).$$

/it.....

it follows that if $\phi = f(x, z)$ is a solution of equation (10) then so is $\phi' = y.f(x, z)$.

We conclude that if ϕ is the induced velocity potential corresponding to a certain incidence distribution which is independent of y, $g(x)$ say, then ϕ' is the induced velocity potential corresponding to an incidence distribution $w' = y.g(x)$. It follows that in order to find the lift distribution corresponding to $w' = y.g(x)$, we can in fact calculate the precise lift distribution corresponding to $w = g(x)$, by Ackeret's theory, and multiply the result by y.

2). Inverse square variation of spanwise velocity distribution

We may express the spanwise velocity distribution relevant to the present problem and given equation (1) in the approximate form

$$w_c = - \frac{P}{y_o^2} - Q , \quad \text{(see fig.6)}.$$

Using this expression, it can be shown that

$$\phi_1 = - \frac{P}{y} . \frac{x + c_1}{\sqrt{\beta^2 y^2 - (x + c_1)^2}} - \frac{Q}{\beta} (x + c_1)$$

$$\Delta p = -\rho V \left[\frac{P \beta^2 y}{\left\{ \beta^2 y^2 - (x + c_1)^2 \right\}^{3/2}} + \frac{Q}{\beta} \right]$$

which, on integrating over the same rectangular area as previously (see case 1), yields an increment in lift

$$\Delta L = \frac{2 \rho V c}{\beta} (y_2 - y_1) \left[\frac{P \beta}{c(y_2 - y_1)} \sin^{-1} \left\{ \frac{c}{\beta y_1} \sqrt{1 - \left(\frac{c}{\beta y_2} \right)^2} \right. \right.$$

$$\left. \left. - \frac{c}{\beta y_2} \sqrt{1 - \left(\frac{c}{\beta y_1} \right)^2} \right\} + Q \right]$$

which is such that for small values of $c/\beta y_1$ and $c/\beta y_2$

$$\Delta L \rightarrow \frac{2 \rho V c}{\beta} (y_2 - y_1) \left(\frac{P}{y_1 y_2} + Q \right) .$$

This limiting value is identical with that obtained by the approximate theory given in the body of this report which, with the present velocity distribution, predicts a lift increment

$$\Delta L = \frac{2 \rho Vc}{\beta} \int_{y_1}^{y_2} \left(\frac{P}{y^2} + Q \right) dy$$

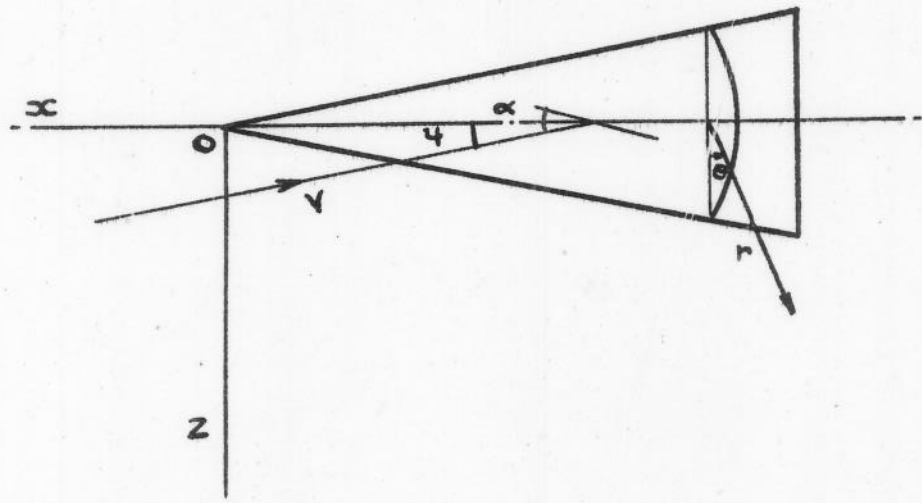$$= \frac{2 \rho Vc}{\beta} (y_2 - y_1) \left(\frac{P}{y_1 y_2} + Q \right).$$
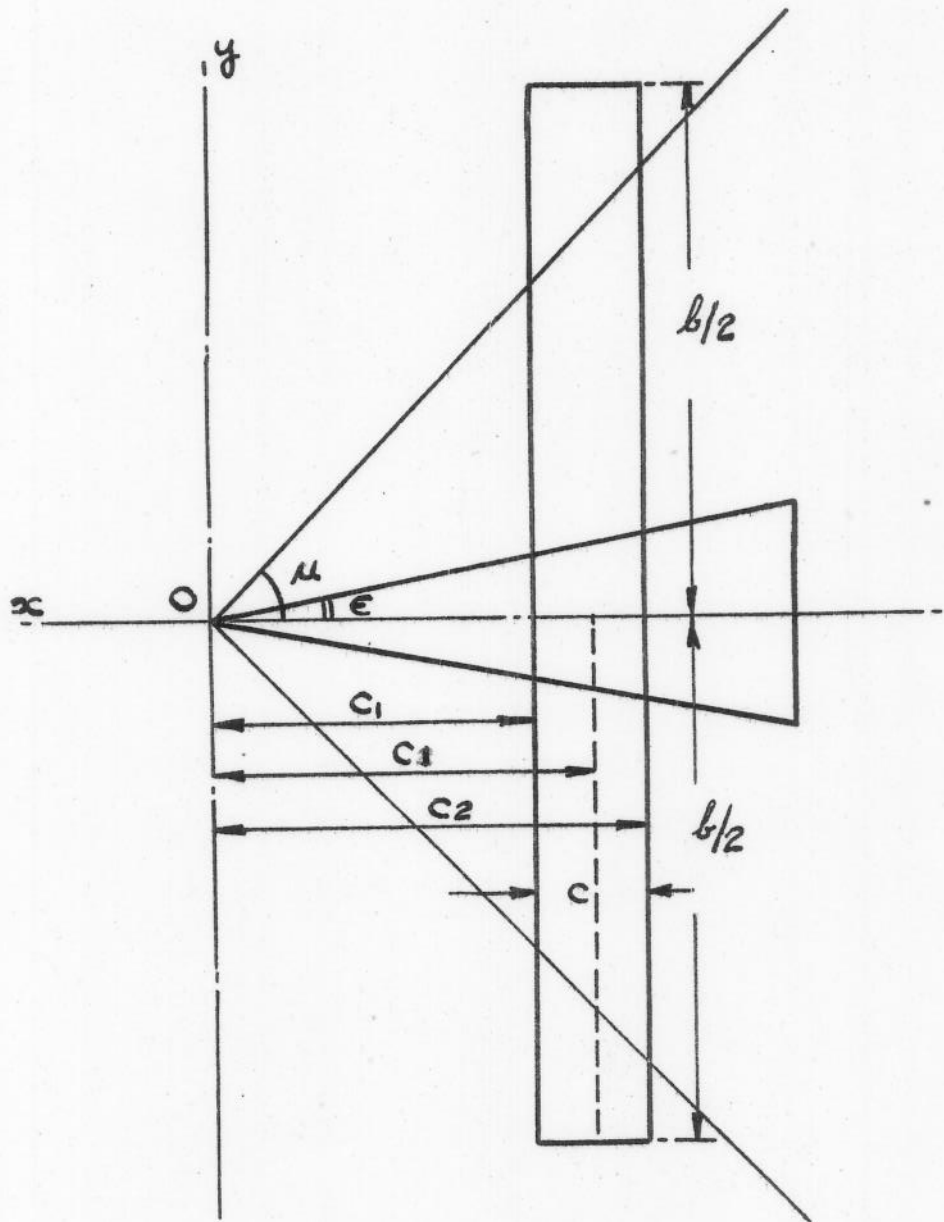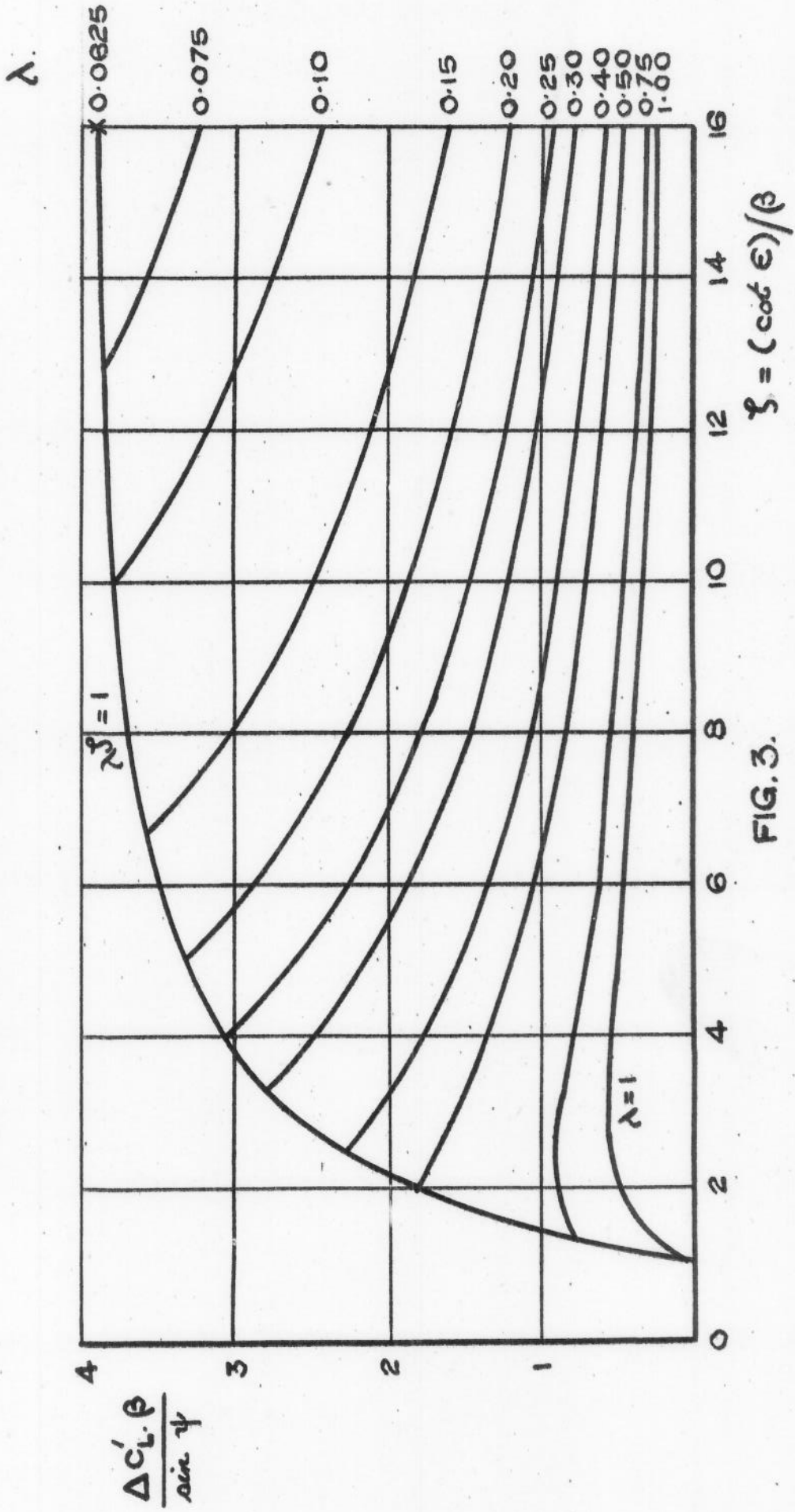

FIGURE .1.

FIGURE .2.

VARIATION OF INDUCED WING LIFT COEFFICIENT WITH
CONE ANGLE FOR DIFFERENT WING POSITIONS.

FIG.3.

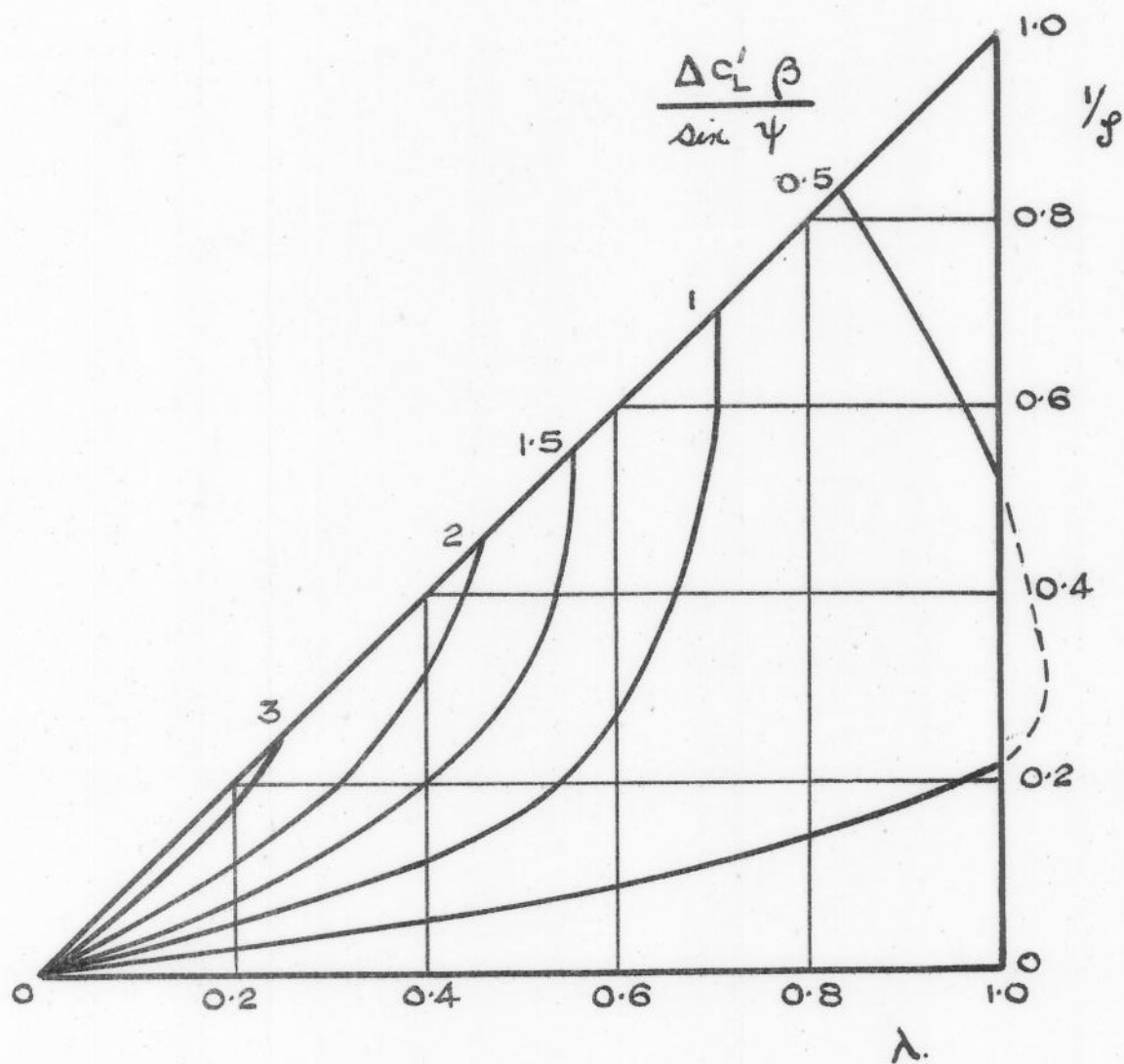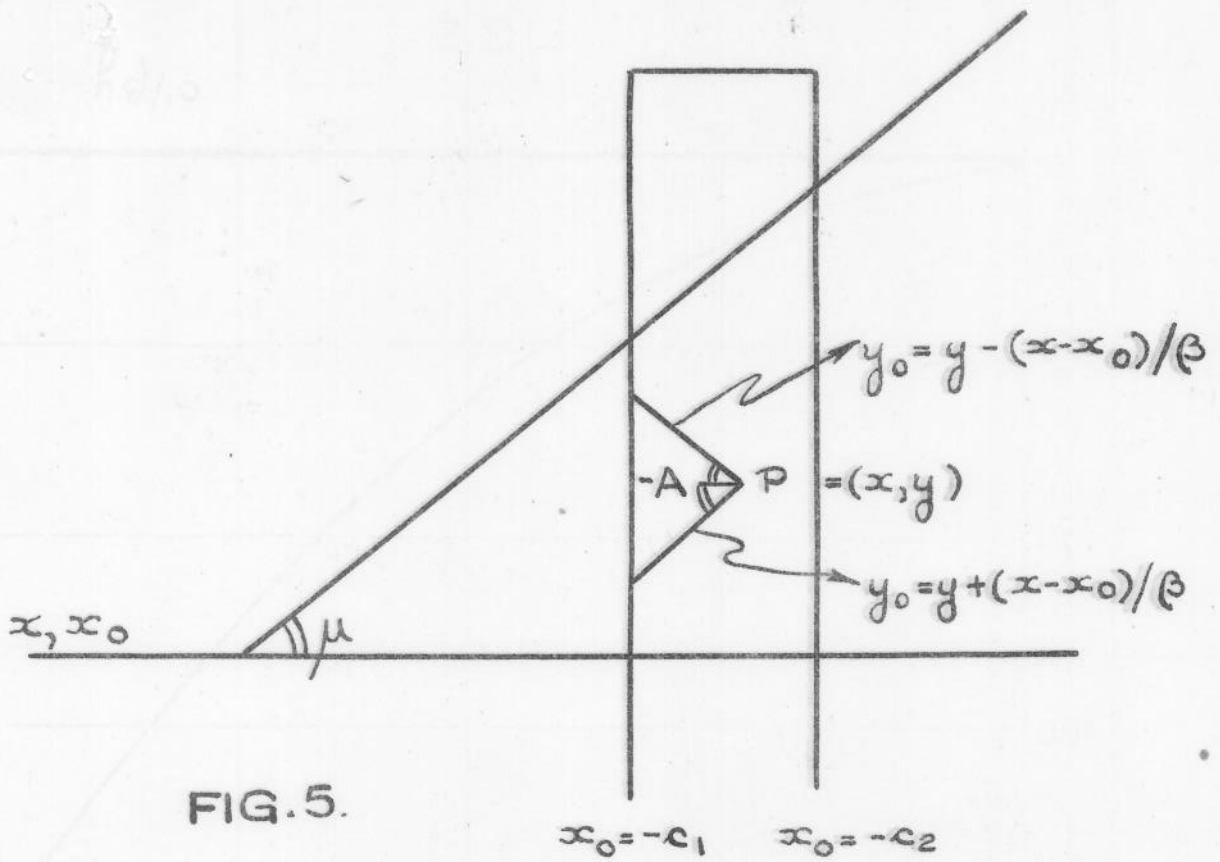CURVES OF CONSTANT $\Delta c_L' \cdot \beta / \sin \psi$
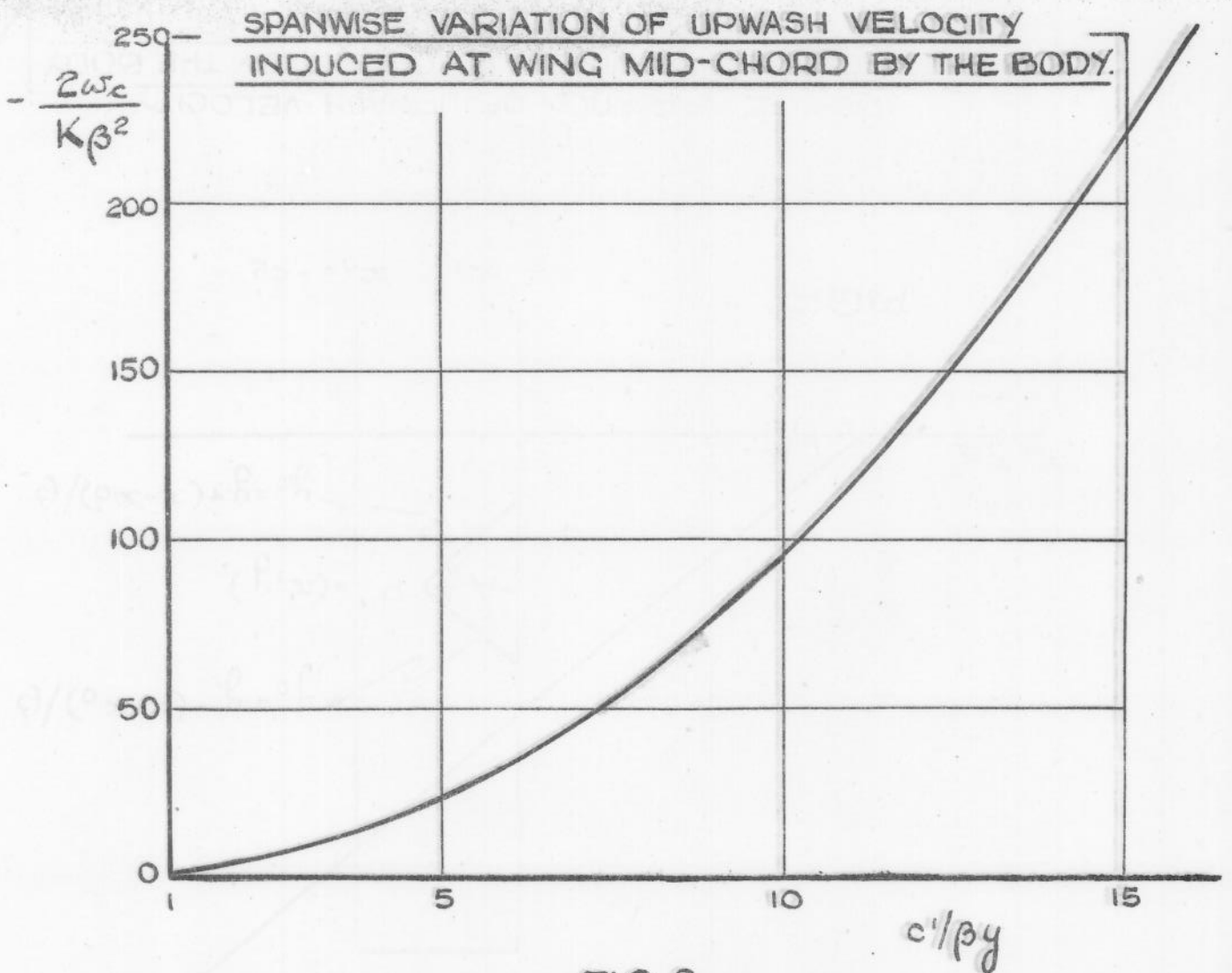

FIG.4.

$$y_0 = y - (x - x_0)/\beta$$

$-A$ P $= (x, y)$

$$y_0 = y + (x - x_0)/\beta$$

x, x_0 μ

FIG. 5.

$x_0 = -c_1$ $x_0 = -c_2$

SPANWISE VARIATION OF UPWASH VELOCITY
INDUCED AT WING MID-CHORD BY THE BODY.

$-\dfrac{2\omega_c}{K\beta^2}$

$c'/\beta y$

FIG. 6.