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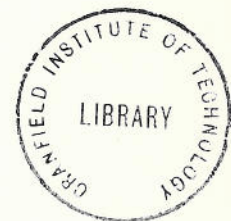
CRANFIELD INSTITUTE OF TECHNOLOGY

Department of Aircraft Design

DESIGN CHARTS FOR CARBON FIBRE COMPOSITES

by

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SUMMARY

This report contains charts of elastic properties and buckling coefficients of a simply supported compression panel, based on theory of multi layer plates of orthotropic material, for a typical carbon fibre composite. In addition the optimum orientation of plies, of a three ply system, is considered for a corrugated compression panel together with the modifications necessary for other panel shapes. The computer programmes used are contained in the Appendix.

Work carried out as part fulfilment of MinTech contract no. PD/28/040 'Application of Carbon Fibre Composite to an Airbus'.

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## NOTATION

|                  |   |
|------------------|---|
| $D_{1,2}$        | flexural rigidity (eq.4a)                           |
| $E$              | elastic modulus                                     |
| $F$              | efficiency factor (eq.5n)                           |
| $G$              | shear modulus                                       |
| $K$              | buckling coefficient                                |
| $K'$             | buckling coefficient of composite plate (eq.4d)     |
| $L$              | strut length  |
| $M$              | panel geometry parameter (eq.5l)                    |
| $N$              | $a/b(D_2/D_1)^{1/4}$                                |
| $R$              | angle ply thickness/total thickness                 |
| $S$              | measured shear strength of single ply.              |
| $V_f$            | filament volume fraction                            |
| $X$              | measured longitudinal strength of single ply        |
| $Y$              | measured transverse strength of single ply          |
| $Z$              | material parameter (eq.5l)                          |
| $a_{11}, a_{22}$ | flexibility coefficients of single layer (eq.2c)    |
| $b_{11}, b_{22}$ | stiffness coefficients of single layer (eq.2b)      |
| $a$              | plate length  |
| $b$              | plate width   |
| $r_{11}, r_{22}$ | flexibility coefficients of total composite (eq.3b) |
| $t$              | plate thickness                                     |
| $\bar{t}$        | effective thickness (eq.5d)                         |
| $w$              | end load/in.  |
| $\epsilon$       | direct strain                                       |
| $\gamma$         | shear strain  |
| $\theta$         | ply angle (fig.2)                                   |
| $\phi$           | corrugation angle (fig.9)                           |
| $\mu$            | Poisson's ratio                                     |
| $\sigma$         | direct stress                                       |
| $\tau$           | shear stress  |



Notation ctd.

Subscripts

|                         |                                  |
|-------------------------|----------------------------------|
| A                       | applied                          |
| C                       | compression                      |
| T                       | tension                          |
| CR                      | buckling                         |
| LI                      | local instability                |
| LW                      | long wave                        |
| f                       | flank                            |
| c                       | crest                            |
| x,y,z                   | applied stress axes              |
| $\alpha, \beta, \gamma$ | filamentary axes                 |
| o                       | longitudinal axis of single ply. |

## 1. Introduction

The increasing availability of high strength and stiffness, lightweight filaments and in particular carbon fibre has led to a study of the potential structural weight saving with this new material. In the course of this work the need for methods of selecting the most suitable orientation of multi-layer composites, for various types of loading, became apparent. Techniques similar to those used for predicting the behaviour of plywood and glass reinforced plastics have been used.

The results are presented as follows,

- i) graphs of elastic and strength properties together with a computer programme with which it is possible to select layups suitable for combined loading conditions
- ii) a chart of buckling coefficients for a simply supported panel, together with the modification necessary for other edge conditions
- iii) a design chart for a corrugated compression panel.

The material chosen for the study was a high modulus carbon fibre composite with a 60 per cent filament volume. However the results can be modified for other materials.

## 2.0 Elastic Properties

Methods of predicting the elastic properties of a single uni-directional laminate from the properties of its constituents, are available, but all rely on a knowledge of the void content, straightness of filaments, etc. Data regarding these properties is practically non-existent for carbon composites hence the measured elastic properties of a single sheet of composite have been used. The fibre content chosen was 60 per cent by volume as this appears to be approaching a working maximum, further increases resulting in a marked reduction in transverse strength and stiffness.

The properties used were as follows:-

Material - Fibre: Type I High Modulus, treated  
Resin: Epoxy

$$V_f = 0.60$$

$$E_\alpha = 30 \times 10^6 \text{ lb/in}^2$$

$$E_\beta = 1.1 \times 10^6 \text{ lb/in}^2$$

$$G_{\alpha\beta} = 0.7 \times 10^6 \text{ lb/in}^2$$

$$\mu_{\alpha\beta} = 0.3$$

Methods of determining the elastic properties of multi-directional laminates are well established (ref.1) and have been used with success for plywoods and glass laminates.

## 2.1 General relationship for orthotropic laminates

Each layer has three moduli of elasticity in the direction of the three axes (see fig.1) one of which is parallel to the filaments, three moduli of rigidity  $G_{\alpha\beta}$ ,  $G_{\beta\alpha}$ ,  $G_{\gamma\alpha}$  and six Poisson's ratios, two associated with each axis,  $\mu_{\alpha\beta}$ , etc. The latter are not independent but are associated as follows

$$\left. \begin{aligned} \frac{\mu_{\alpha\beta}}{E_\alpha} &= \frac{\mu_{\beta\alpha}}{E_\beta} \\ \frac{\mu_{\alpha\gamma}}{E_\alpha} &= \frac{\mu_{\gamma\alpha}}{E_\gamma} \\ \frac{\mu_{\beta\gamma}}{E_\beta} &= \frac{\mu_{\gamma\beta}}{E_\gamma} \end{aligned} \right\} \dots (2a)$$

It thus requires nine independent properties to completely define the elastic behaviour of an anisotropic material. For sheet materials it is normal to assume (ref.1) that



$$E_\gamma = E_\beta, G_{\beta\gamma} = G_{\alpha\gamma} = G_{\alpha\beta}, \mu_{\beta\gamma} = \mu_{\gamma\beta} = \mu_{\alpha\gamma}$$

this reduces the required number of elastic constants to  $E_\alpha, E_\beta, G_{\alpha\beta}, \mu_{\alpha\beta}$ .

## 2.2 Stress-strain relationship

Consider now the effect of rotating the filamentary axes,  $\alpha, \beta$  through an angle  $\theta$  with respect to the axes of applied stress,  $x, y$  (see fig.2).

It can be shown that (ref.1)

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = [b_{ij}] \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} \quad \dots (2b)$$

$$\text{where } b_{11} = \frac{1}{\lambda} [E_\alpha \cos^4 \theta + E_\beta \sin^4 \theta + \sin^2 \theta \cos^2 \theta (2E_\alpha \mu_{\beta\alpha} + 4\lambda G_{\alpha\beta})]$$

$$b_{22} = \frac{1}{\lambda} [E_\beta \cos^4 \theta + E_\alpha \sin^4 \theta + \sin^2 \theta \cos^2 \theta (2E_\alpha \mu_{\beta\alpha} + 4\lambda G_{\alpha\beta})]$$

$$b_{33} = \frac{1}{\lambda} [\sin^2 \theta \cos^2 \theta (E_\alpha + E_\beta - 2E_\alpha \mu_{\beta\alpha}) + \lambda G_{\alpha\beta} (\cos^2 \theta - \sin^2 \theta)^2]$$

$$b_{21} = b_{12} = \frac{1}{\lambda} [\sin^2 \theta \cos^2 \theta (E_\alpha + E_\beta - 4\lambda G_{\alpha\beta}) + E_\alpha \mu_{\beta\alpha} (\cos^4 \theta + \sin^4 \theta)]$$

$$b_{31} = b_{13} = \frac{1}{\lambda} [\sin^3 \theta \cos \theta (E_\beta - E_\alpha \mu_{\beta\alpha} - 2\lambda G_{\alpha\beta}) - \sin \theta \cos^3 \theta (E_\alpha - E_\alpha \mu_{\beta\alpha} - 2\lambda G_{\alpha\beta})]$$

$$b_{32} = b_{23} = \frac{1}{\lambda} [\sin \theta \cos^3 \theta (E_\beta - E_\alpha \mu_{\beta\alpha} - 2\lambda G_{\alpha\beta}) - \sin^3 \theta \cos \theta (E_\alpha - E_\alpha \mu_{\beta\alpha} - 2\lambda G_{\alpha\beta})]$$

$$\text{where } \lambda = 1 - \mu_{\beta\alpha} \mu_{\alpha\beta}$$

and also by inversion

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} = [a_{ij}] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = [b_{ij}]^{-1} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \quad \dots (2c)$$



$$\text{where } \mu_{xy} = -\frac{a_{12}}{a_{11}}; \quad \mu_{yx} = -\frac{a_{12}}{a_{22}}$$

$$E_x = \frac{1}{a_{11}}; \quad E_y = \frac{1}{a_{22}}; \quad \sigma_{xy} = \frac{1}{a_{33}}$$

### 2.3 Laminated composites

Only 'balanced' laminates are considered, that is plates that have a symmetrical distribution of plies about the median plane. The plates do not therefore twist when subjected to in plane forces or bending moments. (See fig.2)

#### a) Plate Stretching

$$b_{11} \text{ COMP} = \frac{1}{t} \sum_{i=1}^{i=n} b_{11_i} t_i \quad \dots (2d)$$

etc.

#### b) Plate Bending

$$b_{11} \text{ COMP} = \frac{1}{I} \sum_{i=1}^{i=n} b_{11_i} I_i \quad \dots (2e)$$

etc.

Inversion of the  $b_{ij}$  matrix for the composite will yield the composite elastic moduli.

For this study the number of layers has been assumed to be large, as thin prepregs (.001") are available and the effect of asymmetry could be minimised. Consequently eq.(2d) has been used to calculate the laminate properties. This was achieved with the use of an ICL 1905 computer together with a simple program in JEAN language. The results are tabulated for various values of cross ply angle ( $\theta$ ) and proportion of cross plies (R), (App.1). The results are also plotted in figs. 3, 4 and 5. It will be noted that with  $R = 1.0$  (i.e. all cross plies) at small angles (say  $10^\circ$ )  $\mu_{xy} > 1.0$ . This is verified by Cox (ref.2) and although it is not calculated would be accompanied by an appropriate Poisson's ratio  $\mu_{xz}$  through thickness of the laminate in order that the volume of the plate does not change. This is also borne out by Rothwell (ref.3) for a similar layup.

### 3.0 Strength of Multi Layer Composites

Methods of predicting the strength of uni-directional sheets based on the constituent properties have been suggested (ref.4 ). However for practical purposes the values obtained are dubious in that they rely on a knowledge of the void content, arrangement of fibres, etc. Consequently measured unidirectional sheet strength properties have been used in this analysis. Very little experimental data is available on the behaviour of carbon composites loaded at axes other than the filamentary axes.

The data used is based on the same material as that used in Appendix 1 and the measured strength values used were as follows

$$\begin{aligned} X_T &= 150,000 \text{ lb/in}^2 \text{ tension} \\ X_C &= 120,000 \text{ lb/in}^2 \text{ compression} \\ Y &= 6,000 \text{ lb/in}^2 \text{ tension and compression} \\ S &= 8,000 \text{ lb/in}^2. \end{aligned}$$

### 3.1 Theoretical analysis

Failure criteria are discussed in detail by TSAI (ref.4 ) and by Chamis (ref.5 ). The most effective method appears to be that proposed by Hill (ref.6 ) for an anisotropic material.

The criterion used is

$$K = \frac{X}{\sqrt{\sigma_\alpha^2 - \sigma_\alpha \cdot \sigma_\beta + \frac{X^2}{Y^2} \cdot \sigma_\beta^2 + \frac{X^2}{S^2} \cdot \tau_{\alpha\beta}^2}} \quad \dots (3a)$$

where X, Y, and S are the measured longitudinal transverse and shear strengths respectively.

Consider now the stresses applied to a multi layer composite.

$$\begin{aligned} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix}_{\text{COMP}} &= \begin{bmatrix} b_{ij} \end{bmatrix}_{\text{COMP}}^{-1} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} a_{ij} \end{bmatrix}_{\text{COMP}} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \\ &= \begin{bmatrix} r_{ij} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \quad (3b) \end{aligned}$$



Then using the strains calculated above stresses in a layer of the composite can be calculated

$$\therefore \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_{\text{LAYER}} = \begin{bmatrix} b_{ij} \end{bmatrix}_{\text{LAYER}} \begin{bmatrix} r_{ij} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_{\text{COMP}} \dots (3c)$$

In order that these may be applied to the failure criterion then these stresses must be transferred to the filamentary axes of the layer, hence:

$$\begin{bmatrix} \sigma_\alpha \\ \sigma_\beta \\ \tau_{\alpha\beta} \end{bmatrix}_{\text{LAYER}} = \begin{bmatrix} mn \end{bmatrix} \begin{bmatrix} b_{ij} \end{bmatrix}_{\text{LAYER}} \begin{bmatrix} r_{ij} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_{\text{COMP}} \quad (3d)$$

where

$$\begin{bmatrix} mn \end{bmatrix} = \begin{bmatrix} \cos^2\theta, \sin^2\theta, -2\sin\theta\cos\theta \\ \sin^2\theta, \cos^2\theta, 2\sin\theta\cos\theta \\ \sin\theta\cos\theta, -\sin\theta\cos\theta, (\cos^2\theta - \sin^2\theta) \end{bmatrix}$$

These stresses can then be substituted into the chosen strength criterion and the strength factor K determined. Failure will obviously occur in one of the layers and if required further analysis could be carried out to determine final strength of the laminate without the failed layer. This latter exercise has not been attempted as it was felt that for components subjected to a complex stress system almost invariably the remaining layers would be incapable of supporting the stresses imposed and total failure would occur. For simple tension however it might be possible to use the initial failure as a proof load but it is very unlikely to be of practical value.

The analysis has been carried out for various ratios of applied stresses and the results are shown in figs. 6 and 7 for simple tension and compression.

The analysis was achieved by use of an ICL 1905 and a Fortran IV computer program. This is listed in App.2. As the data and programs are on cards, substitution of new data and deletion of WRITE statements where not required is simple. Using this program and by substitution of appropriate values of  $\sigma_x$ ,  $\sigma_y$ , and  $\tau_{xy}$  (i.e. applied stress system) it is possible to determine the best layup, based on maximum strength, for a 3-fibre system. The program is being modified to include other types of layup (e.g. a 4-fibre system) and to allow for possible assymetry.

#### 4.0 Buckling of Plates

The theoretical analysis of the buckling of anisotropic plates in compression and shear is well established (ref.7) and in the case of plywood and G.R.P. plates has been verified experimentally (ref. 8, 9). Experimental testing has been carried out at the College as part of student theses (ref.10, 11) and results indicate the theory is applicable to C.F. composites.

#### 4.1 Buckling in compression of simply supported orthotropic plates

An expression for the buckling stress of an orthotropic plate is derived in ref.12 as shown below.

$$\sigma_{CR} = \frac{2\pi^2}{b^2t} \left[ (D_1 D_2)^{1/2} + D_3 \right] \quad \dots (4a)$$

where  $D_1$  = flexural rigidity corresponding to bending moment  $M_x$

$$= (EI)_x / \lambda$$

$D_2$  = flexural rigidity corresponding to bending moment  $M_y$

$$= (EI)_y / \lambda$$

$$D_3 = \frac{1}{2}(\mu_{xy} \cdot D_2 + \mu_{yx} \cdot D_1) + 2(GI)_{xy}$$

where  $(GI)_{xy}$  is the average torsional rigidity of the plate.

$$\lambda = 1 - \mu_{xy} \cdot \mu_{yx}$$

For a thin plate, thickness  $t$ , these stiffnesses may be expressed as (ref.12)

$$D_1 = E_x \cdot t^3 / 12\lambda$$

$$D_2 = E_y \cdot t^3 / 12\lambda \quad \dots (4b)$$

$$D_3 = t^3 (\mu_{xy} \cdot E_y + \mu_{yx} \cdot E_x) / 24\lambda + t^3 G_{xy} / 6$$

Then substituting (4b) into (4a) gives

$$\sigma_{CR} = \frac{\pi^2}{6\lambda} \left[ (E_x \cdot E_y)^{1/2} + \mu_{xy} \cdot E_y / 2 + \mu_{yx} \cdot E_x / 2 + 2\lambda G_{xy} \right] \times \left[ \frac{t}{b} \right]^2 \quad \dots (4c)$$

$$\text{or } \sigma_{CR} = E_{xo} (K') \left( \frac{t}{b} \right)^2 \quad \dots (4d)$$



where  $E_{xo}$  = longitudinal modulus of a single unidirectional sheet.

$$K' = \frac{1}{E_{xo}} \cdot \frac{\pi^2}{6\lambda} \left[ (E_x \cdot E_y)^{1/2} + \mu_{xy} E_y/2 + \mu_{yx} E_x/2 + 2\lambda G_{xy} \right]$$

The expression  $K'$  has been evaluated in the JEAN programme used in App.1 for plates with a balanced layup and the results are plotted in fig.8. It can be seen that the optimum layup for plate buckling occurs when  $R = 1.0$  (all angle ply),  $\theta = 45^\circ$ . However the permissible compressive stress for this layup is only 15,000 psi. compared with a maximum possible of 120,000 psi for unidirectional material.

It can be shown that the optimum stress level,

$$\sigma_{OPT} = \sqrt[3]{E_{xo} \cdot K' \left( \frac{P}{b^2} \right)^{2/3}} \quad \dots (4e)$$

where  $P$  = applied compressive load

$\frac{P}{b^2}$  = structural index.

As the structural index is increased it will be necessary to introduce an increasing number of axial plies or angle plies at a small angle,  $\theta$ , in order to increase the optimum stress level. By superimposing lines of constant stress from fig.9 on the chart for buckling coefficient,  $K'$  (fig.9) a 'path to follow' from the optimum layup (i.e.  $R = 1.0$ ,  $\theta = 45^\circ$ ) may be determined. This is shown as a dotted line on fig.8.

Fig.10 shows optimum stress level vs. structural index for carbon fibre composite compared with steel and aluminium alloy.

The buckling chart, fig.8 may also be used to determine the buckling coefficient for plates with different edge conditions and aspect ratios. Wittrick (ref.13) has suggested that the buckling curves in ref.14 which give buckling coefficient  $K$  versus  $a/b$  (panel aspect ratio) for various edge conditions may be used by substituting a value

$$N = \frac{a}{b} \cdot \left( \frac{D_2}{D_1} \right)^{1/4} \quad \text{for aspect ratio } a/b$$

Then buckling stress,

$$\sigma_{CR} = \frac{K_R \cdot A_e \cdot S.}{3.62} \cdot K' \left( \frac{t}{b} \right)^2 \quad \dots (4f)$$

## 5.0 Design of Wide Compression Panels

Optimum design procedures for wide panels subjected to compressive loading almost invariably establish a design criteria that all modes of buckling occur simultaneously. The procedure adopted in this section is similar in that local and long wave instability modes are assumed to be coincident. The types of panels considered do not buckle in the torsional mode and in fact types of construction prone to this mode of buckling appear to be unattractive in filamentary composites for most purposes.

### 5.1 Corrugated compression panels

Consider the buckling of a corrugated compression panel (see fig.11), the modes of buckling to be anticipated are

- i) local buckling of plate elements
- ii) long wave buckling of whole panel.

For i) the plate is assumed to be simply supported and infinitely long, although if necessary the interaction effect with the adjacent plates could be included and for ii) ends of the panel are assumed to be simply supported.

In this initial analysis the column is assumed to be made from a constant thickness sheet where

$$t_c = t_f = t; \quad b_c = b_f = b \quad \dots (5a)$$

Then local buckling stress of plate,

$$\sigma_{LI} = E_{xo}(K')\left(\frac{t}{b}\right)^2 \quad \dots (5b)$$

where  $E_{xo}$  and  $(K')$  are defined in 4.1.

Long wave buckling stress of panel,

$$\sigma_{LW} = \frac{\pi^2 E_x \cdot I}{L^2 \bar{t}} \quad \dots (5c)$$

$$\text{where } \bar{t} = \frac{2t}{(1 + \cos\phi)} \quad \dots (5d)$$

$$I = \left[ 2bt\left(\frac{h}{2}\right)^2 + \frac{2t}{\sin\phi} \cdot \frac{h^3}{12} \right] \cdot \frac{1}{2b(1 + \cos\phi)} \quad \dots (5e)$$

for  $0 < \phi < 120^\circ$

$$h = b \sin\phi \quad \dots (5f)$$

Substituting (5f) into (5e) we obtain

$$I = \frac{1}{3} \frac{b^2 t \sin^2 \phi}{(1 + \cos\phi)} \quad \dots (5g)$$



and by substituting (5g) and (5d) into (5e) we obtain

$$\sigma_{LW} = \frac{\pi^2 \cdot E_x \cdot \sin^2 \phi}{6} \cdot \frac{b^2}{L^2} \quad \dots (5h)$$

now applied stress,

$$\sigma_A = \frac{\omega}{t} = \frac{\omega(h \cos \phi)}{2t} \quad \dots (5i)$$

As all buckling modes occur simultaneously

$$\sigma_{LI} = \sigma_{LW} = \sigma_A = \sigma \quad \dots (5j)$$

and by examining equations (5b), (5c) and (5h) we can show

$$\begin{aligned} \sigma^4 = \sigma_{LI} \cdot \sigma_{LW} \cdot \sigma_A^2 &= \frac{\pi^2 \cdot E_x \sin^2 \phi}{6} \cdot \frac{b^2}{L^2} \cdot E_{xo}(K') \left(\frac{t}{b}\right)^2 \\ &\times \frac{\omega^2}{4t^2} (1 + \cos \phi)^2 \end{aligned}$$

Rearranging this becomes

$$\sigma^4 = \frac{\pi^2}{24} [E_x \cdot E_{xo}(K')] [\sin^2 \phi (1 + \cos \phi)^2] \left[\frac{\omega}{L}\right]^2 \quad \dots (5k)$$

or

$$\sigma^4 = \frac{\pi^2}{24} \cdot E_{xo}^2 \cdot Z \cdot M \cdot \left[\frac{\omega}{L}\right]^2 \quad \dots (5l)$$

where  $Z = \frac{E_x \cdot E_{xo} \cdot K'}{E_{xo}^2}$

$$M = \sin^2 \phi (1 + \cos \phi)^2$$

By examining eq.(5l) it can be seen that the parameters for material properties, Z and cross sectional geometry, M are independent and for a given loading index,  $\omega/L$  may be maximised separately in order to achieve the maximum stress,  $\sigma$  and hence the lightest panel. The maximum value of M occurs when  $\phi = 60^\circ$  and substituting this value into eq.(5l) yields

$$\sigma = 0.914 \sqrt[4]{E_{x0}^2 \cdot Z} \sqrt{\omega/L} \quad \dots (5m)$$

The material property parameter, Z has been evaluated in the JEAN programme in App.1 and the results are shown in fig.12. It can be seen that the maximum value of Z occurs when  $R = 0.33$ ,  $\theta = 55$ . This 'optimum' orientation will only apply up to the stress level that can be achieved by this particular configuration. In order to increase the stress level (i.e. at a higher loading index,  $\omega/L$ ) then the layu must be altered and the 'path to follow' from the optimum is obtained as in 4.0 by superposition of lines of constant allowable stress and the path is shown on fig.12. By following this path a graph of optimum stress level,  $\sigma_{opt}$  versus loading index,  $\omega/L$  may be constructed and is shown in fig.13 in comparison with aluminium alloy.

When the material considered is isotropic i.e.  $E_x = E_{x0} = E$  etc. eq.(5m) becomes

$$\sigma = F \sqrt{E} \sqrt{\omega/L} \quad \dots (5n)$$

where  $F$  = Efficiency Factor (= 1.26 for corrugated panel) and this agrees with the expression obtained by Emero and Spunt (Ref.15)

For other cross sectional shapes it is suggested that eq.(5m) can be modified as follows

$$\sigma = 0.914 \sqrt[4]{E_{x0}^2 \cdot Z} \cdot \sqrt{\frac{\omega}{L}} \cdot \frac{F'}{1.26} \quad \dots (5p)$$

where  $F'$  = Efficiency Factor of panel (see table 1 )  
However this will only apply to panels prone to similar modes of failure (i.e. no torsional instability)



## 6.0 Conclusions

Before attempting to use these highly anisotropic materials careful consideration must be given not only to the type, magnitude and variations in the applied loadings but also to the design specifications. Merely attempting to substitute carbon fibre for metal and expecting exactly similar performance will not realise the full potential of the material.

The methods presented should be useful as a guide for designers faced with selecting fibre orientations for the chosen material. They will also serve as a guideline for similar highly anisotropic materials as the buckling charts are presented in terms of the longitudinal elastic modulus of a single ply.

It is proposed to extend the work and to present a family of charts to include materials with less marked anisotropy eg. glass fibre. If required the methods and computer programme are also suitable with little modification for use with composites composed of layers of different materials.

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APPENDIX 1 - Evaluation of Elastic Properties and Buckling Coefficients

THIS IS A JEAN PROGRAMME FOR EVALUATING THE BIJ MATRIX

```
1.0 TYPE "B11"
1.1 SET Z=M* $\cos(A)^4+N*\sin(A)^4$ 
1.2 SET Y=[Z+SIN(A) $^2*\cos(A)^2*(2*M*Q+4*S*R)$ ]/S
1.3 TYPE Y
2.0 TYPE "B22"
2.1 SET X=N* $\cos(A)^4+M*\sin(A)^4$ 
2.2 SET W=[X+SIN(A) $^2*\cos(A)^2*(2*M*Q+4*S*R)$ ]/S
2.3 TYPE W
3.0 TYPE "B33"
3.1 SET V=SIN(A) $^2*\cos(A)^2*(M+N-2*M*Q)$ 
3.2 SET U=[V+S*R*( $\cos(A)^4-2*\sin(A)^2*\cos(A)^2+\sin(A)^4$ )]/S
3.3 TYPE U
4.0 TYPE "B12"
4.1 SET C=SIN(A) $^2*\cos(A)^2*(M+N-4*S*R)$ 
4.2 SET D=[C+M*Q*( $\cos(A)^4+\sin(A)^4$ )]/S
4.3 TYPE D
5.0 TYPE "B13"
5.1 SET E=SIN(A) $^3*\cos(A)*(N-M*Q-2*S*R)$ 
5.2 SET F=[E-SIN(A)* $\cos(A)^3*(M-M*Q-2*S*R)$ ]/S
5.3 TYPE F
6.0 TYPE "B32"
6.1 SET G=SIN(A)* $\cos(A)^3*(N-M*Q-2*S*R)$ 
6.2 SET H=[G-SIN(A) $^3*\cos(A)*(M-M*Q-2*S*R)$ ]/S
6.3 TYPE H
```



# Results of JEAN Programme I

| $\theta$   | $B_{11}$    | $B_{22}$    | $B_{33}$    | $B_{12}$    | $B_{13}$         | $B_{23}$        |
|------------|-------------|-------------|-------------|-------------|------------------|-----------------|
| $0^\circ$  | 30.09932778 | 1.103642019 | .7          | .3310926056 | 0                | 0               |
| $10^\circ$ | 28.41373704 | 1.166705089 | 1.511263838 | 1.142356444 | -4.708206227     | -2.50348073     |
| $20^\circ$ | 23.84202089 | 1.630036936 | 3.565455987 | 3.196548593 | -8.074434353     | -1.244599418    |
| $30^\circ$ | 17.64900923 | 3.151166349 | 5.90139711  | 5.532489716 | -9.28077814      | -3.274722096    |
| $40^\circ$ | 11.3929345  | 6.357886508 | 7.426074396 | 7.057167001 | -8.324782427     | -5.952805646    |
| $50^\circ$ | 6.357886509 | 11.3929345  | 7.426074396 | 7.057167001 | -5.952805646     | -8.324782426    |
| $60^\circ$ | 3.151166349 | 17.64900923 | 5.90139711  | 5.532489717 | -3.274722096     | -9.28077814     |
| $70^\circ$ | 1.630036937 | 23.84202089 | 3.565455988 | 3.196548593 | -1.244599419     | -8.074434354    |
| $80^\circ$ | 1.166705089 | 28.41373703 | 1.511263839 | 1.142356444 | -2.503480731     | -4.708206228    |
| $90^\circ$ | 1.103642019 | 30.09932778 | .7          | .3310926056 | -4.302697535&-11 | -1.945331443&-9 |
| $45^\circ$ | 8.666288753 | 8.666288752 | 7.635196147 | 7.266288753 | -7.248921441     | -7.248921441    |

-SET M=30.0  $E_\alpha$   
 -SET N=1.1  $E_\beta$   
 -SET Q=0.011  $\mu\beta_\alpha$   
 -SET R=0.7  $G_\alpha\beta$



JEAN Programme 2 - Buckling Coefficients

|  |                     |
|--|---------------------|
| 2.00 SET $M=A+(E-A)*R$                             | A = B11             |
| 2.01 SET $N=B+(F-B)*R$                             | B = B22             |
| 2.02 SET $O=C+(G-C)*R$                             | C = B33 0° Ply      |
| 2.03 SET $P=D+(H-D)*R$                             | D = B12             |
| 2.04 SET $Q=M*N*O-P^2*O$                           |                     |
| 2.05 SET $S=N*O/Q$                                 | E = B11             |
| 2.06 SET $T=M*O/Q$                                 | F = B22             |
| 2.07 SET $U=(M*N-P^2)/Q$                           | G = B33 Cross Ply   |
| 2.08 SET $V=P*O/Q$                                 | H = B12             |
| 2.09 TYPE R IN FORM 1                              |                     |
| 2.10 SET $W=1/S$                                   | R = Cross Ply Ratio |
| 2.11 SET $X=1/T$                                   |                     |
| 2.12 SET $Y=1/U$                                   |                     |
| 2.13 SET $I=V/S$                                   |                     |
| 2.14 SET $J=V/T$                                   |                     |
| 2.15 TYPE W,X,Y,I,J IN FORM 2                      |                     |
| 2.16 SET $K=[W*0.5*X*0.5+I*X/2+J*W/2+2*Y*(1-I*J)]$ |                     |
| 2.17 SET $K=K*1.6499/(1-I*J)$                      |                     |
| 2.19 SET $Q=K/30$                                  | Q = K               |
| 2.20 TYPE Q IN FORM 3                              |                     |
| 2.21 SET $Z=W*K/900$                               |                     |
| 2.22 TYPE Z IN FORM 4                              |                     |

Set of Typical Results for Jean Programme 2

```

*SET E=28.414
*SET F=1.1667
*SET G=1.5113
*SET H=1.1424
*DO PART 2 FOR R=0(0.1)1.0
  .000
30.000  1.100  .7000  .3000  .0110
  .4122
  .4122
  .100
29.777  1.104  .7811  .3714  .0138
  .4256
  .4224
  .200
29.544  1.108  .8623  .4420  .0166
  .4390
  .4323
  .300
29.299  1.111  .9434  .5118  .0194
  .4523
  .4418
  .400
29.044  1.114  1.0245  .5808  .0223
  .4657
  .4509
  .500
28.778  1.117  1.1057  .6490  .0252
  .4791
  .4596
  .600
28.502  1.118  1.1868  .7165  .0281
  .4924
  .4678
  .700
28.215  1.120  1.2679  .7833  .0311
  .5058
  .4757
  .800
27.919  1.121  1.3490  .8493  .0341
  .5191
  .4831
  .900
27.612  1.121  1.4302  .9146  .0371
  .5324
  .4900
1.000
27.295  1.121  1.5113  .9792  .0402
  .5457
  .4965

```

# APPENDIX 2 - FORTRAN Programme for Composite Strength

```

MASTER COMPOSITE STRENGTH
WRITE(2,1)
1 FORMAT(28H * YELLOW COMPOSITE STRENGTH)
READ(1,2)EA,EB,G,PB,BL,B110,B220,B330,B120
C EA IS EALPHA EB IS EBETA G IS SHEAR MOD PB IS MUBA BL IS LAMBDA
C SUFFIX 0 IN B TERMS REFERS TO ANGLE THETA
2 FORMAT(9F0.0)
READ(1,3)SX,SY,IXY
3 FORMAT(3F0.0)
DIMENSION R(10)
C R IS CROSS PLY RATIO
DIMENSION T(10)
C T IS ANGLE THETA IN RADIANS
READ(1,4)R
4 FORMAT(10F0.0)
READ(1,5)T
5 FORMAT(10F0.0)
WRITE(2,7)SX,SY,IXY
C SX,SY,IXY ARE APPLIED STRESSES USE 1000 AS MAX
7 FORMAT(3F7.0)
DO 61 I=1,10
DO 62 N=1,10
B11I=EA*COS(T(I))**4+EB*SIN(T(I))**4
B11I=(B11I+(2.*EA*PB+4.*BL*G)*SIN(T(I))**2*COS(T(I))**2)/BL
B22I=EB*COS(T(I))**4+EA*SIN(T(I))**4
B22I=(B22I+(2.*EA*PB+4.*BL*G)*SIN(T(I))**2*COS(T(I))**2)/BL
B33I=(EA+EB-2*EA*PB)*SIN(T(I))**2*COS(T(I))**2
B33I=(B33I+BL*G*(COS(T(I))**2-SIN(T(I))**2)**2)/BL
B12I=(EA+EB-4.*BL*G)*SIN(T(I))**2*COS(T(I))**2
B12I=(B12I+EA*PB*(COS(T(I))**4+SIN(T(I))**4))/BL
B13I=(EB-EA*PB-2.*BL*G)*SIN(T(I))**3*COS(T(I))
B13I=(B13I-(EA-EA*PB-2.*BL*G)*SIN(T(I))*COS(T(I))**3)/BL
B23I=(EB-EA*PB-2.*BL*G)*SIN(T(I))*COS(T(I))**3
B23I=(B23I-(EA-EA*PB-2.*BL*G)*SIN(T(I))*COS(T(I))**3)/BL
C SUFFIX S REFERS TO TOTAL COMPOSITE
B11S=(1.-R(N))*B110+R(N)*B11I
B22S=(1.-R(N))*B220+R(N)*B22I
B33S=(1.-R(N))*B330+R(N)*B33I
B12S=(1.-R(N))*B120+R(N)*B12I
S=B11S*B22S-B33S-B12S**2*B33S
R11=B22S*B33S/S
R22=B11S*B33S/S
R33=(B11S*B22S-B12S**2)/S
R12=(-B12S*B33S)/S
EX=R11*SX+R12*SY
EY=R12*SX+R22*SY
EXY=R33*IXY
C SX0,SY0,IXY0 ARE STRESSES IN THE 0 LAYER
SX0=B110*EX+B120*EY
SY0=B120*EX+B220*EY
IXY0=B330*EY
WRITE(2,49)SX0,SY0,IXY0
49 FORMAT(3F10.0)
IF (SX0)2,50,50
C F01 IS STRENGTH FACTOR F02 F03 F04 F05 F06 F07 F08 F09 F10 F11 F12 F13 F14 F15 F16 F17 F18 F19 F20 F21 F22 F23 F24 F25 F26 F27 F28 F29 F30 F31 F32 F33 F34 F35 F36 F37 F38 F39 F40 F41 F42 F43 F44 F45 F46 F47 F48 F49 F50 F51 F52 F53 F54 F55 F56 F57 F58 F59 F60 F61 F62 F63 F64 F65 F66 F67 F68 F69 F70 F71 F72 F73 F74 F75 F76 F77 F78 F79 F80 F81 F82 F83 F84 F85 F86 F87 F88 F89 F90 F91 F92 F93 F94 F95 F96 F97 F98 F99 F100 F101 F102 F103 F104 F105 F106 F107 F108 F109 F110 F111 F112 F113 F114 F115 F116 F117 F118 F119 F120 F121 F122 F123 F124 F125 F126 F127 F128 F129 F130 F131 F132 F133 F134 F135 F136 F137 F138 F139 F140 F141 F142 F143 F144 F145 F146 F147 F148 F149 F150 F151 F152 F153 F154 F155 F156 F157 F158 F159 F160 F161 F162 F163 F164 F165 F166 F167 F168 F169 F170 F171 F172 F173 F174 F175 F176 F177 F178 F179 F180 F181 F182 F183 F184 F185 F186 F187 F188 F189 F190 F191 F192 F193 F194 F195 F196 F197 F198 F199 F200 F201 F202 F203 F204 F205 F206 F207 F208 F209 F210 F211 F212 F213 F214 F215 F216 F217 F218 F219 F220 F221 F222 F223 F224 F225 F226 F227 F228 F229 F230 F231 F232 F233 F234 F235 F236 F237 F238 F239 F240 F241 F242 F243 F244 F245 F246 F247 F248 F249 F250 F251 F252 F253 F254 F255 F256 F257 F258 F259 F260 F261 F262 F263 F264 F265 F266 F267 F268 F269 F270 F271 F272 F273 F274 F275 F276 F277 F278 F279 F280 F281 F282 F283 F284 F285 F286 F287 F288 F289 F290 F291 F292 F293 F294 F295 F296 F297 F298 F299 F300 F301 F302 F303 F304 F305 F306 F307 F308 F309 F310 F311 F312 F313 F314 F315 F316 F317 F318 F319 F320 F321 F322 F323 F324 F325 F326 F327 F328 F329 F330 F331 F332 F333 F334 F335 F336 F337 F338 F339 F340 F341 F342 F343 F344 F345 F346 F347 F348 F349 F350 F351 F352 F353 F354 F355 F356 F357 F358 F359 F360 F361 F362 F363 F364 F365 F366 F367 F368 F369 F370 F371 F372 F373 F374 F375 F376 F377 F378 F379 F380 F381 F382 F383 F384 F385 F386 F387 F388 F389 F390 F391 F392 F393 F394 F395 F396 F397 F398 F399 F400 F401 F402 F403 F404 F405 F406 F407 F408 F409 F410 F411 F412 F413 F414 F415 F416 F417 F418 F419 F420 F421 F422 F423 F424 F425 F426 F427 F428 F429 F430 F431 F432 F433 F434 F435 F436 F437 F438 F439 F440 F441 F442 F443 F444 F445 F446 F447 F448 F449 F450 F451 F452 F453 F454 F455 F456 F457 F458 F459 F460 F461 F462 F463 F464 F465 F466 F467 F468 F469 F470 F471 F472 F473 F474 F475 F476 F477 F478 F479 F480 F481 F482 F483 F484 F485 F486 F487 F488 F489 F490 F491 F492 F493 F494 F495 F496 F497 F498 F499 F500 F501 F502 F503 F504 F505 F506 F507 F508 F509 F510 F511 F512 F513 F514 F515 F516 F517 F518 F519 F520 F521 F522 F523 F524 F525 F526 F527 F528 F529 F530 F531 F532 F533 F534 F535 F536 F537 F538 F539 F540 F541 F542 F543 F544 F545 F546 F547 F548 F549 F550 F551 F552 F553 F554 F555 F556 F557 F558 F559 F560 F561 F562 F563 F564 F565 F566 F567 F568 F569 F570 F571 F572 F573 F574 F575 F576 F577 F578 F579 F580 F581 F582 F583 F584 F585 F586 F587 F588 F589 F590 F591 F592 F593 F594 F595 F596 F597 F598 F599 F600 F601 F602 F603 F604 F605 F606 F607 F608 F609 F610 F611 F612 F613 F614 F615 F616 F617 F618 F619 F620 F621 F622 F623 F624 F625 F626 F627 F628 F629 F630 F631 F632 F633 F634 F635 F636 F637 F638 F639 F640 F641 F642 F643 F644 F645 F646 F647 F648 F649 F650 F651 F652 F653 F654 F655 F656 F657 F658 F659 F660 F661 F662 F663 F664 F665 F666 F667 F668 F669 F670 F671 F672 F673 F674 F675 F676 F677 F678 F679 F680 F681 F682 F683 F684 F685 F686 F687 F688 F689 F690 F691 F692 F693 F694 F695 F696 F697 F698 F699 F700 F701 F702 F703 F704 F705 F706 F707 F708 F709 F710 F711 F712 F713 F714 F715 F716 F717 F718 F719 F720 F721 F722 F723 F724 F725 F726 F727 F728 F729 F730 F731 F732 F733 F734 F735 F736 F737 F738 F739 F740 F741 F742 F743 F744 F745 F746 F747 F748 F749 F750 F751 F752 F753 F754 F755 F756 F757 F758 F759 F760 F761 F762 F763 F764 F765 F766 F767 F768 F769 F770 F771 F772 F773 F774 F775 F776 F777 F778 F779 F780 F781 F782 F783 F784 F785 F786 F787 F788 F789 F790 F791 F792 F793 F794 F795 F796 F797 F798 F799 F800 F801 F802 F803 F804 F805 F806 F807 F808 F809 F810 F811 F812 F813 F814 F815 F816 F817 F818 F819 F820 F821 F822 F823 F824 F825 F826 F827 F828 F829 F830 F831 F832 F833 F834 F835 F836 F837 F838 F839 F840 F841 F842 F843 F844 F845 F846 F847 F848 F849 F850 F851 F852 F853 F854 F855 F856 F857 F858 F859 F860 F861 F862 F863 F864 F865 F866 F867 F868 F869 F870 F871 F872 F873 F874 F875 F876 F877 F878 F879 F880 F881 F882 F883 F884 F885 F886 F887 F888 F889 F890 F891 F892 F893 F894 F895 F896 F897 F898 F899 F900 F901 F902 F903 F904 F905 F906 F907 F908 F909 F910 F911 F912 F913 F914 F915 F916 F917 F918 F919 F920 F921 F922 F923 F924 F925 F926 F927 F928 F929 F930 F931 F932 F933 F934 F935 F936 F937 F938 F939 F940 F941 F942 F943 F944 F945 F946 F947 F948 F949 F950 F951 F952 F953 F954 F955 F956 F957 F958 F959 F960 F961 F962 F963 F964 F965 F966 F967 F968 F969 F970 F971 F972 F973 F974 F975 F976 F977 F978 F979 F980 F981 F982 F983 F984 F985 F986 F987 F988 F989 F990 F991 F992 F993 F994 F995 F996 F997 F998 F999 F1000 F1001 F1002 F1003 F1004 F1005 F1006 F1007 F1008 F1009 F1010 F1011 F1012 F1013 F1014 F1015 F1016 F1017 F1018 F1019 F1020 F1021 F1022 F1023 F1024 F1025 F1026 F1027 F1028 F1029 F1030 F1031 F1032 F1033 F1034 F1035 F1036 F1037 F1038 F1039 F1040 F1041 F1042 F1043 F1044 F1045 F1046 F1047 F1048 F1049 F1050 F1051 F1052 F1053 F1054 F1055 F1056 F1057 F1058 F1059 F1060 F1061 F1062 F1063 F1064 F1065 F1066 F1067 F1068 F1069 F1070 F1071 F1072 F1073 F1074 F1075 F1076 F1077 F1078 F1079 F1080 F1081 F1082 F1083 F1084 F1085 F1086 F1087 F1088 F1089 F1090 F1091 F1092 F1093 F1094 F1095 F1096 F1097 F1098 F1099 F1100 F1101 F1102 F1103 F1104 F1105 F1106 F1107 F1108 F1109 F1110 F1111 F1112 F1113 F1114 F1115 F1116 F1117 F1118 F1119 F1120 F1121 F1122 F1123 F1124 F1125 F1126 F1127 F1128 F1129 F1130 F1131 F1132 F1133 F1134 F1135 F1136 F1137 F1138 F1139 F1140 F1141 F1142 F1143 F1144 F1145 F1146 F1147 F1148 F1149 F1150 F1151 F1152 F1153 F1154 F1155 F1156 F1157 F1158 F1159 F1160 F1161 F1162 F1163 F1164 F1165 F1166 F1167 F1168 F1169 F1170 F1171 F1172 F1173 F1174 F1175 F1176 F1177 F1178 F1179 F1180 F1181 F1182 F1183 F1184 F1185 F1186 F1187 F1188 F1189 F1190 F1191 F1192 F1193 F1194 F1195 F1196 F1197 F1198 F1199 F1200 F1201 F1202 F1203 F1204 F1205 F1206 F1207 F1208 F1209 F1210 F1211 F1212 F1213 F1214 F1215 F1216 F1217 F1218 F1219 F1220 F1221 F1222 F1223 F1224 F1225 F1226 F1227 F1228 F1229 F1230 F1231 F1232 F1233 F1234 F1235 F1236 F1237 F1238 F1239 F1240 F1241 F1242 F1243 F1244 F1245 F1246 F1247 F1248 F1249 F1250 F1251 F1252 F1253 F1254 F1255 F1256 F1257 F1258 F1259 F1260 F1261 F1262 F1263 F1264 F1265 F1266 F1267 F1268 F1269 F1270 F1271 F1272 F1273 F1274 F1275 F1276 F1277 F1278 F1279 F1280 F1281 F1282 F1283 F1284 F1285 F1286 F1287 F1288 F1289 F1290 F1291 F1292 F1293 F1294 F1295 F1296 F1297 F1298 F1299 F1300 F1301 F1302 F1303 F1304 F1305 F1306 F1307 F1308 F1309 F1310 F1311 F1312 F1313 F1314 F1315 F1316 F1317 F1318 F1319 F1320 F1321 F1322 F1323 F1324 F1325 F1326 F1327 F1328 F1329 F1330 F1331 F1332 F1333 F1334 F1335 F1336 F1337 F1338 F1339 F1340 F1341 F1342 F1343 F1344 F1345 F1346 F1347 F1348 F1349 F1350 F1351 F1352 F1353 F1354 F1355 F1356 F1357 F1358 F1359 F1360 F1361 F1362 F1363 F1364 F1365 F1366 F1367 F1368 F1369 F1370 F1371 F1372 F1373 F1374 F1375 F1376 F1377 F1378 F1379 F1380 F1381 F1382 F1383 F1384 F1385 F1386 F1387 F1388 F1389 F1390 F1391 F1392 F1393 F1394 F1395 F1396 F1397 F1398 F1399 F1400 F1401 F1402 F1403 F1404 F1405 F1406 F1407 F1408 F1409 F1410 F1411 F1412 F1413 F1414 F1415 F1416 F1417 F1418 F1419 F1420 F1421 F1422 F1423 F1424 F1425 F1426 F1427 F1428 F1429 F1430 F1431 F1432 F1433 F1434 F1435 F1436 F1437 F1438 F1439 F1440 F1441 F1442 F1443 F1444 F1445 F1446 F1447 F1448 F1449 F1450 F1451 F1452 F1453 F1454 F1455 F1456 F1457 F1458 F1459 F1460 F1461 F1462 F1463 F1464 F1465 F1466 F1467 F1468 F1469 F1470 F1471 F1472 F1473 F1474 F1475 F1476 F1477 F1478 F1479 F1480 F1481 F1482 F1483 F1484 F1485 F1486 F1487 F1488 F1489 F1490 F1491 F1492 F1493 F1494 F1495 F1496 F1497 F1498 F1499 F1500 F1501 F1502 F1503 F1504 F1505 F1506 F1507 F1508 F1509 F1510 F1511 F1512 F1513 F1514 F1515 F1516 F1517 F1518 F1519 F1520 F1521 F1522 F1523 F1524 F1525 F1526 F1527 F1528 F1529 F1530 F1531 F1532 F1533 F1534 F1535 F1536 F1537 F1538 F1539 F1540 F1541 F1542 F1543 F1544 F1545 F1546 F1547 F1548 F1549 F1550 F1551 F1552 F1553 F1554 F1555 F1556 F1557 F1558 F1559 F1560 F1561 F1562 F1563 F1564 F1565 F1566 F1567 F1568 F1569 F1570 F1571 F1572 F1573 F1574 F1575 F1576 F1577 F1578 F1579 F1580 F1581 F1582 F1583 F1584 F1585 F1586 F1587 F1588 F1589 F1590 F1591 F1592 F1593 F1594 F1595 F1596 F1597 F1598 F1599 F1600 F1601 F1602 F1603 F1604 F1605 F1606 F1607 F1608 F1609 F1610 F1611 F1612 F1613 F1614 F1615 F1616 F1617 F1618 F1619 F1620 F1621 F1622 F1623 F1624 F1625 F1626 F1627 F1628 F1629 F1630 F1631 F1632 F1633 F1634 F1635 F1636 F1637 F1638 F1639 F1640 F1641 F1642 F1643 F1644 F1645 F1646 F1647 F1648 F1649 F1650 F1651 F1652 F1653 F1654 F1655 F1656 F1657 F1658 F1659 F1660 F1661 F1662 F1663 F1664 F1665 F1666 F1667 F1668 F1669 F1670 F1671 F1672 F1673 F1674 F1675 F1676 F1677 F1678 F1679 F1680 F1681 F1682 F1683 F1684 F1685 F1686 F1687 F1688 F1689 F1690 F1691 F1692 F1693 F1694 F1695 F1696 F1697 F1698 F1699 F1700 F1701 F1702 F1703 F1704 F1705 F1706 F1707 F1708 F1709 F1710 F1711 F1712 F1713 F1714 F1715 F1716 F1717 F1718 F1719 F1720 F1721 F1722 F1723 F1724 F1725 F1726 F1727 F1728 F1729 F1730 F1731 F1732 F1733 F1734 F1735 F1736 F1737 F1738 F1739 F1740 F1741 F1742 F1743 F1744 F1745 F1746 F1747 F1748 F1749 F1750 F1751 F1752 F1753 F1754 F1755 F1756 F1757 F1758 F1759 F1760 F1761 F1762 F1763 F1764 F1765 F1766 F1767 F1768 F1769 F1770 F1771 F1772 F1773 F1774 F1775 F1776 F1777 F1778 F1779 F1780 F1781 F1782 F1783 F1784 F1785 F1786 F1787 F1788 F1789 F1790 F1791 F1792 F1793 F1794 F1795 F1796 F1797 F1798 F1799 F1800 F1801 F1802 F1803 F1804 F1805 F1806 F1807 F1808 F1809 F1810 F1811 F1812 F1813 F1814 F1815 F1816 F1817 F1818 F1819 F1820 F1821 F1822 F1823 F1824 F1825 F1826 F1827 F1828 F1829 F1830 F1831 F1832 F1833 F1834 F1835 F1836 F1837 F1838 F1839 F1840 F1841 F1842 F1843 F1844 F1845 F1846 F1847 F1848 F1849 F1850 F1851 F1852 F1853 F1854 F1855 F1856 F1857 F1858 F1859 F1860 F1861 F1862 F1863 F1864 F1865 F1866 F1867 F1868 F1869 F1870 F1871 F1872 F1873 F1874 F1875 F1876 F1877 F1878 F1879 F1880 F1881 F1882 F1883 F1884 F1885 F1886 F1887 F1888 F1889 F1890 F1891 F1892 F1893 F1894 F1895 F1896 F1897 F1898 F1899 F1900 F1901 F1902 F1903 F1904 F1905 F1906 F1907 F1908 F1909 F1910 F1911 F1912 F1913 F1914 F1915 F1916 F1917 F1918 F1919 F1920 F1921 F1922 F1923 F1924 F1925 F1926 F1927 F1928 F1929 F1930 F1931 F1932 F1933 F1934 F1935 F1936 F1937 F1938 F1939 F1940 F1941 F1942 F1943 F1944 F1945 F1946 F1947 F1948 F1949 F1950 F1951 F1952 F1953 F1954 F1955 F1956 F1957 F1958 F1959 F1960 F1961 F1962 F1963 F1964 F1965 F1966 F1967 F1968 F1969 F1970 F1971 F1972 F1973 F1974 F1975 F1976 F1977 F1978 F1979 F1980 F1981 F1982 F1983 F1984 F1985 F1986 F1987 F1988 F1989 F1990 F1991 F1992 F1993 F1994 F1995 F1996 F1997 F1998 F1999 F2000 F2001 F2002 F2003 F2004 F2005 F2006 F2007 F2008 F2009 F2010 F2011 F2012 F2013 F2014 F2015 F2016 F2017 F2018 F2019 F2020 F2021 F2022 F2023 F2024 F2025 F2026 F2027 F2028 F2029 F2030 F2031 F2032 F2033 F2034 F2035 F2036 F2037 F2038 F2039 F2040 F2041 F2042 F2043 F2044 F2045 F2046 F2047 F2048 F2049 F2050 F2051 F2052 F2053 F2054 F2055 F2056 F2057 F2058 F2059 F2060 F2061 F2062 F2063 F2064 F2065 F2066 F2067 F2068 F2069 F2070 F2071 F2072 F2073 F2074 F2075 F2076 F2077 F2078 F2079 F2080 F2081 F2082 F2083 F2084 F2085 F2086 F2087 F2088 F2089 F2090 F2091 F2092 F2093 F2094 F2095 F2096 F2097 F2098 F2099 F2100 F2101 F2102 F2103 F2104 F2105 F2106 F2107 F2108 F2109 F2110 F2111 F2112 F2113 F2114 F2115 F2116 F2117 F2118 F2119 F2120 F2121 F2122 F2123 F2124 F2125 F2126 F2127 F2128 F2129 F2130 F2131 F2132 F2133 F2134 F2135 F2136 F2137 F2138 F2139 F2140 F2141 F2142 F2143 F2144 F2145 F2146 F2147 F2148 F2149 F2150 F2151 F2152 F2153 F2154 F2155 F2156 F2157 F2158 F2159 F2160 F2161 F2162 F2163 F2164 F2165 F2166 F2167 F2168 F2169 F2170 F2171 F2172 F2173 F2174 F2175 F2176 F2177 F2178 F2179 F2180 F2181 F2182 F2183 F2184 F2185 F2186 F2187 F2188 F2189 F2190 F2191 F2192 F2193 F2194 F2195 F2196 F2197 F2198 F2199 F2200 F2201 F2202 F2203 F2204 F2205 F2206 F2207 F2208 F2209 F2210 F2211 F2212 F2213 F2214 F2215 F2216 F2217 F2218 F2219 F2220 F2221 F2222 F2223 F2224 F2225 F2226 F2227 F2228 F2229 F2230 F2231 F2232 F2233 F2234 F2235 F2236 F2237 F2238 F2239 F2240 F2241 F2242 F2243 F2244 F2245 F2246 F2247 F2248 F2249 F2250 F2251 F2252 F2253 F2254 F2255 F2256 F2257 F2258 F2259 F2260 F2261 F2262 F2263 F2264 F2265 F2266 F2267 F2268 F2269 F2270 F2271 F2272 F2273 F2274 F2275 F2276 F2277 F2278 F2279 F2280 F2281 F2282 F2283 F2284 F2285 F2286 F2287 F2288 F2289 F2290 F2291 F2292 F2293 F2294 F2295 F2296 F2297 F2298 F2299 F2300 F2301 F2302 F2303 F2304 F2305 F2306 F2307 F2308 F2309 F2310 F2311 F2312 F2313 F2314 F2315 F2316 F2317 F2318 F2319 F2320 F2321 F2322 F2323 F2324 F2325 F2326 F2327 F2328 F2329 F2330 F2331 F2332 F2333 F2334 F2335 F2336 F2337 F2338 F2339 F2340 F2341 F2342 F2343 F2344 F2345 F2346 F2347 F2348 F2349 F2350 F2351 F2352 F2353 F2354 F2355 F2356 F2357 F2358 F2359 F2360 F2361 F2362 F2363 F2364 F2365 F2366 F2367 F2368 F2369 F2370 F2371 F2372 F2373 F2374 F2375 F2376 F2377 F2378 F2379 F2380 F2381 F2382 F2383 F2384 F2385 F2386 F2387 F2388 F2389 F2390 F2391 F2392 F2393 F2394 F2395 F2396 F2397 F2398 F2399 F2400 F2401 F2402 F2403 F2404 F2405 F2406 F2407 F2408 F2409 F2410 F2411 F2412 F2413 F2414 F2415 F2416 F2417 F2418 F2419 F2420 F2421 F2422 F2423 F2424 F2425 F2426 F2427 F2428 F2429 F2430 F2431 F2432 F2433 F2434 F2435 F2436 F2437 F2438 F2439 F2440 F2441 F2442 F2443 F2444 F2445 F2446 F2447 F2448 F2449 F2450 F2451 F2452 F2453 F2454 F2455 F2456 F2457 F2458 F2459 F2460 F2461 F2462 F2463 F2464 F2465 F2466 F2467 F2468 F2469 F2470 F2471 F2472 F2473 F2474 F
```



```
50 FOT=150./SQRT(SX0**2-SX0*SY0+625.*SY0**2+352.*TXY0**2)
WRITE(2,51)FOT
51 FORMAT(5H FOT=,3PF10.0)
GO TO 54
52 FOC=120./SQRT(SX0**2-SX0*SY0+400.*SY0**2+225.*TXY0**2)
WRITE(2,53)FOC
53 FORMAT(5H FOC=,3PF10.0)
54 SXT=B111*EX+B121*EY+B131*EXY
SYT=B121*EX+B221*EY+B231*EXY
TXYT=B131*EX+B231*EY+B331*EXY
C SXT,SYT,TXYT ARE STRESSES IN CROSS PLY LAYER - 0 Positive.
WRITE(2,47)SXT,SYT,TXYT
47 FORMAT(3F10.0)
SA=COS(T(1))**2*SXT+SIN(T(1))**2*SYT-2.*SIN(T(1))*COS(T(1))*TXYT
SB=SIN(T(1))**2*SXT+COS(T(1))**2*SYT+2.*SIN(T(1))*COS(T(1))*TXYT
TAB=SIN(T(1))*COS(T(1))*SXT-SIN(T(1))*COS(T(1))*SYT
TAB=TAB+(COS(T(1))**2-SIN(T(1))**2)*TXYT
C SA,SB,TAB ARE STRESSES IN CROSS PLY REFERED TO FILAMENT AXES
WRITE(2,48)SA,SB,TAB
48 FORMAT(3F10.0)
IF (SA)57,55,55
55 FIT=150./SQRT(SA**2-SA*SB+625.*SB**2+352.*TAB**2)
WRITE(2,56)FIT
56 FORMAT(5H FIT=,3PF10.0)
GO TO 59
57 FIC=120./SQRT(SA**2-SA*SB+400.*SB**2+255.*TAB**2)
WRITE(2,58)FIC
58 FORMAT(5H FIC=,3PF10.0)
59 WRITE(2,60)((N),T(I))
60 FORMAT(1F6.3,1F8.4)
62 CONTINUE
61 CONTINUE
STOP
END
```

END OF SEGMENT, LENGTH 989, NAME COMPOSITESTRENGTH

Typical Set of Results

R TELLOW COMPOSITE STRENGTH

For  $\sigma_x = 1000$ .

$\sigma_y = 0$ .

$\tau_{xy} = 0$ .

$\theta = 10^\circ$

|       |        |       |  |
|-------|--------|-------|--|
| 1000. | 0.     | 0.    |  |
| 1007. | -3.    | 0.    |  |
| FOT=  | 148.   |       |  |
| 940.  | 24.    | -155. |  |
| 965.  | -2.    | 11.   |  |
| FIT=  | 152.   |       |  |
| 0.100 | 0.1745 |       |  |
| 1014. | -5.    | 0.    |  |
| FOT=  | 146.   |       |  |
| 945.  | 21.    | -156. |  |
| 970.  | -4.    | 12.   |  |
| FIT=  | 150.   |       |  |
| 0.200 | 0.1745 |       |  |
| 1021. | -8.    | 0.    |  |
| FOT=  | 144.   |       |  |
| 950.  | 19.    | -156. |  |
| 975.  | -7.    | 12.   |  |
| FIT=  | 147.   |       |  |
| 0.300 | 0.1745 |       |  |
| 1030. | -11.   | 0.    |  |
| FOT=  | 140.   |       |  |
| 955.  | 10.    | -157. |  |
| 981.  | -9.    | 13.   |  |
| FIT=  | 144.   |       |  |
| 0.400 | 0.1745 |       |  |
| 1038. | -13.   | 0.    |  |
| FOT=  | 137.   |       |  |
| 962.  | 15.    | -158. |  |
| 987.  | -12.   | 14.   |  |
| FIT=  | 140.   |       |  |
| 0.500 | 0.1745 |       |  |
| 1048. | -16.   | 0.    |  |
| FOT=  | 135.   |       |  |
| 968.  | 11.    | -159. |  |
| 994.  | -15.   | 14.   |  |
| FIT=  | 136.   |       |  |
| 0.600 | 0.1745 |       |  |
| 1058. | -19.   | 0.    |  |
| FOT=  | 129.   |       |  |
| 975.  | 8.     | -160. |  |
| 1001. | -17.   | 15.   |  |
| FIT=  | 132.   |       |  |
| 0.700 | 0.1745 |       |  |
| 1068. | -22.   | 0.    |  |
| FOT=  | 124.   |       |  |
| 983.  | 5.     | -161. |  |
| 1009. | -20.   | 16.   |  |
| FIT=  | 128.   |       |  |
| 0.800 | 0.1745 |       |  |
| 1079. | -25.   | 0.    |  |
| FOT=  | 120.   |       |  |
| 991.  | 3.     | -162. |  |
| 1017. | -25.   | 17.   |  |
| FIT=  | 125.   |       |  |
| 0.900 | 0.1745 |       |  |
| 1091. | -27.   | 0.    |  |
| FOT=  | 115.   |       |  |
| 1000. | 0.     | -163. |  |
| 1026. | -25.   | 17.   |  |
| FIT=  | 119.   |       |  |
| 1.000 | 0.1745 |       |  |

TABLE 1: Efficiency Factors for Wide Columns  
(Derived from ref.15, Emero and Spunt)

$$\sigma_{OPT} = F \cdot \sqrt{E} \sqrt{\frac{\omega}{L}}$$

| TYPE                                  | F    |
|---------------------------------------|------|
| Plain corrugation                     | 1.26 |
| Trapezoidal corrugated, semi sandwich | 0.83 |
| Truss core, semi sandwich             | 0.83 |
| Semi trap. corrugated semi sandwich   | 0.85 |
| Top hat stiffened                     | 0.96 |
| Truss core corrugation                | 1.07 |
| Semicircle corrugation                | 0.84 |
| Truss core sandwich                   | 0.78 |



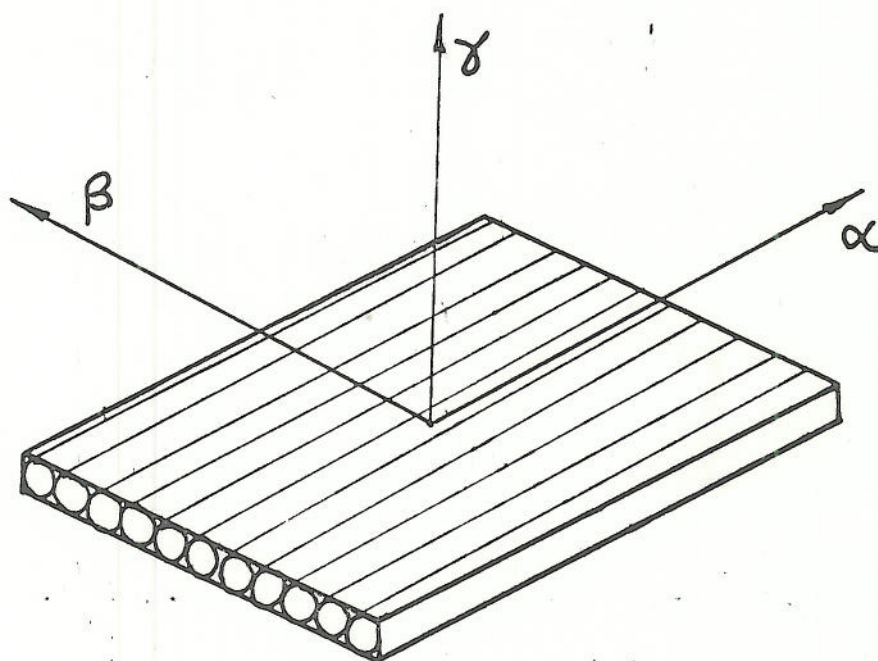


FIG. 1. SINGLE UNIDIRECTIONAL PLY

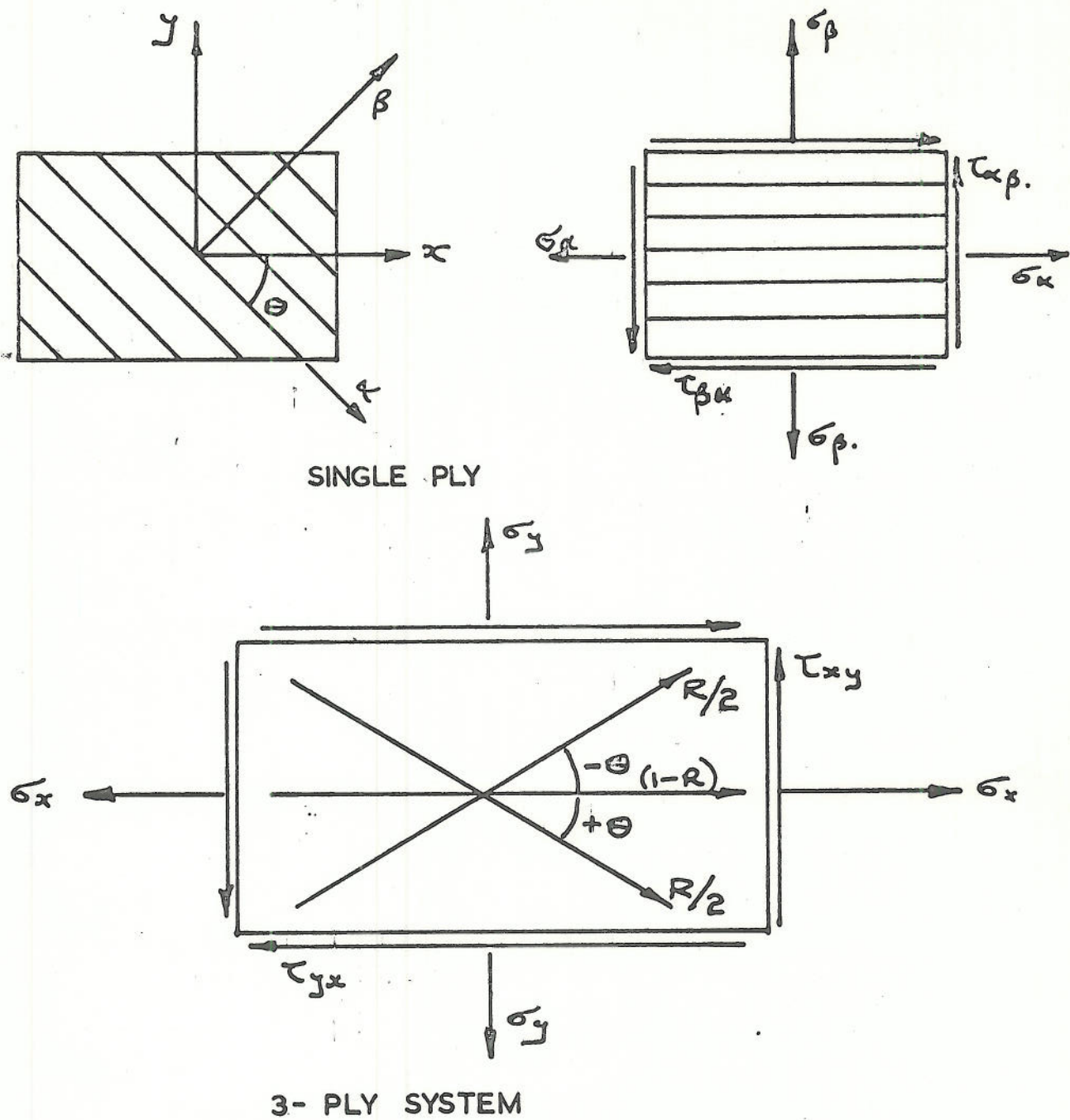


FIG. 2. SIGN CONVENTION



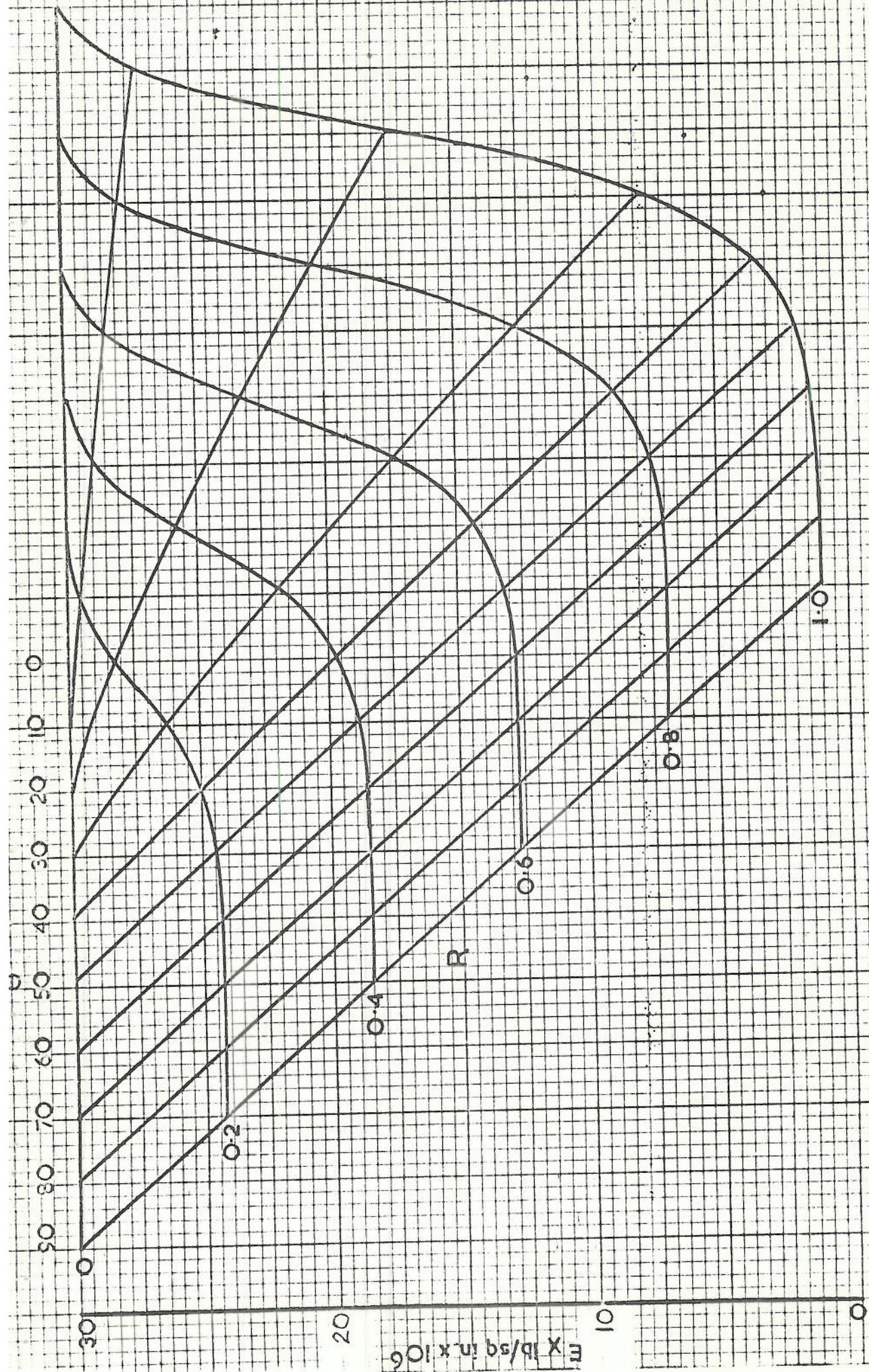


FIG 3. LONGITUDINAL ELASTIC MODULUS, EX.



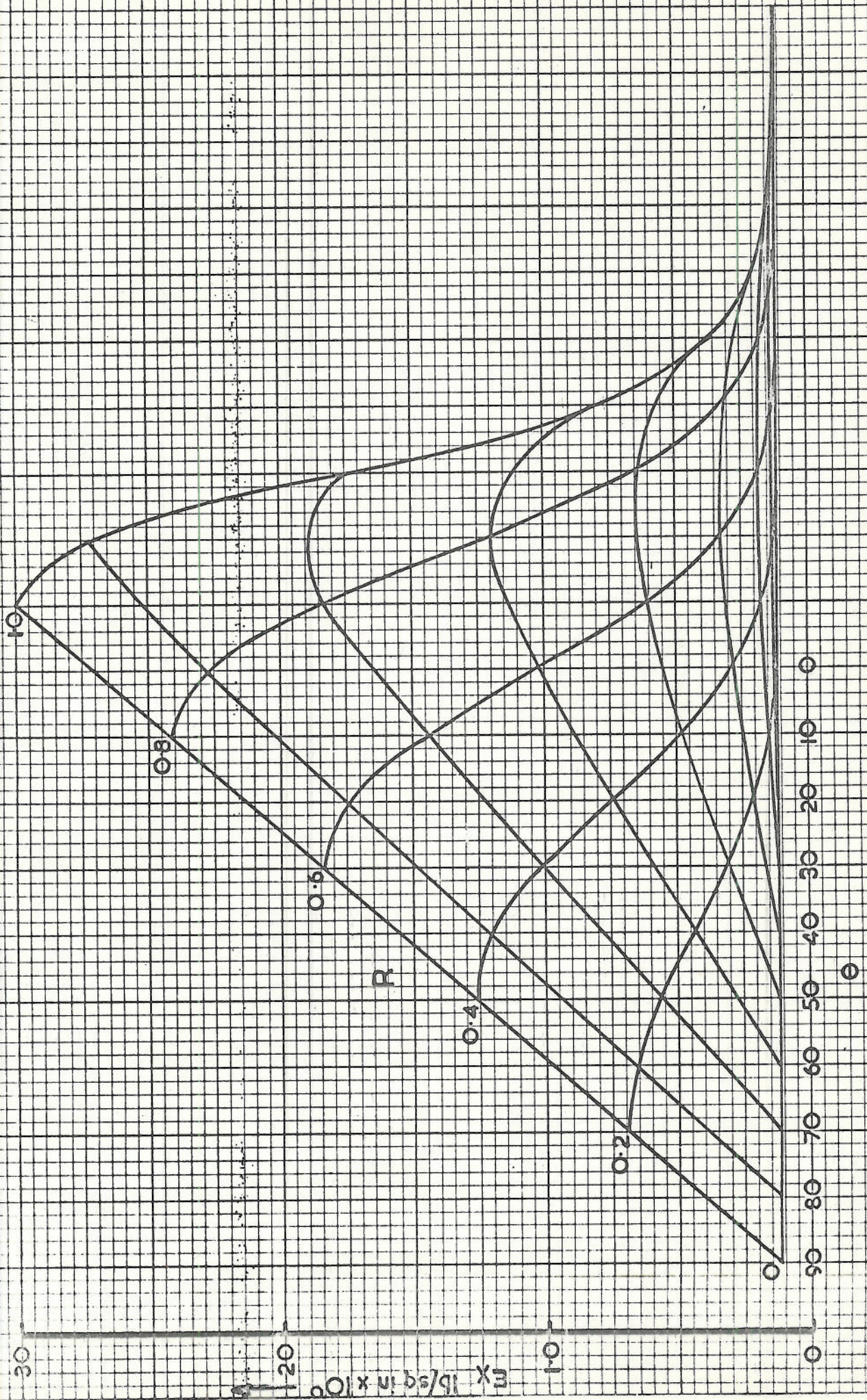


FIG 4. TRANSVERSE ELASTIC MODULUS,  $E_T$



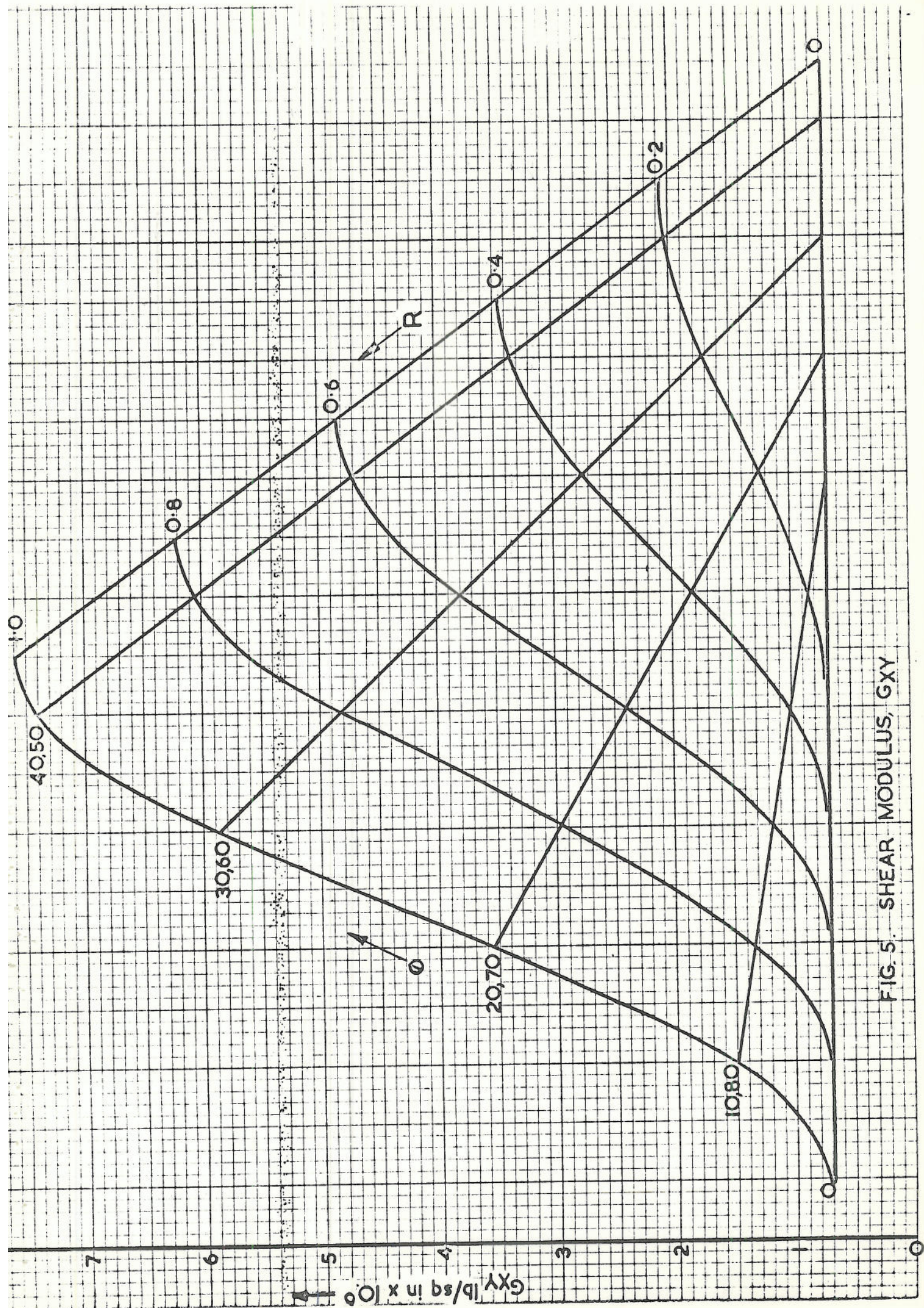


FIG. 5. SHEAR MODULUS,  $G_{XY}$



FIG. 6. STRENGTH CHART  
(longitudinal stress only)

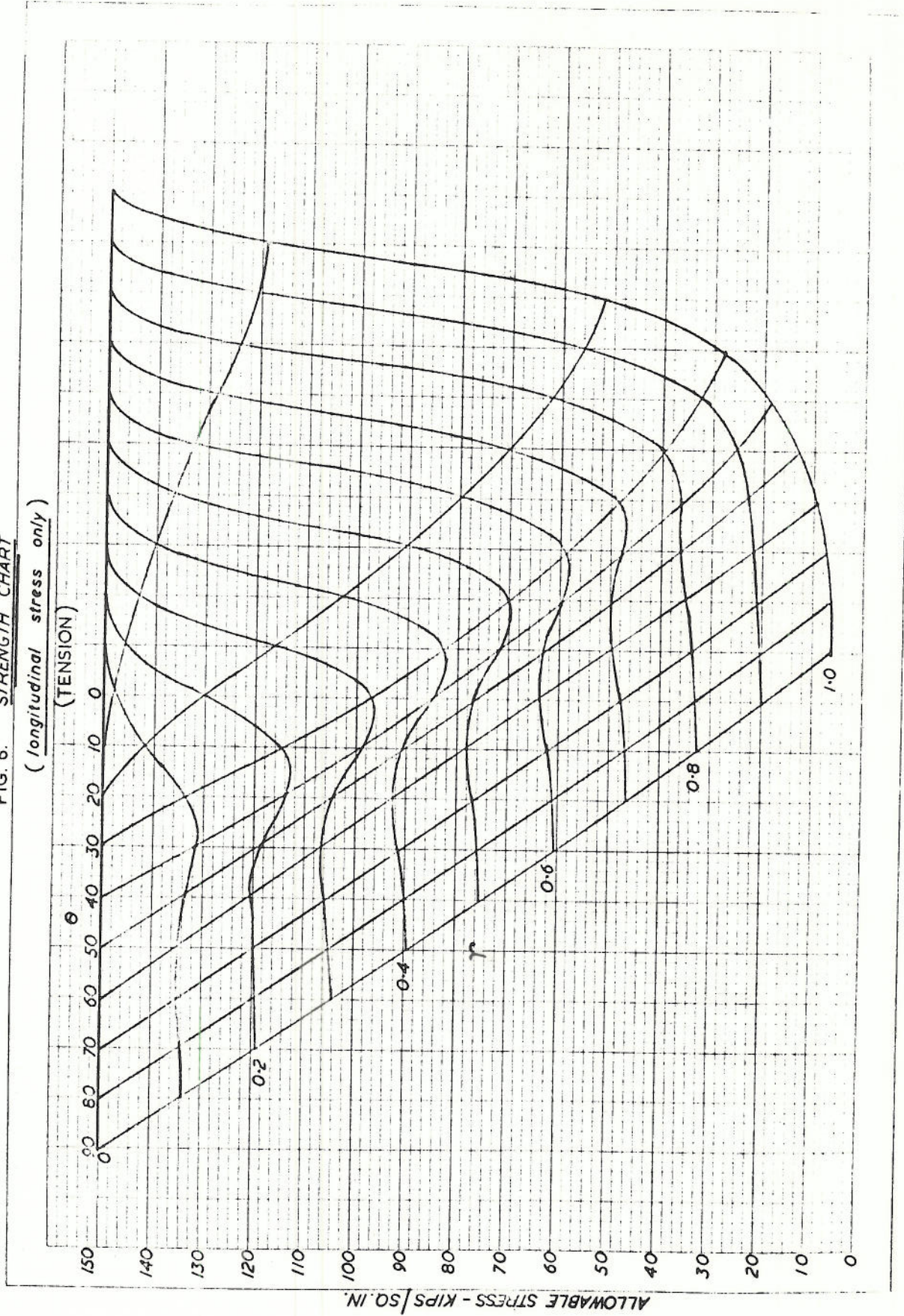
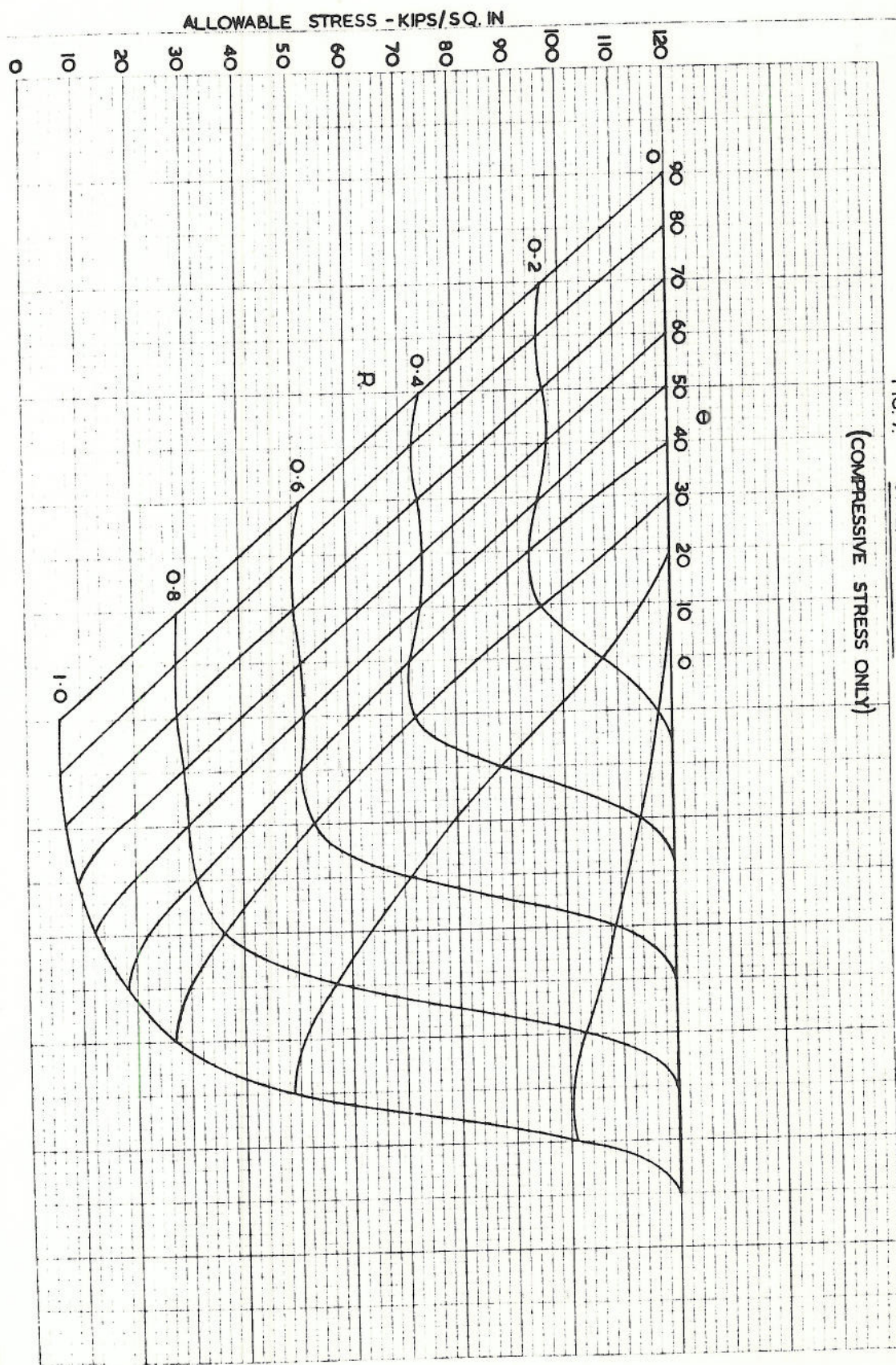




FIG. 7. STRENGTH CHART

(COMPRESSIVE STRESS ONLY)



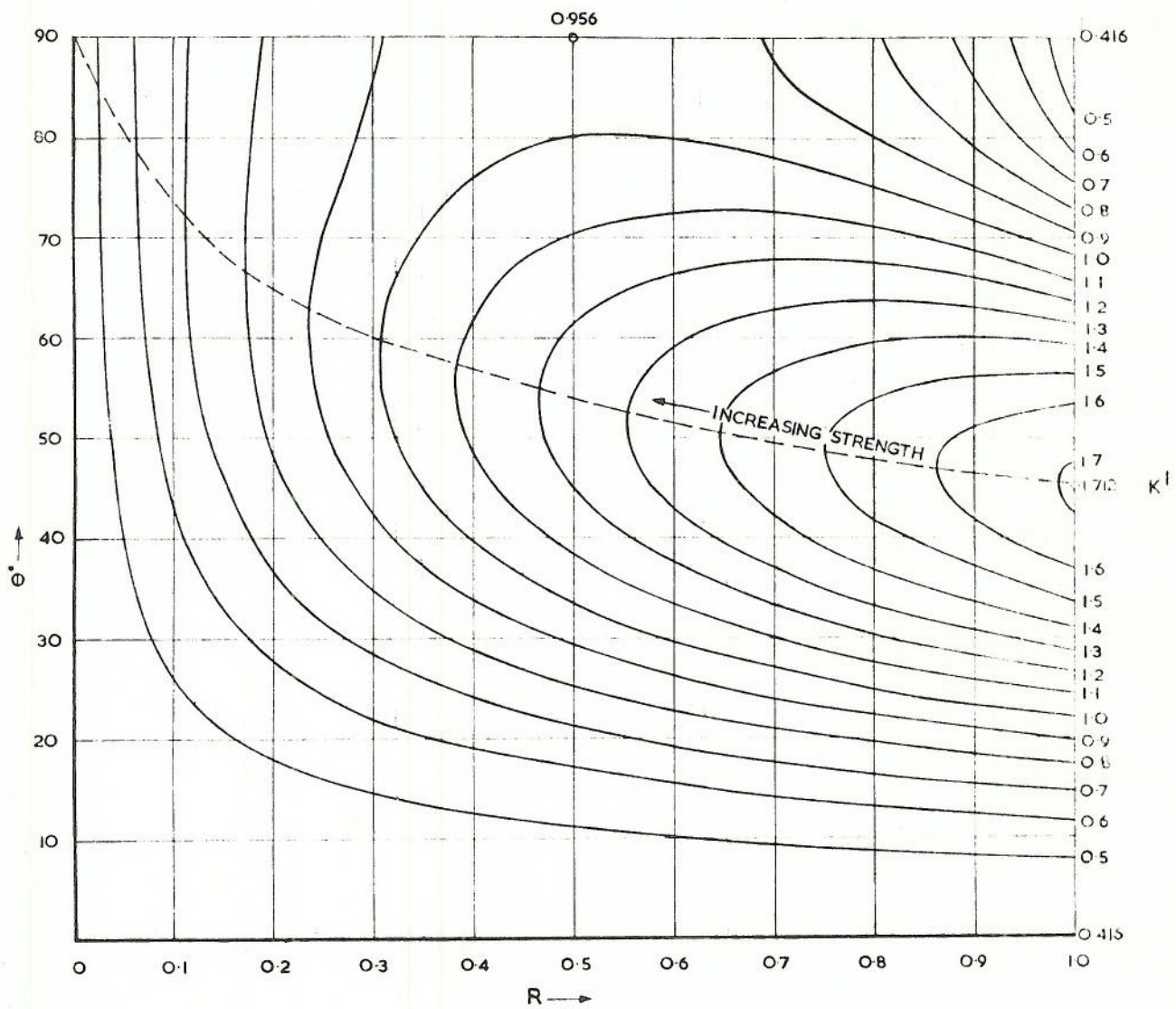


FIG. 8. SIMPLY SUPPORTED COMPRESSION PANEL, BUCKLING COEFFICIENTS



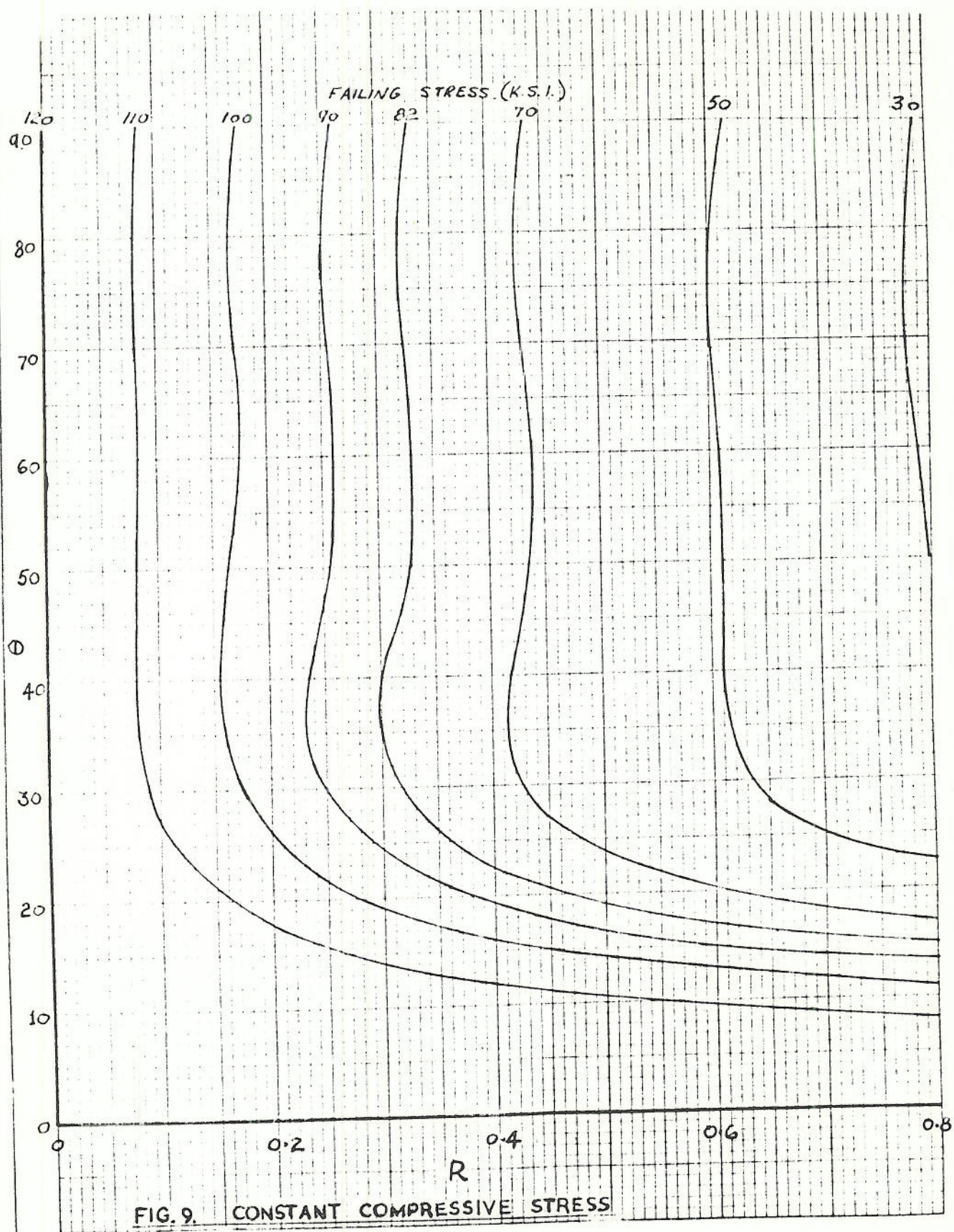
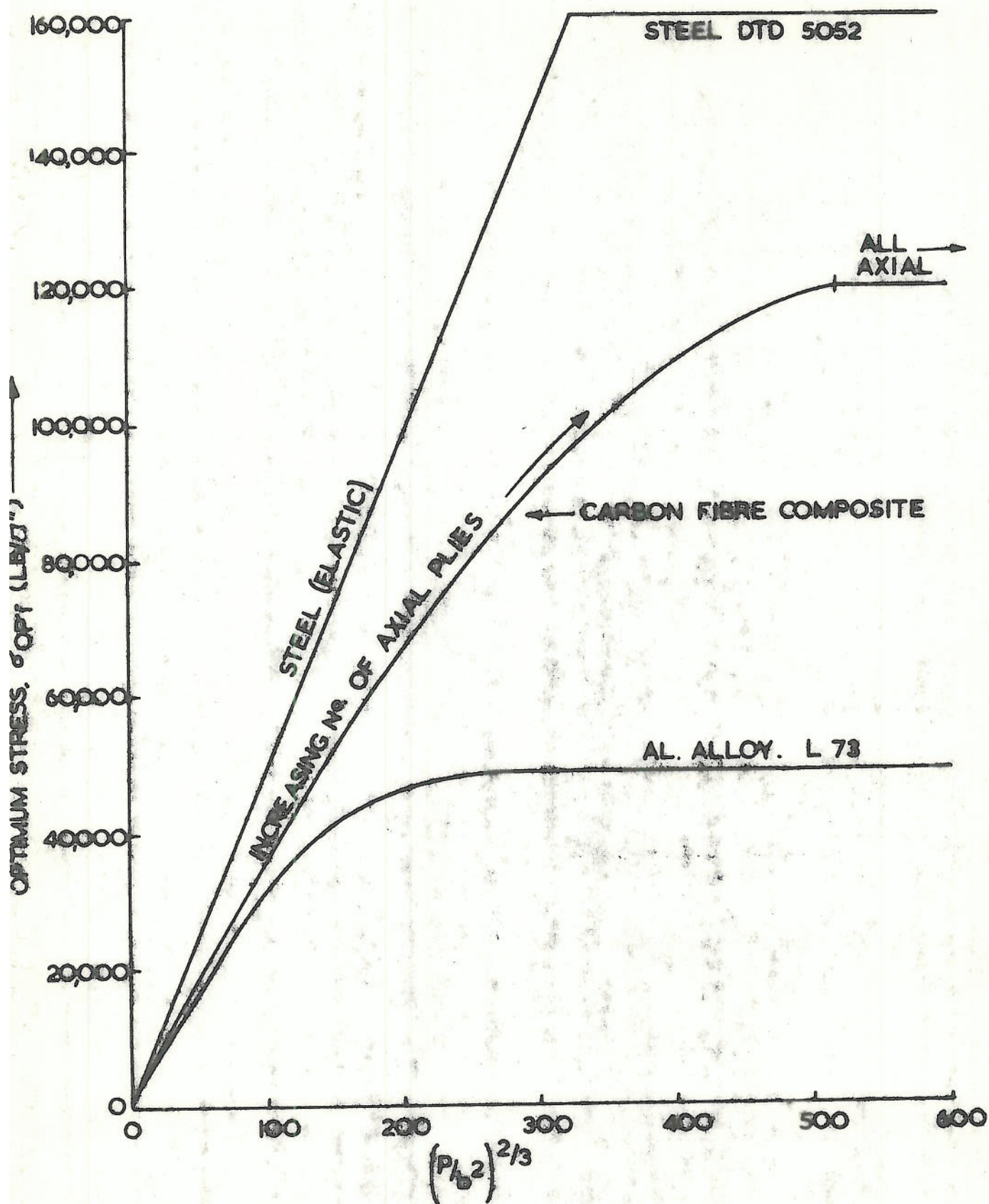




FIG. 10. OPTIMUM STRESS LEVEL FOR S.S. PLATE IN COMPRESSION



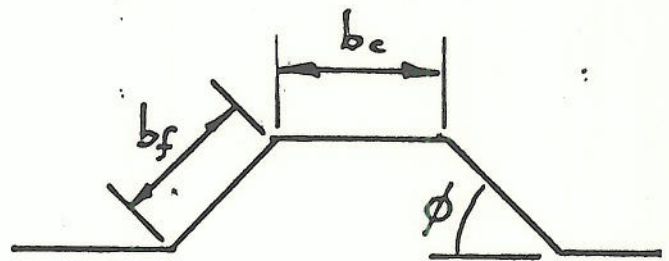
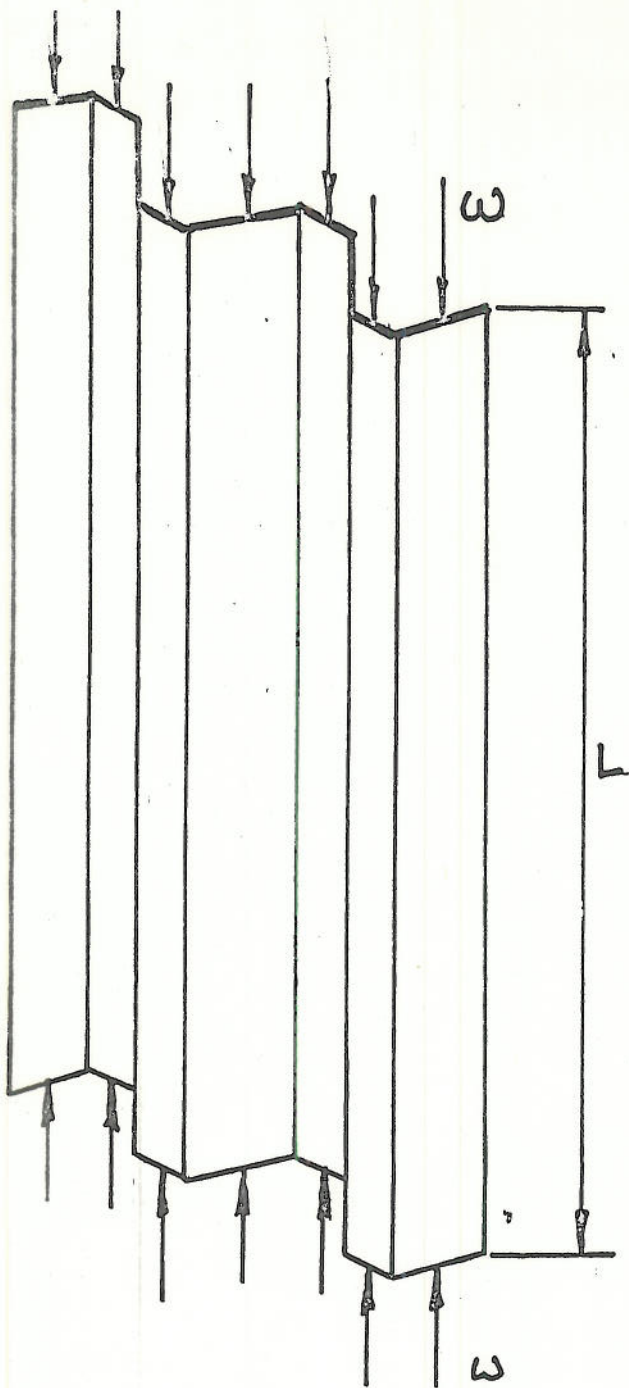


FIG. II. CORRUGATED COMPRESSION PANEL

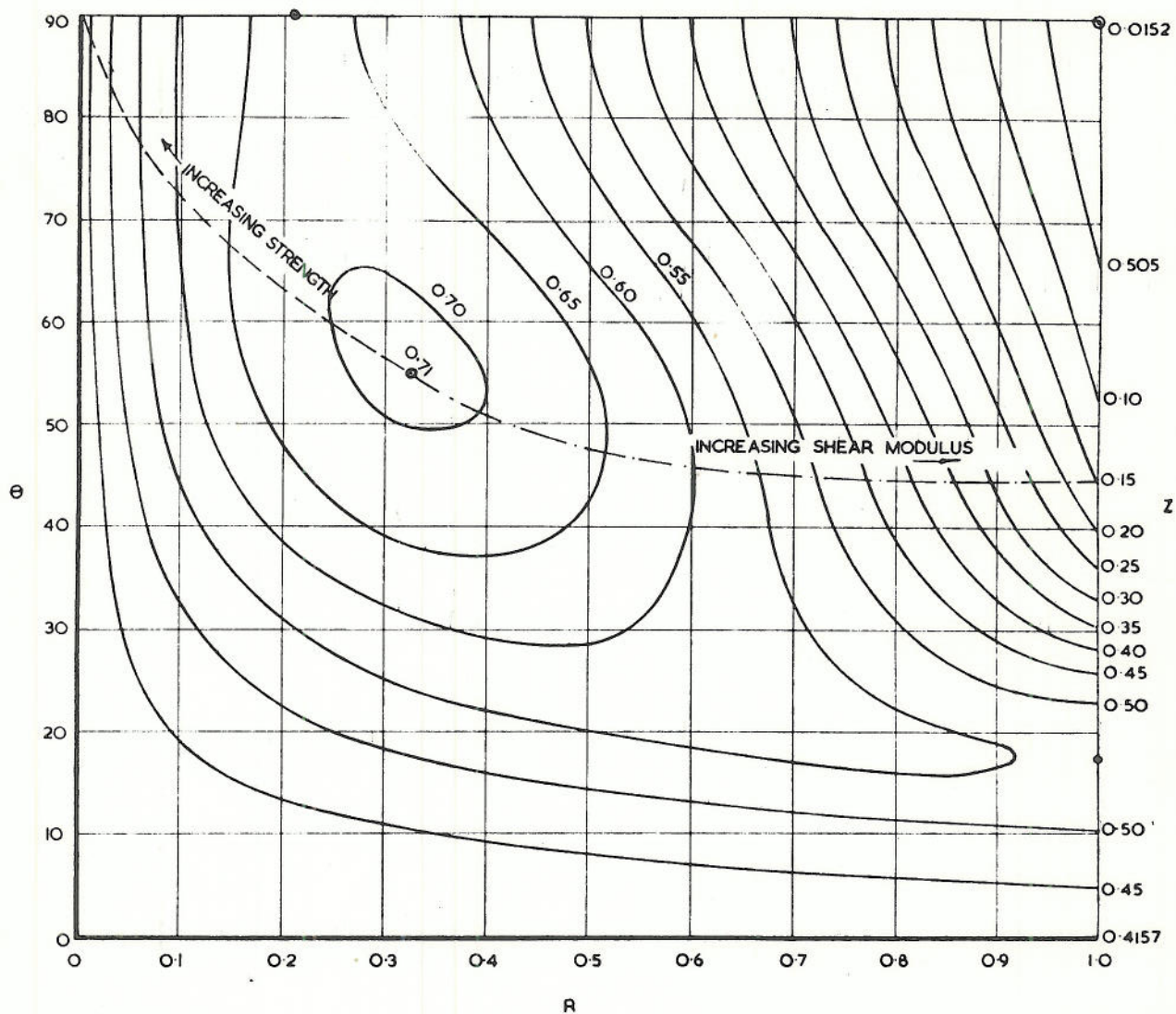


FIG J2, CORRUGATED COMPRESSION PANEL, BUCKLING CHART



FIG 13.4

OPTIMUM STRESS FOR CORRUGATED COMPRESSIVE

NEL

R=0

INCREASING No  
AXIAL PLIES

(OPTIMUM) LAYUP

R=0.325  
θ=55°

CARBON FIBRE

ALUMINUM ALLOY

L73

σ OPT KIPS PER sq in

$\sqrt{w/L}$

