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The Effect of Time-Window Constraints and Fleet Size on the Cost of a Distribution Operation

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For My Sins..............

ABSTRACT

Adopting a Continuous Space Modelling-type scenario of no detailed data being available at a customer-specific level, and on the basis, therefore, of basic information on delivery-area size, total number of locations to be visited and average road-speeds etc., quantitative expressions are derived for,

1. the relationship between the number of vehicles operating from a central depot and the total fleet mileage that is required to visit a set of locations, and,

2. the effect of time-window constraints on the total cost of a similar operation.

These expressions are derived using a simulation-based methodology, involving the setting-up of a computer program which both generates Travelling-Salesman tours and provides information on these tours at a detailed, disaggregated level. In the time-constrained context, it was necessary to develop a heuristic route-building procedure for solving Travelling-Salesman Problems due to the algorithmic difficulties posed by time-windows.

ACKNOWLEDGEMENTS

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1. INTRODUCTION AND DISCUSSION OF PROBLEM-AREA

CHAPTER 1

INTRODUCTION: THE DISTRIBUTION PROBLEM

The general subject-matter of this thesis is provided by the Distribution Problem, a term that refers to a group of related problems that commonly involve the dispatching of goods, people or services from one or more points of origin to a set of specified locations. In the real world, this problem manifests itself in the industrial context of distributing finished goods or raw materials from a central supply point to a population of consumers, either directly or via a smaller number of production points or depots. Since it is the movement of items over space that is the main activity here, the fundamental problem is obviously one of transportation, but the wider issue of distribution also involves the storage of goods within the system in warehouses, and also the question of whether the process of distribution should include a "break-of-bulk" stage, (ie. whether the goods being transported should be divided into smaller consignments for subsequent carriage by smaller vehicles).

The Transportation Problem is another term that may be used generally to refer to a class of problems whose general objective is to distribute goods optimally, given an existing network of depots. This type of problem invariably involves employing the optimum vehicle fleet in terms of the number and carrying-capacity of vehicles used, and usually includes the task of minimising the total distance travelled by this fleet, subject of the maintenance of a minimum level of service.

The distance-minimisation stage is associated with a rather specialised body of literature concerned with The Travelling-Salesman Problem, which will be discussed, in several of its formulations, later on in this chapter.

The transportation component of distribution can be seen in several different contexts, including the mass transportation of people as well as goods, and the context within which the problem is set will obviously influence the objectives and constraints that are involved. For example, in freight transport the vehicle-fleet employed will be required to dispatch goods to a set of locations in order to satisfy a periodic demand, and often be expected to do so on a particular day of each week, or even at a specified time of day. The major limiting factors on an operation will normally be the length of the working day and the maximum legal speed and carrying-capacity of the vehicles, and it is within the framework of these constraints, along with the required frequency and quality of service, that costs may be minimised.

In the area of passenger transport, however, the question of the level of service provided becomes more important. In some cases, as discussed later, the minimisation of a measure representing the total inconvenience caused to passengers becomes a major objective. However, passenger-orientated problems are still restricted by similar time, speed and capacity constraints to those experienced with freight operations, and the issue of cost minimisation is rarely disregarded, despite the greater emphasis placed on customer-satisfaction.

The problem of distribution is clearly an important one, since the cost of this activity accounts for a substantial proportion of the retail-price of any manufactured item; subsequent chapters will demonstrate that, by close examination of the cost-functions and key parameters of distribution operations, it is possible to significantly reduce these costs.

A more detailed summary of the precise research objectives of the thesis, along with an outline of the cost components and system characteristics that are to be examined, will appear in Chapter 2; the following section, meanwhile, describes the Distribution Problem, with its different formulations and sub-problems, in greater detail.

1.1. The Major Components of the Distribution Problem

Figure 1.1. is a taxonomic diagram, in as much as its purpose is one of classification, whose role here is to define the relationships between each of the various components of the Distribution Problem, which may be divided into three main parts:-

1. the Warehousing Problem, which covers all issues associated with the management of people, equipment and flows <u>within</u> a depot or warehouse,
2. the Depot Location Problem, dealing with the location and siting of one or a network of depots or warehouses,

and 3. the Routing & Scheduling Problem, which focuses on the size, nature and management of the fleet of vehicles operating from one or more operating centres.

It is the routing & scheduling component of the problem that is of interest here, and Figure 1.1. divides routing & scheduling into the four formulations of the problem that are most commonly found in the literature: the Travelling-Salesman Problem, the Dial-a-Ride Problem, the Chinese Postman Problem and the School Bus Problem.

1.2. Routing & Scheduling Problems

Both Figure 1.1. and Figure 1.2. serve to describe the relationship of Routing & Scheduling to the other components of the Distribution Problem. Figure 1.2. goes on to define a

Figure 1.1. Taxonomic diagram of the Distribution Problem

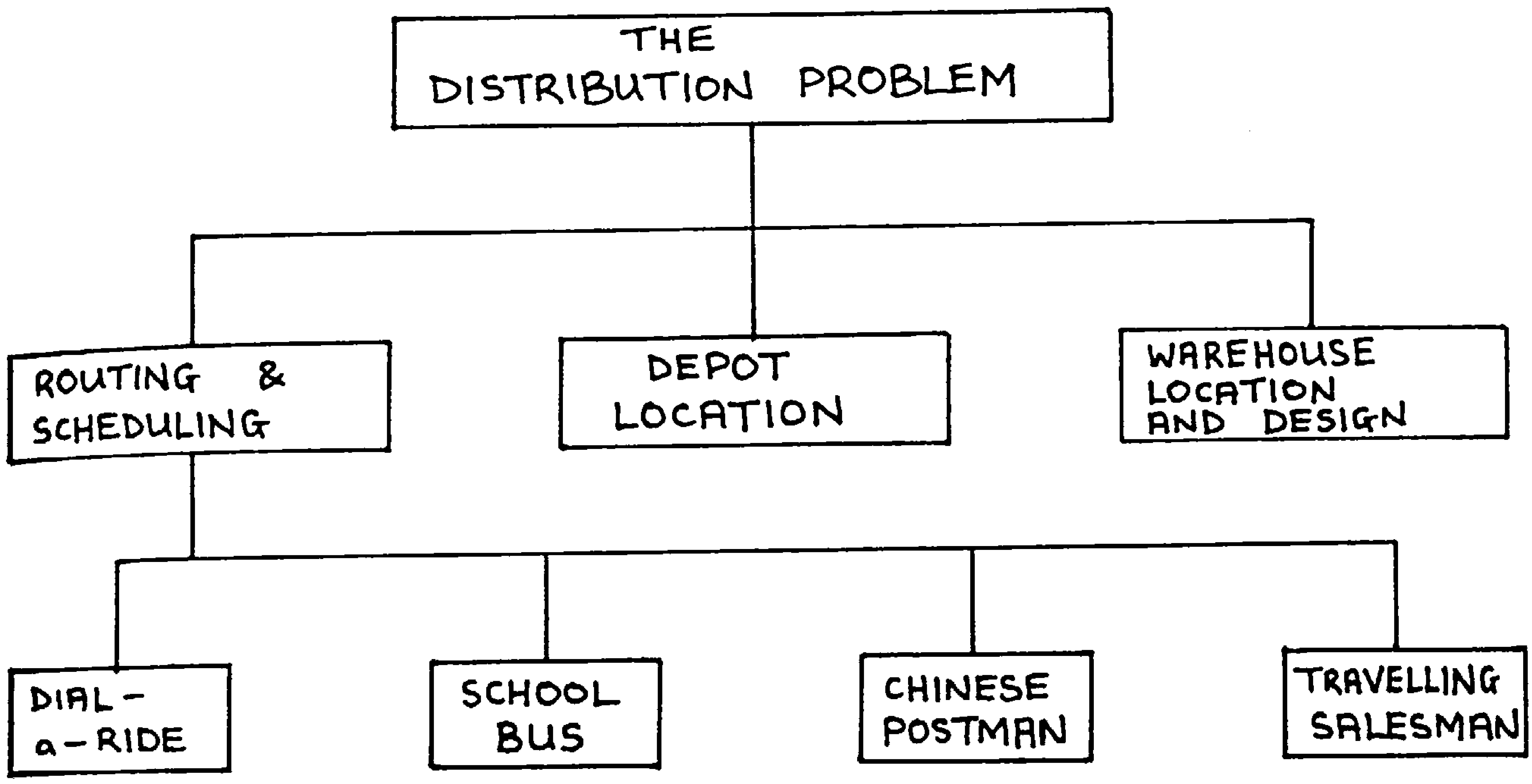
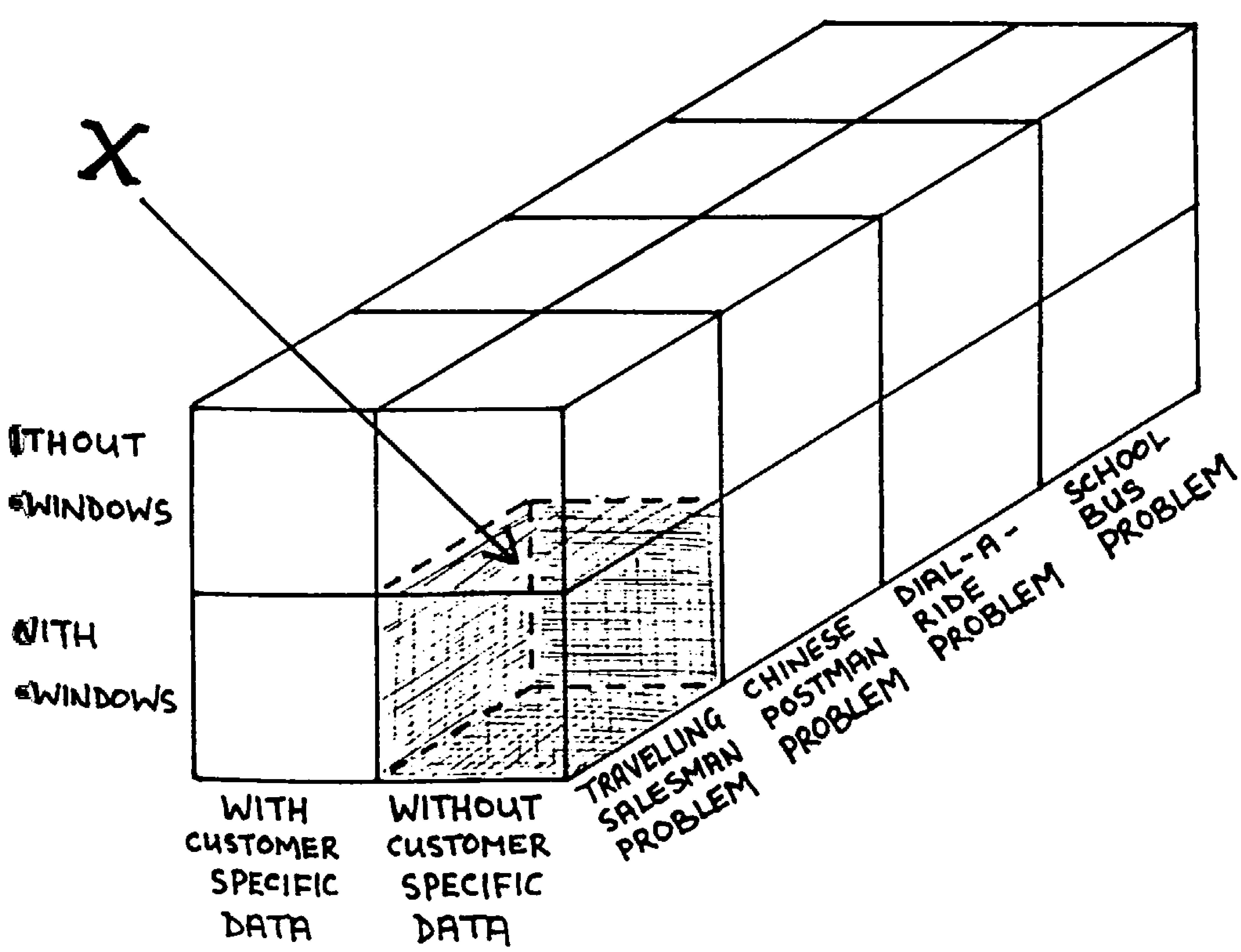


Figure 1.2. Definition of the subject-area



taxonomic space in terms of the following three dimensions:-

(a) the type of routing & scheduling problem addressed,
(b) the availability or unavailability of details on the location and demand-characteristics of individual customers,
and (c) the presence or absence of time-window constraints.

It should be stressed that Figure 1.2. is by no means intended to be a totally comprehensive taxonomic framework within which all work relating to routing & scheduling problems may be classified; such a model would, of course, need to be multi-dimensional, and therefore be impossible to portray diagrammatically. However, it is useful to use such a framework as a basis for discussion, as it enables quite a diverse subject-area to be presented as a set of more specialised areas of inquiry. Furthermore, the above dimensions, particularly dimensions "(b)" and "(c)", highlight the aspects of the current research that give originality to the thesis, and emphasise the importance here of estimating the effects of time-windows on the behaviour of the major parameters of distribution operations, especially in the absence of customer-specific data.

1.2.1. The Formulations of the Routing & Scheduling Problem

This dimension of the framework shown in Figure 1.2. illustrates the variety of different formulations of the problem of distributing goods, people or services to a set of locations, each of which may either deal purely with a routing problem, or deal specifically or simultaneously with the scheduling of vehicles and crews. The Routing Problem on its own is typically mainly concerned with distance-minimisation in the absence of time-window constraints, whilst the task of scheduling is rather more orientated towards the question of time-constraints, whether these constraints are imposed by customers or suppliers, in the form of restrictions on times of delivery & collection or by drivers' hours limitations. Both routing and scheduling entail the order in which customers on a route are visited, and are, in practice, normally dealt with together; the Classical Travelling-Salesman Problem, however, is mainly concerned with the minimum distance that is required to serve a given set of customers.

The Travelling-Salesman Problem.

One of the earliest, and certainly one of the most commonly-referenced, attempts at offering a solution to the Travelling-Salesman Problem was presented by Clarke & Wright(1) in 1963; in this paper, the "Savings Method" is outlined, an

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(1) CLARKE, G., and WRIGHT, J.W., "Scheduling of vehicles from a central depot to a number of delivery points." OPERATIONS RESEARCH. Vol. 11., P.568. (1963).

iterative procedure which produces an optimum, or near-optimum, route through a set of points, starting and finishing at a central depot. This method has subsequently been used, refined and developed by a number of researchers. A more detailed description and discussion of the Savings Method can be found in Chapter 7 .

The Dial-a-Ride Problem.

Rather more complex solutions are required to distribution problems involving scheduling, whether it is goods or people that need to be transported. In the context of passenger transport, there is an extensive body of literature which deals with the Dial-a-Ride Problem, a theoretical problem named after a shared-ride scheme in which users state a desired time and location for both pick-up and delivery. Clearly, a solution for such a customer-orientated problem would be rather unrealistic if scheduling in terms of time were not considered, although Stein, D.M.,(2), for instance, defines pick-up and delivery-times as decision-variables which are imposed upon each customer in order to maximise bus-utilisation, rather than as constraints. However, despite the importance of time-constraints in this problem, the usefulness of dial-a-ride-orientated algorithms to the Travelling-Salesman Problem is limited, since each passenger has two stated "arrival-times"; usually, in the context of freight distribution, the problem involves just one time-window per location during each delivery-cycle. This important distinction means that the Many-to-Many Dial-a-Ride Problem is algorithmically quite different to the One-to-Many Distribution Problem. Furthermore, the former's strong emphasis on passengers' needs means that customer-service considerations must be directly incorporated into the objective function of the model, a requirement that has led to various attempts at quantifying the concept of "customer-inconvenience". For example, the main stated objective of Sexton & Bodin's, (3), Mathematical Programming formulation is to schedule a single vehicle so as to minimise the total of each customer's "Excess Ride Time" (defined as actual ride-time less the minimum feasible ride-time) plus "Delivery-Time Deviation" (desired delivery-time less actual delivery-time). Similarly, Hung et al seek to minimise the number of vehicles utilised whilst maintaining a pre-determined level of customer-service, (4),

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(2) STEIN, D.M., "Scheduling Dial-a-Ride Transportation Systems". Transportation Science (1978), Vol.12., No.3, P.P. 232-49

(3) SEXTON, T.R., and BODIN, L., "Optimising Single Vehicle Many-to-Many Operations with Desired Delivery Times: I. Scheduling." Transportation Science, (1985a), Vol.19., P.P. 378-410.

(4) HUNG, H., CHAPMAN, R., HALL, W., and NEIGUT, E., A heuristic algorithm for routing and scheduling dial-a-ride vehicles. (Presented at the ORSA/TIMS 1982 Joint National Meeting in San Diego, Oct. 1982).

and Psaraftis minimises a weighted sum of travel-time and "customer-dissatisfaction", (this measure being assumed to be a linear function of the waiting and riding times of each customer), (5). Despite the importance of customer-service to freight distribution operations in the real world, there is, as yet, no evidence of customer-inconvenience, or a similar variable, being used in algorithms to provide a solution to a Travelling-Salesman Problem.

The Chinese Postman Problem.

Another problem based on distance-minimisation is the Chinese Postman Problem. Whereas the theoretical salesman is required to visit each of a set of points, the postman in these problems must traverse every "street" in a street-network, so that the latter problem deals with Arc Routing, as opposed to Node Routing. The Chinese Postman Problem is so-called because one of its earliest formulations was presented in Chinese Mathematics in 1962, (6). Despite the conceptual similarity between this formulation and the Travelling-Salesman Problem, their solutions require totally different algorithms. The Chinese Postman Problem will be discussed no further here but more detailed descriptions of this problem have been put forward by both Bodin et al, (7), and Minieka, (8).

The School Bus Problem.

There are certain contexts in which the routing & scheduling of vehicles is merely part of a wider Location-Allocation Problem, usually in connection with the location and dispersal of public services; an example of this is the School Bus Problem, in which the given fixed variables are the number and location of schools, the number and location of bus-stops associated with certain schools, the number of children assigned to each bus-stop and the starting - and finishing-times of the schools.

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(5) PSARAFTIS, H.N., "A Dynamic Programming Solution to the single vehicle many-to-many immediate request Dial-a-Ride Problem." Transportation Science, (May, 1980), Vol.14., No.2, P.P. 130-54.
(6) MEI-KO, K., "Graphic Programming using odd or even points." Chinese Mathematics, (1962), Vol.1., P.P.278-87.
(7) BODIN, L.D., GOLDEN, B.L., ASSAD, A.A., and BALL, M.O., "Routing & Scheduling of vehicles and crews: The State of the Art". Computers and Operations Research, (1983), Vol.10., No.2, P.P. 63-211.
(8) MINIEKA, E., "The Chinese Postman Problem for mixed networks." Management Science, (1979), Vol.25., P.P. 643-8.

The time-window component of this problem is the timing of the pick-up and delivery of children at their bus-stop and school, the main objective being to minimise the cost of the whole operation, which is a function of both the number of vehicles used and the running cost of the fleet. Among the researchers who have addressed this problem are Swersey & Ballard, (9), who assume that the bus-routes for each school are given, Bennett & Gazis, (10), and Bodin & Berman, (11).

1.2.2. The availability or unavailability of customer-specific data

There are many examples of algorithms which are designed to give optimal or near-optimal solutions to Routing & Scheduling problems, given information on the location and demand-characteristics of a set of customers; where such details are not available, key cost components and other important parameters of distribution systems may be estimated using an approach known as Continuous Space Modelling.

This approach, of which one of the earliest examples is Kantorovich's use of the technique in 1942, to describe the movement of bulk materials, (12), sets out to give accurate estimates of cost components using simple, analytical expressions derived from basic parameters such as the density of customers and the size of the distribution-area etc.. In other words, customers may be located anywhere within the study-area, which is regarded for the purpose of the analysis to be both continuous and homogeneous, without affecting the resulting estimates; it is the dimensions and overall characteristics of the delivery-zone and its customers that are of importance.

Eilon et al, (13), use Continuous Space Modelling to estimate the length of a travelling-salesman tour in a square

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(9) SWERSEY, A., and BALLARD, W., Scheduling School Buses (Yale School of Organisation and Management Technical Report, Yale University, 1982).
(10) BENNETT, B., and GAZIS, D., "School bus routing by computer." Transportation Research, (Dec., 1972), Vol.6., No.4, P.P. 317-25.
(11) BODIN, L., and BERMAN, L., "Routing and Scheduling of school buses by computer." Transportation Science, (1979), Vol.13., No.2, P.P. 113-29.
(12) KANTOROVITCH, L., "On the translocation of masses." Comptes Rendus (Doklady) de l' Academie des Sciences de l'URSS, (1942), Vol.37., No.s 7-8, P.P. 199-201.
(13) EILON, S., WATSON-GANDY, C.D.T., and CHRISTOFIDES, N., Distribution management: mathematical modelling and practical analysis. (Griffin, London, 1971).

zone of given dimensions as a function of the number of customers that may be visited during one vehicle-tour, the area of the delivery-zone and the aggregate distance between the depot and each of the locations that require a delivery; when customers are randomly located in the delivery-zone, the latter parameter is, of course, a direct function of area-size. This work is discussed in greater detail in Chapter 4.

Another example is provided by Blumenfeld & Beckmann, who derive simple expressions for both the number of destination stops per load, and the distance travelled per load, from information regarding the average density of locations and the average and variance of demand. Although the equations are, by the authors' admission, merely generalised estimates,

"........for the cost of distributing freight under general conditions........"

they are useful in as much as they are convenient to use whilst still maintaining a good level of accuracy, (14).

Similar work using Continuous Space Modelling has been carried out by Daganzo, who calculates total distance travelled per drop from vehicle-capacity, the number of customers to be visited and a parameter representing the average of the distances from the depot to any random point in the delivery-area, (15).

The same author also explores the impact of zone-shape on the expected length of travelling-salesman tours, thus illustrating the usefulness of deriving such analytical expressions for estimating the consequences of changing a given variable on the cost components of a distribution operation, (16).

Daganzo applies the same concept of continuous space to the Many-to-One Demand-Responsive Transportation Problem, a problem similar to that of Dial-a-Ride, involving a single vehicle picking up passengers from a "rendez-vous" point and

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(14) BLUMENFELD, D.S., and BECKMANN, M.J., "Use of Continuous Space Modelling to estimate freight distribution costs." Transportation Research., Vol.19A., No.2., P.P. 173-87.(Mar.,1985).
(15) DAGANZO, C.F., "The distance travelled to visit N points with a maximum of C stops per vehicle: a manual tour-building strategy and case study." Research Report, Institute of Transportation Studies, University of California. (Aug-Sep., 1982).
(16) DAGANZO, C.F., "The length of tours in zones of different shapes." Transportation Research - B., Vol.18B., No.2., P.P. 135-45. (1984).

distributing them to a random set of destinations, before picking up a fresh load of randomly-located passengers on the way back to the rendez-vous point; the vehicle is constrained to return to this point periodically at fixed times. A typical application for this problem is a shared-ride taxi-service operating from an airport or railway station, (17). In this paper, Daganzo refers to one of his own unpublished pieces of work in which he proposes a simple analytical model for the Many-to-Many Dial-a-Ride Problem itself, (18).

These applications of Continuous Space Modelling suggest that the same approach can also be used to tackle the other types of Routing & Scheduling Problem outlined above.

1.2.3. The presence or absence of time-window constraints

In the real world, the routing of a fleet of vehicles is invariably conditioned by time-constraints of one type or another; this, of course, introduces a divergence from the original Travelling-Salesman Problem, which is primarily concerned with minimising the distance travelled through a given set of points, regardless of time restrictions.

The most obvious type of time constraint involved is the limit on the working-day, as vehicles are often required to return to the depot at the end of each day's work; even if it is feasible for a driver and vehicle to continue away from their operating centre for one or more nights, there are still limits on the aggregate and continuous time during which a driver may drive.

But the type of time-constraint that has received most attention from researchers in this area is the "time-window." A time-window, in the context of freight distribution, may be defined as the time-period during which a visit may be made to a given location for the purposes of delivery or collection; the two times that define the span of each time-window will normally be specified by the customer.

The presence or absence of time-windows will obviously influence both the formulation of the particular problem and the choice of algorithm. The extent to which the total mileage travelled by a fleet is increased as a result of time-windows depends on how severe these restrictions are, in terms of either the width of the time-band during which delivery or collection may be made, or the percentage of outlets that specify such limitations.

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(17) DAGANZO, C.F., HENDRICKSON, C., and WILSON, N.H.M., "An approximate analytic model of many-to-one demand responsive transportation systems." (Aug. 1977).
(18) DAGANZO, C.F., An analytic model of Many-to-Many Dial-a-Ride transportation sytems. (Unpublished manuscript, 1974).

Time constraints must also be taken into account when considering other types of routing & scheduling problem. For example, in the context of the Dial-a-Ride Problem, time-constraints exist in the form of customers' desired time of arrival at their destination; although each customer need not necessarily arrive at the precise time preferred, a major objective of the problem is to minimise the sum of deviations from the stated delivery-time of all customers. Furthermore, the time at which each customer should be picked up is determined in order that Total Customer Excess Ride-Time should be minimised, so that the overall objective of dial-a-ride problems is to minimise total inconvenience for the whole population of customers. In other words, a time-window is, in this example, a precise time that is stated by the customer, but which need not necessarily be complied with, so that the time at which a vehicle arrives at a given location is a decision-variable.

The time-windows considered in the current thesis, however, are defined by time-limits within which a delivery must be made to a certain customer, and these time-limits act as fixed constraints on the final solution.

Of the relatively few references that deal specifically with time-windows in the context of freight operations, it is common for time-window constraints to be added to a problem after the generation of an initial feasible solution. For example, the approach used by Baker is to derive an initial solution, using a technique based on Nearest Neighbour Analysis, in the absence of time-window constraints, and to then impose time-windows at a second stage, (19). Savelsberg uses a similar two-stage method when he employs route-improvement procedures to reduce both total travel-time and the total time taken to complete a tour, having initially generated a feasible solution, (20).

1.3. Summary

The foregoing sections have provided a general definition of the Distribution Problem, and have described both the major sub-problems of which it consists, and the main formulations of the more specific Routing & Scheduling Problem. This problem-area is conveniently summarised by the taxonomic diagrams of Figures 1.1. and 1.2., and, referring to the latter, it is the area of interest that is indicated by the

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(19) BAKER, E.K., "Vehicle routing with time-window constraints." The Logistics and Transportation Review. Vol.18., No.4. (1982).
(20) SAVELSBERG, M.W.P., Local search in routing problems with Time-Windows. (Report OS-R8409, Centre for Mathematics and Computer Science, Amesterdam, 1984).

symbol "X" that provides a focus for the current thesis. In other words, the general aim, here, is to analyse the Travelling-Salesman Problem with Time-Windows using the "Continuous Space Modelling" approach that is employed by Blumenfeld & Beckmann (14), Daganzo (15, 16, 17, 18), and others.

Chapter 2 now goes on to present a more detailed statement of the thesis's research objectives, along with a description of the common features of the various problem-definitions that are used in subsequent chapters. This objectives statement is then placed in the context of the objectives pursued by other researchers in this field. Finally, a summary is given of the thesis's structure and of how this structure relates to the objectives that have been set.

CHAPTER 2

THESIS OBJECTIVES AND METHODOLOGY

2.1. Thesis Objectives

The central aim of the research is to explore how a computerised simulation model may be used to measure the effect of,

1. the number of vehicles that make up the vehicle fleet, and,
2. the severity of constraints on delivery-times,

on the cost of a hypothetical delivery task. This task is to deliver a uniform consignment of goods, using a uniform fleet of vehicles, to each of a set of customers. These customers are located within a delivery-area of known dimensions, and each one specifies a "time-window" during which a delivery must be made.

A simulation model is constructed in order to provide a simplified representation of a distribution system. This model includes a number of assumptions, particularly about the nature of the area within which goods are to be distributed. Generally, these assumptions correspond to those associated with Continuous Space Modelling, (which has already been discussed in Chapter 1). The most important of these are:-

1. The customer-locations requiring a delivery are distributed irregularly throughout the delivery-area with the location of individual customers being unspecified.
2. The delivery-area is, for all intents and purposes, homogeneous, so that vehicle speed within this area is constant regardless of the direction of travel. No information is given on the details of a road network; therefore, the straight-line distance between two customer locations is taken to be equivalent to the "road distance" between them.
3. The delivery task must be accomplished from a single depot, using a uniform fleet of vehicles.

A formal problem definition is presented in Section 2.2.. Once the simulation procedure has calculated the mileage that is required for making a delivery to each customer-location, this distance figure is then converted to cost using vehicle cost tables published by Commercial Motor magazine. These costings, along with the assumptions that underlie them are discussed in depth in Section 3.3..

The creation of such a model makes it possible to exercise complete control in the hypothetical system that is provided, so that a particular parameter of interest may be made variable whilst all other parameters are held constant. The general methodology pursued here is to observe the way in which the Total Cost of making deliveries to a given set of customer-locations is affected when controlled changes are made to, first, the size of the vehicle fleet, and then to the severity of the time-window constraints. The objective then is to derive quantative expressions to describe the observed relationships. The purpose of this process of running a simulation model to generate such equations is to produce predictive tools that can estimate, say, the minimum length of a Travelling-Salesman tour required to visit a given set of customer-locations within a delivery-area of given size and shape. Obviously, other assumptions, such as average road-speeds within this area, also have to be made. Although the applicability of the models predictions is initially restricted to the types of operation described in the problem definition of Section 2.2., the technique of simulation allows the flexibility of being able to elaborate on these basic assumptions. For example, a constraint on the length of the working day may be added to the model so that more than one vehicle is required for all deliveries to be made before the end of the day. In other words, the model has an aptitude for impact analysis, predicting the impact of a change in working practices, in this example, on the Total Cost of an operation. In particular, the current research focuses on the effects of changes in both the size of the fleet and the constraints that are imposed on delivery-times at demand-points.

The research also aims to contribute to the existing body of knowledge that is associated with Continuous Space Modelling, a methodology that has been used extensively for research at a tactical/strategic level. Continuous Space Modelling has already been described in Section 1.2.2., but it might be useful to clarify the precise meaning of the terms "tactical" and "strategic", since they have a variety of interpretations when applied to planning, policy analysis and research methods. This is dealt with in the following sub-section.

2.1.1. The Distinction between Strategic, Tactical and Operational Planning.

The distinction between "strategic", "tactical" and "operational" levels of activity is well documented.

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(1) CHRISTOPHER, M.G., Strategy of distribution management effective logistics management. (Gower, 1985).
(2) WALLER, A.G., "Use and location of depots." in GATTORNA, J., Handbook of physical distribution management, (Third Edition). (Gower, 1970).
(3) BOWERSOX, D.J., Logistical Management: A systems integration of physical distribution management and materials management, (Second Edition). (McMillan, 1978).

Christopher (1) is specific in describing these three terms as,

"........distribution planning horizons........"

and Waller (2) portrays them as forming a,

".........hierarchy of decision levels........."

whilst making specific reference to depots and warehousing. Bowersox (3) takes a more general view when he refers to the three terms as a,

".....classification of planning situations...."

and suggests that the relevant criteria when using this means of categorisation are,

".....the nature of asset commitment, the time duration of the plan, the likelihood of implementation".

the time duration element of these criteria is clearly important. All three of the authors quoted above put forward their views on the time horizons that are associated with operational, tactical and strategic decision-making activities. These views are summarized in Table 2.1.. This table reveals a broad agreement within this small sample of authors on the operational and strategic time horizons, but it is interesting that it is only Waller, who bases his classification on the number of times that each type of decision would normally be reviewed, is specific about the time-horizon that is associated with the tactical planning level. Christopher, for example, defines a two-tiered hierarchy, in which strategic planning, (or "resource planning"), adopts

Table 2.1. Views as to the time horizons that correspond to operational, tactical and strategic decision-making

	TIME HORIZON		
AUTHOR	OPERATIONAL	TACTICAL	STRATEGIC
Bowersox	Up to 1 yr	Adaptation within strategic horizon	5 to 10 yrs
Christopher	1 year	———	5 + years
Waller	Review many times each year	Review once a year	Review every 3 to 5 years

whatever time horizon enables all resources to be considered variable, whilst, with operational decision-making, resources are effectively fixed. Similarly, Bowersox suggests that the tactical planning level shares the same time horizon as the strategic level, but that the former is concerned with the setting up of contingency plans, support requirements and procedures for adaptation which are to be used by decision-makers working in an operational environment. Bowersox goes on to argue that tactical planning may also be regarded as being short-term, since it is,

"........event orientated........"

All three authors agree that, whatever the precise definition of the terms, the three planning levels described above are mutually-dependant. Both Bowersox and Waller point out that operational and tactical activities take place within a structure that has already been established at a strategic level, whilst Christopher suggests that the development of decision support systems and more systematic management techniques has enabled the products of strategic planning to be used increasingly at an operational level. A trend that is exemplified in the recent growth in the use of "desk-top" software for Routing & Scheduling and stock control tasks etc..

An alternative set of definitions is offered by Ballou (4), who, whilst making no attempt to define the time horizon that is associated with each term, distinguishes between strategic planning, which he describes as,

"........deciding in a broad sense what the overall system configuration should be for distribution."

and tactical planning, which is considered to be concerned with the efficient use of facilities, equipment or fleet. Ballou goes on to suggest that tactical planning is not bound to a specific time horizon, and that it may even be carried out on a daily basis. Ballou's definition of decision-making at an operation level is somewhat dismissive in that he describes is as being,

".....a process for developing logistical policy and plans to handle routin or regularly anticipated management action........."

Slater (5), with reference to load planning, suggests that tactical decision-making is associated with a time horizon of a few months, often in the presence of seasonal variations, whilst decision-making at an operational level involves the planning of routes on a day-to-day basis. He describes strategic planning as dealing with matters such as depot location, fleet size & composition, changes in the length of driver's working day, change in customers' order patterns and opening hours.

The conclusion from the limited literature survey described above is that there is no agreement on what constitutes planning, or a decision, at a tactical level; in fact, some authors do not recognise a tactical tier between the strategic and operational levels at all. There is,

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(4) BALLOU R.H., Basic business logistics - transportation, materials management, physical distribution, (Second Edition)

(5) SLATER, A.G., "Load Planning." in GATTORNA, J. Handbook of physical distribution management, (Third Edition) (Gower, 1970).

however, certainly a distinction made between operational activities and those which fall into the broad category of "tactical/strategic planning".

The following sub-section considers how the current thesis might be described, in relation to the foregoing discussion.

2.1.2. The planning and decision-making level that corresponds to the current thesis.

First of all, it should be made clear that this research does not consider the effect of decisions that are made at an operational level of activity. In fact, it may be argued that an approach at this level is precluded by the use of Continuous Space Modelling through out the thesis, since this is a technique that regards a delivery zone as being a "Black Box" of known size within which no information is available on location, order-size or time-window width at a customer-specific level; the products of this research, therefore, are more likely to be of relevance to strategic decisions such as the optimum size of the vehicle fleet etc., than of direct use in the day-to-day task of routing and scheduling vehicles, for example. Although the Travelling Salesman Problem provides a central theme for the research, no attempt will be made to produce a model or algorithm that might be used by, say, an operations manager for his daily duties.

In other words, the parameters of an operation that are dealt with here, such as fleet-size, are not flexible at an operation level, or within an operational time-scale. Furthermore, the time-horizon assumed for analysing the impact on Total Cost of changes in working-hours restrictions, policy on overnight stays, or customers' policies on delivery-time restrictions, etc. would relate to the tactical/strategic level of planning outlined in the foregoing discussion.

Another feature of the modelling methodology is that simulation enables all parameters to be variable, with the facility to change even the size of delivery zone. Using Christopher's definition of a strategic planning horizon being one at which all resources are variable, therefore, (SEE sub-section 2.1.1.), it may be argued that the findings of the research are applicable at this strategic level. Furthermore, the number of vehicles in the fleet, the variable that is shown to be the most important influence on Total Cost in both Part 2 and Part 3 of the thesis, is set at the strategic planning stage. Also, due to the substantial capital commitment that is involved with a change in fleet size, this is not a decision that is reviewed on a regular basis.

To further strengthen the argument that the current research deals with the planning of distribution operations at a strategic level, Slater has already been quoted, (again in Section 2.1.1.), as using Fleet size, drivers' working hours and customers' opening times (ie. time-windows) as being examples of variables and constraints that provide the subject-matter for strategic planning.

It is difficult to categorise the thesis in terms of it being either strategic or tactical, because of the difference of opinion that exists as to the precise definition of the latter term. Taking Ballou's view that tactical planning involves optimising resources, however, the current research may be defined as tactical, considering the emphasis that is placed on finding the optimum fleet size for a given task, and on the minimisation of both mileage and cost. On the other hand, the research does not focus on contingency and support mechanics, which form the basis for Bowersox's definition of tactical planning. It may be argued that the subject-matter of the thesis is not bound by any particular time-horizon since, whereas fleet size is clearly a variable that must be planned on a long-term basis, a change in customers' opening times or a company's policy on allowing drivers to make overnight stays may occur at short notice, and so will be associated with a far shorter time scale. Furthermore, the part of the analysis that examines routing and scheduling might be described as tactical, since the algorithm employed in this task, or the zoning technique used, might be changed at short notice and with no effect on capital input.

Because there is a range of issues addressed here, the thesis can not readily be placed in one category or the other, and there is arguably little point in trying to make a firm distinction between tactical and strategic levels. It may be reemphasised, however, that the central theme of the thesis is the effect of various decision variables and constraints on fleet size, and so the findings of such research will clearly be mainly applicable to strategic planning activities.

2.1.3. Reasons for focusing on fleet size and time-windows.

Ideally, the ultimate objective of such analysis is to develop an interactive model of a distribution system, which describes the relationships between all of the variables that have a significant effect on Total Distribution Cost, and which may be applied to any physical distribution context. It is clearly not feasible to try to achieve this with the current research, and so it has been necessary to be selective in choosing the particular relationships that are to be examined; the thesis therefore focuses on the effect on Total Cost of the number of vehicles in the fleet, and the severity of time constraints that are imposed on the fleet's deliveries.

One of the reasons for concentrating on these two variables is that their effect on distribution cost has so far received little attention from researchers in this field and subsequent chapters will demonstrate that both have a considerable influence on Total Cost.

There is a fundamental difference between fleet-size and time-windows in the way which they relate to Total Cost, since, whereas timing constraints are taken to be an external influence on a distribution system, that is largely beyond the control of the operator, fleet-size is a decision-variable which is related to other aspects of the relevant distribution system. For example, there is a fundamental trade-off between the average carrying-capacity of a fleet's vehicles, (denoted later as "x"), and the number of vehicles used, ("n"); the way in which fleet size influences distribution cost via its effect on "x" is examined in some detail in Chapter 3. Similarly, Chapter 5, which considers the influence of factors relating to drivers' working-hours, explains how the number of vehicles used is once again the major variable that affects Total Cost, whilst Chapter 4 focuses on the way in which fleet size itself influences distribution costs through its effect on Total Fleet Mileage.

The structure of the thesis is described in greater detail in Section 2.5.; Section 2.2., meanwhile, describes the basic problem formulation that is used throughout.

2.2. Problem Definition

The problem-definition that is used as a basis for analysis often differs in detail throughout the thesis, and so relevant variations in constraints and assumptions are outlined at the start of each section, as and where appropriate.

However, these precise formulations of the Distribution Problem have many common features. The most important of these features is that they are based on the fundamental problem of distributing a consignment of goods from a centrally-located depot to a set of randomly-distributed demand-points at the lowest possible cost, given a set of constraints. Again, many of the assumptions that are made correspond to those associated with the Continuous Space Modelling technique, in that it is assumed that there is no infomation available on the location of individual customers, which are distributed within a homogeneous area in which precise details of the road network are unimportant, and where road-speeds are uniform in all directions. It is also generally assumed that the level of demand is the same for each customer, and that the fleet of vehicles operating from the depot is also uniform in terms of capacity. The information that is given in each section usually concerns the number of customers requiring a delivery, the size of the circular, or square, area in which they are located and data referring to the temporal constraints in which the fleet must operate, (eg. the maximum permissible length of a working-day, average road-speeds, amount of time required at each location, etc.).

The first objective in the context of this problem formulation is to deliver the required consignment of goods to the set of locations at the lowest cost possible. In order to achieve the wider objectives outlined in Section 2.1., it is then necessary to estimate the Total Cost of an optimum solution to this Distribution Problem, and then to examine the way in which this Total Cost estimate varies in response to changes in both the assumptions that are made, and value of certain parameters. A more detailed statement of these objectives already appears earlier on in this chapter; Section 2.3. now goes on to compare these objectives with those of other researchers in this field.

2.3. Comparison of Thesis Objectives with those of Other Researchers

The research-objectives outlined above have most in common with those of Daganzo and Blumenfeld & Beckmann, whose use of Continuous Space Modelling to develop formulae that might be used as analytical tools has already been discussed in Chapter 1, (referred to in that chapter as references (15), (16), (17), (18), and (14), respectively).

Far more researchers have as their objective the development of a distance-minimising, tour-building algorithm for solving Travelling-Salesman Problems, such as Held & Karp (6). Also, because of the computational complexity of such problems, particularly when a large number of customers is involved, references concentrating on the implementation of route-building or route-improvement algorithms have appeared. Such work presents algorithms which, despite being able to derive only near-optimum tours, have the advantage of requiring substantially less computer time or capacity than similar precedures that are designed to produce optimum solutions. For example, Golden, Magnanti & Nguyen (7),

> "......consider heuristic algorithms for vehicle routing.... ....presenting modifications and extensions which permit problems.....to be solved in a matter of seconds."

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(6) HELD, M., and KARP, R.M., " The Travelling-Salesman Problem and Minimum Spanning Trees, Part II." Mathematical Programming, (1971), Vol.1., No.1, P.P.6-25.
(7) GOLDEN, B.L., MAGNANTI, T.L., and NGUYEN, H.A., "Implementing vehicle routing algorithms." Networks, (1977), Vol.7., P.P.113-48.

and Baker, Schaffer & Solomon (8) suggest streamlined route-improvement procedures for the Vehicle Routing Problem with Time-Windows, which lead to,

> ".......significant increases in algorithmic efficiency with minimal deterioration in solution quality."

There is also a section of the literature whose objectives are confined to one particular sub-problem. For instance, both Ball et al (9) and White(10) focus on issues concerning the size of a vehicle-fleet, whilst Kirby(11) and Wyatt (12) consider methods for establishing optimum fleet-size.

The Fleet-Size Problem is closely related to the more general Vehicle Loading Problem, which deals with the allocation of a set of consignments to a fleet of vehicles of known size, the objective being to minimise the number of vehicles used; a thorough discussion of this problem appears in Chapter 10 of Eilon et al (13)

Golden et al (14) go a step further to consider both the optimum fleet-size and the optimum mix of leased and owned vehicles, formulating the Fleet Mix Problem as a Mathematical Program, and Gould (15) uses Linear Programming for the same purpose.

Having outlined some of the different objectives that are pursued by other researchers, in relation to the Distribution Problem, it is also possible to put the current

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(8) BAKER, E.K., SCHAFFER, J.R., and SOLOMON, M.M., Vehicle Routing & Scheduling problems with time-window constraints: efficient implementations of solution improvement procedures. (Unpublished paper, Oct. 1986).
(9) BALL, M.O., GOLDEN, B.A., ASSAD A.A., and BODIN, L.D., "Planning for truck fleet-size in the presence of a common-carrier option." Decision Sciences., (1983), Vol.14., P.P.103-117.
(10) WHITE, G.O., "An easy lower bound on the number of trucks needed to service a set of destinations." OMEGA, The International Journal of Management Science., Vol.8., No.3., (1980), P.P.385-7.
(11) KIRBY D., "Is your fleet the right size?" Operational Research Quarterly, (1959), Vol.10., P.252.
(12) WYATT, J.K., "Optimal Fleet Size." Operational Research Quarterly, (1961), Vol.12., P.P.186-7.
(13) EILON, S., WATSON-GANDY, C.D.T., and CHRISTOFIDES, N., Distribution Management: mathematical modelling and practical analysis. (Griffin, London, 1971).
(14) GOLDEN, B.L., MAGNANTI, T.L., and NGUYEN, H.A., (1977). Op cit.
(15) GOULD, J., "The size and composition of a road transport fleet." Operational Research Quarterly, (1969), Vol.20., P.P.81-92.

thesis into perspective with existing literature by examining the methodology that is used here. This is the aim of the following section.

2.4. Methodology

There are three main ways in which the thesis may be described, vis-a-vis the methodology that is used: 1. The central technique employed for generating data is that of stochastic simulation, as opposed to empirical observation or the analysis of existing data associated with actual distribution systems. 2. The particular modelling technique used has much in common with what has already been referred to here as Continuous Space Modelling. 3. All of the algorithms that are used for deriving "Optimum" solutions are "heuristic" in nature, rather than "exact", (both of these terms will be defined later).

The simulation aspect of the methodology used refers to the generation of vehicle-tours using a computerised Travelling-Salesman algorithm - this algorithm is used iteratively to generate a number of hypothetical vehicle-tours in order to estimate the length of a set of tours given basic data on the number of vehicles used, the size of the delivery-area, the number of locations visited etc.. The fundamental research-strategy here is that of building a computerised model of a distribution operation, in which all of the relationships between important variables are quantified. Having set up such a model that is adequately representative of an actual operation, observations may be made as to the effect of changes in the value of key variables and/or in the nature of important constraints, on factors such as Total Distribution Cost, Total Fleet Mileage etc.. In other words, once such a model has been established, inferences as to the nature and behaviour of actual systems may be made on the basis of the results yielded by this model. Apart from the facility of being able to quantitatively predict the effect of various changes in selected parameters, the use of simulation enables the researcher to produce a number of solutions for the same problem, so that each series of iterations may yield, not only a mean result, but also an indication of the variance that is associated with this mean figure. The advantages of being able to measure this variance are described and illustrated in greater detail in Chapter 4.

A major characteristic of the way in which distribution systems are modelled, here, is the fact that it is assumed that no information is available on the location of individual customers, and that the area in which these customers are situated is a featureless plain in which "road"-speeds are uniform in each direction. This is, of course, a characteristic of Continuous Space Modelling, which is described in greater length in Chapter 1.

Having described the algorithms used to generate Travelling-Salesman solutions as being "heuristic", as opposed to "exact", it is necessary to provide a more detailed definition of these terms. The distinction between these two approaches is quite neatly presented by Rand (16), who when referring to the methodological approaches available for Depot Location studies, portrays this choice as a trade-off involving the acceptance of sub-optimum solutions to Travelling-Salesman-type problems as a result of using less complex procedures for solving them. Rand illustrates this trade-off by means of the 2-dimensional taxonomic space reproduced in Figure 2.1., which defines a continuum that ranges from approaches that seek to derive optimum solutions using exact methods, to those that arrive at more approximate solutions by less complex means. For example, as Figure 2.1. suggests, the "Route-Optimisation" approach seeks to develop a very complex procedure that will guarantee an absolute optimum solution, (eg. Held & Karp (17)), but, because of the level of sophistication of the search procedure involved, this can only be achieved if a relatively simple cost function is used. On the other hand, "Heuristic" procedures employ a more detailed and realistic cost function, at the expense of complexity in the search procedure. Above, and to the left of, "heuristics" in the continuum of Figure 2.1. are "Simulation Models", whose cost function is even closer to reality, with the accompanying simplification of the search procedure.

Whilst assessing the position of the current research in Rand's continuum, it is important to point out that the creation of exact algorithms is not the central aim of the thesis, (SEE Section 2.1. for a precise statement of objectives). The development of an algorithm for constructing Travelling-Salesman tours in the presence of time-windows, in Chapter 6, is made necessary by the absence of a suitable existing algorithm under such conditions, and, in common with the Savings-based method used in Chapter 4, is merely a means to the end of generating a sample of vehicle tours on which further analysis may be carried out. In other words, rather than the production of optimum tours, what is of importance here is that the route-building algorithm employed should enable a parameter of interest, (eg. delivery-area size), to be varied whilst all others are held constant, in order to examine the effect of this parameter on, say, Total Fleet Mileage. A model that produces near-optimum tours is therefore adequate for the purposes of the current analysis.

It should also be noted that no attempt is made to simulate an actual distribution system in detail, so that the methodology employed here cannot be described as being purely simulation; in fact, it is assumed that there is a very simple situation of a delivery-area in which vehicle speed is

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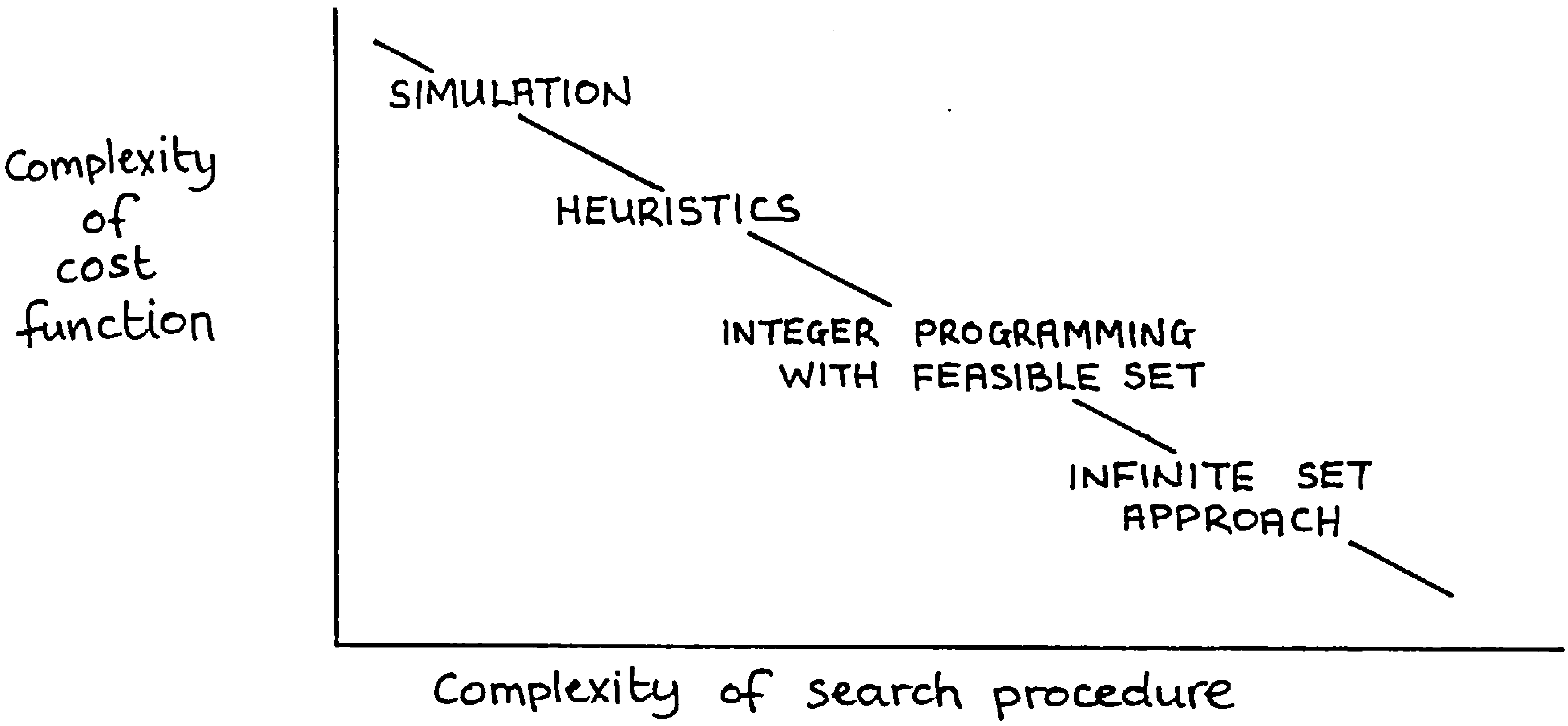
(16) RAND, G.K., "Methodological choices in Depot Location studies," Operational Research, Vol.27, No.1, (1974).
(17) HELD, M.M., and KARP, R.M., op cit

Figure 2.1. Rand's continuum of methodological approaches



(After: Rand, G.K. (11))

uniform in all directions, and all cost-data that are used are based on very generalised cost-tables (18). Referring, again, to Figure 2.1., therefore, it can be argued that, since each Travelling-Salesman tour "simulated" merely represents a simple, hypothetical tour within a homogeneous space, the thesis, in Rand's continuum, can by no means be described as occupying a position at the extreme "Simulation" end of the scale, due to the nature of the cost-function that is used.

An example of the type of tour-building algorithm that is used here for deriving Travelling-Salesman solutions is the "Savings Method", developed by Clarke & Wright (19); although it may be argued that this algorithm falls short of producing optimum solutions, the "near-optimum" tours provided by this method are adequate for the purpose of exploring the way in which Total Fleet Mileage varies in response to changes in other key parameters.

As with the particular problem-formulation adopted, details of the methods used, along with a fuller description of the relevant model that is constructed, will be included in the text as and where appropriate. Section 2.5. now outlines some alternative algorithms for solving the Travelling-Salesman Problem that might have been used, and discusses the reasons for adopting a savings-based method, similar to that of Clarke & Wright, in the simulation program.

2.5. The Savings Method and Alternative Routing Algorithms

The travelling-Salesman Problem may be formulated in a symmetric or an asymmetric context. The difference is that the symmetric version of the problems assumes that the cost of travelling between, say, locations i and j is the same regardless of whether the direction of travel is i-to-j or j-to-i; this "cost" may be measured simply in terms of travel-time. The asymmetric Travelling-Salesman Problem, on the other hand, does not include this assumption, so that a matrix of the costs of travelling between pairs of locations would be asymmetrical. Algorithms for solving the asymmetric case are therefore more complex than those designed for the simpler case, and require more computer time and computer storage space. In this section, however, the discussion is confined to algorithms for the symmetric version of the problem.

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(18) The cost-tables used were published by <u>Commercial Motor</u> magazine in1982.

(19) CLARKE, G., and WRIGHT, J.W., "Scheduling of vehicles from a central depot for a number of delivery-points." <u>Operations Research</u>. Vol.11., P.568., (1963).

There is no theoretical difficulty in finding exact solutions to the Travelling-Salesman Problem. For each set of points that is generated, there is a finite set of tours which pass through these points; the length, or cost, of each tour may be calculated and the tour incurring the lowest cost may be identified as the "optimal" tour. In practice, however, even with problems involving a relatively small number of locations, the number of possible tours makes such a complete enumeration of all options impractical, even with the use of a computer. The total number of tours that might pass through a set of C customer locations may be calculated using the simple formula,

$$(C - 1)!$$

For example, when there are 3 locations to be visited, only 2 tours are possible, and when C = 4, there are six alternatives, (since 3! = (3x2) = 6). In simple cases such as these, there is no problem in quantatively evaluating all alternatives, but a problem involving 10 customer-locations has 9!, or 362880, possible solutions, whilst one with 15 locations has (8.7 x 10) solutions. The computer capacity that is required for complete enameration of problems involving substantially more customer-locations is therefore extremely large. This limitation as to the number of locations that can be dealt with by algorithms that yield exact solutions underlines the need for heuristic methods that can produce near-optimum tours using far less computer time and storage space.

Procedures for solving the Travelling-Salesman Problem fall into one of four categories:-

1. Partial enumeration. This method, by definition, can not guarantee an exact solution, since not all possible routes are enumerated. This is because some links between locations are eliminated before the enumeration stage of the procedure in order to reduce the computational size of the problem. For example, this might be done by excluding links which are obviously not likely to form part of the optimal tour, so that a complete enumeration may be carried out on the reduced set of possible links that remain.

2. Sequential tour-building. Both exact and approximate algorithms may be employed with this strategy. The method involves first selecting the customer location that is "closest" to the depot (in terms of distance, time or cost, etc.), as the first location that should be visited in the tour. The next step is to select the point that is nearest to the customer location, using the same criterion, in order to form the second link in the tour. This process continues until it is no longer possible to add another location to the tour and return to the depot without violating a time or mileage constraint on the length of a tour. Most commonly this involves the constraint on the length of the working-day,

or legislative limitations on drivers' hours. If no further additions to the tour are feasible, the tour must be completed by making a link from the last customer-location to be added to the depot. Should there be some customers that have not yet received a delivery, then a new tour must be initiated, using the same procedure. A number of methods may be employed for selecting the next customer-location to be added to the tour, from simple Nearest Neighbour Analysis to exact techniques such as branch & bound procedures and dynamic programming.

3. Tour-to-Tour Improvement. Methods falling into this category begin by generating an initial, feasible solution which is then improved upon according to a set procedure. This might, for instance, involve removing one or more locations from one tour and replacing them with other locations, or entail reversing the order in which locations are visited. The former method of tour improvement is used by Lin (20), who describes a tour as being "r-optimal" if no improvement can be obtained by replacing r of its links with r alternative links, (21). Usually, a 3-optimal, or even a 2-optimal, tour will have a high probability of being the optimal solution to a problem. An exact solution can only be guaranteed, however, if the tour is found to be "C-optimal", (C representing the number of locations that must be visited). Since such procedures rarely evaluate a tour beyond the 3-optimal stage, all algorithms based on tour-to-tour improvement, or "local optimisation" are approximate.

4. Sub-tour Contraction. This method begins with a matrix of the distance, time or cost that separates each pair of customer locations, each of which is a potential link that might be included in the minimum-cost optimal tour. If all lowest-cost links are made until all locations have been linked to at least one other, then a solution to the "Assignment Problem" will have been produced. At this stage, however, the "solution" will not constitute a tour, but a series of sub-tours. The next step is therefore to eliminate the sub-tours so that a single tour is produced. Unlike tour-to-tour improvement, this strategy may be used to give exact solutions, as demonstrated by both Eastman (22) and Shapiro (23), using "Branch & Bound" algorithms.

Branch & bound is one of the methods discussed in the following sub-section on algorithms that have been used to derive exact solutions to the Travelling-Salesman Problem.

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(20) LIN, S., "Computer solutions of the Travelling-Salesman Problem". Bell Systems Technical Journal., Vol.44., p.p.2245-69, (1965).
(21) Lin's notation is used here; his r should not be confused with the r used in Chapters 6 & 7 of the thesis.
(22) EASTMAN, W.L., Linear programming with pattern constraints. (Harvard University Ph.D. thesis, 1958).
(23) SHAPIRO, D., Algorithms for the solution of the optimal cost travelling-salesman problem. (Washington University Sc.D. thesis, 1956).

2.5. Exact algorithms

An example of the use of a branch & bound, or "tree-search), procedure is provided by Little et al (24). The method is based on the division of the set of all possible tours into a number of sub-sets, a process that is analogous to the branching of a tree. The procedure begins with the making of a link between two customer locations, (a link from, say, point i to point j). The forming of such a link has two implications: 1. The set of all possible tours may be divided into two sub-sets: those tours that contain the i-to-j link, and those that do not. 2. The number of feasible links to j from i may now be eliminated, along with any link that may cause a sub-tour to be formed, (such as j-to-i, in this example). The matrix of all possible links between customer locations is thus "reduced" whenever a "branch" is made. A lower bound on, (ie. the minimum value of), the length of the least-cost optimal tour within each of these sub-sets is then calculated, (ways in which the lower bound may be estimated are outlined in sub-section 2.5.2.). Each sub-set that is created by the selection of a link from one location to another may, in turn, be divided, by the same process, into two sub-sets. In this way, more and more, smaller and smaller, sub-sets of tours are formed. There is a point in this branching procedure at which the links that have been made form a tour, whose length, or cost, may be measured. If the length of this tour is less than, or equal to, the lower bounds of all sub-sets of tours that are to be branched from, then this tour is the optimal tour that is the solution to the Travelling-Salesman Problem. If this is not the case, then the search procedure continues from a sub-set whose lower bound is less than the length of the initial tour; the search ends when a tour - the optimal tour - is found whose length is less than or equal to the lower bounds of all the sub-tours that remain "unbranched".

Obviously, a tree-search of this type requires a great deal of computer-time and storage space, which increases considerably as the number of customer-locations involved is increased. The same comment may be applied to dynamic programming, a technique used extensively in operations research, which involves the solving of a recursive equation to find the minimum cost of each stage of a problem. In the context of routing & scheduling problems, the technique consists of finding all feasible tours through a set of locations, and the cost of each. Held & Karp (25) and Bellman (26) are among those who have used dynamic programming for solving the Travelling-Salesman Problem.

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(24) LITTLE, J.D.C., MURTY, K.G., SWEENEY, D.W., and KAREL, C., "An algorithm for the Travelling-Salesman Problem". Operations Research, Vol.11., p.p.979-89, (1963).
(25) HELD, M.M., and KARP, R.M., op cit
(26) BELLMAN, R., "Dynamic Programming treatment of the travelling salesman problem". Jour. Assoc. Computer Mech., Vol.9., p.p.61-3, (1962).

In the context of the solutions produced for the purposes of the current thesis, however, exactness is by no means crucial. The simulation process used here involves the solving of a large number of Travelling-Salesman Problems, and so it is of greater importance that the algorithm used should be economical in terms of both computer time and computer storage space, whilst still producing near-optimal solutions. For this reason, the discussion will now be focused on alternative heuristic algorithms that might have been used.

2.5.2. Heuristic methods

The heuristic of Lin (20) has already been outlined above, in the section dealing with methodologies based on tour-to-tour improvement. Reiter & Sherman (27) employ a similar method. Having formed and initial feasible tour, one of the points through which the tour passes is removed from the tour; the objective is then to insert this point into the sequence of links that remains in the position that maximises the improvement of the overall tour. This procedure is repeated with each point, and then takes place with each linked pair of points. When it is no longer possible to reduce the total length of the tour, then the tour is said, using the same terminology used by Lin, to be "2-optimal". Similarly, "3-optimality" may be achieved by successively removing and replacing linked chains of 3 customer-locations.

The "Sweep Algorithm", proposed by Gillett & Miller (28) is an example of a two-stage procedure, since customer locations are first sorted into clusters and then sequenced into tours at a secondary stage. The first phase begins with the selection of the location, i, that has the smallest angle with the depot, (a customer location situated "due North" of the depot, for example, has an angle of zero degrees). The "sweep" continues with locations having the smallest angle with the depot, (j,k,l.....etc.), being added to the cluster until no more locations may be included without the violation of either distance or capacity constraint. At this point, the formation of a new cluster begins, and the process continues until every customer location has been assigned to a cluster. A shortest-path algorithm may then be employed for solving the Travelling-Salesman Problems that are presented by each cluster of points. In order to test whether the initial solution arrived at can be improved upon, the whole process may be repeated, with the "sweep" procedure beginning with j, the location that was originally the second point to be selected.

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(27) REITER, S., and SHERMAN, G., "Discrete optimising." S.I.A.M. Review., Vol.13., p.p.864-89, (1965).

(28) GILLETT, B.E., and MILLER, L.R., "A heuristic algorithm for the vehicle-dispatch problem." Operations Research, Vol.22., p.p.340-9, (1974).

With the central depot acting as a pivot, the imaginary axis has effectively been rotated clockwise, so that point j's angle with the depot is now zero degrees and point i is the last point to be "swept". The procedure may be repeated as many times as there are customer locations in the problem definition. One advantage of the Sweep Algorithm is that it is effective for problems with up to 250 locations, but, according to Ballou & Agarwal (29),

> "Time windows on stops are not well handled by this method."

Tyagi (30) presents a similar two-stage methodology, which may also be described as a "cluster first route second" procedure. Nearest Neighbour Analysis is used to sequentially group customer locations into clusters, each of which constitutes a separate Travelling-Salesman Problem.

Some researchers have proposed methods which, though based on the Savings principle, differ from the procedure originally described by Clarke & Wright, (31). For example, Tillman & Cochran (32) employ a different method for deciding which links should be made. Whereas Clarke & Wright's method is for links to be made in order of greatest distance-saving, Tillman & Cochran consider connected pairs of links, so that the search procedure calculates the combined cost of making two choices in sequence. This principle may be extended further so that the total cost of making three connected links is calculated at each stage. At this point, however, the number of possible 3-location chains becomes quite large, as do the demands that are made on computer time and storage space.

Gaskell (33) differs from Clarke & Wright in the way in which savings are calculated. Gaskell's method is an attempt to counteract the tendency of the Savings Method to favour links between locations that are both close to each other and remote from the depot, thus giving rise to rather circumferential routes. This tendency is suggested by the

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(29) BALLOU, R.H., and AGARWAL, Y.K., "A performance comparison of several popular algorithms for vehicle routing and scheduling." Journal of Business Logistics. Vol.9., no.1, p.p.51-65, (1988).

(30) TYAGI, M., "A practical method for the truck dispatching problem." Journal of the Operations Research Society of Japan. Vol.10., p.p.76-92, (1968).

(31) CLARKE, G., and WRIGHT, J.W., op cit

(32) TILLMAN, F.A., and COCHRAN, H., "A heuristic approach for solving the delivery problem." Journal of Industrial Engineering. Vol.19., p.354, (1968).

(33) GASKELL, T.J., "Bases for vehicle fleet scheduling." Operational Research Quarterly. Vol.18., p.281, (1967).

very nature of the savings formula,

$$S_{ij} = d_{oi} + d_{oj} - d_{ij} \qquad (E.2.1.)$$

Where, S_{ij} = the distance saving from linking points i & j,
d_{oi} = the distance from point i to the depot, O,
and, d_{ij} = the distance between points i and j.

In order to encourage more "radical" links, Gaskell proposes following formula which discriminates in favour of links between locations that are close together, and against those whose joining together yields the greatest saving,

$$A_{ij} = S_{ij}\ (c + (c_{oi} - c_{oj}) - c_{ij}) \qquad (E.2.2.)$$

Where, A_{ij} = the modified measure of the desirability of a link,
and, c = the average distance of all locations from the depot.

The formula $(S_{ij} - c_{ij})$ is also suggested as an alternative measure that might be used. Since heuristic algorithms, by definition, are liable to produce less-than optimum solutions to routing problems, it is useful to be able to estimate the difference between the algorithm's solution and the optimum. One way in which the cost of an optimum solution may be estimated is using a formula devised by Beardwood, Halton and Hammersley, (34). They suggest that the shortest distance that is required to pass through a set of points within an area of known size is,

$$K.a^{o.5}.\ C^{0.5}. \qquad (E.2.3.)$$

Where, K = a constant that has a value of approximately 0.75,
a = the size of the area,
and, C = is the number of points through which the route must pass.

There are several other ways in which a "lower bound" on the cost of an optimal tour may be estimated. This may, for example, be done by calculating the "shortest Spanning Tree"; this is the minimum total length of all the links that are required to join all customer-locations, which does not necessarily have to be a tour. The Shortest Spanning Tree may be calculated using an algorithm designed by Kruskal (35). Similarly, the sum of the distances from

(34) BEARDWOOD, J., HALTON, J.H., and HAMMERSLEY, J.M., "The Shortest Path through many points." Proceedings of the Cambridge Philosophical Society, Vol.55., p.299, (1959).
(35) KRUSKAL, J.B., "On the shortest spanning subtree of a graph and the travelling salesman problem." Proceedings of the American Mathematical Society, Vol.2., p.p.48-50, (1956).

each location to its nearest neighbour and next-nearest neighbour may be considered as the lower bound on the length of a tour, and may be calculated as follows,

$$\text{"Sum of the shortest links"} = \tfrac{1}{2} \left(\quad (d_{j1} + d_{j2}) \right) \qquad \text{(E.2.4.)}$$

where, d_{j1} and d_{j2} are the distances of j's nearest, and next-nearest, neighbour, respectively.

The sum of these distances is divided by 2 in order to eliminate double counting, (36).

An alternative strategy for evaluating heuristic algorithms is to compare their performances, both with one another and with exact procedures, when applied to a range of hypothetical test problems. The results of this type of research, where the savings method has been one of the heuristics tested, are reviewed in the following sub-section.

<u>2.5.3. The performance of the savings method compared with other algorithms</u>

A recent evaluation of heuristic algorithms is provided by Ballou & Agarwal (37), who compare the performance of the savings, sweep and cluster (38) heuristics when used to solve a number of test problems. These tests are carried out using five types of point configurations, described as "random", "cluster", "West Coast", "sector" and "urban-rural"; these are illustrated in Figure 2.2.. Ten or twelve problems for each of these configurations is devised, with 20 customer locations each time. In addition to the heuristics mentioned above, an exact method is used for deriving optimum, or near-optimum, solutions to each problem; the purpose of this is to provide a means of comparing the results of the approximate methods with the optimum solution. The exact algorithm used is set partitioning, a method based on integer programming. Although set partitioning is prone to becoming computationally complex when dealing with large problems, which is a characteristic of exact methods, the 20 customer locations that were involved in the test problems where insufficient to create any difficulties. A summary of results, for all five configuration-types, is presented in Table 2.2.. Clearly, the savings method emerges as the most

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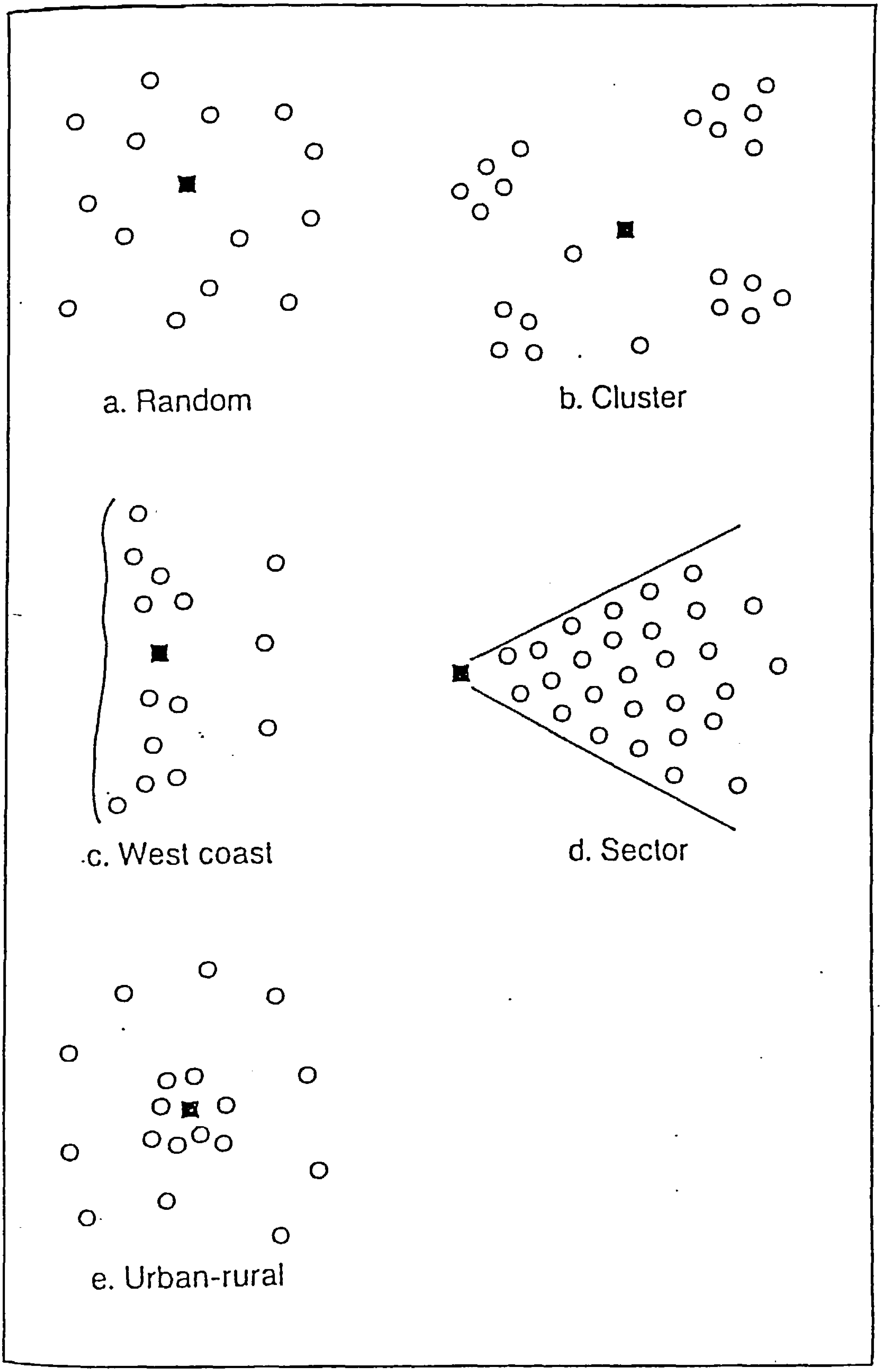
(36) EILON, S., WATSON-GANDY, C.D.T., and CHRISTOFIDES, N., <u>op cit</u>.

(37) BALLOU, R.H., and AGARWAL, Y.K., "A performance comparison of several popular algorithms for vehicle routing and scheduling." <u>Journal of Business Logistics</u>, Vol.9., p.p.51-65 (1988).

(38) The "cluster method" used involves a two stage "cluster first route second" procedure, with Nearest Neighbour Analysis employed initial clusters, (although the details of the method are not the same as those of Tyagi's (<u>op cit</u>)).

Figure 2.2. Ballou's test problem point configurations.



(After: Ballou, R.H., and Agarwal, Y.K.. (37)).

effective of the heuristics tested, producing solutions that varied from the optimum by less than 2%, on average, compared with average figures of 12.1% and 14.3% for the cluster method and sweep method, respectively. For each problem type, the savings method produced an optimum solution for at least one problem, something that was never achieved by either of the other heuristics, and the worst performance by the savings method in the entire experiment was a solution that differed from the optimum by 13.3%. Ballou & Agarwal also point out that the savings method produced solutions requiring more vehicles than the optimum solution on only 8% of occasions.

Table 2.2. Summary of test results, (percentage deviation from optimum solutions

	Savings Method			Cluster Method			Sweep Method		
Problem type	Min	Ave	Max	Min	Ave	Max	Min	Ave	Max
WEST COAST	0.0	2.7	8.9	1.5	16.5	46.4	5.6	13.3	29.8
URBAN RURAL	0.0	2.7	2.8	4.6	14.5	40.1	2.7	22.7	39.4
CLUSTER	0.0	1.0	3.0	2.1	9.8	16.9	1.5	9.0	20.6
RANDOM	0.0	2.8	10.2	4.6	10.0	14.3	0.2	10.2	18.2
SECTOR	0.0	2.6	13.3	2.8	21.8	47.7	18.1	29.5	47.5
OVERALL	0.0	1.9	13.3	1.5	12.1	47.7	0.2	14.3	47.5

(After: Ballou & Agarwal (37)).

Ballou has followed up this work by repeating the experiment with a larger number of customer locations, (39). The same point configurations are used, but with 50 or 100 locations for each problem. It was therefore not feasible to use an exact method to produce optimum solutions and so the savings method was simply compared with modified versions of the cluster and sweep methods. Again, the savings method out-performed both of these, since, on average, the cluster method gave solutions that were 7.9% longer than the savings method, whilst the sweep method produced solutions that were 5.9% longer. With reference to the question of whether the savings method would be as effective on problems involving a larger number of locations, Ballou concludes that,

"........there is no reason to believe that the savings method did not perform as well as it did in the previous study".

(39) BALLOU, R.H.,"A continued comparison of several popular algorithms for vehicle routing and scheduling." Journal of Business Logistics, Vol.11., No.1., p.p.111-126, (1990).

Herlihy, Butler & Pitts (40) performed similar tests on commercially available computer packages, using test problems of 100 customer locations, and found that packages based on the savings method produced solutions that were approximately 6% longer than the best known solutions, (the tests were based on widely used test problems, first suggested by Eilon et al (41)). It was with these test problems that Eilon et al had earlier found that, with small problems involving 10 customer locations, the Clarke & Wright savings heuristic gave solutions that were only approximately 3.2% longer than the optimum.

2.5.4. Conclusions on the use of the savings method for generating travelling-salesman solutions

The role of the routing algorithms used within the simulation procedure, for the current thesis, is to produce solutions to travelling-salesman problems which approximate the optimum, but which do not require excessive amounts of computer time and storage space. Because the process of simulation requires that many routing problems are solved, then solution time is extremely important, especially as 100 customer locations are involved in each problem. The sheer size of the routing problem in any case precludes the use of an exact algorithm.

The savings method consists of an extremely simple formula, and is widely used in commercial routing packages, although there are some criticisms that might be made of it. The fact it has an inherent bias towards the linking of points that are both close to each other and remote from the depot, thus encouraging rather circuitous, circumferential routes, has already been mentioned, and Gaskell's modification is an attempt to counteract this tendency, (33). Clarke & Wright's procedure for constructing tours, based on savings, is another aspect of the methodology that is open to criticism. This is because links between points are made in descending order of savings value, so that the linkage that yields the largest distance saving is made first, and so on. The problem with this is that the remainder of the tour-building process is affected by this first link; initially, of course, this is because there are some links that are impossible due to the fact that these two points are irreversably linked together. In a situation where an

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(40) HERLIHY, P., BUTLER, M., and PITTS, E., Estimation of energy and cost savings arising from rationalisation of milk assembly operations. (EEC report EUR 9272 EN, 1984).

(41) EILON, S., WATSON-GANDY, C.D.T., and CHRISTOFIDES, N., op cit.

optimum solution may have been derived if the link that yields the second-, or third-highest, saving had been made first, it is clear that the tactics of achieving the greatest saving with the first link would ensure that the eventual solution must be sub-optimal. Tilman's method of considering the desirability of selecting connected pairs of links is an attempt to overcome this problem, (32), although the general criticism of Clarke & Wright here is that their method is not interactive, in the sense that it does not return to decisions that have been made in order to consider whether the eventual solution might be improved.

In spite of these criticisms, however, the work of those who have used test problems to evaluate the performance of heuristic methods has shown that the savings method, not only out-performs rival heuristics, but also produces solutions that are demonstrably near to the optimum. The use of the savings method for producing tours for the current research therefore appears to be fully justified.

The most commonly cited weakness of the savings algorithm is that its effectiveness is substantially reduced when time-windows and other constraints are added to the problem formulation. This is not relevant to the current discussion, since the savings method is no longer utilised when time-window constraints are considered, in Chapters 6 and 7. This issue is dealt with in Chapter 6. Meanwhile, Section 2.6. outlines the structure and content of the thesis.

2.6. Summary of the Structure and Contents of the Thesis

The analytical portion of the thesis is divided into two parts: Part 2, which deals with both the direct and indirect effects of fleet-size, a decision variable, on distribution costs, and Part 3, which considers the "external" influence of time-constraints, imposed by customers, on the cost of an operation.

Part 2 begins with Chapter 3, which has as its major theme the dilemma of whether to deliver an order of goods using a large fleet of small vehicles, or a smaller fleet made up of vehicles with the largest carrying-capacity permissible, (given that the problem formulation constrains the fleet to being made up of vehicles of the same size). The central concept of this chapter is "Economies of Scale of Transport", which deals with the fundamental trade-off between fleet-size and vehicle carrying-capacity. This trade-off is associated with a similar problem, of which a hypothetical example is given in this chapter, which involves the choice between delivering a weekly order of goods to a number of customer-locations in one day using one large truck, and employing a smaller vehicle for the same task on every day of the week.

Chapter 4 considers specifically the spatial implications of the fleet-size decision, by examining the precise relationship that exists between the number of vehicles that operate from a depot and Total Fleet Mileage. As well as producing analytical equations to describe this relationship, this chapter also includes a description of an alternative method for estimating the total mileage of a fleet, given basic data such as the number of vehicles used, the number of customer-locations that require a visit, the maximum number of drops that may be made in a day, the size of the delivery-area etc..

Chapter 5 considers those decisions and constraints that concern the management of drivers and their hours of work. The two main topics included here are the impact of scheduling drivers to make overnight stays away from the operating -centre, instead of them being constrained to return before the end of each working-day, and the effect of altering the constraint on the number of hours that each driver may work in a day. The discussion of the overnight stay question also relates to the parts of Chapter 3 which deal with the Economies of Scale of Transport and the fundamental trade-off between fleet-size and vehicle-size.

Chapters 6 and 7 focus upon the times that are laid down, if any, within which a delivery may be made at a location, (ie. "time-window"). Although such time-constraints may exist in many forms, the particular scenario that is assumed in these chapters is one in which each customer specifies one time-window during which deliveries may be made, whose width is the same at each location, and which may be fixed at any time within the working-day. Because no algorithm currently exists for constructing Travelling-Salesman tours in the presence of such constraints, a considerable part of this section of the thesis is devoted to describing how such an algorithm was developed in order to make it possible to make estimates of vehicle-tour length in such circumstances.

Chapter 7 discusses the output that is obtained from many iterations of the resulting program, and goes on to compare these results with those derived using alternative route-building algorithms. To provide a comparison with the analysis of the relationship between fleet-size and Total Fleet Mileage described in Chapter 4, this chapter also includes consideration of the same relationship in the presence of time-window constraints.

2.7. Summary of Introduction

After Chapter 1's general discussion of the Distribution Problem, with its various formulations and components, in which the broad area of interest of the thesis is defined, Chapter 2 has focused on the main objectives that are pursued in the following text, and has described in greater detail the relevant problem definition. The current chapter also outlines the methodologies that are used for meeting these objectives, and the way in which the findings of the subsequent research are structured and presented.

Another theme of this introductory section has been to, not only define the problem-area dealt with by the thesis, but to also set the research into the context of work already carried out by other researchers in this field. The discussion falls short, however, of attempting to make an exhaustive literature review of the subject-area; comprehensive surveys have already been carried out by Bodin et al (42) and by Bodin & Golden (43).

(42) BODIN, L.D., GOLDEN, B.L., ASSAD, A.A., and BALL, M.O., "Special issue - Routing & Scheduling of vehicles and crews - the State of the Art." Comput. & Operations Research, (1983), Vol.10., No.2., P.P..63-211.
(43) BODIN, L.D., and GOLDEN, B.L., "Classification in vehicle Routing & Scheduling." Networks, (1981), Vol.11., No.2., P.P.97-108.

2. THE EFFECT OF FLEET SIZE ON THE COST OF A DISTRIBUTION OPERATION

CHAPTER 3

FLEET SIZE AND VEHICLE SIZE: THE ECONOMIES OF SCALE OF TRANSPORT

The concept of reducing the unit cost of a business or industrial concern by increasing the size of either the entire operation or of certain units of production, is based on the economic principle of Economies of Scale. The basis of this principle is the distinction between variable, or running, costs, which increase, often proportionately, as the size of an operation increases, and fixed costs, often referred to as "overheads", which remain constant regardless of the level of production etc.. As an example, consider an industrial firm that manufactures the product "A". As the output of this firm increases, the fact that fixed costs remain constant means that the total of fixed and variable costs of producing each unit of "A" rises less than proportionately, so that the average cost of production, (the cost of producing each individual unit of "A"), actually decreases.

Similar reductions in the unit cost of transport can be made in road haulage operations, encouraging the carriage of freight by road in increasingly large vehicles; evidence of the advantages of Economies of Scale in physical distribution is the fact that the road haulage industry in the U.K. has recently continually called for an increase in the maximum permissible weight of vehicles on British roads from the current legal maximum of 38 tonnes. In the context of road freight, the most important fixed costs are drivers' wages, licences, taxes, insurance, rent and rates for operating centres and administrative costs, whilst the main variable costs involved are for fuel, maintenance, lubricants, tyres, depreciation and overtime costs. Variable and Fixed costs will be referred to here as Running Costs and Standing Costs respectively.

The remainder of this chapter sets out to investigate the nature and extent of the Economies of Scale that are associated with distribution operations; first by disaggregating Total Cost and examining the behaviour of its major components in response to changes in the scale of an operation, and then by deriving expressions that describe the relationship between Total Cost and both vehicle-size and fleet-size.

3.1. The Existence of Economies of Scale in Road Haulage Operations

In Road Haulage operations, there are two main ways in which Economies of Scale might be derived; firstly, transport costs per mile tend to increase less than proportionately with increasing distance and, secondly, the unit costs of transportation will be reduced as vehicle-size increases.

3.1.1. Economies of Scale with Distance

The existence of Economies of Scale for transportation costs with increasing distance is one of the main conclusions of Daganzo & Newell, who consider not only the distance-orientated cost of transport but also inventory costs at the depot and the amount of time that goods spend in vehicles, although these economies are apparently only effective up to a certain size of delivery-area, (1).

Economies of Scale with distance for any individual vehicle is, of course, to be expected, since, whereas Running Costs will increase proportionately with mileage, Standing Cost will, by definition, remain constant, so that Total Transport Cost will increase less than proportionately with distance.

This view of the nature of transport costs is confirmed by Scott, who states that,

> "....It is a well-known feature of probably the vast majority of real transport systems that transport costs are in fact rarely directly proportional to distance."

and goes on to suggest that the unit costs of transport are of the general form,

$$a.D^b \qquad \text{(E.3.1.)}$$

where, D = distance,
and, a & b are parameters.

The parameter "b", according to Scott, will normally have a value of less than 1.0 , thus allowing Economies of Scale as distance increases; Figure 3.1., adapted from Scott, illustrates the possible shape of this trade-off between unit cost and distance, (2).

3.1.2. Economies of Scale with increasing vehicle size

By way of comparison, it is interesting to plot transport costs per mile against vehicles' carrying-capacity. The resulting curve, Figure 3.2., might be compared with Figure 3.1.; although the former curve is by no means a smooth one,

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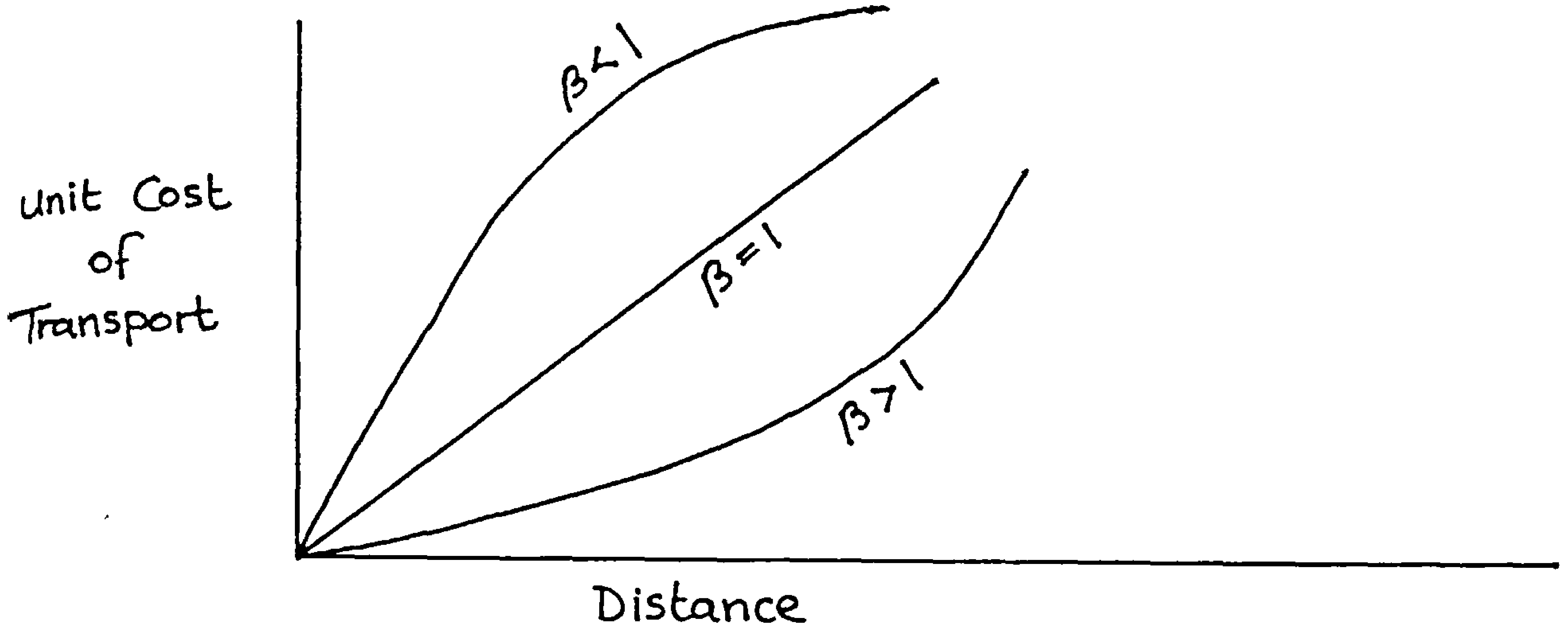
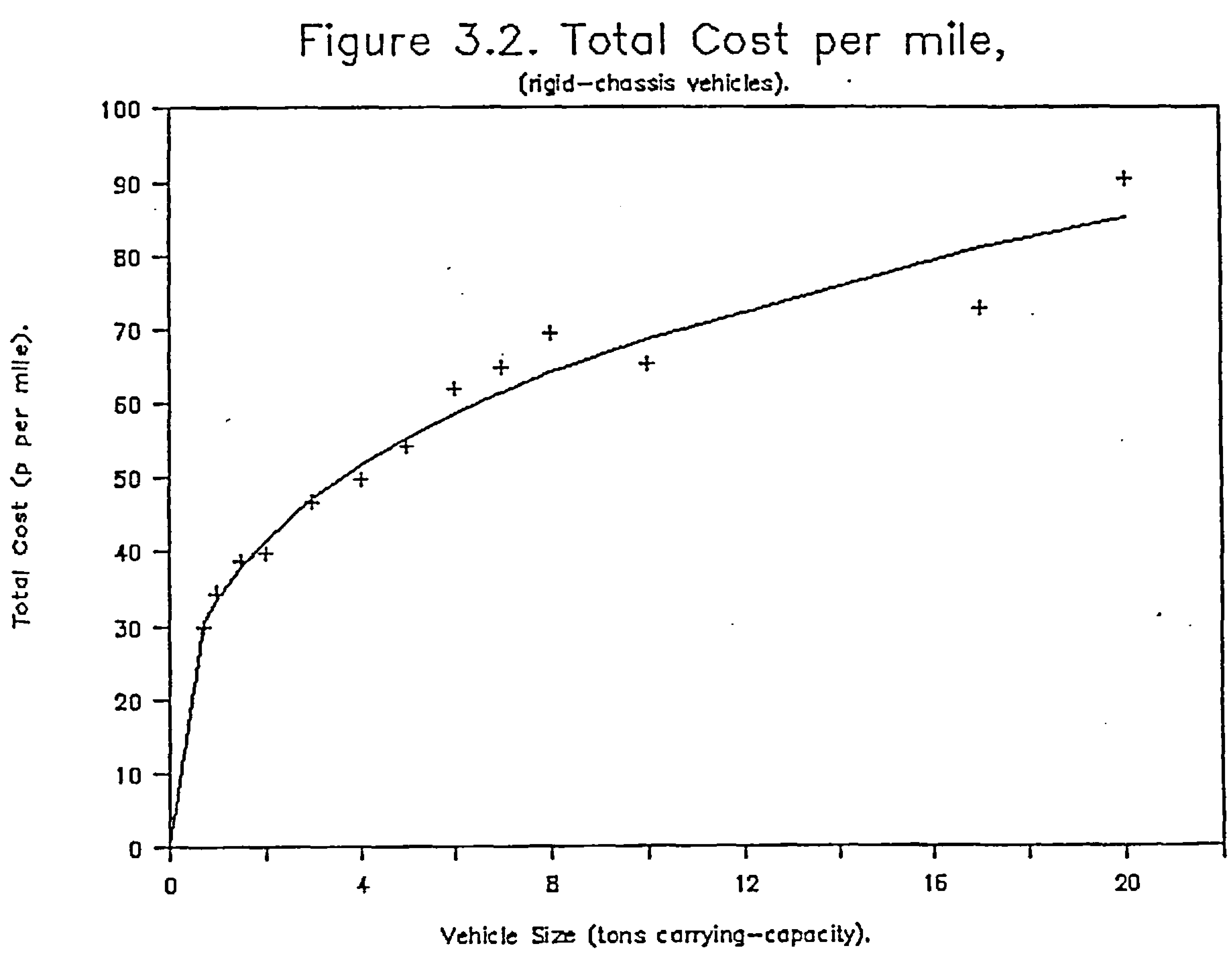
(1) DAGANZO, C.F., and NEWELL, G.F., "Physical distribution from a warehouse: vehicle coverage and inventory levels." in Transportation Research. Special Issue: Transportation Systems and Logistics, (Part B: Methodological). Vol.19b., (Oct. 1985), No.5., P.P.397-407.
(2) SCOTT, A.J., Combinatorial programming, spatial analysis and planning. (London,1971).

Figure 3.1. Scott's trade-off between the unit cost of transport and distance



(After Scott, A.J. (1))



(and reasons for irregularities in this distribution will be discussed in a later section), there is still evidence of a definite tapering off of this curve, suggesting that Economies of Scale may be derived with increasing vehicle size as well as with increasing distance. It is also interesting to note here that the cost of fuel per mile, according to Commercial Motor cost-tables, also increases less than proportionately with vehicle size, (SEE Figure 3.3.).

Figure 3.4. demonstrates how the distribution in Figure 3.2. may be expressed in the form,

$$a.x^{b} \qquad \text{(E.3.2.)}$$

where, x = vehicle size,
and, a & b are parameters,

a variation on the general formula proposed by Scott, (2). After simple Linear Regression Analysis on the transformed data, yielding an "R^2" value as high as 0·973, the resulting equation of the regression-line is,

$$\log. TC = 1{\cdot}525 + 0{\cdot}312 \log. x \qquad \text{(E.3.3.)}$$

where, TC = Total Cost per mile, (in £'s).
and, x = carrying-capacity of vehicle, (in tons).

This equation may be rewritten as,

$$TC = 33{\cdot}4965x^{0\cdot312} \qquad \text{(E.3.4)}$$

and is represented by the continuous curve shown in Figure 3.2.. It should be noted that this curve of predicted unit costs actually passes through "the origin"; of course, this situation will not occur in practice, as fixed operating costs ensure that cost per mile will never fall below a certain level, regardless of the carrying-capacity of a vehicle. In some cases, a problem would be created by the use of a curve passing through the origin of a graph to describe a distribution whose points will never meet the x-axis, since the variable described by the y-axis will inevitably be underestimated for small values of x. This is not the case with Figure 3.2., however, as equation E.3.4. makes a very close estimate of the unit costs of the smallest vehicles, so that the acknowledged absurdity of cost-estimates for vehicles of much less than 15cwt carrying-capacity presents no problem in this context.

The value of the parameter b in equation E.3.4. is 0·312, which agrees with Scott's assertion that, in most transportation contexts, b will be less than 1·0, and once again confirms the existence of Economies of Scale as vehicle size increases.

Figure 3.3. Fuel Cost per mile,
(rigid-chassis vehicles).

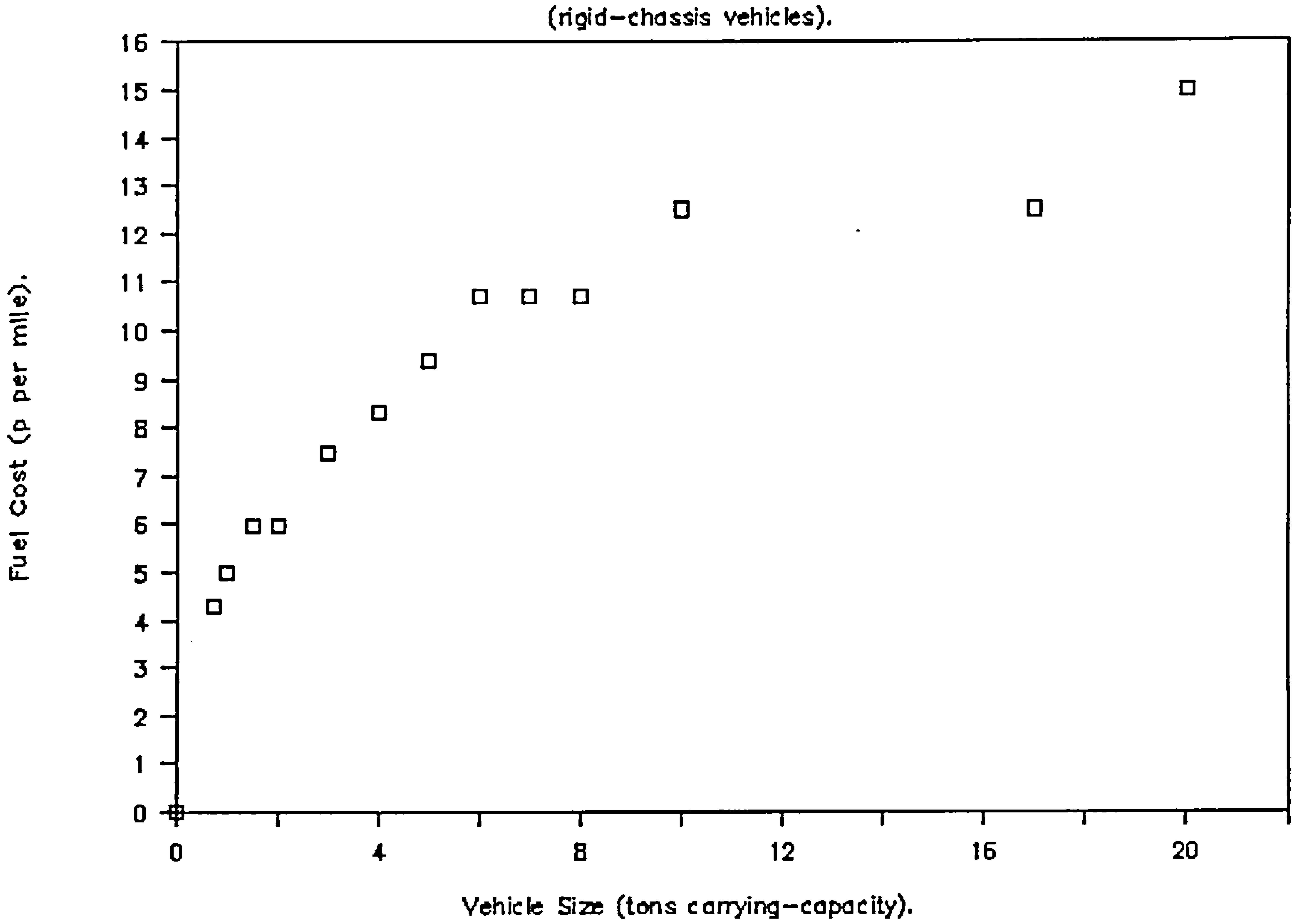
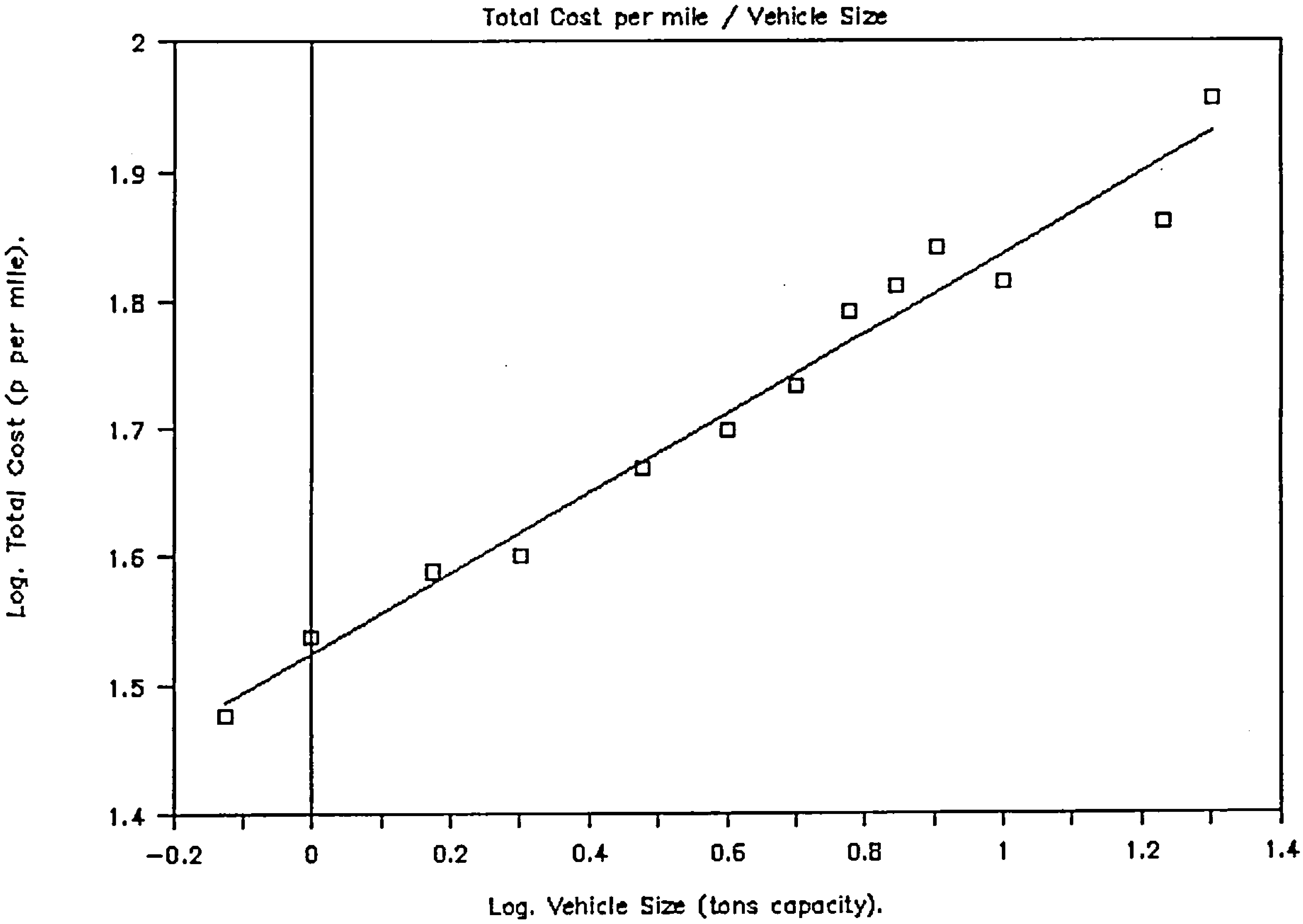


Figure 3.4. LOGARITHMIC REGRESSION
Total Cost per mile / Vehicle Size



It is on this type of Economies of Scale that the thesis will now concentrate.

3.2. The Economies of Scale of Transport with Increasing Vehicle Size

Throughout the following discussion, all data referring to transportation cost is based on cost tables published by Commercial Motor magazine, an example of which may be found in Appendix A. Closer analysis of these figures reveals an interesting relationship between Running Cost per ton per mile, (having previously considered unit-costs in terms of cost per ton-mile), and the carrying-capacity of a vehicle, (so that cost per ton actually represents cost per ton of carrying-capacity). For example, Figure 3.5. shows this relationship for rigid vehicles of between 0·75 tons and 20 tons carrying-capacity, and Figure 3.6. illustrates a similar curve for Standing Cost per ton per mile, again for rigid-chassis vehicles; because Commercial Motor cost tables express Standing Cost in terms of £ per week, it is assumed that each vehicle travels 1000 miles each week, so that Running Cost and Standing Cost could be expressed in the same units. Figure 3.7. shows the similar shape of the Total Cost curve.

To illustrate the closeness of the relationship between vehicle carrying-capacity and unit cost, Figure 3.8. graphs the results of a logarithmic transformation of the data on which the above figures are based; again, there is a "log.-linear" relationship, enabling the relationship between unit-cost and vehicle size to be described quite accurately with regression-lines. Table 3.1. summarises the results of the regression analysis performed on this transformed data, and includes results of the same analysis for articulated vehicles of between 10 tons and 22 tons carrying-capacity. The figures of interest in Table 3.1. are the coefficients of log. x, (ie. the powers by which x is raised, eg. -0·727 in the case of the Running Cost of rigid vehicles), since these represent the elasticity of the cost estimates provided by these equations to changes in vehicle size. These elasticities are slightly smaller for arctics than for rigid vehicles, but this may again be accounted for by the absence of data on articulated vehicles of less than 10 tons carrying-capacity. Nevertheless, the figures contained in Table 3.1. provide a useful means of estimating the unit cost of transport as a function of vehicle carrying-capacity.

Of course, it should always be borne in mind that the results of the above analysis are all derived from the basic data contained in the cost tables published by Commercial Motor magazine; the following section therefore examines this data-source in greater detail.

Figure 3.5. Running Cost per ton-mile.
(rigid-chassis vehicles).

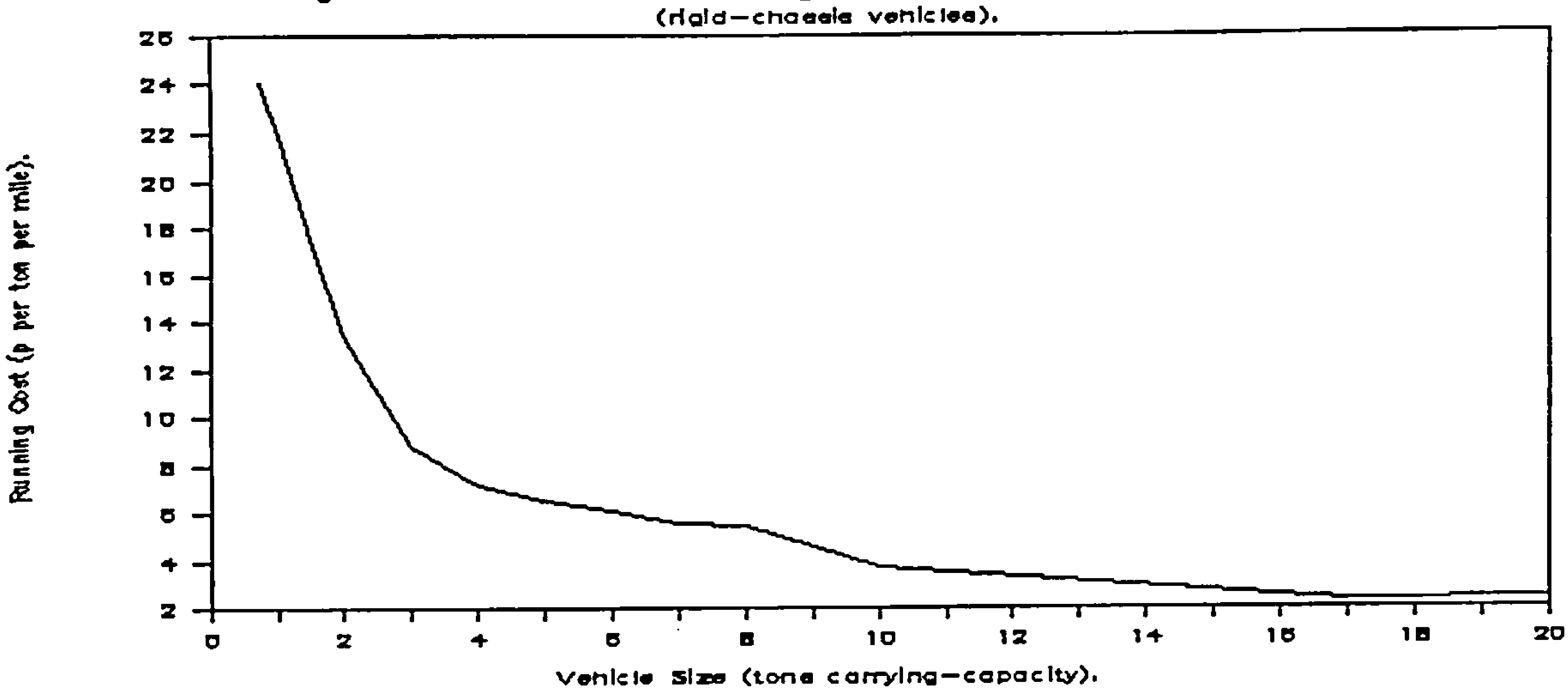


Figure 3.6. Standing Cost per ton-mile.
(rigid-chassis vehicles).

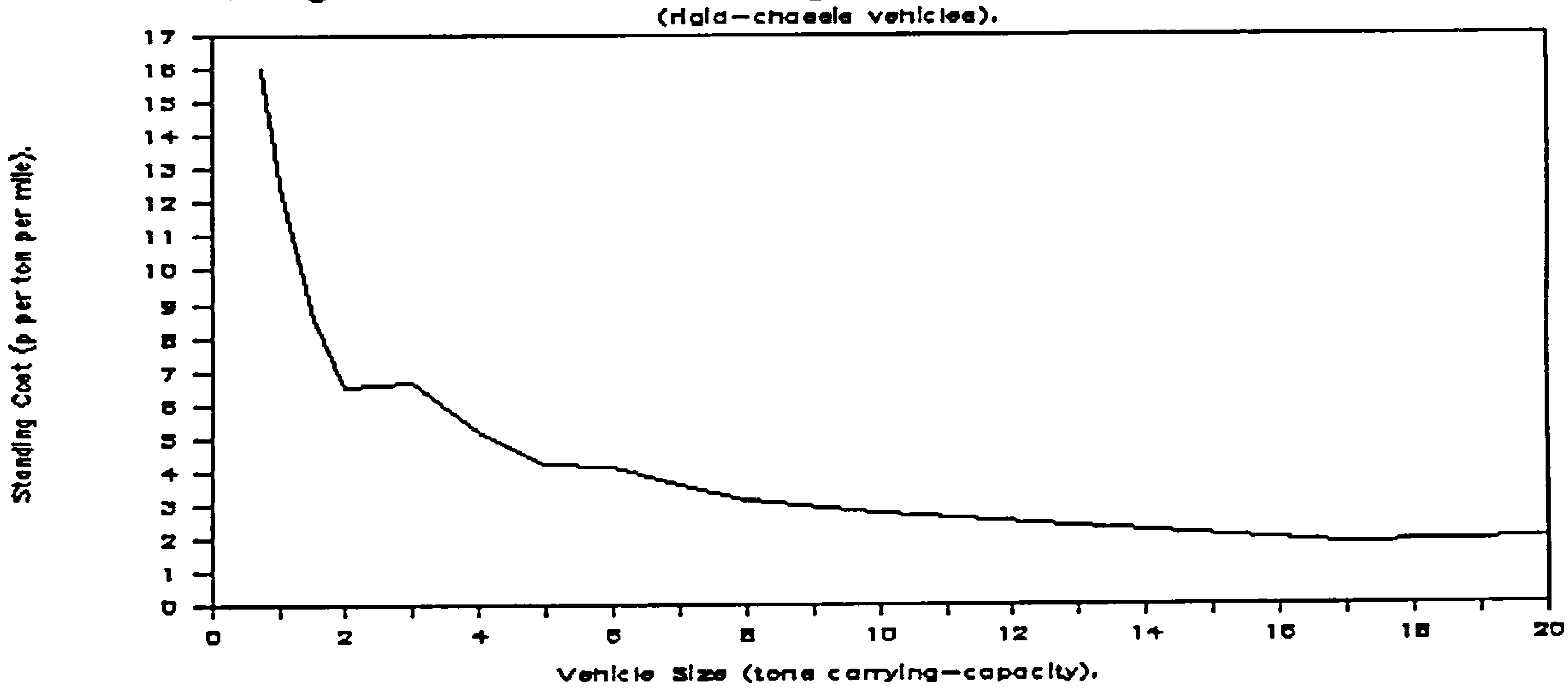


Figure 3.7. Total Cost per ton-mile.
(rigid-chassis vehicles).

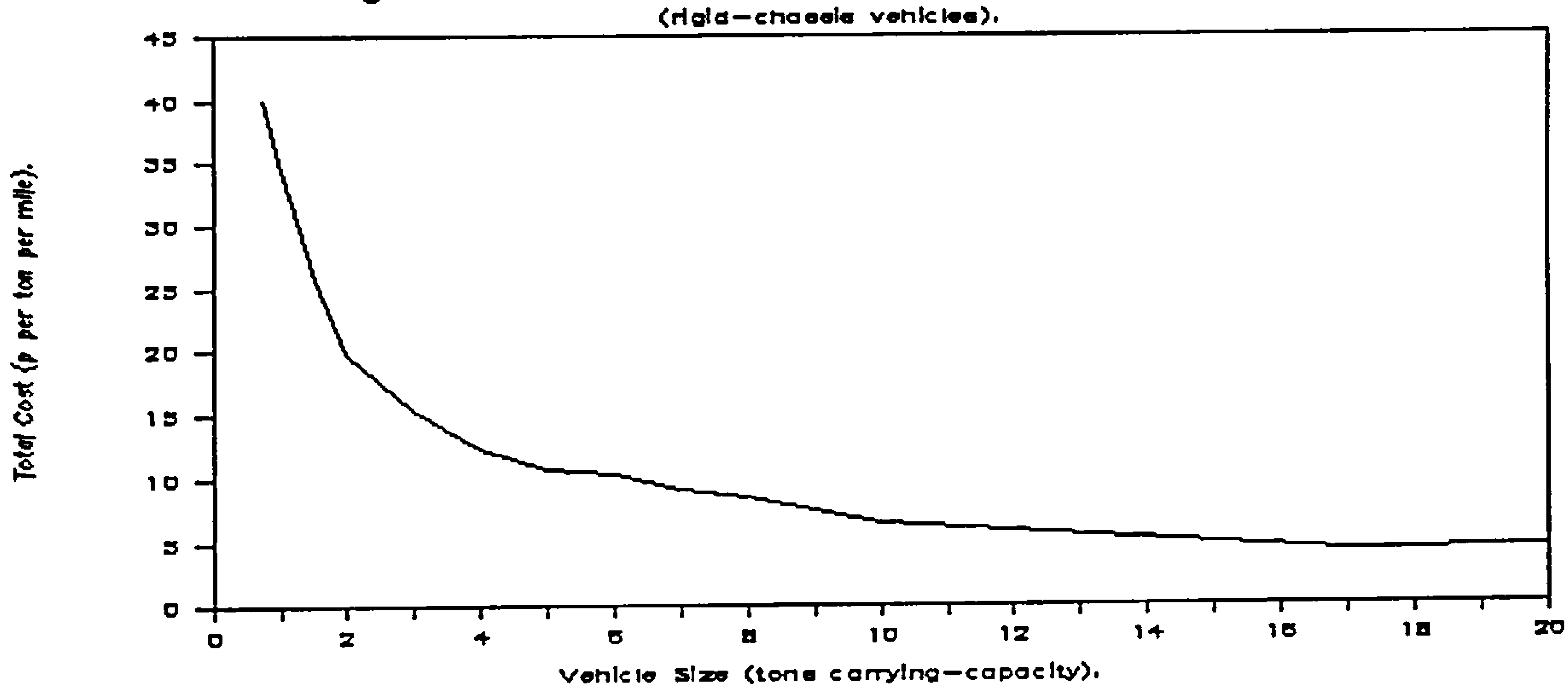
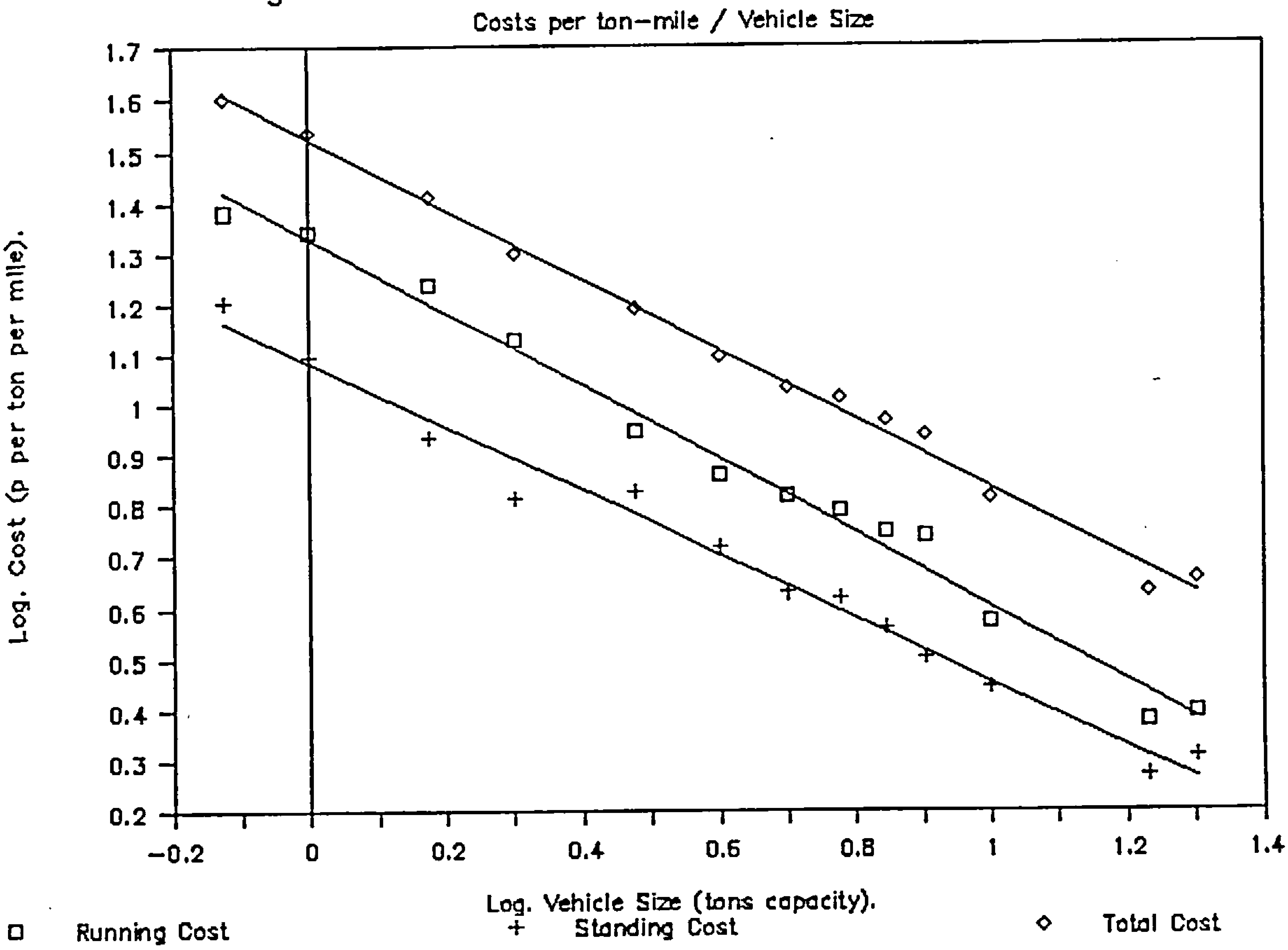


Figure 3.8. LOGARITHMIC REGRESSIONS



<u>Table 3.1. Expressions describing transport costs in pence per ton per mile</u>

	Running Cost	Standing Cost
Rigid-Chassis Vehicles	$21 \cdot 3796\ x^{-0 \cdot 727}$	$12.106\ x^{-0 \cdot 629}$
Articulated Vehicles	$11 \cdot 885\ x^{-0 \cdot 5}$	$9 \cdot 8175\ x^{-0 \cdot 535}$

3.3. Discussion of the Basic Data

3.3.1. Data-Sources Available

The cost figures used in this thesis were published by Commercial Motor magazine in 1982 with the intention of providing operators with a tool for costing their transport operations. These costs are based on a survey conducted by this trade magazine in conjunction with the Mercedez-Benz Transport Consultancy. Although this data is, at the time of writing, several years out of date, there is every reason to trust that these figures were as accurate as it is reasonable to expect them to be at the time that they were published, so that these cost tables may be accepted as a reliable basis for analysis. Cost data that appears in following sections therefore corresponds to 1982 prices, but these figures may be converted to 1988 levels by simply multiplying them by a factor, as described in Appendix A.

These tables are not the only source of costing information available, since the Freight Transport Association also publishes figures based on a survey of its own, (SEE Table 3.2.). Unfortunately, direct comparison of the two data-sources is difficult, as the F.T.A. categorises vehicles according to Gross Vehicle Weight, whilst Commercial Motor tables differentiate according to carrying-capacity, and, more importantly, the F.T.A.'s tables provide information on only five different sizes of rigid vehicle and three categories of articulated vehicle. Also, whereas Commercial Motor consider vehicles with a specific carrying-capacity, F.T.A. tables publish average figures for vehicles within a certain GVW range, (SEE Table 3.2.).

Despite these differences, graphs of F.T.A. figures, (SEE Figures 3.9.1., 3.9.2. & 3.9.3.), confirm the presence of Economies of Scale, since transport costs certainly increase less than proportionately with increasing vehicle capacity. For the purpose of Figure 3.9., where a range of vehicle sizes is quoted, the largest Gross Vehicle Weight is taken. It should be noted, here, that these figures are in metric tonnes, as opposed to the imperial tons used by Commercial Motor; furthermore, as the F.T.A. publishes standing costs in terms of £ per year, it was again assumed that each vehicle travels 1000 miles per week, and Commercial Motor's assumption that each vehicle is available for 45 weeks in a year was also adopted so that standing cost could be expressed in £ per week.

It is noticeable from Table 3.2. that F.T.A. data omits Rent & Rates and Wages from Standing Cost calculations; this is explained by the fact that "Driver Costs" and general "Administration Costs" are added to vehicle costs separately, to give "Total Costs per year". As figure 3.9.3. shows, the main effect of including these costs is to create a marked difference between the cost of vehicles of 7·5 tonnes GVW and

Table 3.2. Cost figures published by the Freight Transport Association

tonnes gvw	£ per year			TOTAL STANDING COST	p per mile			TOTAL RUNNING COST	TOTAL COST
	V.E.D.	Insurance	Depreciation		Fuel	Tyres	Maintenance		
≤ 3·5 (petrol)	106	142	1161	1409	10·58	0·63	7·66	18·87	5 499
≤ 3·5 (diesel)	105	168	1638	1911	6·58	0·76	4·84	12·18	5 858
7 - 5	130	172	2498	2800	9·52	1·07	13·02	23·61	19 087
9 - 13	370	169	2500	3039	10·86	1·32	13·36	25·54	22 656
13 - 16	992	207	3449	4648	12·53	1·6	9·06	23·19	27 269
≤ 24 (artic)	972	257	3650	4879	16·04	1·21	10·93	28·18	31 143
32·52 (artic)	2450	224	4689	7363	18·56	2·87	11·34	32·77	41 583
38 (artic)	3018	282	5494	8794	19·86	5·03	9·23	34·12	47 351

Figure 3.9.1. Standing Cost per week,
(FTA figures).

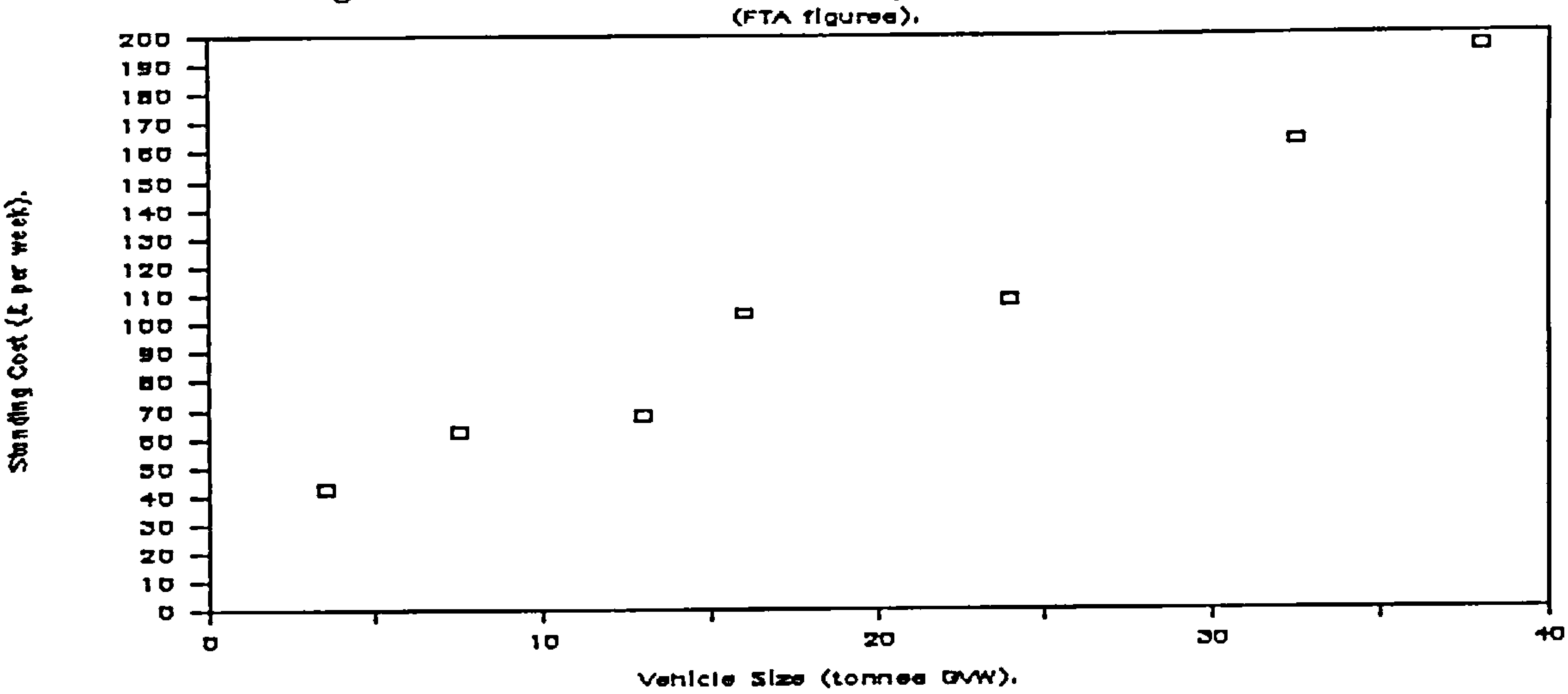


Figure 3.9.2. Running Cost per mile,
(FTA figures).

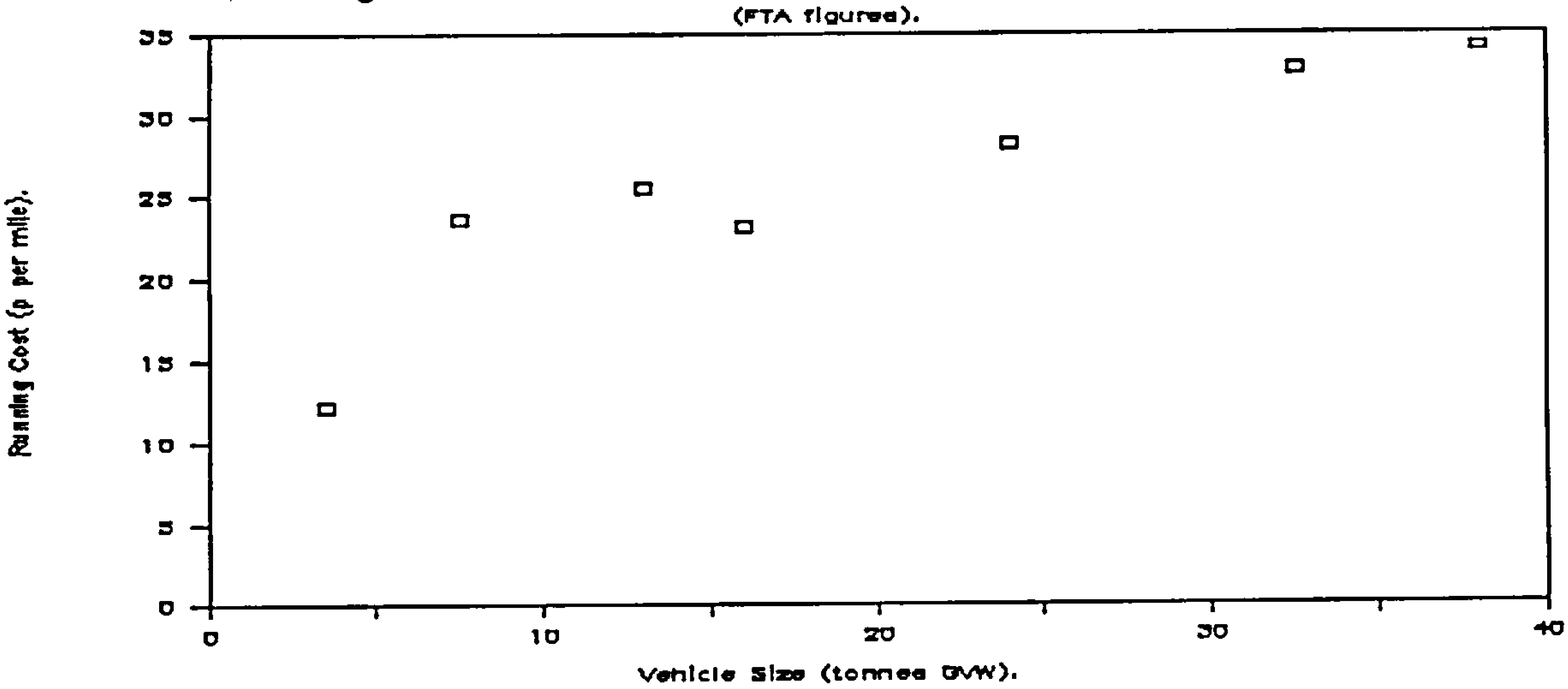
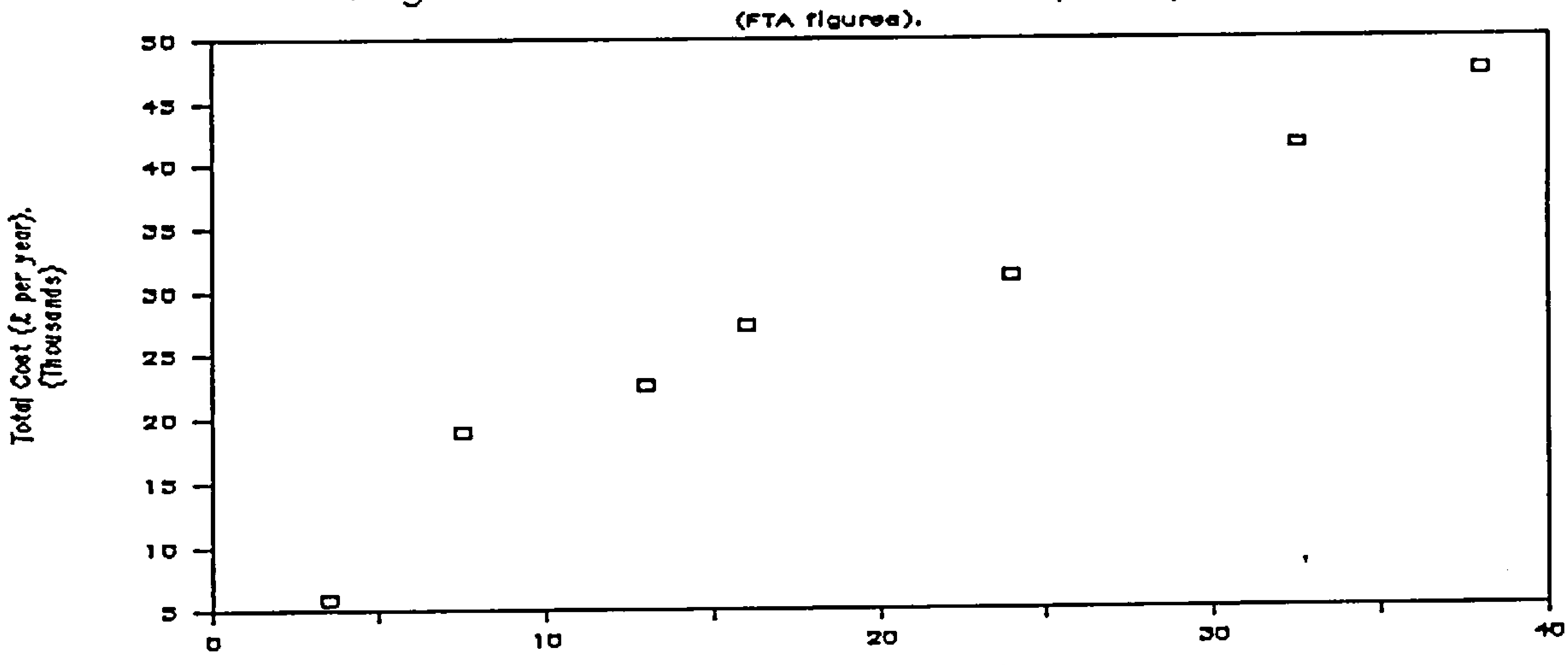


Figure 3.9.3. Total Cost per year,
(FTA figures).



those of 3.5. tonnes GVW or less; this is primarily due to the much lower wage-costs involved with smaller vehicles, which is reemphasised in Figure 3.11.3.. It is also interesting that, whereas Commercial Motor considers depreciation to be a running cost, the F.T.A. includes this factor as a Standing Cost, although this difference does not significantly affect Figure 3.9..

Both data-sources are based on separate surveys of actual operators, so differences in detail will inevitably exist; for the purposes of the rest of the analysis, all costs quoted will have originated from Commercial Motor cost tables published in 1982.

3.3.2. Disaggregation of Basic Data

The Total Cost curve of Figure 3.2. is made up of Running Cost and Standing Cost, whose individual cost per mile curves are shown in Figure 3.10.; the relationship between vehicle size and Standing and Running Cost per ton-mile is illustrated in Figure 3.5. and Figure 3.6. respectively, and has already been referred to in the previous section. Both Figure 3.2. and Figure 3.10. show irregularities in their respective curves, which might be explained by closer examination of the behaviour of the individual costs by which Standing Cost and Running Cost are defined. In the case of Standing Cost, the main reason for discontinuities in the curve is the stepped nature of the structure of licence-fees, wages and insurance charges, with rent & rates and interest, by contrast, rising uniformly with increasing vehicle size, (SEE Figures 3.11.1. to 3.11.5.). Figures for licence costs are here based on the Unladen Weight of vehicles, and NOT on Gross Vehicle Weight and number of axles, which are the criteria on which Vehicle Excise Duty has been calculated since October 1982; Figure 3.11.1. distinguishes three licencing levels for vehicles with a carrying-capacity of 1.5 tons or less, 2 tons to 8 tons and 10 tons or more. Wages and insurance costs have a similar structure, the main cut-off points being at 2,5,6 and 10 tons carrying-capacity, (SEE Figures 3.11.2. and 3.11.3.), and it is the difference between the wages and insurance for a vehicle of 2 tons carrying-capacity and one of 3 tons carrying-capacity that mainly accounts for the discontinuity at this vehicle size in the Standing Cost curve of Figure 3.10.. The structure of wages and insurance charges is also largely responsible for the increase in Standing Costs from a 5 ton vehicle to a 6 ton vehicle.

In all cases, Standing Cost per week is calculated by adding up the total cost of each cost element for a year and then dividing by 45, since it is assumed that each vehicle is operational for this number of weeks in a year, which takes into account drivers' holidays and maintenance time.

Figures 3.12.1. to 3.12.5. also show that most of the

Figure 3.10. Costs per mile,
(rigid-chassis vehicles).

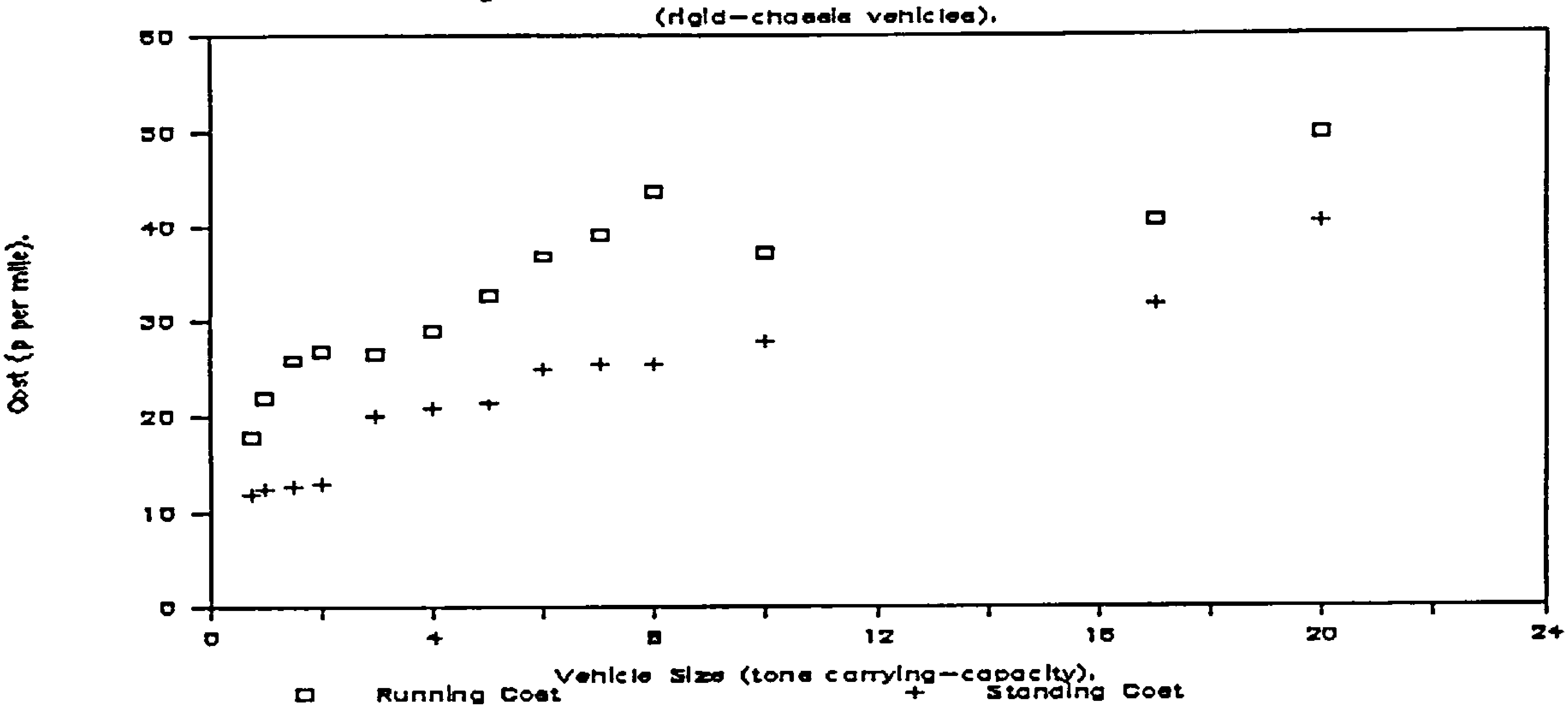


Figure 3.11.1. Licence costs,
(rigid-chassis vehicles).

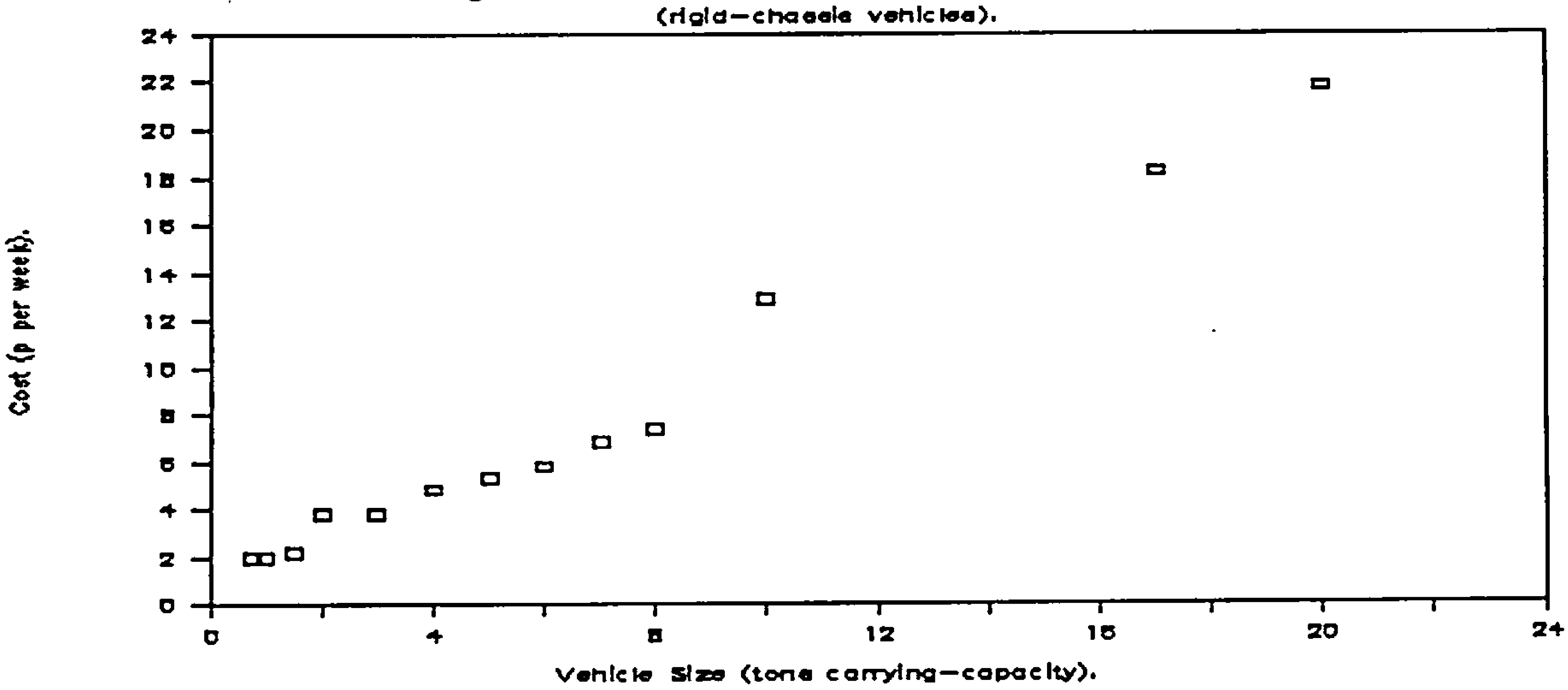


Figure 3.11.2. Insurance costs,
(rigid-chassis vehicles).

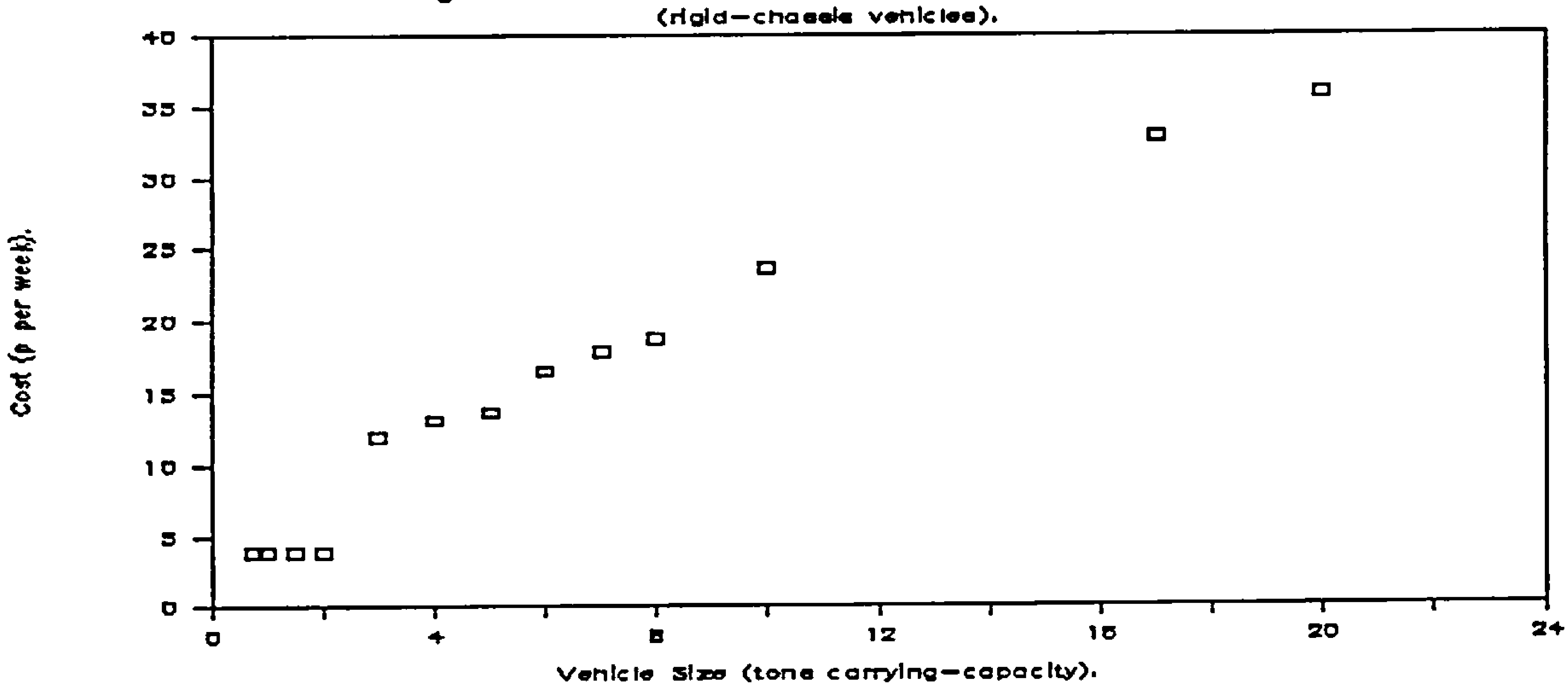


Figure 3.11.3. Wage costs,
(rigid-chassis vehicles).

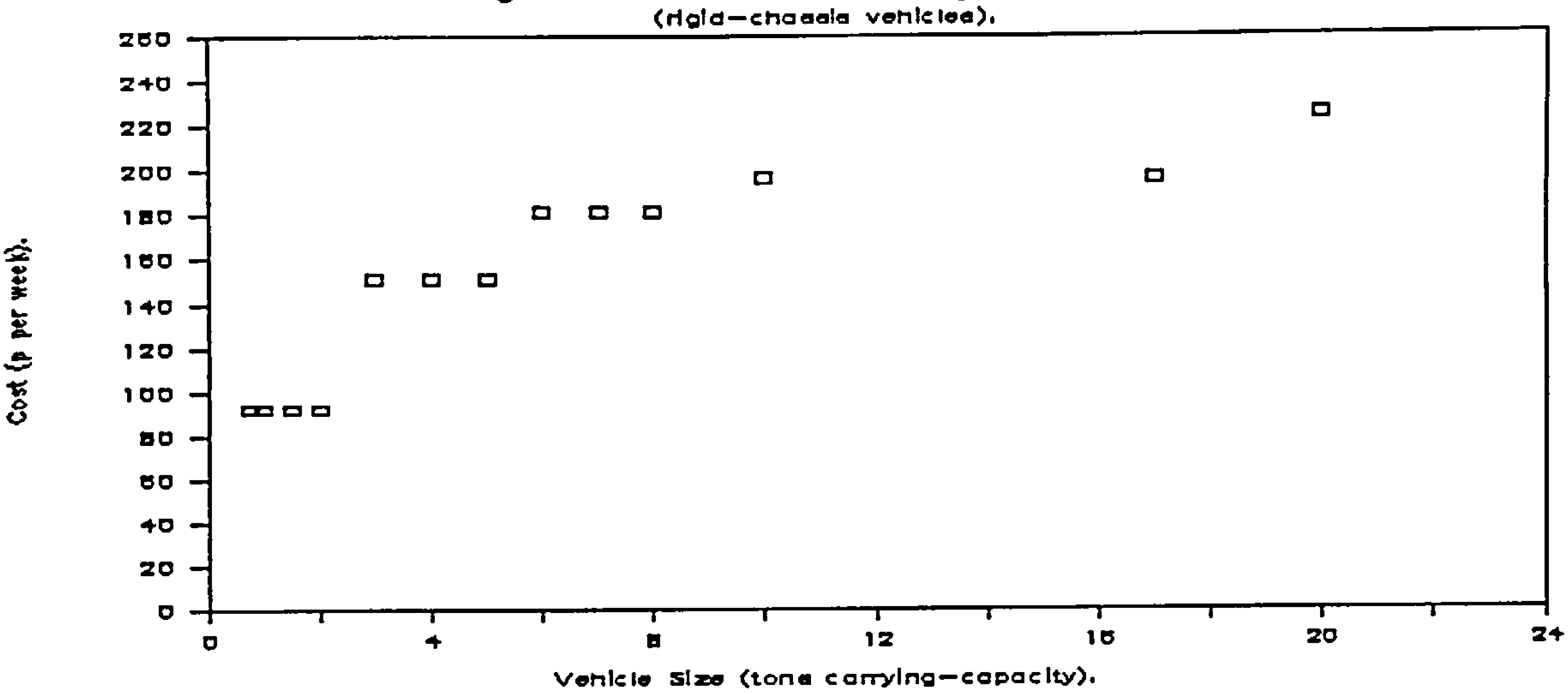


Figure 3.11.4. Rent & rates costs,
(rigid-chassis vehicles).

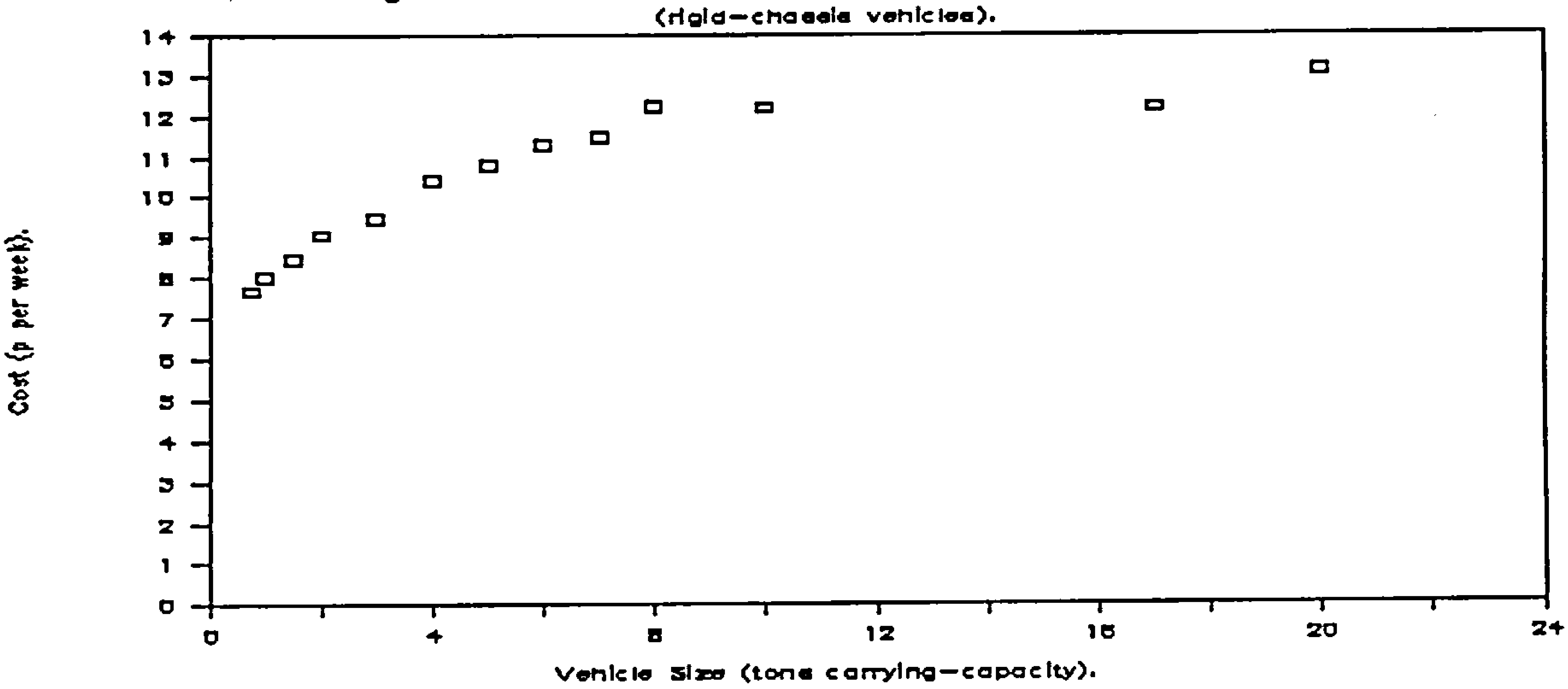


Figure 3.11.5. Interest costs,
(rigid-chassis vehicles).

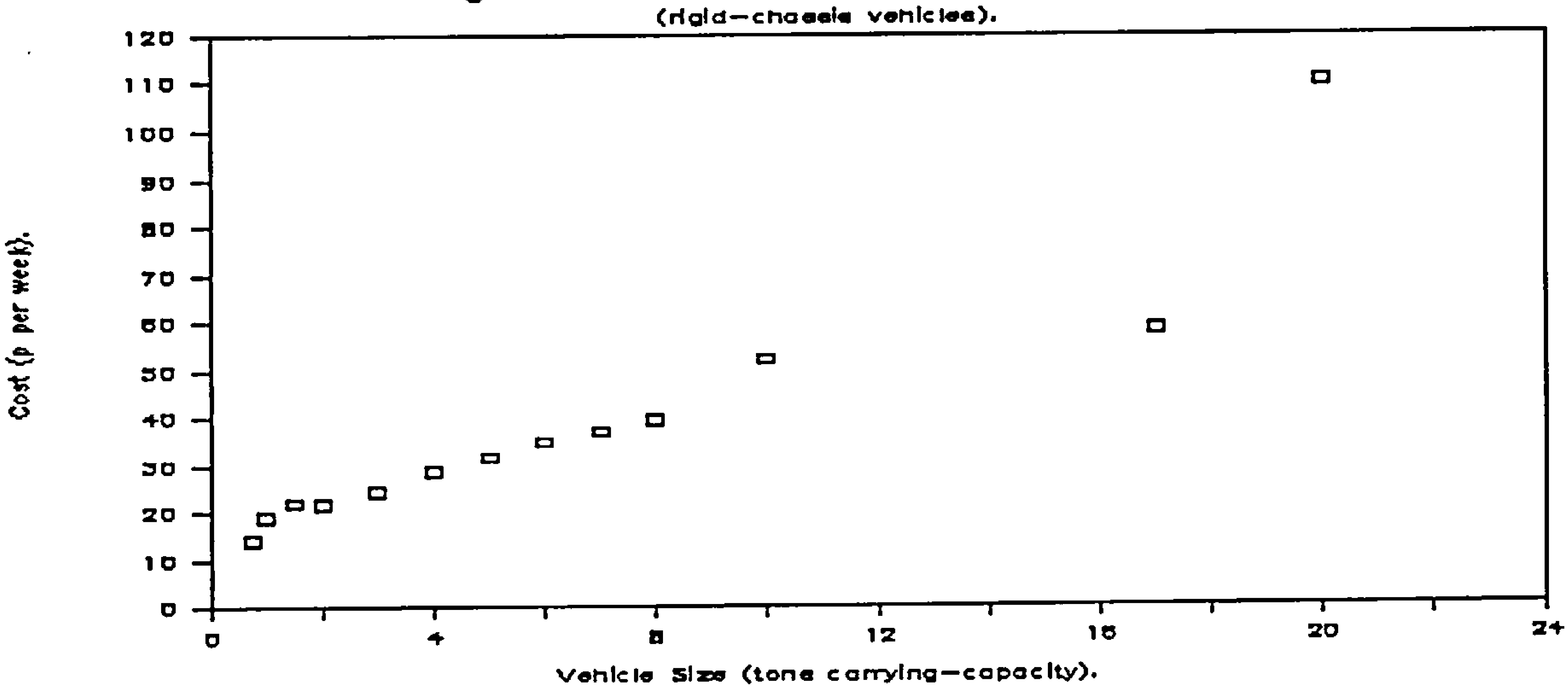


Figure 3.12.1. Fuel costs,
(rigid-chassis vehicles).

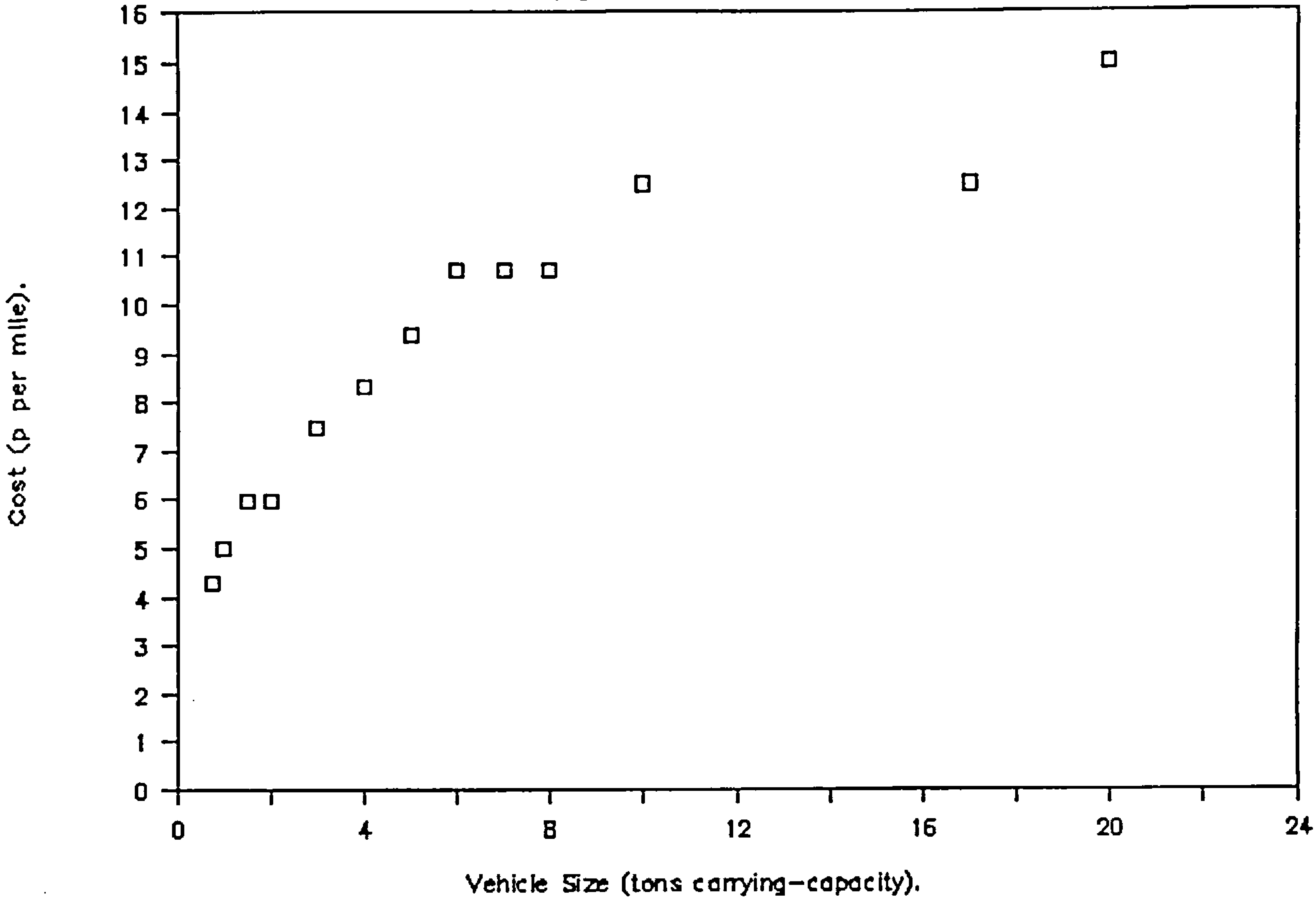


Figure 3.12.2. Maintenance costs,
(rigid-chassis vehicles).

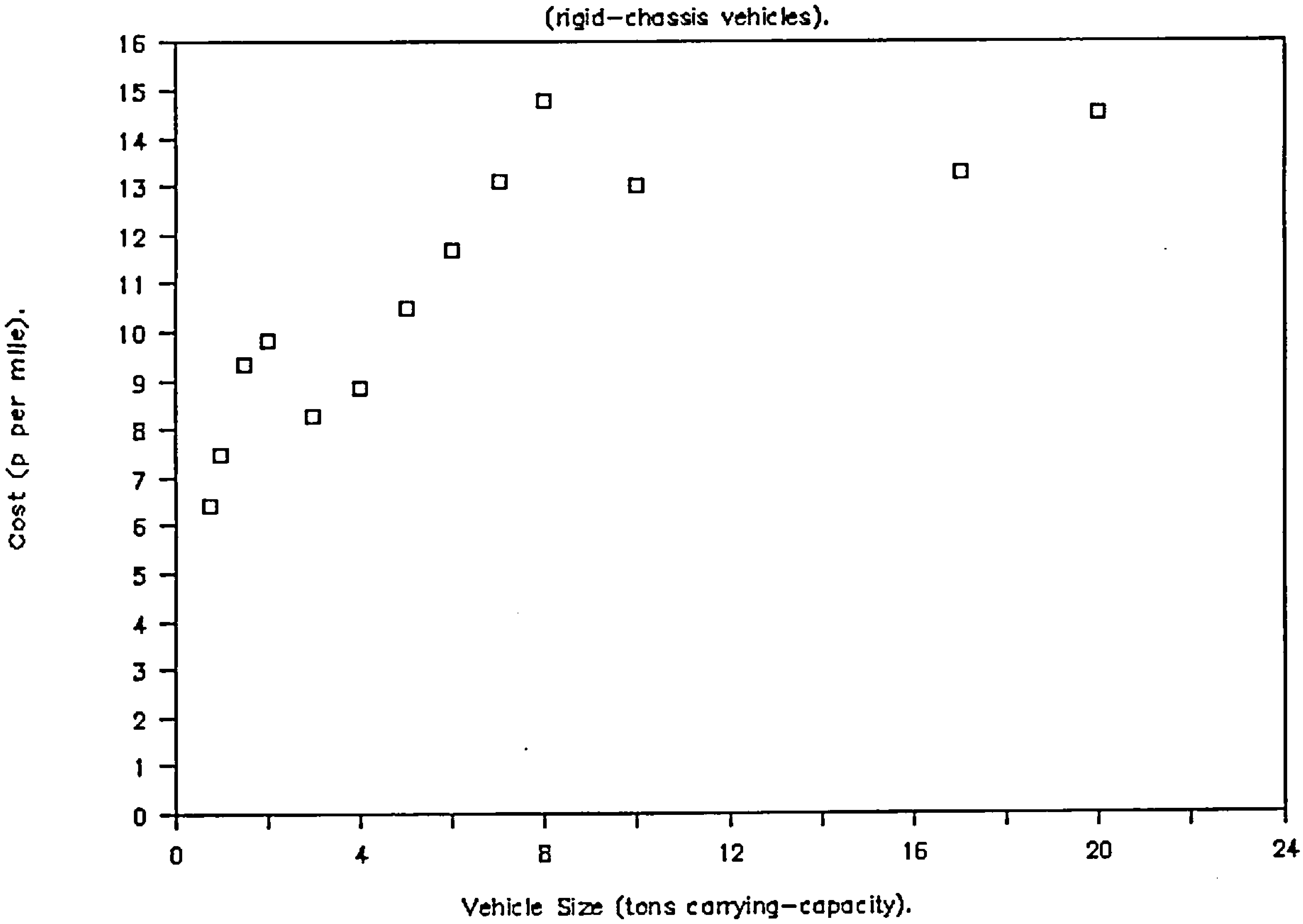


Figure 3.12.3. Lubricant costs,
(rigid-chassis vehicles).

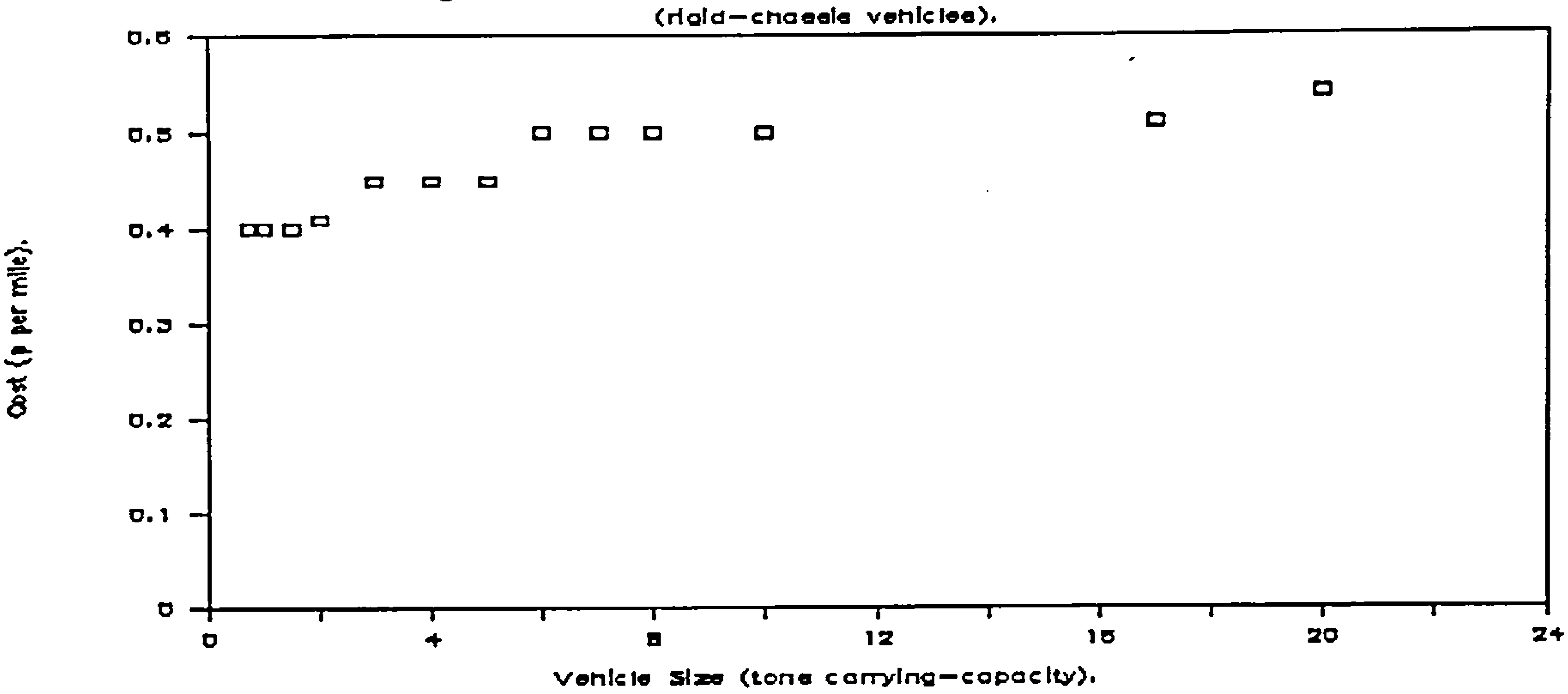


Figure 3.12.4. Tyre costs,
(rigid-chassis vehicles).

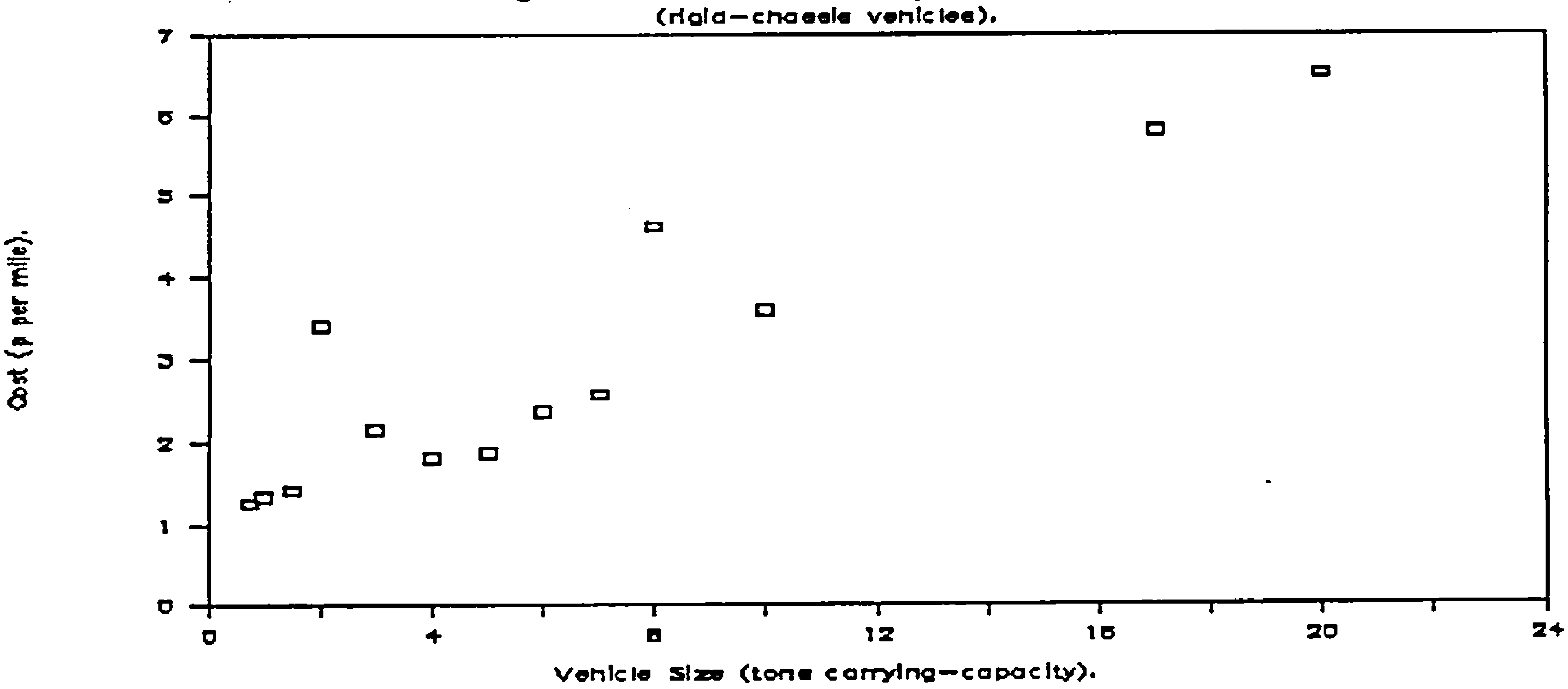
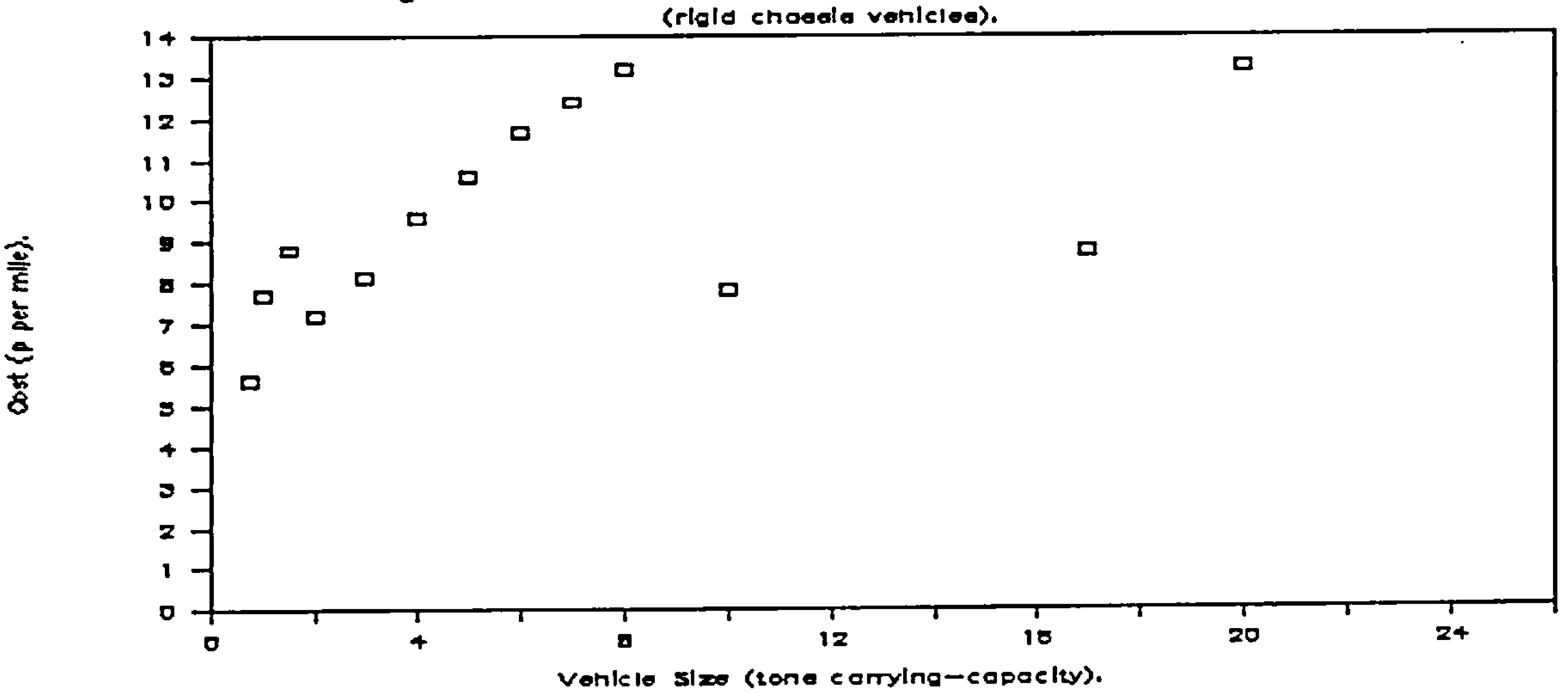


Figure 3.12.5. Depreciation costs,
(rigid chassis vehicles).



components of Running Cost have separate curves for vehicles with carrying-capacities of less than 3 tons, 3 tons to 8 tons, and over 8 tons, and this is reflected in the overall curve for Running Cost shown in Figure 3.10.; the exception is the relatively smooth curve for fuel costs per mile, calculated from a base-price of £1·50 per gallon. For the cost of maintenance, tyres and depreciation, it is noticeable that the unit cost of a 10 ton vehicle is lower than that of an 8 ton vehicle, (SEE Figures 3.12.2., 3.12.4. and 3.12.5.). One explanation of this is that the Commercial Motor tables are based on the assumption that the 8 ton vehicle will have a life of 90,000 miles, whilst the initially slightly more expensive 10 ton vehicle will last for 200,000 miles; this means that depreciation, based on a five-year vehicle life-span with the cost of tyres and the vehicle's residual value subtracted from the list-price, will be less for the 10 ton vehicle. Similarly, the tyres on an 8 ton vehicle are assumed to have a life of 35,000 miles as opposed to 45,000 miles in the case of a 10 ton vehicle; no explanation is given for this particular difference by Commercial Motor, although all running costs are based on a previous empirical survey with the final figure adjusted to account for inflation.

Also, the difference between the Running Cost of a 2 ton and a 3 ton vehicle is partially explained by the fact that a set of new tyres (excluding the spare) for a 2 ton vehicle is assumed to cost £852 as opposed to £540 for a 3 ton vehicle, (SEE Figure 3.12.4.), and depreciation of a 2 ton vehicle with a life of 90,000 miles is substantially less than that of a 1.5 ton vehicle with a life of only 75,000 miles, (SEE Figure 3.12.5.).

In contrast, the cost of lubricants is virtually constant, regardless of vehicle size, and is in any case only a fraction of overall Running Cost, ranging from 0.4p per mile for a 15cwt van to 0.54p per mile for a 20 ton lorry.

Despite these details of the behaviour of individual cost components in response to changes in vehicle size, the only major kink in what is otherwise a relatively smooth Total Cost per mile curve, (SEE Figure 3.2.), occurs between vehicles of 8 tons and 10 tons carrying-capacity.

Having disaggregated the raw cost data in such a way, the next logical step is to try to establish which, if any, of the cost components discussed above are mainly responsible for the Economies of Scale observed. This might be done by calculating the percentage of Total Cost attributable to each cost component for each vehicle size; these percentages are presented in Table 3.3.. There are very few trends in evidence in this table, and only the percentage of Total Cost accounted for by rent & rates declines consistently as vehicle size increases; also, insurance and licensing costs, both fixed costs, have a tendency to increase in terms of their percentage share of Total Cost as vehicle size increases.

Table 3.3. The percentage of Total Cost attributable to each cost component

Vehicle Size (tons)	Wages	Insurance	Rent	Licences	Interest	Tyres	Maintenance	Fuel	Depreciation
0·75	30·7	1·28	2·56	0·67	4·71	4·2	21·4	14·3	18·8
1	26·75	1·11	2·33	0·58	5·60	3·9	21·7	14·5	22·4
1·5	23·78	0·99	2·18	0·57	5·69	3·6	23·9	15·5	22·7
2	23·08	0·96	2·27	0·97	5·42	8·5	24·6	15·0	18·1
3	32·47	2·56	2·03	0·83	5·25	4·6	17·7	16·1	17·5
4	30·36	2·63	2·08	0·97	5·77	3·6	17·8	16·7	19·2
5	28·01	2·52	2·00	0·99	5·88	3·4	19·4	17·3	19·6
6	29·33	2·67	1·82	0·94	5·65	3·8	18·9	17·3	18·8
7	28·03	2·77	1·77	1·06	5·74	4·0	20·2	16·5	19·1
8	26·17	2·71	1·76	1·07	5·71	6·7	21·3	15·4	19·0
10	30·06	3·61	1·87	1·97	7·99	5·5	19·9	19·1	12·0
17	27·03	4·52	1·68	2·31	8·04	7·9	18·3	17·2	12·0
20	24·87	3·97	1·45	2·41	12·20	7·2	16·1	16·6	14·6

Figure 3.13.1. Wages costs per ton,
(rigid chassis vehicles).

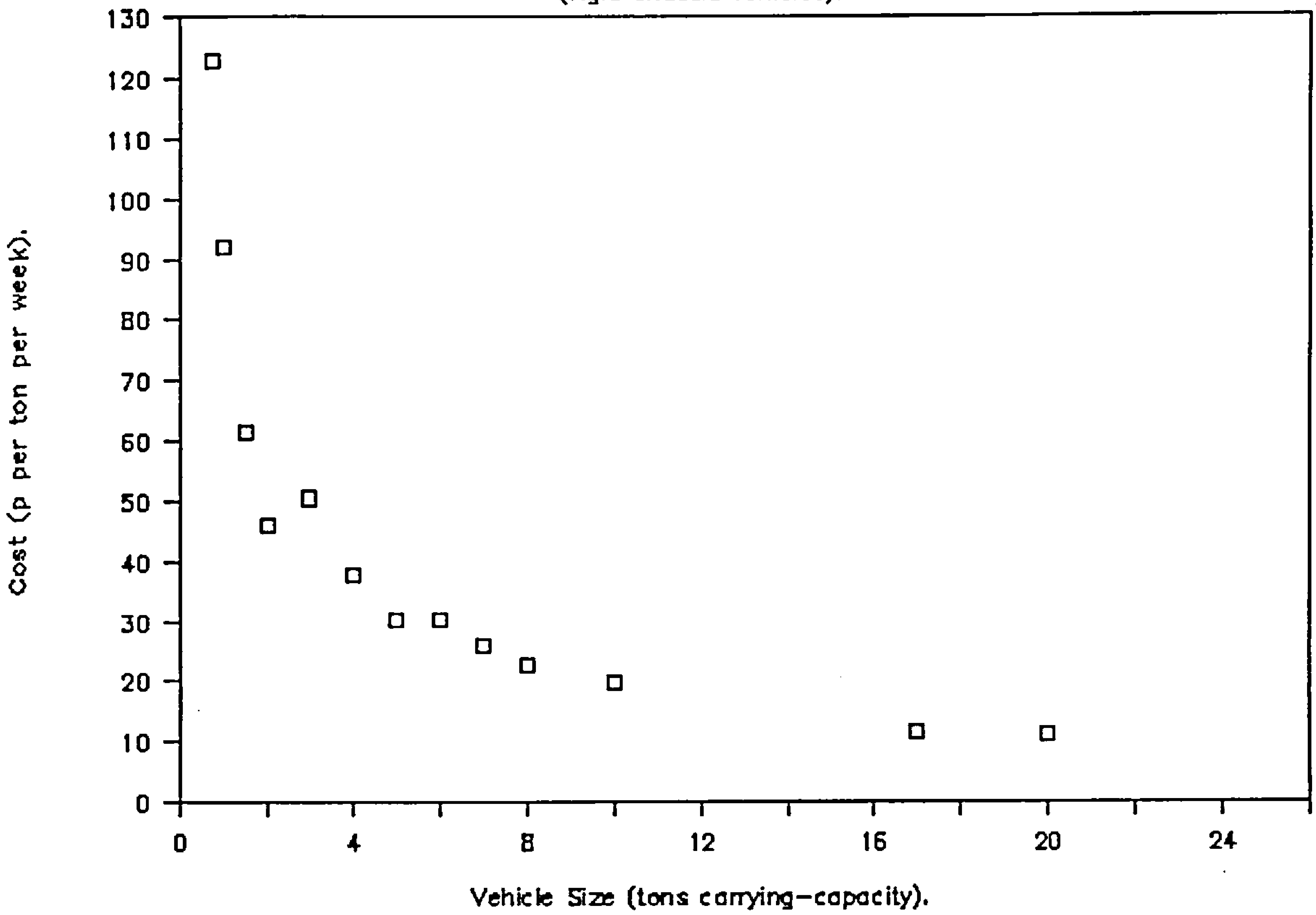


Figure 3.13.2. Maintenance cost per ton
(rigid chassis vehicles).

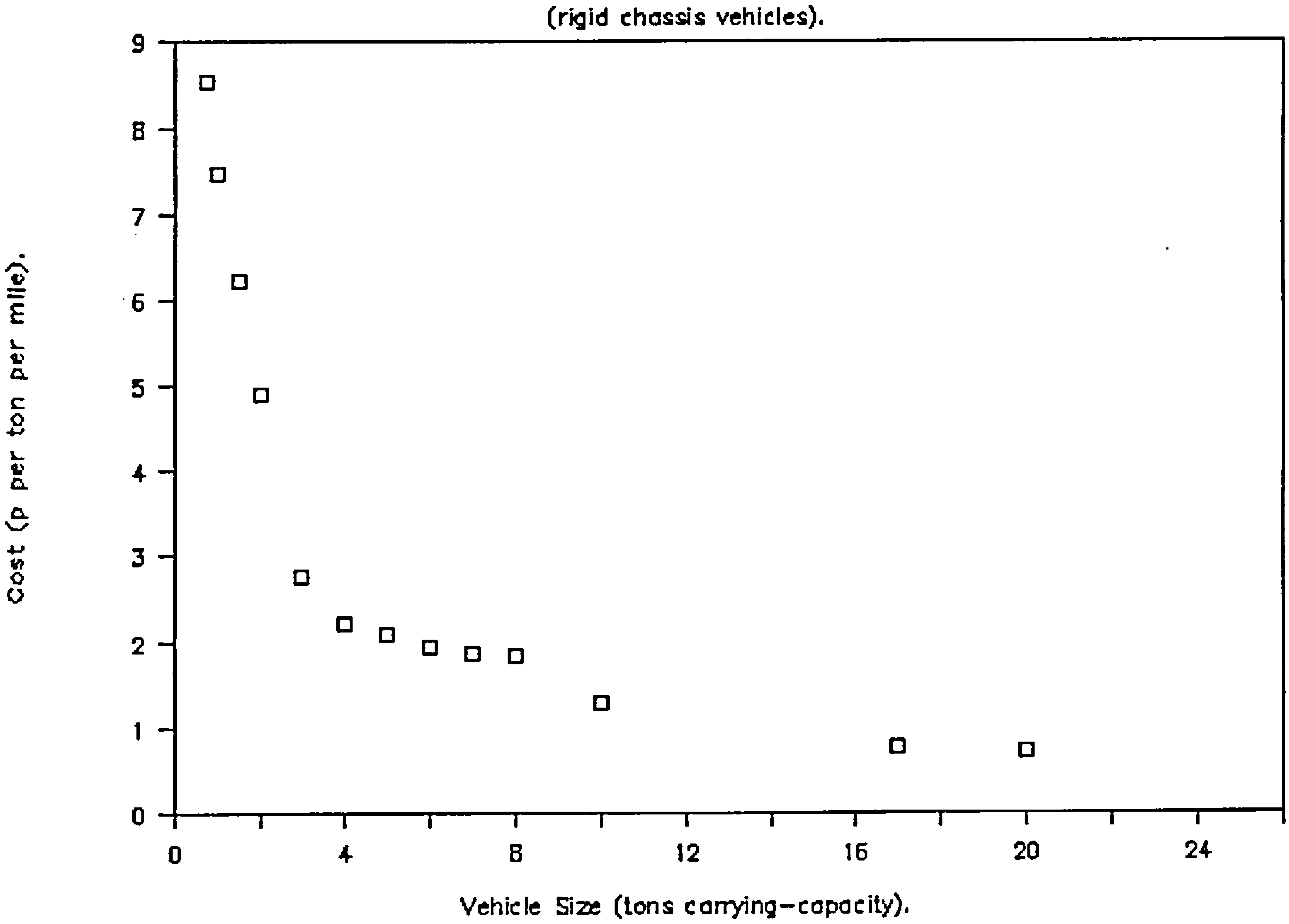


Figure 3.13.3. Fuel costs per ton,
(rigid chassis vehicles).

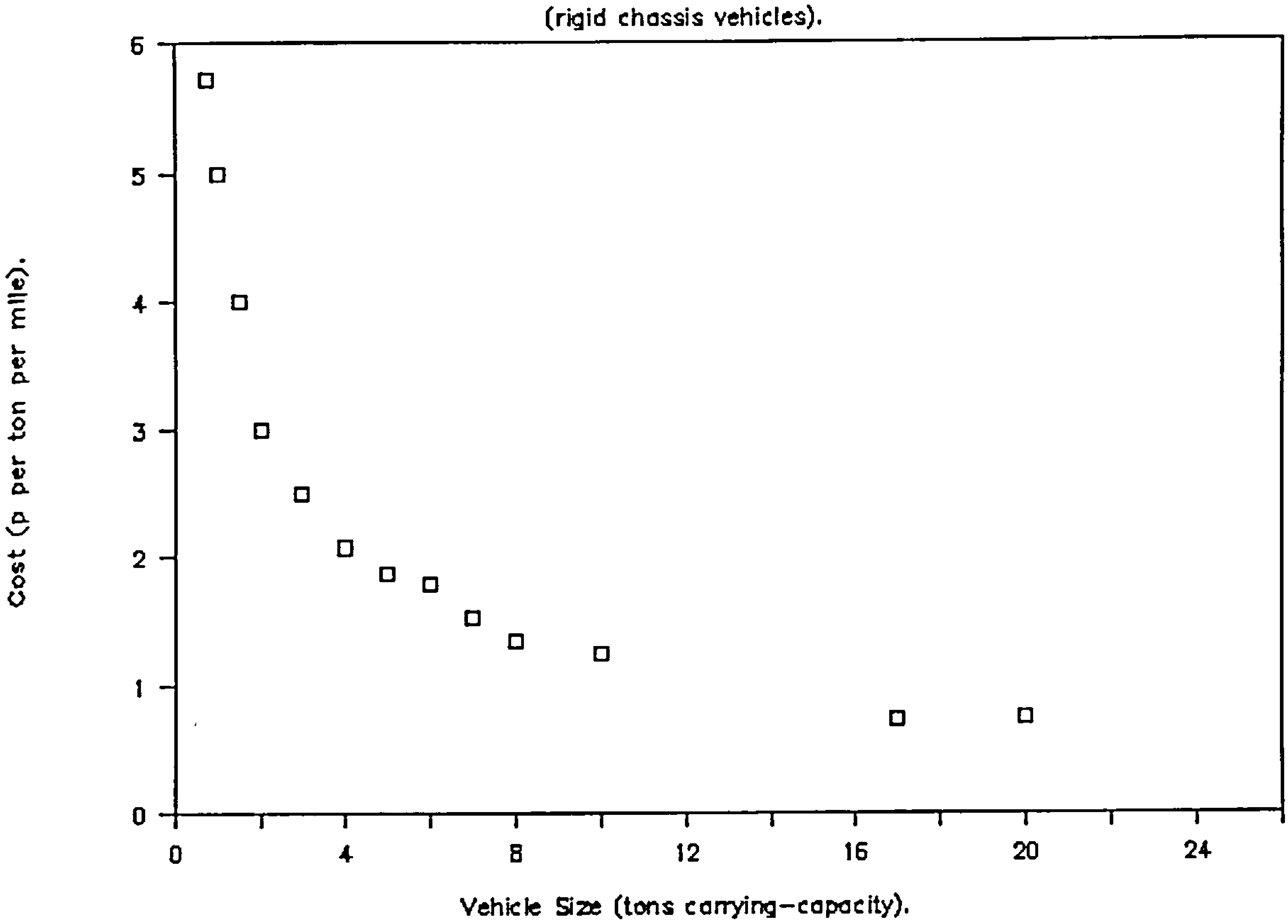
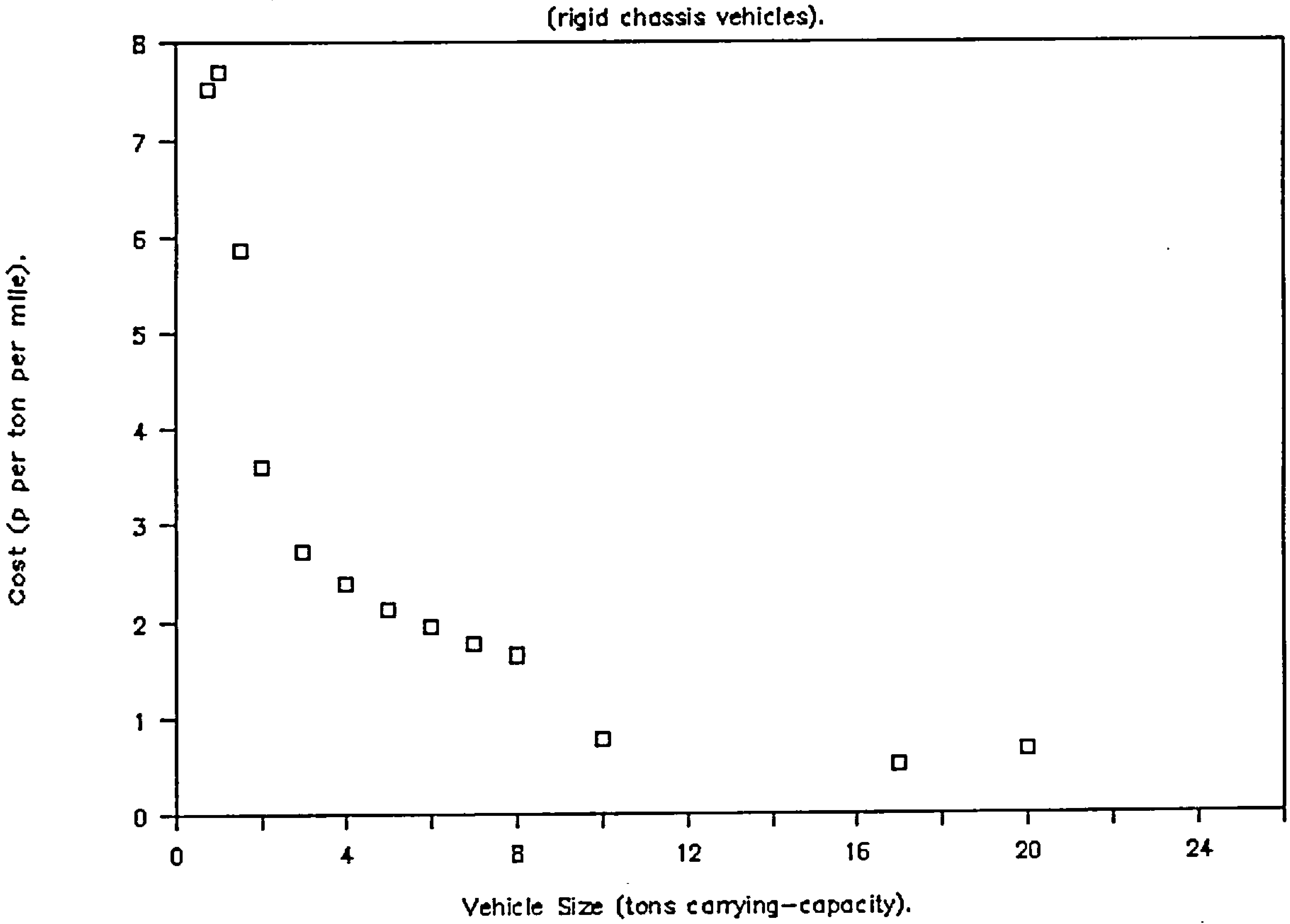
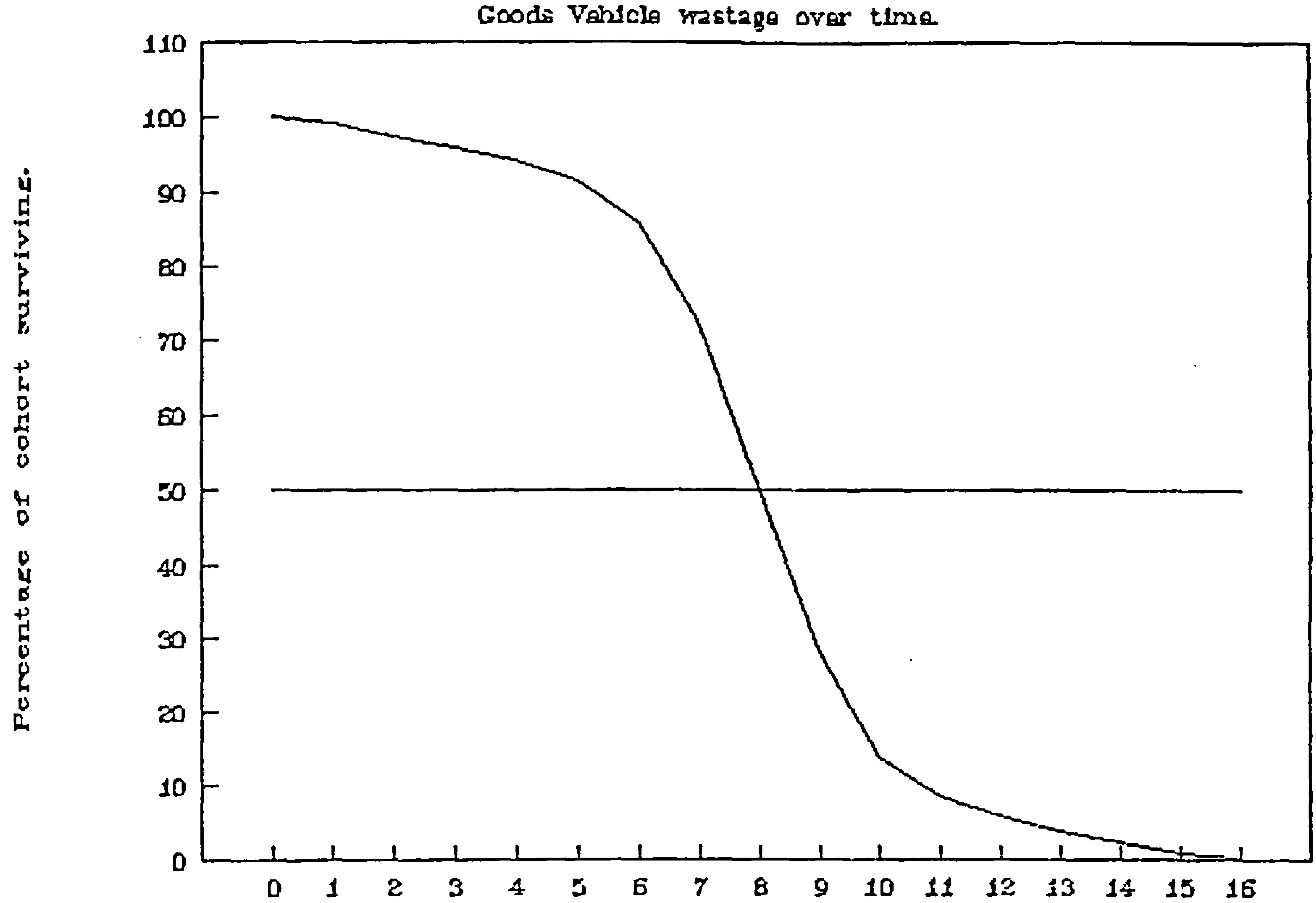


Figure 3.13.4.Depreciation cost per ton
(rigid chassis vehicles).



However, these three components together account for only about 4% to 7% of Total Cost, and so have relatively little overall effect. The behaviour of Wage Costs, the largest component of Total Cost, is worthy of note, since their percentage contribution to Total Cost decreases within certain ranges of vehicle size for which wages are constant, (SEE Figure 3.11.3.), but then increase markedly when a higher wage-bracket is reached, (eg. a driver's weekly wage rises from £92·15 to £151·53 when vehicle carrying-capacity increases from 2 tons to 3 tons; SEE Appendix A.). The overall effect, as Table 3.3. shows, is for wages as a percentage of Total Cost to fluctuate between 23·08% and 30·73% for all sizes of rigid-chassis vehicles. As wage-costs make up such a substantial proportion of Total Cost, it is useful to plot the relationship between the unit cost of wages - measured in pence per mile per ton of carrying-capacity, and assuming that a vehicle travels 1000 miles per week, (SEE Section 3.4.) - and vehicle-size, (also measured in terms of tons carrying-capacity). Figure 3.13.1. reveals that the shape of the resulting distribution is, not surprisingly, very similar to that of Total Cost per ton-mile, as shown in Figure 3.7.. Figures 3.13.2. to 3.13.4. show similar curves for the three largest components of Running Costs - maintenance, fuel and depreciation respectively.

Figure 3.14. Assumed rate of Heavy
Goods Vehicle wastage over time.



3.3.3. The Sensitivity of Cost Functions to the Assumptions Underlying the Data

Having derived cost-functions based on cost data published by Commercial Motor, and having descibed the various components that go to make up Running Cost and Standing Cost, it is now necessary to look beyond the raw data to consider some of the assumptions and methods of calculation on which this information is founded. Table 3.3. has shown that depreciation, wages, fuel and maintenance are the largest components of cost, together accounting for over 90% of Total Cost, and these will be dealt with in turn.

(a) Depreciation and interest are considered separately by Commercial Motor. A crude method of straight-line depreciation is used, and a life-span of 5 years for all vehicles is adopted. Depreciation per mile is calculated by simply dividing the list price of the vehicle, (less the cost of tyres and the vehicle's residual value), by an assumed mileage life; therefore, for a 0.75-ton van costing £4240, which is taken to have a life of 75000 miles, the cost of depreciation is 5·65p/mile.

Notwithstanding the fact that a more accurate method of depreciation might have been used, it is the assumption that each vehicle has a life of 5 years that is most significant. This assumption may be tested by consulting consecutive editions of the Department of Transport's Transport Statistics Great Britain publication, (3). This source contains information on the total number of Heavy Goods Vehicles Licensed in the U.K. on the 1st of September each year. These figures are disaggregated in terms of vehicles' year of first registration. In other words, it is possible to calculate the number of "survivors" from each vehicle-cohort each year. Conversely, the number of vehicles in each cohort that do not survive from one year to the next can also be deduced. It is reasonable to assume, here, that the mean life-span of vehicles coincides with the median of this distribution, so that, if the percentage of HGV's surviving from each cohort is plotted against time, an S-shaped curve will be produced whose mid-point coincides with the point at which the number of vehicle "deaths" is at a peak, (SEE Figure 3.14.). This means that the mid-point in this curve, the time after which exactly 50% of the cohort remains, may be used as a measure of the average life-span of the cohort's vehicles.

The data for all HGV's registered in 1981, for example, are shown in Table 3.4.. These figures show a general increase in vehicle "deaths" over time, although 57.5% of the cohort

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(3) Transport Statistics Great Britain (Department of Transport, September 1985 to September 1990).

Table 3.4. Wastage of HGV's first registered in 1981

Years after first Registration	1	2	3	4	5	6	7	8
Number Remaining	38.7	38.3	37.4	35.8	33.2	30.4	26.6	22.3
Percentage Remaining	100	99.0	96.6	92.5	85.8	78.6	68.7	57.6

remained for an eighth year,(after which no further data are yet available). It is therefore necessary to extrapolate from incomplete curves such as this, in order to predict the number of years after first registration at which 50% of the cohort has failed to survive. The curves for vehicles first registered between 1981 and 1984, inclusive, are shown in Figure 3.15.; Figures 3.16.1. and 3.16.2. show the respective curves for vehicles of more than 20 tons gross vehicle weight and for those of 20 tons g.v.w. or less.

Despite all these curves being incomplete, it appears from these graphs that 50% of each cohort will have disappeared from the record by year 8. The evidence of Figures 3.16.1. and 3.16.2., however, suggests that vehicles of over 20 tonnes g.v.w. have a slightly longer life than smaller vehicles, but, for the purpose of this exercise, it will be assumed from these data that the average life-span of a Heavy Goods Vehicle is 8 years.

Mileage life assumptions made by Commercial Motor may also be contrasted with Department of Transport statistics. This time, weight categories used by the information source (4) are rigid-chassis vehicles with a g.v.w. of less than 7.5. tons, and "rigids" with a G.V.W. of 7.5 to 17 tons. The publication reveals that, in 1985, 146707 vehicles in the lower weight category covered 2952 million kilometres; the distance travelled per vehicle is therefore 20122km for the year, (or 12503 miles). If a vehicle lasts for 8 years, then its mileage life is 100029 miles. This compares with Commercial Motor's assumptions of mileage life being 75000 to 90000 miles for vehicles in this weight category. Similarly, according to the DoT source, 156560 vehicles in the higher weight category travelled 5066 million kilometres in 1985. Using the same logic, the mileage life of these vehicles may be estimated at 160855 miles, with Commercial Motor's assumptions ranging from 90000 to 200000 miles.

The way in which depreciation is calculated may also be criticised. Adopting the philosophy that depreciation is

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(4) JOHNSON, F., The Transport of Goods by road in Great Britain 1985. (Department of Transport Statistics Bulletin (86) 23, Aug. 1986).

Figure 3.15. HGV wastage over time,

(all vehicles).

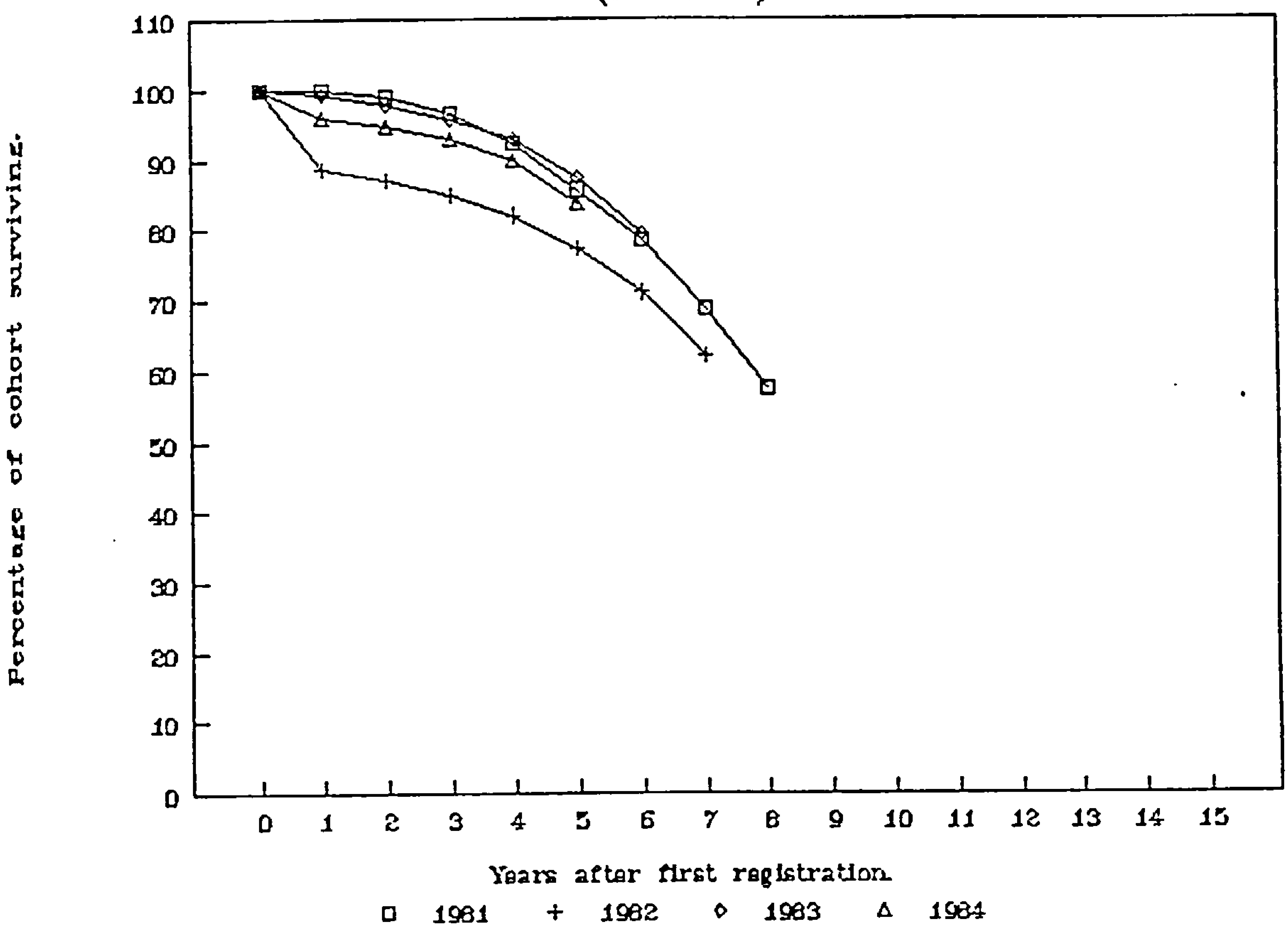


Figure 3.16.1. HGV wastage over time,

(vehicles over 20 tons g.v.w.).

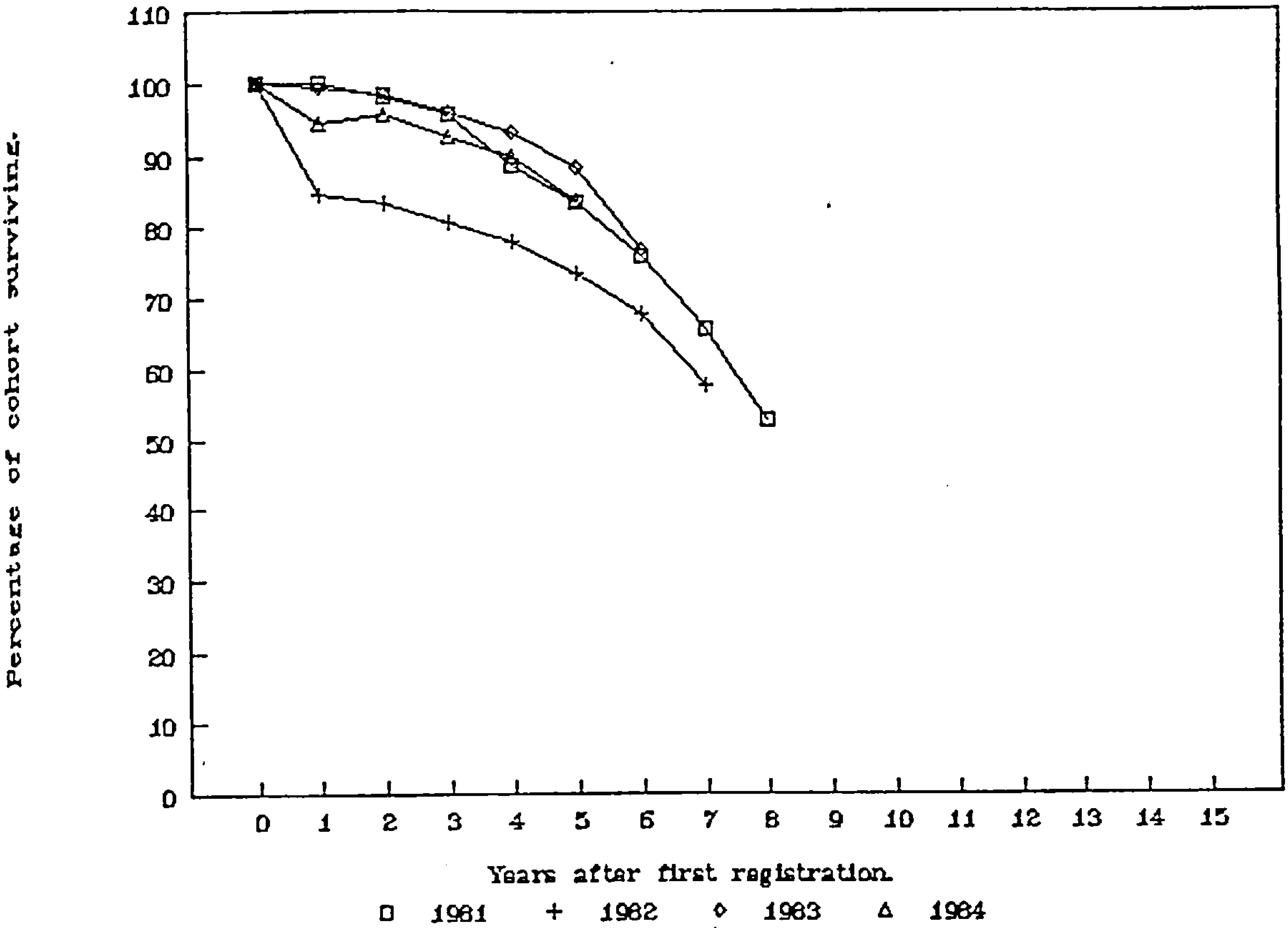
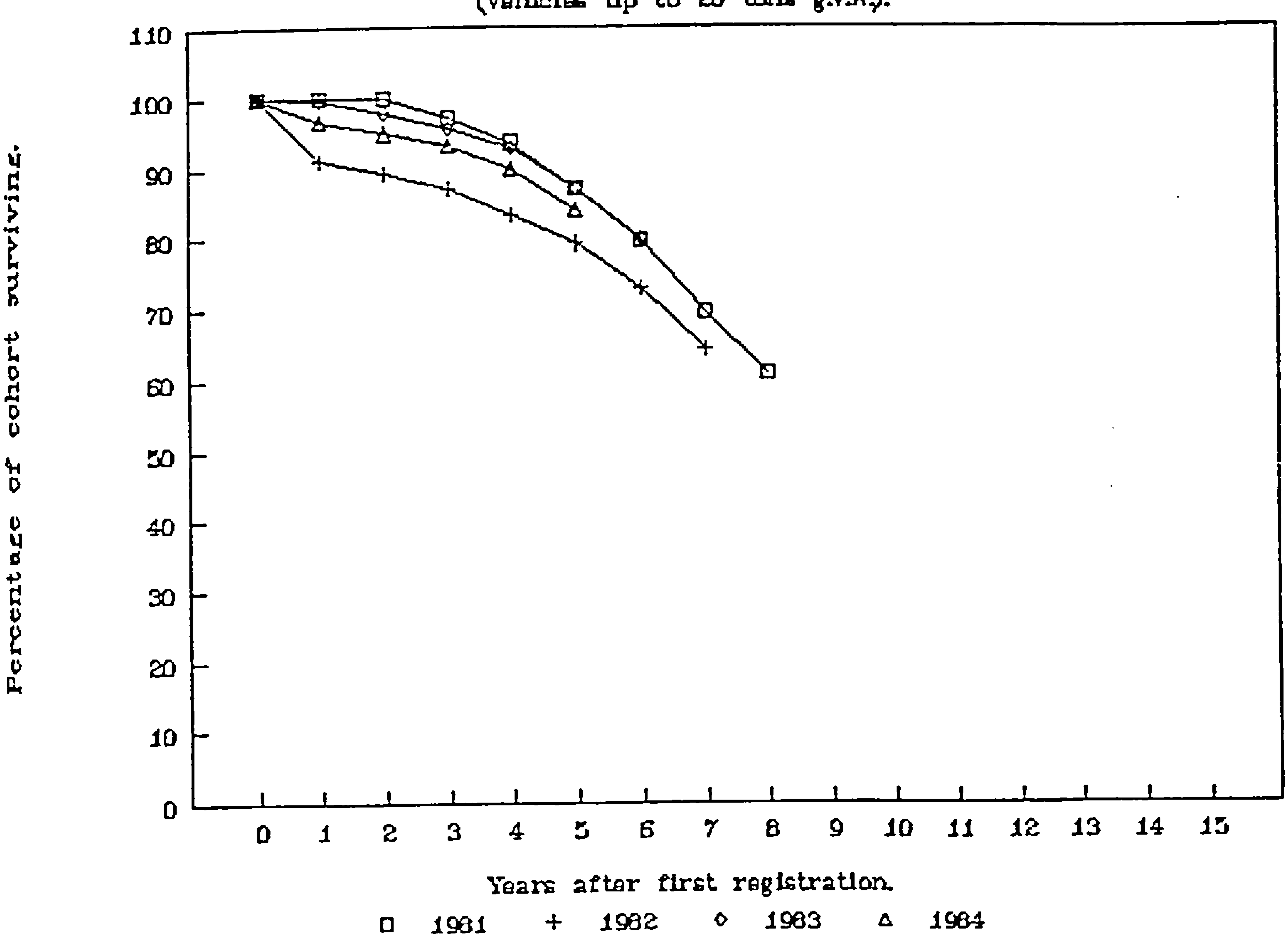


Figure 3.16.2. HGV wastage over time,

(vehicles up to 20 tons g.v.w.).



equivalent to the amount of money that must be put aside each year in order to purchase a replacement item of capital, a straight-line method of calculation is used, with depreciation cost ultimately expressed in pence per mile. Interest is then treated separately. This is regarded as a standing cost, and is simply 15% of the vehicle's list price divided by the number of weeks in the year. Interest is therefore assumed by Commercial Motor to be constant throughout the life of the vehicle. A more realistic alternative might be to calculate the depreciation and interest associated with a vehicle simultaneously. This may be done by finding the "Average Annual Equivalent Value" of the sum of money that is required for purchasing the vehicle; this measure may be calculated by means of a standard formula. For example, in the case of a 0.75-ton van, the average annual equivalent value of the £4240 purchase price, at 15% interest, is £944.88 per year, (using the aforementioned assumption of an 8-year life-span). This is equivalent to £21.00 per week. Using also the above estimate of a 100029 mile life of such a vehicle, over an 8-year life-span, this works out as 7.56p per mile for depreciation and interest combined. These figures may be compared with those published by Commercial Motor for 1982 of £14.13 per week interest and 5.65p per mile depreciation. Using this publication's own assumption about a 0.75-ton van's life-span of 75000 miles and 5 years, whether depreciation and interest combined are

Interest	= £14.13 per week	or	4.24p per mile
Depreciation	= £18.83 per week	or	5.65p per mile
TOTAL	= £32.96 per week	or	9.89p per mile

regarded as a Standing Cost or a Running Cost, therefore, the Commercial Motor Figures are considerably higher than the alternative figures, based on the same initial outlay, that are devised in this section. This discrepancy may certainly be attributed to a large extent to the fact that the former cost estimates are based on the assumption of a 5-year vehicle life.

Alternative estimates of depreciation/interest cost may be made for the entire range of vehicle sizes - these estimates are shown in Figure 3.17.1., expressed in terms of cost per ton-mile. Figure 3.17.2. compares this new set of figures with Commercial Motor's figures for depreciation and interest combined. The latter shows that there is little difference overall in the two curves produced; Commercial Motor's estimates tend to be substantially higher only for the vehicles with carrying-capacities of 0.75 to 1.5 tons and of 6 to 8 tons. For 10-, 17- and 20-ton vehicles, Commercial Motor Figures are actually lower, mainly because of the high mileage lives that this publication assumes for these vehicles. (These are 200000 miles for 10-ton and 17-ton vehicles, and 250000 miles for 20-ton vehicles).

Similarly, the revised estimates may be tested for their sensitivity to changes in interest rates and vehicle

Figure 3.17.1. "Depreciation/interest",
(alternative estimates).

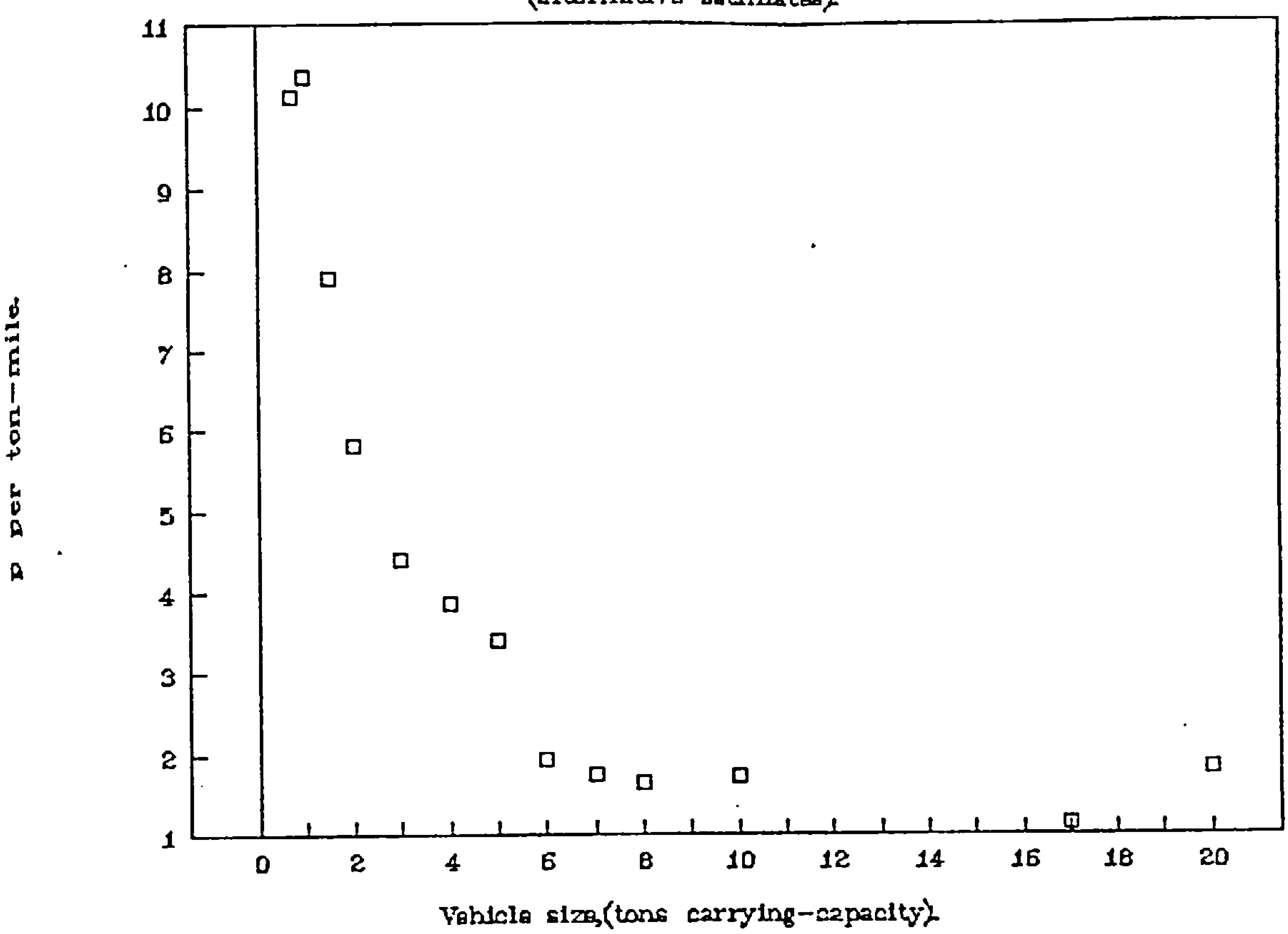
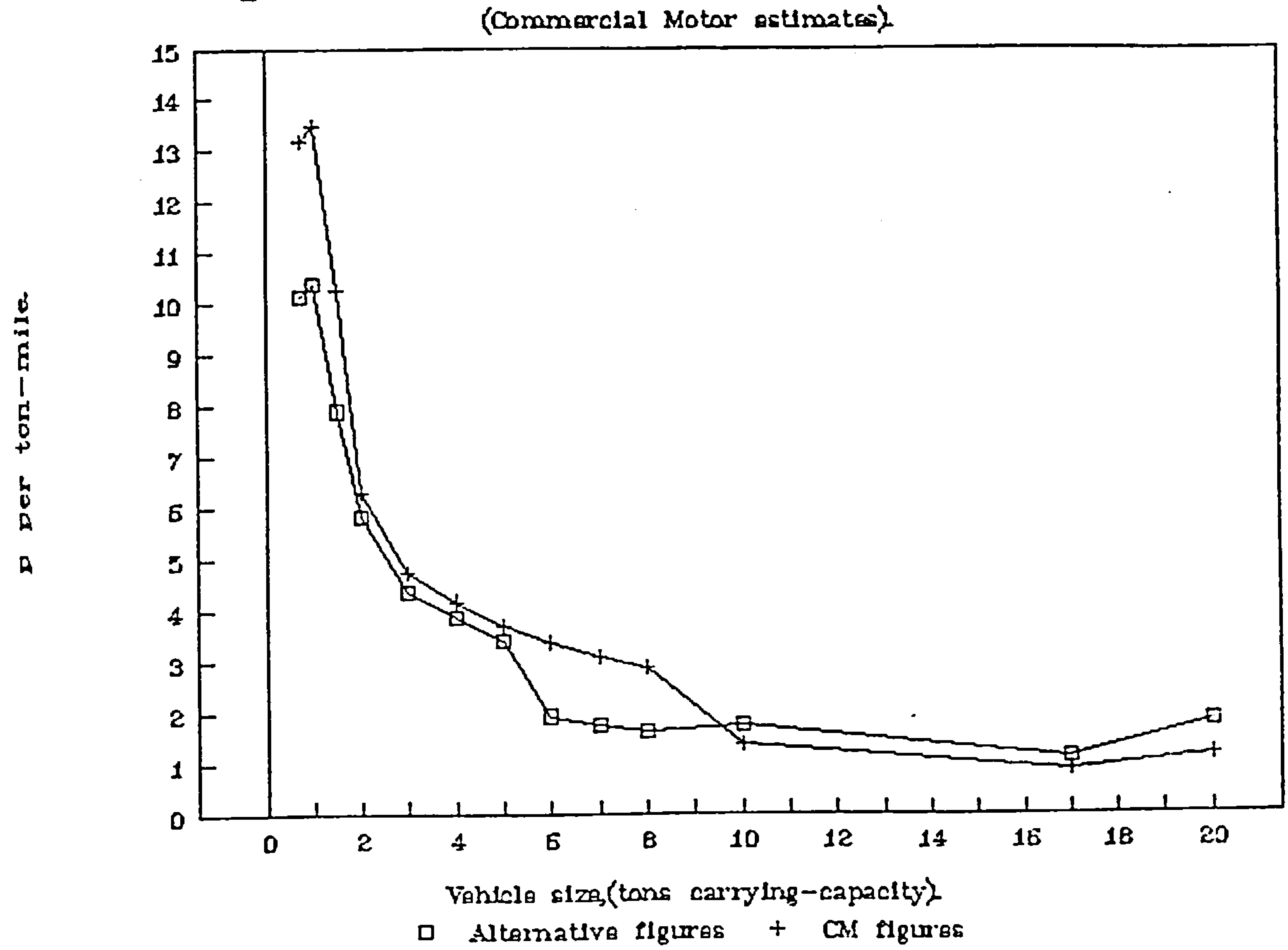


Figure 3.17.2. "Depreciation/interest",
(Commercial Motor estimates).



life-spans. In the above calculations, the rate of interest has been taken to be 15%, because this is a factor that can fluctuate considerably, and Figure 3.18. compares depreciation/interest per mile curves using interest rates of 10%, 15% and 20%. There is little difference in the shape of these curves, but a fluctuation of just 5% in interest rate makes a difference of 2p to 3p per mile for most sizes of vehicle. Because the assumed vehicle life of 8 years is only an approximation, and because this will in any case vary a great deal between vehicles, a similar exercise was carried out using vehicle life estimates of 7 years and 9 years, (mileage was held constant and the interest rate was fixed at 15%). The three resulting curves are shown in Figure 3.19., and the impact of altering the vehicle life-span by 1 year is clearly less than that produced by changing the interest rate by 5%.

(b) Wage Cost per week estimates appearing in Commercial Motor tables are inclusive of National Insurance charges, employer's liability insurance, holiday and subsistence allowances and pension fund payments, but exclude clothing and laundry allowances, productivity bonuses and travelling-time payments. Obviously, these estimates are national averages, and do not reflect the fact that wage levels vary considerably geographically. This variation is highlighted by alternative costings published by the Freight Transport Association (5), which are based on voluntary returns from a cross-section of contributors operating throughout the U.K. According to this sample, basic wages for HGV drivers are 5% above the national average, gross wages are 12% above and the number of hours worked during the week is less than average; gross pay per hour is thus 20% above the national average. Conversely, basic pay and gross pay per hour in Scotland are respectively 8% and 13% below the national average. Added to these regional differences is the fact that earnings in the main centres of population in each region are usually greater than in rural areas.

The FTA's costings confirm that wage costs increase with vehicle size according to a stepped progression that relates to the class of HGV licence that is required to drive different types of vehicle, (the stepped structure used by Commercial Motor is illustrated in Figure 3.11.3.). The FTA's vehicle size categories in this publication are as follows,

Light Rigid Vehicles - No HGV licence required
(less than 7.5 tonnes g.v.w.)

Medium Rigid Vehicles - Class 3 HGV licence required
(7.5 to 17 tonnes g.v.w.)

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(5) The Managers Guide to Distribution Costs, 1990. (Freight Transport Association, Jan. 1990).

Figure 3.18. The effect of changing interest rates

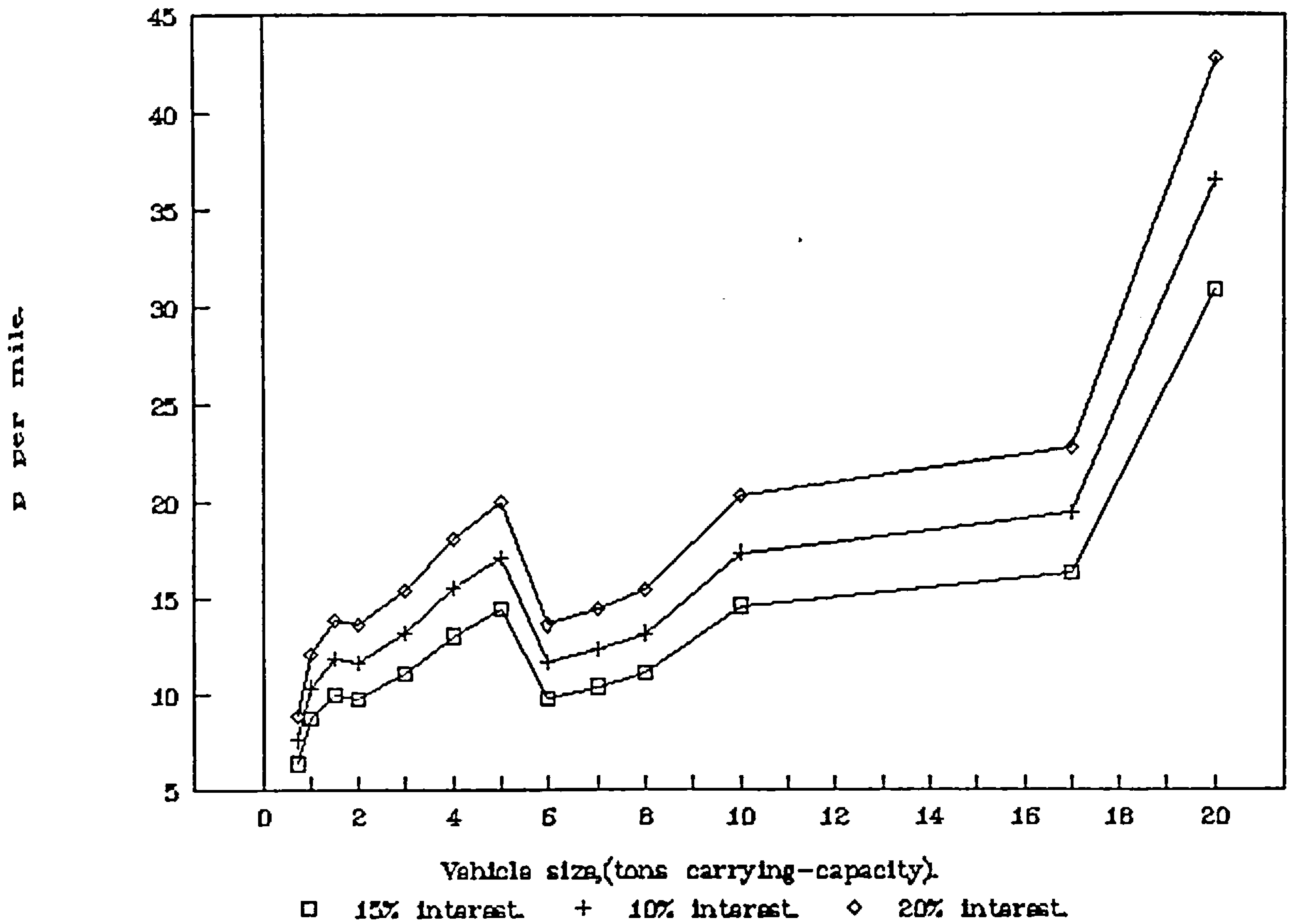
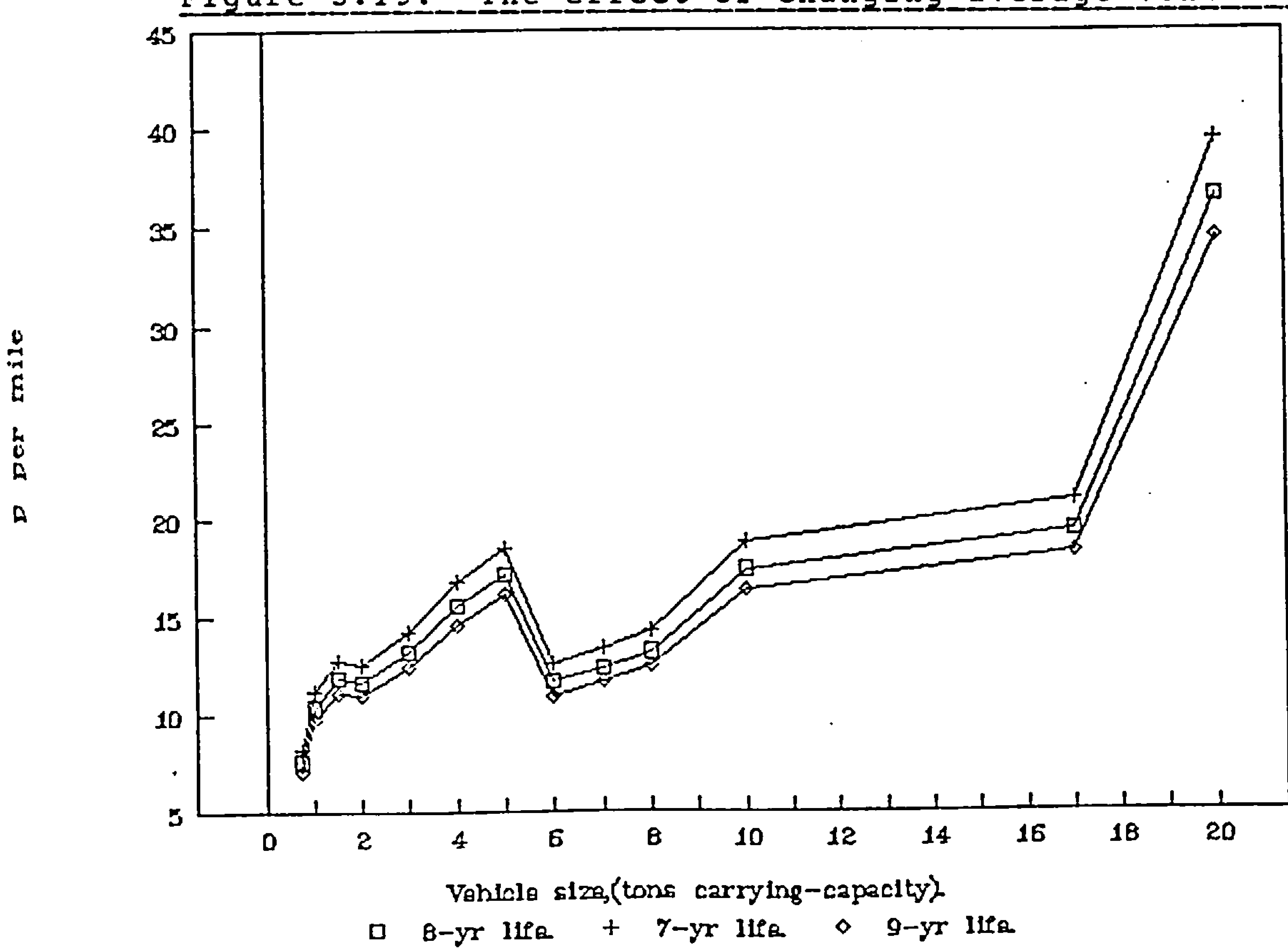


Figure 3.19. The effect of changing average vehicle life-spa



Heavy Rigid Vehicles - Class 2 HGV licence required (more than 17 tonnes g.v.w.)

Articulated Vehicles - Class 1 HGV licence required (All sizes).

This categorisation is slightly at variance with Commercial Motor's five-tier structure, which specifies three wage-levels for vehicles with a carrying capacity of 8 tons or less.

There is little point in comparing absolute figures published by the two organisations, since FTA costings refer to January 1990 levels whilst the Commercial Motor figures used are from 1982. The main reason for consulting a second source of costing information, however, is that the Freight Transport Association publishes, not just average figures, but also the minimum and maximum, and upper and lower quartile (6) values of the sample. In other words, FTA tables provide information on the variance of the data around the mean values; these figures are shown in Table 3.5.. The anomaly in this Table is the set of costings for light rigid vehicles, since the upper quartile value is actually less than the mean; this indicates that at least 75% of the sample of contributors reported the

Table 3.5. Gross Wage Cost Estimates for January 1990

Vehicle Class	Minimum	Lower Quartile (Difference from mean)	Mean	Upper Quartile (Difference from mean)	Maximum
Light Rigid (less than 7.5 tonnes g.v.w.)	154.37	154.37 (-3.0%)	159.20	154.37 (-3.0%)	232.25
Medium Rigid (7.5. to 17 tonnes g.v.w)	148.18	177.71 (-12.6%)	203.26	243.11 (+19.6%)	323.96
Heavy Rigid (more than 17 tonnes g.v.w)	200.00	214.00 (-12.7%)	245.09	267.53 (+9.2%)	281.00

(Source: Freight Transport Association)

(6) The lower quartile value is the value that is midway between the minimum and the median of the distribution; similarly, the upper quartile lies between the median and the maximum.

same figure - £154.37 per week - as the gross wage for a driver. The figure for "medium rigids" and "heavy rigids" include the percentage deviation from the mean of both the lower and upper quartile values, and, in the absence of information on the standard deviation of the distribution, these figures may be regarded as good estimates of the variance of wage costs. Using these percentages it is possible to make estimates as to the quartile values associated with <u>Commercial Motor</u> cost data, in order to calculate the effect of varying wage levels between these limits on Total Cost. This, however, is unnecessary, as Table 3.3. already provides information on the percentage of Total Cost that is accounted for by wages, and so this percentage may be multiplied by the percentage variation from the mean wage level, to give the percentage variation in Total Cost.

For example, FTA figures suggest that, with a 0.75-ton van, the maximum variation in wage level that can take place between the upper and lower quartiles is for gross wages to fall to a minimum level of £154.37 per week - this constitutes a reduction of 3% from the average wage of a driver of this type of vehicle. Since Table 3.3. reveals that 30.7% of the cost of a 0.75-ton van is accounted for by wages, the effect on the Total Cost estimate of adopting this lower wage estimate is 0.92%, (since 0.03 x 0.307 = 0.0092). Similarly, using the upper quartile figure instead of the mean value for a 20-ton rigid vehicle, the effect on the Total Cost estimate would be a 2.3% increase, (since 0.092 x 0.2487 = 0.0229). The maximum impact of using a quartile value instead of the mean is with a 10-ton rigid vehicle; adopting the upper quartile figure of £243.11 per week would increase the corresponding Total Cost estimate by 5.89%. It may be concluded, therefore, that the Total Cost Function is not particularly sensitive to variation in the assumed level of wage costs within the quartile range of the FTA's figures.

(c) <u>Fuel and maintenance</u> are the other main components of Running Cost. As they are both very closely linked to vehicle mileage, they may be dealt with together; separate graphs of fuel and maintenance cost against vehicle size appear in Figures 3.12.1. and 3.13.2, respectively. Both of these graphs show a definite trend for cost per mile to increase as the size of the vehicle increases.

The cost of maintenance is dependent on the regularity of maintenance checks and servicing, which in turn affects the number of man-hours that are assigned to this task each week. Maintenance cost also depends on whether this work is carried out on the operator's premises, or whether it is done under contract by another company. In the case of <u>Commercial Motor's</u> costings, it is assumed that there is

> "...the highest standard of maintenance, including servicing, repairs and washing. The costs produced are calculated from those incurred by operators using their own facilities and those charged by private garages."

Among the factors that affect the level of maintenance cost are the wage levels of fitters and other workshop staff and the price of vehicle parts.

Fuel cost per mile is a product of fuel consumption and fuel price. Both are very volatile and vary both regionally and temporally in response to a number of political and economic events. The rate of fuel consumption is sensitive to both the type and make of vehicle and to the individual driver. The factor that has a major influence on both fuel consumption and maintenance cost is the way in which the vehicle is employed, particularly the number of miles that are driven each year. Generally, a vehicle with a high annual mileage will have a lower unit cost than one which does fewer miles, usually because the latter is involved in more urban, "stop-start" activities.

The Freight Transport Association deals with this "level of usage factor" by deducing three separate figures, from the information provided by its panel of contributors, for each cost factor - these are the average figure for both "high mileage" and "low mileage" operators, and the overall average. The criteria for high and low mileage vary according to vehicle size. For example, for a diesel-powered "car-derived" van, low mileage estimates refer to vehicles that travel approximately 18000 miles per year, whilst the average mileage per vehicle of a high mileage operator is 30000 miles; the overall average for the FTA's sample in this vehicle class is 24000 miles. At the other end of the scale, the corresponding figures for a 38-ton articulated lorry are 50000 miles, 80000 miles and 66000 miles, respectively. Having categorised contributors according to the average mileage that is covered by one of their vehicles each year, average figures for maintenance cost, fuel consumption and other major components of distribution cost are calculated for each group of contributors.

The FTA's fuel price estimate is based on both the retail price of diesel and the bulk purchase price, (which is substantially cheaper). On the assumption that 80% of all fuel purchases by the panel will be made in bulk, the cost of fuel per gallon is assumed to be the sum of 80% of the bulk purchase price and 20% of the retail price.

The figures published for 3.5-tonne diesel vans are fairly typical of the FTA costing as a whole, and these are displayed in Table 3.6.. The three columns of this table show considerable differences in cost per mile estimates. Fuel cost per mile, for example, varies between 8.1% and 10% either side of the overall mean figure depending on whether high mileage or low mileage is assumed, and maintenance cost per mile varies from +30.1% (low mileage) and -21.9% (high mileage) of the mean. The Total Cost per mile figures also show considerable divergence from the overall sample mean (from +37.7% to -20%), although it should be stressed here that the range

Table 3.6. Running Cost and Total Cost estimates for a 3.5-tonne diesel van, Jan.1990

	Low Mileage	High Mileage	Overall Mean
Annual Mileage	15000	40000	26000
Fuel Consumption (miles/gal)	25	30	27
Fuel Cost (p/mile)	5.62	4.68	5.2
Maintenance Cost (p/mile)	5.4	3.24	4.15
Total Running Cost (p/mile)	11.75	8.43	9.92
Total Cost (p/mile)	28.64	16.87	21.1
Total Cost (£/year)	4294	6746	5485

(Source: Freight Transport Association)

of assumed annual mileages is also quite wide, (the lower and higher mileage estimates being respectively -42.3% and +53.8% of the mean). Furthermore, the variations in Total Cost are a result of a number of variables changing in response to differences in assumed vehicle usage. If a single cost component, such as fuel cost, should change in isolation, as a direct result of an increase in the price of derv, for example, then the effect on Total Cost would be relatively minor. Given that Table 3.3. indicates that fuel costs account for between 14.3% and 19.1% of Total Cost, the variations in fuel cost of 10% or less shown in Table 3.6. can themselves have little impact on Total Cost estimates.

The FTA's use of an upper and lower mileage estimate does, however, illustrate the extent to which operating costs may be influenced by vehicle usage and the nature of an operator's work. This indication of the possible variance of Running Cost estimates is particularly useful in view of the fact that Commercial Motor's statistics do not include such information.

The purpose of this section has been to more closely examine the Commercial Motor cost data that are used for the research. This is important, since these figures form the basis of all the cost functions that appear in this and subsequent chapters. After a comparison of these data with those of an alternative source, namely the Freight Transport Association, the costings published by Commercial Motor have been disaggregated in order to assess the effect of each cost component on the Total Cost of a vehicle. The latter exercise has revealed that wages, fuel, maintenance and depreciation/interest account for over 90% of this cost, and so the assumptions that underlie these cost estimates have been examined in greater detail in sub-section 3.3.3.. More particularly, the importance of the assumption made as to the average life-span of Heavy Goods Vehicles has been evaluated, along with the effect of fluctuating interest rates and drivers; wage-levels on published costings. Alternative figures have also been produced for the cost of depreciation, using a different method of calculation to the simple straight-line method employed in Commercial Motor. The following section, however, in which analytical expressions are developed for estimating Total Cost as a function of vehicle size, utilises vehicle cost data as originally published by the latter publication.

3.4. Estimating Total Cost as a Function of Vehicle Size

The equations derived in Section 3.1. for Total Cost as a function of vehicle size, (eg. Equation E.3.4.), estimate costs per mile per ton of carrying-capacity available for vehicles of different sizes, but these figures may be readily adjusted to provide an estimate of Total Cost per week, given a fleet of n vehicles travelling m miles each week. Firstly, the Running Cost formula must be multiplied by the number of miles actually travelled per week by the fleet, and Standing Costs, having been multiplied by the assumed weekly travel-distance of 1,000 miles, should be further multiplied by the number of vehicles used. Finally, the whole equation should be multiplied by vehicle size, (assuming a uniform fleet of vehicles), so that the modified equation for Total Cost per week of rigid vehicles is now,

$$TC = x[n.12106x^{-0\cdot 629} + m.21\cdot 3796x^{-0\cdot 727}] \qquad (E.3.5.)$$

or,

$$TC = [n.12106x^{0\cdot 371}] + [m.21\cdot 3796x^{0\cdot 273}] \qquad (E.3.6.)$$

where, TC = Total Cost per week, (£)
x = vehicle carrying-capacity, (tons),
m = weekly fleet mileage,
and, n = number of vehicles.

At this stage of the analysis, it is assumed that there is a very simple situation of a fleet of vehicles travelling from a supply-point to a demand-point and back, so that the length of a round-trip is twice the distance between these two points. Therefore, assuming that the vehicles can always be fully laden,

$$m = \frac{td}{x} \qquad (E.3.7.)$$

where, t = tonnage to be delivered each week,
and, d = the length of a round-trip.

Obviously, the assumption that vehicles are always fully laden is not realistic, and the consequences of the integer effect experienced in reality will be discussed later.

However, substituting equation E.3.7. into E.3.8. produces the equation,

$$TC = [n.12106x^{0 \cdot 371}] + [td.21 \cdot 3796x^{-0 \cdot 727}] \quad (E.3.8.)$$

One problem with this equation is that the first term of Equation E.3.8., the expression for Standing Cost, contains two terms - n and x - that are inter-related, for as the carrying-capacity of each vehicle increases, the number of vehicles required in the fleet to transport the same weekly tonnage of goods will decrease, and vice versa. To overcome this difficulty, it is possible to express n as a function of x, since the maximum number of round-trips required to deliver a given order each week can be expressed as (t/x), assuming a uniform fleet of vehicles always operating at full capacity. Therefore, on the basis of a five-day week,

$$n = \frac{t}{5x} \quad (E.3.9.)$$

(However, it should be stressed here that the value of n and (t/x) must always be rounded UP to the nearest whole number, as it is absurd to think in terms of using a fraction of a vehicle, and the same integer effect also applies to the number of round-trips made). Substituting this expression for n, E.3.8. may be written as,

$$TC = [\frac{t}{5x} \cdot 12106x^{0 \cdot 371}] + [td.21 \cdot 3796x^{-0 \cdot 727}]$$

or,

$$TC = [\frac{t}{5} . 12106x^{-0 \cdot 629}] + [td.21 \cdot 3796x^{-0 \cdot 727}] \quad (E.3.10.)$$

The main independent variable in this equation is "td", which represents the total ton-mileage required for a given distribution task. In this very simple example, in which a fleet of vehicles shuttles between one supply-point and a single destination, so that d is simply twice the distance between these two points, td serves as a convenient expression for the scale of an operation. Obviously, an increase in t or d will cause x or n, or both, to also be increased, which will, in turn, raise Total Cost.

The shape of the Total Cost curve as td increases, in this simple situation, is illustrated by Figure 3.14.; the figures on which this graph is based were calculated using both equation E.3.10., for rigid-chassis vehicles, and the following expression for articulated vehicles,

$$TC = [\frac{t}{5} \cdot 9817{\cdot}5x^{-0{\cdot}535}] + [td.11{\cdot}885x^{-0{\cdot}5}] \quad (E.3.11.)$$

with the added assumption that the length of each round-trip, d, is 100 miles. A full list of the results of these calculations is presented in Table 3.7..

The major constraints involved here are that no more than one round-trip per vehicle per day is possible, and that vehicle carrying-capacity, x, is a discrete variable whose value may only correspond to the vehicle-sizes quoted in the Commercial Motor cost tables on which the analysis is based. These vehicle-sizes are 0·75, 1, 1·5, 2, 3, 4, 5, 6, 7, 8, 10, 17 and 20 tons capacity for rigid-chassis vehicles, and 10, 12, 14, 16, 18 and 22 tons capacity for articulated vehicles; in addition to these, it is assumed that "artics" of 25, 30, 35 and 38 tons are also available.

For each value of td, the carrying-capacity of each vehicle, of what is assumed to be a uniform fleet, is calculated using the following formula,

$$x = \frac{t}{5n} \quad (E.3.12.)$$

where the smallest value of n that does not cause x to exceed the maximum carrying-capacity of 38 tons is used. For example, when t=250 tons per week, the calculated value of x, using Equation E.3.12., is 50 tons when n=1. As this vehicle size is infeasible, n must be increased to 2, so that the value of x, using the same formula, becomes 25 tons.

Again, in all cases x must be rounded UP to the nearest feasible vehicle-size; where either an articulated or rigid-chassis vehicle may be used, the cheaper alternative is adopted, for the purposes of Table 3.7..

The discontinuities in the Total Cost curve of Figure 3.20. indicate the points at which an extra vehicle must be added to the fleet; Figure 3.21. graphs the carrying-capacity of each vehicle as weekly ton-mileage changes. Figure 3.22. shows the relationship between Average Cost, (in pence per ton-mile), and td, and illustrates once again the Economies of Scale that have been discussed throughout this chapter, showing that Average Cost declines as vehicle-size increases. As this unit cost per ton per mile is a direct function of x, as Equations E.3.10. and E.3.11. suggest, the discontinuous curve shown here is, in fact, more or less the inverse of Figure 3.15., and also bears a close resemblance to Figure 3.7., which is discussed in Section 3.2..

Another important feature of Figure 3.22. is the way in which the Average Cost curve bottoms out once the maximum vehicle-capacity of 38 tons is reached, which re-emphasises the fact that the Economies of Scale resulting from the use of a fleet of vehicles arise as a consequence of increasing vehicle-size, and not due to the increase in ton-mileage itself!

3.5. A Formal Proof that Economies of Scale Exist in Road Haulage Operations

When the parameters t and d are held constant, so that x, and therefore n, become the independent variables in the Total Cost equation, the relationship between Total Cost and vehicle-size is as shown by the curve in Figure 3.23.; the results of the calculations on which this graph is based are contained in Table 3.8.. In this example, t is fixed at 190 tons per week, and the length of a round-trip is 100 miles. For each feasible vehicle-size, the number of vehicles required is calculated using Equation E.3.9. - again, rounding UP to the nearest whole number to take account of the obvious integer effects - and the Total Cost per week of using both rigid and articulated vehicles calculated with Equations E.3.10. and E.3.11., respectively.

Figure 3.23. shows that the Total Cost curve continues to decline as vehicle carrying-capacity increases, so there is certainly visual evidence of Economies of Scale in this graph. However, the x-axis here extends to only 38 tons carrying-capacity, as this is the current maximum permissible weight for Heavy Goods Vehicles on UK roads, and so it is not clear from Figure 3.23. whether the observed Total Cost curve shows a truly downward-sloping cost function or merely the first part of a "U"-shaped curve.

The fact that the former explanation is valid here, may be demonstrated algebraically, since Equation E.3.6. may be written in the following general form,

Figure 3.20. Total Cost/td, (using Equations E.3.10. & E.3.11.).

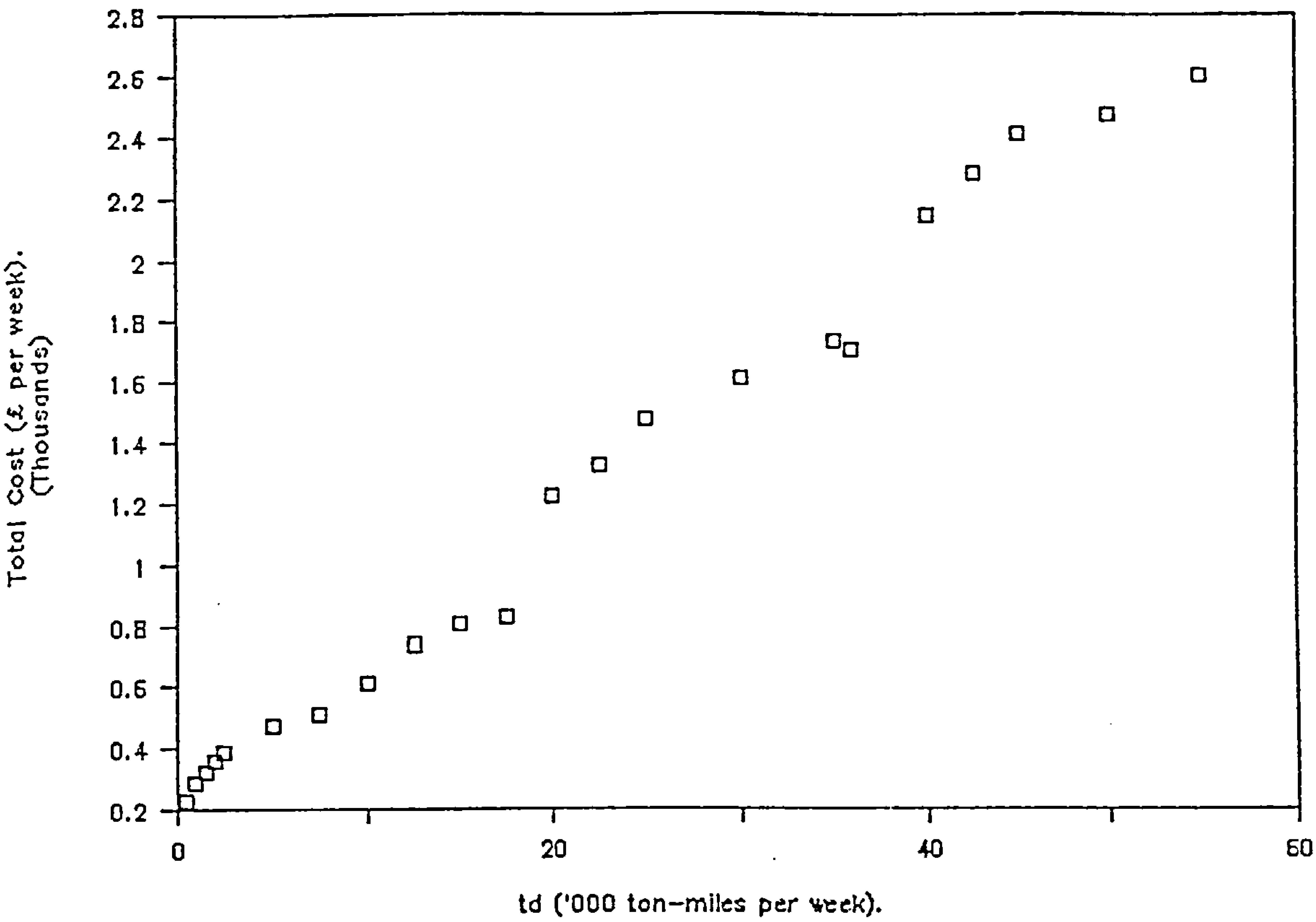


Figure 3.21. x/td, (using Equations E.3.10. & E.3.11.).

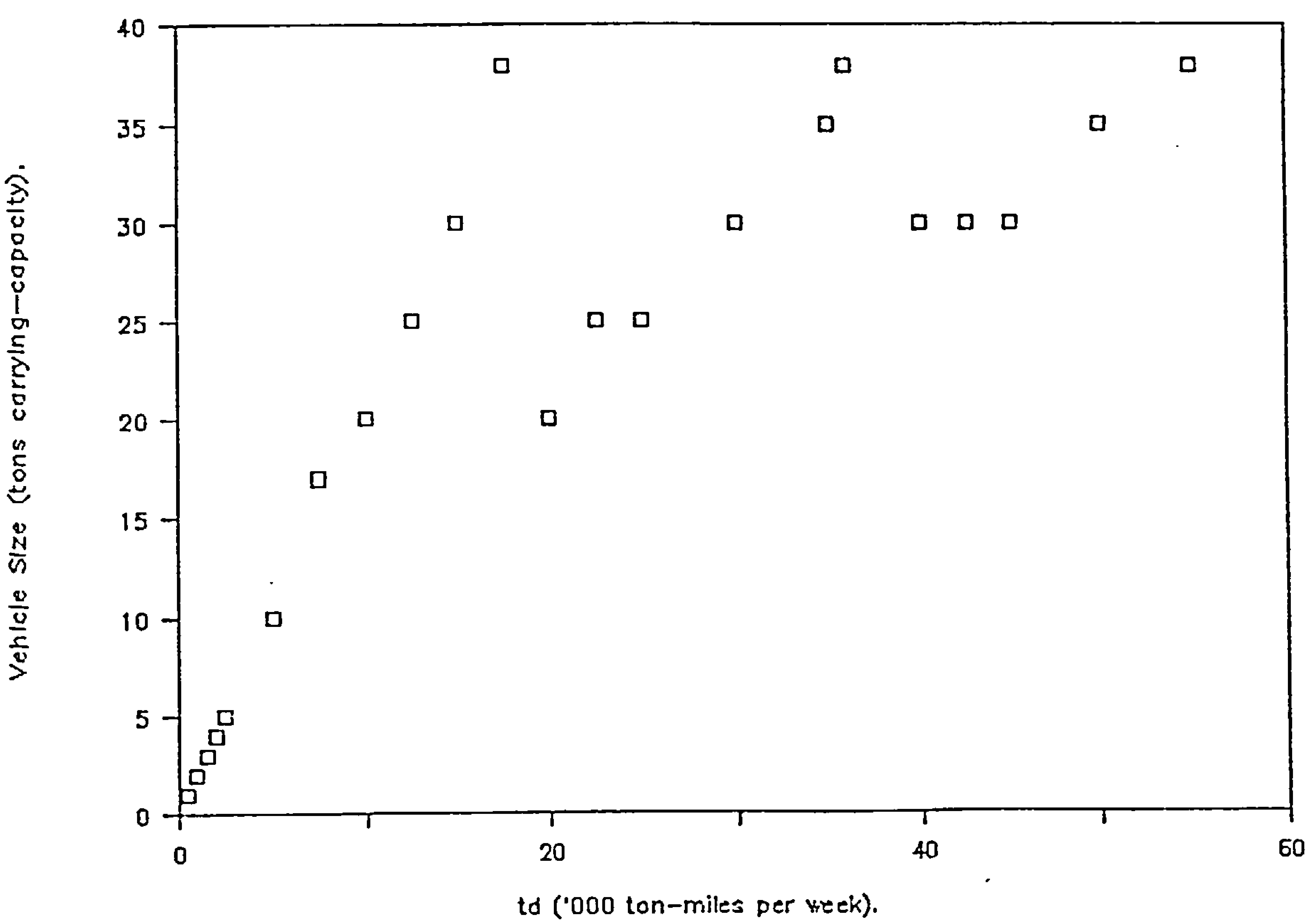


Figure 3.22. Average Cost/td, (using Equations E.3.10. & E.3.11.).

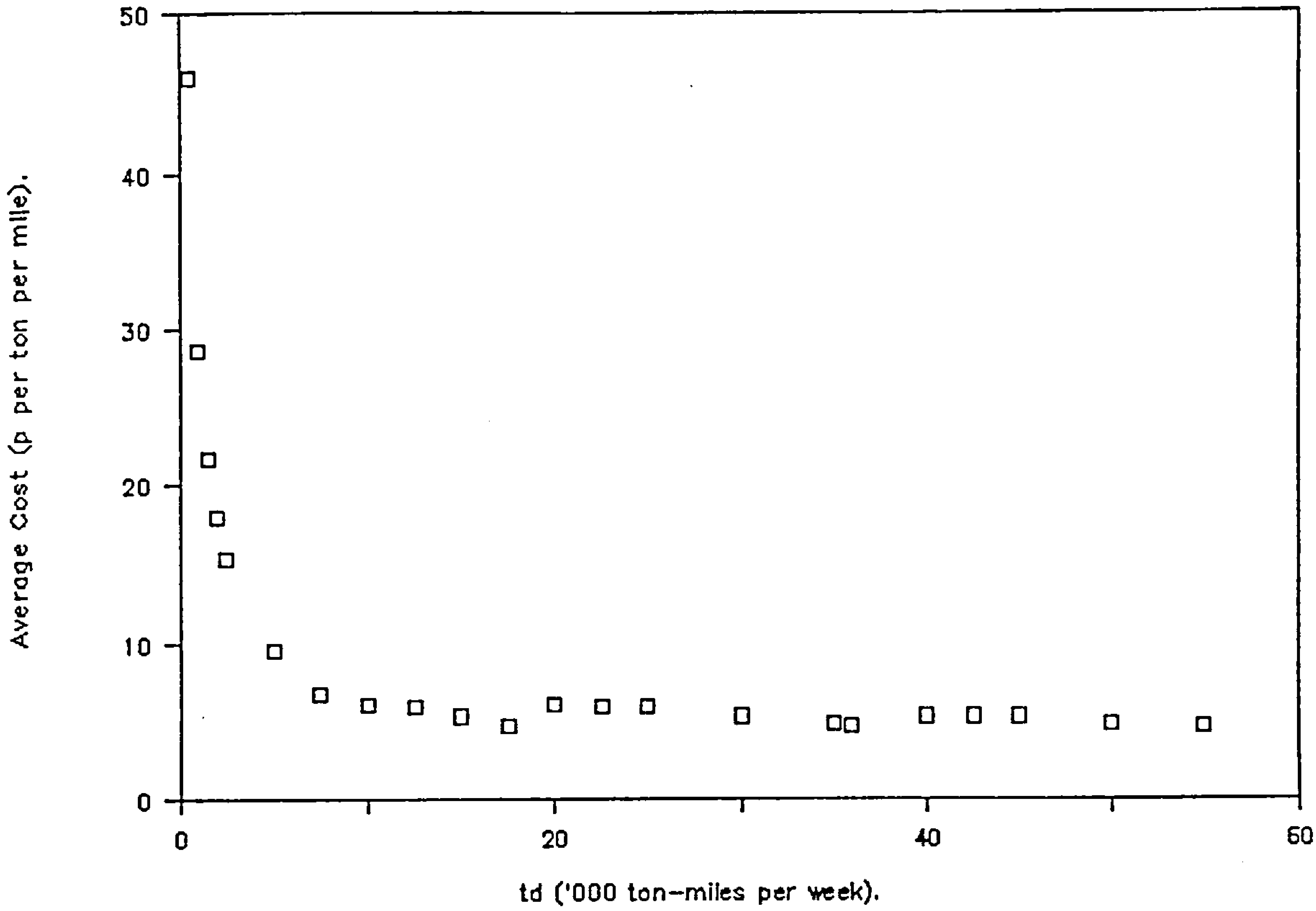
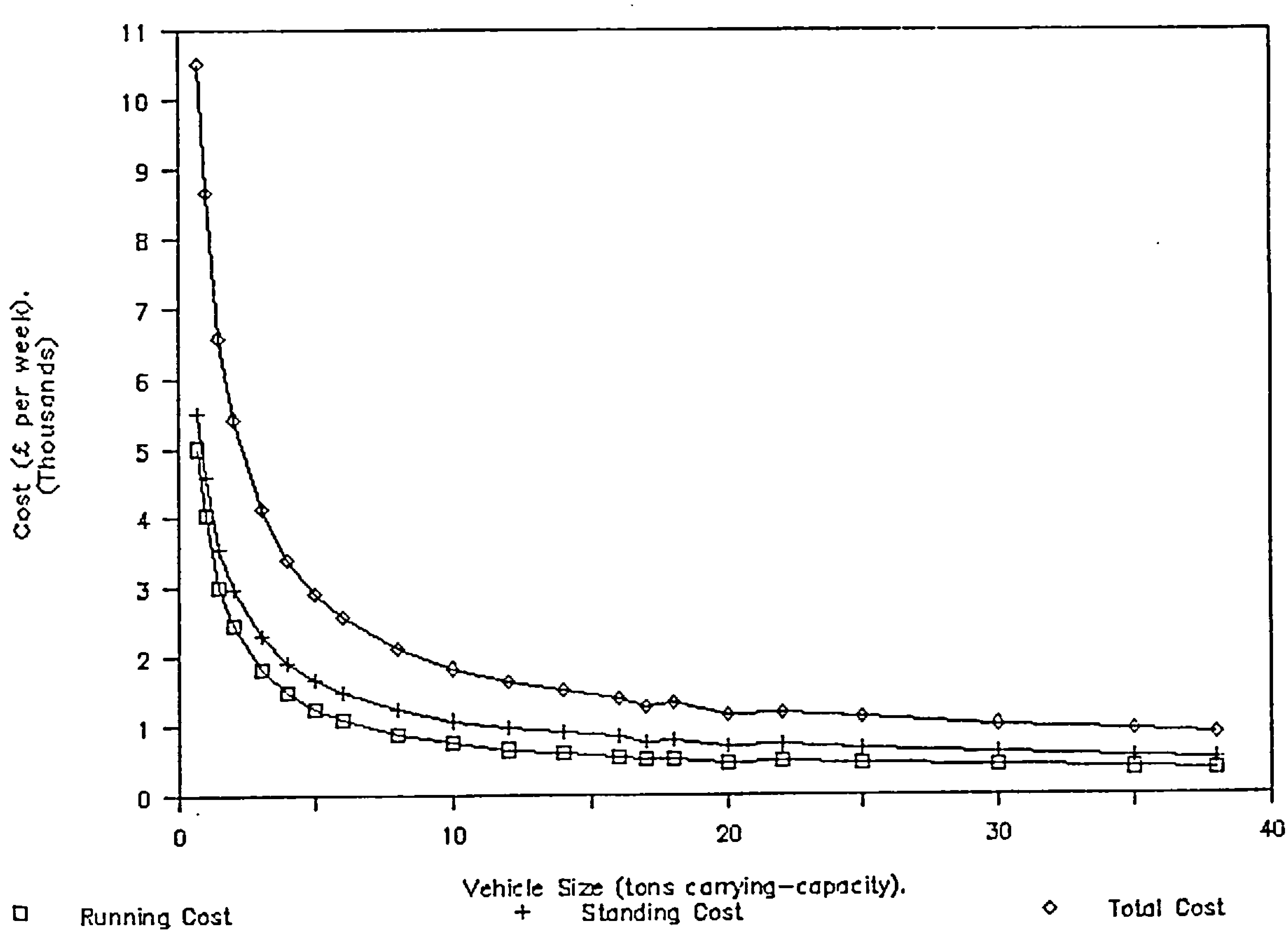


Figure 3.23. Costs per week/x, (using Equations E.3.9. to E.3.11.).



$$TC = [n.A.x^{a}] + [m.B\ x^{b}] \qquad (E.3.13.)$$

where, A & a are parameters pertaining to Standing Cost,
and, B & b are parameters pertaining to Running Cost.

Substituting Equations E.3.7. and E.3.9. into this expression,

$$TC = [\frac{t}{5} . A.x^{a-1}] + [td . B.x^{b-1}] \qquad (E.3.14.)$$

Since the parameters a & b both have a value of less than 1, (SEE Table 3.1.), the coefficient of x in the two parts of E.3.14. will be negative in both cases. For this equation to describe a "U"-shaped curve, these two coefficients of x would have to have different signs; in such a situation, the expression for Total Cost could be differentiated and the optimum value of x, which would correspond to the lowest point of the Total Cost curve, could thus be calculated. However, Table 3.1. confirms that both Running Cost and Standing Cost have a negative elasticity with vehicle-size, ensuring that Total Cost will continue to decline as the value of x increases.

The conclusion that may be drawn from this finding is that, when faced with the decision of whether to perform a delivery-task using a small fleet of large vehicles or a larger fleet of smaller vehicles, the former alternative will always be the lowest-cost operation, due to the fundamental behaviour of Running Cost and Standing Cost per ton-mile. The following section provides a hypothetical numerical illustration of the implications of these findings.

3.6. A Numerical Illustration

Consider a very simple situation in which 20 customers, distributed at random within a square delivery-zone, each require a delivery of goods amounting to exactly 1 ton each day; given that it is possible, from the point of view of both vehicle-capacity and time, to visit every customer-location in one day, the problem is one of selecting the vehicle-fleet that will perform this task at the lowest cost.

To mention just two options, the deliveries may be made by either one 20-ton vehicle visiting each customer every day, or by a fleet of five 4-ton vehicles serving 4 locations each. The main trade-off here, in terms of cost, is between the lower cost PER VEHICLE of a larger fleet of small vehicles, and the fact that more vehicles in such a fleet will incur Standing Costs, such as insurance and licences etc.. Furthermore, it will be demonstrated later in this section that the distance covered by a fleet in visiting a given set of locations increases in proportion with the number of vehicles employed.

Table 3.9. summarises the estimated cost of a variety of options, assuming that rigids are to be used in each case; all of these estimates are therefore calculated using Equation E.3.10.. The distance travelled by the fleet in a day is estimated using the following empirical formula, a formula that will be discussed in greater detail, along with other spatial aspects of distribution operations, in Chapter 4,

$$\text{Total Fleet Mileage} = a[5{\cdot}308 + 0{\cdot}566n] \qquad (E.3.15.)$$

where, a = length of one side of the square delivery-zone, (miles).

The main assumption associated with this formula is that the delivery-area in question is a homogeneous square of dimensions (axa), served by a fleet of vehicles operating from a single depot located at the centre. This expression merely describes the way in which the total distance travelled by the fleet - in a day, in this case - increases as the number of vehicles used increases. For the purposes of Table 3.9., it is assumed that a=50 miles.

The figures confirm that the lowest-cost means of delivering the required tonnage of goods each day is to employ the largest vehicles available, and therefore the smallest possible fleet. As Figure 3. 24. clearly shows, this is because the reduction in Running Cost achieved by selecting smaller vehicles, in spite of the consequent increase in Total Fleet Mileage, is insufficient to off-set increased Standing Costs resulting from the rise in the number of vehicles and drivers that need to be employed; as a result the Total Cost curve continues to go upwards as fleet-size increases. Furthermore, when the x-axis of Figure 3. 24. is extended to include very small vehicle-sizes, it becomes apparent that the Running Cost curve is, in fact, "U"-shaped, which further supports the assertion that costs will continue to rise as n increases.

The "U"-shaped nature of the Running Cost curve is not unexpected, in view of the nature of the Running Cost formula. To recapitulate, the relevant part of Equation E.3.10. here is,

$$\text{Running Cost} = td.21{\cdot}3796x^{-0{\cdot}727} \qquad (E.3.16.)$$

Since the ton-mileage per week (t) and the value 11·885 are both constants, the two variable terms in this expression are $x^{-0{\cdot}727}$ and d, the average distance per tour. As vehicle-size increases, the term $x^{-0{\cdot}727}$ will <u>decrease</u> due to the negative power of the coefficient, whilst the value of d will <u>increase</u>; this is because the use of larger vehicles implies a reduction in fleet-size, when t is constant, so that the <u>average</u> length of each vehicle-tour - which is the

Figure 3.24. Total Cost per week/n, (numerical illustration: a=50).

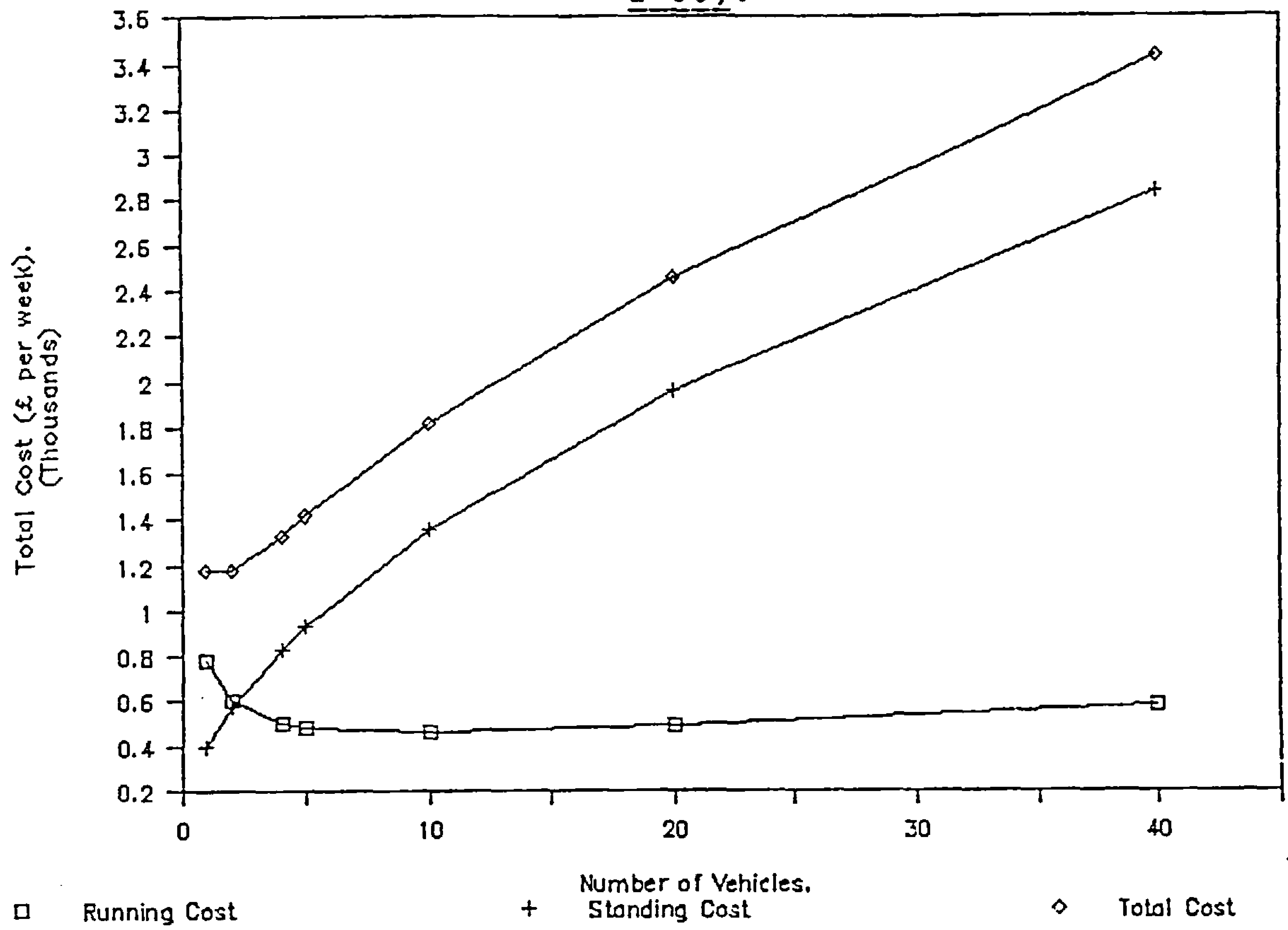
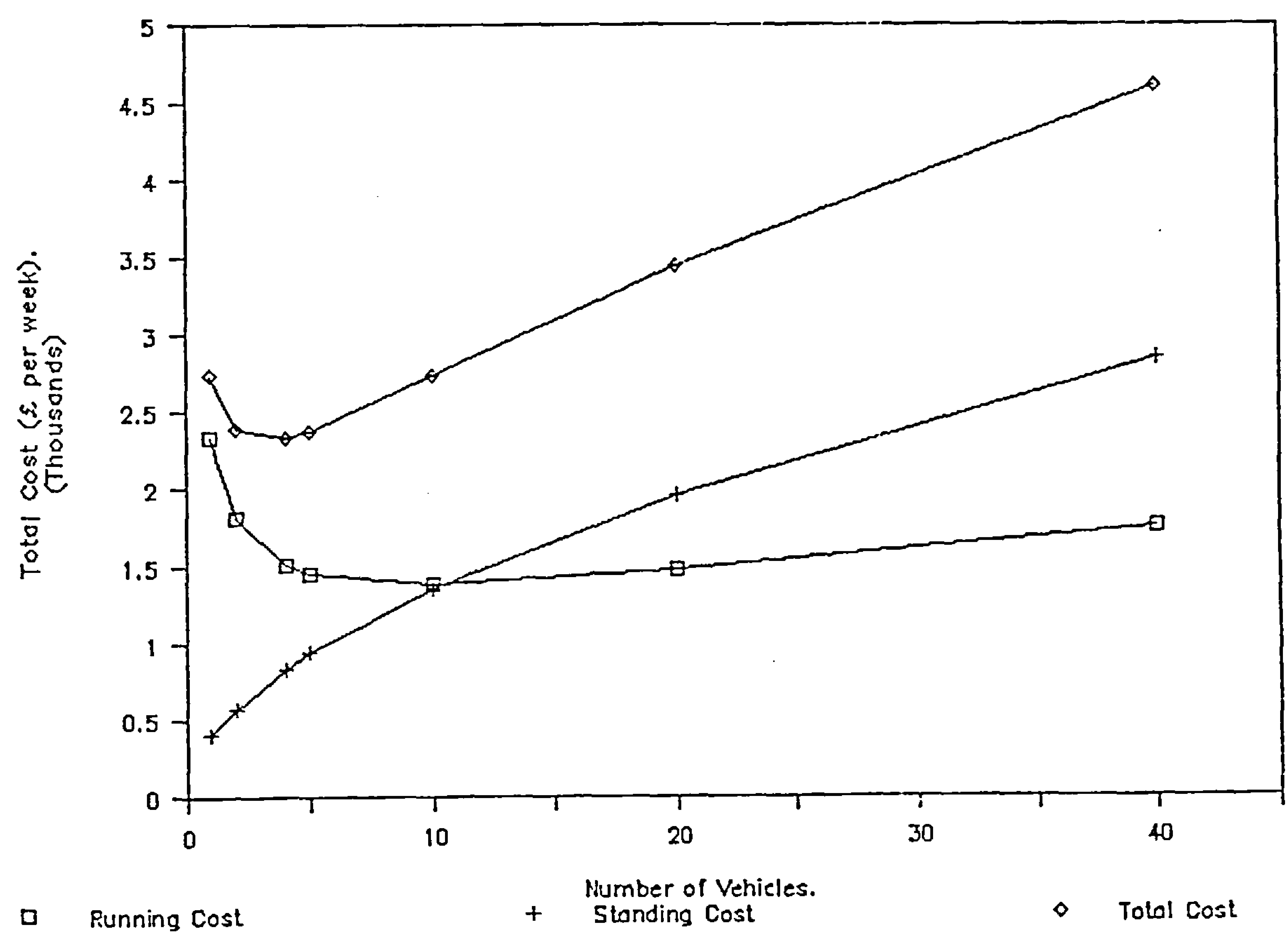


Figure 3.25. Total Cost per week/n, (numerical illustration: a=150).



accurate definition of d now that the length of a round-trip is no longer fixed - decreases. Algebraic confirmation that d is reduced as x increases is given by the fact that the formula for average tour-distance is simply Equation E.3.15. divided by n, so that,

$$d = \frac{a[5\cdot308 + 0\cdot566n]}{n} \qquad (E.3.17.).$$

It is the fact that the two variable components of the Running Cost formula behave in opposite ways in response to changes in vehicle-size that causes the Running Cost curve to be "U"-shaped.

The same conclusion must be drawn if Equation E.3.16. is re-written in the form,

$$\text{Runnning Cost} = \frac{td}{x} \cdot 21\cdot3796\ x^{0\cdot273} \qquad (E.3.18.)$$

In this case, the $x^{0\cdot273}$ term will clearly increase as vehicles become larger, whilst $[\frac{td}{x}]$, which now represents Total Daily Fleet Mileage, will decrease due to the reduction in the number of vehicles and vehicle-tours.

In simple terms, Running Cost is fundamentally a function of the cost of operating a vehicle per mile, which increases when larger vehicles are used, and of the mileage travelled by a fleet, which decreases with increasing x due to the accompanying reduction in n.

One implication of a "U"-shaped Running Cost curve is that, in circumstances where Running Costs are large in relation to Standing Costs, the Total Cost curve may also be "U"-shaped. This is not the case with Figure 3.24 ., but the situation may change after a sensitivity analysis is performed on the data of Table 3.9.. For example, if the delivery-area in question is enlarged, so that "a" increases from 50 miles to 150 miles, Running Costs will undergo a three-fold increase also, causing the Running Cost curve to be raised. The revised cost calculations, when a=150 miles, are contained in Table 3.9. and are illustrated by Figure 3.25 ., which confirms that the Total Cost curve is, in this case, distinctly "U"-shaped.

It should be noted, here, that although this numerical example provides an instance in which, for small values of n, Total Cost can actually be increased when the size of the fleet is reduced, this in no way invalidates or contradicts the proof of the presence of Economies of Scale in road haulage, presented in Section 3.5.. This is because the latter section refers to a situation in which the length of a round-trip is fixed, whilst the introduction of a spatial perspective means that d, being a direct function of n, is effected by changes in x. It is the interaction between x and d, therefore, that determines the extent to which Economies of Scale may occur in this situation, particularly the effect that vehicle-size has on the term "x^b" in Equation E.3.13., compared with its effect on the average length of a round-trip. It has already been pointed out in this section that the extent to which this "U"-shaped Running Cost curve influences the shape of the Total Cost curve is dependent on the relative importance of Running Cost and Standing Cost, which, in turn, involves factors such as the size of the delivery-area, fuel costs, average vehicle speed etc..

3.7. Conclusion

The main characteristic of a vehicle-fleet considered in this chapter has been vehicle-size, measured in terms of tons carrying-capacity, and in particular the interaction between vehicle-size and the number of vehicles employed. The major conclusion to arise from the preceding discussion is that Economies of Scale may be derived from a reduction in the numerical size of a fleet, and the use of a smaller number of larger vehicles. The existence of such economies has been both demonstrated graphically, (eg. Figure 3.7., Figure 3.22. etc.), and proven algebraically given the characteristics of the data provided by Commercial Motor magazine.

Throughout most of this chapter, the discussion has focused upon a very simplistic formulation of the Distribution Problem, involving the transportation of a given tonnage of goods from an origin to a single destination, using a uniform fleet of vehicles. The purpose of using such an uncomplicated example of a distribution operation has primarily been to simplify the fleet mileage variable, so that the interaction between the number of vehicles used and vehicle carrying-capacity could be examined in the absence of factors such as the shape of individual vehicles' delivery-zones and the average length of vehicle tours etc.. The impact of such spatial considerations, as a result of changes in the number of vehicles used, will now be discussed in Chapter 4.

CHAPTER 4

THE SPATIAL IMPLICATIONS OF FLEET-SIZE

In Chapter 3 it was assumed that the extent of a fleet's movement in space is to shuttle between one source and one destination; this simplistic situation was used in order to focus attention on the issue of Economies of Scale in transport. Consideration is now given to the more realistic scenario of a set of randomly distributed customers, and variables such as the size of the delivery-area, the number of customers served and the number of stops per vehicle-trip now replace the concept of weekly ton-mileage, (td), as indicants of the scale of a distribution operation. Less emphasis will be placed on factors such as carrying-capacity and loading of vehicles in this chapter, since the main concern, here, is with the relationship between the number of vehicles used in an operation and Total Fleet Mileage.

The distance travelled by the fleet will obviously be influenced by the shape of the delivery-area, the spatial distribution of customers and the location of the depot within this area, but as all of these factors may be viewed as being part of the environment in which a fleet must operate, the effect of such factors will be considered in Chapter 6.

What is of interest in this chapter is the way in which the total distance that a fleet covers changes, purely as a response to variations in the number of vehicles used; in other words, Chapter 4 focuses on the relationship between the variables n and m.

4.1. Formulation of the Problem and Discussion of the Assumptions Made

The problem consists of making an accurate estimate of the minimum distance required to visit each of a set of customers located within a square delivery-zone from a centrally-located depot; although the precise co-ordinates of these customers within this continuous, homogeneous, Euclidean space are unknown, it is to be assumed that their distribution is, for all intents and purposes, random. This is, of course, the general formulation for the well-documented Travelling-Salesman Problem, although much Travelling-Salesman-type research concentrates on the distance required for <u>one vehicle</u> to visit all the relevant locations. In this analysis, it is assumed that, when more than one vehicle is being operated, the delivery-area is divided into n non-intersecting segments, whose boundaries intersect at the depot to form an angle of (360/n) degrees, (SEE Figure 4.1.). For example, if 2 vehicles operate from the depot, then the delivery-area will simply be divided into 2 halves; if a fleet of 10 vehicles is used, then it is assumed that each vehicle operates within a 36-degree "wedge"-shaped zone.

There is no particular reason for using a square delivery-area. Although it may seem more logical to think of a delivery-area in a homogeneous plain as being circular, the boundaries of this area are of little importance; the inclusion of such boundaries here merely acknowledges the fact that there is a maximum distance that vehicles may travel from the operating centre. This limitation on the range of each vehicle is imposed by the requirement that each driver should return to the depot before the end of the working-day. The size of the delivery-area is therefore effectively defined, in reality, by factors such as average vehicle speed and the average time that a vehicle spends at each location, although neither of these factors is considered at this stage of the analysis.

However, one implication of assuming that a fleet operates within an "a x a" square is that not all of the n sectors are of equal size, so that the average distance travelled by this fleet is slightly greater than would be the case if the delivery-area were a circle of diameter "a". (This is self-evident, since the area of such a circle is only 0·7857a). The impact that area-shape has on Fleet Mileage will be discussed in greater detail in Chapter 6.

The method for dividing a square delivery-area up into a set of n sectors is certainly worthy of further discussion here, for despite the fact that a wedge-like sector may seem to be a fair approximation of the nature of a vehicle-tour found in reality, it may still be argued that alternative shapes might be just as appropriate.

For example, when n=4 there is a dilemma as to whether to assume four square or triangular sectors; these alternatives are summarised in Figure 4.2. as "(a)" and "(b)"; this diagram has been reproduced from Eilon, Watson-Gandy and Christofides, (1971), (1). Eilon et al investigate the difference between the length of vehicle-tours in these two sector types, and their conclusion, whose notation has been altered for the sake of clarity, is that,

> ".......the length of a tour through points distributed in a sector such as "(a)" has been found to be only 5% greater than the length of an equivalent tour in "(b)" in the case where n=4, ...and even less for larger values of n."

(Eilon et al, 1971). Throughout the current thesis, whenever a fleet is comprised of 4 vehicles, it is assumed that each operates within a delivery-zone of the type "(b)", shown in Figure 4.2..

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(1) EILON, S., WATSON-GANDY, C.D.T., and CHRISTOFIDES, N., <u>Distribution Management: Mathematical Modelling and Practical Analysis.</u> (Griffen, London,1971).

Sector shape is also an important issue when n is large. To take a rather extreme example, when n=48, the angle of each segment would be only $7\frac{1}{2}$-degrees, which would give rise to the absurd pattern on delivery-sectors shown in Figure 4.3.1.; with such a large fleet, the assumption of non-overlapping segments is no longer realistic,especially when the number of stops per vehicle is also large. In such cases, a more efficient routing strategy might be to superimpose a series of concentric rings onto a set of larger segments, and so produce a pattern of zones similar to that illustrated by Figure 4.3.2.. The extent to which there is justification for adopting such a sectoring strategy may depend upon the extent of spatial clustering of the customers requiring a delivery, but since a random distribution of customers is assumed at this stage, this particular aspect of the sectoring question will also be considered in Chapter 9. Nevertheless, the issue of which of the strategies illustrated in Figure 4.3. should be adopted remains relevant to this section, since it is likely that the number of vehicles in a fleet plays an important part in determining the point at which the assumption of a system of wedge-like sectors, (as in Figure 4.1.), becomes unrealistic. Once such a strategy is abandoned, and some vehicle delivery-zones no longer extend to the depot, a vehicle-trip must be regarded as a two-stage process, consisting of both the journeys from the depot to the first and last customers to be served, and the total of the distance between each stop on the route. This distinction between what might be termed "stem distance" and "delivery distance", respectively, will be discussed in more detail in Setion 4.3..

Daganzo, when referring to tour-distances involving visits to remote clusters of locations, uses the term "line haul"and "detour distance" to refer to these two parts of a vehicle-tour, (SEE Figure 4.4.). He describes a trade-off between the use of delivery-zones that are compact, but at some distance from the depot, and more "slender" zones that are contiguous to the depot; increasing the slenderness, or "elongation"of a formerly compact sector will reduce the line-haul portion of the round-trip, but at the expense of increasing the detour distance. Daganzo's conclusion is that wedge-shaped sectors which have contact with the depot should be used when,

$$s > \frac{4C}{P} \qquad \text{(E.4.1.)}$$

where, C = the number of stops per vehicle,
P = the number of customers served by the fleet,
and, s is a "slenderness factor".

The value of the "Slenderness Factor", s, is defined as an index based on the length and breadth of a rectangle that might be used to approximate the size and shape of a sector,

Figure 4.1. Assumed sectoring strategy in a square delivery-zone, (n=10)

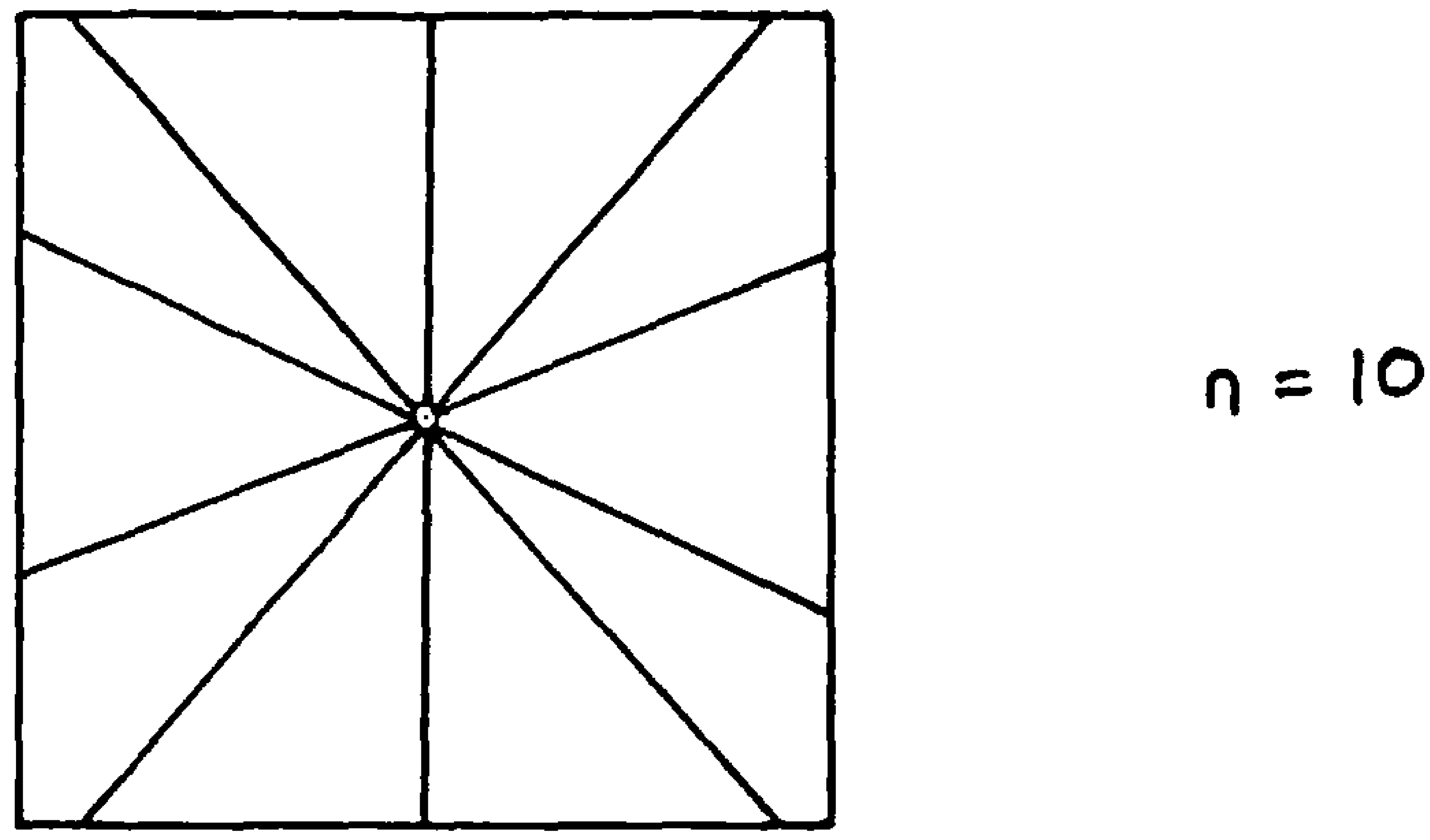
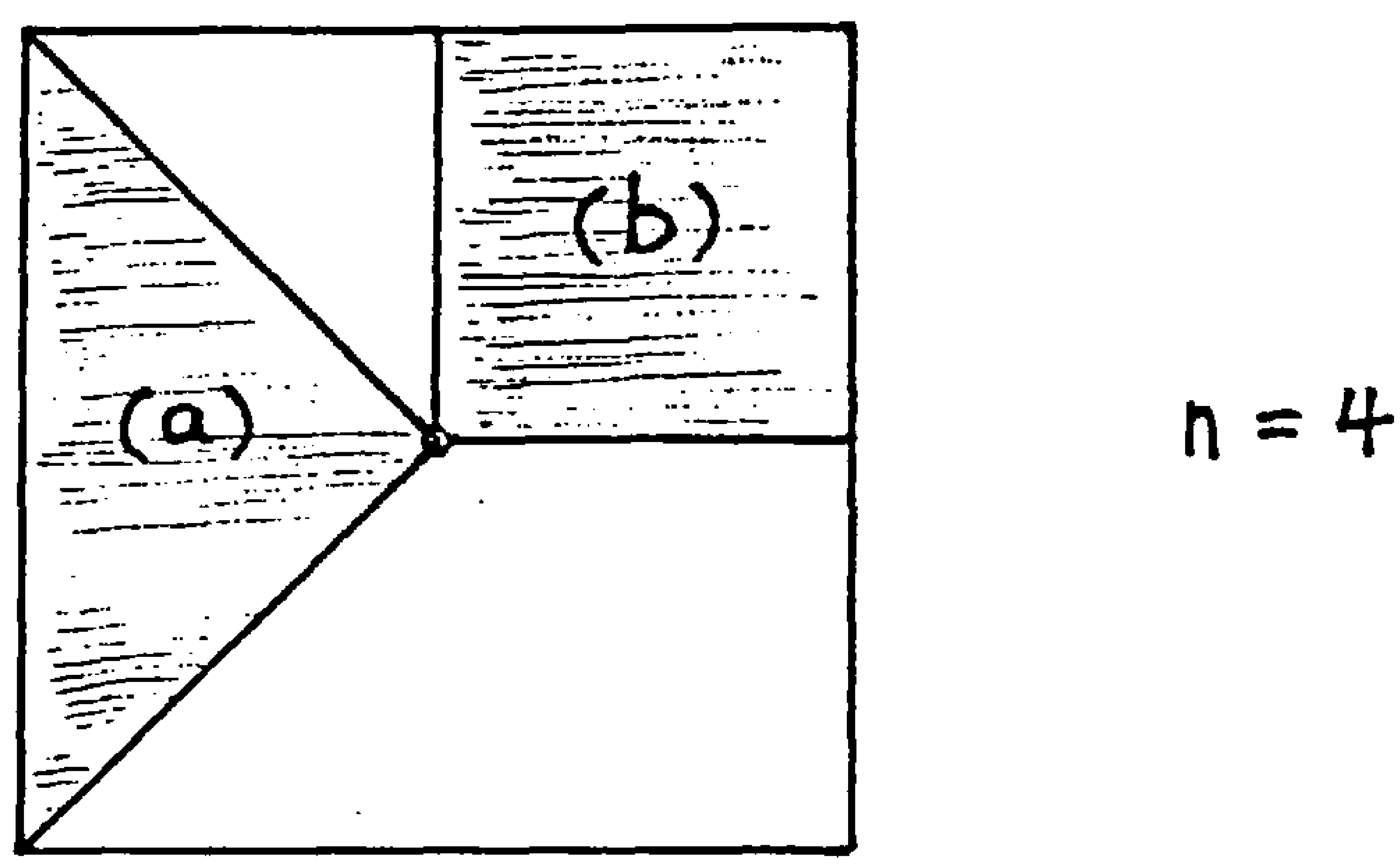


Figure 4.2. Alternative sectoring strategies when n=4



(After: Eilon, S., et al (1))

Figure 4.3. Alternative sectoring strategies when n=48.

Figure 4.3.1.

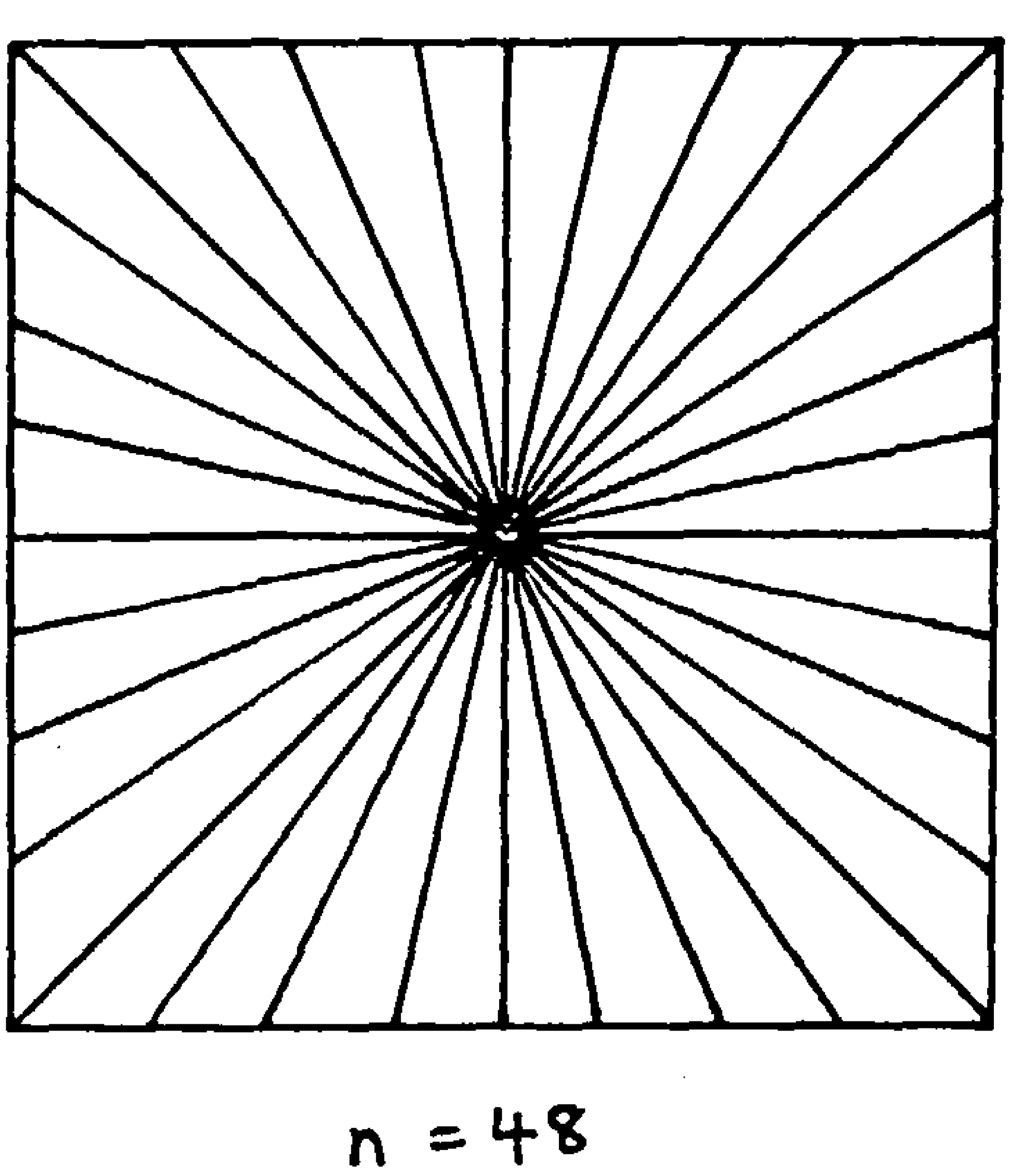
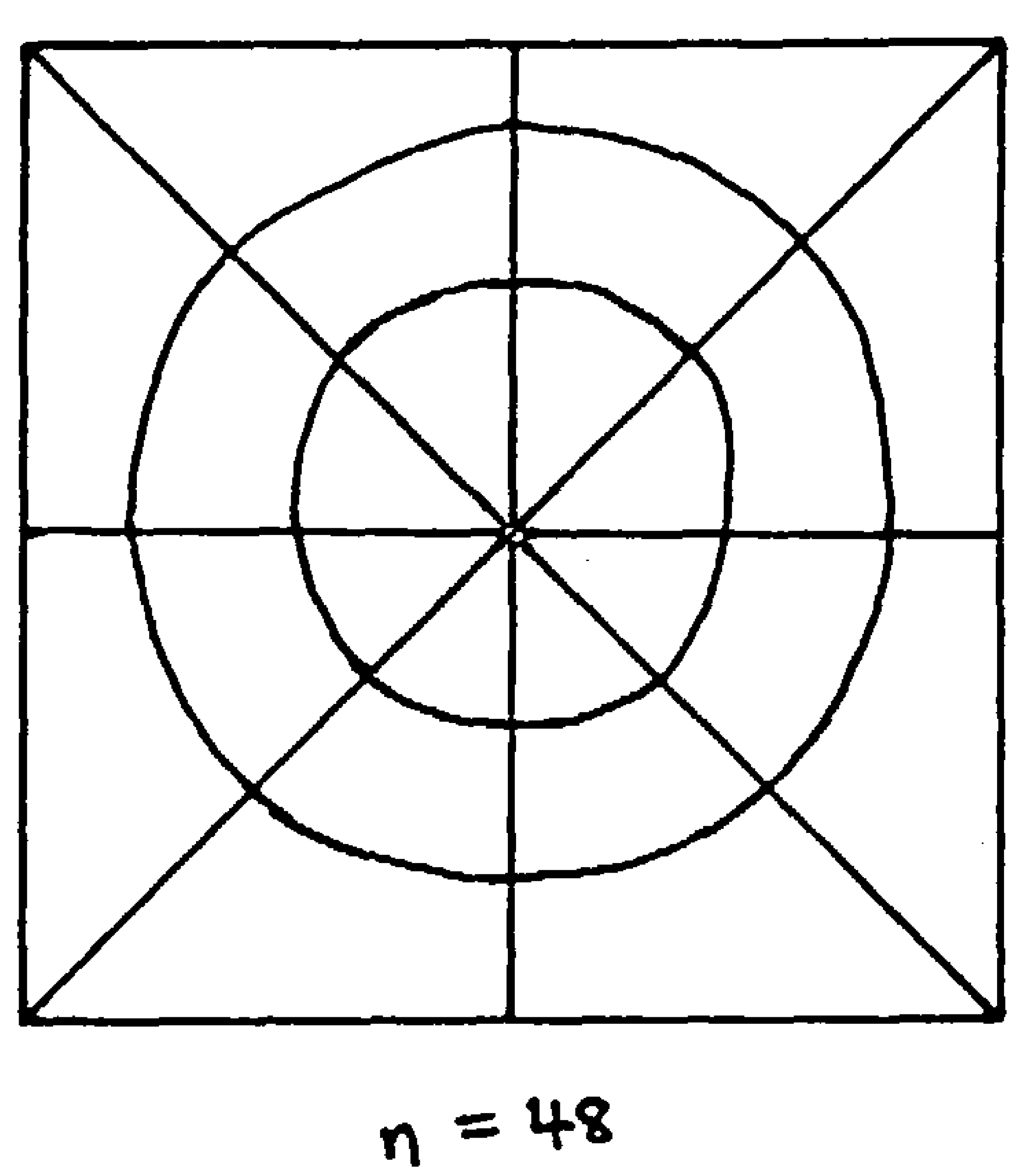


Figure 4.3.2.



the length of the rectangle's shortest side being divided by that of the longest side, (SEE Figure 4.5.). For example, if the rectangle measures 4 miles by 1 mile, then the sector's Slenderness Factor is 0·25; similarly, a square sector will have the maximum s-value of 1·0, (2).

Despite the importance of sector-type to the relationship between Total Fleet Mileage and the number of vehicles employed, the point at which one sectioning strategy becomes more appropriate that another will not be pursued further in this section. Since the number of vehicles involved in the following calculations is rarely so high as to make vehicle delivery-zones unrealistically slim, the delivery-area for current purposes is invariably divided into n wedge-like segments.

4.2. A Simple Formula for Total Fleet Mileage

In order to establish an expression to describe the relationship between the variables m and n, a simple method was used: a set of customer-locations was generated using random numbers and travelling-salesman tours were constructed manually for selected numbers of vehicles ranging from 1 to 15 inclusive, the Total Fleet Mileage being measured in each case. Although this is an extremely crude and laborious method, the results provided an interesting comparison with those obtained using more sophisticated techniques, and, with relatively few customer-locations per vehicle-tour, it was not difficult to arrive at optimal or near-optimal solutions almost immediately using manual methods.

The results of this tour-building exercise are presented in Table 4.1.; although the process was repeated many times for each fleet-size, only the average results are shown in this table, for the sake of brevity. The figures here are based on the assumption that the length of each side of the square delivery-area is equal to 1 unit, (ie. a=1), so that all distances are, in fact, multiples of a. It should also be noted here that, since few of the values of n considered divide exactly into 50, (the total number of customers to be served each week), the number of points generated each time for a given fleet-size often varied. For example, for a fleet of 11 vehicles, 6 vehicle-tours involved 5 customers and 5 tours involved 4, and when n=14, 7 tours included 4 customers and 7 tours only 3; in both cases, the intention was to reflect the fact that, with a total of 50 locations to visit, not all vehicles in a fleet will be scheduled to make the same number of stops each day. The figures in Table 4.1. are presented graphically in Figure 4.6.; clearly, the relationship between Total Fleet Mileage and the number of vehicles used is linear.

Simple Regression Analysis on the data reveals that this relationship may be described by the equation,

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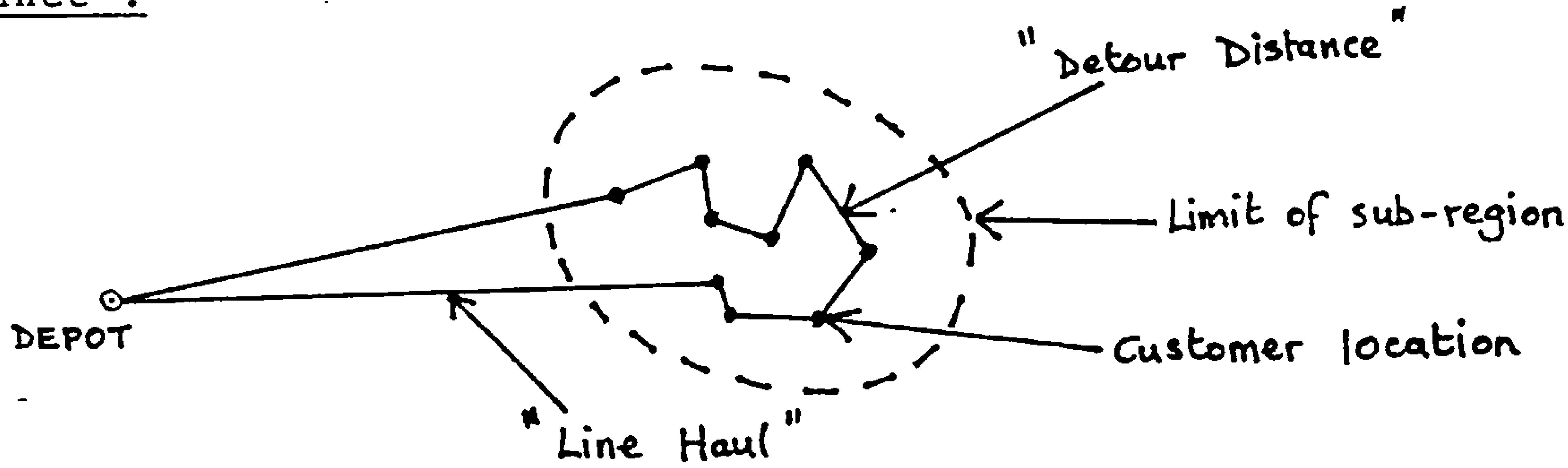
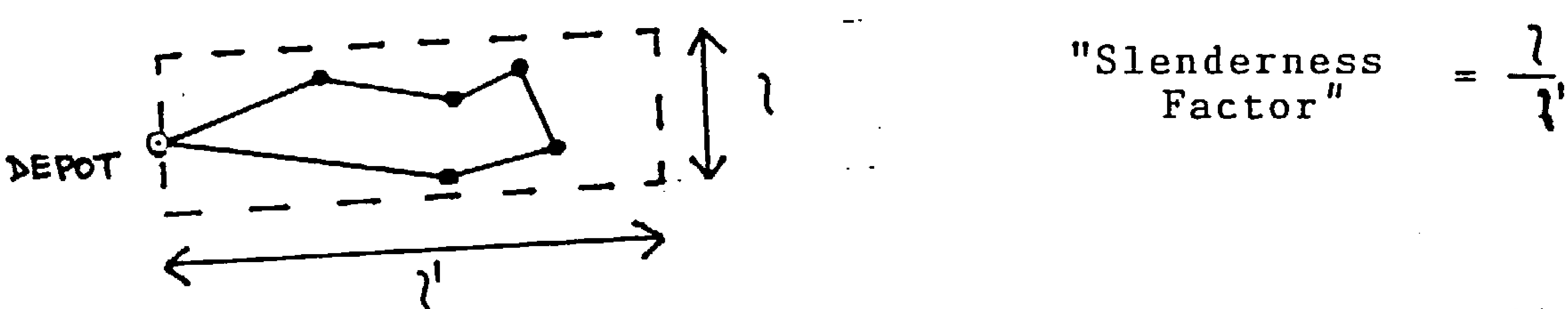
(2) DAGANZO, C.F., "The distance travelled to visit N points with a maximum of C stops per vehicle: A manual tour-building strategy and Case Study." Research Report Institute of Transportation Studies, University of California. (Aug. - Sep., 1982).

Figure 4.4. Daganzo's definition of "Line Haul" and "Detour Distance".



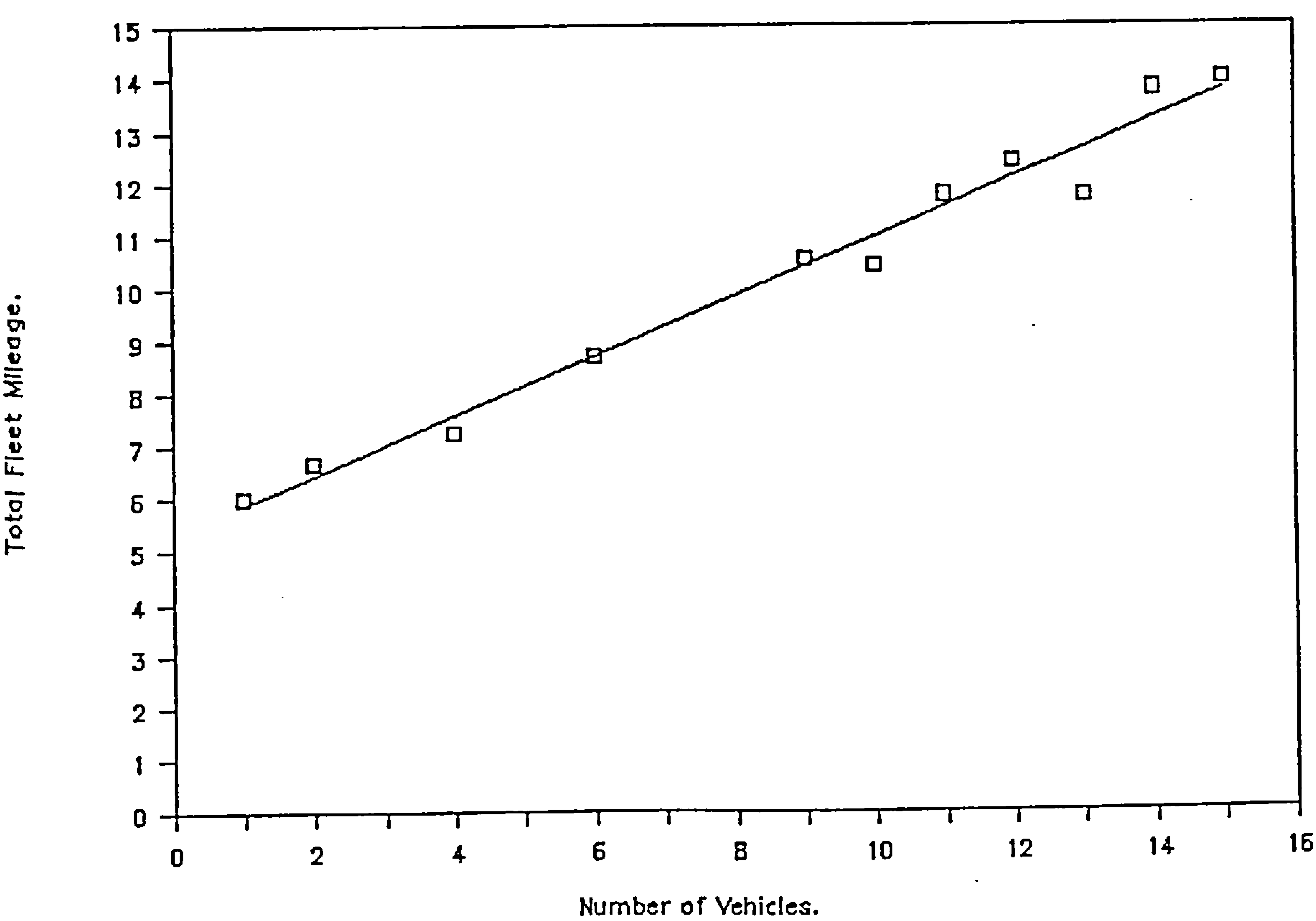
(After: Daganzo, C.F., (2))

Figure 4.5. Daganzo's definition of the "Slenderness Factor".



(After: Daganzo, C.F. (2))

Figure 4.6. Results of manual tour-building Ex.. (P=50, a=1 (square))



$y=5{\cdot}308 + 0{\cdot}566x$, ($R^2=0{\cdot}976$); in other words, using the assumption that a=1, for the purpose of this exercise, the following empirical formula for Total Fleet Mileage may be put forward,

$$\text{Total Fleet Mileage} = a(5{\cdot}308 + 0{\cdot}566n) \quad \text{(where } C=50\text{)} \qquad (E.4.2.)$$

The manually-derived figures of Table 4.1. closely match those obtained from computer simulations of the same problem-formulation; Table 4.2. compares the two sets of figures for values of n of up to 15, which are also plotted in the graph of Figure 4.7..

The computer-generated results were produced by means of a travelling-salesman program based on the Savings Method, although Table 4.2. suggests that the tours constructed by this algorithm are not, in this case, significantly superior to those derived manually; this is, perhaps, not surprising in view of the simplicity of the routing problems involved in this analysis.

The main limitation of Equation E.4.2. is that this formula is only applicable to a situation where there is a population of 50 customers in the delivery-area; this is confirmed by Table 4.3., which shows the average length of computer-generated travelling-salesman tours within the same square area, when the number of locations to be visited is 20 and 100. It is obvious from this Table that an expression for Total Fleet Mileage as a function of both the number of vehicles employed <u>and</u> the number of customers served would be far more useful; such an expression will be developed in Section 4.3., and the effect of the total number of customers served on Total Fleet Mileage will then be considered in greater detail.

An alternative formula for estimating the total distance travelled by a fleet of vehicles is suggested by Eilon et al, and is reproduced here in the form of Equation E.4.3.; again, some of the variables in this expression have been altered, so that they should correspond to the notation that has been used previously,

$$m_d = \frac{A.D_r}{C} + B.a^{0{\cdot}5}.D_r^{\,0{\cdot}5} \qquad (E.4.3.)$$

where, D_r = the sum of the radial distances from the depot,
C = the number of stops per vehicle,
A and B are constants
and, m_d = Total Fleet Mileage per day.

Total Fleet Mileage.

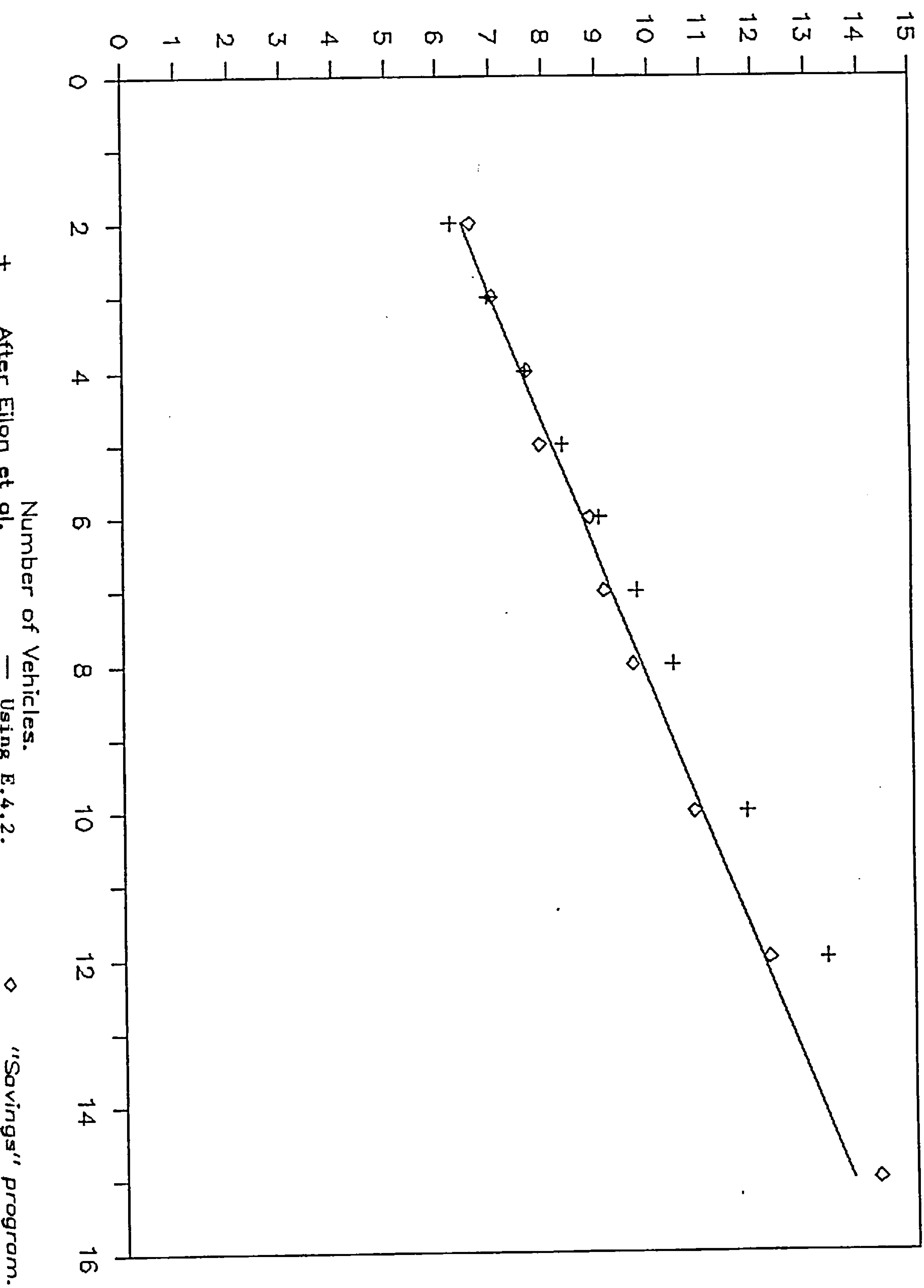


Figure 4.7. Estimates of Total Fleet Mileage. (P=50, a=1 (square))

Table 4.3. Results of computerised tour-building exercise, varying the number of customers to be served. (a=1 (square))

	Average Total Fleet Mileage		
Nº. of vehicles	P=20	P=50	P=100
1	4·068		
2	4·684	6·617	
3	5·029	7·015	9·217
4	5·488	7·680	9·536
5	6·327	7·920	9·591
6	6·630	8·855	10·497
7	7·430	9·130	11·578
8	—	9·668	12·243
10	8·418	10·830	13·294
12		12·232	14·206
14			15·589
15		14·274	

It should be emphasised that Total Fleet Mileage in this case, "m_d", represents the distance travelled by the fleet in <u>one day</u>, so that this variable should not be confused with the variable "m" in Equation E.3.7. which represents a <u>5-day</u> distance figure. The parameter "C" describes the number of stops made by each vehicle per tour, and is invariably an integer when used elsewhere in this thesis; however, Eilon et al describe their equivalent variable as,

> "......the average value of the maximum number of customers that can be supplied on one route.........."

not specifying whether the constraint determining what this maximum number of deliveries may be is a function of vehicle-capacity or some time factor. Nevertheless, this variable is merely the discrete equivalent of the integer "C", or,

$$\frac{P}{n} \qquad (E.4.4.)$$

where, P = the total number of customers to be served.

The key independent variable in Equation E.4.4. is D_r, which represents the sum of the distances of all the customer-locations in the delivery-area from the depot; Eilon et al's major research-objective is to examine the relationships between this variable, Total Fleet Mileage and the number of stops per vehicle-tour. An additional objective is to assess the effect of the centrality of the depot's location within the delivery-area on the value of "m_d", and Eilon et al's findings on this subject are outlined in reference (1). The value of the constants "A" and "B", when the depot is centrally-located, are empirically found to be 1·8 and 1·1 respectively; Equation 4.3. therefore becomes,

$$m_d = \frac{1 \cdot 8 . D_r}{C} + 1 \cdot 1 . a^{0 \cdot 5} . D_r^{\ 0 \cdot 5} \qquad (E.4.5.)$$

It is quite valid to compare the estimates of fleet mileage calculated from Equations 4.5. and 4.2., since Eilon et al make the same key assumptions, described above, of a square delivery-area, a set of randomly-located customers and wedge-shaped, non-overlapping vehicle-tours. The aspect in which Eilon et al's estimates differ is in the algorithm employed to generate travelling-salesman tours - rather than rely on manual tour-building or the Savings Method, a "3-Optimal" scheduling algorithm is used. A fuller description of this algorithm, along with a description of the Principle of r-Optimality, also appears in Eilon (1), but it is sufficient here to note that this method involves the generation of an initial feasible route through a given set of points, followed by a reduction in the length of this tour by replacing 3 of the existing links with 3 alternative links, (Eilon et al).

Estimates derived from Equation E.4.5. are presented in Table 4.2., alongside the figures produced using both manually-constructed and computer-generated tours. When making calculations from Equation E.4.5., the value of D_r was based on a finding from the results of the entire computerised tour-building exercise, this being that the average distance of all customer-locations from a centrally-located depot in a square delivery-area is 0·36478 when a=1; D_r may be readily calculated by multiplying this figure by the number of customers to be served.

The three sets of estimates contained in Table 4.2. are very similar, and this is re-emphasised by Figure 4.7., which shows a graph of the three corresponding distributions. One interesting feature of Figure 4.7. is that the estimates given by E.4.5. tend to be less than those derived by simulation for lower values of n and higher for larger fleets, so that the line on the graph referring to Eilon et al's figures has a slightly steeper slope. This phenomenon was also found when estimates using the same equation were compared with the results of computer simulations with 20, and then 100, customers.

Clearly, a major advantage of using Eilon et al's equation as opposed to the expression of E.4.2. is that the former expression may be applied to situations involving any number of customers, the latter referring solely to the problem of delivering to 50 locations. The following section therefore describes the development of a formula for calculating Total Fleet Mileage that takes account of both changes in the size of the fleet of vehicles employed and variations in the number of customers served, by expressing tour-distance as a function of both "n" and "C".

4.3. The Development of a Formula for Estimating the Total Distance Travelled to Serve any Number of Customers

Throughout this section it will be shown that the total distance travelled by a fleet of vehicles is a function, not only of the number of vehicles employed in the fleet, but also the total number of locations that are to be visited; it has already been pointed out that, because of the influence of the latter, Equation E.4.2., on its own, is not an adequate tool for estimating Total Fleet Mileage. What is important, here, is the combination of the variables n & P to determine the number of trips, C, which must be made by each vehicle.

The fresh aspect of the following discussion is the introduction of a distinction between the two components of a travelling-salesman vehicle-tour: "Stem Distance" and "Delivery Distance". This dichotomy is analogous to Daganzo's distinction between "Line-Haul" and "Detour Distance", referred to earlier in this chapter and illustrated in Figure 4.4., which he uses to differentiate between the separation of a sub-region or cluster of customers from the depot, and the total mileage travelled within that sub-region, (Daganzo, 1982). Stem Distance and Delivery Distance differ from the concepts defined by Daganzo only in as much as they refer to parts of a vehicle-tour that serves customers located at any distance from the depot; Stem Distance is the sum of the distances from the depot of the first and last customers to be served in each tour, whilst Delivery Distance is the distance required for the vehicle to travel from the first customer to the last and is thus directly equivalent to "Detour Distance". Both of these components of a vehicle tour are affected by the number of locations that a vehicle must visit, but in different ways; for this reason, Stem Distance and Delivery Distance will be discussed separately.

4.3.1. Stem Distance

Stem Distance accounts for a significant proportion of a vehicle-fleet's Total Mileage, since each individual vehicle has to make two "stem journeys" each day at the beginning and end of every delivery-round. Obviously, the importance of Stem Distance relative to Delivery Distance depends on the size of the vehicle-fleet, since Total Stem Distance is directly related to n; what is less clear is the relationship between n and Stem Distance per Vehicle! This depends greatly upon the method used for constructing travelling-salesman tours, since the choice of algorithm will directly influence which customers are likely to be visited first and last in each route.

For example, one method of routing might be to begin a vehicle-tour at the nearest location to the depot, and then continue to include in the tour the next-nearest location until the constraint on the length of a working-day demands that the vehicle should return to the depot. Using this crude technique, the "stem journey" to the first customer will always be relatively short - for as long as the locations closest to the depot remain available - but the distance from the last customer-location on the route to the depot would be more or less random!

In the most simple situation, in which it can be assumed that it is purely a random process as to which customers link directly to the depot, then Stem Distance may be estimated using the parameter D_r, which has been defined previously in this chapter as the average distance of all customer locations from the depot. In this case, Stem Distance per vehicle is approximately $2.D_r$, so that Total Stem Distance for the fleet may be calculated as $n(2.D_r)$. The value of D_r is simply a function of the size of the delivery-area, although it is necessary to point out that delivery-area shape may also have an influence on this parameter.

From all of the computerised tour-building exercises conducted in order to produce the data on which this thesis is based, it was found that the average value of D_r for a square delivery-area, whose sides are each "a" units in length, is approximately 0·3648a units, whilst the average D_r value of a circular zone which si "a" in diameter is approximately 0·3315a. The main reason for the discrepancy in these two figures is the fact that the square zone does, of course, cover a larger area. The figures are also both rather below Daganzo's estimate that D_r in a square zone is roughly 0·383a, (Daganzo, 1982).

However, the parameter D_r has limited relevance to this section, since, in the real world, customers visited first and last by each vehicle will not be at random distances from the depot; it is more likely that customer locations directly connected to the depot in actual vehicle tours, whether these tours are constructed manually or by computer, will be located rather closer to the depot than D_r. Furthermore, the computer program that is used here to contruct travelling-salesman tours is based on the Savings Method, an algorithm whose solution tend to begin and end with points that are located close to the depot; the reason for this is evident from the savings formula, reproduced here as Equation E.4.6., which calculates the distance that is saved by linking a pair of locations in the same tour rather than visiting them both separately,

$$S = a+b-x \qquad (E.4.6.)$$

where, S = "savings value" of linking a pair of points,
x = the distance separating the two points in question,
and, a & b are the respective distances of the two points from the depot.

The Savings Method involves the linking of pairs of customer-locations to form vehicle-tours, starting with the pair having the highest savings value and continuing to connect pairs of points in descending order of savings value. This procedure continues until either all customers requiring a delivery are included in one large vehicle-tour, or time or capacity contraints dictate that a tour may include no more locations. It is clear from Equation E.4.6. that pairs of locations far away from the depot will become connected before less remote pairs that are the same distance apart, so that points located closest to the depot are most likely to be linked directly to it. For this reason, Stem Distance will always tend to be less than D_r.

Even when employing the Savings algorithm, Total Stem Distance will also very according to the particular problem formulation and the idiosyncrasies of the method used. For instance, in this chapter, when the size of the fleet, n, is greater than 1, Total Fleet Mileage is calculated by generating a random set of P/n co-ordinates within a segment whose angle is 360/n degrees at the depot, and then multiplying all distances by n. In this case, Total Stem Distance is most often the sum of the distances from the depot of the two locations that are closest to this depot, multiplied by n. However, if n tours were built, from P customer-locations distributed throughout the entire delivery-area, then a different figure for Total Stem Distance would result; this is because customers that are located further and further away from the depot would be at the start and/or finish of a vehicle-tour. For example, if 50 customers are to be served by a fleet of 10 vehicles, then even assuming that it is always the least remote locations that link directly to the depot, the average length of the 20 stem-journeys made each day will not be the same as the corresponding figure derived by the method that is actually used in this analysis, (as described previously).

It may be concluded, therefore, that, using the Savings Method to construct hypothetical vehicle-tours, the parameter D_r can not be employed as an adequate estimate of Stem Distance; since it is by no means guaranteed that this algorithm will always link the nearest locations directly to the depot in every tour, the most satisfactory way to investigate the nature of Stem Distance is to do so by simulation.

Because of the nature of the Savings Method, it is not difficult to isolate the stem-journey component of Total Fleet Mileage once a program for constructing travelling-salesman tours has been created. Having generated a set of P randomly-scattered points, (SEE Figure 4.8. for a detailed summary of the procedure used), the program calculates Total Fleet Mileage as being equal to $(2.P.D_r)$; in other words, it is initially assumed that all customers are served by a single vehicle that returns to the depot after visiting each location, so that Total Distance is equal to Stem Distance and Delivery Distance is zero. When the two locations with the highest savings value are connected to form the first tour, the distance that is thus saved, (ie. the savings value associated with this pair of locations), is subtracted from the existing figure for Total Fleet Mileage, (ie. from $2.P.D_r$). This process continues until all customer-locations are combined in a continuous tour, at which point Delivery Distance is the sum of all the savings values that are subtracted from the original $2.P.D_r$ figure; Stem Distance must therefore represent the difference between Delivery Distance and the figure that is eventually calculated for Total Distance. Overall figures for Stem, Delivery and Total Distance, as fleet-size increases, are presented in Table 4.4. and illustrated by Figure 4.9., the major assumptions here being that the delivery-area is a square of dimensions 1 x 1, and that P=100. This graph emphasises the importance of Stem Distance as a component of Total Fleet Mileage, since Delivery Distance is virtually

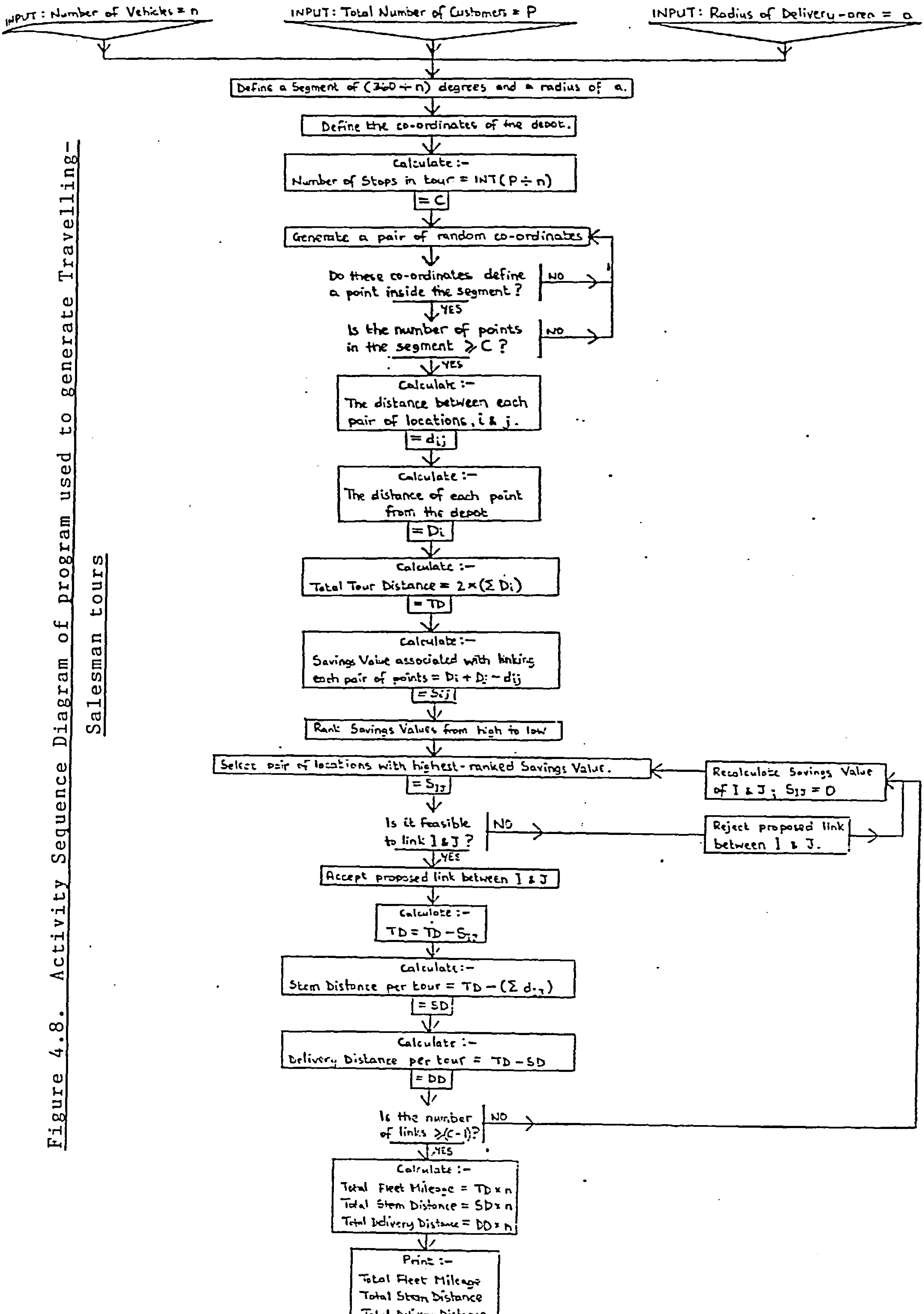


Figure 4.8. Activity Sequence Diagram of program used to generate Travelling-Salesman tours

constant as n changes. The flatness of the Delivery Distance curve is, to some extent, a reflection of the particular method employed here to generate tours; if an alternative method were used that involved the building of n tours to serve P customers within the entire "a x a" delivery-area, it would be reasonable to expect that Delivery Distance would decrease slightly as fleet size increased, because the number of stem journeys would rise at the expense of the number of links between customer-locations. However, using the current method of building tours to serve C customers in a delivery zone 1/n of the size of the larger area, Total Delivery Distance remains virtually constant, as Figure 4.9. shows.

The growing dominance of Stem Distance over Delivery Distance as fleet size increases is reemphasised by Figure 4.10. and Table 4.5., which show Stem Distance as a percentage of Total Distance.

Because of the particular method used to produce the figures contained in Table 4.4., it is perhaps more useful to examine distances per vehicle trip, since all estimates of distance for the total fleet here are merely figures per vehicle multiplied by n; figures for distances per vehicle-trip are presented in Table 4.6. and plotted together in Figure 4.11.. What is interesting in this graph is the way in which Total Distance per vehicle is disaggregated in terms of both Stem and Delivery Distance, since Stem Distance per vehicle increases with fleet-size, whilst, at the same time, Delivery Distance actually declines. In other words, although the rather linear relationship between Total Fleet Mileage and fleet size shown in Figure 4.9. tends to suggest that Total Distance increases rather uniformly as n is increased, Figure 4.11. indicates that the processes affecting Fleet Mileage are far more complex, since Stem Distance and Delivery Distance are influenced by changes in n in different ways.

To complicate the situation further, with a fixed population of customers to visit - in this case, P is fixed at 100 - an increase in the number of vehicles used affects distances both by reducing the size of the area in which each vehicle must operate and by decreasing the number of locations that each vehicle must visit.

To examine these two processes in turn, in terms of the way in which they affect Stem Distance first of all, it would seem reasonable to hypothesise that the width, or "slenderness", of a vehicle's delivery-sector should not, on its own, be expected to significantly change Stem Distance. This is borne out by the figures in Table 4.7., (SEE also Figure 4.12.), figures that were obtained by varying the number of vehicles used, and thus changing the angle of each vehicle's delivery-segment, whilst keeping the number of locations visited in each round-trip, C, constant. This graph suggests that there is, indeed, no effect on Stem Distance per vehicle as a result of changes in n itself.

Figure 4.9. Disaggregated distance figures. (P=100, a=1 (square))

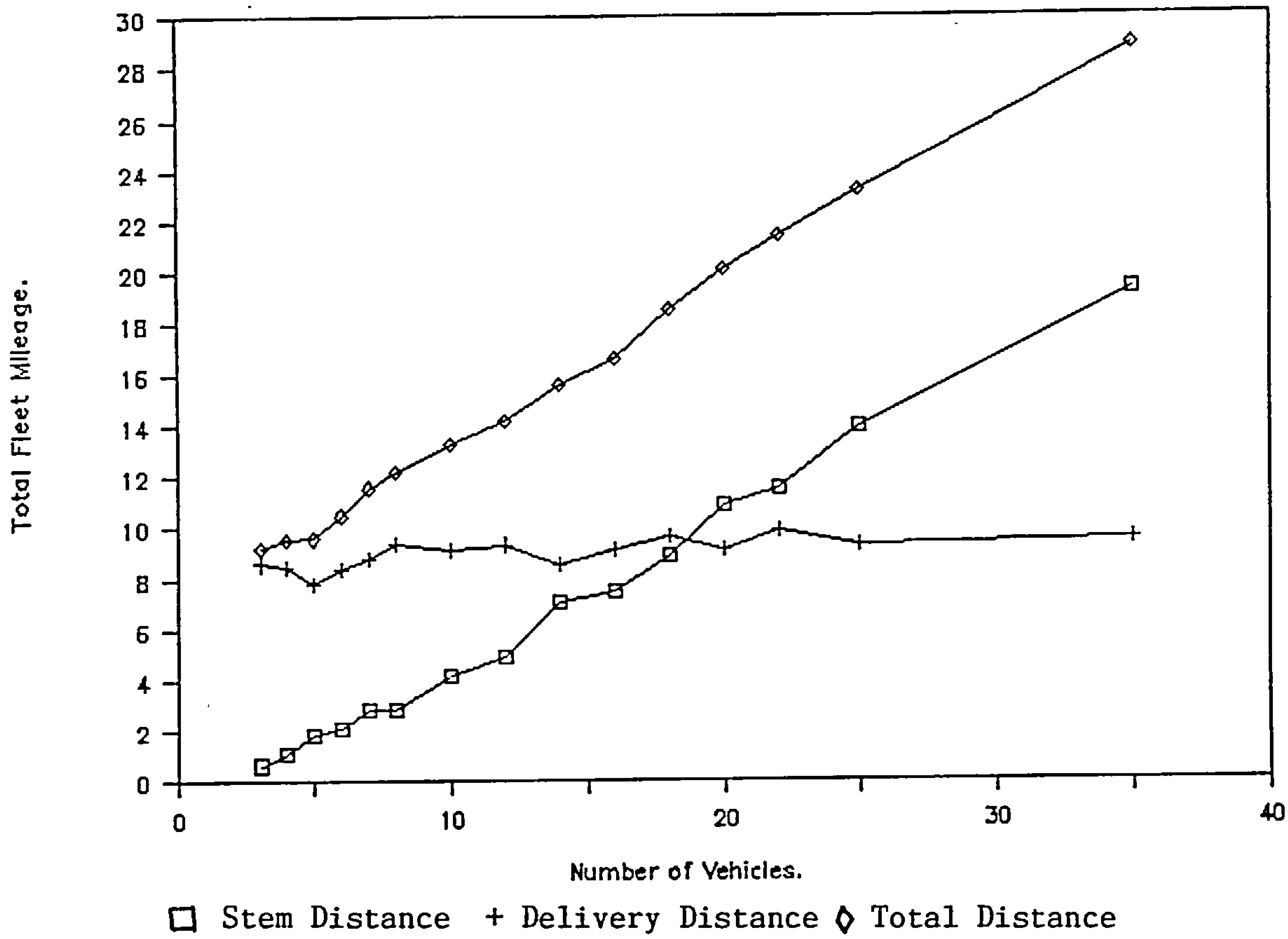
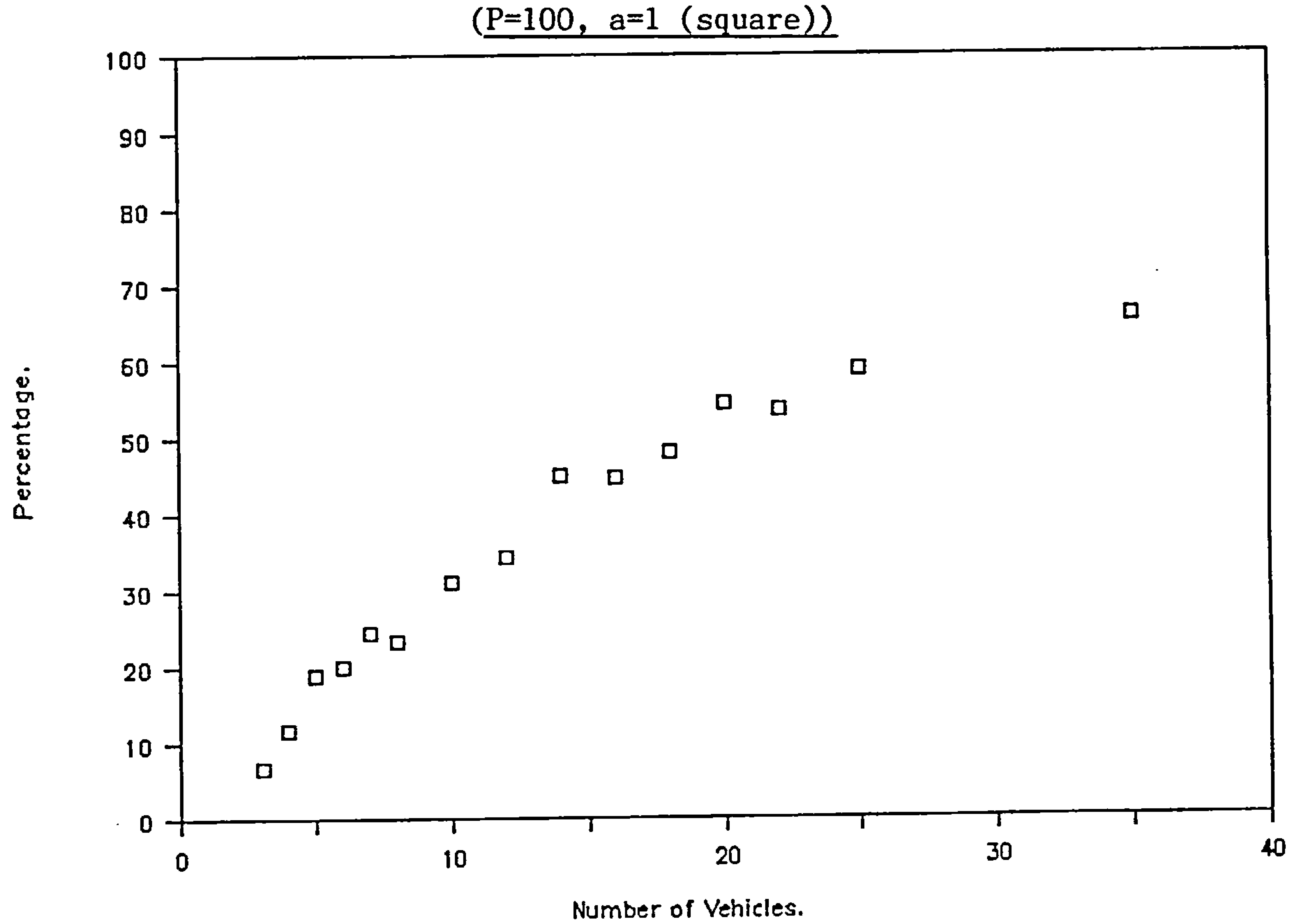


Figure 4.10. Stem Distance as a percentage of Total Distance (P=100, a=1 (square))



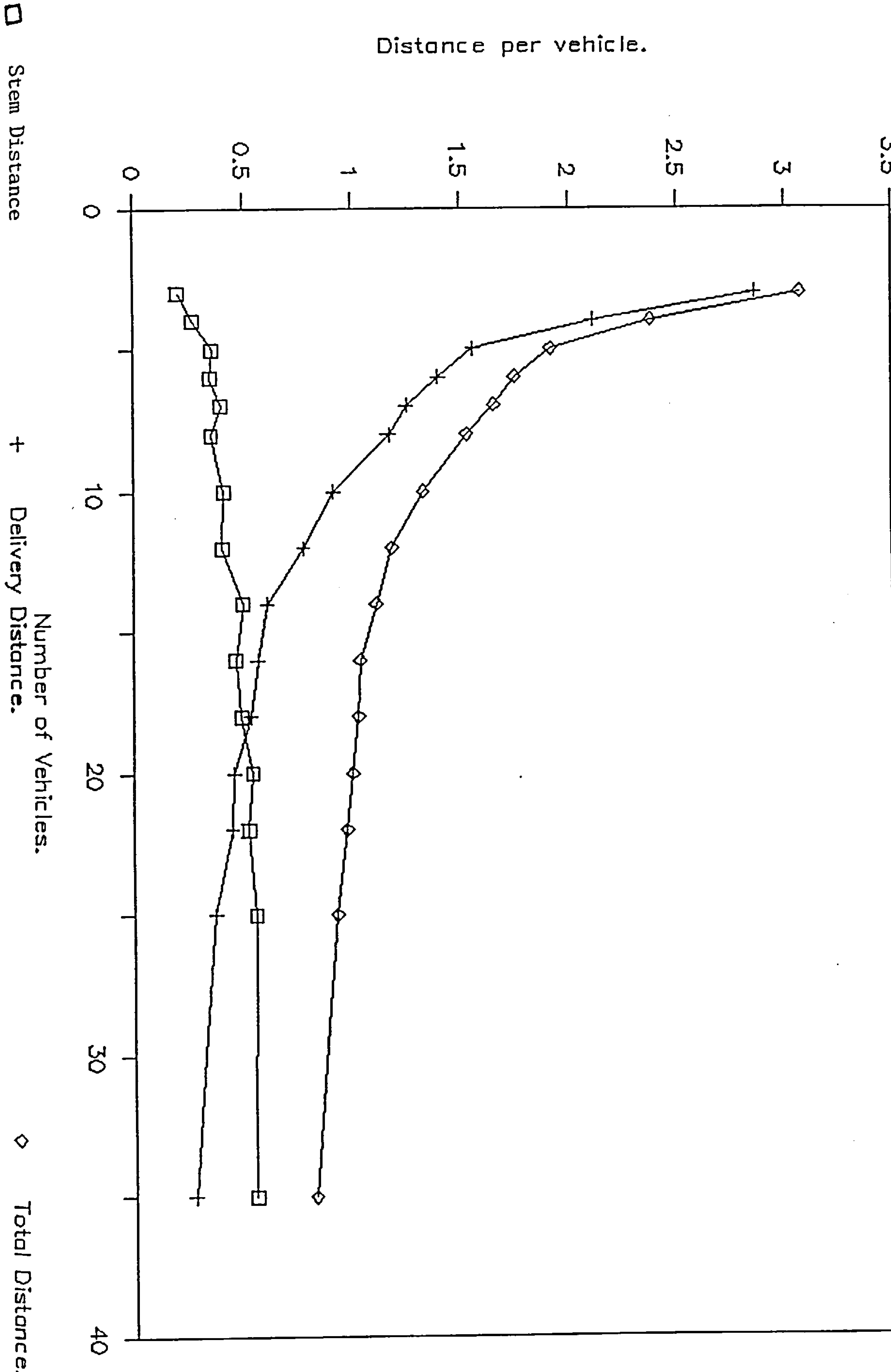


Figure 4.11. Disaggregated distance
(P=100,a=1(square)).

It should be noted, here, that Figure 4.12. presents results that were obtained using a circular area. This is because of the distorting effect that a square area might have on results when n is a key parameter. It is sufficient here to refer to Figures 4.13.1. and 4.13.2., which compare the relationship between the parameter D_r and n for both a square and a circular delivery-area, (SEE also Table 4.8.). These figures clearly show that, with a square delivery-area, the average distance of all customer-co-ordinates from the depot increases substantially for low values of n, whereas the corresponding D_r figures with a circular delivery-area approximate the average value of 0·33155, regardless of fleet-size. The inconsistency shown in Figure 4.13.1. is attributable to the corners of the a x a square, which are excluded when a circle of diameter a is used instead; as it is reasonable to expect Stem Distances to be distorted in a similar fashio when n is small, all subsequent results of simulation discussed in this chapter are founded on the assumption of a circular area.

Having established that fleet-size does not affect Stem Distance per vehicle by reducing the size of the delivery-sector, (SEE Figure 4.12.), the next step in the analysis is to explore the extent to which n does so indirectly by changing the number of locations visited by each vehicle. This was achieved in similar fashion, by changing the value of C whilst holding the size of the delivery-sector constant; Figure 4.14. graphs the resulting Stem Distance per vehicle figures, and these are presented in Table 4.9.. Notice, here, that Stem Distance per vehicle when C=1 is recorded as being 0·663. This figure is based not on the results of a series of simulation trials, but on the fact that, in the hypothetical situation of a vehicle having only one location to visit, Delivery Distance per vehicle is zero, whilst Stem Distance per vehicle is simply twice the value of D_r; the figure of 0·663 in Table 4.9. is therefore derived by multiplying 0·33155 by 2.

In this case, the value of n is 20, so that the angle of the delivery-sector at the depot is 18 degrees, although similar experiments, using different values of n, confirmed the earlier finding that fleet-size has no direct influence on Stem Distance per vehicle, as almost the same curve as that shown in Figure 4.14. was produced each time. An example of this is provided by Figure 4.15., which shows the corresponding results when n=4.

The general conclusion that may be drawn from Figures 4.12., 4.14., and 4.15. is that, regardless of the delivery-sector's size, it is solely the number of stops that must be made by a vehicle that determines how far from the depot the nearest customers will be located, (accepting that it is to and from these nearest locations that stem journeys will be

Figure 4.12. Stem Distance per vehicle with C fixed. (C=20, a=1 (circle))

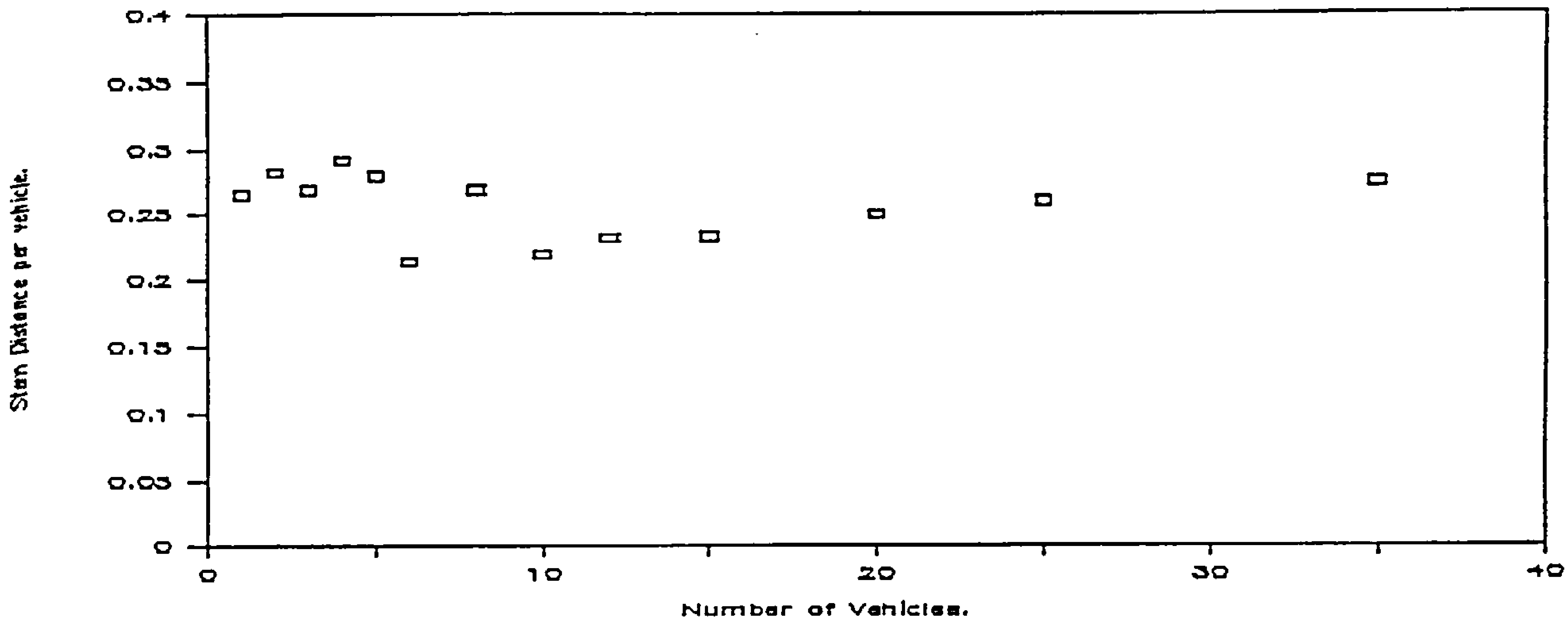


Figure 4.13.1. Relationship between D_r and Fleet Size (a=1 (square))

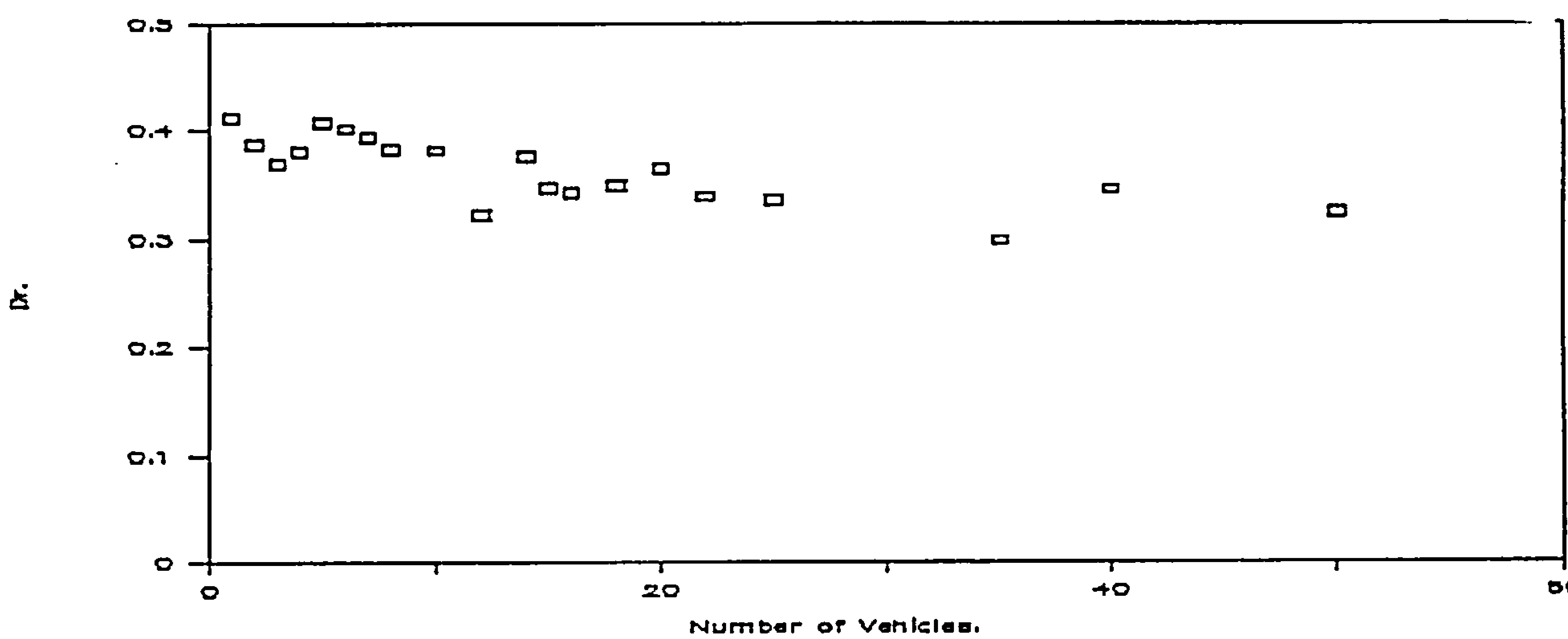


Figure 4.13.2. Relationship between D_r and Fleet Size (a=1 (circle))

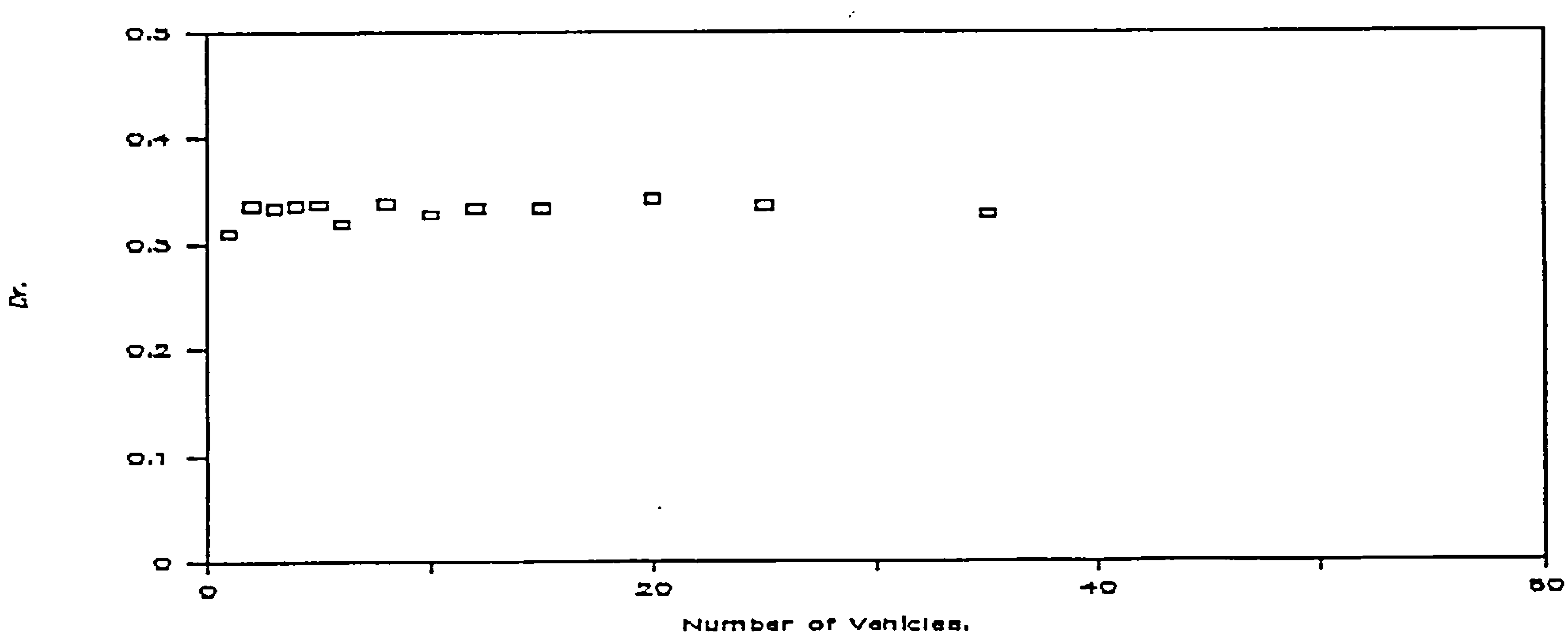


Figure 4.14. Stem Distance per vehicle

with n fixed. (n=20,a=1(circle)).

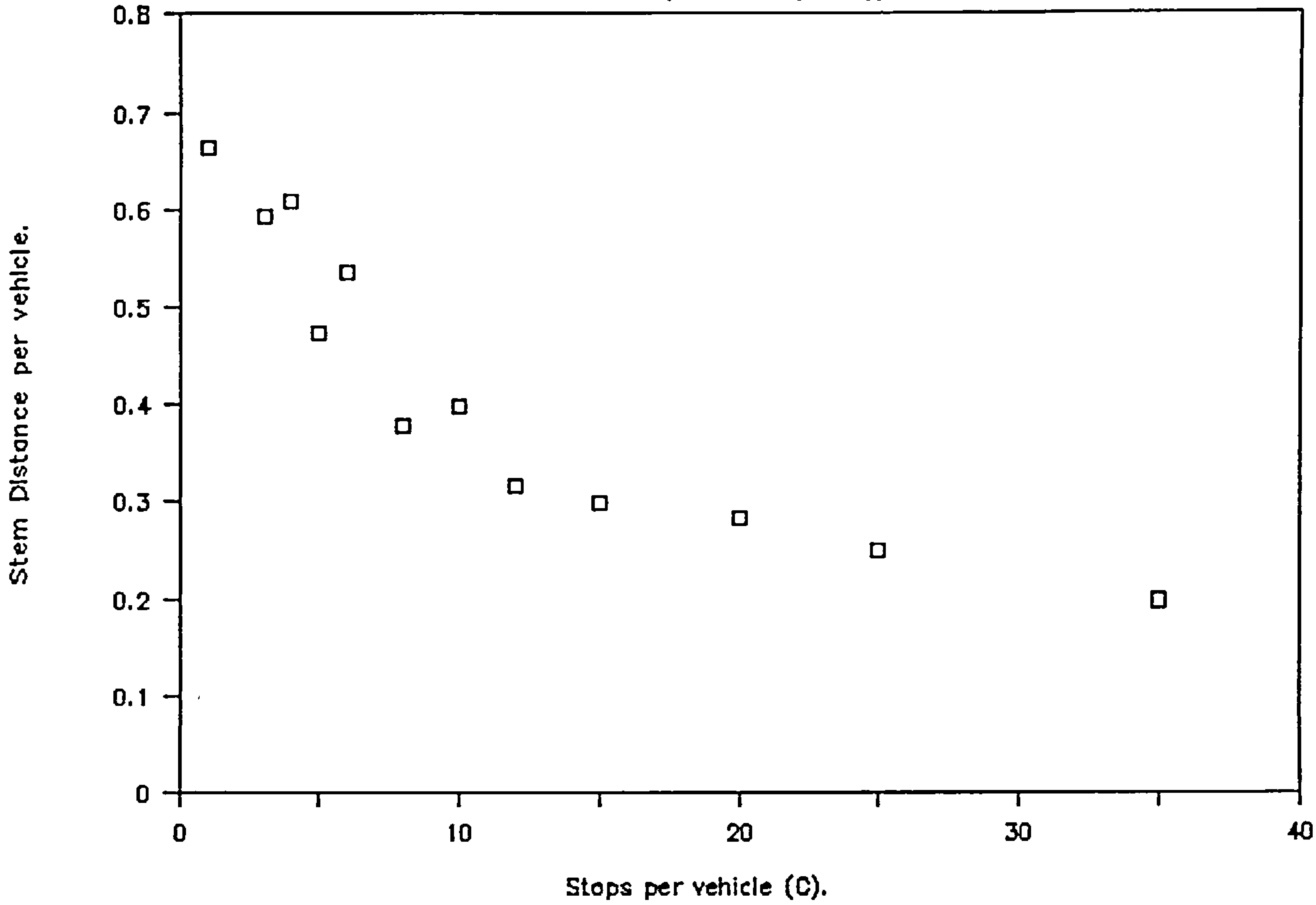
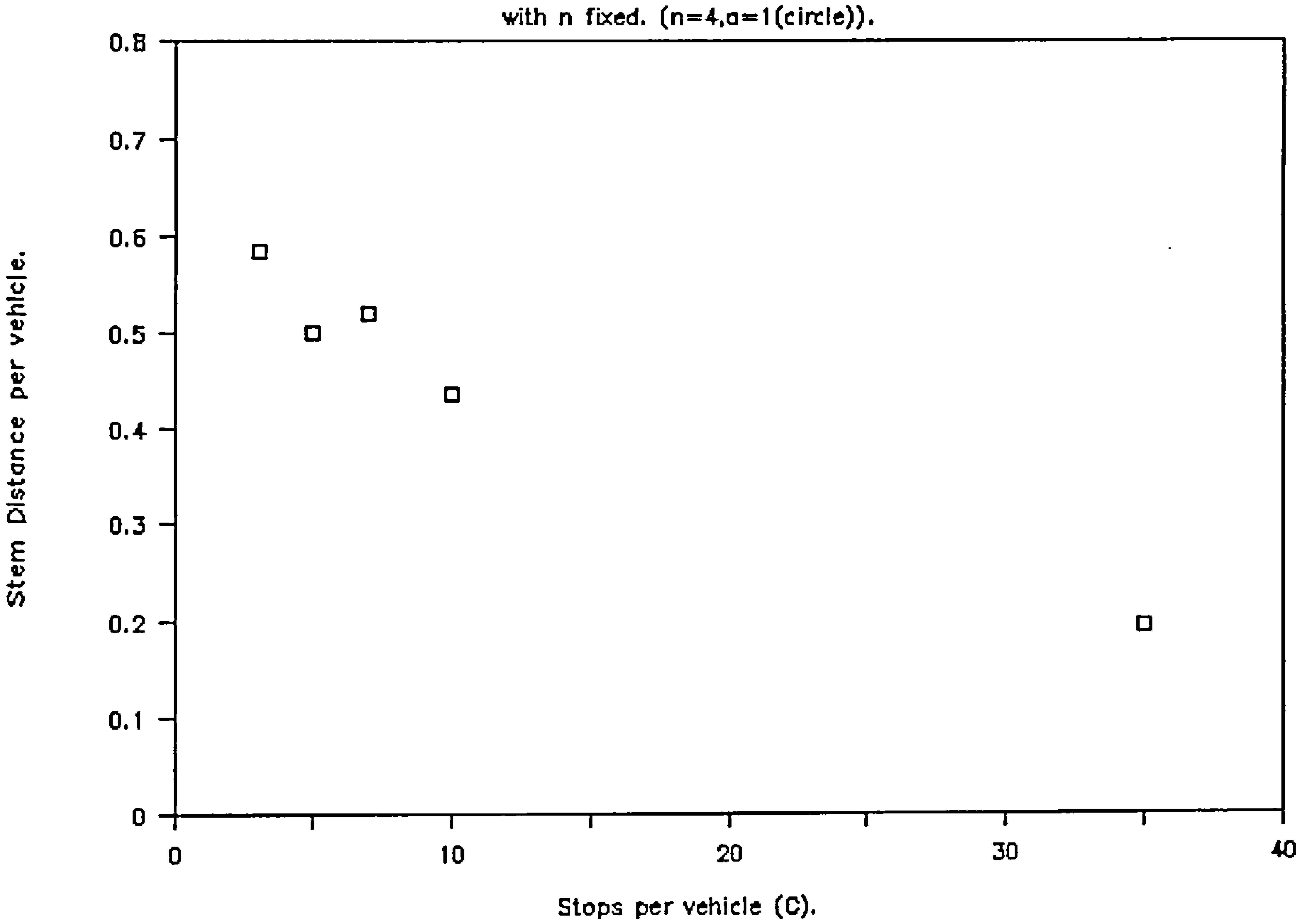


Figure 4.15. Stem Distance per vehicle

with n fixed. (n=4,a=1(circle)).



made). This finding is, of course, perfectly logical, since, if a depot is looked upon as being just one of (C+1) points located in a circle, which just happens to occupy a central position, then Stem Distance is simply a reflection of the average spacing of the (C+1) points, which, in turn, is a direct function of the density of these points. Obviously, as (C+1) increases, so the density of the points within the circle will increase and Stem Distance will be reduced. The evidence presented here suggests that the implications for Stem Distance will be the same whether these points are distributed throughout the whole of the delivery-area or within a limited section, (although the same would, of course, not be true for Delivery Distance per vehicle). Figure 4.16. provides a numerical illustration of this explanation; Stem Distance in both Figure 4.16.1. and 4.16.2., is identical, since the two distributions are the same both in terms of the distance of each point from the depot and, more importantly, in terms of the probability of such a distribution of points being produced by a random co-ordinate generator. (This is because, during the computerised construction of tours, random co-ordinates are actually generated for the whole delivery-area, and when the value of n causes attention to be focused upon a smaller segment of this area, all of those points which lie outside this segment are rejected).

The main implication of the above findings is that Stem Distance per vehicle may be estimated purely on the basis of the number of customers served in each vehicle-tour, to which it is inversely related; a summary of the relationship of C to the other key parameters involved here is provided by Figure 4.17..

Obtaining an expression for Stem Distance per vehicle as a function of C may be readily achieved by Regression Analysis. This analysis was carried out on the data contained in Table 4.9.; Figure 4.14. reveals a curved distribution when these figures are plotted on a graph, and so a semi-logarithmic transformation was performed on them in order to make it possible to describe the relationship between Stem Distance per vehicle and C with a linear regression-line. The resulting graph is shown as Figure 4.19., and the relevant data is contained in Table 4.10.. After Regression Analysis on this partially transformed data, the following expressions were derived,

$$\text{Stem Distance per vehicle} = a(0{\cdot}7285 - 0{\cdot}3418 \log.C) \qquad \text{(E.4.7.)}$$

and, therefore, $(R^2 = 0{\cdot}9059)$

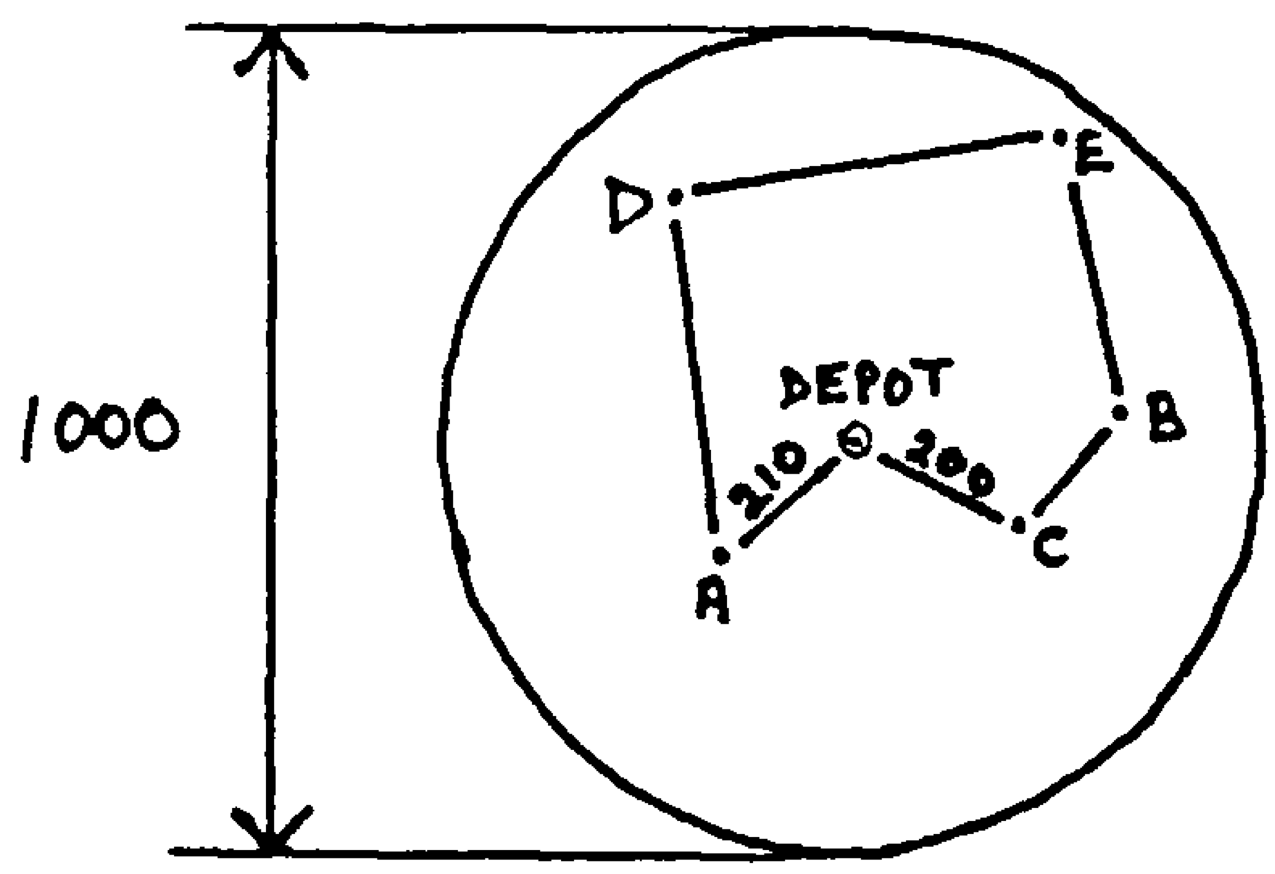
$$\text{Total Stem Distance} = n.a(0{\cdot}7285 - 0{\cdot}3418 \log.C) \qquad \text{(E.4.8.)}$$

where a = the diameter of the delivery-area.

Table 4.11. and Figure 4.18. compare the predicted values of

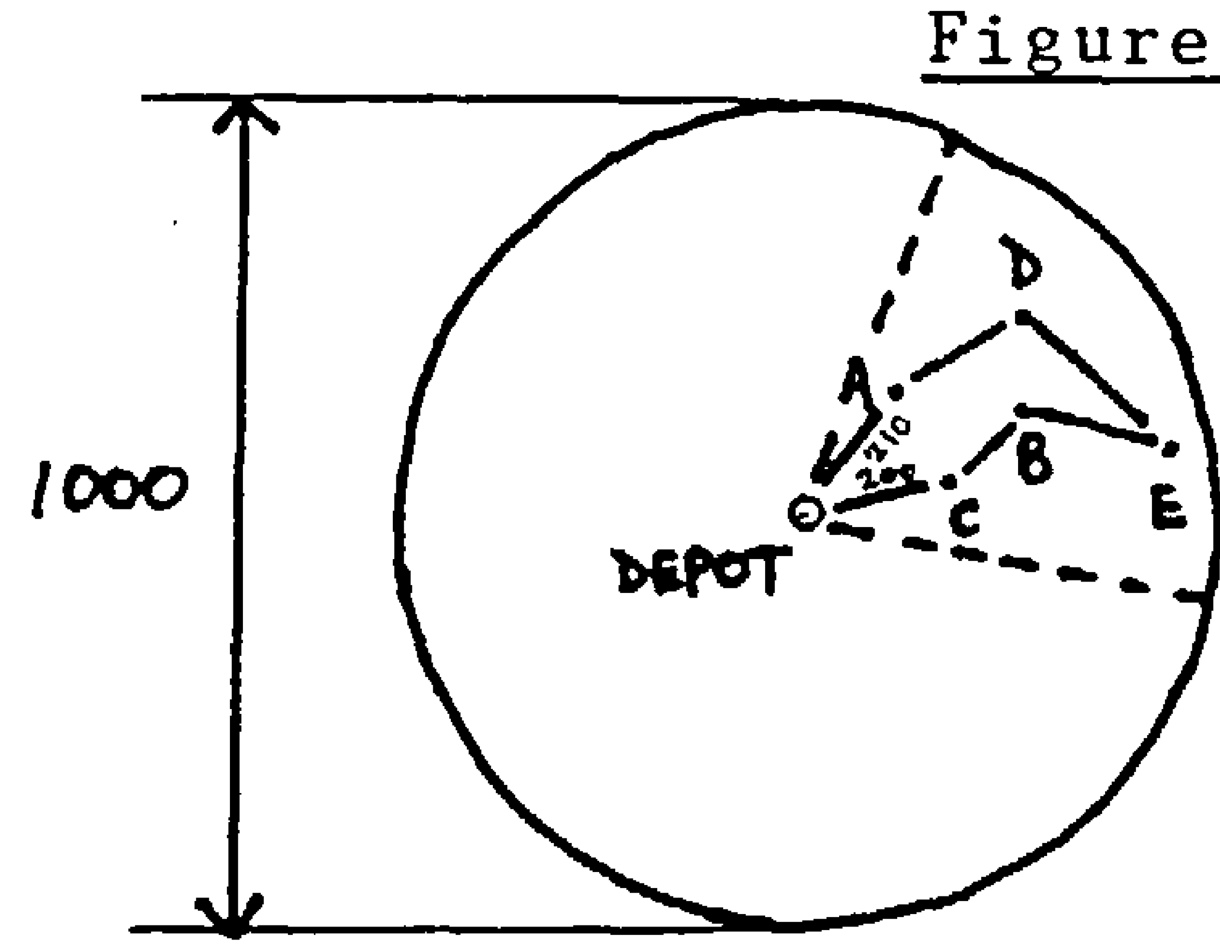
<u>Figure 4.16. Illustration of the effect of reducing vehicle-sector size on Stem Distance and Delivery Distance per vehicle</u>

<u>Figure 4.16.1.</u>



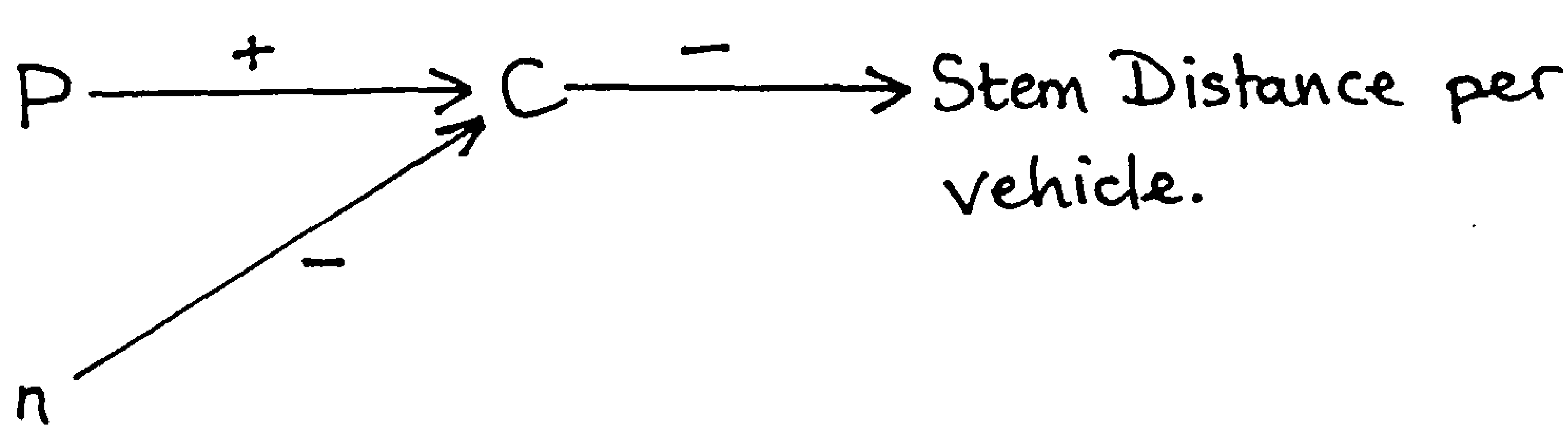
P=5
n=1
C=5
Stem distance
per vehicle = 410

<u>Figure 4.16.2.</u>



P=40
n=8
C=5
Stem Distance
per vehicle=410

<u>Figure 4.17. The mechanisms affecting Stem Distance per vehicle</u>



where, P = Total Number of Customers to be visited,
C = Number of Customers served per Vehicle-Tour,
and, n = Number of Vehicles.

Figure 4.18.SEMI-LOGARITHMIC REGRESSION

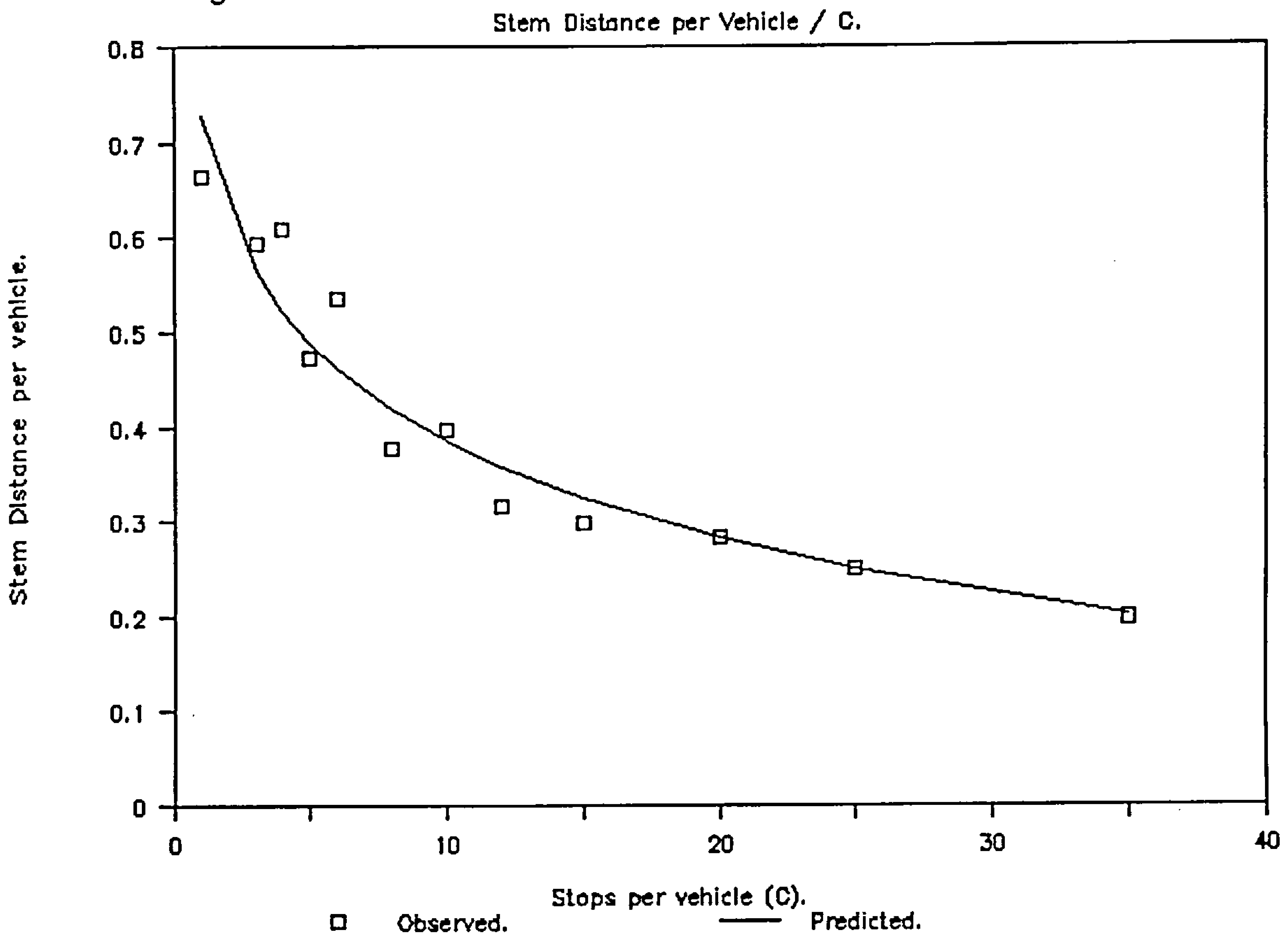
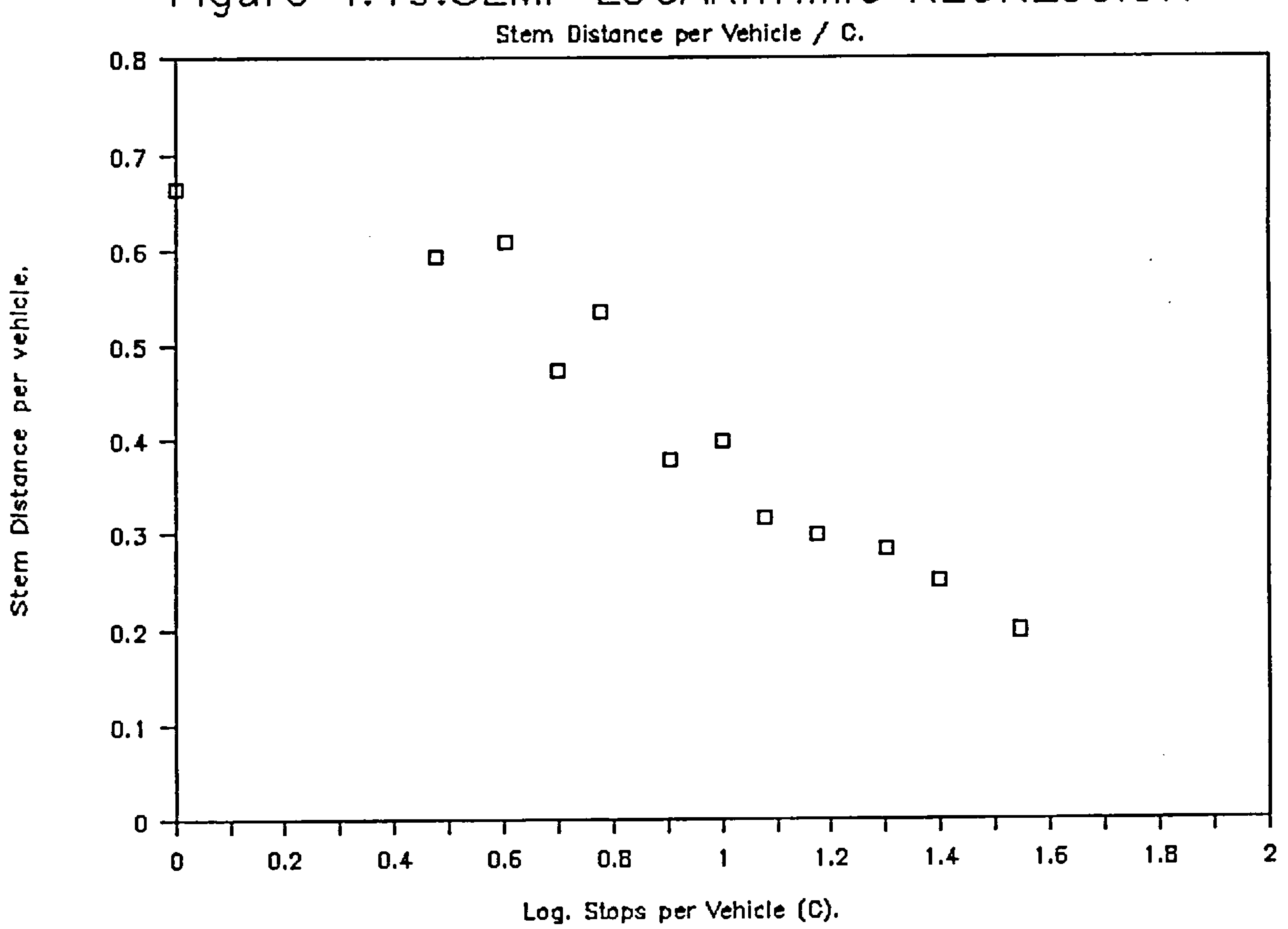


Figure 4.19.SEMI-LOGARITHMIC REGRESSION



Stem Distance per vehicle, as calculated from Equation E.4.7., with the results of simulations from which this equation was developed; on the evidence of Figure 4.18., it would seem that the regression-line described by Equation E.4.7. is an accurate enough representation of observed Stem Distances.

The derivation of Equations E.4.7. and E.4.8. is relatively straightforward, owing to the fact that it has been established that only the number of points visited per vehicle directly affects the value of Stem Distance per tour. However, the development of an expression for Delivery Distance, the other component of Total Fleet Mileage, is more complex, since it is influenced by both the number of stops made per vehicle and the size of the vehicle's delivery-zone; the following section sets out to quantify the effect of both of these parameters.

4.3.2. Delivery Distance

The simple definition of Delivery Distance is the distance that is travelled by each vehicle between the first and last customer to be served in each tour; in other words, Total Delivery Distance is Total Fleet Mileage less Total Stem Distance. It has been noted in the previous section that the effect of increasing the value of C, on its own, is to reduce Stem Distance per vehicle, and thus, when n is constant, to reduce Total Stem Distance. It must be concluded, therefore, that it is changes in Delivery Distance that are mainly responsible for the differences in Total Fleet Mileage brought about by changes in P that are observed in the data contained in Table 4.3.. This is quite plausible, since it would seem to naturally follow that an increase in the number of customer-locations to be visited will lead to an increase in the total distance travelled between stops. Close examination of the results of a separate series of computer-based routing exercises, however, reveals that there are several factors that influence Total Delivery Distance.

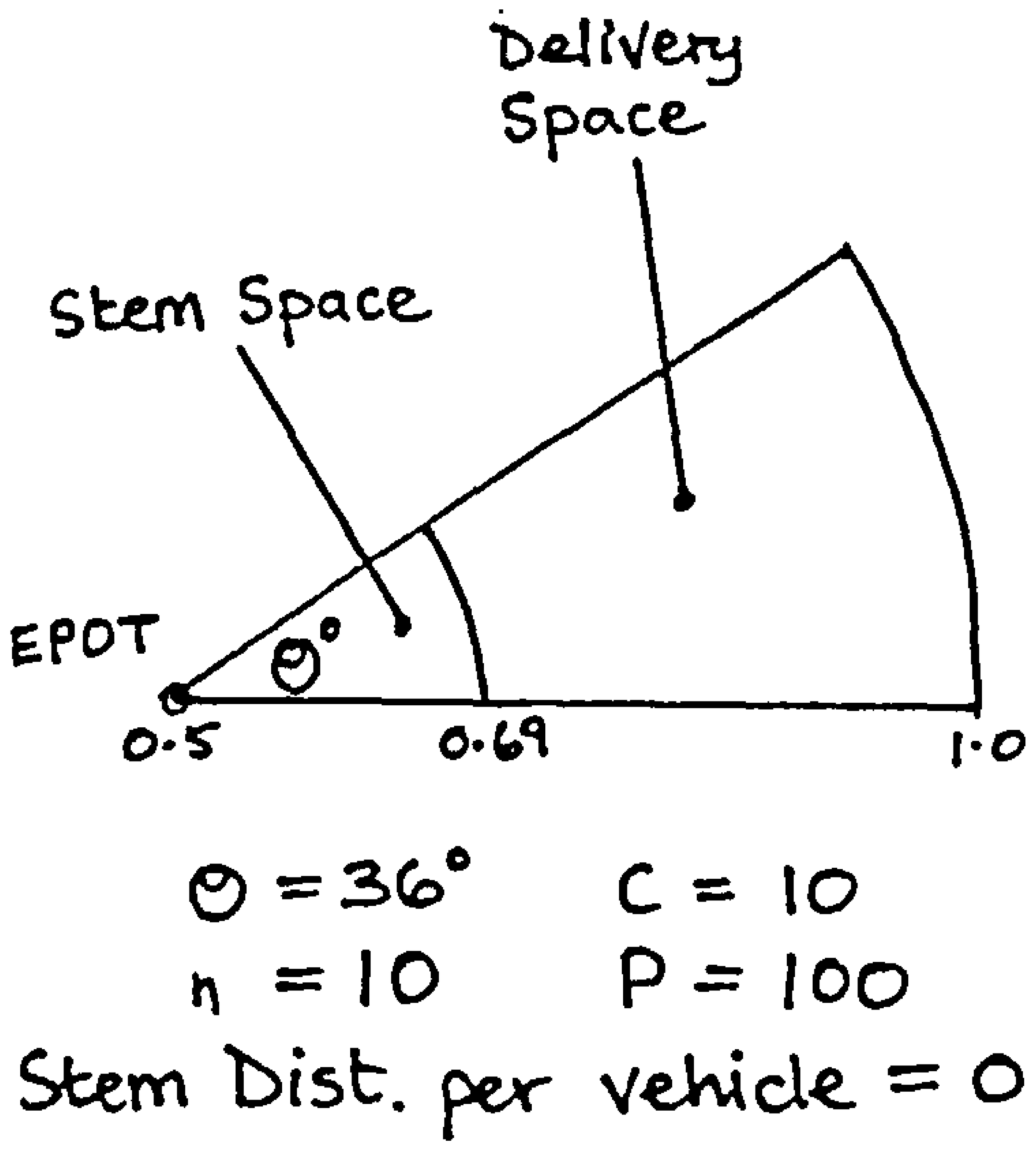
To focus the discussion, once again, on individual vehicle-tours, there are three main influences acting on Delivery Distance per vehicle when the total population of customers to be served is fixed. Firstly, as the number of vehicles employed increases, the average Delivery Distance required for each vehicle-tour will obviously be reduced, because each one will involve fewer stops, (ie. C will decrease). A further implication of changes in C involves the way in which this parameter affects the relationship between Stem Distance and Delivery Distance; it has already been established that average Stem Distance per vehicle is inversely related to C, and so an increase in the distance between the depot and the two customer-locations that are linked directly to it would mean that the remaining space in the vehicle's delivery-segment, within which all the remaining customers requiring a visit are located, will be reduced as a result. Figure 4.20. provides a numerical illustration of the balance between what might be termed "Stem Space" and "Delivery Space", two areas of a vehicle's delivery-sector that may be distinguished by a boundary-line whose distance from the depot corresponds to average Stem Distance per vehicle. In the example of Figure 4.20., n is fixed at 10, (so that the angle of each vehicle's delivery sector is 36 degrees, at the depot), and in the two instances shown here, the value of C is 10 and 4, respectively. Using Equation E.4.7., and assuming that the diameter of the circular delivery-area is 1, Stem Distance may be estimated as being approximately 0·52 when C=4, and roughly 0·39 when C=10; as the diagram shows, the space labelled "Delivery Space" is larger for the higher value of C.

The space in which deliveries are made is also reduced in response to an increase in n due to the reduction in the overall size of each vehicle's delivery-area. The combined effect of diminishing delivery-space and the reduction in the number of locations to be visited within this space, both brought about by an increase in fleet-size, is to produce the Delivery Distance per vehicle curve shown in Figure 4.11..

In order to make an accurate estimate of Delivery Distance, it is necessary to disaggregate the vehicle-tour even further, so that attention is focused upon the distance between consecutive stops on a tour. The "average length of inter-nodal links", as this distance may be termed, (denoted as i here), is obviously related to such measures as density and the mean distance to each customer's nearest neighbour, although there is an important distinction to be made between i and the latter.

In the very simple case where each vehicle visits only two customer-locations, (so that C=2), i will be equal to the Average Distance to a Nearest Neighbour, but this is clearly not the case when C is large, since not all points in a tour will be linked directly to their nearest neighbour; furthermore, although every location has, by definition, only <u>one</u> nearest neighbour, all bar those at the beginning or end of a tour will be linked to <u>two</u> other locations! Figure 4.21. illustrates this point by means of a simple tour through a set of six locations that are arranged in two clusters of three; the points of each cluster are equidistant from one another, so that each set of three forms an equilateral triangle. Although the distance to each point's nearest neighbour is the same in each case, and despite the fact that every point is linked in the tour to a nearest neighbour, the average length of the 5 inter-nodal links in this example, i, is clearly greater than the average nearest neighbour distance, because of the space between the clusters. The Average Distance to Nearest Neighbour parameter can more usefully be regarded as the <u>minimum</u> value of i, and will, in turn, be a function of both the density of points within each vehicle's delivery-segment and, as Figure 4.21. indicates, the degree of clustering of the points.

Figure 4.20. "Stem Space" and "Delivery Space". (a=1)



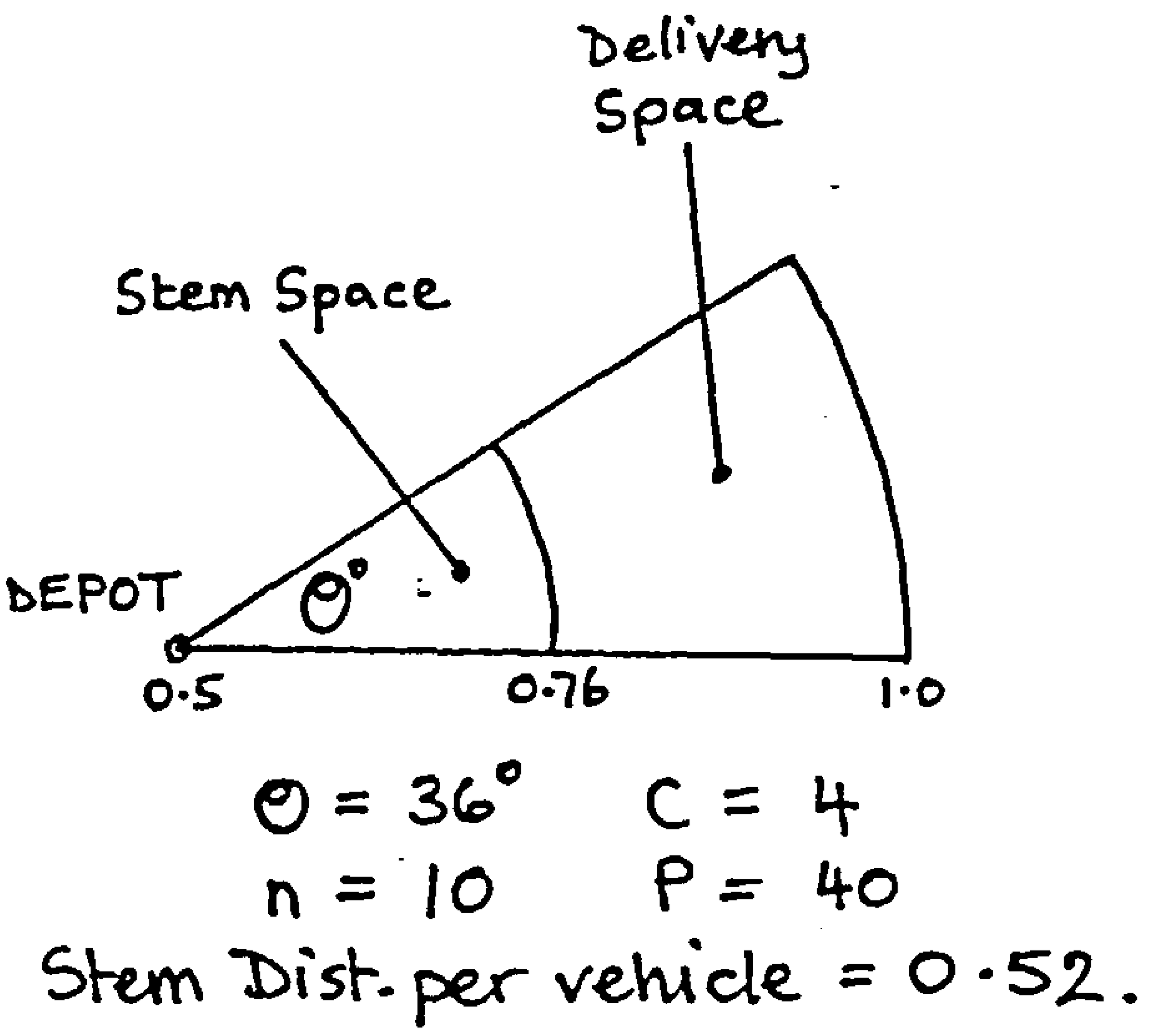


Figure 4.21. Hypothetical vehicle-tour

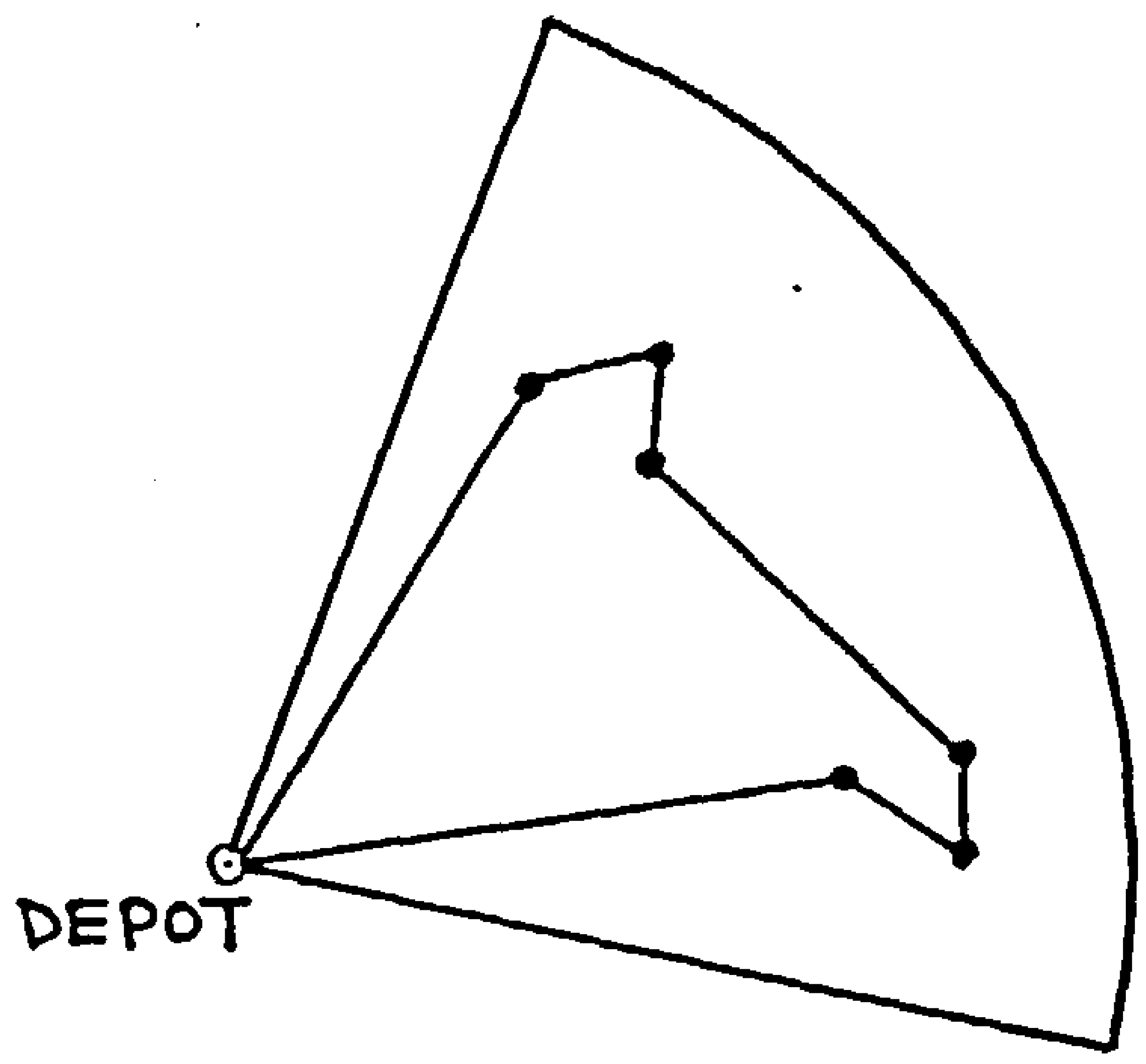


Table 4.12. Variation in the number of inter-nodal links, as n and C change. (P=100)

n	C	Inter-nodal links	n	C	Inter-nodal links
1	100	99	12	8	84
2	50	98	14	7	84
3	33	96	16	6	80
4	25	96	18	6	90
5	20	95	20	5	80
6	17	96	25	4	75
7	14	91	35	3	70
8	12	88	50	2	50
10	10	90			

(N.B. Number of inter-nodal links = n (C-1))

Although it is true that i will always bear some relationship to indicants such as density and the average spacing of points, the value of i will invariably differ from these other measurements because it is a product of the tour-building process, and not merely a characteristic of a given distribution of points. For this reason, i will also be influenced by the choice of algorithm to be used in the construction of tours. If the average length of inter-nodal links were estimated on the basis of density or Nearest Neighbour data, then these estimates would ignore the fact that any tour-building algorithm is constrained to eliminate "sub-tours" when it links points together, in order to define a continuous route through a given set of points. This may be illustrated by referring once again to Figure 4.21.; if linkages were made based purely on the Nearest Neighbour criterion, then the final "solution" to this particular routing problem would be two separate sub-tours each consisting of three locations. The necessary addition of a constraint to avoid the formation of such sub-tours immediately causes the value of i to exceed the Average Distance to Nearest Neighbour figure of the set of points, and the difference between these two parameters will increase as the distribution of points becomes more clustered. Obviously, when a tour-building exercise involving the generation of random co-ordinates is repeated many times, as is the case here, it is to be expected that point distributions will show a degree of clustering on at least some occasions; therefore, the average value of i over many iterations of a Travelling-Salesman-type program will certainly significantly exceed density and other density-related parameters, which will be constant for each repetition.

For these reasons, it is far more satisfactory to base estimates of Delivery Distance on i than on indicants of overall density or dispersal of points.

The calculation of i is not merely Total Delivery Distance divided by the total number of customers served, as the number of inter-nodal links making up a fleet's total tour-distance declines as n increases, due to the rise in the total number of stem-journeys, (SEE Table 4.12.). In fact, for each individual vehicle-tour, the number of inter-nodal linkages is always one less than C, so that i may be calculated as,

$$i = \frac{\text{Total Fleet Mileage} - \text{Total Stem Distance}}{n(C-1)} \qquad (E.4.9.).$$

The term (C-1) is, of course, always an integer; although this integer effect certainly has an effect on the content of Table 4.12., particularly when n is small, the tendency for the total number of inter-nodal links to decline as n increases is attributable to the fact that more and more potential links between customer-locations are replaced by return journeys to the depot.

Having identified i as being the smallest unit to which a set of vehicle-tours may be disaggregated, and having established Equation E.4.9. as a means of calculating the value of this parameter, the next stage of the analysis involves the relationship between i and both n and C, a relationship that may be used as the basis of a model for estimating Total Delivery Distance.

Evidence of the way in which i varies with n is shown in Table 4.13., and displayed graphically in Figure 4.22.. These figures show that there's a definite trend for i to increase as n increases; this is, perhaps, surprising, since both density and Average Distance to Nearest Neighbour are not at all affected by fleet-size! The reason for this tendency is that, as n increases and causes both a decrease in the size of each vehicle's delivery-sector and a reduction in the number of points that each vehicle must visit, the value of (C-1) is reduced to a greater extent than the average Delivery Distance within the sector. In terms of Equation E.4.10.,

$$i = \frac{\text{Delivery Distance per vehicle}}{(C-1)} \qquad \text{(E.4.10.)}$$

which is simply an alternative way of expressing Equation E.4.9., a reduction of fleet-size leads to a greater decrease in the denominator of this expression than in the numerator, so that the overall effect of enlarging the fleet is to increase the value of i. This algebraic explanation is supported by Figure 4.23., which shows that, for smaller values of n, the slope of the (C-1) curve is, in fact, steeper than that of the Delivery Distance per vehicle curve.

The relative impact of the main forces that influence the behaviour of i - changes in the size of each vehicle's delivery-sector and in the number of locations that are visited in each tour - may be examined by once again observing the effect of one of these parameters on i whilst the other is held constant, (and vice versa). Figure 4.24., for instance, shows the relationship between i and n with C fixed at 20 stops per vehicle. The fact that the graph in Figure 4.24. illustrates a distinctly curved relationship is interesting, in view of the fact that i would be expected to be closely related to density, since the relationship of density to fleet-size, when C is fixed, is linear! The validity of the latter assertion may be demonstrated algebraically, using the simple example of a circular delivery-area whose radius is equal to 1 unit of distance; its area,

Figure 4.22. Relationship between i and Fleet Size (P=100, a=1 (circle))

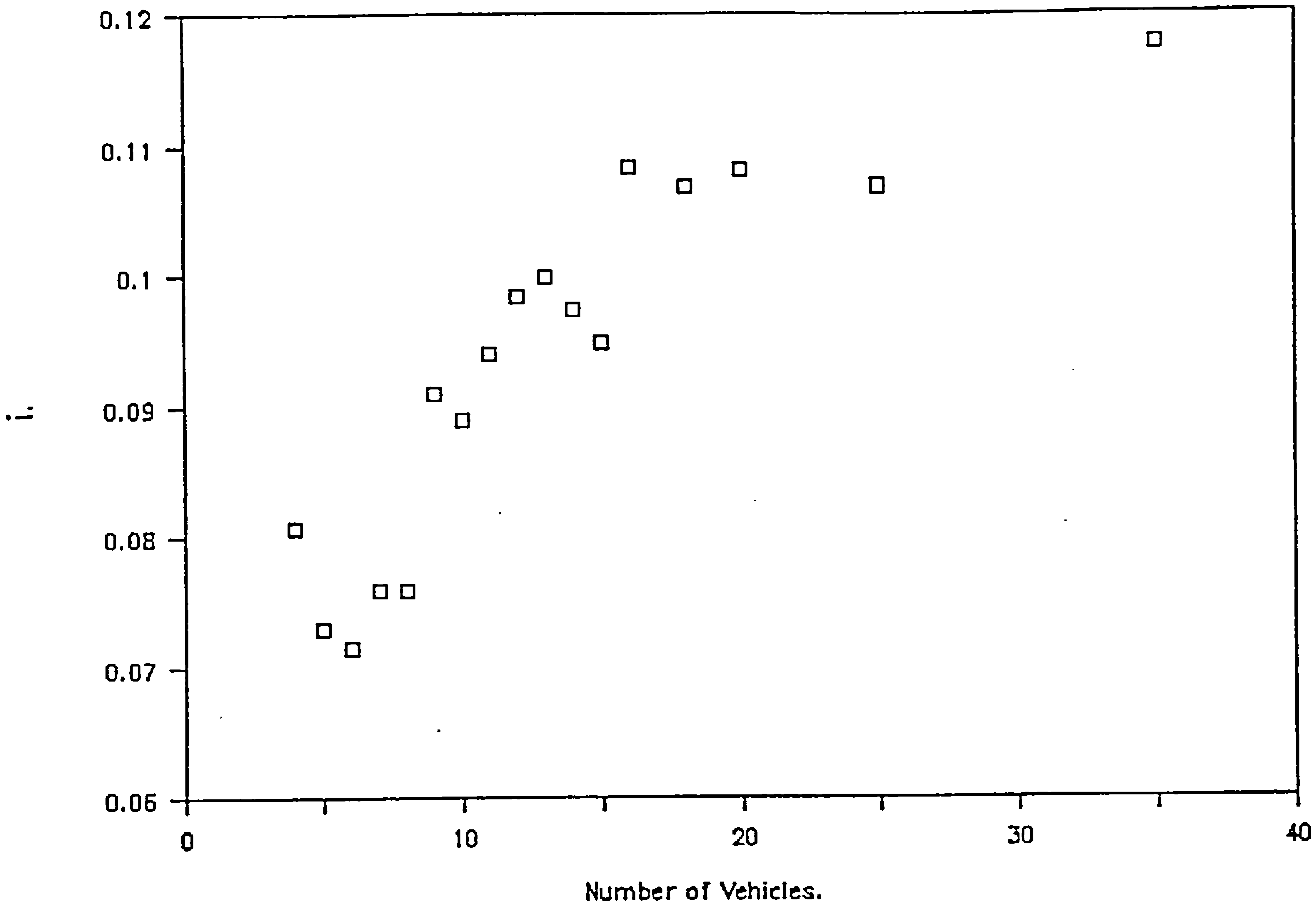
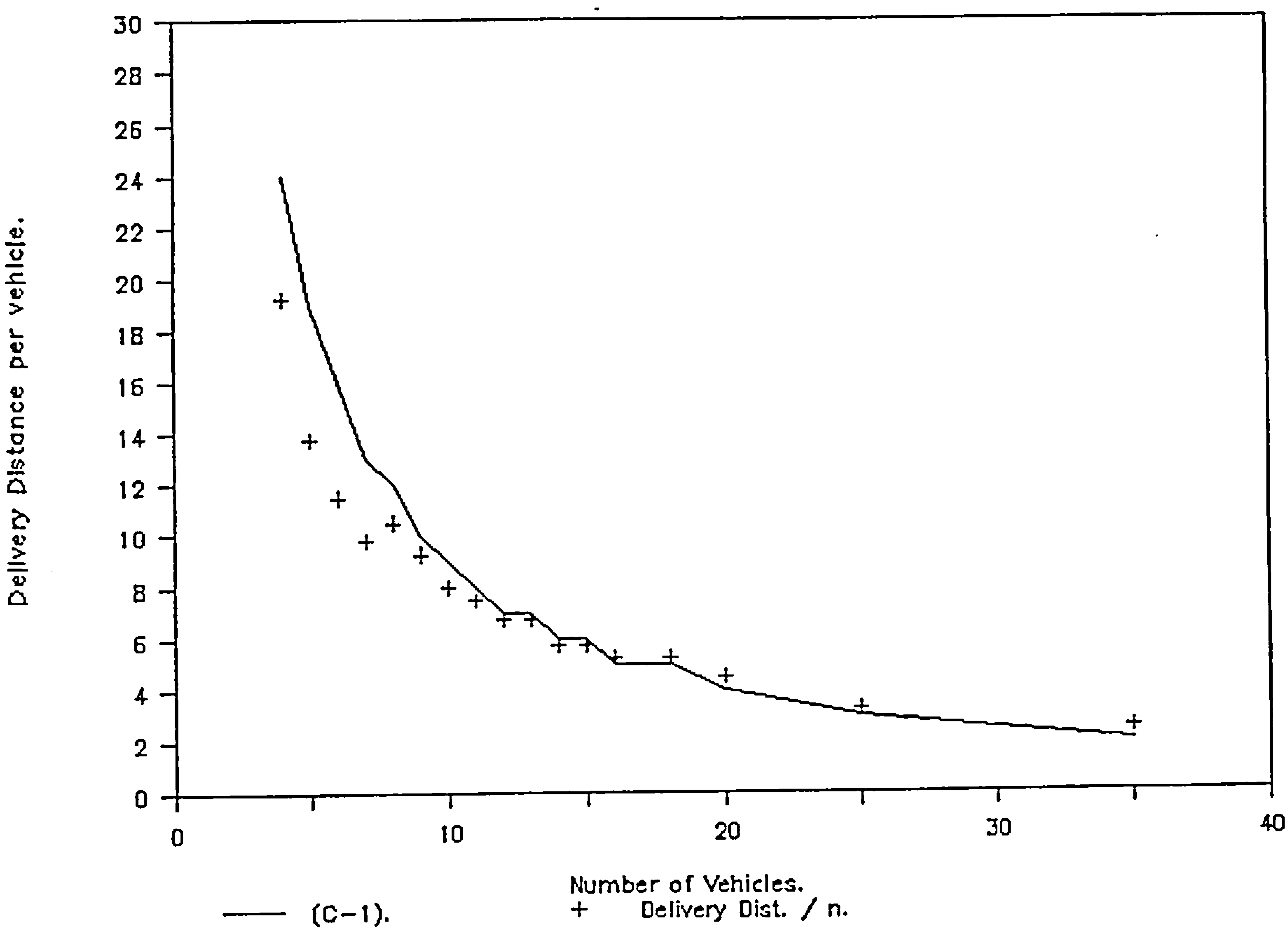


Figure 4.23. Comparing Delivery Distance per vehicle and (C-1) as n changes. (P=100, a=10)



$$\pi r^2$$

is therefore,

$$\pi$$

the area of each vehicle-segment,

$$\frac{\pi}{n}$$

and the density of points within this segment,

$$\frac{C}{\left(\frac{\pi}{n}\right)}$$

This may be written as,

Density of segment $= \dfrac{C.7.n.}{22}$ (E.4.11.)

and, as C is fixed at 20 in this case, the density of both the entire delivery-area and each individual segment is 6·3636n. The linearity of the density - fleet-size relationship is to be expected, since the density of a fixed set of points will obviously double if the area in which they are distributed is reduced by half!

The contrasting curved nature of the i - fleet size relationship, as shown in Figure 4.24., however, may be transformed into a more linear form using logarithms, and the graph resulting from this logarithmic transformation appears as Figure 4.25.. Using the same technique as is used to derive an expression for Stem Distance, described earlier in this chapter, a regression-line may be fitted to this distribution; the equation for this line is,

$$\text{Log.}i = -0{\cdot}1854 - 0{\cdot}382 \text{ Log.}n \qquad \text{(E.4.12.)}$$

so that, when C=20 and a=1,

$$i = 0{\cdot}15325n^{-0{\cdot}382} \qquad \text{(E.4.13.)}$$

The predicted values for i derived from this expression are compared with those observed empirically in Figure 4.26. and Table 4.14., and it is apparent from this table that Equation 4.13. tends to under-estimate i for values of n greater than 10 and less than 4, and to provide over-estimates for values of n within this range. The residuals of this regression analysis therefore display a degree of autocorrelation, a fact which should be taken into account when this expression is used as part of a model for estimating fleet distances.

Figure 4.24. Relationship between i and n,

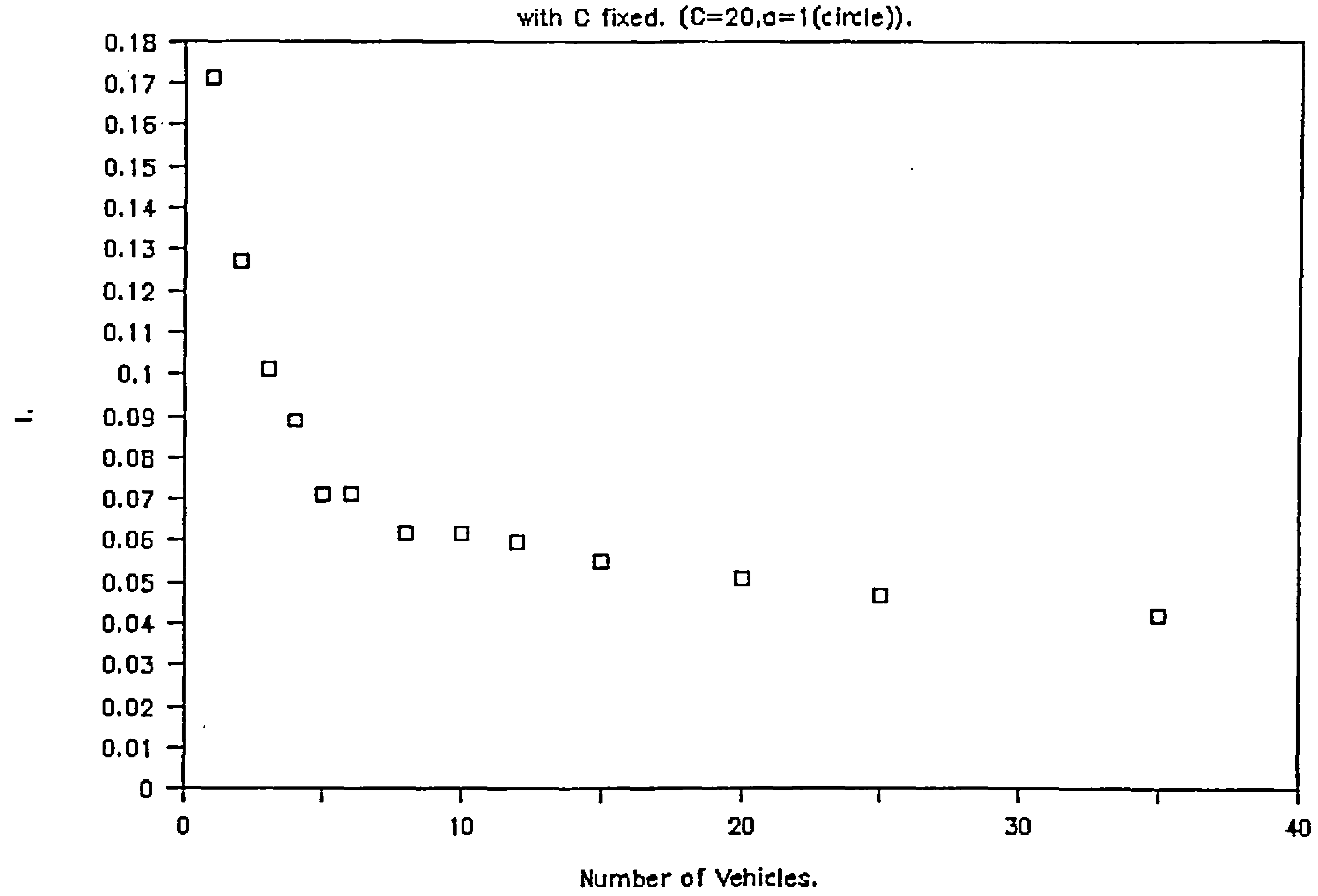
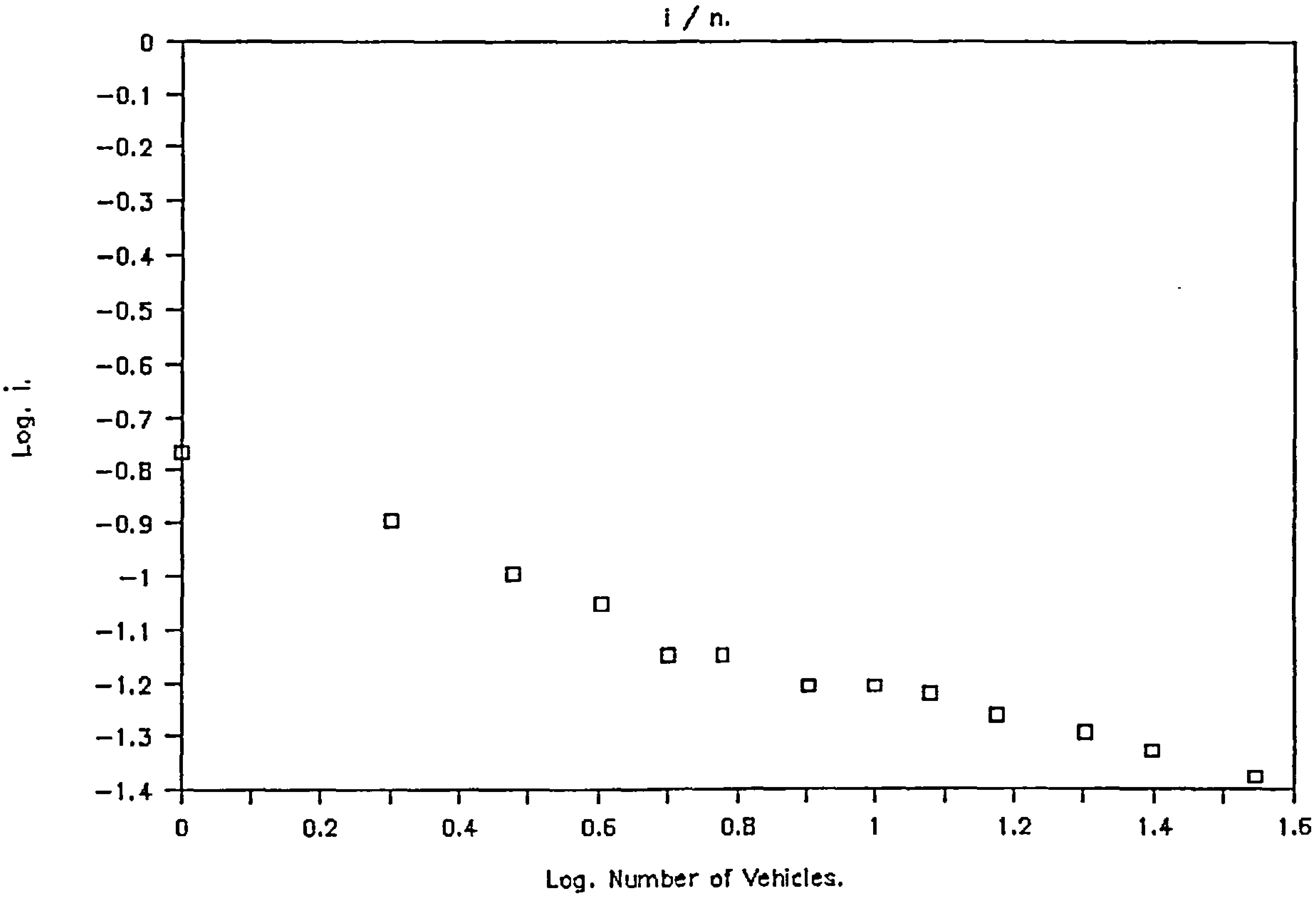


Figure 4.25. LOGARITHMIC REGRESSION



However, these residuals are mainly very small, which suggests that Equation E.4.13. does, nevertheless, provide a very good approximation of the i-values that were observed by simulation, and so may be accepted as an adequate tool for predictive purposes.

The same procedure was carried out using fixed values of C of first 15 and 10; the resulting expressions for i as a function of fleet-size are respectively,

$$i = 0{\cdot}16904n^{-0{\cdot}3615} \qquad (E.4.14.)$$

and,

$$i = 0{\cdot}20963n^{-0{\cdot}3573} \qquad (E.4.15.)$$

The distributions of observed and predicted values of i are plotted in Figure 4.27. when C is fixed at 15, (with the corresponding figures presented in Table 4.15.), and in Figure 4.28. when C=10, (SEE also Table 4.16.). Systematic correlation of residuals is again in evidence in both cases, but, again, Figures 4.27. and 4.28. suggest that the predicted values of i closely approximate those derived from simulations.

Comparing Equations E.4.13., E.4.14. and E.4.15., it is noticeable that the coefficient of n in each case is similar, although it is to be expected that the constant in each equation should increase as the value of C is reduced, (for reasons that are explained earlier in this Section). Taking these three equations together, it is possible to perform a Multiple Regression Analysis on all three, and thus derive an expression for i with both n and C variable; the resulting equation is,

$$i = 0{\cdot}7379.n^{-0{\cdot}3638}\ C^{-0{\cdot}5394} \quad (R^2=0{\cdot}95) \qquad (E.4.16.)$$

The coefficient of C here, $-0{\cdot}5394$, is very similar to that found for the relationship between i and C with n held constant at 20, (so that different-sized sets of customer-locations were generated within an 18-degree segment). The full expression for i arising from these simulations is,

$$i = (0{\cdot}27146\ .\ C^{-0{\cdot}5672})a \qquad (E.4.17.)$$

Figure 4.29., which includes both the observed and predicted values of i as C changes, (SEE also Table 4.17.), again shows that there is a close association between the distribution of points plotted and the regression-line fitted to it, although the largest residuals are for small values of C; this reflects the fact that inconsistences in the empirical results are more likely to occur as the number of points generated for each iteration of the route-building exercise decreases.

Figure 4.26. Observed and predicted i
with C fixed. (C=20,a=1(circle)).

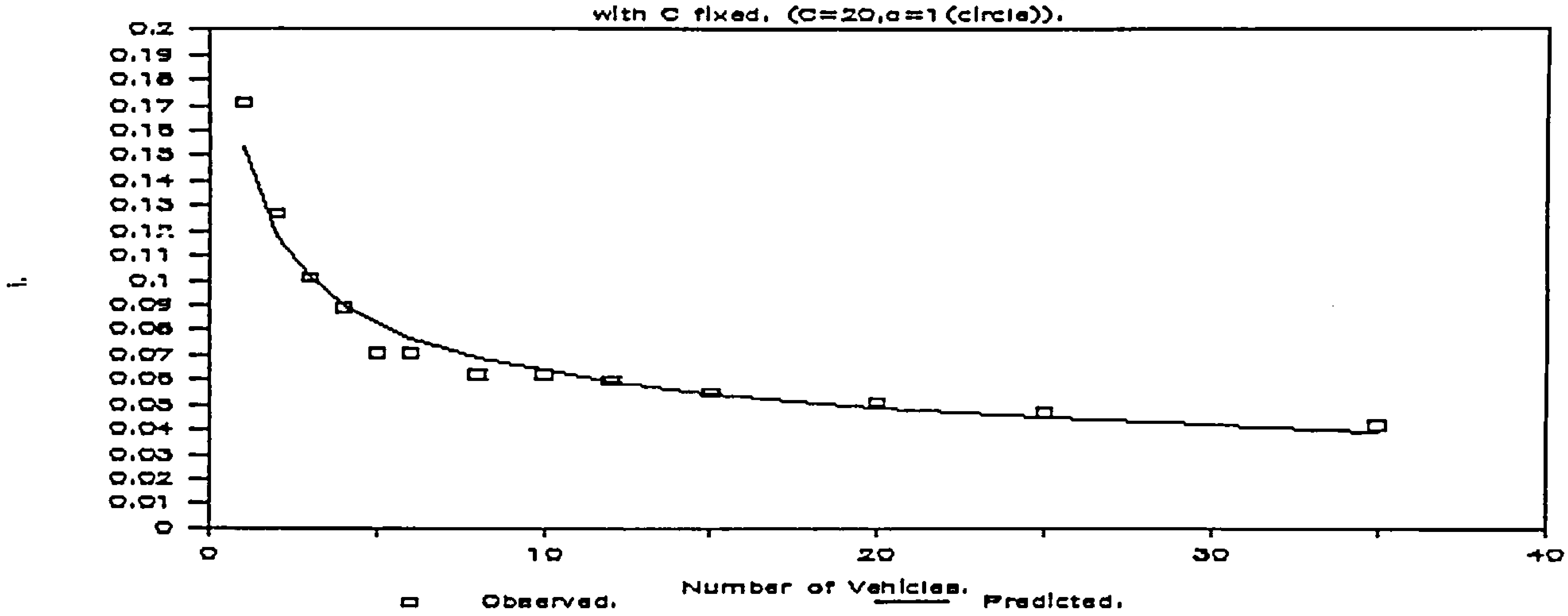


Figure 4.27. Observed and predicted i
with C fixed. (C=15,a=1(circle)).

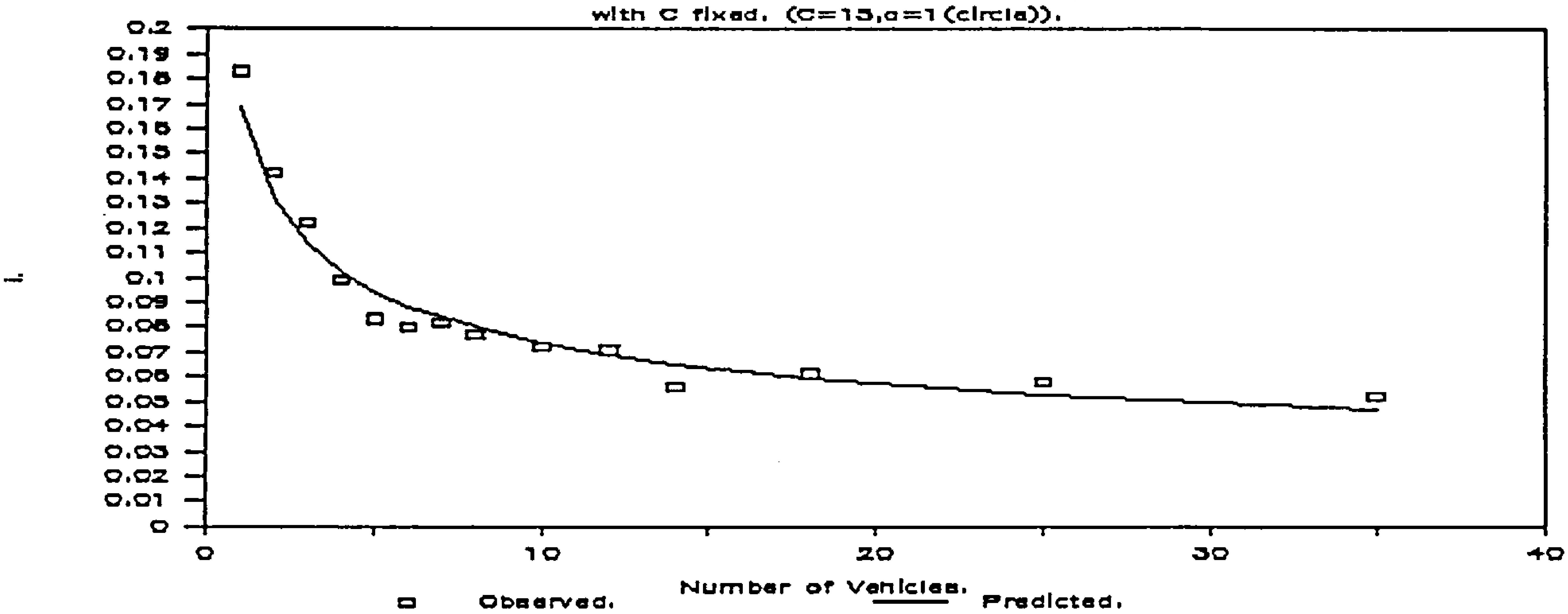
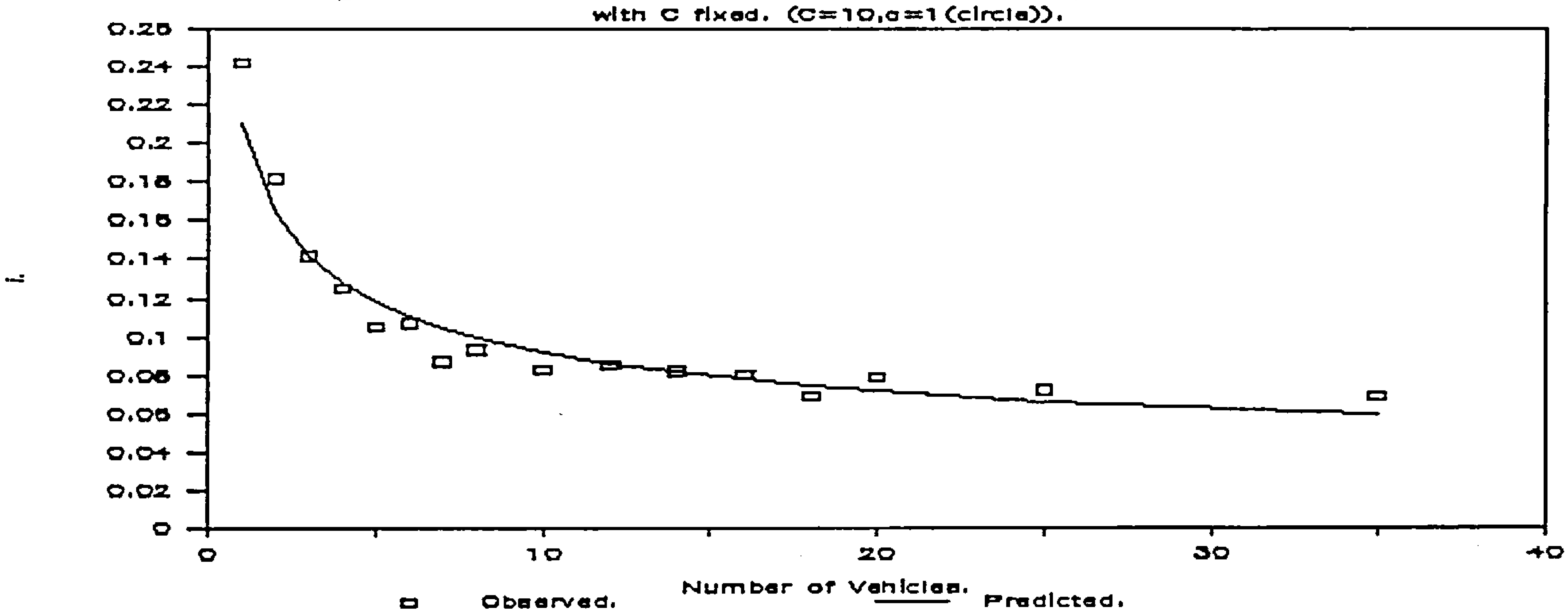


Figure 4.28. Observed and predicted i
with C fixed. (C=10,a=1(circle)).



But in order to test the effectiveness of Equation E.4.16. as a tool for estimating the value of i, given n and C, it is obviously necessary to compare predictions made from this equation with observed i-values derived with both n and C variable. Such a comparison is provided by Table 4.18., whose figures are based on the assumption that P is fixed at 100, and a, the diameter of the circular delivery-area, is 1. In the corresponding diagram, Figure 4.30., the predicted values of i do not form a smooth curve, and appear, instead as a series of discrete points on the graph; this is purely because of the effect of C being an integer variable.

From Equation E.4.10., it may be deduced,

$$\text{Delivery Distance} = i\,[n(C-1)] \qquad (E.4.18.)$$

and so the final step required to produce a formula for Delivery Distance as a function of n and C is to combine Equations E.4.18. and E.4.16., so that,

$$\text{Total Delivery Distance} = a[(0\cdot7379.n^{-0\cdot3638}.C^{-0\cdot5394})(n(C-1))]$$

$$\therefore\ \text{Total Delivery Distance} = a[(0\cdot7379.n^{0\cdot6362}.C^{-0\cdot5394})(C-1)] \qquad (E.4.19.)$$

4.3.3. An expression for Total Fleet Mileage as a Function of n and C

Having developed equations for both Stem Distance and Delivery Distance as a function of n and C in preceding sections, the task of deriving an expression for estimating Total Fleet Mileage now only requires the simultaneous use of Equations E.4.8. and E.4.9. to produce the following expression,

Total Fleet Mileage

$$= a[((0\cdot7379.n^{0\cdot6362}.C^{-0\cdot5394})(C-1)) + (n(0\cdot802147 - 0\cdot4115\ \log.C))] \qquad (E.4.20.)$$

Table 4.19. presents a full list of both observed and predicted values of Total Distance, Stem Distance and Delivery Distance, when P is 100 and the diameter of the circular delivery-area is 1, and these figures are displayed graphically in Figures 4.31.1., 4.31.2. and 4.31.3.. Despite the fact that the regression-lines shown in figures 4.27., 4.28. and 4.29. do not, as the residuals indicate, exactly fit the distributions that they describe, Figure 4.31. nevertheless suggests that the equation for Total Fleet Mileage, E.4.20., that is derived directly from the associated regression equations, appears to serve as an adequate predictive tool. The two regression analyses that go to make up Equation E.4.20., Equations E.4.7.

Figure 4.29. Observed and predicted i

with n fixed. (n=20,a=1(circle)).

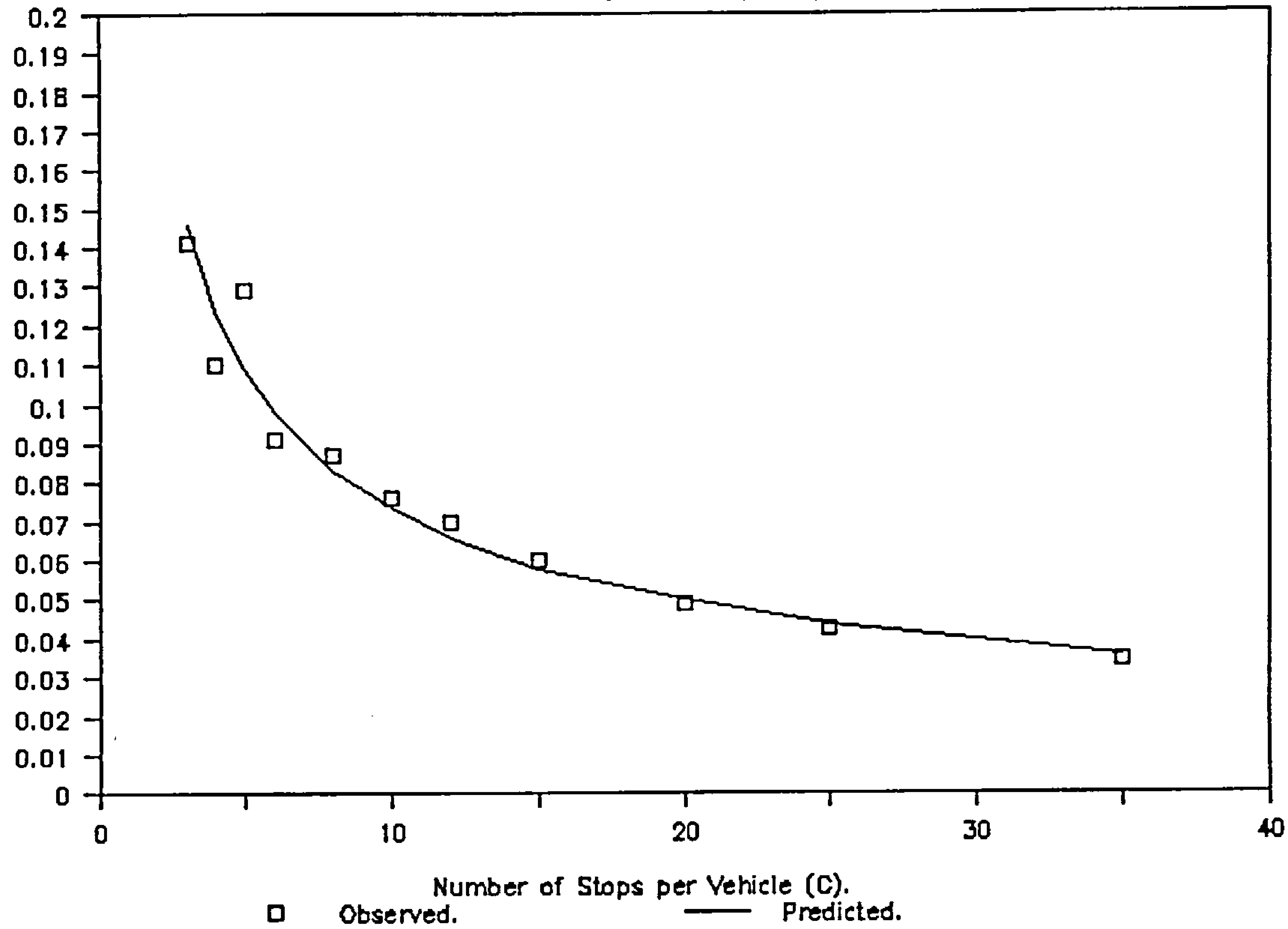


Figure 4.30. Observed and predicted i

with n and C variable. (P=100,a=1)).

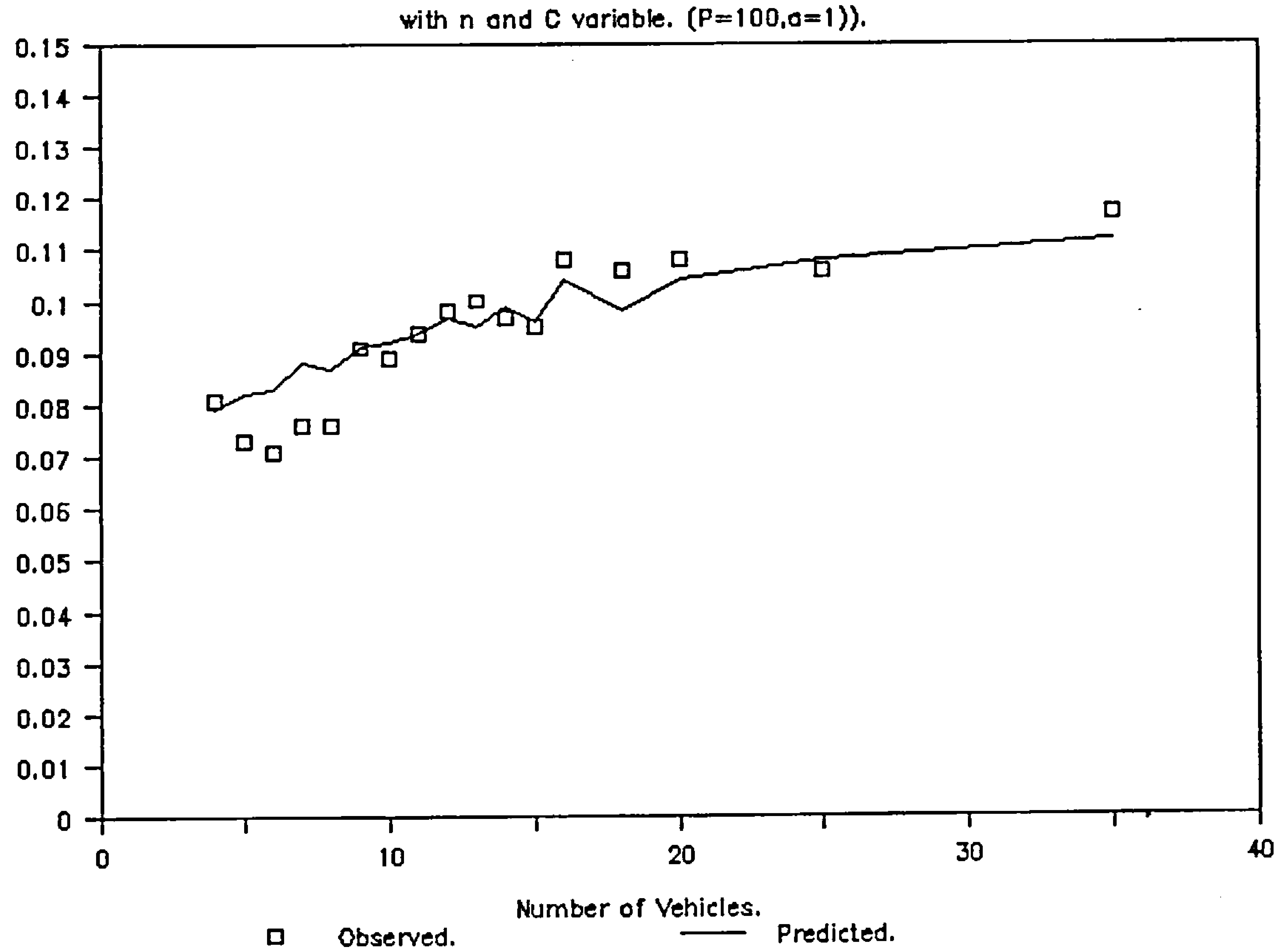


Figure 4.31.1. Stem Distance
with n and C variable. (P=100,a=1)).

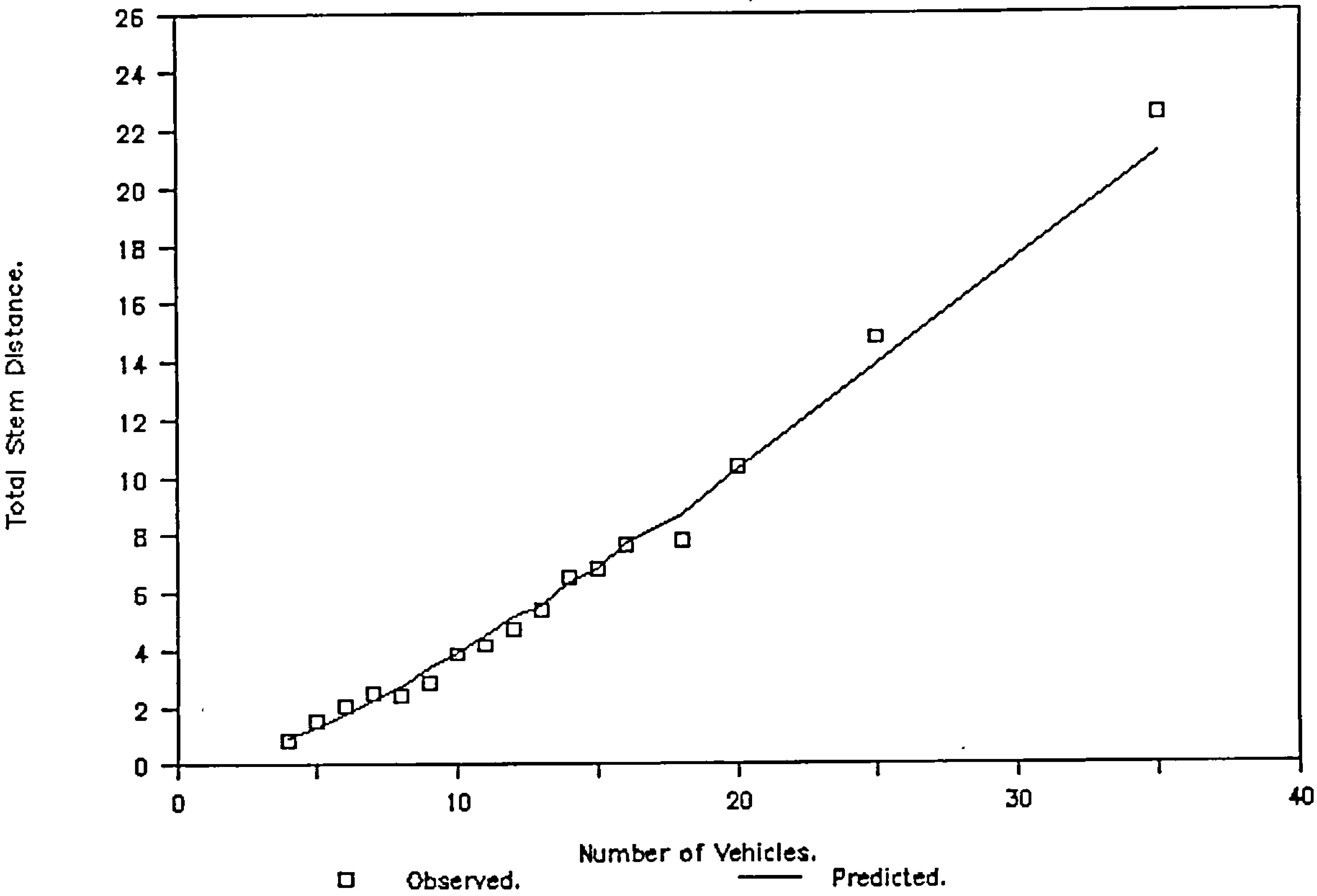
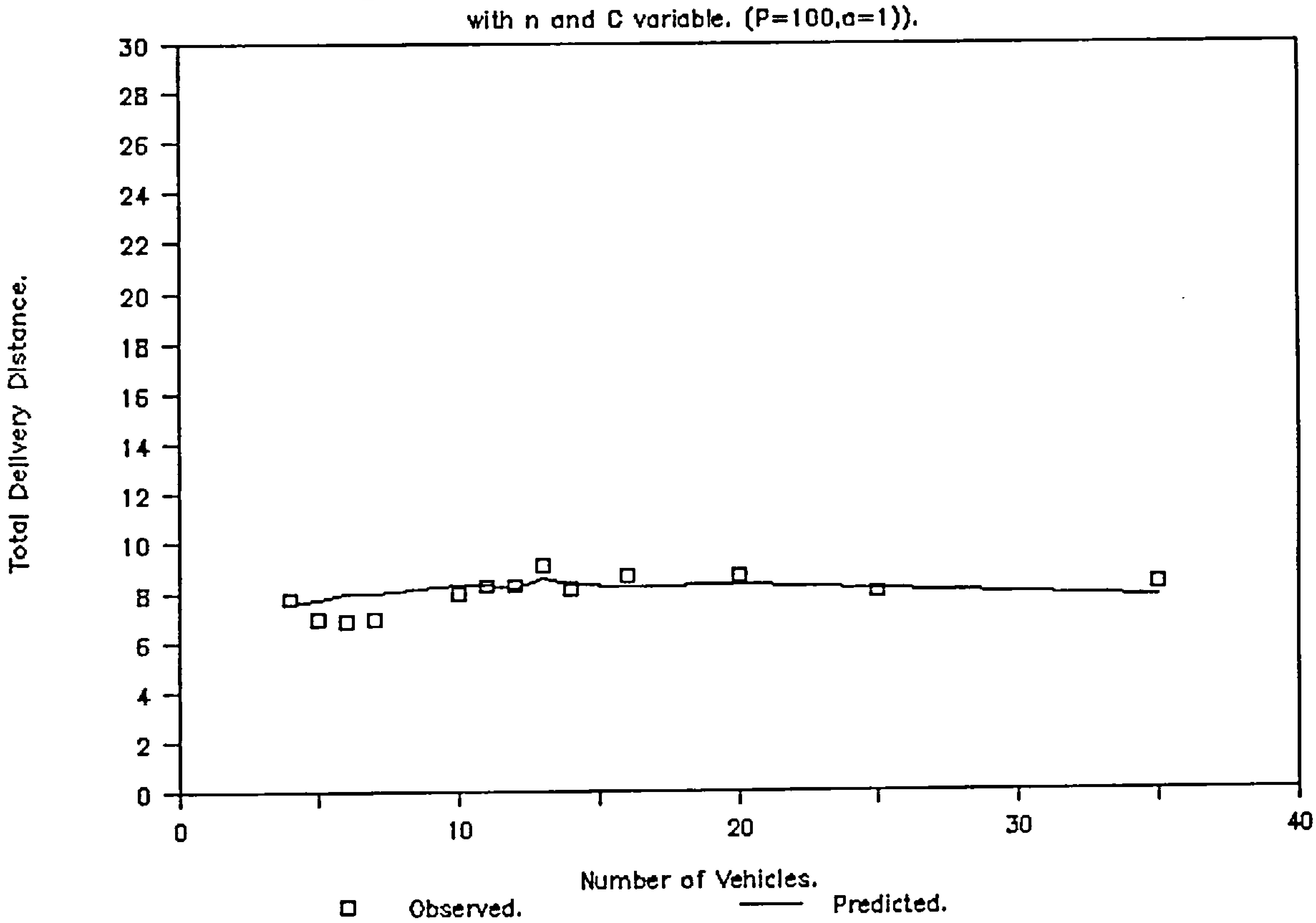


Figure 4.31.2. Delivery Distance
with n and C variable. (P=100,a=1)).



Total Distance.

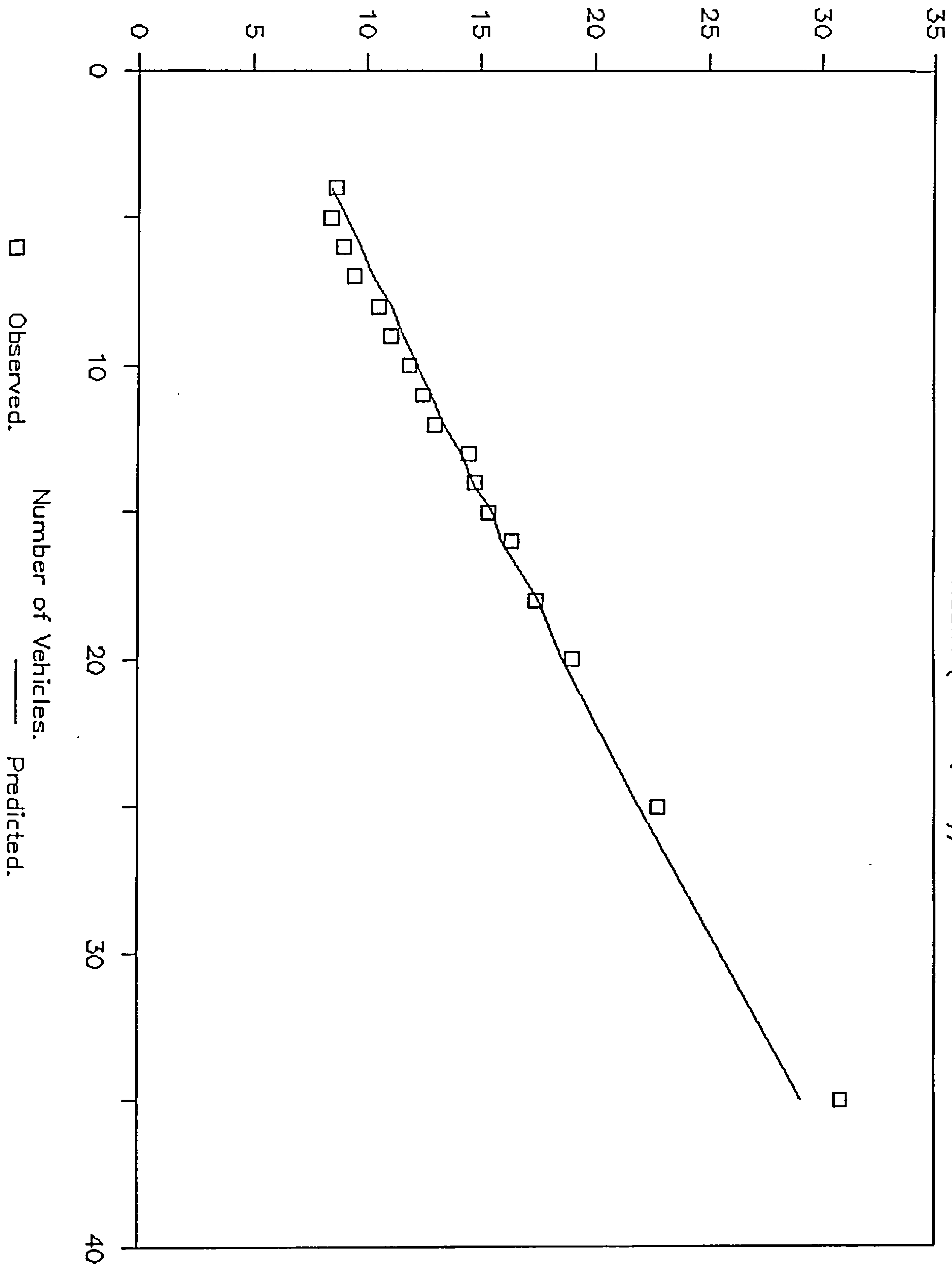


Figure 4.31.3. Total Distance
with n and c variable. (P=100,a=1).

and E.4.16., produced R^2 values of 0·9389 and 0·95 respectively, which confirms that the variables n and C account for nearly all of the variation in the distances observed by simulation.

4.4. An Expression for Total Fleet Mileage as a Function of P and k.

The previous sections in this chapter have concentrated on the combined effect on Total Fleet Mileage of fleet-size and the average number of stops made in each vehicle-tour. What these two parameters have in common is that they might both be described as "decision variables"; given a fixed population of customers, P, the value of C will be a direct function of the number of vehicles used.

In a real world situation, it is most often the case that an operator wants to estimate the cost of a distribution operation on the basis of "external" constraints imposed by the characteristics of both the population of customers that are to be served and the environment in which they are situated. This change of emphasis may be accommodated by manipulating Equation E.4.20. so that Total Fleet Mileage is expressed as a function of the parameters P and k, where k is the maximum value of C; this maximum may be determined either by constraints on the amount of time available for making deliveries, or by constraints related to vehicle carrying-capacity. Whichever is the case, it is realistic to assume that the number of stops made in each vehicle-trip will always, in practice, be at, or near, the maximum permissible, so that, for the purposes of the present discussion, C=k. Since fleet-size may be expressed as,

$$\frac{P}{k}$$

the variable n may be eliminated from Equation E.4.20., so that,

$$TFM = a[(0\cdot7379\left[\frac{P}{k}\right]^{0\cdot6362}.k^{-0\cdot5394})(k-1)]$$

$$+[\frac{P}{k}(0\cdot802147 - 0\cdot4115 \log.k)]$$

This may be rewritten as,

$$TFM = a[((0\cdot7379.P^{0\cdot6362}.k^{-1\cdot1756})(k-1)] + [\frac{P}{k}(0\cdot802147 - 0\cdot4115 \log.k))] \qquad (E.4.21.)$$

Total Fleet Mileage is now estimated purely as a function of area-size, the number of customers to be served and the maximum number of delivery-drops that may be made per vehicle-tour. Because these three factors are external conditions that are imposed upon a system, it may seem more logical to have included this discussion in Part 3; however, since much of the research carried out elsewhere relating to the topics covered in this chapter tends to utilise such variables, it is felt that it is important here to express Total Fleet Mileage as a function of P and k, in order to facilitate comparisons with alternative formulae which will be discussed in Section 4.6..

A method for estimating the value of k is described in Chapter 5, with the effect of the average time spent at each location introduced in Chapter 9.

Tables 4.20. to 4.23., together with Figures 4.32. to 4.36., illustrate the effect that the parameters P and k have on Total Fleet Mileage according to estimates derived from Equation E.4.21.. In Table 4.20., estimates of mileage as a function of n and C for selected values of P are included, in order to show the extent to which figures calculated from Equation E.4.20. differ from those that are a function of P and k. The values of n and C used here are derived by assuming that n is always the integer of (P/k), with C the consequent discrete value of (P/n). Table 4.20. indicates that there is a discrepancy between the two sets of estimates produced from these equations when C is substantially less than k. An extreme example of the difficulty created when the value of P is small, is the hypothetical case in which a single customer is served from the depot; in this instance, the value of k in Equation E.4.21. is still taken to be 10, whilst C can, in fact, only be 1, and the value of P/k, which replaces the term n in E.4.20., is calculated as 0·1, when it is obvious that the number of vehicles used must be an integer variable! The result of this anomaly, as Table 4.20. reveals, is the absurdity of Delivery Distance being estimated at 0·443 units, when it is clearly zero when P=1, and the Stem Distance estimate is unrealisticly small at 0·0391 units. Equation E.4.21. should therefore be accompanied by the condition that it may only be usefully employed when C, and, of course, P, are not significantly less than the constraint k, (bearing in mind that C is the average number of stops made in each tour). This condition is implied in the initial C=k assumption that appears in the derivation of Equation E.4.21., (SEE above).

Estimates of mileage as a function of P and k in Table 4.20. are founded on the assumption that (P/k) is a discrete variable, despite the fact that the equivalent parameter in Equation E.4.20., n, is clearly an integer; this is so that graphs produced from Equation E.4.21., such as those shown in Figure 4.32., represent the true shape of the relationships illustrated, without the interference of complicating factors such as the effects of integers. If integers are to be

Figure 4.32.1. Estimates of Stem Distance, Delivery Distance & Total Distance, using E.4.21., with k fixed. (k=10, a=1 (circle))

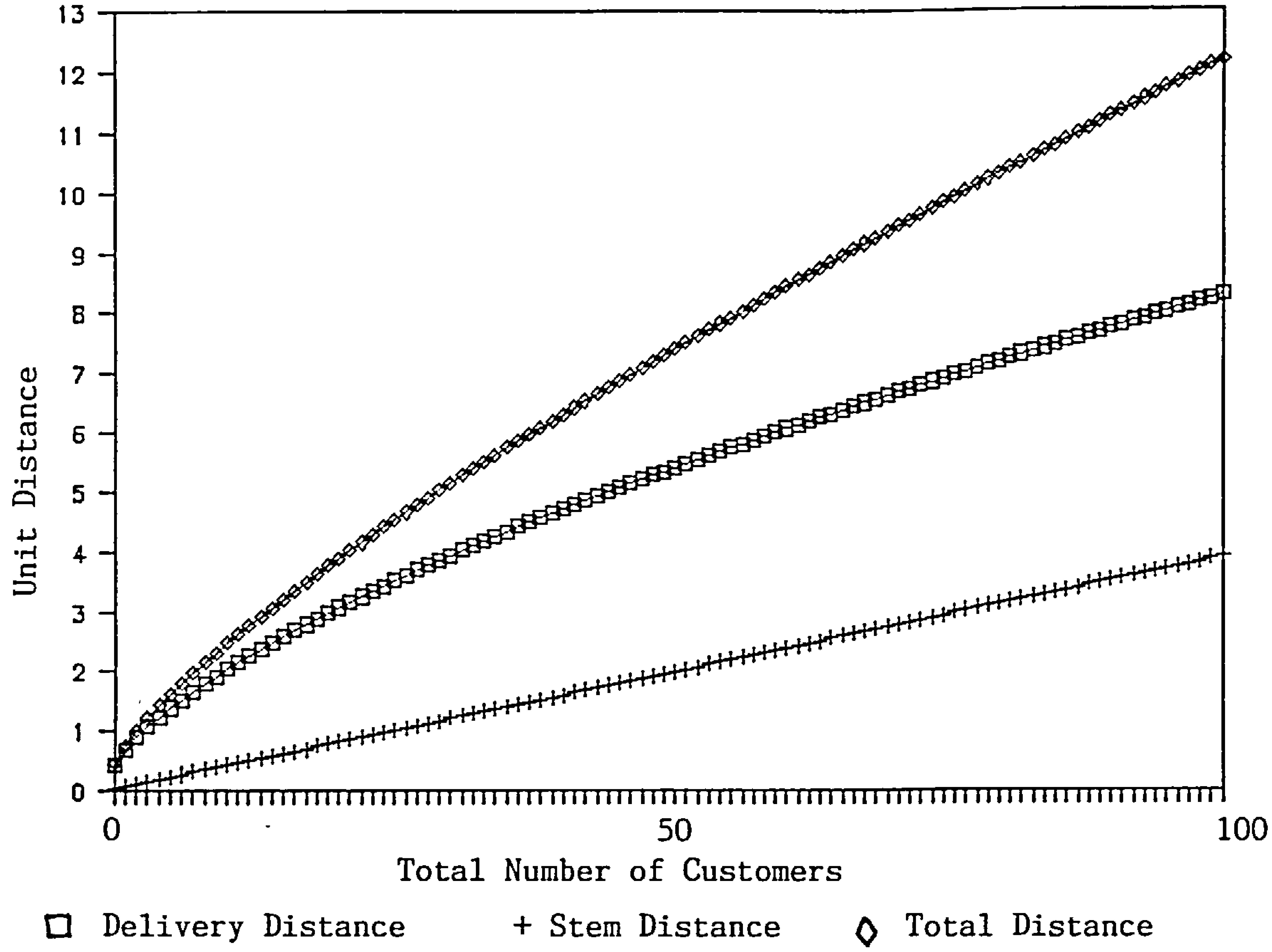
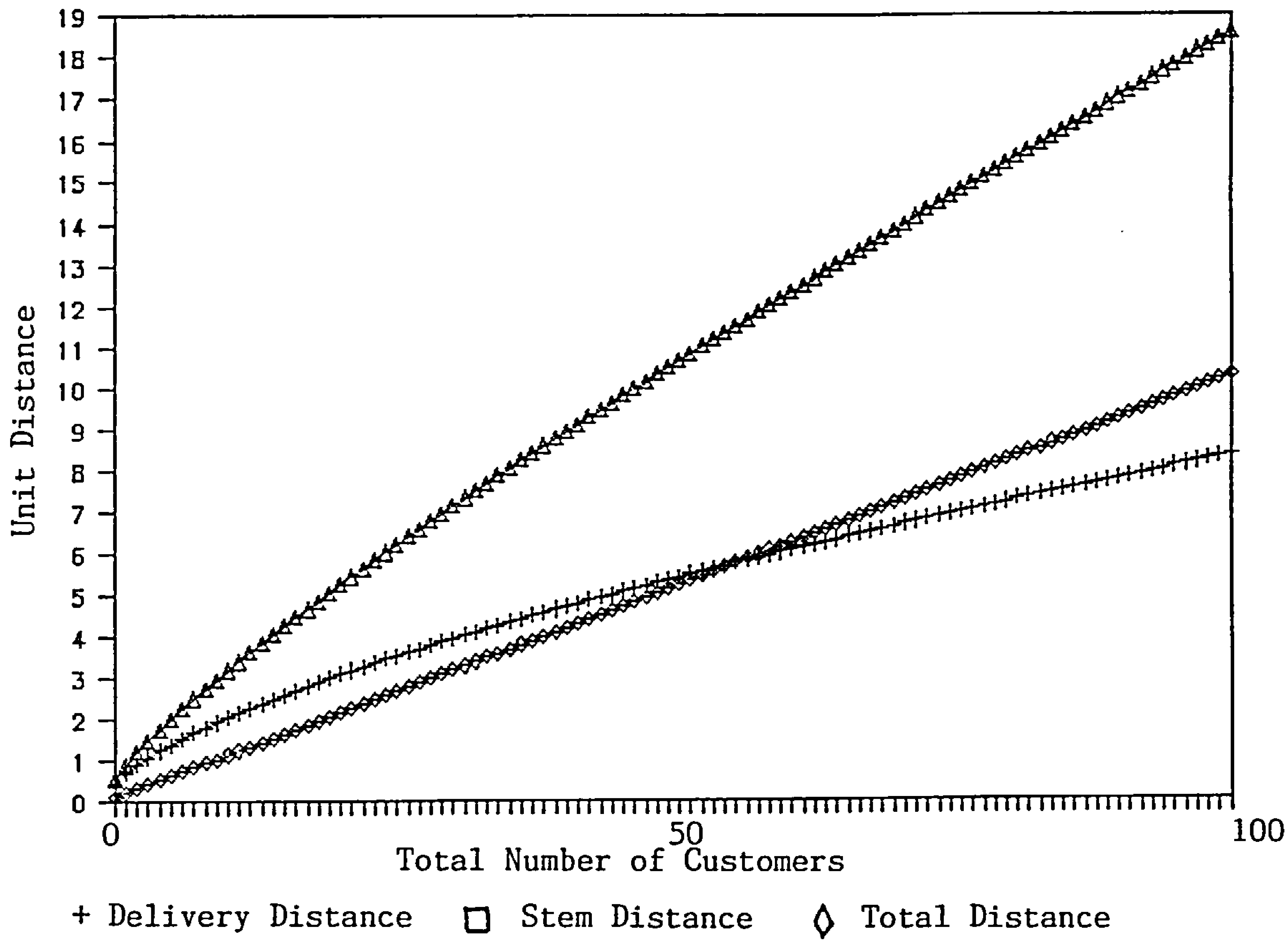


Figure 4.32.2. Estimates of Stem Distance, Delivery Distance & Total Distance, using E.4.21., with k fixed. (k=5, a=1 (circle))



retained, then the appropriate equation for Total Fleet Mileage as a function of P and k, which is again developed from E.4.20., is,

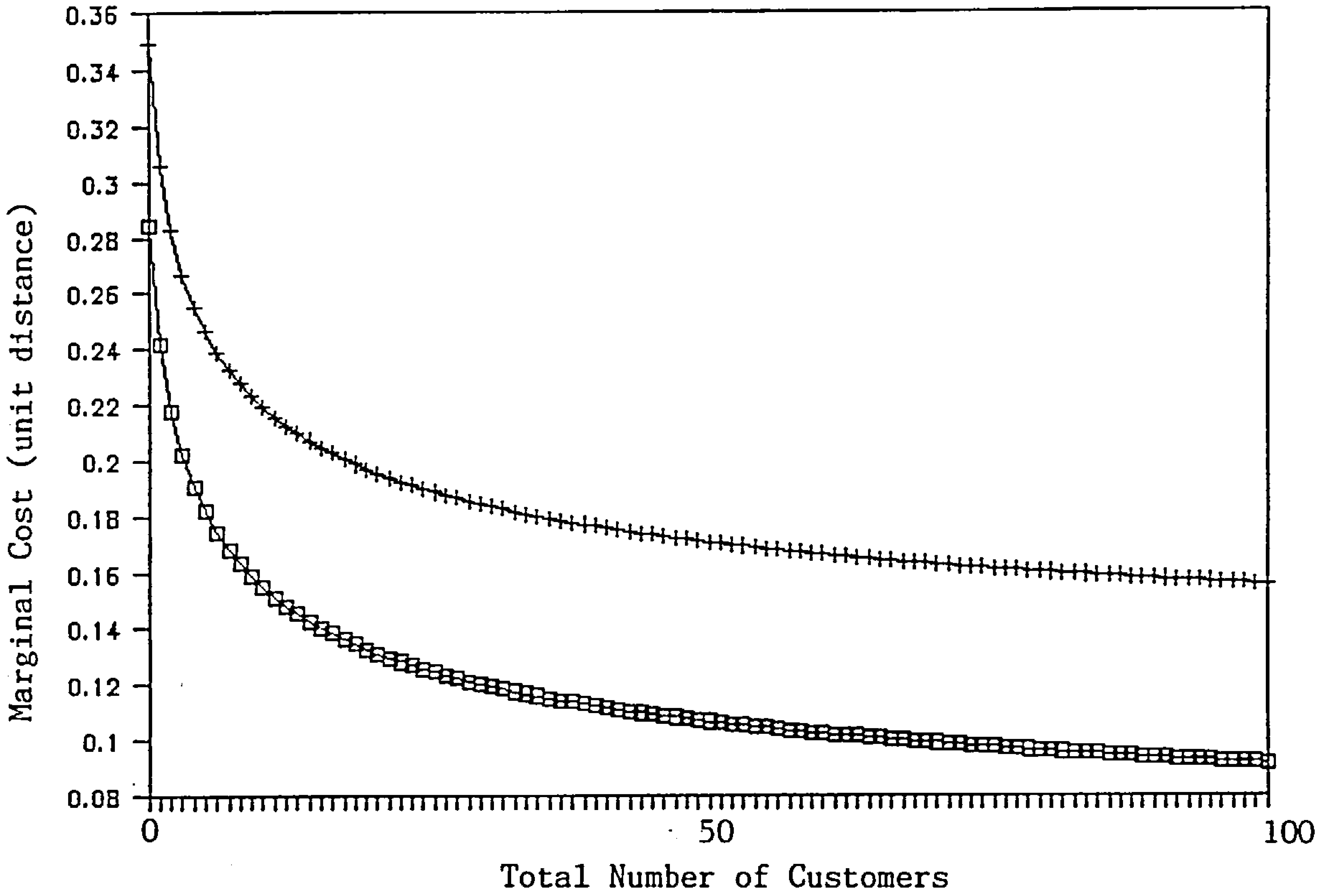
$$TFM = [(0.7379.(INT \left[\frac{P^{0\cdot6362}}{k^{0\cdot6362}} \right]) .k^{-0\cdot5394})(k-1)]$$

$$+ [INT\left(\frac{P}{k}\right)(0\cdot802147 - 0\cdot4115 \log.k)] \qquad (E.4.22.)$$

This is a far more cumbersome expression than E.4.21., and Figure 4.32. indicates that the effect of integers on estimates of distance are, in any case, minimal. Furthermore, the problems caused by differences between the values of C and k, described above, only arise when the total number of customers to be served, P, is small - a situation that is unlikely to occur with this type of problem in the real world - and so there is good reason to accept Equation E.4.21. as the most appropriate tool for estimating Total Fleet Mileage.

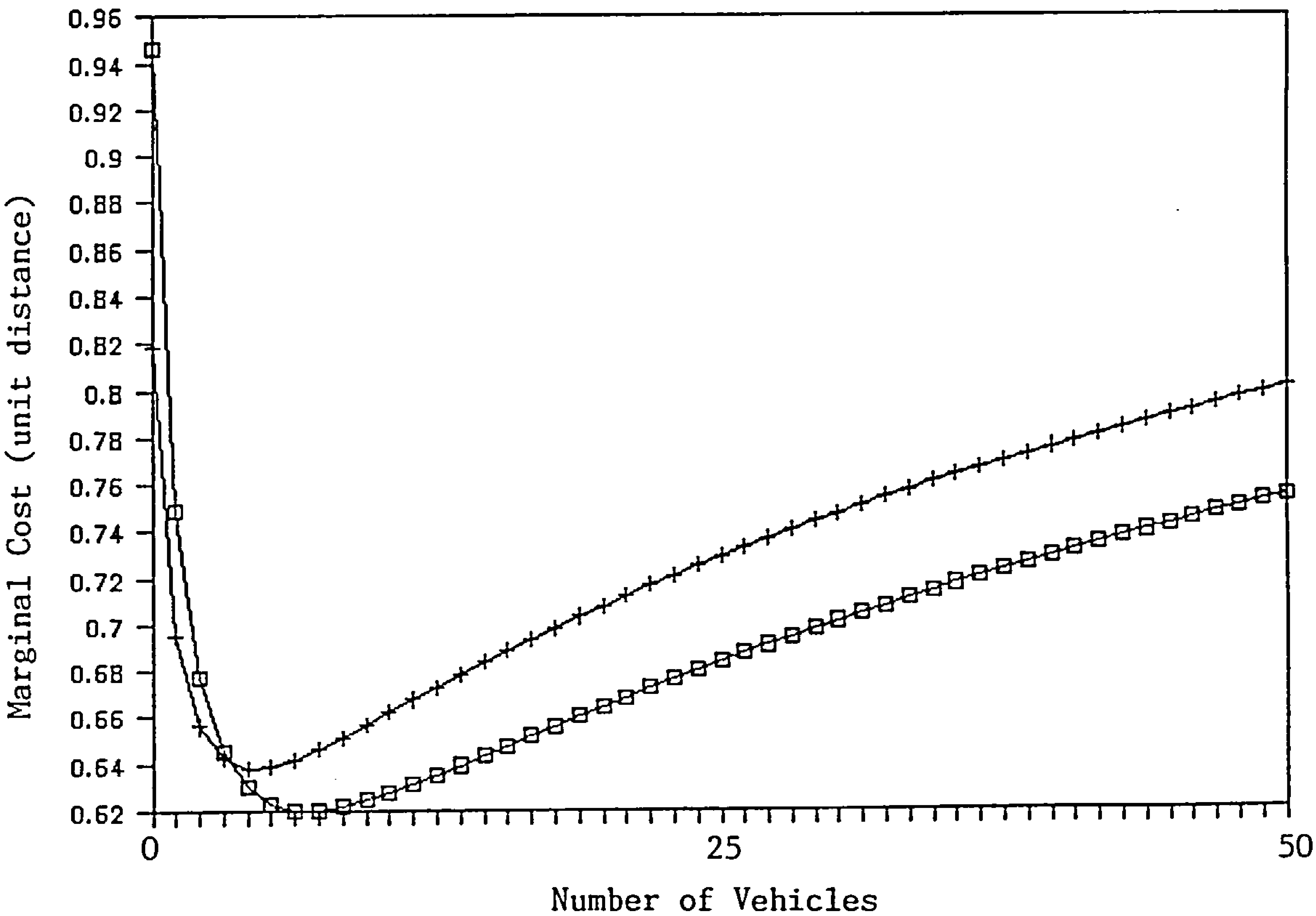
It is interesting to compare the relationship between fleet mileages and P, as shown in Figure 4.32., with corresponding relationships involving n, (SEE Figure 4.31.). In both cases, Total Stem Distance rises uniformly, which is mainly attributable to the fact that the number of stem-journeys is increased by both P and n, although there are obvious differences in the shape of the two Delivery Distance curves, and also in the effect that P and n have on <u>Total</u> Fleet Mileage. Whereas an addition to the number of vehicles that are to serve a fixed population of customers makes no difference to the Delivery Distance curve, Figure 4.31.2. clearly indicates that this component of Total Fleet Mileage increases substantially with P; this is to be expected, since any increase in the number of locations that are to be visited leads to a rise in the total number of between-customer links. As both Stem Distance and Delivery Distance increase with P, it follows that Total Fleet Mileage also rises; this is confirmed by Figure 4.32.. Although the Total Mileage curve in Figure 4.31.3. is virtually linear, the corresponding line with P as the independent variable is noticeably non-linear; this observation is confirmed by Figure 4.33.1., (SEE also Table 4.21.1.), which shows the Marginal Cost, in terms of distance, of adding an extra customer-location to the exisiting population, P - Figure 4.33.2., for the sake of comparison, graphs the marginal cost of adding an extra <u>vehicle</u> to the fleet whilst P is held constant at 100. In Table 4.21.1., the Marginal Cost value is calculated by subtracting the Total Fleet Mileage for each value of P from the Total Fleet Mileage figure of the next-highest P-value; a similar calculation is performed for consecutive n-values in order to arrive at Table 4.21.2..

Figure 4.33.1. Marginal Cost, in terms of distance travelled, as a function of P, with k fixed, using E.4.21. (a=1 (circle))



+ K=5 □ K=10

Figure 4.33.2. Marginal Cost, in terms of distance travelled, as a function of n, with P fixed, using E.4.20. (a=1 (circle))



+ P=50 □ P=100

Figure 4.33.2. is particularly interesting, due to the fact that the shape of the Marginal Cost curve shown here is completely different to that described in Figure 4.33.1.; whereas Marginal Cost is a "U"-shaped function of the number of vehicles used, the extra Fleet Mileage required to visit one additional customer-location becomes increasingly less significant. The "U"-shaped nature of Figure 4.33.2.'s curve is not detectable from the seemingly linear Total Fleet Mileage curve of Figure 4.31.3.; Figure 4.35., which illustrates the shape of Stem Distance, Delivery Distance and Total Fleet Mileage curves that are predicted using Equation E.4.20. in the absence of integer effects, does, however, reveal that both the Stem Distance and Delivery Distance curves are not completely straight, which causes a small bend in the Total Distance curve for small values of n. Nevertheless, these three curves are virtually identical to the straight lines of Figure 4.43., which are drawn using Equation E.4.27., (SEE Section 4.6. for a discussion of this expression). Despite this close similarity between the nature of the aggregate distance-estimates shown in Figures 4.35. and 4.43., an indication of the fundamental differences that exist between the respective equations used, along with an explanation of the erratic behaviour of the Marginal Cost curves of Figure 4.33.2., is provided by Figure 4.34.. This graph disaggregates the overall Marginal Cost curve derived from Equation E.4.20., to show both the extra Stem Distance and Delivery Distance incurred by adding one vehicle to the fleet. Clearly, these curves contrast with the corresponding graph that would be derived using Equation E.4.27.; Table 4.28. reveals that the Marginal Cost of both Stem Distance and Delivery Distance is constant, at approximately 0·66 and zero, respectively.

It should be stressed that the impact of Equation E.4.20.'s "U"-shaped Marginal Cost curve on predicted Total Fleet Mileage is relatively slight, and is, in any case, only effective when n is small. Nevertheless, Figure 4.34. draws attention to the structural idiosyncrasies of Equation 4.20., which are not otherwise apparent from aggregate figures, and also serves to further highlight the interaction between Stem Distance and Delivery Distance.

Figure 4.36., (SEE also Table 4.23.), illustrates the way in which the <u>percentage</u> of Total Fleet Mileage that is attributable to Stem Distance changes as P increases with k fixed at 5 stops, and may be compared with Figure 4.10., which shows the corresponding graph when n is the independent variable. The two graphs are similar in as much as Stem Distance as a percentage of Total Fleet Mileage increases with both P and n, but at an ever-decreasing rate. Closer examination of the curves, however, reveals that both the actual figure for this percentage, and the rate of increase, are greater for increasing fleet-size than for an expanding population of customers. This is because, whereas an addition to the size of the vehicle-fleet increases Stem Distance with virtually no change to Delivery Distance, an increase in P will lead to an increase in both components of

Figure 4.34. Disaggregated MC curves

with P fixed. (P=100,a=1(circle)).

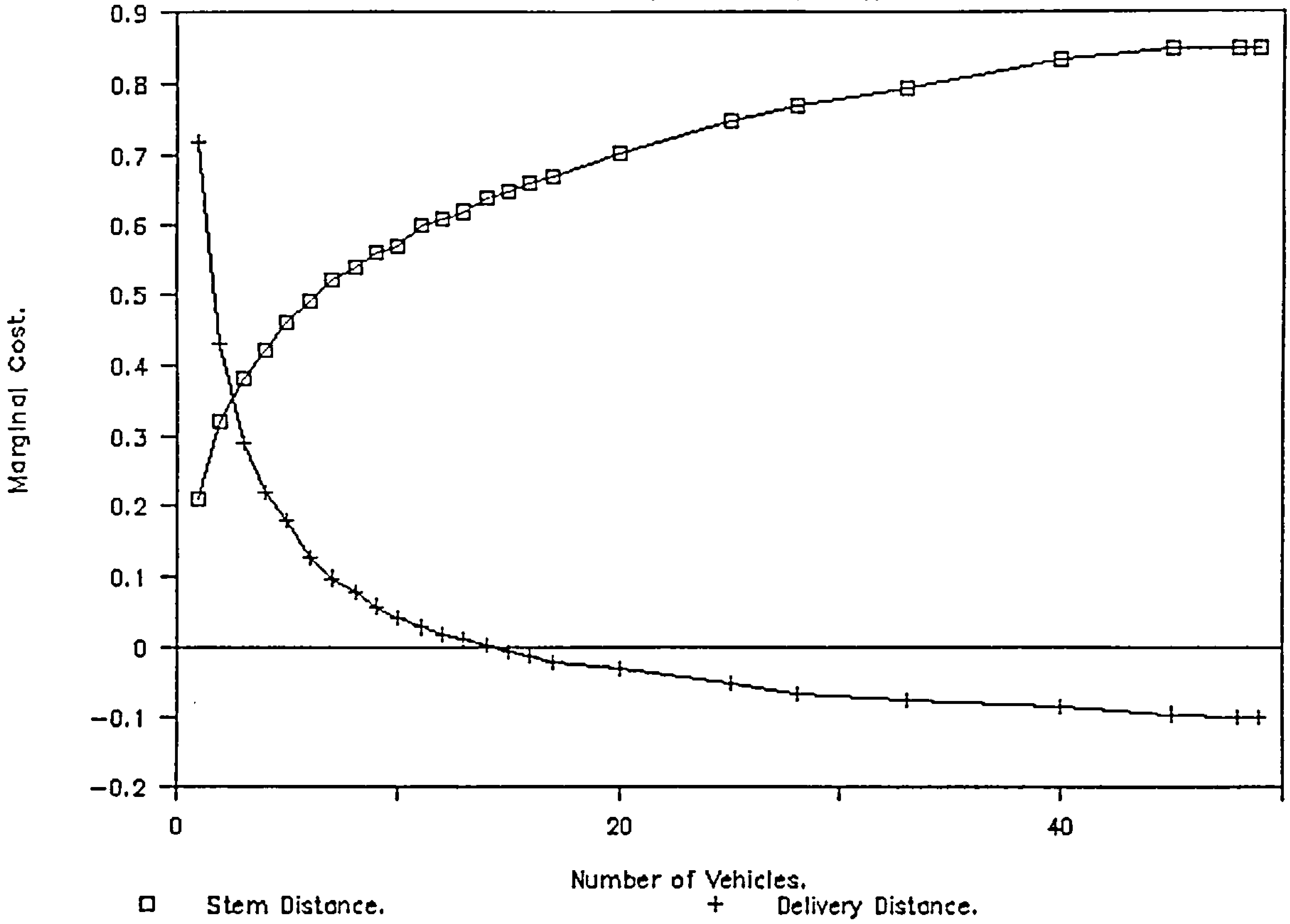


Figure 4.35. Estimates of Stem Distance, Delivery Distance and Total Fleet Mileage, with P fixed, using Equation E.4.20.

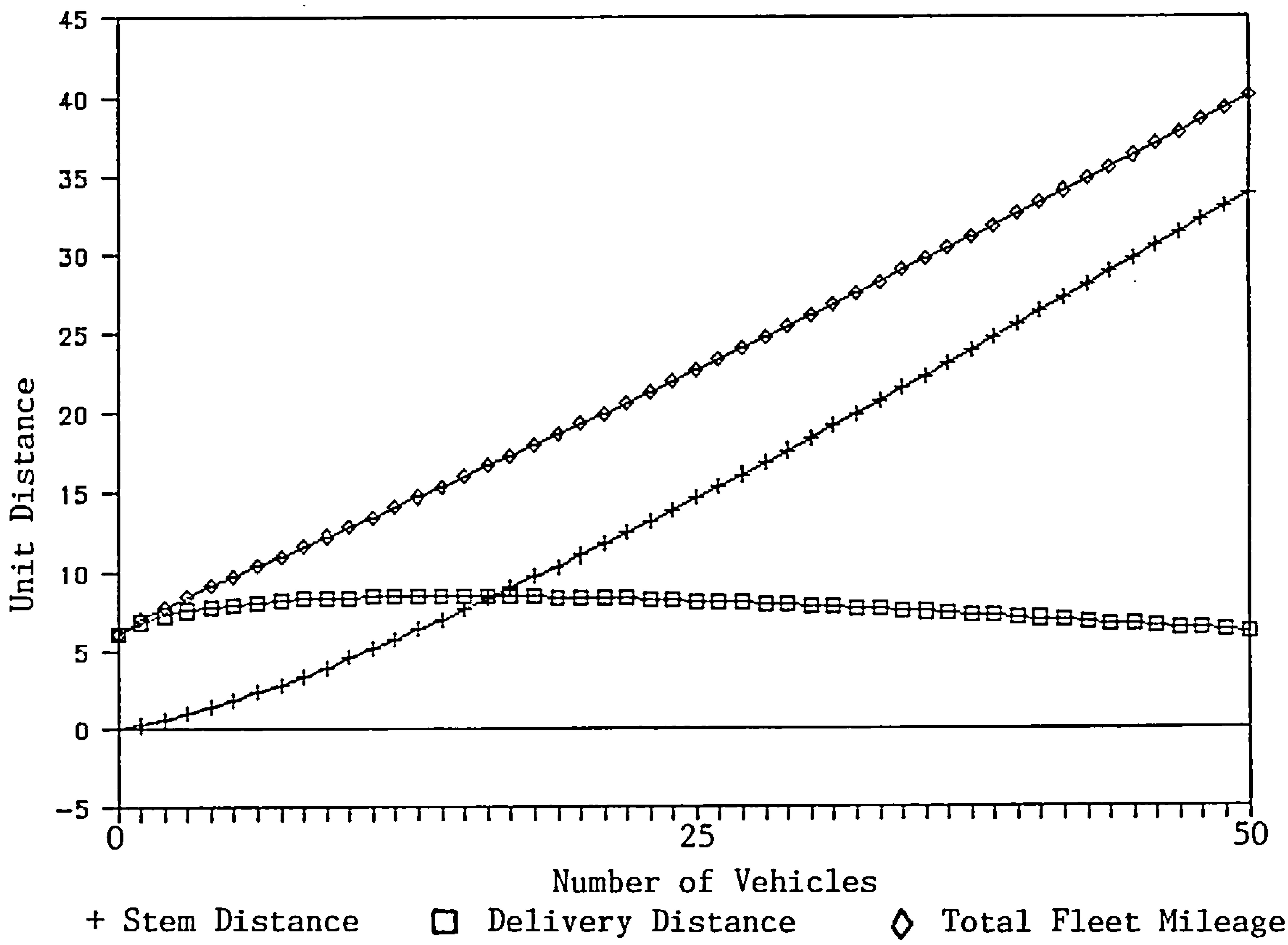
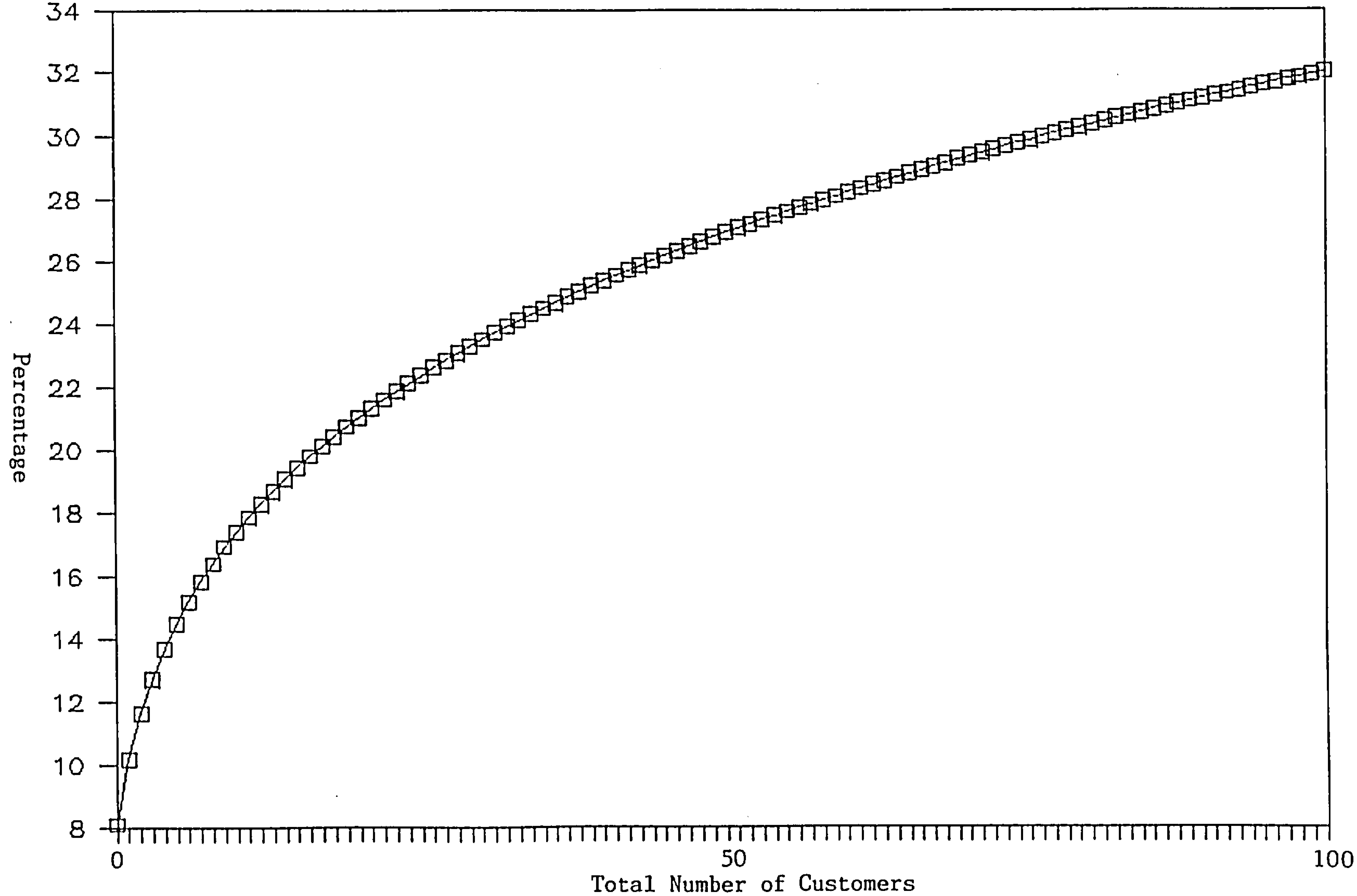


Figure 4.36. Stem Distance as a percentage of Total Fleet Mileage, as estimated from Equation E.4.21.. (k=10 a=1 (circle))



Total Fleet Mileage, (as Figures 4.32.1. and 4.32.2. confirm). Another difference between the two percentage graphs is that Figure 4.36. shows a smoother curve than the one plotted in Figure 4.10.; this is merely because the figures presented in Table 4.23. are estimates obtained directly from Equation E.4.21., whilst those associated with Figure 4.10. are observed average distances from computer simulations, (SEE Table 4.5.).

4.5. The Presence of Variance in the Results of Simulations

The data on which all distance estimates appearing here are founded, are derived from regression analyses that have been carried out on figures that are themselves made up of the means of many iterations of a simulation program; for this reason, the results published here are inevitably associated with a certain amount of variance. In other words, each predicted value of, say, Total Fleet Mileage, given n and C for example, is not a deterministic statement of what distance a fleet of vehicles will always travel, given certain stated conditions; it is rather an approximation of the mean of a range of values within which the Total Mileage of the fleet would fall on any one simulation. This is to say that each estimate is part of a distribution of values, which has both a mean and a Standard Deviation. The shape and range of this distribution is important, since the amount of confidence that may be placed in an estimate from an equation will decline as the spread of values that such an estimate represents widens. This type of variance, which may vary as a function of the nature of the constraints imposed on the model, is an integral part of simulation, and is a result of the generation of random numbers; this should not be confused with the variance of residuals either side of the regression-line. The latter measure rather gives an indication of the "goodness of fit" of the regression-line to the distribution that it describes, and is quantified by the R^2-value that accompanies each of the above equations. This type of variation in the results is of less interest, since this error will gradually disappear as the number of iterations of the simulation program carried out increases.

What is important is to be able to qualify distance estimates with an indication of the extent to which actual values will vary either side of what is really a mean figure, particularly as it is by no means certain that distance figures resulting from simulations will be normally distributed about this mean.

There are various ways of doing this - the following description of one such method uses, as an example, the set of estimates generated of Stem Distance per Vehicle as a function of C; this data has been chosen because it has already been shown in Section 4.3. that this measure varies only with C, and is therefore independent of n. The shape of this relationship is similar to that shown in Figure 4.18., although all figures appearing in this section are based on the assumption of a square delivery-area, due to the availability of a larger body of data. For the purposes of the following numerical example, three values of C are considered, these being 5,10 and 20. The distributions of the results of all the simulations that contributed to the estimate of Stem Distance per Vehicle for each of these C-values, are illustrated in Figure 4.37., and the relevant statistics are contained in Table 4.22.. The histograms shown here, suggest that the actual distance figures produced by simulation are more or less normally distributed around the mean value for each value of C, although there appears to be a marked tendency for the distributions to become increasingly lopsided as C increases. This is confirmed by the skewness values given in Table 4.22.; although the value for when C=5 is virtually zero, the distributions of values when C=10 and 20 are clearly slightly positively-skewed, indicating that the majority of simulations in each case yielded distance estimates that were lower than the sample mean. A Normal Distribution would have a skewness coefficient of zero, but the coefficients contained in Table 4.22. are not sufficiently greater than zero for any serious doubt to be cast on the validity of accepting the mean value of each series of simulations as a reliable estimate of Stem Distance per Vehicle in each case.

The statistics for kurtosis, also shown in Table 4.22., give some indication of the extent to which the distributions "peak". A Normal Distribution has a kurtosis of 3·0, so that the distribution plotted in Figure 4.37.1. is rather flatter than "Normal", whilst the corresponding histogram for C-values of 10 and 20 are shown to be slightly peaked, or "leptokurtic". Again, the kurtosis figures displayed in Table 4.22. do not diverge sufficiently from those that would be expected with a Normal Distribution to cast doubt upon the validity of distance-estimates associated with such distributions.

Perhaps the most useful statistics appearing in Table 4.22. are the Standard Deviation figures - Standard Deviation being simply the square root of variance - as these provide a good indication of the amount of "spread" that exists in the original data, either side of an estimate. The Standard Deviations of the distributions increase as C decreases; this is to be expected, since the likelihood of obtaining an "erratic" result from a single simulation will obviously be reduced as the number of locations visited per vehicle-tour is increased.

When used in conjunction with the mean value of a set of simulations, Standard Deviation may enable approximate confidence limits to be placed around an estimate, and, at the same time, illustrate the way in which variance changes for different values of the independent variable. For example, given a normally-distributed set of values, 68·2% of these will lie within a range of 1·0 Standard Deviations either side of the mean; similarly, 95·4% of these will fall within 2·0 Standard Deviations of the mean, and 99·7% within

Figure 4.37.1. Value-distribution of Stem Distance per vehicle

(C=5, a=1 (square))

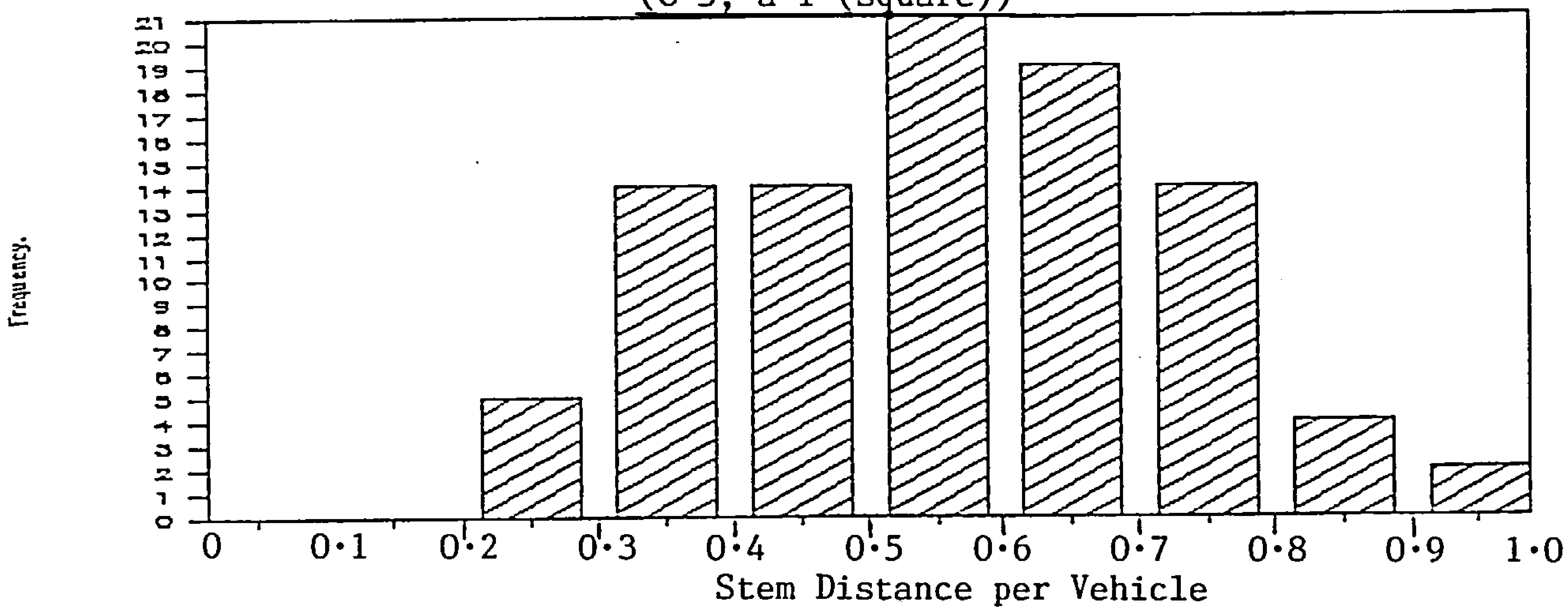


Figure 4.37.2. Value-distribution of Stem Distance per vehicle

(C=10, a=1 (square))

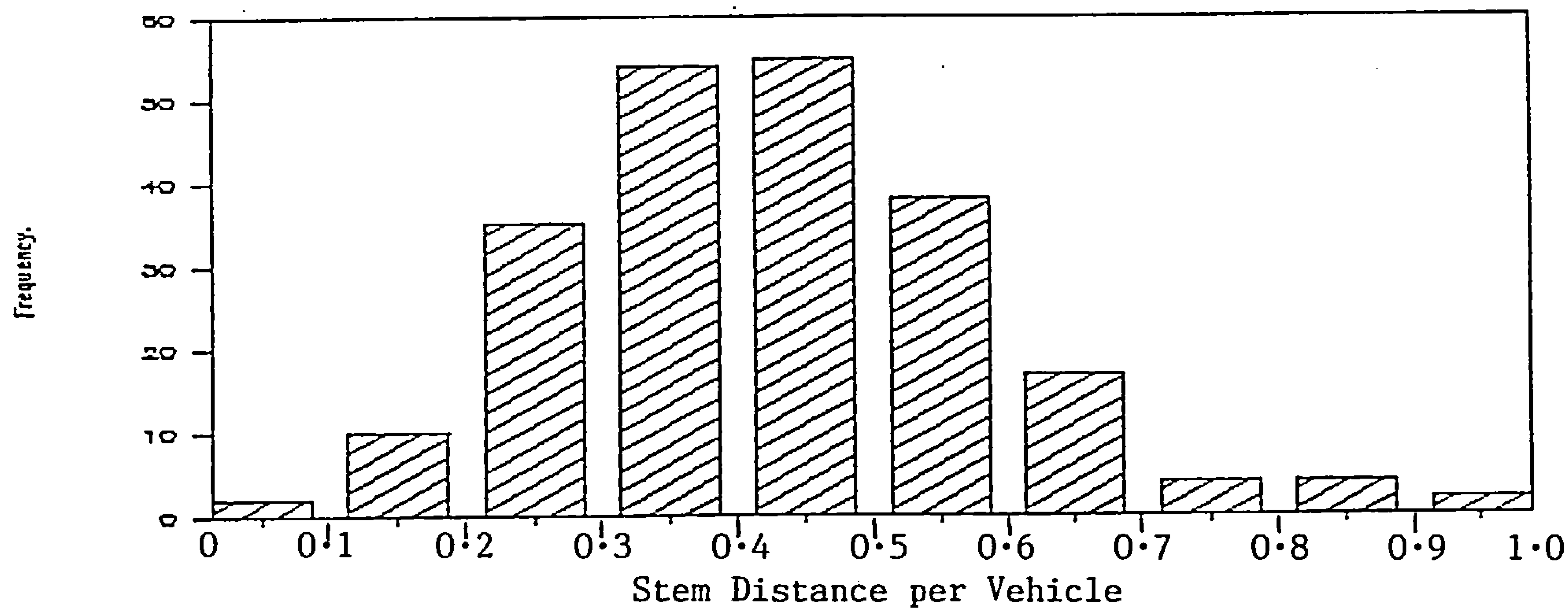
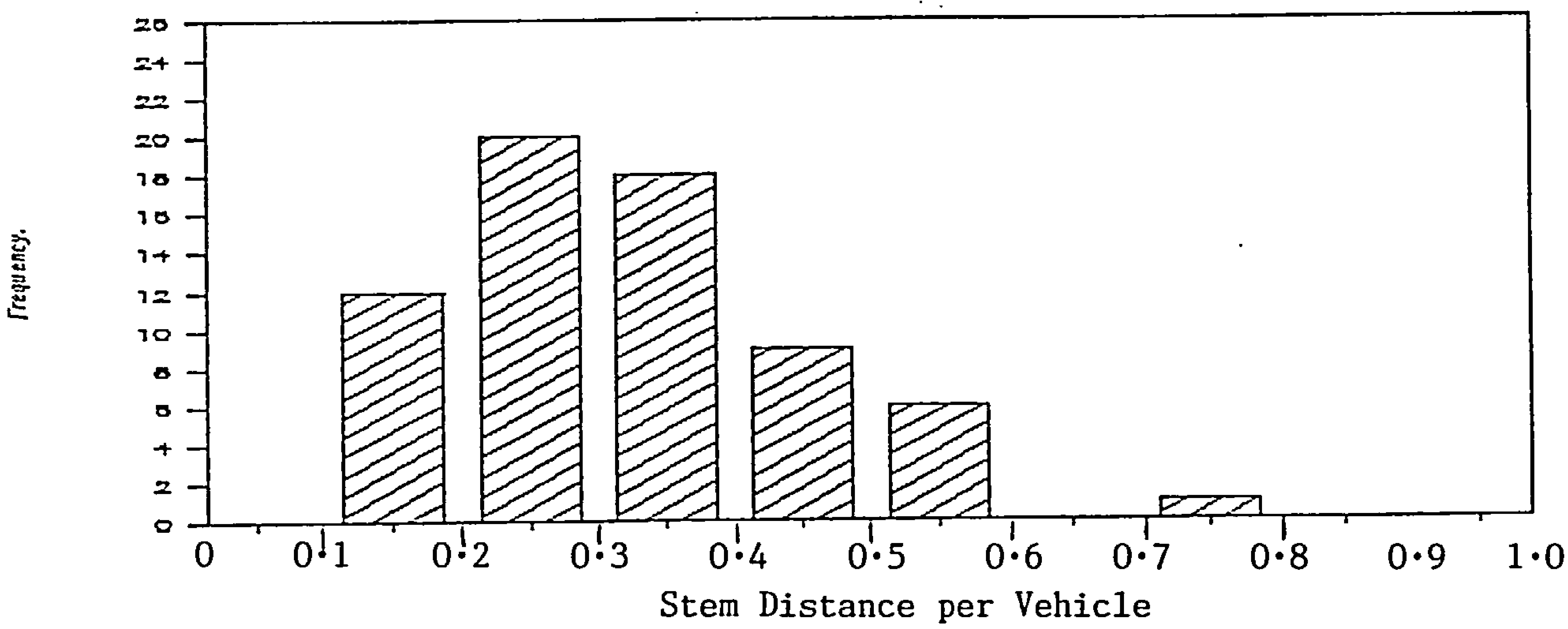


Figure 4.37.3. Value-distribution of Stem Distance per vehicle

(C=20, a=1 (square))



3·0 Standard Deviations. Using these figures, referring again to the information contained in Table 4.22., it may be deduced that there is a 0·682 probability that the Stem Distance per Vehicle figure resulting from any given simulation will have a value within one Standard Deviation of the mean; in other words, when C=5, this is the probability that any of the simulation results, taken at random, will yield a value of between 396·06 and 726·94. This calculation may be repeated for any value of C, so that confidence limits may be applied to a set of estimates of mean values, although it should be noted here that this is not the same thing as placing confidence intervals around a regression-line, as a regression-line does not necessarily coincide with the mean of each set of simulations. Figure 4.38. shows the results of performing this exercise on the <u>three</u> values of C featured in Table 4.22., (although it is quite possible to do these calculations for any value of the independent variable). The three lines linking these three sets of points have been extended to illustrate the way in which the range of values defined by these limits, which may be fixed to correspond to any chosen level of confidence, becomes smaller as C increases. The range shown in Figure 4.38. appears to be quite wide, particularly as 68·2% is a low level of confidence in relation to those that might normally be used in a real world context, although it should be pointed out that the probability of the result of a given simulation having a particular value within such a range increases the closer this value is to the mean.

It would be unwise to draw too many firm conclusions from what is a relatively small sample, here, in terms of both the numberof C-values considered, and the total number of simulations involved, (SEE Table 4.22.). The numerical example above rather serves as an illustration of how statistics such as Skewness, Kurtosis and Standard Deviation may be used to test the validity of an estimate that is derived from a series of simulation exercises. Such a statistical check would, in fact, appear to be quite necessary, due to the inevitability of the presence of some variance in the results.

4.6. Comparison of Findings with Work of Other Researchers

Most of the published work by other researchers that relates to the topics dealt with in this chapter, is associated with the Continuous Space Modelling approach to the analysis of distribution systems; this approach has already been described in some detail in Chapter 1. Much of the research discussed in this section obtains a substantial amount of its data from sources that are described in the literature as "empirical" although such information is invariably derived by means of computerised simulation rather than by direct observation of real-world systems. However, there is no evidence of any attention being paid to the variance associated with such data. Nevertheless, equations for Total Fleet Mileage proposed elsewhere often have much in common with those developed earlier in this chapter, in as

Figure 4.38. Application of confidence limits to mean value. (a=1000)

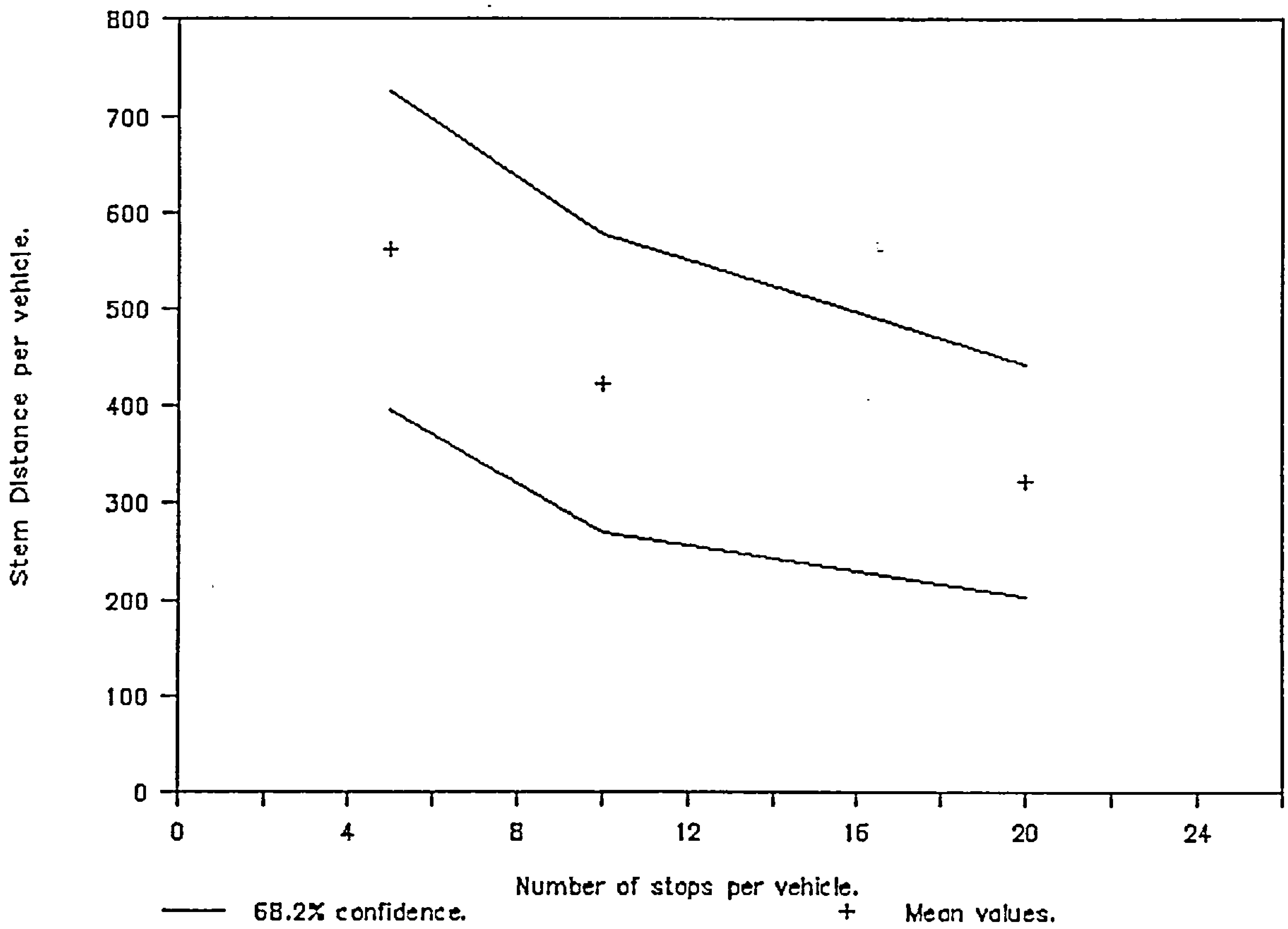


Table 4.22. Dispersion statistics of Stem Distance per vehicle as a function of C

	C=5	C=10	C=20
Mean	561·5	423·4	322·8
Standard Deviation	165·44	153·25	120·26
Skewness	0·018	0·515	0·691
Kurtosis	2·384	3·592	3·289
Number of observations	93	223	67

much as they involve similar parameters, and are often the product of a similar approach to the problem.

For example, the formula that Daganzo proposes for Fleet Mileage per customer-location, (Daganzo, 1982), (which was originally published by Eilon et al, (Eilon Watson-Gandy & Chrisofides, 1982)), is relevant to the findings described above, not only in the way in which it uses the variables P and C, but also in as much as a distinction is clearly made between what is described as "Line Haul" and "Detour Distance". The analogy that exists between these two terms, and Stem Distance and Delivery Distance, has already been described in Sections 4.2. and 4.3.. The expression in the form presented by Daganzo, although the precise notation has been changed for the sake of clarity, is as follows,

$$\frac{TFM}{P} = \frac{1 \cdot 8.D_r}{C} + \frac{1 \cdot 8.D_r}{\sqrt{P}} \qquad (E.4.23.)$$

with the first term representing Stem Distance per Point and the second term Delivery Distance per Point. Daganzo relates the variable C to vehicle carrying-capacity, although it has already been argued in earlier sections that this parameter can be used as a general indicant of the maximum number of stops that may be made per vehicle-trip. Therefore, C in this equation can be inter-changed with k, and the whole of E.4.23. may be multiplied by P in order to convert this expression to a formulation for Total Fleet Mileage. Equation E.4.23. thus becomes,

$$TFM = P \left[\frac{1 \cdot 8.D_r}{k} + \frac{1 \cdot 8.D_r}{\sqrt{P}} \right]$$

and so,

$$TFM = 1 \cdot 8.D_r \left[\frac{P}{K} + \sqrt{P} \right] \qquad (E.4.24.)$$

This form of the expression makes comparisons with E.4.21. far easier, although it should be noted that E.4.24. is applicable to tours made within a square delivery-area, whilst the latter equation is based on the assumption of a circular area.

The main difference between Daganzo's formula, and those developed in this chapter, is Daganzo's use of the parameter D_r in the calculation of both components of Fleet Mileage. In the context of Stem Distance, it can be deduced from Equation E.4.24. that this distance may be estimated as being $(1 \cdot 8.D_r.n)$, since n=P/C, so that Stem Distance per vehicle is $1 \cdot 8.D_r$, regardless of the value of C. In other words, this expression assumes that, no matter how many customer-locations are included in a tour, the average distance from the depot of both the first and the last location to be visited by each vehicle will always be 0·9 times the value of D_r; this means that Total Stem Distance will always be a direct function of n! Since the results of simulations have shown in this chapter that the average length of a stem-journey is reduced as C increases, Daganzo's estimate must be regarded as being a very crude one! Furthermore, the term $(1{\cdot}8.D_r.n)$ will clearly increasingly overestimate Stem Distance as the value of C becomes larger - this is illustrated by Table 4.24. and Figure 4.39., which compare estimates of Stem Distance per vehicle obtained from Equation E.4.20., (SEE Table 4.19.), with the value $(1{\cdot}8.D_r)$. These figures are based on the assumption that the total population of customers, P, is fixed at 100, and that the parameters n and C are variable; as it is assumed that the delivery-area is a circle of 1 distance-unit in diameter, $(1{\cdot}8.D_r)$ is fixed at 0·5968. As the graph shows, when C=1, both Equation E.4.20. and Equation E.4.23. actually underestimate Stem Distance, since average Stem Distance per vehicle, in this situation, is obviously $2.D_r$, or 0·6631!

The Delivery Distance component of Fleet Mileage, according to Equation E.4.23., is merely a function of D_r and P, with D_r, in this case, being a direct function of a; (Daganzo himself estimates D_r in a square area to be 0·382a, (Daganzo, 1982), which is slightly greater than the figure of 0·3648a, suggested by computer simulations). The term D_r therefore acts as a constant reflecting the size of the delivery-area. When P is fixed, therefore, the estimate of Delivery Distance will remain constant - this is confirmed by Figure 4.9., despite the fact that Equation E.4.20. describes Delivery Distance as a complex function of both n and C. Figure 4.40., (SEE also Table 4.25.), summarises the distance estimates that are calculated from Equation E.4.23., and the shape of the curves that are illustrated here are very much like those of Figure 4.9.. In much the same way, figures calculated from Equation E.4.24., which expresses Total Fleet Mileage as a function of P and k, may be compared with the estimates that are presented in Table 4.20. and graphed in Figure 4.32.; Figure 4.41. shows that the general shape of the curves derived using, first E.4.24., and then E.4.21., is similar in as much as Stem Distance rises linearly with increasing P whilst Delivery Distance, and therefore Total Fleet Mileage, are seen to have a non-linear relationship with P. However, closer observation of the two sets of estimates, (SEE Tables 4.26. and 4.21.1.), reveals that, as anticipated, Equation E.4.24. substantially overestimates Stem Distance, which leads, in turn, to overestimates of Total Fleet Mileage, according to the evidence of the results obtained from computer simulations. This error becomes more and more pronounced as P increases. Figures for Delivery Distance using Equations E.4.21. and E.4.24. are far less disparate, although there is a definite tendency for the

Figure 4.39. Stem Distance per vehicle

using E.4.20. and (1.8.Dr).(P=100,a=1).

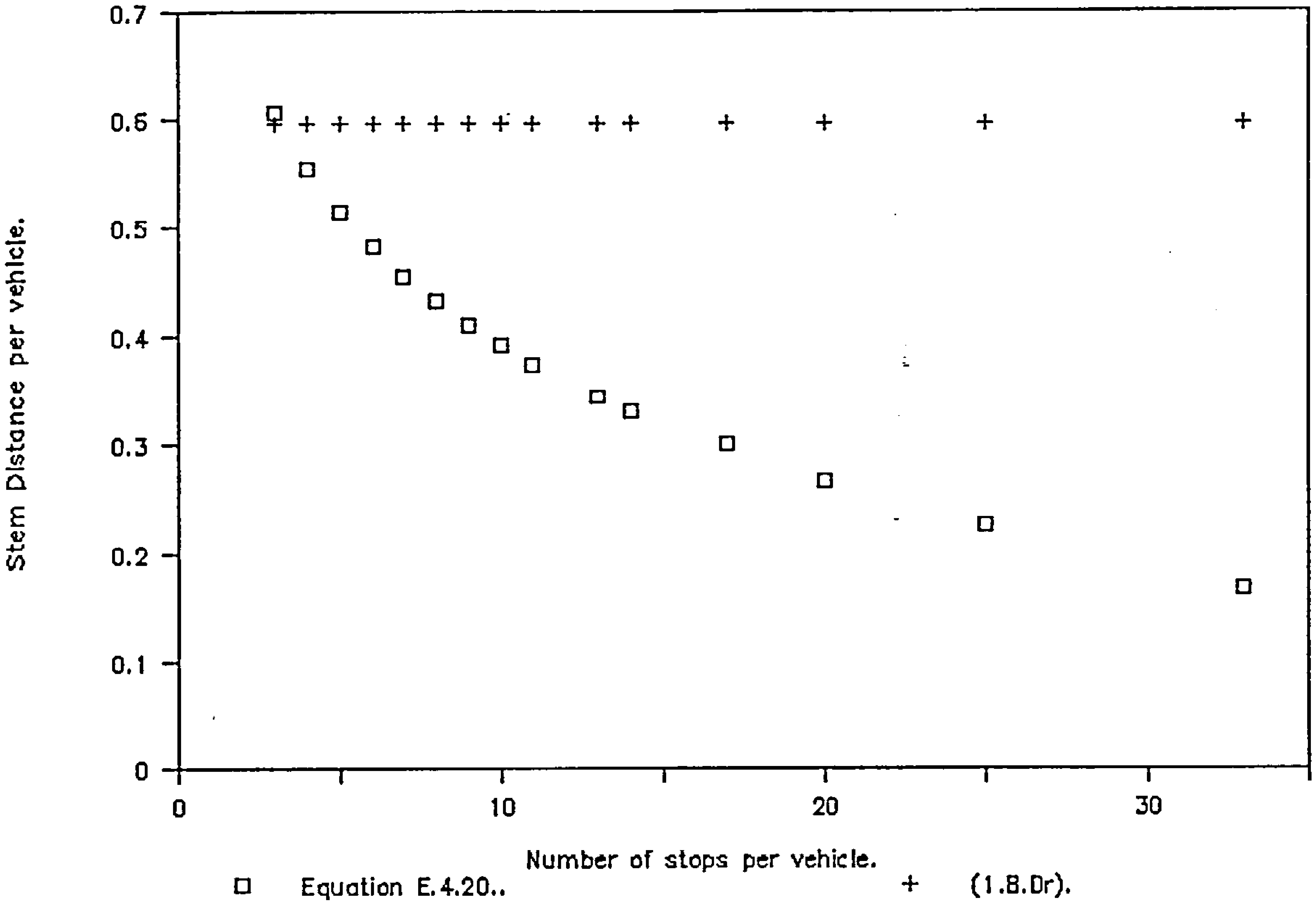
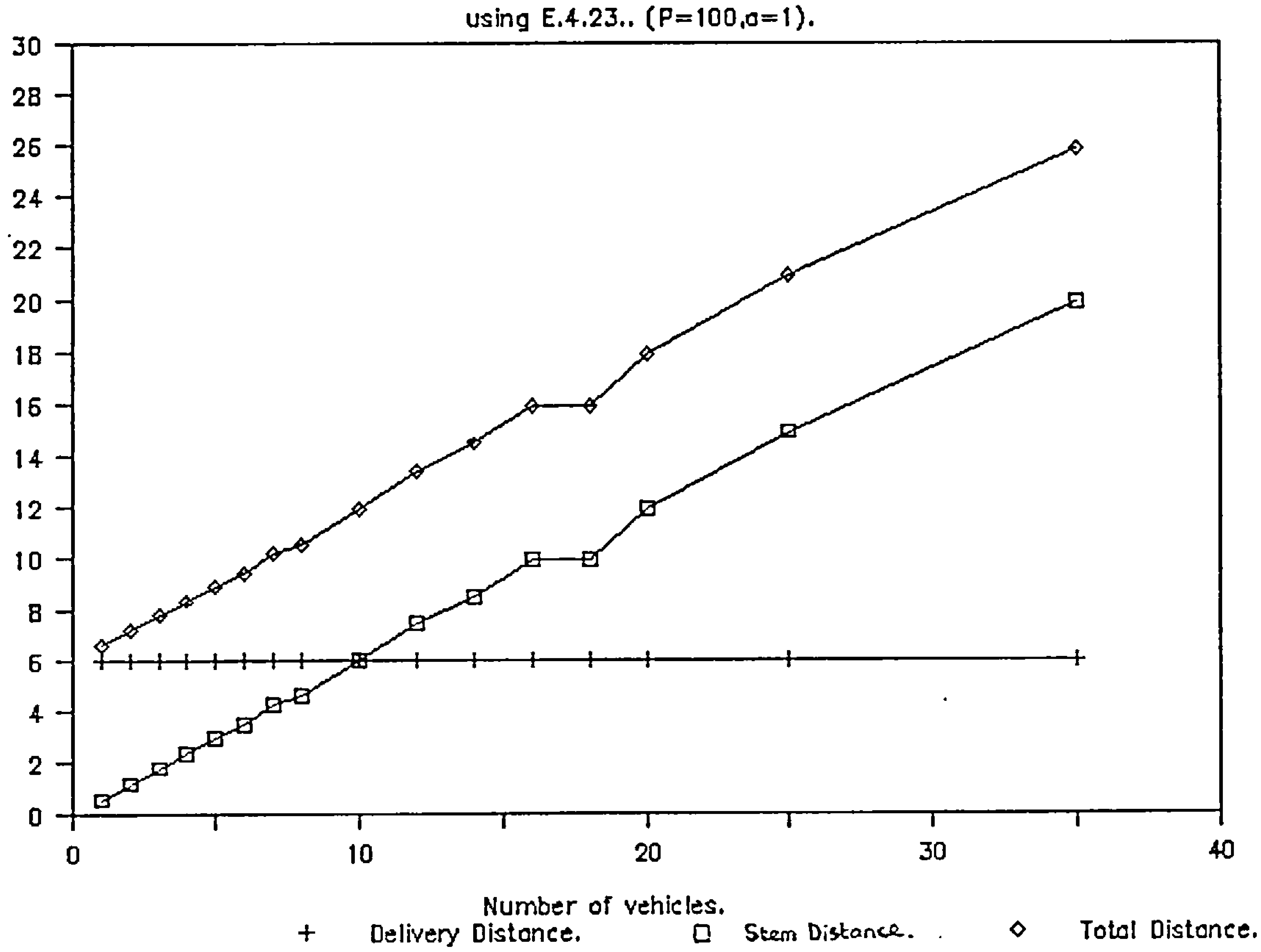


Figure 4.40. Distance estimates, with P fixed

using E.4.23.. (P=100,a=1).



latter expression to underestimate. The Marginal Cost figures presented in Table 4.27., and plotted in Figure 4.42., reflect the non-linear nature of the Delivery Distance and Total Fleet Mileage curves of Figure 4.41., and follow the general shape of the Marginal Cost curves shown in Figure 4.33.1..

Another expression for Total Fleet Mileage which makes a clear distinction between Stem Distance and Delivery Distance is proposed by Daganzo & Newell, (3), who, in the course of exploring the trade-off between transportation costs and costs related to the storage of goods in vehicles, utilise the following expression,

$$\frac{TFM}{P} = \frac{2.D_r}{C} + \frac{0 \cdot 57}{\sqrt{\delta}}$$

Again, the equation appears in the literature as a formula for distance per point, but this expression may readily be rearranged so that it may be used as an equation for Total Mileage,

$$TFM = \frac{2.D_r.P}{C} + \frac{0 \cdot 57.P}{\sqrt{\delta}} \qquad (E.4.25.)$$

The Stem Distance component of this expression is almost identical to that in Equation 4.23., except that the constant in this term is 2·0 rather than 1·8; the effect of this difference is to exaggerate the extent to which Stem Distance is overestimated for high values of P!

The Delivery Distance term in this equation, however, is different, since it is expressed as a function of P and the density of customers in the delivery-area; the use of density in this context illustrates an alternative means of representing the average spacing between points to the parameter i. One advantage with using density is that it may itself be expressed in terms of P, as density is simply P/A. With a square delivery-area, when making calculations on the basis of "unit area", there is no problem, as, with a 1 by 1 square, A=a=1, so that density is equal to P. With a circular area, however, the situation is slightly more complex, since,

$$A = \frac{22.r^2}{7}$$

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(3) DAGANZO, C.F., and NEWELL, G.F., "Physical Distribution from a Warehouse: vehicle coverage and inventory levels". in Transportation Research Special Issue: Transportation Systems and Logistics, (Part B: Methodological). Vol 19b., (Oct., 1985), No.5., P.P. 397-407.

The value of r for a delivery-area of diameter 1 is 0·5, so that,

$$A = \frac{22}{28} + 0{\cdot}7857$$

Therefore, the density of the circular area is,

$$\delta = \frac{P}{0{\cdot}7857} = 1{\cdot}2727.P. \qquad (E.4.26.)$$

Substituting Equation E.4.26. into E.4.25., and using the empirical finding that D_r=0·33155 for a circular delivery-area,

$$TFM = \frac{0{\cdot}6631.P.}{C} + \frac{0{\cdot}57.P}{\sqrt{1{\cdot}2727.P}}$$

Rearranging this expression,

$$TFM = \frac{0{\cdot}6631.P.}{C} + 0{\cdot}57.P.1{\cdot}2727^{-0{\cdot}5}.P^{-0{\cdot}5}$$

So that,

$$TFM = \frac{0{\cdot}6631.P.}{C} + 0{\cdot}5052.P^{0{\cdot}5} \qquad (E.4.27.)$$

Again, distance estimates using this formula may be calculated and compared with those generated using alternative expressions; the relevant figures are presented in Tables 4.28. and 4.29., and illustrated graphically in Figures 4.43. and 4.44.. The ways in which these estimates differ from those calculated from Equation E.4.20., with P fixed at 100, (SEE Table 4.20.), are that Stem Distance figures are slightly higher whilst estimates of Delivery Distance, on the other hand, are slightly lower. In contrast to Figures 4.33.1., 4.33.2. and 4.42., the curve for the Marginal Cost of P, using this equation, would be horizontal! When a constraint on the maximum value of C is imposed, so that P is the main independent variable, the tendency for Stem Distance to be overestimated and for both Delivery Distance and Total Fleet Mileage to be underestimated is repeated.

Blumenfeld and Beckmann, (4), also use the concept of there being two distinct parts of a vehicle-tour, although

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(4) BLUMENFELD, D.E., and BECKMANN, M.J., "Use of Continuous Space Modelling to estimate freight distribution costs". Transportation Research., Vol. 19A., No. 2., (Mar.,1985), P.P.173-87.

Figure 4.41. Estimates of Total Fleet Mileage as a function of P, with k fixed, using Equation E.4.24.. (k=10, a=1 (square))

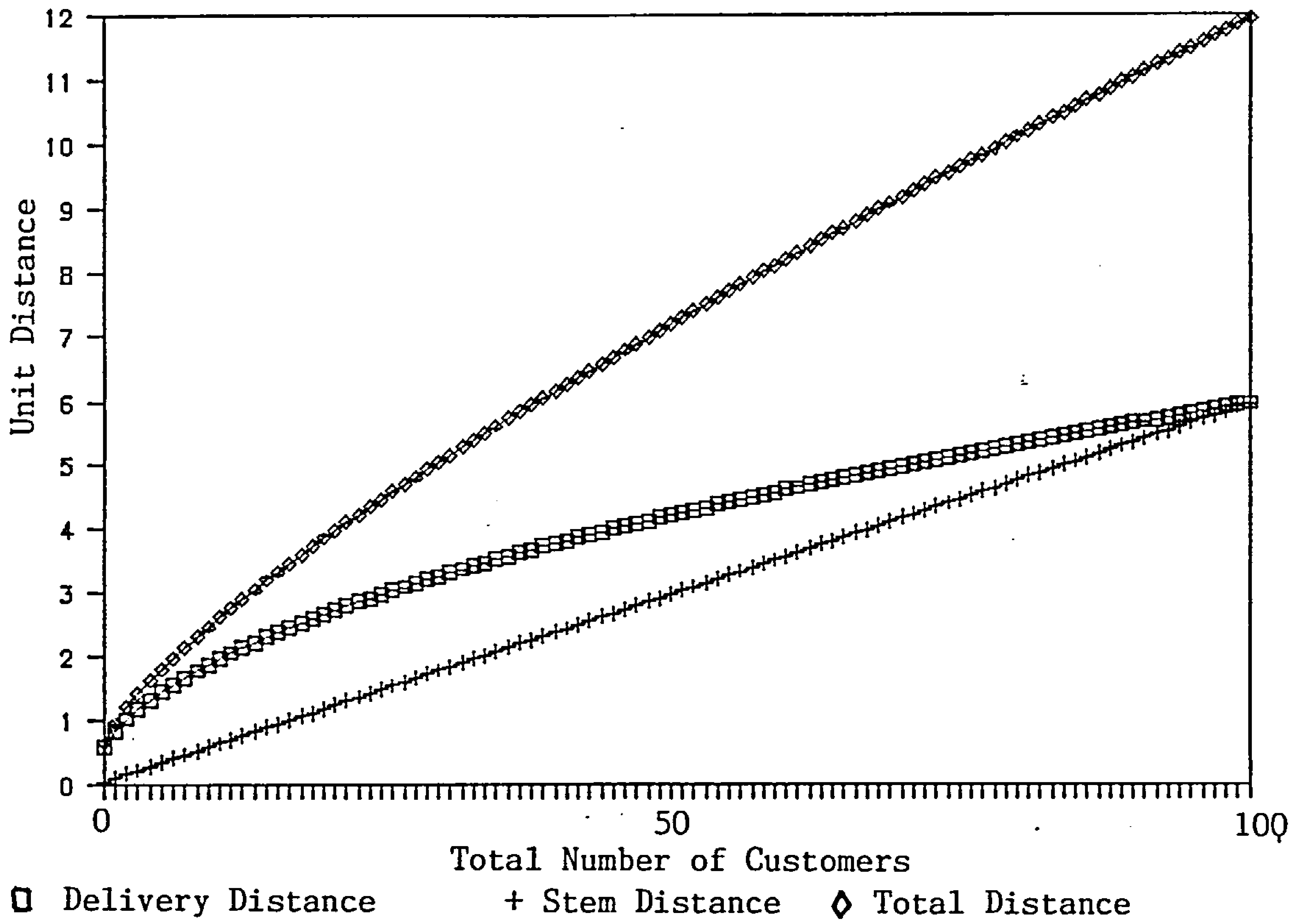


Figure 4.42. Marginal Cost as a function of P, with k fixed, using Equation E.4.24.. (a=1 (square))

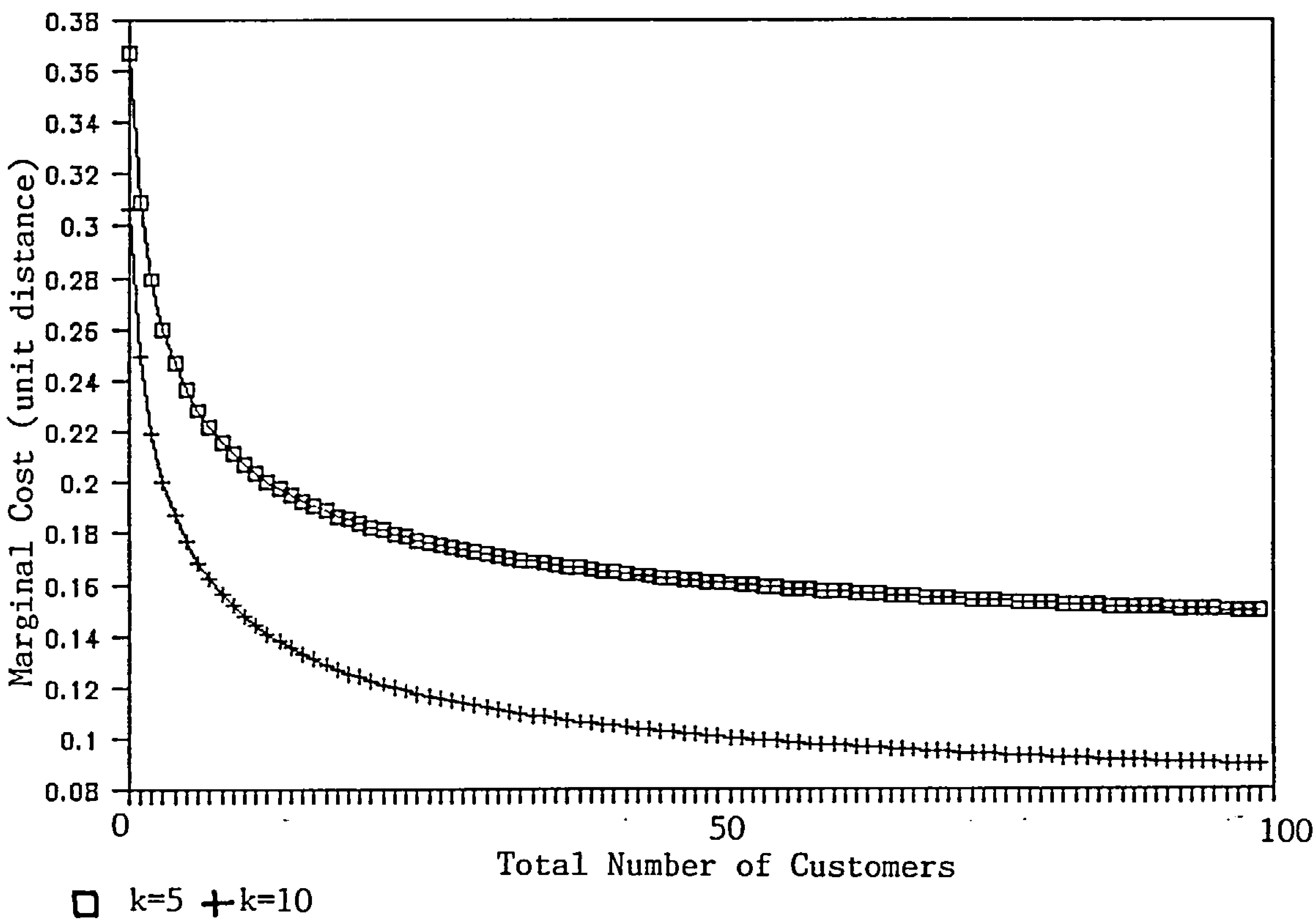


Figure 4.43. Estimates of Total Fleet Mileage as a function of p and C with P fixed, using Equation E.4.27.. (P=100,a =1 (circle))

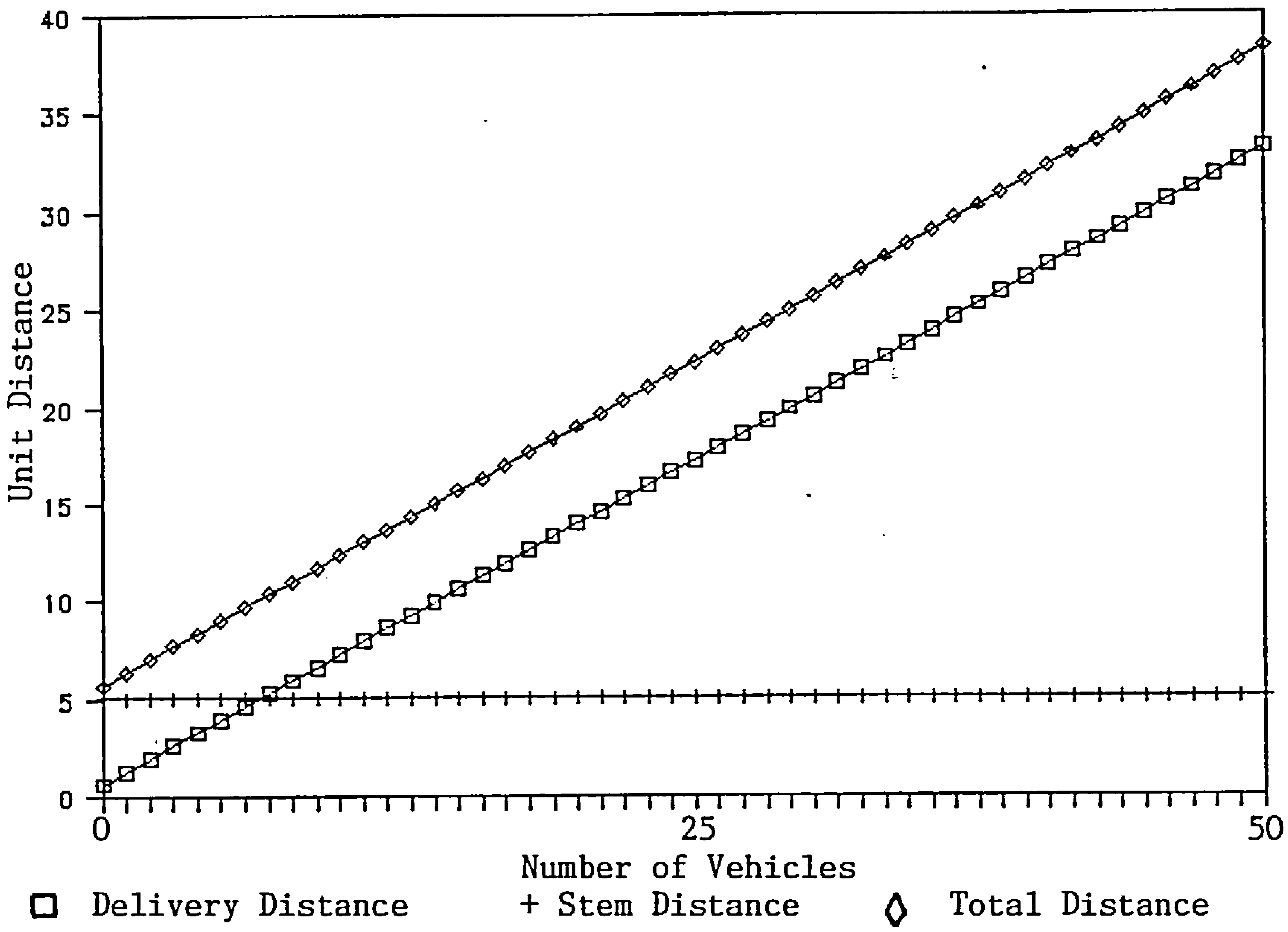
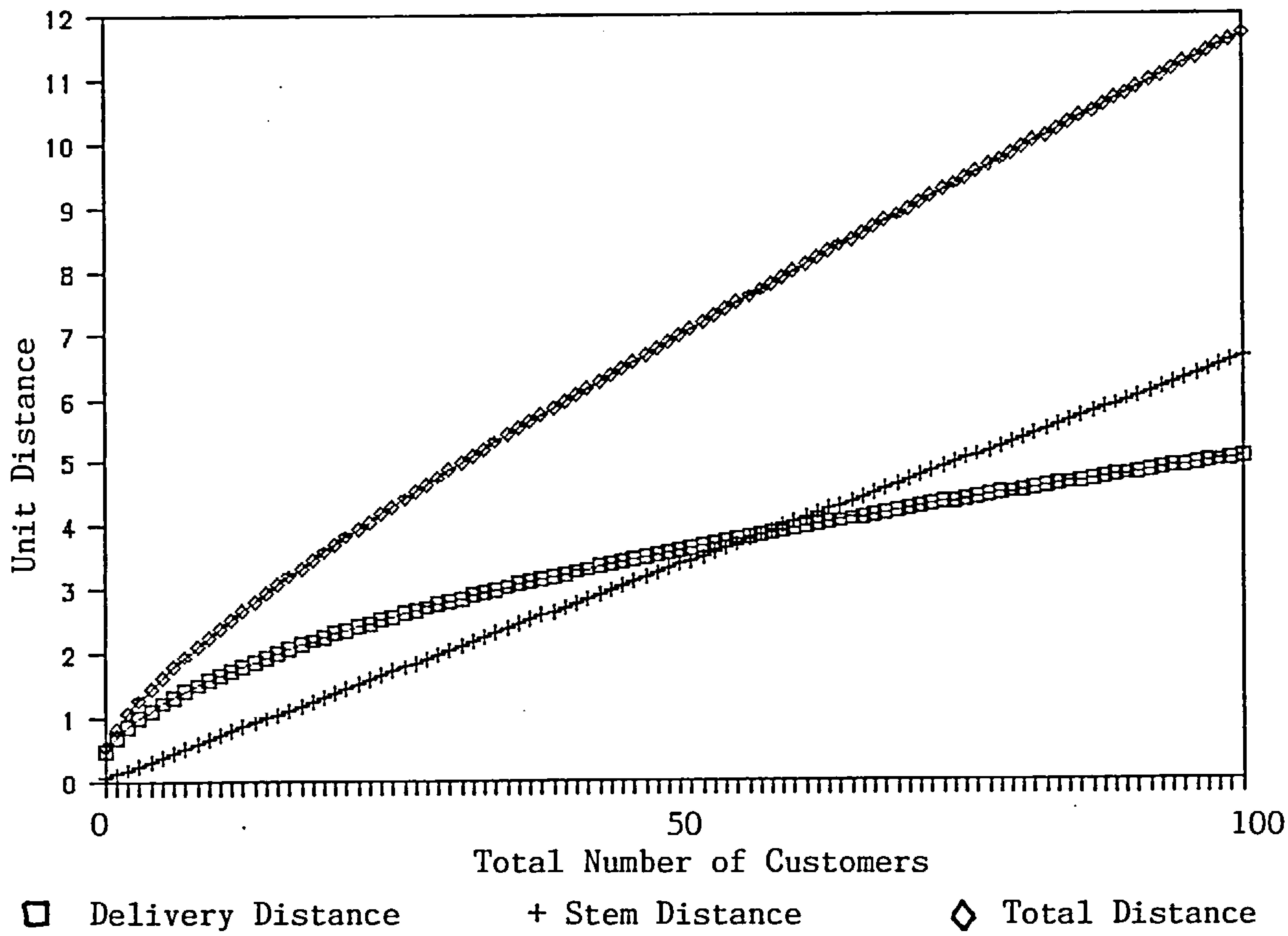


Figure 4.44. Estimates of Stem Distance, Delivery Distance and Total Fleet Mileage as a function of P, with k fixed, using E.4.27. (k=10, a=1 (circle))



their problem formulation involves the distribution of freight to a sub-region, or cluster of locations, that is remote from the depot. The two terms of the simple equation used for "Peddling Distance", which is equivalent to Total Mileage per vehicle-tour,

$$2.D_r + w$$

therefore represent the distance travelled from the depot to the sub-region and back, and the distance covered within this sub-region, respectively, However, analogies may once again be made with the concept of Stem and Delivery Distance used throughout this chapter.

The formula for the distance travelled within each sub-region, (ie. Delivery Distance per Vehicle), is,

$$K\sqrt{\frac{C.A}{n}} \qquad \text{(E.4.28.)}$$

where, K is a constant,
and, A is the area of a delivery-zone whose shape is unspecified.

Since,

$$A = \frac{P}{\delta}$$

and,

$$n = \frac{P}{C}$$

Equation E.4.28. may be rewritten as a function of C, δ and the constant, K,

$$K\sqrt{\frac{C.C}{\delta}}$$

or,

$$\frac{K.C}{\sqrt{\delta}} \qquad \text{(E.4.29.)}$$

Blumenfeld & Beckmann claim that the value of K, according to Daganzo, is approximately 0·6, (5), so that the total length

(5) DAGANZO, C.F., The length of tours in zones of different shapes, (Institute of Transportation Studies, University of California, Berkeley, 1982), Research Report 82-9.

of each vehicle-tour may be expressed as,

$$\frac{TFM}{n} = 2.D_r + \frac{0{\cdot}6.C}{\sqrt{\delta}}$$

Therefore,

$$TFM = n \left[2.D_r + \frac{0{\cdot}6.C}{\sqrt{\delta}} \right] \qquad (E.4.30.)$$

As n = P/C,

$$TFM = \frac{2.D_r.P}{C} + \frac{0{\cdot}6.P}{\sqrt{\delta}}$$

This expression for Total Fleet Mileage is now virtually identical to Equation E.4.25., the only difference being that the latter formula associates a constant of 0·57 with Delivery Distance, instead of 0·6!

4.7. Summary of the Spatial Implications of fleet-size

The main focus of this chapter has been the way in which Total Fleet Mileage changes purely in response to changes in the number of vehicles operating from a single, central depot, both by controlling the number of customer-locations that each vehicle must visit in each round-trip, (C), and by altering the size and shape of the sector in which each vehicle operates. The different effects that n & C have on Stem Distance and Delivery Distance are also demonstrated, and the end-product of efforts to quantify the relationships between these variables is an expression, (E.4.20.), that estimates Total Fleet Mileage as a function of fleet-size and the number of stops per vehicle-tour. Equation E.4.20. is then modified so that Total Fleet Mileage is expressed as a function of the total population of customers served and the maximum number of locations that may be visited each day.

These equations, and all other numerical expressions to be developed in this chapter, are founded upon stochastic simulation, as all distance-figures represent the average outcome of many iterations of a computer-program that builds and measures travelling-salesman tours through sets of randomly-distributed points. This program assumes a deliberately simplistic situation of a uniform fleet of vehicles delivering a fixed consignment of goods to a fixed population of customers situated within a circular, homogeneous delivery-area, so that the only variable that affects Total Fleet Mileage is n. Any further variation in the estimates of distance provided by this program, given a fixed number of customers requiring deliveries and a delivery-area of fixed dimensions, is due to chance variation in the random co-ordinates generated, although the size of this error should be increasingly eliminated as the number of iterations of the program increases.

Because of this acknowledged reliance upon empirical data and Regression Analysis, all estimates made using expressions developed in this chapter are only as accurate as the statistical analyses on which they are based. However, the precise details of such estimates are less important than the insights provided here into the relationships between some of the key cost components of distribution systems and their behaviour as the size of the vehicle-fleet changes. In any case, a discussion and numerical example of how variance in the results of simulation exercises may be dealt with appears in Section 4.5..

The assumption that the hypothetical study-area is a homogeneous plain, which does not cater for the fact that distances will be influenced in the real world by the characteristics of the road network within which the depot is located, is essential for the creation of a situation in which n is the only independent variable that affects the distances in question. Such an assumption has much in common with the continuous Space Modelling approach, whose main objective is to enable generalised estimates of distance to be made in the absence of detailed information on individual customers and idiosyncrasies of the delivery-area.

However, it would be unfair to regard the homogeneous, square, or circular, delivery-zone as being a gross over-simplification of reality, since a set of customers may also be regarded as being distributed in "time space", with their locations within this space being fixed according to their "distance" in time both from the depot and from one another. Again, the effective boundary of the delivery-area will be determined by the length of the working day and drivers' hours restrictions. If road speeds and the degree of connectivity of the road network are more or less uniform in all directions, then a roughly circular shape would provide quite an accurate representation of a real-world delivery-area, whereas, in a more realistic situation in which road-speeds are higher along major roads in, say, four directions from the depot, a square delivery-area may be more appropriate.

One of the main features of the method used to develop an expression for Total Fleet Mileage is the disaggregation of Total Distance into Stem Distance and Delivery Distance, and then eventually down to the level of i, the average distance between successive stops in a tour.

It would have been possible to arrive at a similar expression to Equation 4.20. by means of, say, a Multiple Regression of Total Fleet Mileage figures with first n variable, and then with C variable, using much the same procedure as is used to derive Equation E.4.16. although the disaggregation of Total Fleet Mileage into the various components described in this chapter has several advantages. For instance, graphs such as Figures 4.9., 4.10. and 4.11. demonstrate the importance of Stem Distance in relation to Delivery Distance, and, at the level of distance travelled per vehicle, provide insights into the way in which Stem Distance increases with increasing fleet size whilst Delivery Distance declines.

The value of distinguishing Stem Distance will become apparent in Chapter 5, when the question of whether or not to schedule drivers to make overnight stays will be discussed.

The usefulness of being able to estimate Delivery Distance as a function of n and C, using the parameter i, may be illustrated with a simple numerical example. Consider a situation in which a fleet of 10 vehicles delivers to a population of 50 outlets distributed within a circular area 50 miles in diameter. The value of C in this case is 5. Using Equations E.4.8., E.4.17. and E.4.19., it may be estimated that,

STEM DISTANCE = 244·8 miles
DELIVERY DISTANCE = 268·04 miles
TOTAL DISTANCE = 512·84 miles

If travel-time between locations were to be reduced, due to improved road-speeds, perhaps, to the extent that it becomes possible to accommodate an extra customer-location into each vehicle's daily delivery-round, (so that C=6), the same equations could be used to estimate the amount by which Total Fleet Mileage would increase as a result. The adjusted distance-estimates are as follows,

STEM DISTANCE = 231·26 miles
DELIVERY DISTANCE = 303·67 miles
TOTAL DISTANCE = 534·93 miles

These figures indicate that, as expected, when C increases and n remains the same, it may be anticipated that Stem Distance be reduced slightly, whilst both Delivery Distance and Total Distance will increase. The same analysis may be carried out to estimate the impact of a reduction in the number of deliveries made by each vehicle. This might be caused by a fall in road-speeds, or might simply be due to a general decline in demand for a firm's goods or services; regardless of whether or not this reduction in the value of C is accompanied by a compensatory change in n, the impact of this decrease in the number of locations visited on the fleet's mileage figures may be estimated using the equations developed in this chapter.

The use of the parameter i as the smallest component of a set of Travelling-Salesman tours, contrasts with the work of other researchers in this field, who most often use the overall density of customer-locations to estimate Total Delivery Distance. A detailed comparison of some of their resulting equations with the expressions developed earlier in this chapter appears in Section 4.6..

The importance of fleet-size must ultimately be measured in terms of its effect on overall operating costs. In Chapter 3, n was featured as one of the key parameters influencing both Standing Cost and Running Cost, (SEE Equation E.3.6.), particularly in the way in which it directly affects the variable x, (ie. vehicle carrying-capacity). This chapter has focused upon the ways in which n, both directly and by means of its influence on C, influences the value of m, another key term in Equation E.3.6., and thus Running Cost. As in Chapter 3, it must be concluded that, in order to minimise costs, the size of a fleet, in terms of the number of vehicles employed, should also be minimised, although this is not to suggest that the increased revenue resulting from an increase in the total number of customers served, that has caused fleet-size to be increased, would not offset the accompanying rise in distribution costs.

The number of vehicles used is clearly just one element of the system that affects Total Fleet Mileage, which is itself merely one of many factors that contribute towards the Total Cost of a distribution operation. The impact of other decisions relating to what have been described here as "fleet characteristics" will now be discussed in Chapter 5, beginning with the question of whether vehicles are scheduled to stay away from the operating centre overnight, instead of returning at the end of each working day. The importance of Stem Distance in this context has already been hinted at in this section; the whole subject of overnight stays, as well as other decisions and constraints concerning the drivers of vehicles, will now be dealt with in greater detail.

CHAPTER 5

THE EFFECT OF DRIVERS' HOURS RESTRICTIONS ON TOTAL DISTRIBUTION COST

A key variable in the discussion so far has been k, the maximum number of customer-locations that may be served on each vehicle-tour. In Chapter 4, it is regarded as a constraint which has a fixed, arbitrary value, with no attention given to the factors that determine the value of k; this chapter both defines the variables that influence this constraint, and goes on to measure the impact that such variables have on Total Fleet Mileage.

The amount of time that is required, on average, to visit a set of location will clearly have an effect on the value of k, and will, in turn, be controlled by delivery-area characteristics such as average road-speeds and the average distance between customer-location. The time that is spent on loading, unloading and documentation at each stop also has an important bearing on the number of stops that may be made by each vehicle, and this factor will be dealt with in Chapter 9, but the constraint that appears to be most directly responsible for determining k is the requirement that each vehicle must return to the operating centre at the end of each working day. More precisely, this constraint refers to the limit on the amount of time that drivers may spend driving each day. As the problem formulation here dictates that each vehicle-tour must start and end at the depot, this time-limit effectively puts a restriction on the vehicle's range, since each one may only visit those locations that enable it to return to the depot before the expiry of the permissible driving-period.

In order to measure the impact of this limit on the length of a working-day, an expression for k is developed in the section that follows. In contrast, Section 5.2. then goes on to examine the effect of allowing drivers to make overnight stays away from the operating centre, instead of returning to the depot at the end of each day's driving.

5.1. The Development of an Expression for the Maximum Number of Stops per Vehicle-Tour

To develop such an expression, it is necessary to introduce into the discussion the aspect of time, particularly the average amount of time that is required to complete one daily round-trip. This may readily be achieved by making use of the main variables used in Chapter 4 - Total Fleet Mileage, P and k - with an added assumption concerning average vehicle-speeds within the delivery-area.

For example, the Total Driving Time required to serve a population each day is,

$$\frac{TFM}{S} \quad \text{hours} \qquad (E.5.1.)$$

where, TFM = Total Fleet Mileage,
and, S = average vehicle speed, (in miles per hour).

It has also been established in Chapter 4 that the total number of round-trips undertaken by the fleet each day may be expressed as (P/k), (see Equation E.4.21), so that it may be estimated that,

$$H = \frac{(TFM/S)}{(P/k)}$$

or,

$$H = \frac{TFM}{S} \cdot \frac{k}{P} \qquad (E.5.2.)$$

where, H = the number of driving hours required per vehicle-trip.

This expression may be rearranged so that,

$$k = \frac{S.P.H.}{TFM} \qquad (E.5.3.)$$

although, in this situation, the variable "H" may be used to represent the maximum number of hours available for each vehicle-trip, on the premise that, in practice, an operator will always try to achieve maximum efficiency by fully utilising the driving-time that is available - a similar assumption is made in Section 4.4.

In cases where Total Fleet Mileage is unknown, k may still be estimated by substituting Equation E.4.21. into E.5.3., although, as the former expression is rather cumbersome for algebraic manipulation, it is far more convenient to instead use an expression for Total Fleet Mileage proposed by Daganzo, which is reproduced in Chapter 4 as Equation E.4.24., (1). The maximum number of stops per vehicle per day may thus be expressed as,

$$k = \frac{S.P.H.}{1{\cdot}8\ D_r\ (\frac{P}{k} + \sqrt{P})} \qquad (E.5.4)$$

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(1) DAGANZO, C.F., "The distance travelled to visit N points with a maximum of C stops per vehicle: A manual tour-building strategy and Case Study". Research Report, Institute of Transportation Studies, University of California. (Aug. - Sep., 1982).

This formula may be rearranged as follows,

$$k = \frac{S.P.H.}{\frac{1\cdot 8 \cdot D_r \cdot P}{k} + 1\cdot 8 \cdot D_r \cdot \sqrt{P}}$$

$$\therefore \quad k\left(\frac{1\cdot 8 \cdot D_r \cdot P}{k} + 1\cdot 8 \cdot D_r \cdot \sqrt{P}\right) = S.P.H.$$

$$\therefore \quad \left(1\cdot 8 \cdot D_r \cdot P\right) + \left(1\cdot 8 \cdot D_r \cdot \sqrt{P}.\ k\right) = S.P.H.$$

Therefore,

$$k = \frac{(S.P.H.) - (1\cdot 8 \cdot D_r \cdot P)}{1\cdot 8 \cdot D_r \cdot \sqrt{P}} \qquad (E.5.5.)$$

The form of this equation suggests that there's a linear relationship between the maximum number of stops per vehicle and the limit on daily driving-hours, and this is confirmed by Figure 5.1.; Table 5.1. shows the data on which this graph is based, including both discrete and integer values of k. For the purposes of this exercise, it is assumed that a population of 200 customers is to be served within a circular delivery-area, 100 miles in diameter, in which 20 miles per hour is the average road-speed.

It is perhaps more interesting to consider the impact of H on Total Fleet Mileage; having already developed an expression for k, this is not difficult, as Equation E.5.5. may be substituted into E.4.21.. Incorporating the same fixed values for population size, average vehicle-speed and delivery area size as were used previously, the formula for Total Fleet Mileage becomes,

$$TFM = 100.\left[\left(13\cdot 81648\left(\frac{4000\ H - 11935\cdot 8}{843\cdot 9881}\right)^{-1\cdot 1756}\right) \times \left(\left(\frac{4000\ H - 11935\cdot 8}{843\cdot 9881}\right)^{-1}\right)\right] + \left[\frac{100}{\left(\frac{4000\ H - 11935\cdot 8}{843\cdot 9881}\right)} \times \left(0\cdot 802147 - 0\cdot 4115\ .\ Log.\left(\frac{4000\ H - 11935\cdot 8}{843\cdot 9881}\right)\right)\right]$$

when, a = 100 miles,
P = 200 miles,
and, S = 20 miles per hour.

Estimates, using this equation, of both Stem Distance and Delivery Distance, as well as of Total Fleet Mileage, with H as the independent variable, are contained in Table 5.2., and illustrated in Figures 5.2.1. and 5.2.2..

Figure 5.2.1. suggests that the limit on driving-hours has little effect for H-values of between 7 hours and 11 hours, which may be regarded as the maximum range over which values of H are realistic. In order to show the overall shape of the curves shown in Figure 5.2.1., figures for daily driving-time limits ranging from $3\frac{1}{2}$ hours to 15 hours are plotted in Figure 5.2.2.. The evidence here indicates that, given the assumptions made for this numerical example, varying the value of H only has a really significant influence on Fleet Mileage when the length of the working day is very short; far shorter, in fact, than it is likely to be in the real world. The shape of the Stem Distance curve in Figure 5.2.2. shows that the rise in Total Fleet Mileage when very few hours may be driven each day is primarily due to the increase in the number of round-trips, and therefore vehicles, that are necessary each day.

To support the view that the limit on the length of the working -day does not have a substantial effect on the total distance travelled, Table 5.2. reveals that, even when H is shortened from 11 hours to 7 hours, Total Fleet Mileage only increases from 925·72 miles to 755·44 miles. Expressing this difference in percentage terms, in order to lessen the influence of the assumed values attributed to parameters such as S, a, P & H, this is an increase of only 22·54%; a reduction of H from 11 hours to 9 hours extends TFM by only 8·01%. A policy that may have a greater impact on distance travelled, however, is one which eliminates the need for vehicles to return to the depot at the end of each day's work; the question of the extent to which scheduling drivers to make overnight stays affects Total Fleet Mileage, and therefore Total Distribution Cost, is addressed in the following section.

5.2. The Effect of Allowing Overnight Stays Away from the Operating Centre

The option of allowing drivers to make overnight stays with their vehicle, away from the operating centre, which is not an uncommon practice in the real world, is not considered in the body of literature that is concerned with Travelling-Salesman-Type distribution problems. This is, perhaps, surprising, since the removal of the need for each vehicle-tour to start and finish at a central depot, relaxes a major constraint in the Travelling-Salesman Problem.

The main purpose of adopting overnight stays is to save both time and distance by reducing the number of journeys that are made to and from the depot; in other words, it is

Figure 5.1.Relationship between k and H

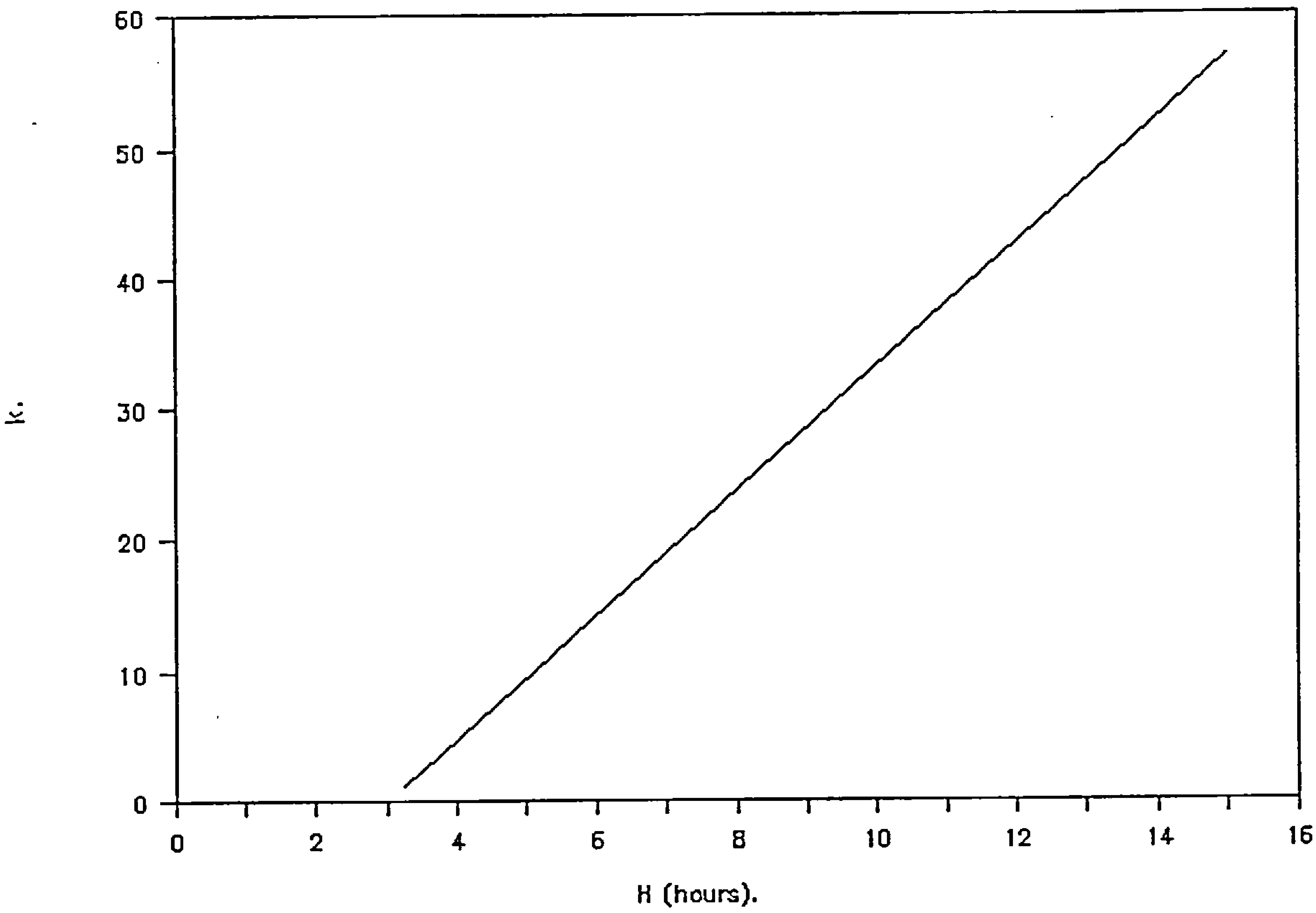


Figure 5.2.1. Disaggregated fleet miles
(H=7....11).

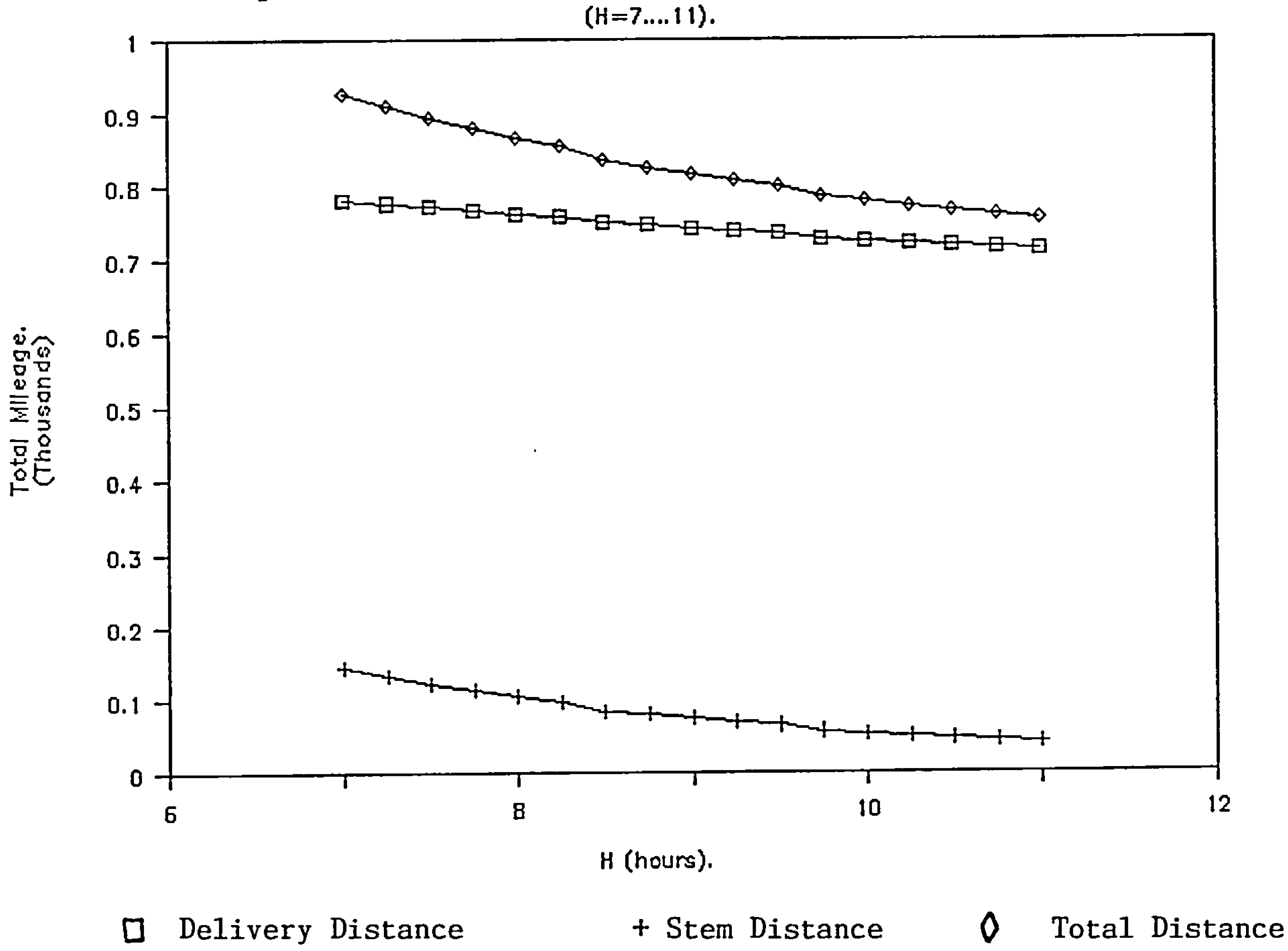


Figure 5.2.2. Disaggregated fleet miles

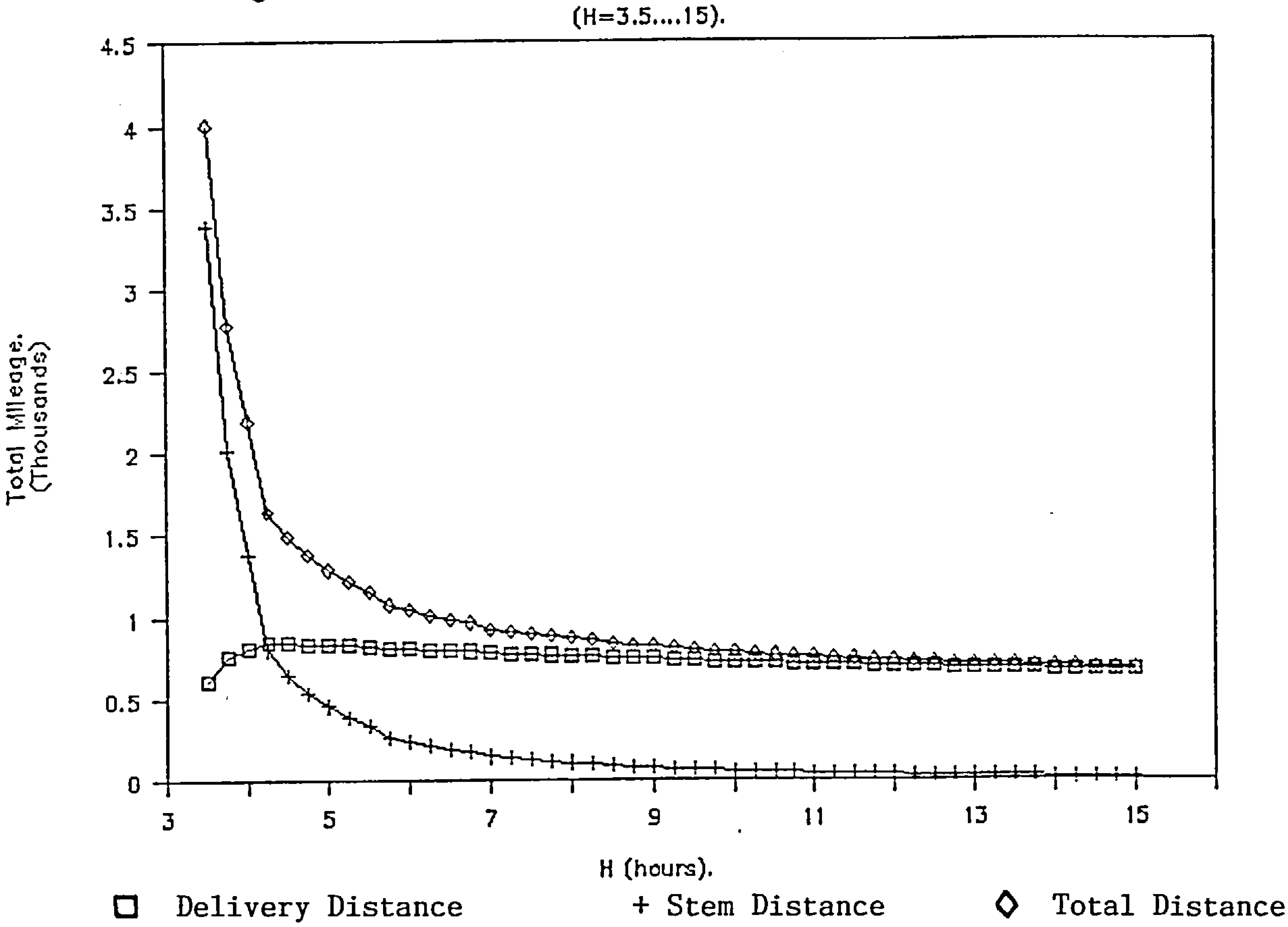
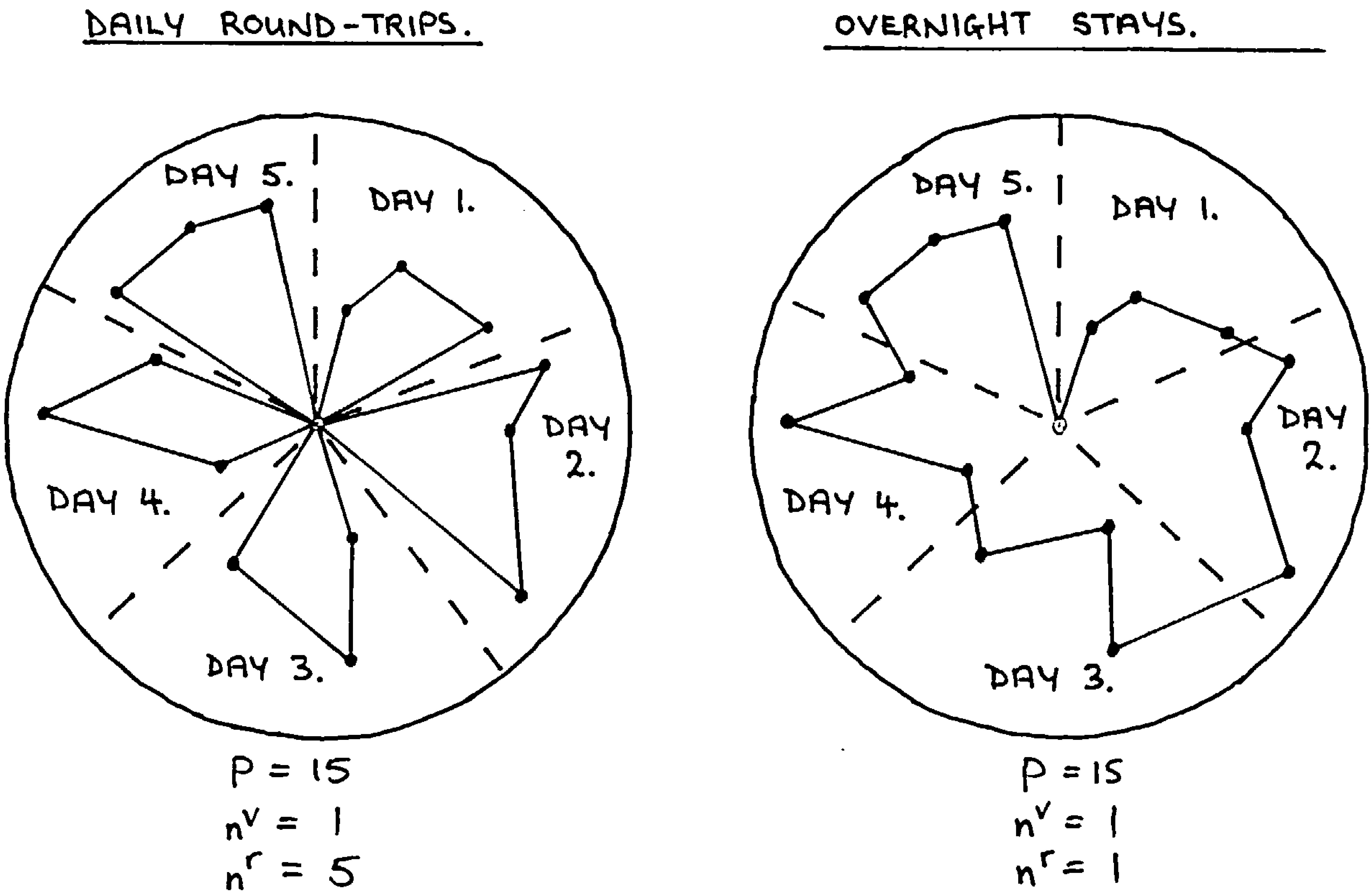


Figure 5.3. Routing alternatives.



mainly Stem Distance that is being saved, although the use of such a system also incurs certain costs. For example, the amount of distance that is saved, as a result of vehicles not having to return to the depot at the end of each working-day, must be traded off against the fact that a fleet of vehicles that returns to the depot fewer times to be loaded will need to have a larger capacity per vehicle; therefore, the unit cost of transport, (both Standing Cost per vehicle and Running Cost per mile), will be higher. The cost of overnight stays themselves should also be considered, which includes not only the overnight payment that must be made to each driver, regardless of whether the vehicles are equipped with a sleeping compartment, but also the overnight parking charge that may be payable, for the vehicle. Furthermore, the extra distance that is travelled in driving to and from a suitable parking site, which will inevitably often require a detour from the vehicle's optimum route, may reduce the overall saving in distance.

Nevertheless, in certain circumstances, the use of overnight stays can lead to substantial cost-savings, which are sufficient to off-set the associated extra costs mentioned above. Apart from the savings that result directly from reductions in Total Fleet Mileage, savings in time may also be made. Possible ways in which such time-savings may be quantified will be discussed at the end of the following sub-section, in which the general problem of whether to use a system of overnight stays is formulated in greater detail.

5.2.1. The simple case of comparing two one-vehicle operations

In order to highlight the trade-offs that are involved in the decision of whether to dispense with the requirement for vehicles to return to the depot at the end of each day, a numerical example is presented here of two alternative types of one-vehicle operation which may be used to satisfy a weekly demand. One of these alternatives is for a vehicle to make separate round-trips, each starting and ending at the depot, on each day of the week, whilst the other is for a vehicle to visit each of the same set of locations consecutively, not returning to the depot until the last of the week's deliveries has been made. Even using a system of overnight stays, of course, a vehicle must return to the depot at sometime, and so it is realistic to regard these alternatives as being, on one hand, a series of five separate vehicle-tours using a vehicle with a capacity of x tons, (assuming a five-day working-week), and, on the other, one weekly round-trip undertaken by a vehicle having a capacity of 5x tons. Figure 5.3. illustrates the alternative routing schemes graphically. In as much as the central trade-off is between increased unit costs and reduced Fleet Mileage, there are analogies here with the numerical example presented in Section 3.6. which illustrates the presence of Economies of Scale in Transport.

The current problem formulation invloves most of the general assumptions used elsewhere in this thesis, so that the set of customers, P, is located at random throughout a homogeneous plain, and that they are to be served from a centrally-located depot using a unifrom fleet of vehicles. However, in order to make it possible for a system of over-night stays to be praciticable, a number of additional assumptions must be made. For example, although it is still assumed that demand does not vary from customer to customer, deliveries must be required on a weekly basis, and both the size of the consignment of goods ordered at each outlet and the location of each customer must be known at the start of each week. Furthermore, it must be assumed that there are no restrictions as to the time and day of delivery at any outlet, so that the vehicle may be optionally routed in the absence of time-window constraints. The problem therefore consists of the comparison between two delivery-systems that are based on vehicle-tours which are assumed to be optimal, and which are designed at the beginning of each week.

The practice of "tramping", whereby a driver sets off from the depot with a consignment of goods with the intention of replacing it with another load at his first destination, is not considered in this section. This is despite the fact that tramping operations require vehicles to be away from their operating centre, often for several days at a time.

Because the current example involves the use of just one vehicle, it must also be assumed that the total weekly demand for goods from the population of customers does not exceed the maximum load that one vehicle may carry. This upper limit may be effectively determined by either value or weight constraints, and so is not necessarily equivalent to the maximum legal weight which may be carried on roads; in any case, this legal limit varies between different countries. For the purposes of this exercise, the maximum value of x is fixed at 30 tons. Finally, it is also necessary to assume that the goods in transit are in no way perishable.

The calculations which follow make use of parameters which, so far, have been excluded from the discussion of both the current chapter and Chapter 4; this is because, in order to calculate the cost of an operation, it is necessary to specify both the Total Weekly Tonnage of goods that need to be delivered, t, and the carrying-capacity of the vehicle, x, that is required. In the example here, it is assumed that each of 15 customers requires a weekly consignment of 0·25 tons of goods, so that this demand may either be met by one 0·75-ton vehicle returning to the depot each day, or by a 4-ton vehicle visiting 3 locations each day and making 4 overnight stays away from the operating centre. In both cases, the value of t is 3·75 tons.

The mileage and cost figures for both vehicles may be calculated using Equations E.3.6. and E.4.20., respectively. Both expressions include the variable n, although, with the assumption that deliveries are made to each customer on a once-a-week basis, this variable must be redefined for the purposes of the Total Fleet Mileage equation. This is because, whereas the number of round-trips made per day was formerly equal to the number of vehicles used, with each day's driving constrained to starting and finishing at the depot, the use of overnight stays means that each day's driving only accounts for a fraction of a completed vehicle-tour, (SEE Figure 5.3.). The number of round-trips per delivery-cycle is therefore dependent on both the number of vehicles used and the number of times that a vehicle is constrained to return to the depot each week. For the purposes of this section, therefore, the term "n" in Equation E.4.20. should be replaced with,

$$n^v . n^r \qquad \text{(E.5.6.)}$$

where, n^v = the number of vehicles in the fleet,
and, n^r = the number of times per week that each vehicle must return to the depot.

In the case of a daily delivery-cycle, the value of $(n^v.n^r)$ is 5, whilst it is 1 when overnight stays are used.

Using this slightly modified version of Equation E.4.20., it can be estimated that the number of miles travelled by a 0·75-ton vehicle making 5 round-trips in a week, assuming a circular delivery-area, 250 miles in diameter, is 1325·2 miles; the number of stops made in each round-trip, C, is 3, in this instance. The same expression estimates that the length of one round-trip which includes all 15 customer-locations, so that C=15, is 678·9 miles. It is clear from this example, therefore, that the savings in Stem Distance, in what is quite a large delivery-area, are substantial when overnight stays are used.

When these weekly mileage figures are incorporated into Equation E.3.6. - where the "n" term still expresses the number of vehicles in the fleet - the cost of using a 0·75-ton vehicle every day is estimated as being £370·73 per week, whilst the cost of the one 4-ton vehicle option, <u>not including</u> the cost of overnight accomodation at this stage, is estimated at £414·39.

These figures suggest that, in a one to one situation, it is, in fact, cheaper to use a vehicle that returns to the depot on each night of the week, although this result is, of course, very much subject to the assumptions made as to the size of the delivery-area, the number of customers served and their level of demand, etc .. The question of whether there are any circumstances in which it may be profitable to use a system of overnight stays, may be answered by increasing the value of "a", since, as Stem Distance is directly proportional to the diameter of the delivery-area, it seems logical to expect that it becomes increasingly expensive to make journeys to and from the depot as this area becomes larger. Table 5.3., whose data are plotted in Figure 5.4., shows the relationship between area-size and Total Cost per week for both operations, and indicates that, when the value of "a" reaches a certain level, it becomes cheaper to permit overnight stays away from the operating centre. Figure 5.4. suggests that, in this example, this value of "a" is approximately 470 miles. At this point, however, with such a large delivery-area, the total distance travelled by the vehicle making one round-trip during the week is estimated at roughly 1300 miles, which, assuming an average vehicle-speed of 20 miles per hour, would take 65 hours to complete! If the limit of a working-week is assumed to be 50 hours, which is consistent with the daily maximum of 10 hours, then the use of overnight stays is not a practical proposition when locations are scattered over an area of this size.

The analysis clearly shows that, for a one-vehicle operation, the distance-saving resulting from the making of overnight stays away from the depot will outweigh the increased unit costs associated with the consequent need to employ a vehicle with a greater capacity, provided that the delivery-area is large enough for Total Stem Distance to be the dominant variable in the calculation of Total Cost. As there is, however, in practice, a limit to the number of miles that may be driven in a day, the value of a at which the overnight stays option becomes the cheaper alternative may well exceed the maximum feasible delivery-area size, given the number of customers that are to be served and average road-speeds.

One important factor that has not yet been considered in very much detail in this section, is the amount of time which may be saved using overnight stays. In the above example, comparing a 0·75-ton vehicle and a 4-ton vehicle, although the use of the former is shown to be the least cost option, it may also be demonstrated that a substantial amount of time is saved by not having to return to the depot at the end of each day. If it is assumed that, in each case, average vehicle speed is 30 miles per hour, then each of the smaller vehicle's daily round-trips takes, on average, 8·83 hours. Similarly, it may be estimated that the 4-ton vehicle will complete a single round-trip in only 22·63 hours, so that the whole week's deliveries can be made in just over two days!

This finding raises the general question as to the connection between time-savings and cost-savings, which is, in turn, related to the principle of the Value of Time. In the numerical illustration above, there is an obvious saving of two nights' accomodation and parking charges, although the real value of saving time is associated with the extra capacity that is created in the system; this is because additional customer-locations may be visited in the time that would otherwise have been spent in travelling to and from the depot. This point may be illustrated using Equation E.5.3.,

Figure 5.4. Total Weekly Cost estimates

(a=250....750).

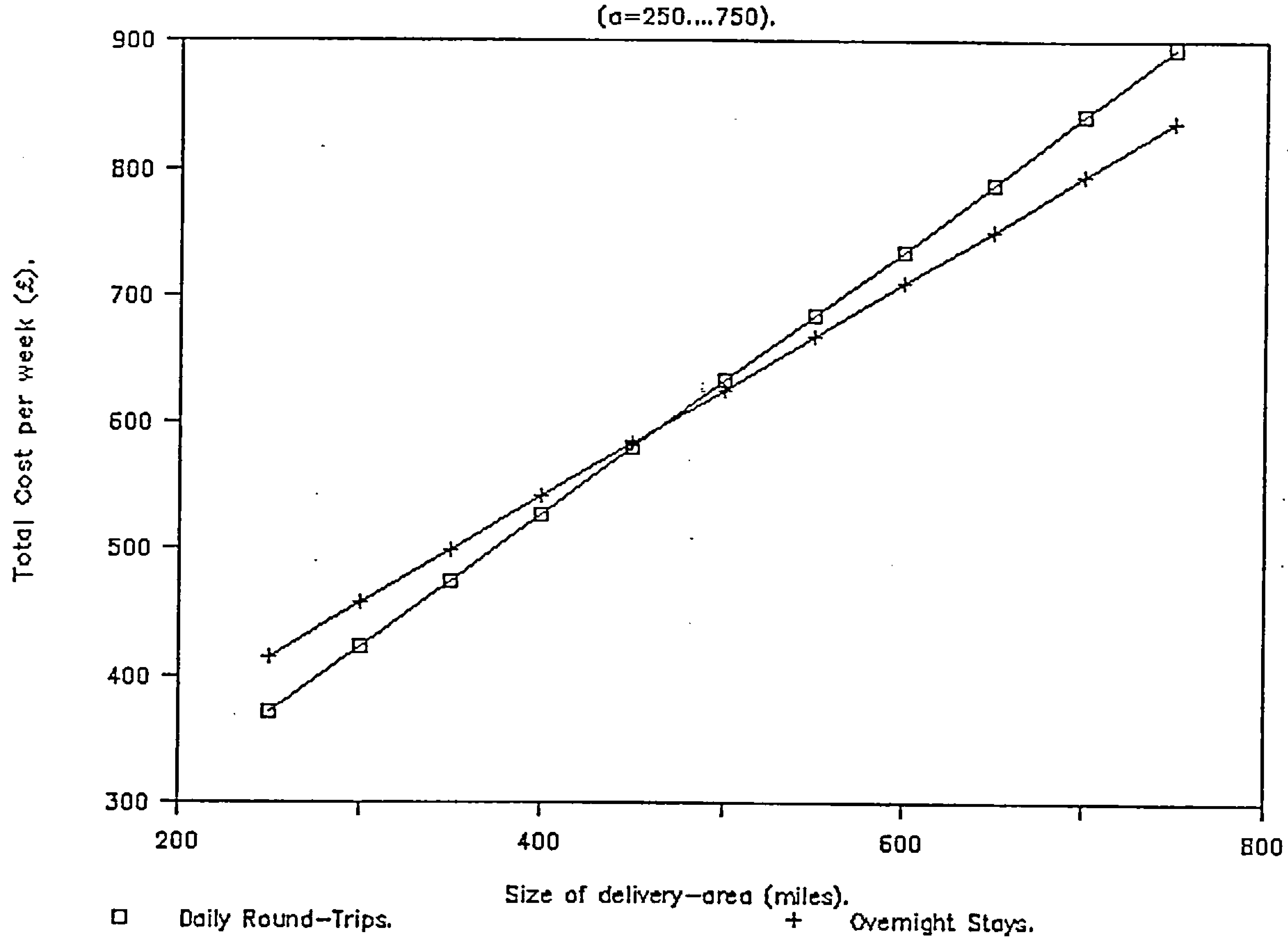


Table 5.3. Total Weekly Cost estimates

a	Daily Round-trips TFM	Daily Round-trips TC (£ per wk)	Overnight Stays TFM	Overnight Stays TC (£ per wk)
250	1325	370·73	679	414·39
300	1590	423·11	815	456·77
350	1855	475·49	950	499·16
400	2120	527·88	1086	541·54
450	2385	580·26	1222	583·92
500	2650	632·65	1358	626·31
550	2915	685·03	1494	668·69
600	3180	737·41	1629	711·08
650	3445	789·80	1765	753·46
700	3711	842·18	1901	795·84
750	3976	894·57	2037	838·23

$$k = \frac{S.P.H.}{TFM} \qquad (E.5.3.)$$

which is discussed in greater detail earlier on in this chapter; this expression suggests that, as Total Fleet Mileage is reduced, mainly due to savings in Stem Distance, the maximum number of stops that may be made in each round-trip, k, will increase, provided, of course, that this increase in k does not cause vehicle-capacity constraints to be exceeded. It is when such improvements in efficiency, in terms of the number of customers that each vehicle may visit each day, actually allow a vehicle and driver to be dispensed with that a real, and tangible, saving is made. Time-savings are frequently quantified using indices that are developed on the basis of the amount of time that must be saved in order to make a measurable saving of this kind, together with the cost-saving that is involved; the whole issue of the Value of Time will be returned to, and discussed more fully, in Chapter 6 .

Such Value of Time indices can not be realistically applied to the current situation under consideration, where two one-vehicle systems are being compared, since there is no way in which a whole vehicle may be saved. Savings resulting from the introduction of overnight stays, in this case, are therefore restricted to those connected with Total Fleet Mileage and overnight accommodation and parking charges. The following section, however goes on to consider the same issues in situations where more than one vehicle is used.

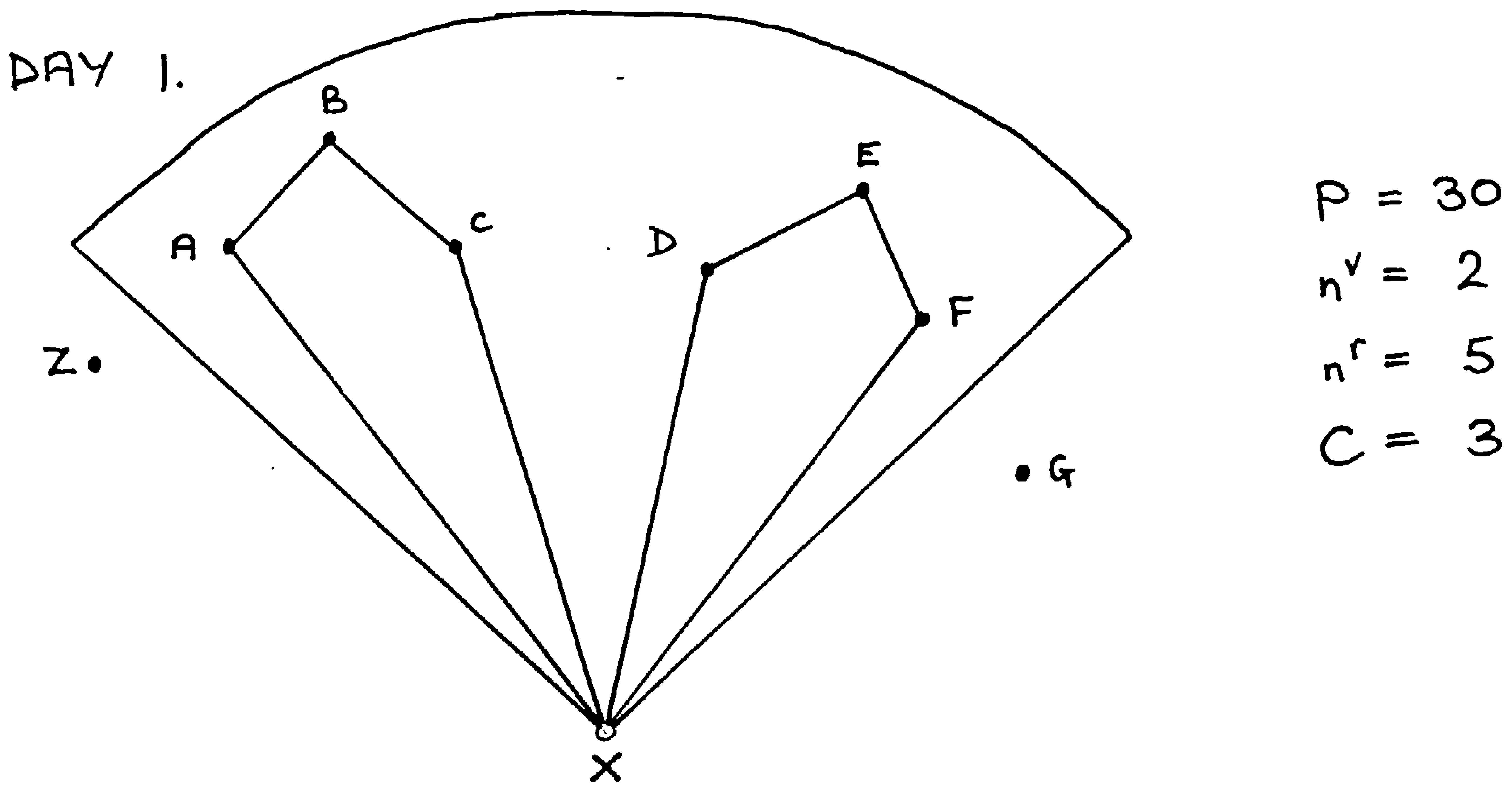
5.2.2. The effect of allowing overnight stays, in the context of a fleet of vehicles

In situations where more than one vehicle is used, the likelihood that savings in time will enable a week's deliveries to be made by a fleet of (n-1) vehicles may be expected to increase as fleet-size increases. This is because the mileage of each individual vehicle, particularly the distance travelled between successive customer-locations, is reduced as the value of n increases, so that the amount of time that must be saved in order to achieve this reduction in fleet-size is correspondingly less. In other words, the amount of time that needs to be saved in order to lower the number of vehicles required, is related to the Delivery Distance that is attributable to each vehicle. Figure 4.11. confirms that this distance does, in fact, decrease as fleet-size increases.

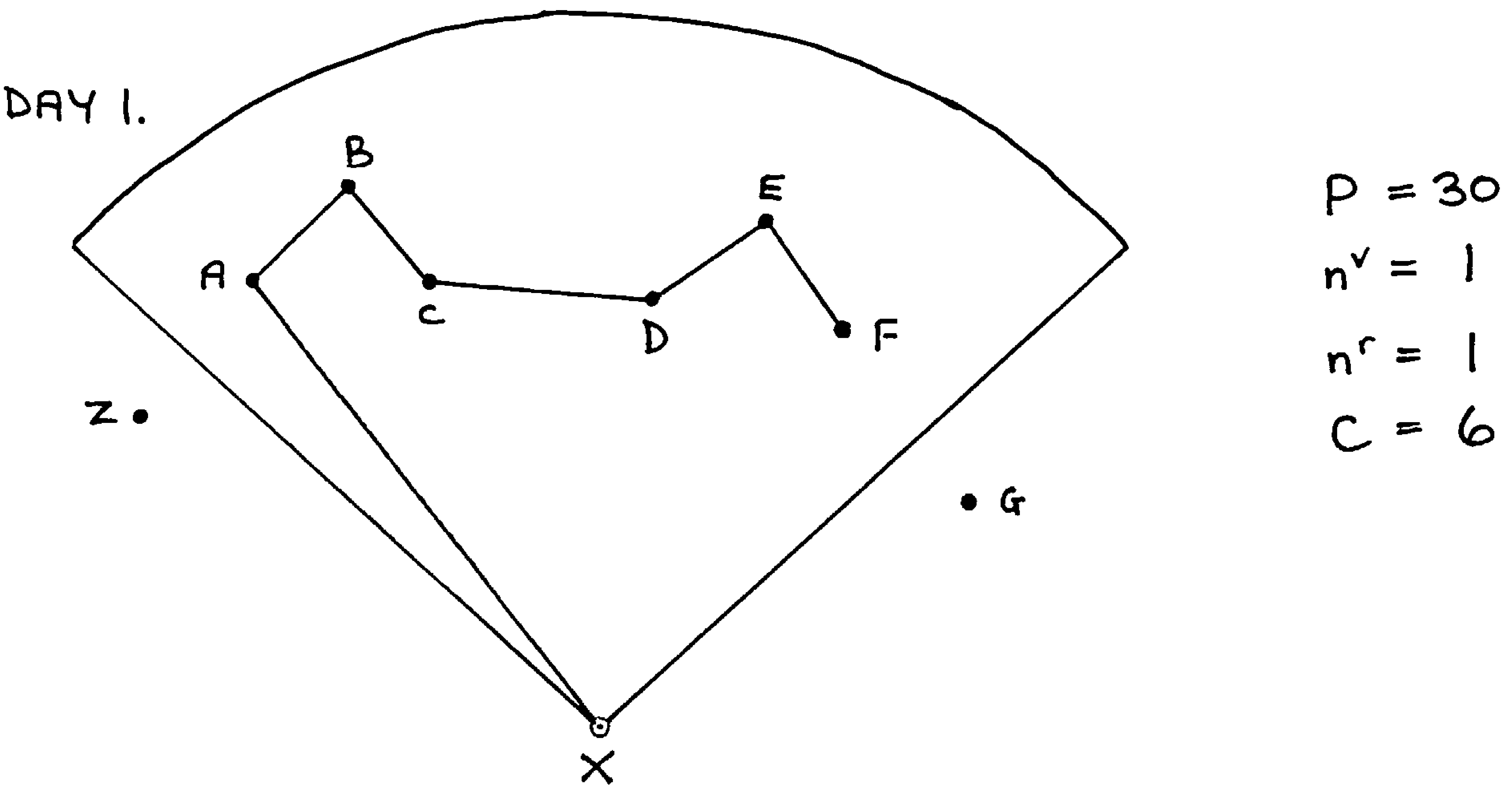
In order to further illustrate this point, it is most convenient to consider a situation in which two vehicles make weekly deliveries to a population of 30 customers, so that each vehicle visits 3 locations each day; an example of how a pair of routes on any one day of the week might look is given in Figure 5.5.1.. In this instance, there is insufficient time available for one vehicle to visit all 6

Figure 5.5. Alternative routes for one day's deliveries using both daily round-trips and overnight stays

5.5.1. Daily round-trips



5.5.2. Overnight stays



locations and to return to the depot, "X", in one day, and so at least two vehicles must be used. What is important here is the relationship between the average length of a stem-journey, (e.g. C-X), and the extra distance that would be travelled by the first vehicle if it were to incorporate into a day's tour the locations currently served by the n^{th}, (i.e. the second), vehicle. In Figure 5.5., whose routes refer to the first day of a weekly cycle, this extra mileage is represented by the route linking points C,D,E and F. If the total tour-distance from C to F is less than, or equal to, the stem journey from C to X, then the round-trips of the two vehicles shown in Figure 5.5.1. may be replaced by the single route of a vehicle making an overnight stay away from the depot, as shown in Figure 5.5.2.. This ivernight stay may be made in the vicinity of location F, or between locations F and G, G being the first location to be visited on the second day of the week. Whether or not all customers can be served by one vehicle on days in the middle of the week depends on the relationship between the average length of two stem journey, examplified by the links A-X and F-X in Figure 5.2.1., and the tour distance from point Z, (the last location to be visited on the previous day), to point F. Similarly, because of the need for the single vehicle to return to the depot at the end of the week, it is the distance of the route between Z and X, via points A to F, that is of importance. Of course, even if there is not sufficient time to allow one vehicle to visit 6 locations on any particular day, it is still possible that a week's deliveries may be made by a single vehicle, provided that the entire tour may be completed within the time permitted for the week.

The foregoing discussion illustrates the most important issues involved in deciding whether a time-saving may result in a tangible cost saving by reducing the number of vehicles required. For practical purposes, of course, there is no need to compare the corresponding routes, with and without the use of overnight stays, for each day's deliveries. What is far easier is to calculate the minimum number of vehicles required, given information on P, a and H etc., using the formulae already developed elsewhere in this thesis.

As an example of how this might be done, consider a situation in which 200 customers are randomly distributed within a circular area, 50 miles in diameter, in which average road-speed is 50 miles per hour. The maximum length of a working-day, for each of 5 days in a week, is 10 hours, whilst the aggregate weekly demand of the whole population of customers is 3·75 tons. To complete the problem formulation, it is necessary to also consider two factors which have not so far been included in numerical examples in this chapter: the cost of overnight accommodation, B, which can be assumed to be fixed at £20 per night, and the time spent at each customer's premises, 1, which may be taken to be a uniform

30 minutes for each stop. It should be stressed here, therefore, that, because of the inclusion of handling and order-processing time at each customer-location, the 10 hour limit on the length of the working-day is effectively a limit on "Total Duty Time", which includes both driving-time and time spent on other activities such as loading and unloading etc..

The first question to be addressed is whether one vehicle, returning to the depot each day, can visit the necessary 40 locations in each daily round-trip without violating the 10 hours duty-time per day constraint. Using Equation E.4.20., and given that C=40 and that $(n^v.n^r) = 5$, the weekly mileage for one vehicle is 715·3 miles, which represents an average of approximately 143 miles for each daily round-trip. With average road-speeds of 50 miles per hour, each day's driving would therefore take just under 3 hours, (171·67 minutes), so that there appears to be no problem with exceeding the maximum value of H purely on the basis of driving-time. It is, however, the total time that a single vehicle would spend on customers' premises during the day which makes it necessary to use at least one extra vehicle; since C=40 and l=30, "total handling time" is 20 hours. With 2 vehicles each visiting 20 location in each round-trip, handling time per day, at 10 hours, is still enough, on its own, to exceed the Daily Duty-Time constraint. With 3 vehicles, when $(n^v.n^r) = 15$ and C=13(to the nearest whole number), the 200 customer-locations may be served using daily round-trips without violating this time-constraint. The calculations which led to this conclusion may be summarised as follows,

---

When $n^v=3$, $(n^v.n^r) = 15$ and INT(C) = 13:-

Total Fleet Mileage per week = 1 200·3 miles
Distance per round-trip = 92·33 miles
Driving-time per round-trip = 110·8 minutes
Handling-time per round-trip = 390 minutes
Duty-time per round-trip = 500·8 minutes
(i.e. approximately 8 hours, 20 minutes).

Similar calculations may be carried out for an alternative system that uses overnight stays away from the depot; the two main differences in this situation are that n^r is now 1, and the time-limit on a round-trip, including the aggregate time spent at all customer-locations, is 50 hours. The following summary indicates that the minimum size of fleet that may be used is one consisting of 3 vehicles, When $n^v = 1$, $(n^v.n^r) = 1$ and C = 200:-

Total Fleet Mileage per week = 414·11 miles
Distance per round-trip = 414·11 miles
Driving-time per round- trip = 496·9 minutes
Handling-time per round-trip = 6000 minutes
Duty-time per round-trip = 6496·9 minutes

When $n^v = 2$, $(n^v.n^r) = 2$ and C = 100:-

Total Fleet Mileage per week = 471·41 miles
Distance per round-trip = 235·7 miles
Driving-time per round-trip = 282·8 minutes
Handling-time per round-trip = 3000 minutes
Duty-time per round-trip = 3282·8 minutes

When $n^v = 3$, $(n^v.n^r) = 3$ and INT(C) = 67:-

Total Fleet Mileage per week = 589·8 miles
Distance per round-trip = 196·6 miles
Driving-time per round-trip =235·9 minutes
Handling-time per round-trip = 2010 minutes
Duty-time per round-trip = 2245·9 minutes
(i.e. approximately 37 hours, 30 minutes).

On the evidence of these figures, therefore - and taking integer effects literally, so that the value of n is always rounded up to the nearest whole number - it must be concluded that it is not possible to reduce fleet-size using overnight stays, in this particular example. However, although the week's deliveries can not be made using only two vehicles, it is noticeable that, in the 3 vehicle option using overnight stays, only 37·5 hours' work is required of each driver, which indicates that there is a substantial amount of unused capacity in the fleet. In the real world, with just over 112 man-hours of work needed each week, it is unlikely that an operator would employ three drivers for 37·5 hours in a 50-hour week and have three vehicle's idle for one day each week. It is far more realistic to expect that, perhaps, two drivers would be employed for a full 50-hour week, with the week's deliveries being completed by a third driver who is employed to work the remaining 12 hours; although the operator would still be faced with having to purchase a third vehicle, the use of part-time staff would certainly change vehicle routing-patterns, and thus Fleet Mileage figures. Another alternative is to extend the working-week of the two existing drivers into a 6th day, and there is also the option of hiring extra capacity to cope with the few deliveries that cannot be made with the two vehicles that the operator owns.

Consideration of aspects such as "Fleet Mix", a subject-area that deals with the balance between owned and hired capacity within a vehicle-fleet, and the extension of the number of man-hours that are available each week by introducing overtime, would considerably alter the formulation of the problem dealt with in this section. Therefore, the conclusion, in this case, if the assumption that the fleet of vehicles is uniform is to be retained, is that there is no cost-saving to be made by waiving the constraint that each vehicle must return to the operating centre at the end of each day; this is because it has already been shown, in Section 5.2.1., that, when fleet-size is the same, the cheaper alternative is to make daily round-trips. Confirmation of this is provided by the following figures, which are derived from Equation E.3.6.,

Using daily round-trips:-

$n^v = 3$ $x = 0{\cdot}75$ tons

Total Fleet Mileage per week = 1200·3 miles

Standing Cost per week = £326·41
Running Cost per week = £237·24
Total Cost per week = £563·65

Using overnight stays:-

$n^v = 3$ $x = 1{\cdot}5$ tons

Total Fleet Mileage per week = 589·8 miles

Standing Cost per week = £422·14
Running Cost per week = £140·86
Cost of overnight stays per week = £240·00
Total Cost per week = £803·00

Again, the question of whether a different conclusion might be arrived at using different assumptions should be addressed. As in Section 5.2.1., it seems logical to expect that increasing the size of the delivery-area is most likely to create a situation in which a system of overnight stays becomes the cheaper option. In order to generate estimates of both distances and costs, with "a" as the important independent variable, a computer program was constructed, based on Equation E.4.20. and E.3.6.. The main aims of this program, which is summarised with the activity-sequence diagram of Figure 5.6., are both to investigate the size of delivery-area that is required for overnight stays away from the depot to make an overall cost-saving, and to estimate the

Figure 5.6. Activity Sequence Diagram of computer program

proportion of the cost of the two systems that is attributable to stem journeys. The assumptions and procedures used in the above calculation are all retained for this analysis.

The first of the aforementioned objectives is largely achieved by Figure 5.7., which shows that the Total Cost curves for the two systems in question intersect when the diameter of the delivery-area is approximately 150 miles; when the value of a is greater than this, given the assumptions made about key variables, it becomes more expensive to make daily journeys to and from the depot. The actual figures on which Figure 5.7. is based, along with all other data relevant to this section of the analysis, are contained in Tables 5.4.1. and 5.4.2..

On the evidence of Figure 5.8., which shows that the curves for fleet-size are very similar in shape to the Total Cost curves of Figure 5.7., the seemingly geometric increase in the cost of making daily round-trips with increasing area-size is very closely related to the resulting rise in the number of vehicles used. Similarly, Figure 5.9.1. suggests, by the parallel nature of the curves that it portrays, that it is the rise in the total cost of making stem journeys that is mainly responsible for the observed increase in Total Cost per week, rather than the increase in Standing Cost caused by having to use more vehicles. This is because, for all values of a, the vehicles involved in daily round-trips are 0·75 ton vans, (SEE Table 5.4.), which have low unit costs, so that the fixed costs associated with having a large number of vehicles are relatively small compared with the extra Running Costs resulting from increases in Total Fleet Mileage. As might be expected, the dominant effect of Running Costs, compared to that of Standing Costs, is exaggerated as the value of "a" becomes larger. Increases in Standing Cost are, however, detectable in Figure 5.9.1. from the irregularities in the Total Cost curve when the diameter of the delivery-area is increased from 125 miles to 150 miles, and from 225 miles to 250 miles etc..

It is interesting to compare this graph with Figure 5.9.2., which plots the three corresponding curves for a system of overnight stays. Running Cost is shown to rise linearly with delivery-area size, whilst increases in Standing Cost caused by changes in fleet-size are again indicated by two small "kinks" in the Total Cost curve, (when the value of a increases to 250 miles and 475 miles, respectively). Predictably, the cost of making stem journeys is far less significant than the cost associated with Delivery Distance, (which is calculated as the difference between Total Running Cost and Stem Distance Cost); this contrasts with Figure 5.9.1., in which the costs incurred by Delivery Distance decline in relation to the cost of Stem Distance. This aspect of the role of stem journeys is further highlighted by Figure 5.10., which plots Stem Distance Cost as a percentage of Total Cost. Clearly, this percentage increases steadily for both daily

Figure 5.7. Total Cost per week of alternative systems. (t=3·75 tons)

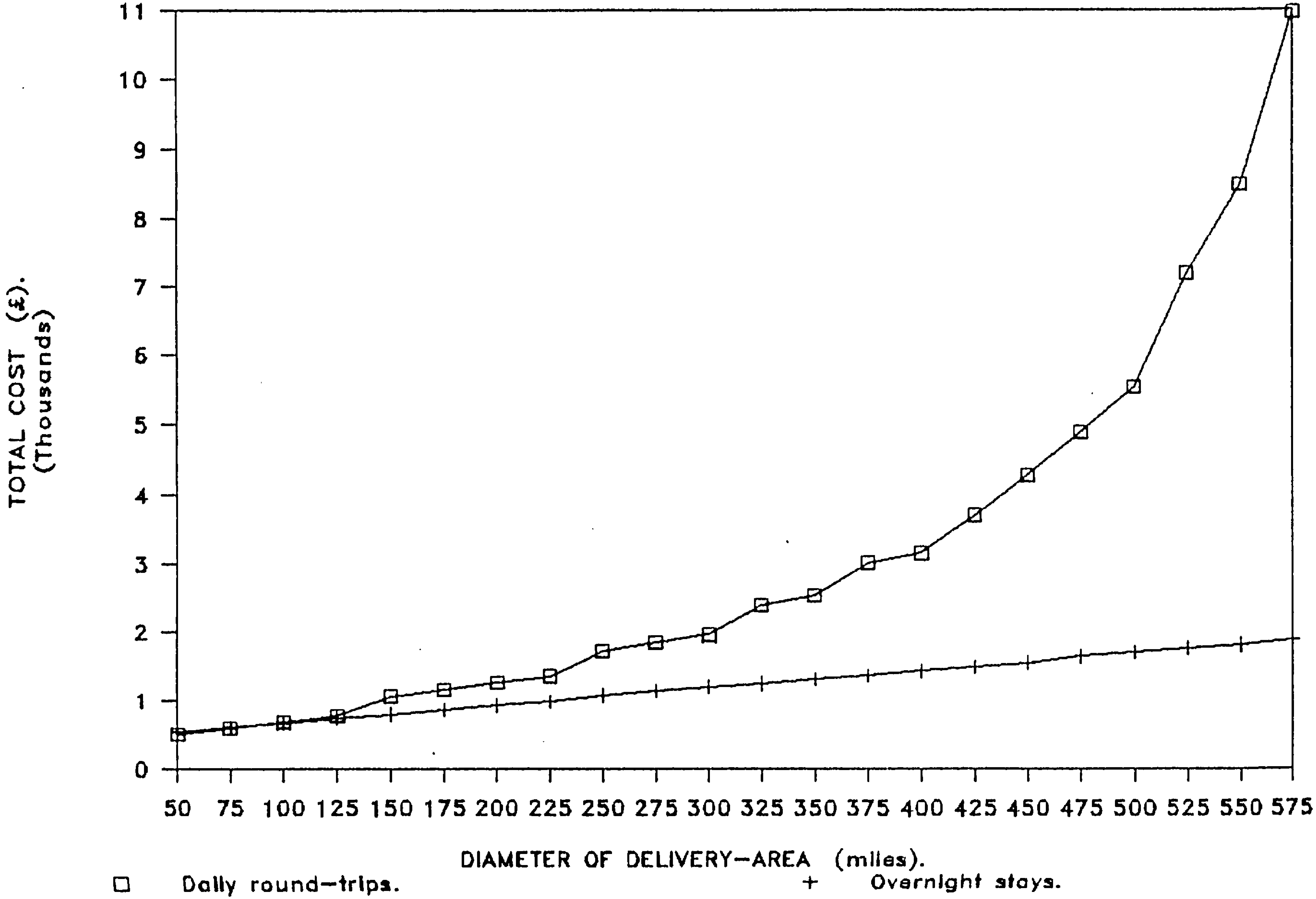


Figure 5.8. Number of vehicles for alternative systems. (t=3·75 tons)

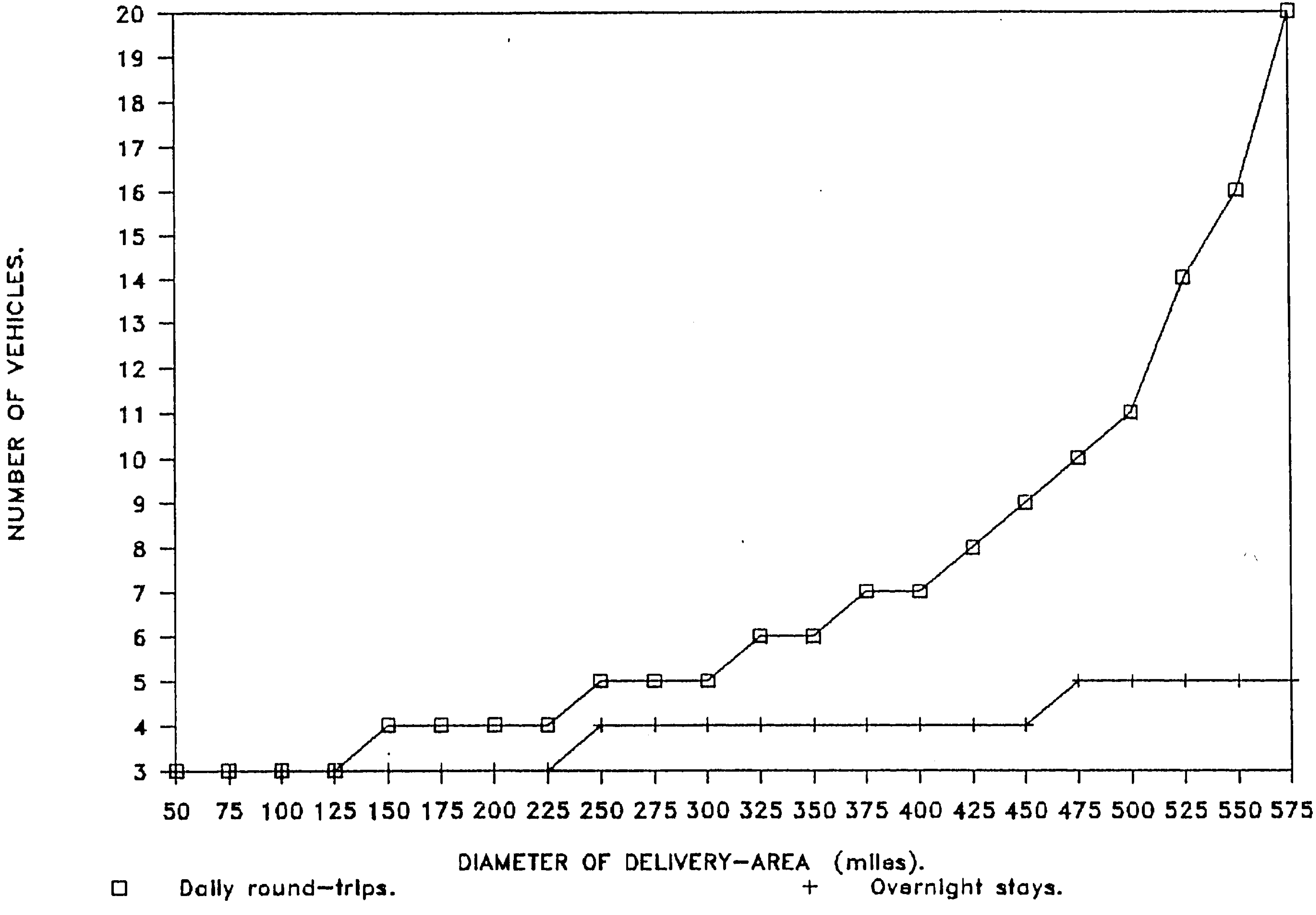


Figure 5.9.1. Disaggregation of weekly costs using daily round-trips. (t=3·75 tons)

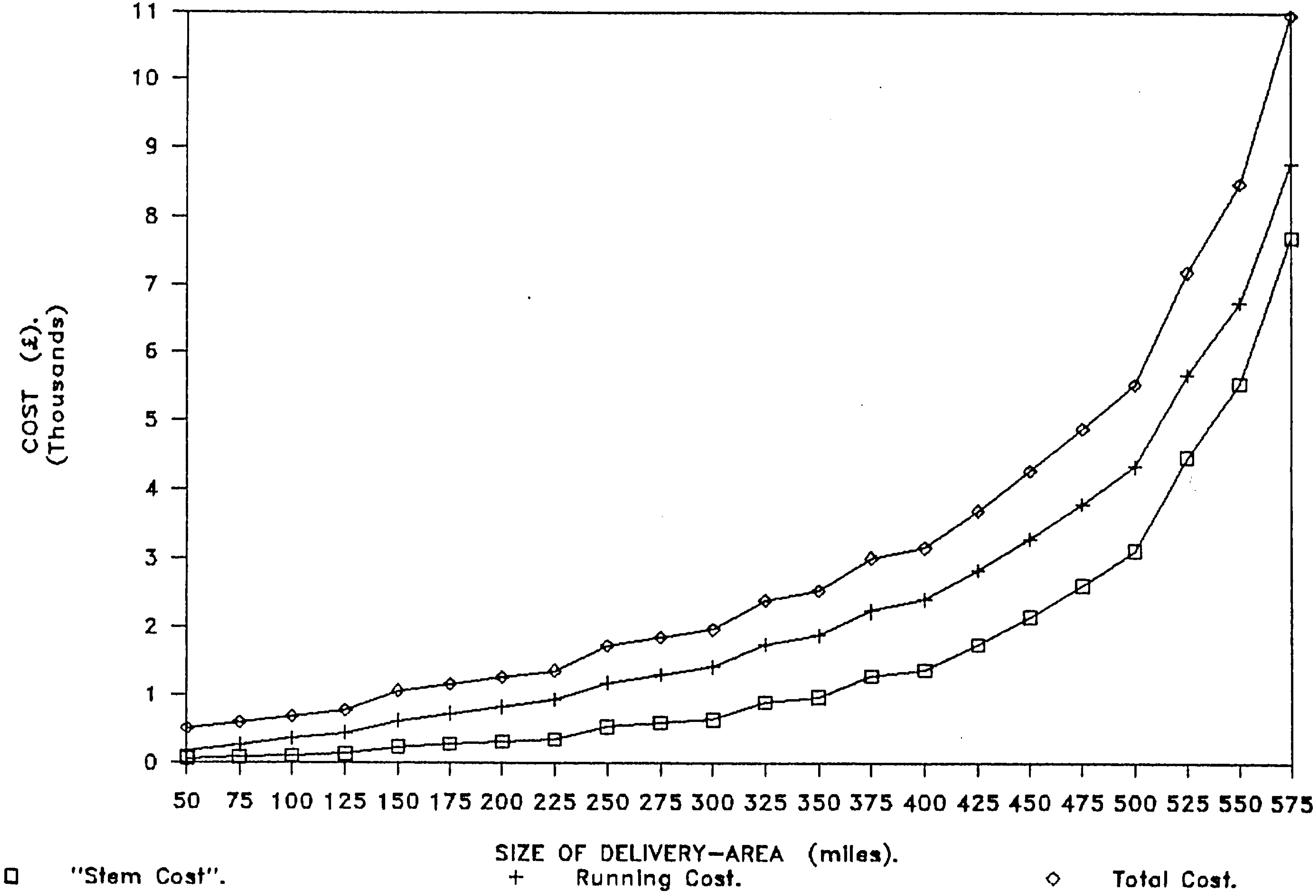


Figure 5.9.2. Disaggregation of weekly costs using overnight stays (t=3·75 tons)

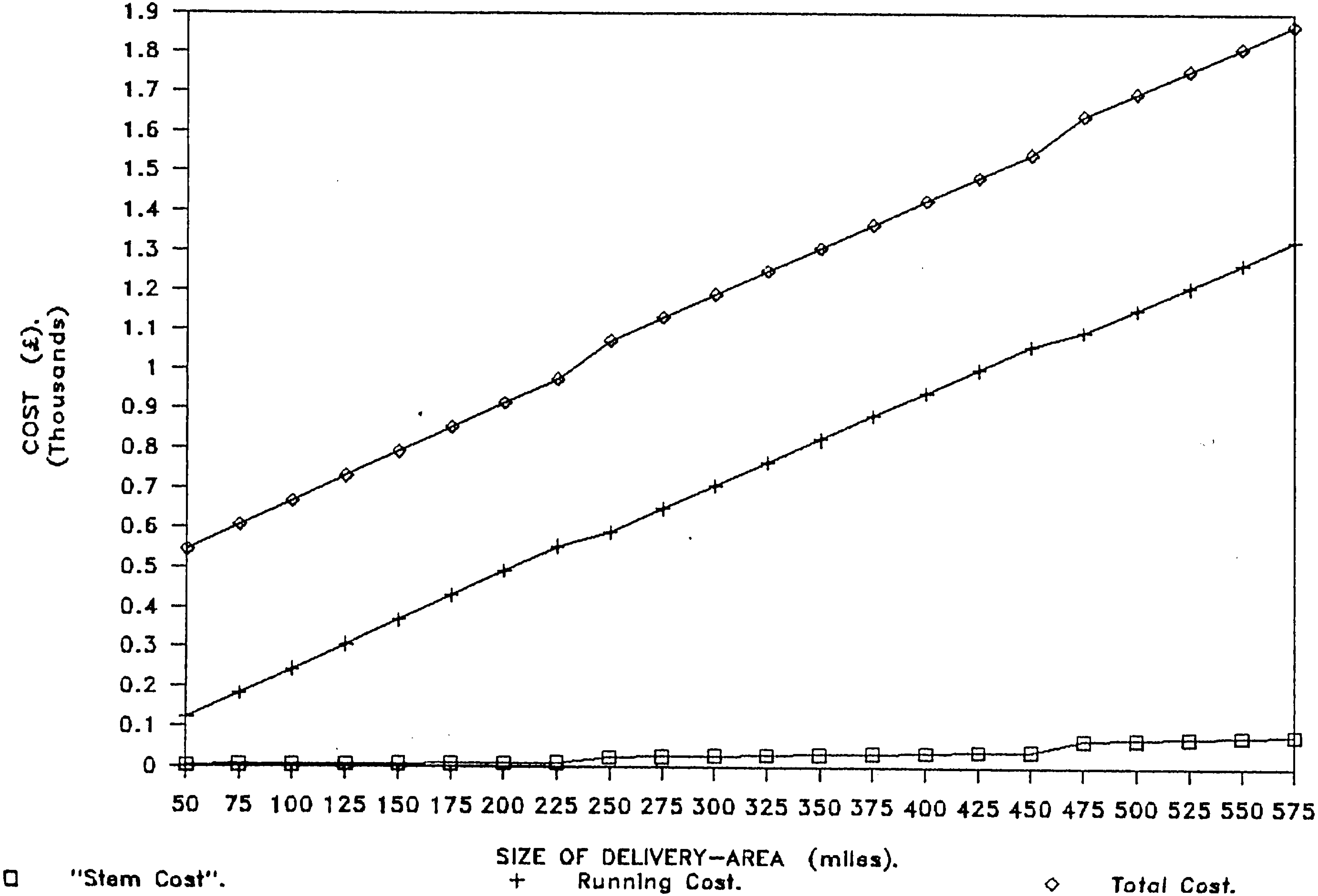
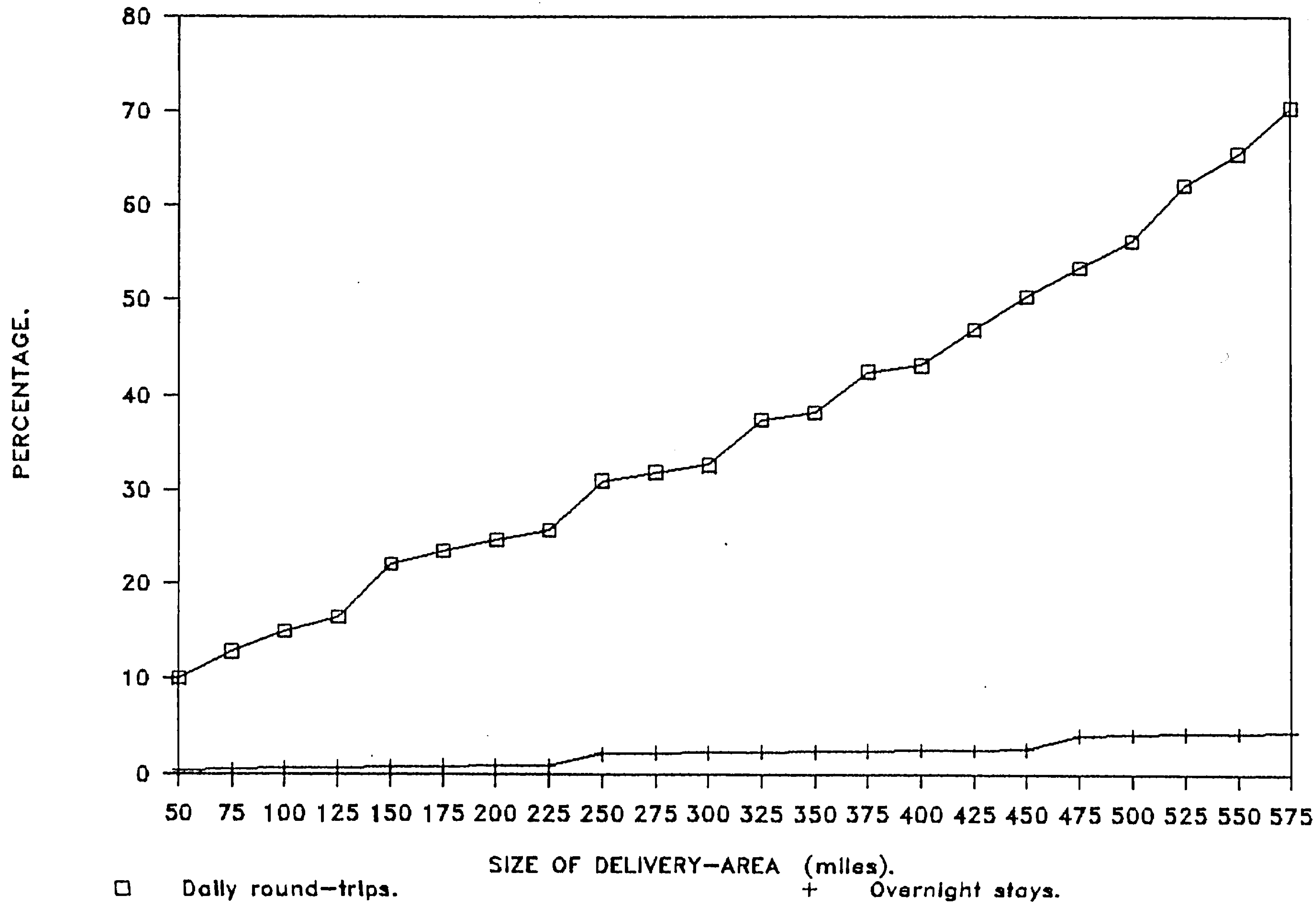


Figure 5.10. Stem Distance cost as a percentage of Total Cost. (t=3·75 tons)



round-trips and overnight stays, but, even when the diameter of the delivery-area exceeds 500 miles, Stem Distance Costs account for less than 5% of the Total Cost of a system that makes use of overnight stays away from the depot.

It has already been pointed out that the Figures contained in Table 5.4., and subsequently illustrated in Figures 5.7. to 5.10., are based on the assumption that aggregate weekly demand, t, is only 3·75 tons. To provide a contrast, Tables 5.5.1. and 5.5.2. show the equivalent figures when t=200 tons, and the corresponding graphs to those of Figure 5.7. to Figure 5.10. are shown in Figures 5.11. to 5.14., respectively.

The first of these, Figure 5.11., reveals that the Total Cost per week curves are similar to those derived when t=3·75. The main differences between Figures 5.7. and 5.11. are that the overnight stays curve both occupies a higher position on the y-axis compared with the daily round-trips curve, and is straighter when t=200. The explanation for both of these differences is that, as Table 5.5. and Figures 5.12.1. and 5.12.2. confirm, due to the general rise in the level of demand of the population of customers, the minimum size of fleet required for a week's deliveries, using overnight stays, is seven 30-ton trucks, (the maximum carrying-capacity permissible under the terms of the current problem - formulation). This has the effect of increasing the value of a, at which it becomes less economical to make daily round-trips, to between 450 and 475 miles; this is because it is generally possible to use fewer vehicles than are required for a system involving overnight stays when the diameter of the delivery-area is less than this figure. Also, as the number and size of vehicles required for weekly round-trips is constant for all values of a, when overnight stays are used, the relevant line in Figure 5.11. is straighter than the one in Figure 5.7., whose unevenness is attributable to changes in n.

Figures 5.13.1. and 5.13.2., which disaggregate the Total Cost curves of Figure 5.11., bear a close resemblance to Figures 5.9.1. and 5.9.2., respectively, except that the scale of the y-axis is larger when t=200. Estimates as to the percentage of Total Cost that is accounted for by Stem Distance Costs are also very similar when t is both 3·75 tons and 200 tons, (SEE Figures 5.10. and 5.14.).

5.3. Summary and Conclusion

The foregoing discussion considers the impact of changes in the amount of time that is available for deliveries to be made each day. In Section 5.1., this is achieved by examining the relationship between the length of the working-day and the number of customer-locations that may be visited each day. The latter variable, C, is, in turn, closely related to fleet-size, n, which has already been shown to influence Total Fleet Mileage, and thus Total Cost.

Figure 5.11. Total Cost per week of alternative systems. (t=200 tons)

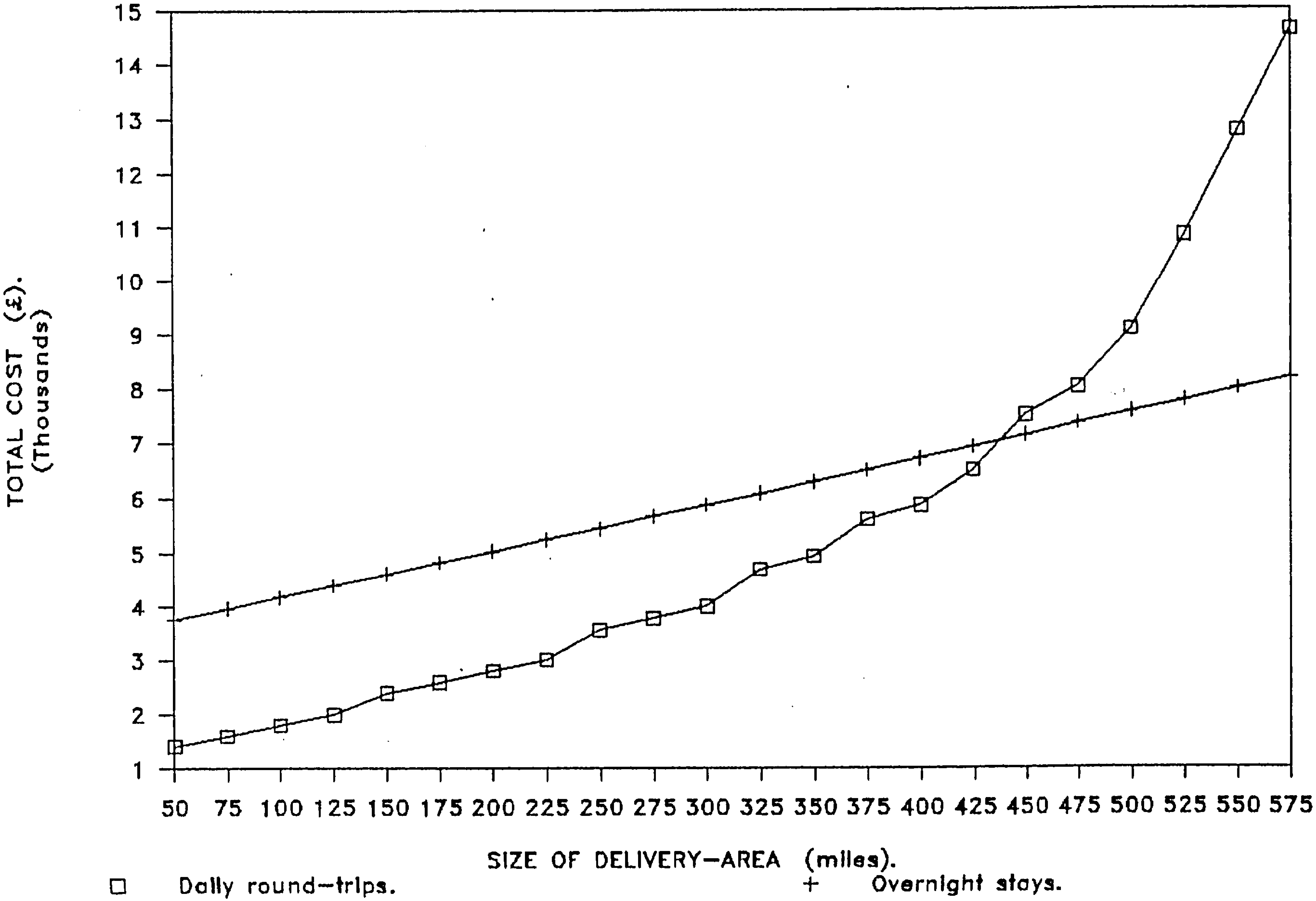


Figure 5.12.1. Number of vehicles required for alternative systems. (t=200 tons)

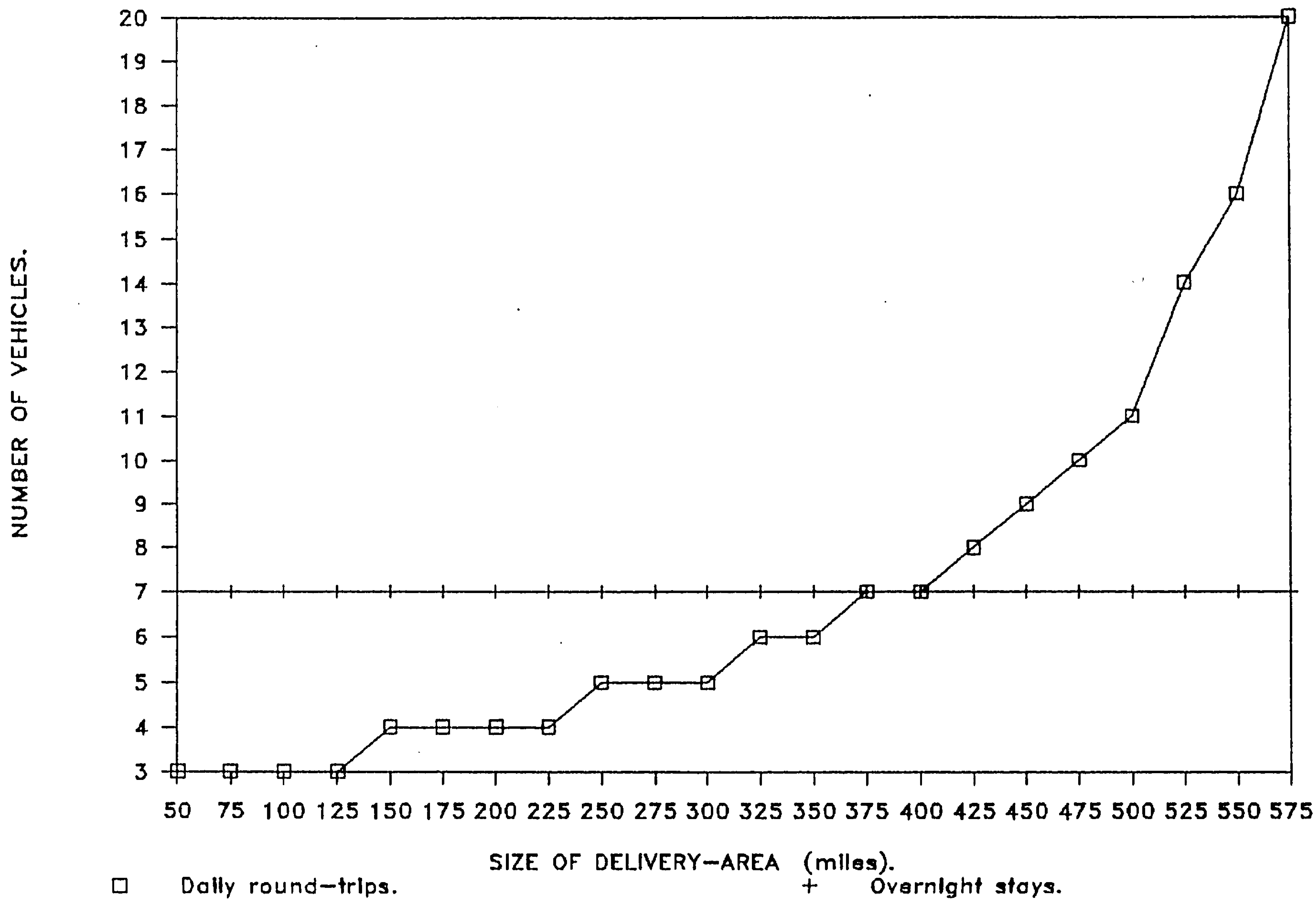


Figure 5.12.2. Vehicle-size for alternative systems. (t=200 tons).

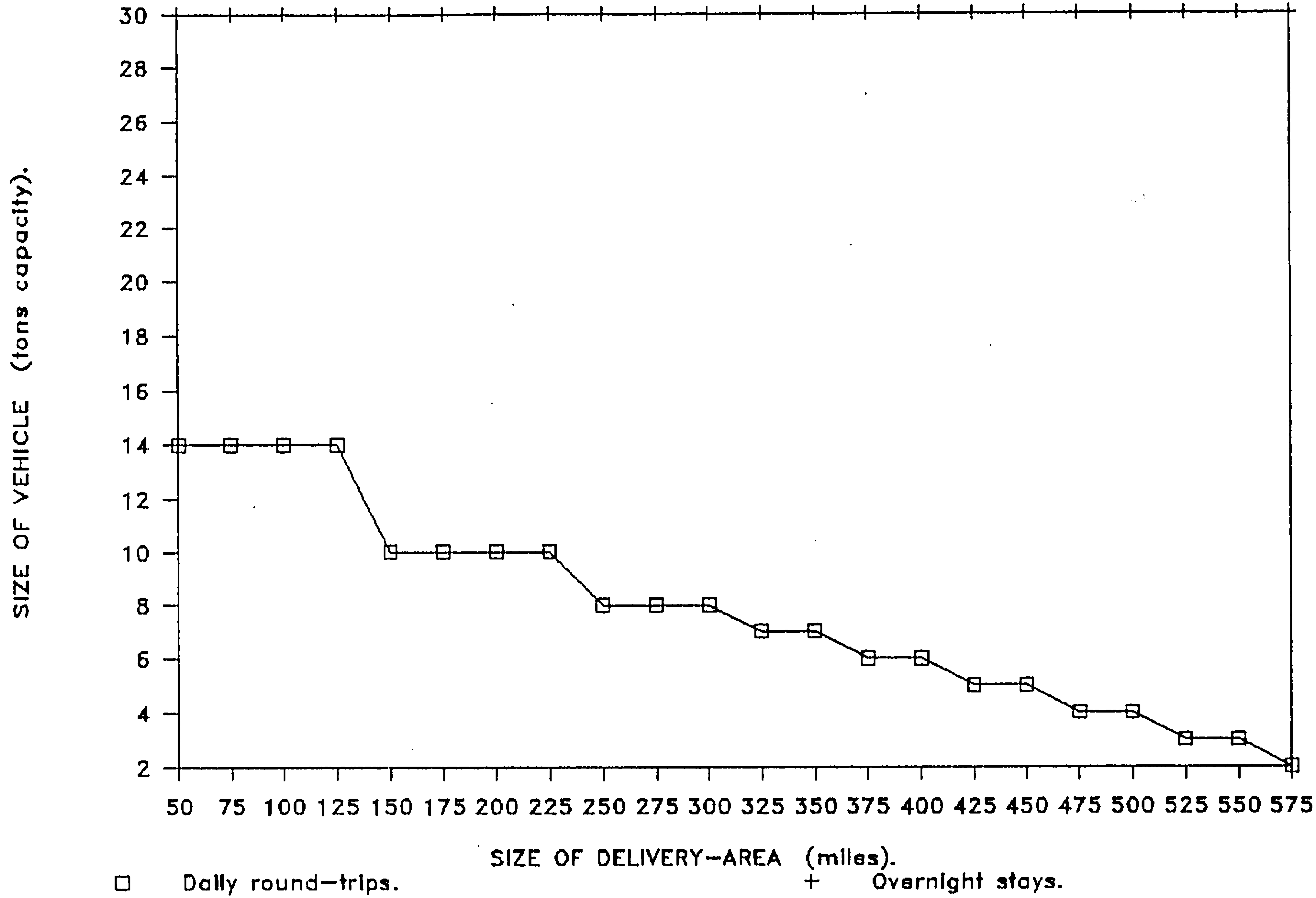


Figure 5.13.1. Disaggregation of weekly costs using daily round-trips. (t=200 tons)

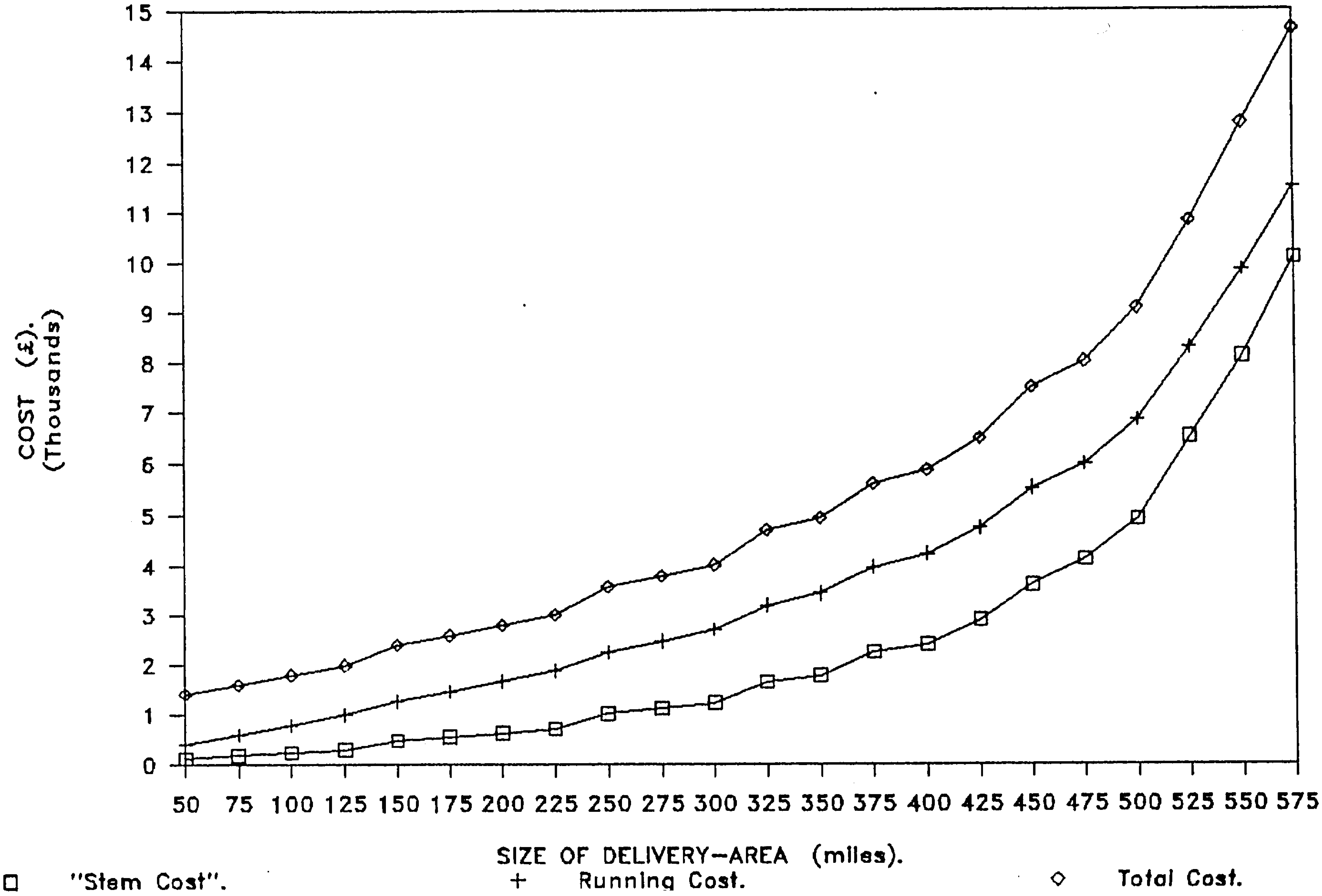


Figure 5.13.2. Disaggregation of weekly costs using overnight stays. (t=200 tons)

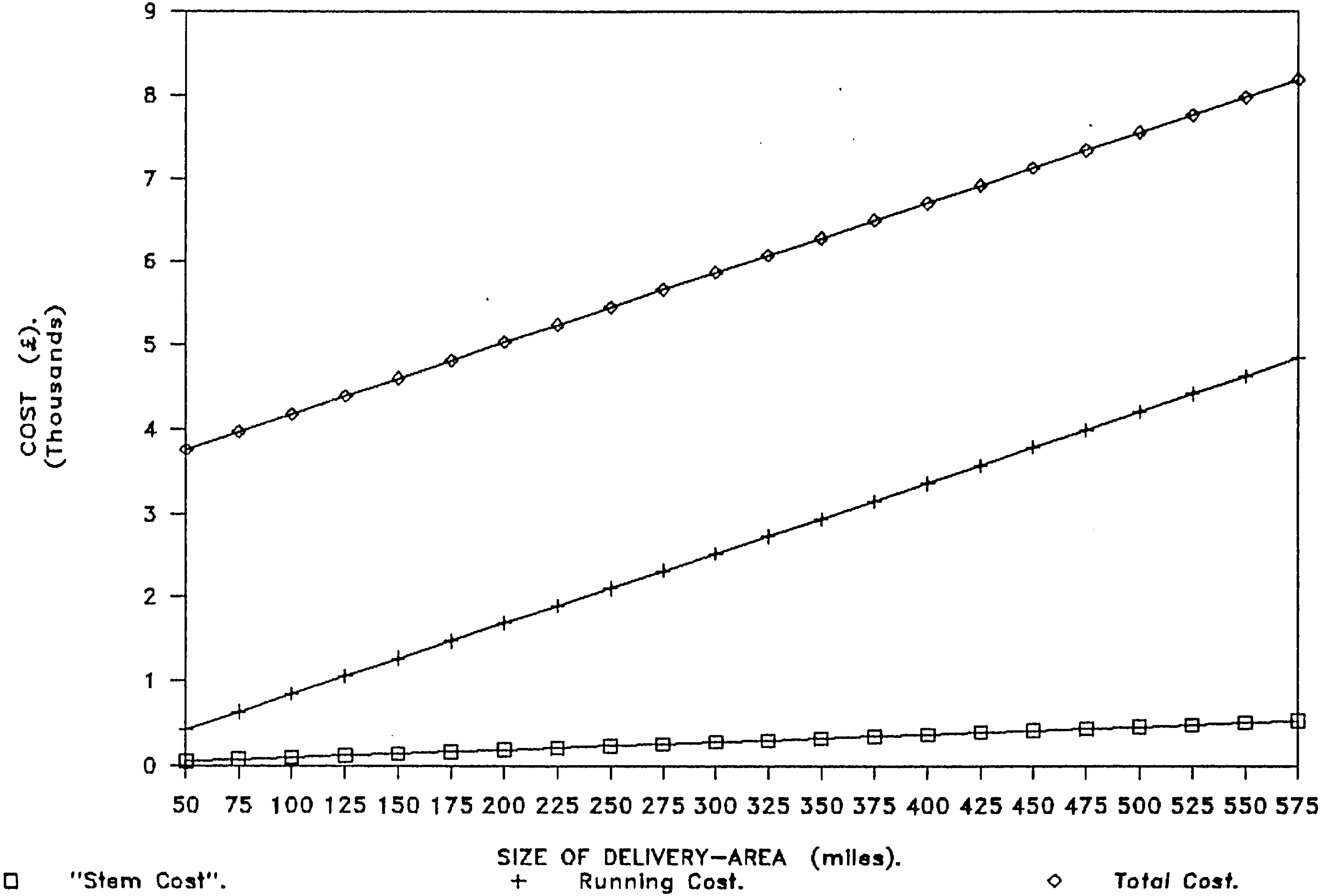
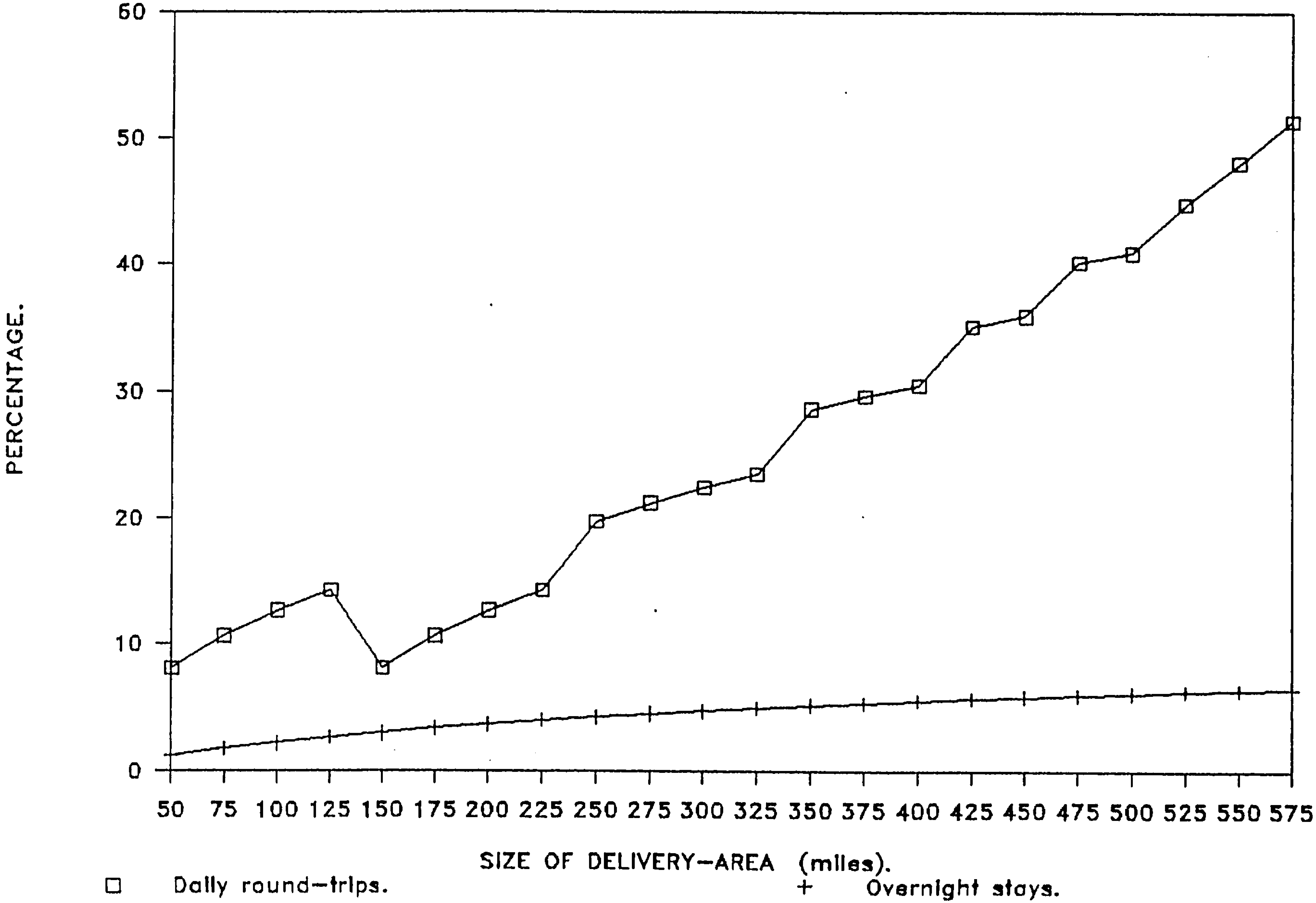


Figure 5.14. Stem Distance Cost as a percentage of Total Cost. (t=200 tons)



The upper limit to the length of a working-day is imposed upon the operator by legislation governing drivers' hours limitations, although Section 5.2. suggests a means of overcoming this constraint, by highlighting the fundamental trade-offs that are involved in the decision as to whether to make deliveries on the basis of one round-trip per vehicle per day, or to allow vehicles and crew to stay away from the depot overnight. Using the latter system, each customer-location is visited once a week in the course of, what is effectively, a single round-trip. It is hypothesised, in Section 5.2.1., that the main potential saving involved in the use of overnight stays is the reduction in the number of stem journeys that are made to and from the depot, and the findings of Section 5.2.2. concerning the cost associated with Stem Distance and its relationship with Total Cost confirm this expectation. The results of calculations in this Chapter, which are mainly based on Equations E.3.6. and E.4.20., strongly suggest that, when the diameter of the delivery-area exceeds a certain critical value, the extra expense incurred through having to both employ a fleet which has higher costs per vehicle, and pay overnight charges for vehicles and crews, may be sufficient to off-set the distance savings derived from not having to return to the operating-centre at the end of each day. The value of "a" at which overnight stays become more cost-effective is found to increase as the total weekly demand of the population of customers increases. The reason for the increase in this critical value, when t=200, has already been attributed to the fact that the Total Cost curve associated with overnight stays occupies a higher position on the y-axis in relation to the corresponding curve for a system involving daily round-trips, (SEE Figures 5.7. and 5.11.). The reason for this may, in turn, be explained by referring to Figures 5.8. and 5.12.1., which show that, in a situation where only daily round-trips are permissible, and when a=50, a fleet of 3 vehicles is required to make a week's deliveries both when t=3·75 and when t=200, (SEE also Tables 5.4. and 5.5.). In contrast, when overnight stays are used, the number of vehicles required, assuming the same delivery-area size, increases from 3 to 7 when the value of t is increased to 200 tons per week; this is a result of the constraint that exists on the carrying-capacity of a vehicle - in this example, the limit on the value of x is 30 tons. As Table 5.5. indicates, an aggregate weekly demand of 200 tons may be delivered using three 14-ton vehicles making daily round-trips, whereas 7 vehicles are required if overnight stays are made.

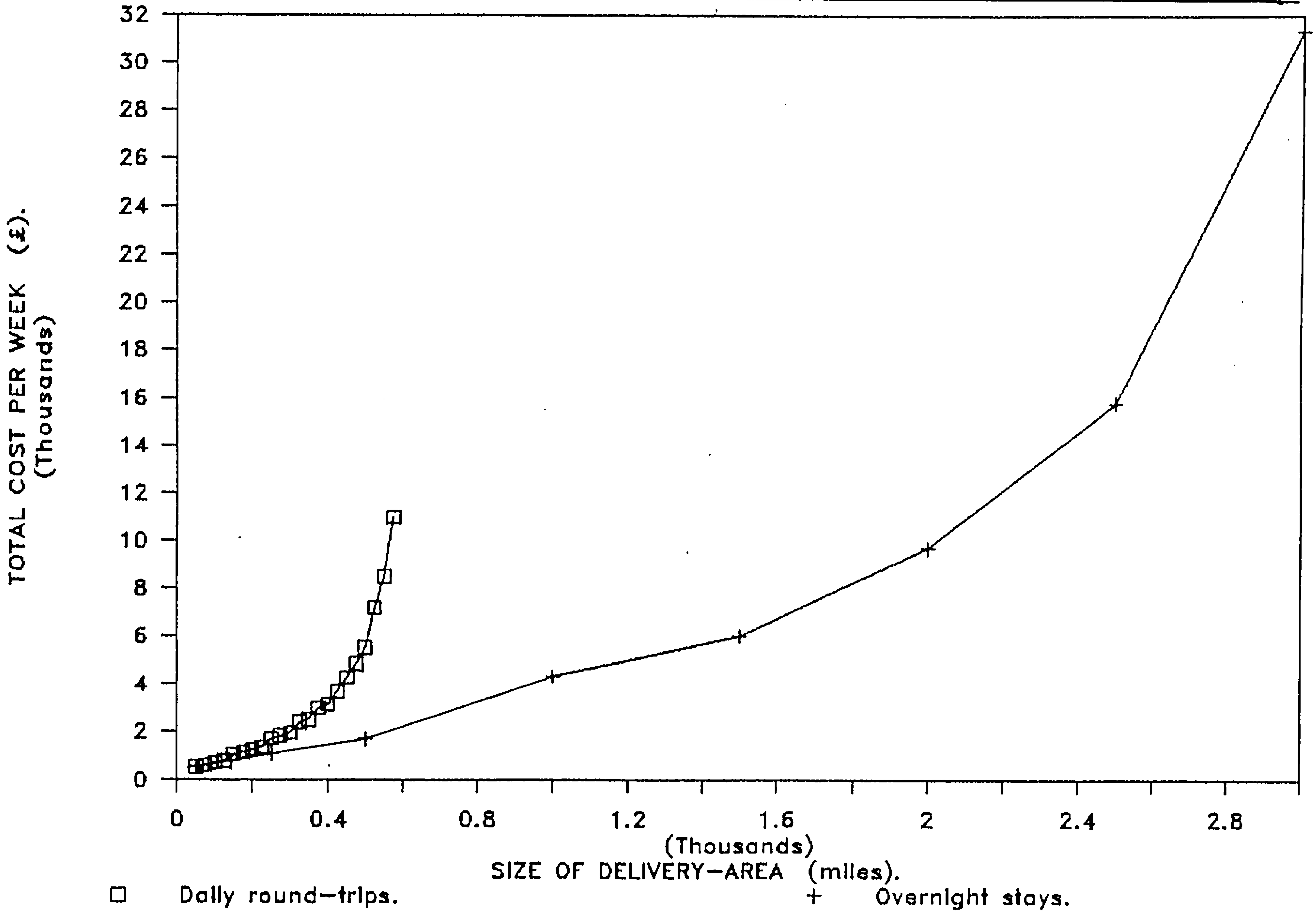
Section 5.2.2. goes on to argue that it is the savings in time resulting from not having to return to the depot at the end of each working-day that provide the main opportunity for cost-reduction, as these time-savings may actually lead to a fall in the number of vehicles that are required for a week's deliveries.

Such a reduction in fleet-size may lead to cost-savings in two ways: Firstly, a decrease in the total number of round-trips made each week will reduce Total Fleet Mileage, and secondly,savings will be made in fixed costs such as drivers' basic wages and vehicle licences etc.. It has also been demonstrated that distance-savings, particularly savings in Stem Distance, become increasingly significant as the size of the delivery-area becomes larger.

The main independent variable used in this analysis is the diameter of the area in which the set of customers is located; this parameter is of importance because of its close relationship with average Stem Distance, and is therefore found to have a major influence on whether it is profitable to adopt a system that involves overnight stays. It is to be expected that other variables that effect the amount of time that is required to deliver goods to a given set of customers, such as average vehicle-speed and the average amount of time that is spent on handling the goods at each location, will have a similar effect on the Total Cost curves of the two types of operation considered.

The relationship between delivery-area size and Total Cost per week is illustrated by the curves in Figures 5.7. and 5.11.. Perhaps the most interesting aspect of these graphs is that the cost curves for daily round-trips appear to rise geometrically with increasing area-size, whilst the corresponding curve for an operation using overnight stays seems to be linear. The reason for this anomaly is shown in Figure 5.15., (SEE also Table 5.6.). This figure reveals the shape of the cost-curve associated with overnight stays, for values of "a" up to 3000 miles, and thus demonstrates that the shape of the entire curve is the same as that for daily round-trips. For the purposes of comparison, the latter curve is superimposed onto the overnight stay cost-curve, in Figure 5.15., and it is clear that the cost of making round-trips that start and finish at the depot each day rises sharply at a far lower value of "a" than the cost of using overnight stays. This rapid increase in Total Cost as the delivery-area becomes larger, is caused by the limit on the length of a working-day raising the minimum number of round-trips that are required at an ever-increasing rate, until the value of "a" at which not even one location can be served from the depot, within the specified time allowed, is reached. This is the point at which each line in Figure 5.15. terminates, (when the calculated value of C is less than 1). An operation that uses overnight stays is, of course, subject to the same type of time-constraint, and Figure 5.15. confirms that the relevant Total Cost Function eventually shows a similar sharp increase, as a result. The difference, here, is that the time-limit that is imposed on a vehicle-tour that occupies a whole week is substantially greater than that which is associated with daily round-trips. Given the set of assumptions made for the purposes of this analysis, the value of "a" at which the weekly time-constraint causes the number of vehicle-trips required to increase rapidly, (i.e. approximately 2000 miles), is absurdly high. For this reason,

Figure 5.15. Total Cost per week of alternative systems for values of a from 50 to 3000 miles. (t=3·75 miles)



for the delivery-area sizes that are considered in Figures 5.7. to 5.14., the cost-curve associated with overnight stays appears to be linear.

3. THE EFFECT OF TIME-WINDOWS ON THE COST OF A DISTRIBUTION OPERATION

CHAPTER 6

THE DEVELOPMENT OF A METHOD FOR ASSESSING THE IMPACT OF TIME-WINDOWS ON DISTRIBUTION COSTS

So far, the discussion of the distance-minimisation aspect of the Distribution Problem has assumed that the only time-constraint that is imposed is the requirement for each vehicle to return to the operating centre before the end of the working-day, (although Chapter 5 considers the special case in which the time-limit on a vehicle's round-trip is extended beyond a day). In reality, however, a delivery to a given location may only be possible at a certain time, or within certain time-limits; the range of time during which a location may be visited is referred to here as a "time-window".

The role of Chapters 6 and 7 is to examine the effect of the presence of time-windows on distribution costs and, in the process, to propose a method for doing this.

The following section outlines the various types of time-window cosntraints that might be imposed upon a distribution operation, and discusses the effect of such limitations on the Travelling-Salesman Problem formulation.

6.1. Time-Windows and their Effect on the Travelling-Salesman Problem

The precise nature of the time-constraints that affect a delivery-round will depend largely on the type of goods, or services, that are being distributed, and will usually be dictated by the customer. For example, it may be specified by some customers that a delivery may only be made in the morning, whilst others may make their premises available for deliveries in the afternoon. A time-window may also be more rigorous, and refer to a period of only a few minutes in which a vehicle must arrive, or, in extreme cases, may involve the scheduling of a consignment of goods to arrive at a location no later than a precise time.

It is also possible for a time-window to be imposed on the initiative of the supplier of the goods, or by the distribution company; an example of how the latter might specify its own timing constraints is in the context of a service industry, such as dry-cleaning or TV repair, where, again, a certain time of collection or administering of the service may be stated. A degree of variation either side of this time may be tolerated by the operator in question, and may be incorporated into its routing & scheduling policy, so that time-windows are, in this case, imposed in accordance with, and are a reflection of, the operator's desired level of customer-service. This type of timing constraint is more commonly seen in connection with demand-responsive passenger transport, where passengers state a preferred time for both pick-up and arrival at their destination; this area of research, involving what is known as the Dial-a-Ride Problem, has already been described briefly in Chapter 1.

Time-Windows may also be asymmetrical, in the sense that a customer or operator will accept earliness far more readily than lateness, meaning that the constraints on the earliest and latest times of delivery at a given location do not allow an equal time-interval either side of the stated time of arrival.

In a real-world situation, of course, it is likely that a set of customers will impose a mixture of the types of time-constraint outlined above.

Clearly, the introduction of any form of time-window into the problem formulation will have an effect on Travelling-Salesman solutions, as the presence of such a constraint at just one location may make it impossible for the routing algorithm employed to arrive at an optimum solution. There are three important ways in which time-windows effect the formulation of the Travelling-Salesman Problem,

1. given assumptions about average road speeds etc., the total number of feasible links that may be made between customer-locations is reduced,

2. the possibility that a vehicle may arrive at a location and have to wait for a time-window to "open", and thus use up a certain amount of time that might otherwise have been used in travelling to another destination, raises the question of a trade-off between saving time and saving distance,

and 3. both "1." and "2." imply that the least-cost scheduling solution to a problem involving time-window constraints will, almost certainly, not be the same as the shortest path through the given set of points, so that the Savings Method, which has been employed to generate Travelling-Salesman tours in all other sections of this thesis, may not be the most appropriate routing algorithm here.

The impact of delivery-time constraints will not just increase Total Running Cost by increasing the average length of a vehicle trip, as the associated increase in travel-time, together with any time that is wasted by vehicles waiting at outlets for time-windows to open, will contribute towards the reduction of the number of locations that may be visited in a day. This lowering of C will, in turn, lead to a rise in the number of vehicles required, and so further increase both Standing Costs and Running Costs.

The magnitude of the effect of time-windows on overall distribution costs will obviously depend on a number of factors, particularly the percentage of an operator's customers that impose such conditions, and the size of these time-windows. The impact of timing-constraints should also be assessed in relation to that of the other parameters that have been discussed in earlier chapters, such as the average time spent at each location, average road-speeds etc.. As the importance of time-windows will also vary according to both the type of constraints in force, and the idiosyncrasies of the set of customers and delivery-area in question, the following section defines the precise formulation of the problem to be addressed in the discussion that follows, and outlines the major assumptions that are made.

6 .2. Problem-Formulation and Discussion of Assumptions

The basic assumptions as to the nature of the problem are the same as those made in previous chapters; in other words, the problem consists of scheduling vehicles to serve a set of customers who are located at random points within a homogeneous delivery-area of known size, from a centrally-located depot. Furthermore, each customer-location must receive a daily visit, although the size of the consignment that must be delivered at each stop has no relevance to this particular problem-formulation. The latter assumption means that vehicle-capacity, x, is a variable of only minor importance here, so that the constraint that has a direct influence on n is the number of customers that may be visited in a day before a vehicle must return to the operating-centre. The subsequent discussion is therefore just as pertinent to the routing & scheduling of vehicles that distribute a service - quite literally travelling-salesman, for example - as it is to that of a fleet of vehicles that delivers an order of goods. Another aspect of the basic problem-formulation used here is the shape of the delivery-area served by the depot; in this case, it is assumed that the fleet operates within a square zone whose area is "a^2", and not within a circle of diameter "a", as it sometimes is in other sections of the thesis.

As far as concerns the time-constraints that are involved, it is assumed that each customer specifies one time-window, which defines the time-period during which a vehicle must arrive, so that handling and documentation procedures may continue after the time-window has "closed". The size of this time-window, (measured by its width, r, in time-space), is the same for each customer, and will always be expressed here in minutes; the mid-point, w_i, of each time-window is, in contrast, assigned a random value representing the time of day at which a delivery must be made. This value must, of course, be feasible, in as much as it should be possible, given the spatial location of each customer, for a vehicle both to reach the premises from the operating centre before the specified dead-line, and to return before the end of the working-day, having spent the necessary time on handling and documentation.

For the purposes of the current example, it is also assumed that each customer-location must be visited before the relevant time-window closes, so that a vehicle must arrive at a time no later than (w_i + 0·5 r). It is necessary to make this point, since, in the real world, it is likely that a trade-off might be made between the cost of arriving at a location a few minutes late, and the extra cost that might be incurred through arriving on-time, (since this may be possible only at the expense of an increase in fleet-size, for example). On the other hand, although no degree of lateness is tolerated under the terms of this problem-formulation, a vehicle is permitted to arrive at a location before a time-window opens, although this will involve a certain amount of "waiting time" during which the vehicle will be idle. The implication of the possibility of some vehicles being scheduled to wait at customer-locations in this manner are dealt with in Section 6.3., which also considers the other ways in which timing constraints complicate the basic Travelling-Salesman Problem.

6. 3. The Complications Caused by Time-Windows, in Relation to Travelling-Salesman Problems

There are three main ways in which time-window constraints affect the process of finding solutions to Travelling-Salesman Problems; two of these, which are discussed in Sections 6.3.1. and 6.3.2., concern the criteria used for measuring the distance between pairs of customer-locations, whilst the third, to be dealt with in Section 6.3.3., is associated with the construction of tours.

6.3.1. The trade-off between saving waiting-time and saving distance

A central problem involved in the routing and scheduling of a fleet of vehicles in the presence of timing constraints is the identification of what constitutes an "optimum solution" in this context. This is because an algorithm which strives merely to minimise distance may produce solutions which cause vehicles to spend a substantial amount of time waiting at customers' premises for time-windows to open, whilst, at the other extreme, a procedure that generates solutions which primarily minimise the amount of "idle time" in the fleet will inevitably not be very successful at reducing Total Fleet Mileage.

The result of both types of inefficiency is that the average time that is spent in serving each customer is greater than it might otherwise be. Because of the limitation on the length of a working-day, this will reduce the number of deliveries that may be made each day, and thus cause the size of the vehicle-fleet required to also be unduly large. Since time is so closely related to cost, in this way, it would seem logical to "plot" the location of each customer in "time-space", although a major flaw in using such a technique is that the value of saving travelling-time is not equivalent to the value of saving waiting-time; this is because a stationary vehicle waiting for a time-window to open is not incurring Running

Costs. It would seem to be even more appropriate, therefore, to weight potential savings in time and distance by expressing the separation of pairs of customer-locations in terms of the combined cost of both the mileage that is covered, and the time that elapses between a vehicle's departure from location i and the start of unloading procedures at location j. The cost of each mile travelled may be estimated from cost tables, in the usual way, although the attachment of a monetary value to a saving in waiting-time introduces the concept of the Value of Time.

This is a concept that is employed in a variety of contexts, and is particularly relevant to transport operations. Time itself, of course, cannot actually be saved. The advantage gained through reducing the time that is taken to perform a given activity lies in the fact that each minute "saved" may be utilised productively elsewhere; if this is not the case, then a time-saving is worthless. In other words, there is an Opportunity Cost associated with each unit of time that is "wasted".

In the context of the current discussion, the value of the time saved during the course of a vehicle's delivery-round, regardless of whether this is a reduction in waiting-time or travelling-time, may be capitalised by this vehicle utilising this time to visit more customer-locations. With a fixed value of P a time-saving may be measured according to the amount by which the fleet-size required to perform a given delivery-task is reduced, so that a fall in the average time taken for each vehicle-trip is worthless unless it is large enough to enable the operator to dispense with at least one vehicle.

The Value of Time is, therefore, generally a function of the time that must be saved for a tangible cost-saving to be made in this way, and the amount, in monetary terms, that is thereby saved. The value of an actual time-saving, in pounds (sterling) per day, may be expressed as follows,

$$vot = c.\frac{s}{s_r} \qquad (E.6.1.)$$

where, vot = the value of the time-saving,
c = the cost-saving resulting from a unit reduction in fleet-size,
s = the amount of time that is actually saved, (in minutes),
and, s_r = the time-saving that is required to enable a reduction in fleet-size (in minutes).

A more generalised expression for the Value of Time is therefore,

$$£ \left[\frac{c . \frac{s}{s_r}}{s} \right] \text{ per minute}$$

so that,

$$\underset{(£ \text{ per min})}{\text{vot}} = \frac{c}{s_r} \qquad (E.6.2.)$$

The problem is now reduced to that of determining the value of the variables "c" and "s_r". The amount of time that may be taken to represent the "workload" of one vehicle is the sum of the average daily travel-time per vehicle, and the average amount of time that each vehicle spends at customers' premises each day. In practice, this figure may not be exactly equivalent to the aggregate amount of time that must be saved from the schedules of all the vehicles in a fleet in order to dispense with one vehicle, although this simple formula may be regarded as being an adequate estimate of s_r for the purposes of this section of the analysis. Assuming that vehicles are fully utilised throughout the working-day, and that each of these days consists of a maximum of 9 hours, then the value of s_r, in minutes, may be assumed to be 540.

The Total Cost saving that accrues as a result of saving s_r minutes is equal to the difference between the cost of operating a fleet of n vehicles, (per day, in this case), and that of operating a fleet of (n+1) vehicles; this is, of course, a reference to the Marginal Cost of n. In Chapter 4 it is demonstrated that the relationship between Total Cost and fleet-size is non-linear, but some of the important assumptions made in that chapter are no longer applicable when time-window constraints are in force. One of these is the assumption of non-intersecting vehicle-tours, which results in the pattern of vehicle delivery-zones shown in Figure 4.1.. Depending on the severity of the timing constraints imposed, each vehicle-route may include customers situated anywhere in the delivery-area; the likelihood of a vehicle operating within a full 360-degree arc around the depot in one day will obviously increase as r is reduced. Chapter 4's conclusions concerning the effect of fleet-size itself on Total Fleet Mileage are therefore inappropriate, given the problem-formulation relevant to this section.

For the purposes of the approximate estimates that are to be made, here, it may be assumed that, notwithstanding the fact that the overheads associated with an operation will be largely independent of n, the value of c may be taken to be equivalent to the cost of operating a single vehicle.

The value of c may actually be calculated using cost tables (1), having made assumptions about the size of vehicle that is to be used; the value of x used for this analysis is 0·75 tons, representing the smallest size of vehicle featured in the cost tables, (SEE Appendix A). This choice of vehicle-size was, however, purely arbitrary. The Standing Cost of one 0·75-ton van is quoted in the tables as being £119·80 per week; as this section deals with a system of daily round-trips, the relevant figure, here, is the daily cost of the van, ie. £23·96 per day, (assuming a 5-day week). As the vehicle's Running Cost per mile is estimated as being £0·1801 per mile, then Running Cost per day is £(0·1801.d), where d is the length, in miles, of one vehicle-trip. Therefore, when x=0·75,

$$\underset{\text{(in £ per day)}}{c} = 23{\cdot}96 + (0{\cdot}1801.d) \qquad \text{(E.6.3.)}$$

and,

$$\underset{\text{(in £ per min.)}}{vot} = \frac{23{\cdot}96 + 0{\cdot}1801.d}{540} \qquad \text{(E.6.4.)}$$

The presence of the term "d" in Equation E.6.4., creates a slight problem, since the average length of daily round-trips, with time-window constraints, is, so far, unknown. This problem was overcome by allocating an empirical value to d, since simulations, using a similar algorithm to that derived in this chapter, performed prior to the set of simulations whose results are to be discussed in Chapter 7, revealed that the average length of each vehicle-trip is roughly 2½ times the value of "a". A description of this alternative method, along with a record of the readings derived from it, also appear in the following chapter, (SEE, particularly, Figure 7.15.). Again, the use of a fixed value for d in Equation E. 6 .4. is not entirely satisfactory, especially as the estimate of "2½.a" is less accurate for smaller r-values. The purpose of this expression for the Value of Time is, however, merely to enable a comparison to be made between the cost of linking alternative pairs of customer-locations, and so it is not vital for this estimate of d to be absolutely precise.

The objective of this section is to develop a means of making a trade-off between saving waiting-time at customer-locations, and saving time spent travelling between successive stops; having established an expression for "vot", it is therefore now necessary to express Running Cost as a function of time. Retaining the assumption that x=0·75 tons, and assuming

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(1) Again, Commercial Motor cost tables from 1982 are used.

that average vehicle-speed, S, is 31·25 miles per hour (2), the Running Cost of the vehicle is £0·1801 per mile; as (31·25/60) miles are travelled in each minute, (ie. 0·5208 miles), then the Running Cost of a 0·75-ton van is £(0·1801 x 0·5208) per minute, (ie. £0·0938 per minute).

The distance between each pair of customer-locations in "cost-space", or, more specifically, in terms of the cost of travelling plus the cost of any time that's spent waiting for a time-window to open, may therefore be measured. The formula for calculating this distance between a pair of locations is,

$$GC_{(\text{in } £)} = 0{\cdot}0938.T_t + T_w \left[\frac{23{\cdot}96 + 0{\cdot}1801{\cdot}d}{540} \right] \qquad (E.6.5.)$$

where, GC = the combined, Generalised Cost of travelling-time and waiting-time,
T_t = travelling-time between locations i and j,
and, T_w = waiting-time at location j.

Using this formula, a (PxP) matrix of distances in Generalised Cost Space may be constructed, although this particular step in the analysis is complicated by the introduction of time-window constraints; this problem is now dealt with in the following section.

6.3.2., The asymmetrical nature of links in Generalised Cost Space

Consider a pair of customer-locations, i and j; under the problem-formulation of the classical Travelling-Salesman Problem it is assumed that both distance and journey-time between these two locations is identical in either direction, although this is not the case when "distances" are measured in terms of Generalised Cost, and when time-windows are involved. The reasons for this may be illustrated with the following example.

Assume that i and j are separated by a 2-minute drive, and that their w-values are 200 minutes and 250 minutes, respectively; it may also be assumed that each time-window is 60 minutes in "width", and that handling-time at both locations is 10 minutes. Clearly, the value of r, in this situation, is large enough, in relation to the time spent at each location and the spatial separation of i and j, to enable travel in either direction. The Generalised Cost of travelling from i to j is, however, different to the equivalent figure for the journey in the opposite direction. It is convenient to illustrate this point by assuming that, in each instance, a

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(2) This figure was originally assumed to be 50 km per hour

vehicle arrives at the first of these locations at the earliest time permitted by the time-window constraints. In other words, when considering a journey in the direction i-j, the arrival-time at i is (w_i - $0{\cdot}5_r$) minutes, (ie. 140 minutes), so that, after handling procedures, the vehicle sets off for j after 150 minutes. Arrival-time at j is therefore 170 minutes, implying that a wait of 20 minutes is necessary, before the time-window at location j "opens". For a journey in the reverse direction, (j to i), the vehicle would leave location j after 200 minutes and arrive at i after 220 minutes, and thus not incur a waiting-time cost. It is clear, therefore, that the Generalised Cost of linking locations i and j in a tour varies according to the direction of travel, although the distance involved is, of course, the same.

In other circumstances, the presence of time-windows may cause travel between a pair of customer-locations to be feasible in one direction and infeasible in the other, thereby imposing a directional constraint on some links.

Such problems caused by time-windows present obvious computational difficulties. A more serious complication, in as much as it creates algorithm problems for the construction of Travelling-Salesman tours, is the uncertainty attached to the time at which a vehicle arrives at each location; it is this problem that is to be discussed in the following section.

6.3.3. The problem of uncertainty concerning arrival-times

The basic methodology for constructing Travelling-Salesman tours used in previous chapters, has begun with the setting-up of a PxP matrix of values, which defines the relative location of each of the given set of P points. Using the criterion of Generalised Cost, however, this is not possible, because the amount of time that a vehicle must stand idle at a location will depend upon the time at which this vehicle departed from the previous location in the tour; at this stage of the analysis, of course, this information is unknown.

This problem may be illustrated by referring again to the numerical example given in Section 6.3.2., concerning the customer-locations i and j, (w_i = 200 minutes, w_j = 250 minutes). The time at which the vehicle departs from i, on a journey to j, may take any value from 150 minutes to 270 minutes, (given that r=60 and l=10), and this time will determine the waiting-time that is necessary, if any, at j. If i is not the first location to be visited in a vehicle-tour, then departure-time from i will be a function of the time at which the vehicle left the previous location on the route, and so on. Without prior knowledge of the sequence in which customers are to be served, therefore, there is no way in which an accurate matrix of Generalised Cost values may be drawn up.

The only context in which a matrix-based algorithm may be employed is where vehicles are constrained to arrive at each location at a precise time, (or earlier), so that the value of r is effectively 1. Similarly, when r is very small, a matrix of more approximate Generalised Cost values may be drawn up; provided that the average time spent at each location can be taken to also be negligibly small, this may be achieved by assuming that the time that each vehicle departs from location i coincides with "w_i", the mid-point of i's time-window. In this situation, the maximum error in the calculation of the vehicle's waiting-time at location j is "+" or "-" the value of r. When the time-windows involved are very narrow, this error may be readily accepted as an inaccuracy in calculation, although, for larger r-values, a matrix of such figures becomes absurd, so that an alternative route construction methodology is required.

A method that is capable of dealing with the complications to the Travelling-Salesman Problem, that are described in this section, is outlined in Section 6.4.1. First, though, attention is turned to the general methodology that is used for this part of the analysis.

6.4. Methodology, in Relation to the Travelling-Salesman Problem with Time-Windows

Given the problem-formulation outlined in Section 6.2., the most important variable relating to time-constraints is r, the width of the time-window specified by each customer, and so it would seem logical to use this measure to represent the overall severity of the constraint imposed upon an operation. An alternative indicant, in this context, might have been the percentage of customers specifying such a retriction, although it has already been stated that it is assumed, here, that a time-window is defined for every location.

In line with much of the analysis described elsewhere in this thesis, the methodology employed to examine the relationship between r and Total Distribution Cost is one of simulation, involving the construction of a computer program to simulate the type of distribution operation described above, with r as the main independent variable. The analysis is based on the hypothesis that, as the width of the time-window at each location is increased from a minimum of 1 minute to a value that is so large that, effectively, timing constraints no longer exist, then restrictions on routing & scheduling are correspondingly relaxed; with Total Fleet Mileage, and therefore Total Fleet Time, both consequently reduced by this lessening in the severity of such constraints, it naturally follows that Total Running Cost and fleet-size will also fall, thus reducing the overall cost of an operation. The rationale for such an argument is self-evident, although the results of these simulations will provide insights into the precise nature of the relationship between r and variables such as Total Fleet Mileage and Total Cost.

The initial stage of such an analysis is to determine the extent to which the imposition of time constraints reduces the set of feasible links that may be made between pairs of customers, given both their spatial distribution and the value of w_i that defines the location of each time-window in time-space. As will be seen later, for the purposes of the simulation exercise, fixed values are used to represent vehicle-speed and the time that is spent at each location, although these values really stand for average figures. It would be more accurate, therefore, to use probability distributions, based on these averages, to represent such variables; whether or not a link between a pair of locations were feasible would then depend upon the probability of the second customer's time-window being "hit". The use of probabilities in this way becomes rather complicated when a succession, or "chain", of links is involved, and so all parameters used in the simulations described below, with the exception of the independent variable, r, and the randomly-generated customer co-ordinates and w_i-values, are fixed.

This provision means that the feasibility of a potential link between any pair of customers may be tested using a simple, algebraic expression, since it is infeasible for a trip to be made from location i to location j if,

$$D > (w_j + 0\cdot5_r) - (w_i - (0\cdot5_r + 1)) \qquad \text{(E. 6 .6.)}$$

where, D = the "time-distance" between locations i and j,
w_i = the centroid of location i's time-window,
w_j = the centroid of location j's time-window,
r = the width of each time-window,
and, 1 = the time spent at each customer-location.

This is because, even if the vehicle in question leaves location i at the earliest possible time, there is insufficient time to reach location j before j's time-window closes. Similar expressions to Equation E. 6 .6. may be developed to ensure that no customer-locations are assigned w_i-values that would make it impossible for the customer to be reached directly from the depot without transgressing the limit on the length of a working-day.

Having checked that all customers may be served from the operating-centre, the next stage of the analysis is to derive the set of routes through the P customer-locations which minimises aggregate Generalised Cost, using Equation E. 6 .5.. A method for achieving this is now presented in the following section.

6 .4.1. A Shortest-Path algorithm for the Travelling-Salesman Problem with Time-Window Constraints

In the absence of the aforementioned complications caused by timing constraints, the process of finding the lowest-cost route, or set of routes, through the given set of points, would begin with the linking together of the pair of customer-locations which share the lowest value in the Generalised Cost matrix. The procedure would then select the pair of customers with the second-lowest value in the matrix, and continue this process until there is no location that is not incorporated into a vehicle-tour; at this point, an optimum, or near-optimum, route, or set of routes, would have been created. The introduction of time-windows, however, for the reasons which have been outlined above, precludes the use of such a procedure by hindering the initial creation of a matrix of Generalised Cost values.

The strategy for building tours that is shown in Figure 6 .1. is "myopic", in the sense that, rather than search through a matrix for the pair of locations that has the smallest value, a route is compiled by scheduling a vehicle to travel next to the "nearest" customer-location to where it happens to be at any given time. In other words, each route is initiated by a link being made from the depot to the depot's nearest customer, which, in the current context, is the one situated the shortest distance away from the depot in Generalised Cost Space. The next step is to add, to this initial link, the nearest location to the first customer to be visited, and so on. This process continues until it is no longer possible to visit another location and still be able to return to the operating centre before the end of the working-day; at this point, the vehicle returns to its origin immediately, and, if there are still customers requiring a visit who have not yet been included in a vehicle's schedule for the day, a fresh tour is commenced, employing the same procedure.

Using this myopic approach, there is little prospect of obtaining an optimum, Shortest-Path solution, either with time-constraints completely absent, or when r is so large as to cause the time-windows in force to be ineffective. Without time-windows, therefore, the method which searches for the smallest value in a matrix, described previously, may be expected to be more successful at finding the shortest path through a set of points; (it is this type of algorithm that is used for building Travelling-Salesman tours for the analysis described in Chapter 4 - SEE Figure 4.8.). The advantage of the myopic procedure, however, is that it eliminates the possibility of travel between a pair of locations being feasible in either direction, and avoids the problem associated with chains of customer-locations that is outlined above. The route-building algorithm described in Figure 6 .1. was therefore adopted as a basis for the computer

Figure 6.1. "Myopic" tour-building strategy

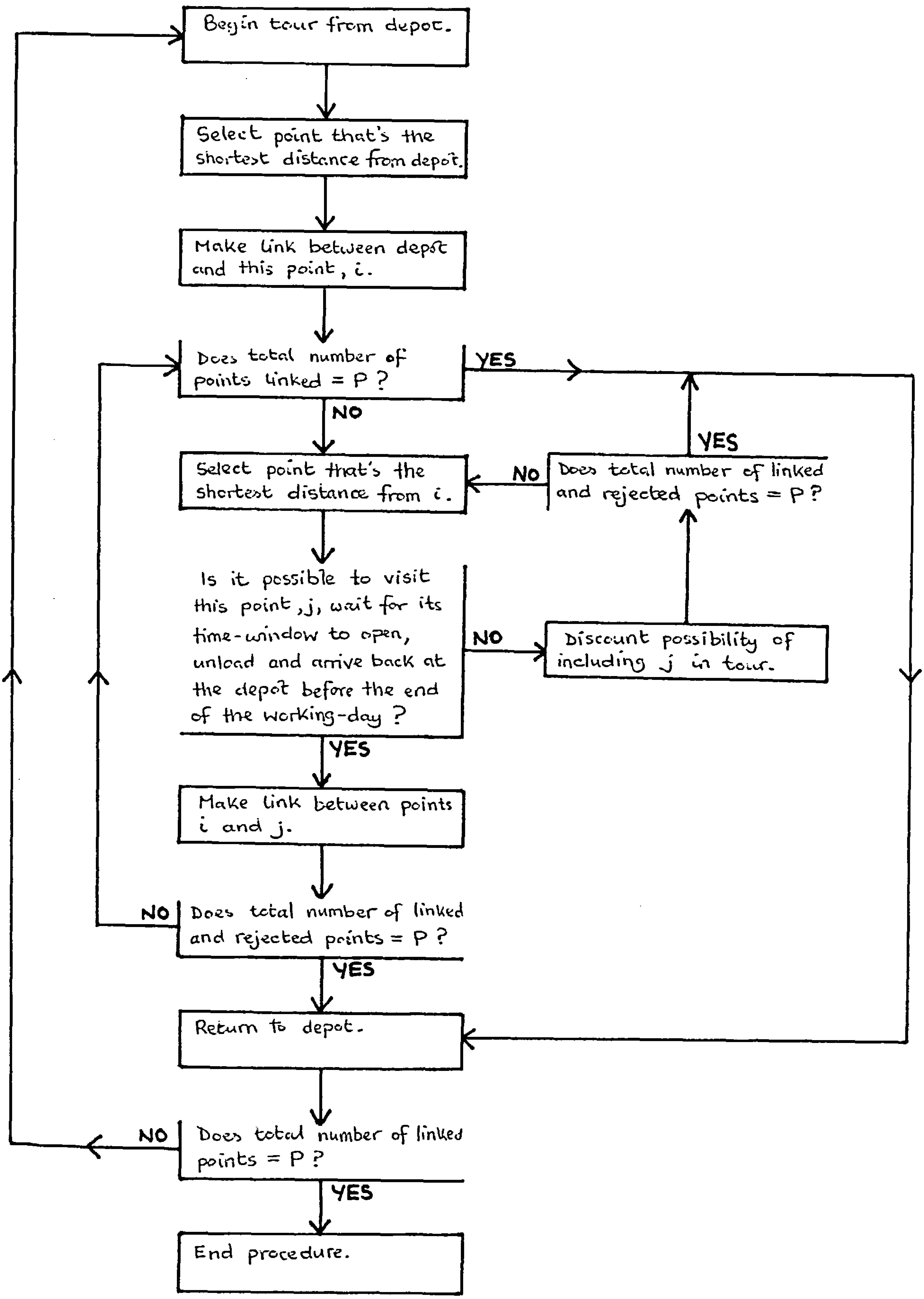


Figure 6.2. Progam used to generate Travelling-Salesman tours in the presence of time-window constraints

program that was used to solve Travelling-Salesman Problems in the presence of time-window constraints.

6.4.2. The Computer Program used to Investigate the Impact of Time-Windows on Distribution Costs

A full description of the steps involved in this program is given by the Activity Sequence Diagram of Figure 6.2.. Simulation exercises, using this procedure, were repeated many times, with the main independent variable involved here, r, being incrementally increased, in order to test the hypothesis that Total Distribution Cost is inversely related to time-window width.

The output generated by this program is presented and discussed in Chapter 7, which also compares the associated findings with those derived using similar methods and algorithms to those described in the current chapter.

CHAPTER 7

THE RELATIONSHIP BETWEEN TIME-WINDOW CONSTRAINTS AND DISTRIBUTION COSTS

The program described in Chapter 6 was designed to produced estimates for fleet size, Total Fleet Mileage and the average length of each round-trip; the figures reproduced in this chapter, for each of these variables, represent average values derived from a number of simulations performed for each value of r. This information was later used to calculate Total Cost per week, again for each value of r.

7.1. The Results of the Simulation Exercises

Two of the major determinants of the Total Cost of an operation are the number of vehicles that are used to make a delivery, and the total mileage covered by the fleet; Figures 7.1. and 7.2., (SEE also Table 7.1.), show the relationship of both of these variables with r. For both of these graphs, and for all subsequent results, (unless otherwise stated), it was assumed that P=100, x=0·75 tons, 1=30 minutes and the maximum length of a working-day is 540 minutes.

Not surprisingly, in view of the fact that Total Fleet Mileage is itself closely related to fleet-size, the distribution shown in these two Figures are very similar in shape. The hypothesis that the number of vehicles required will increase as time-window constraints become more stringent is accepted, with Figure 7.1. showing that this increase in fleet-size accelerates as the value of r approaches 1 minute, the smallest width of time-window considered, here. What is also apparent is that, using the technique developed in Chapter 6, the reduction in n as timing constraints are relaxed stops when the value of r increases to approximately 250 minutes, at which point 9 vehicles appears to be the minimum fleet-size required, given the assumptions outlined above. This bottoming-out of Figure 7.1.'s curve as time-windows become larger contributes to the geometric shape of the relationship between fleet-size and time-window width. Similar comments may be made in connection with Figure 7.2., which plots the average Total Fleet Mileage figures derived from the simulations, although the curve shown here is noticeably shallower than that of Figure 7.1., and appears not to level out until r increases to around 350 minutes.

A parameter that is related to both Total Fleet Mileage and fleet-size is d, the average length of a vehicle-trip; the way in which this variable behaves as time-window width changes is illustrated by Figure 7.3., (SEE also Table 7.1.). The immediate impression given by this graph is that d is, generally, constant for values of r greater than 60 minutes, although closer inspection reveals that there is a very slight tendency for average trip-length to decline as r increases.

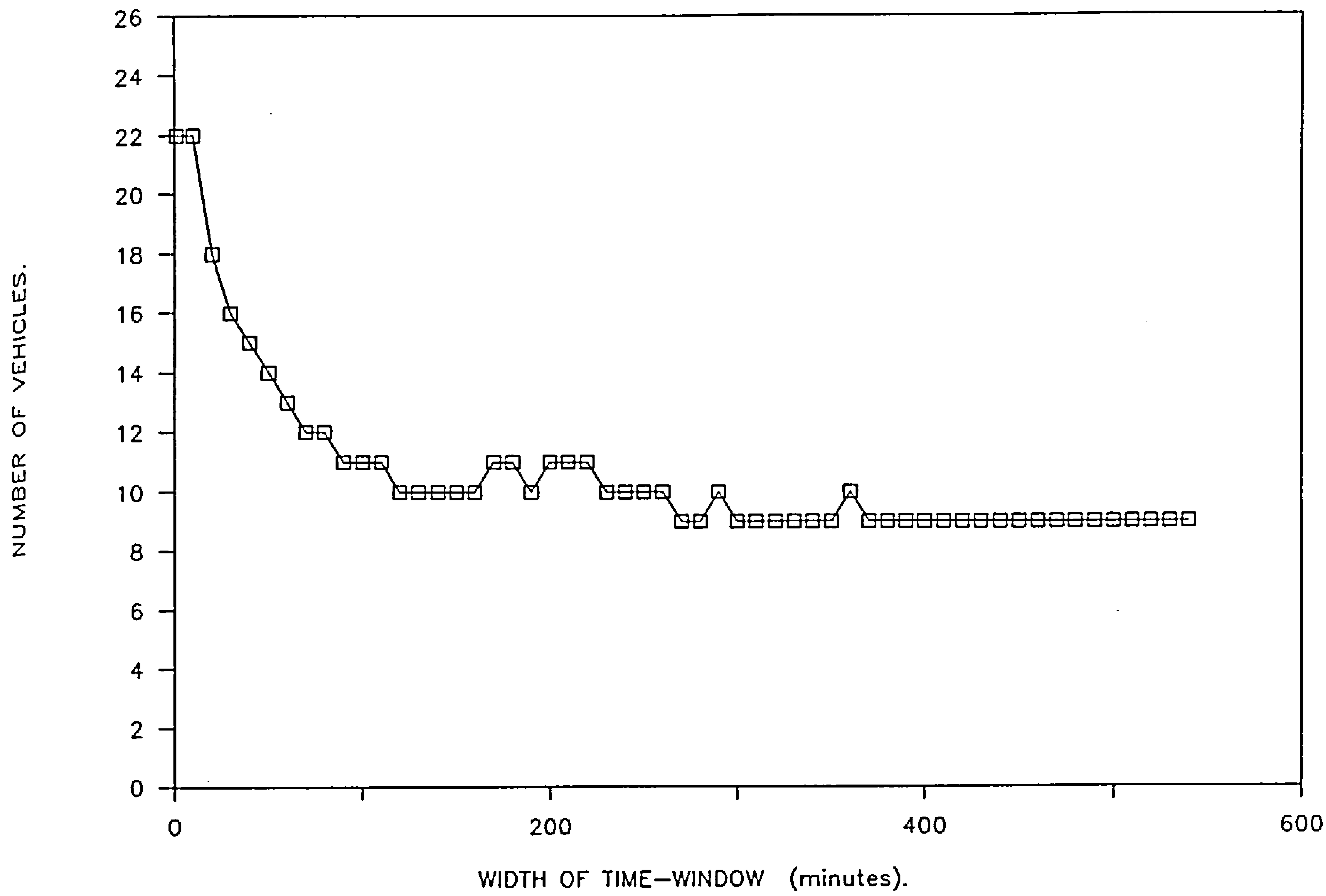


Figure 7.1. The effect of time-windows on the number of vehicles required. (P=100, a=1 (square))

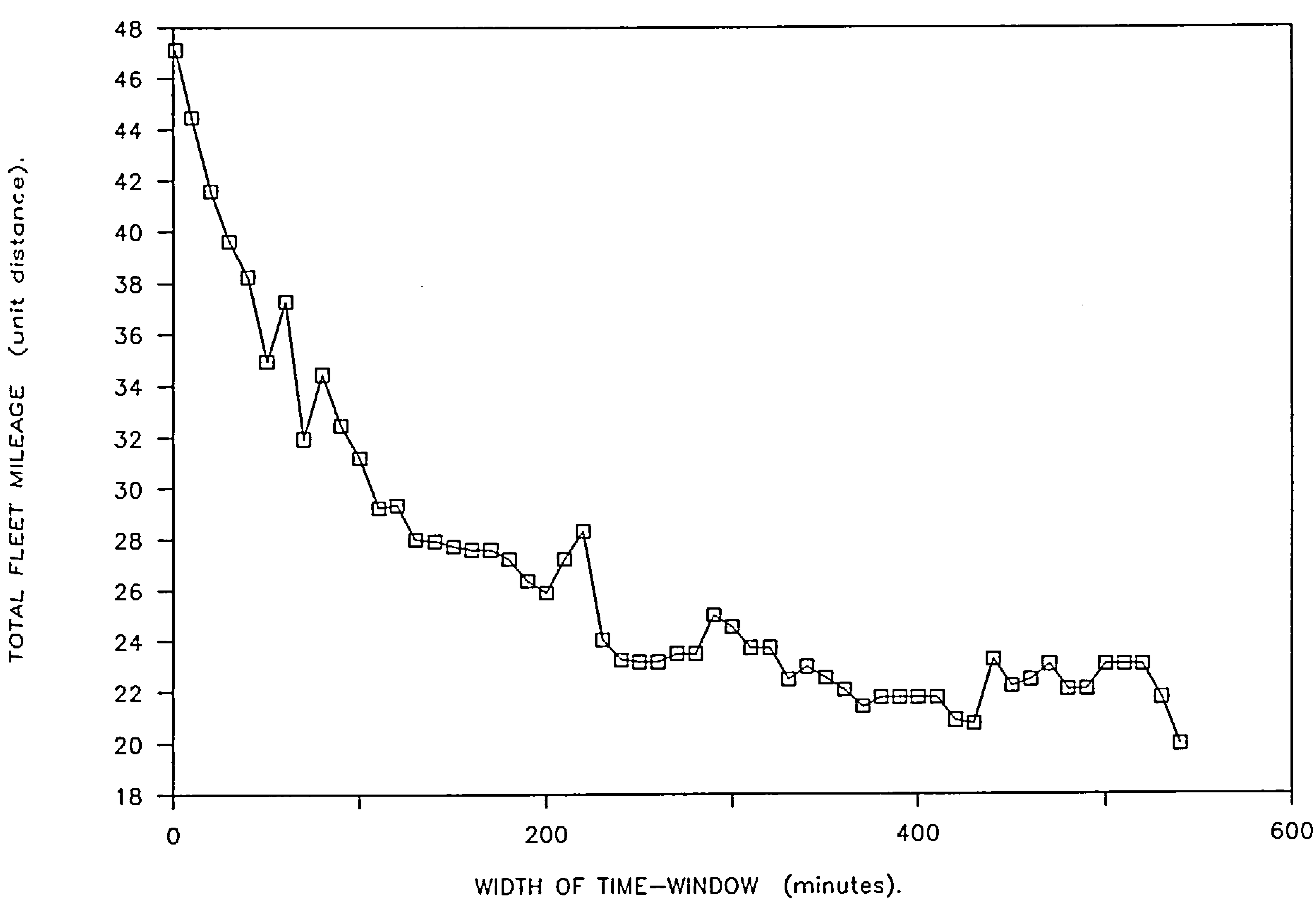


Figure 7.2. The effect of time-windows on Total Fleet Mileage. (P=100, a=1 (square))

For narrower time-windows, however, the general trend is for d to decrease as timing constraints tighten; in terms of the independent variable, r, this fall in d coincides with the portion of Figure 7.1. in which fleet-size rises rapidly. In other words, this reduction in the average length of a round-trip as the value of r declines from about 60 minutes to 1 minute occurs because the corresponding increase in fleet-size, shown in Figure 7.1., is steeper than that of Total Fleet Mileage, (SEE Figure 7.2.).

A clearer picture of the processes that are in operation when time-windows are imposed, is provided by Figure 7.4., which shows the behaviour of the variable "i_d", representing the average distance that is travelled between successive stops, (including the depot), in response to changes in r. Clearly, i_d also increases as time-windows become narrower, especially when the value of r is very small.

The parameter i_d should not be confused with "i", which is used in Chapter 4, (SEE Section 4.3.2., particularly Equation E.4.10.); the latter variable represents the average length of the trips made between customer-locations, so that stem-journeys are excluded. As Stem Distance is unknown in the present context, i_d represents the average length of each of the (C+1) between-customer links which go to make up Delivery Distance, and may be written as,

$$i_d = \frac{d}{(C+1)} \qquad \text{(E.7.1.)}$$

The evidence of Figures 7.1. to 7.4. suggests that the observed increase in Total Fleet Mileage as time-windows become narrower is due mainly to the rise in the number of round-trips required, rather than to increases in the average length of a round-trip. The value of Figure 7.4. is that it shows the way in which the average spacing between successive stops in a vehicle-tour increases as the value of r is reduced, although, at the same time, fewer locations are visited each day, so that the average length of a vehicle-trip is actually decreased.

It should be emphasised that parameters such as C and i_d are average figures for each set of vehicle-tours, and that both will vary from vehicle to vehicle. This is because of the myopic nature of the Shortest-Path algorithm that is used to generate vehicle-tours, (SEE Section 6.4.1.), whereby the first tours to be constructed include as many customer-locations as it is possible to visit before the end of the working-day, so that, as successive tours are built, fewer and fewer locations are available. By definition, therefore, the density of the set of customers that require a visit is lowered as each tour is constructed, which, in turn, increases the i_d-value for each successive vehicle. Towards the end of the Routing

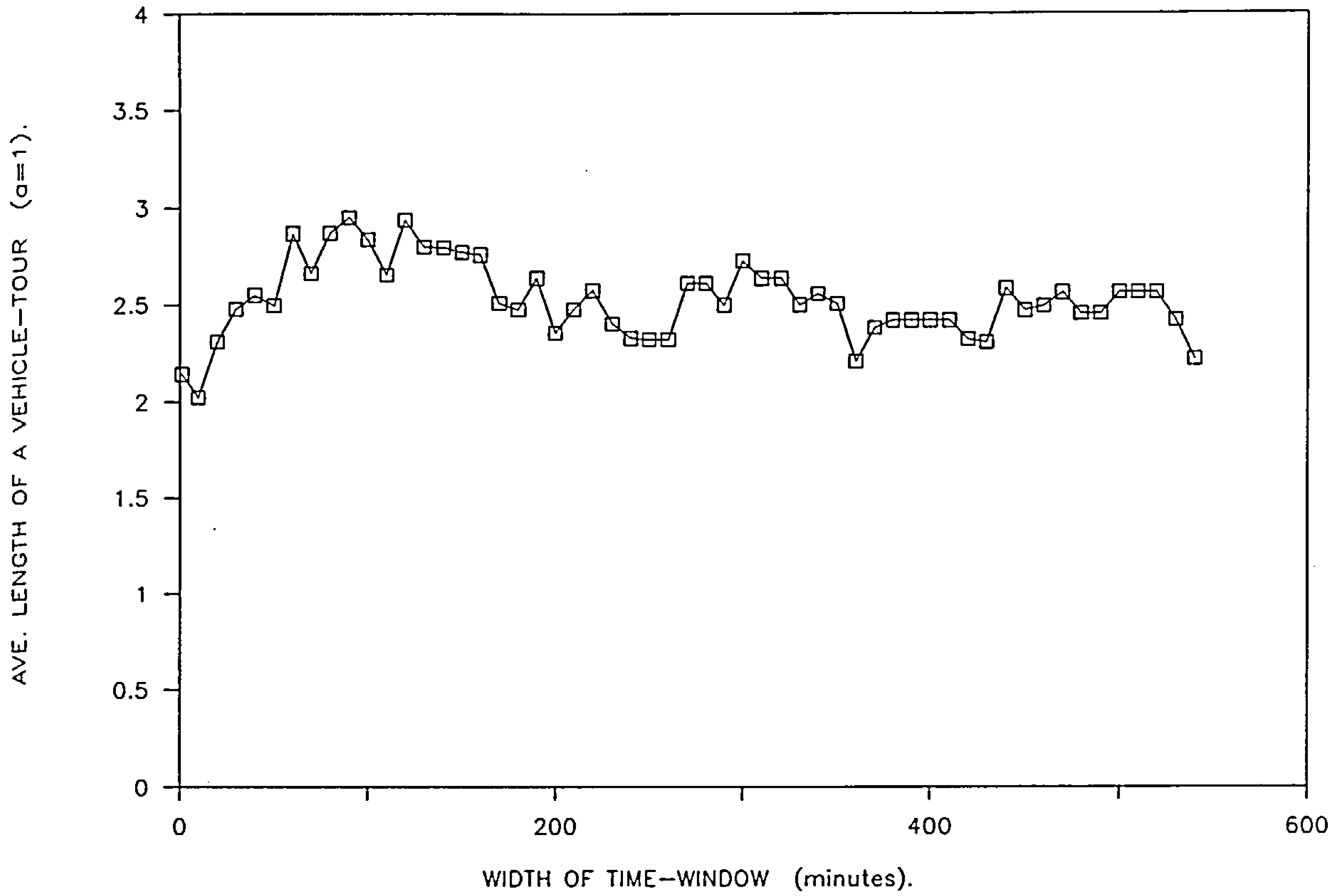


Figure 7.3. The effect of time-windows on average vehicle-tour length. (P=100, a=1 (square))

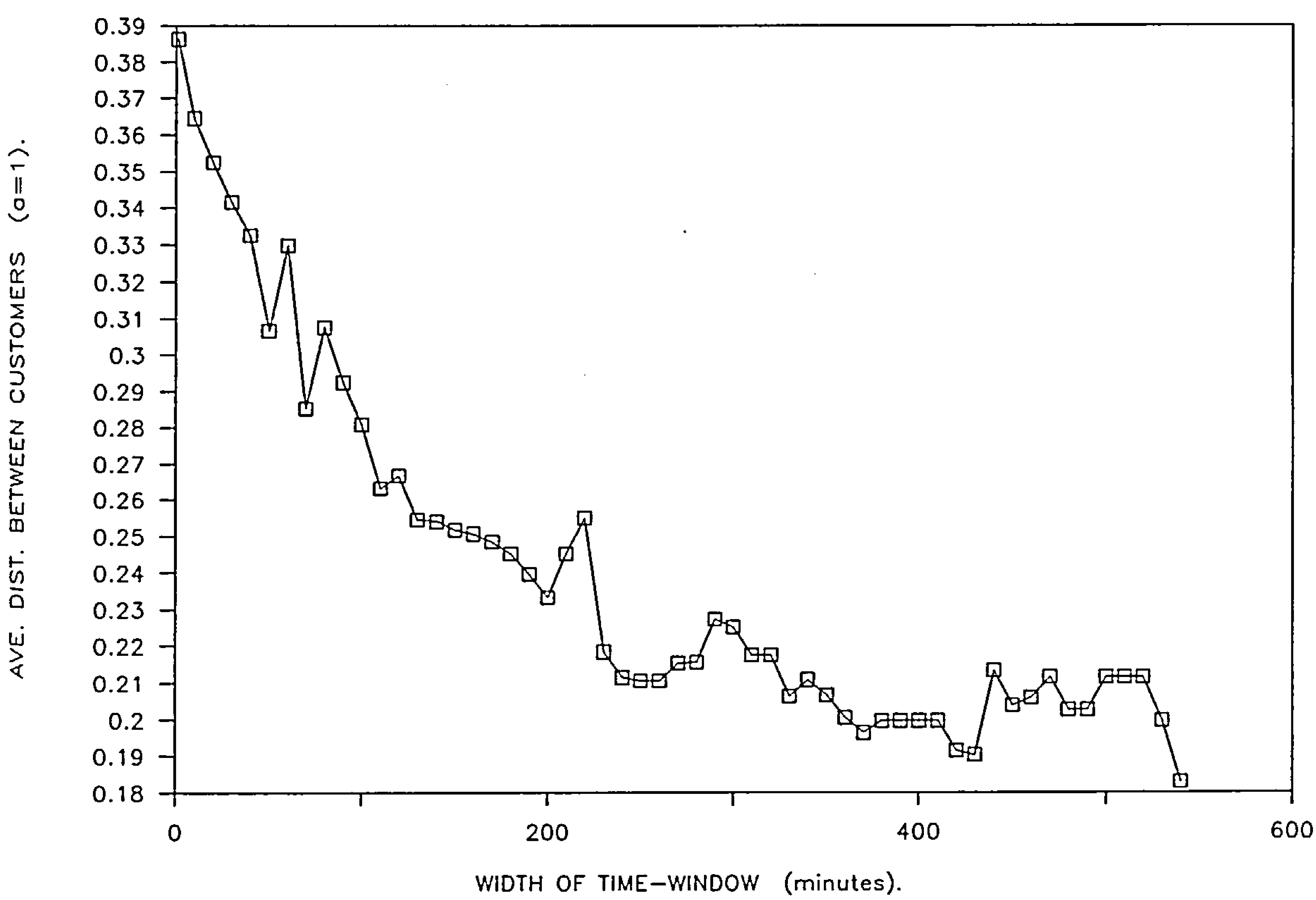


Figure 7.4. The effect of time-windows on the average distance between consecutive customers. (P=100, a=1 (square)).

& Scheduling process, it is possible that the locations remaining to be served may be rather dispersed in terms of their spatial location and/or the time at which they would receive a delivery. Therefore, the number of customers visited, and the average distance that is travelled between each of them, may vary greatly between the first and the last vehicle-tour to be constructed. Again, this problem of having to operate one or more vehicles that are under-utilised, in as much as they serve only a few customers in a daily round-trip, is likely to be exacerbated as the value of r is reduced; this problem is, perhaps, the major drawback associated with the myopic approach to constructing tours that is used here.

In order to test these hypotheses about such variations within each set of vehicle-tours, the program that is described by the flow-diagram of Figure 6.2. was adjusted so that it printed out the number of locations that are visited in each vehicle-tour. From this information, it is possible to produce a set of histograms, for each value of r, which illustrates the decline in the number of customer-locations that are visited, from the first tour to be constructed in each iteration, to the last; an example of six of these histograms is presented in Figure 7.5., (ie. Figures 7.5.1. to 7.5.6.). In each case, the columns of each diagram are arranged in the order in which the relevant vehicle-tour was constructed, the first tour being at the left-hand end of the x-axis. This set of diagrams, providing an example of the structure of one day's tours, for each value of r, confirms both the tendency for the tours formed first to include the most customer-locations, and for this tendency to be accentuated as time-windows become more stringent. In addition, Figures 7.5.4., 7.5.5. and 7.5.6. in particular highlight the problem of the last vehicle to be scheduled having to visit far fewer locations than the average for the fleet, C, even when the time-window constraints imposed are not particularly restrictive.

The general effect of subjecting a fleet of vehicles to increasingly stringent time-constraints, therefore, is to increase the average distance that must be travelled between consecutive deliveries, which, given that there is a limit imposed on the length of a working-day, reduces the number of customers that may be served each day. The consequent increase in the number of vehicles required increases Total Fleet Mileage, primarily, as Chapter 4 demonstrates, due to the extra stem-journeys that are made, but also because of the extra Delivery Distance that is travelled per customer.

All of the results discussed so far have been produced using an algorithm that constructs tours on the basis of the minimisation of the Generalised Cost of visiting each successive location; it is logical, therefore, for the next stage of the analysis to be that of investigating the effect of time-windows on Total Cost. All distance figures quoted

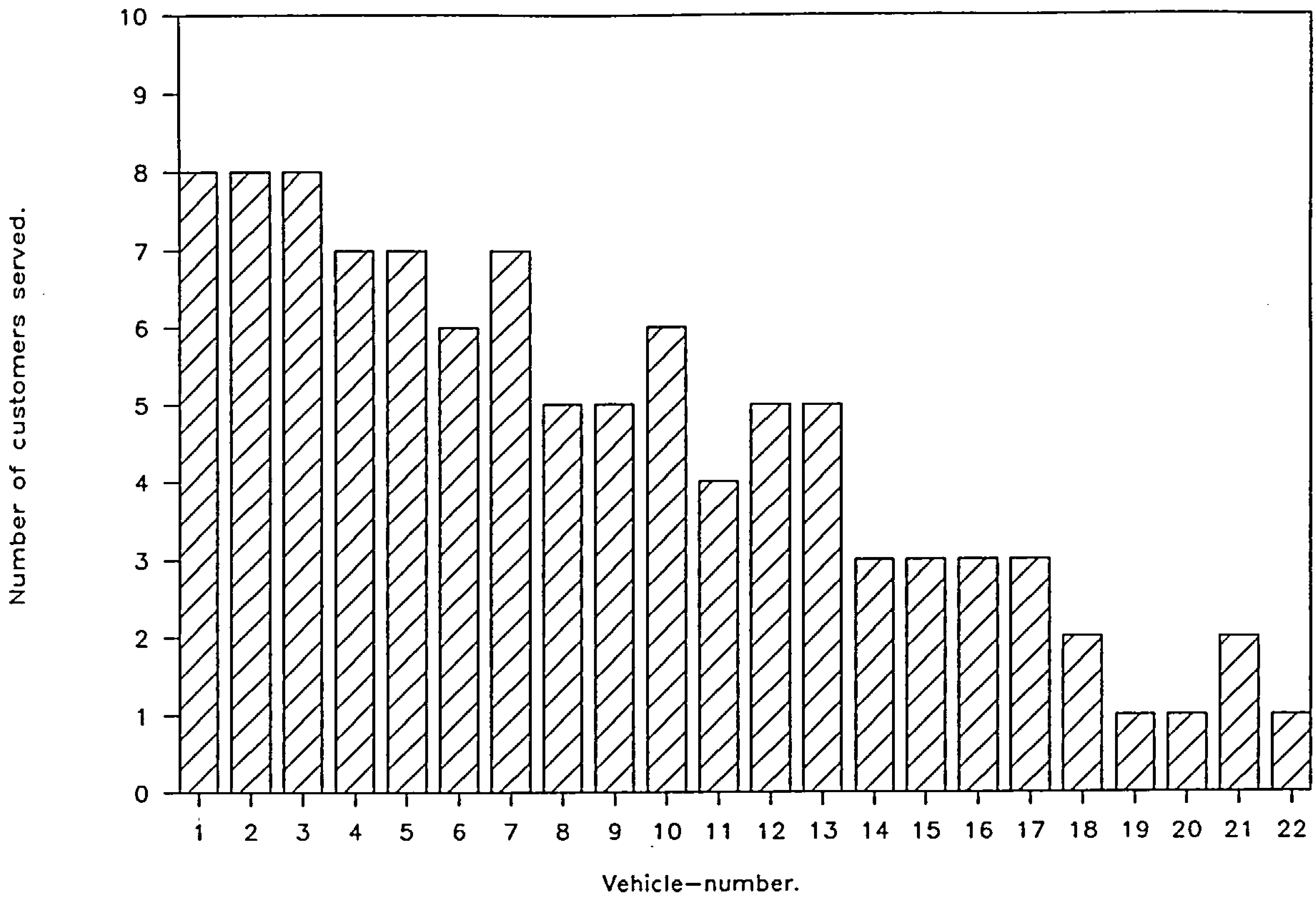


Figure 7.5.1. The number of customers served per vehicle-tour. (r=1)

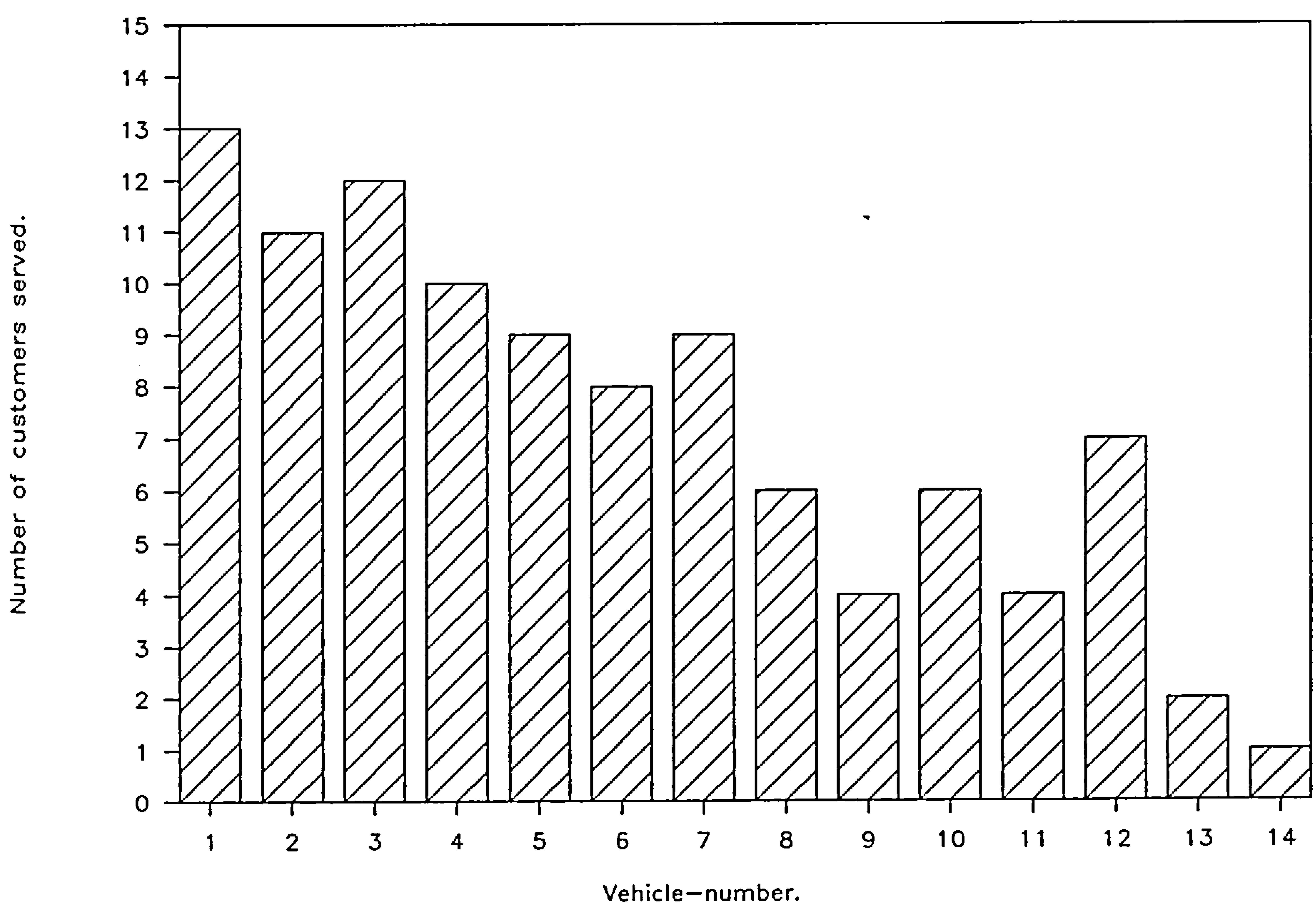


Figure 7.5.2. The number of customers served per vehicle-tour. (r=50)

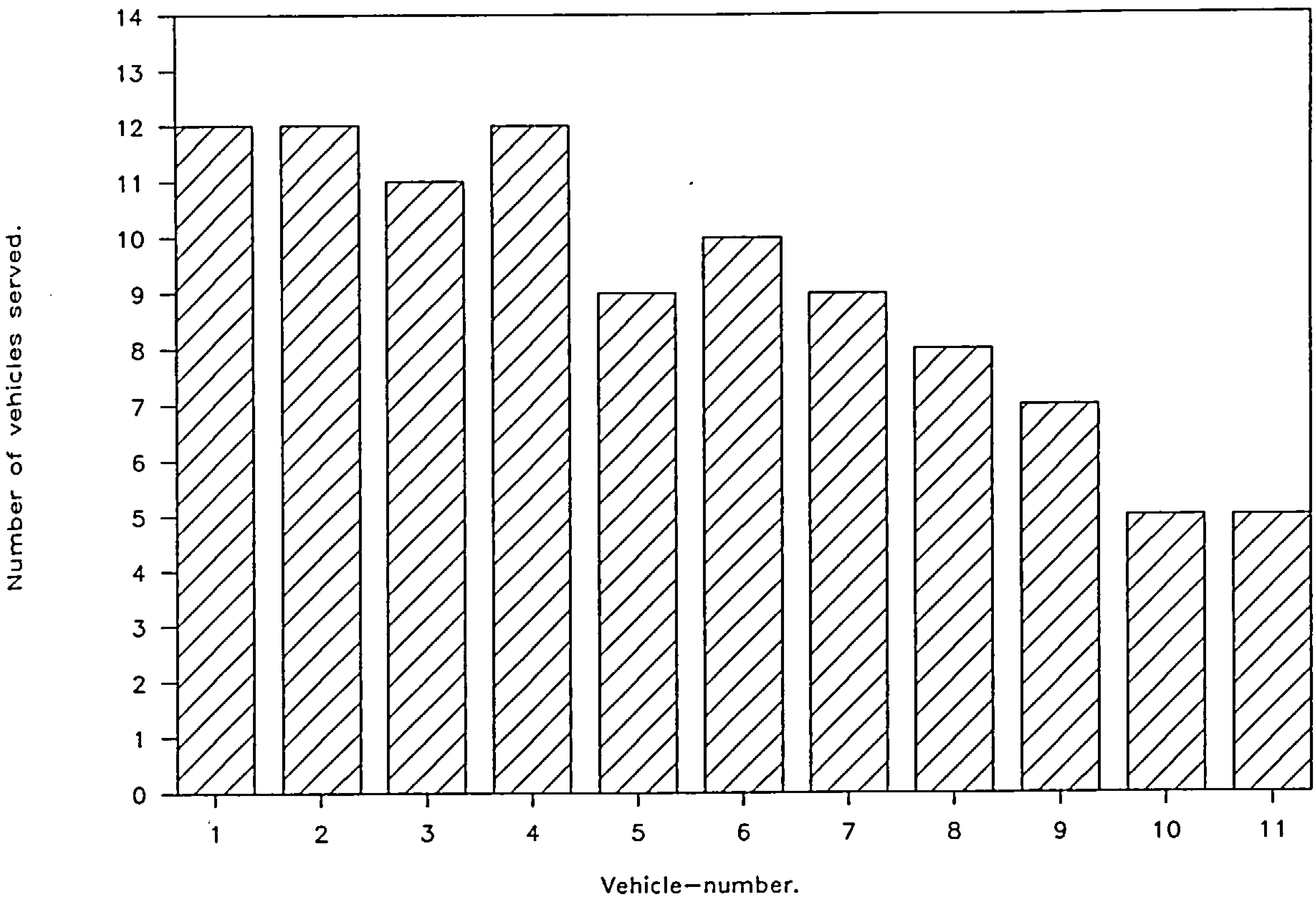


Figure 7.5.3. The number of customers served per vehicle-tour. (r=100)

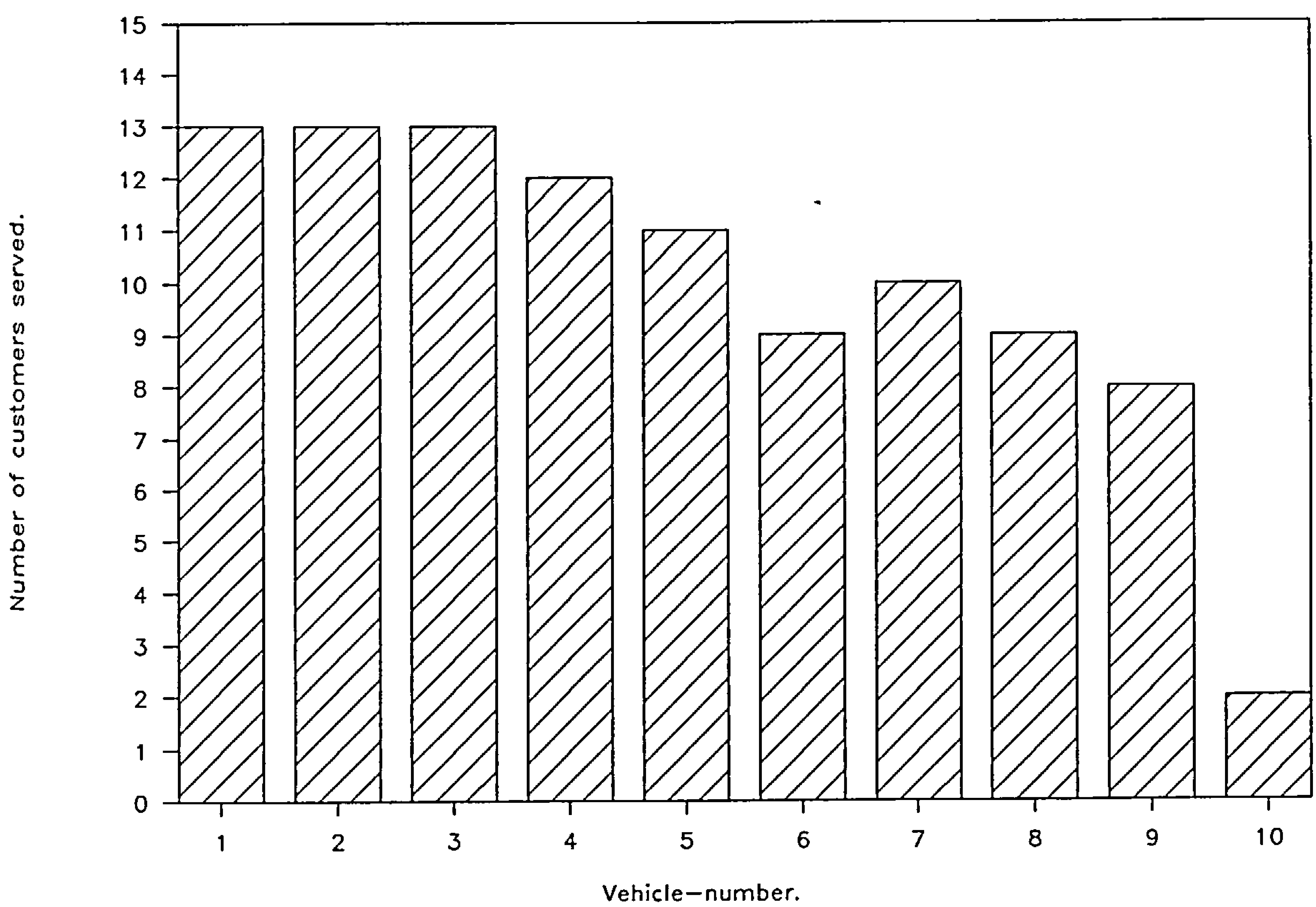


Figure 7.5.4. The number of customers served per vehicle-tour. (r=150)

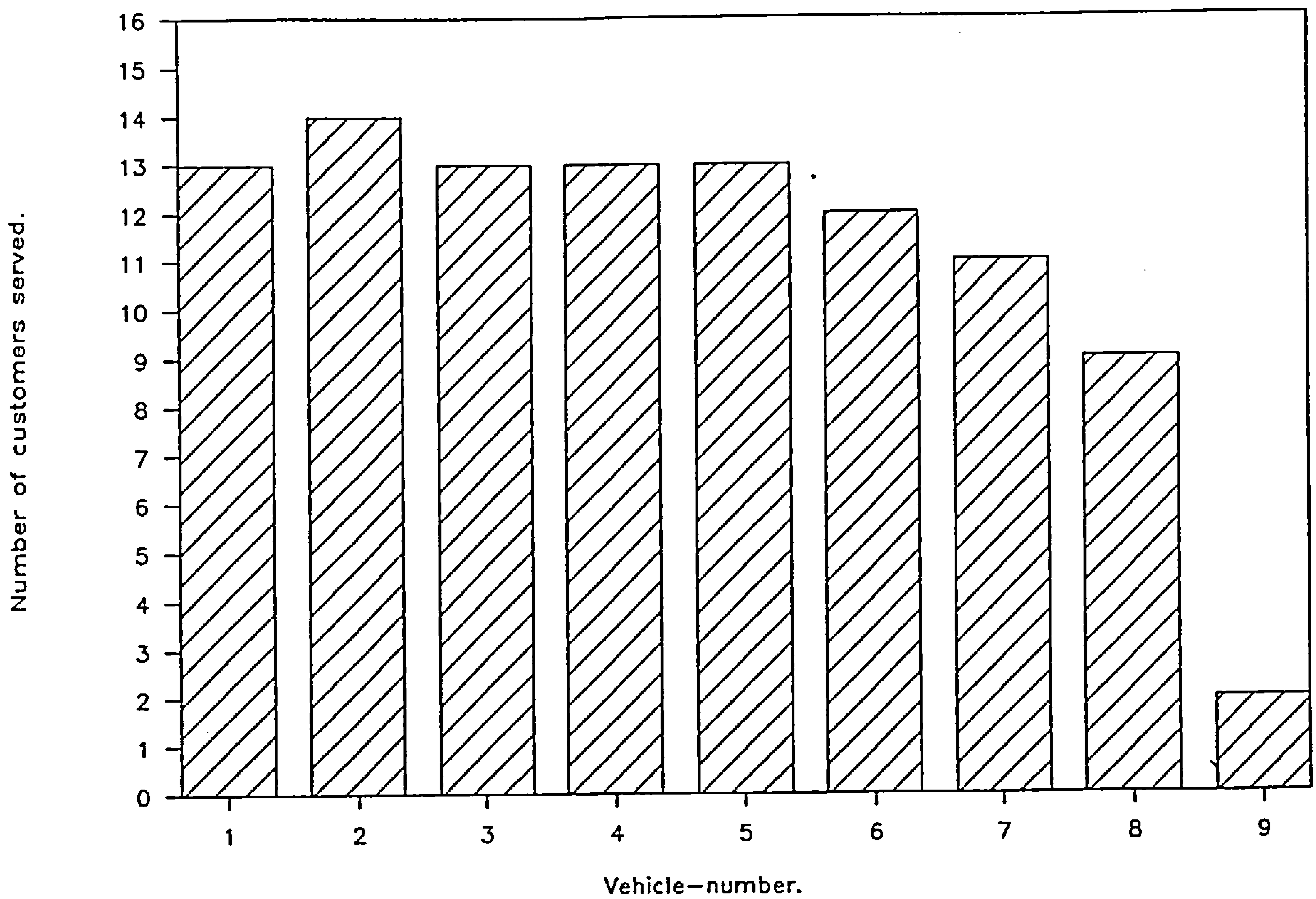


Figure 7.5.5. The number of customers served per vehicle-tour. (r=350)

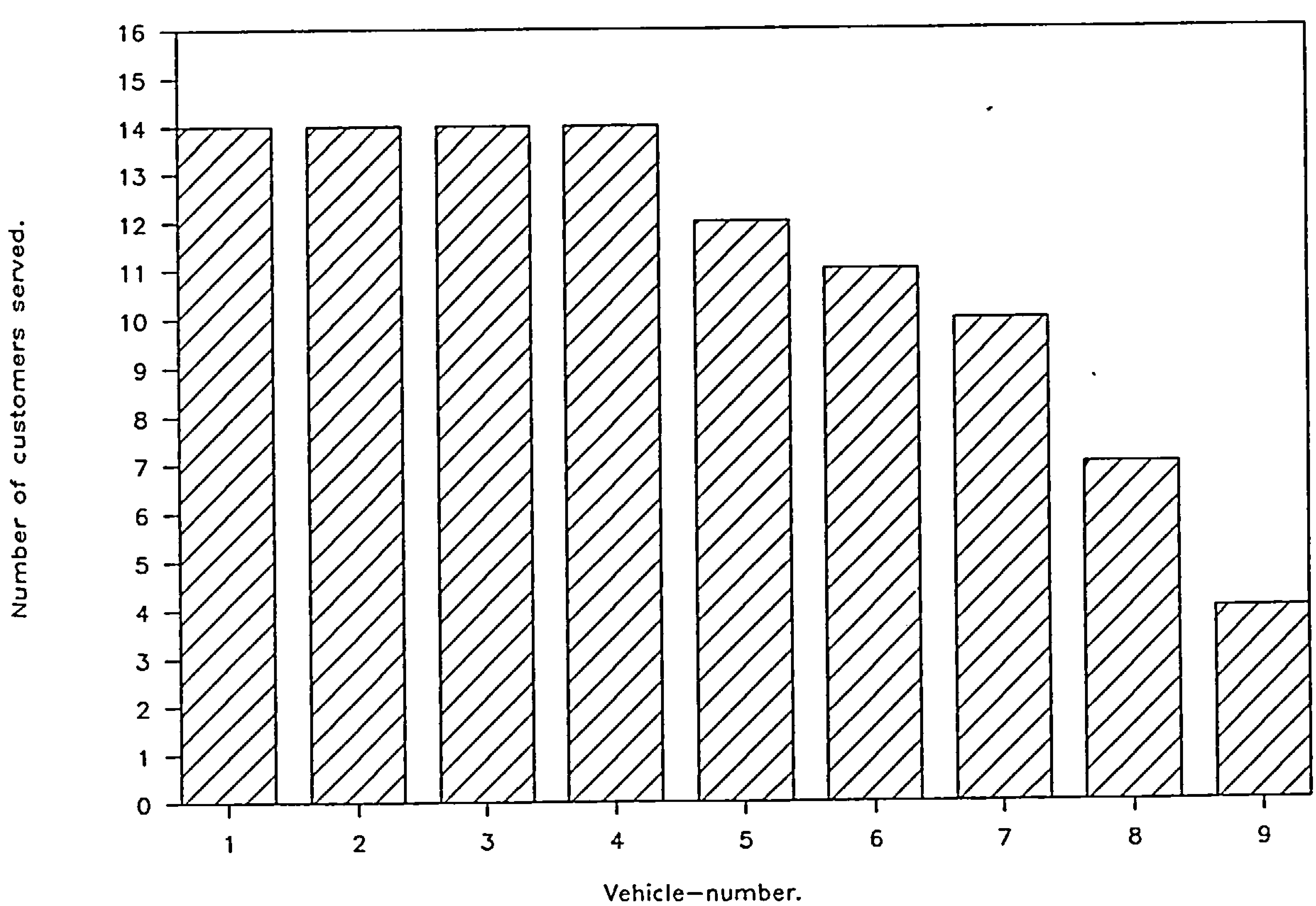


Figure 7.5.6. The number of customers served per vehicle-tour. (r=540)

above are in the form of "unit distance", so that "a", the length of one side of the square delivery-zone, is effectively 1. In order to present realistic cost-estimates, it is now necessary to make assumptions about the size of this area; for the purposes of the current analysis, it is assumed that a=50 miles. Again Commercial Motor cost-tables were used as a basis for these estimates, (SEE Appendix A), the key figures required being the Standing Cost and Running Cost of a 0·75-ton van, (which are £119·80 per week, and £0·1801 per mile, respectively). To get a weekly estimate of Running Cost it is merely necessary to multiply this figure by the appropriate figure for Total Fleet Mileage shown in Table 7.1., then by 50, (the value of "a"), and, finally, by 5, (the number of days in a working-week). The resulting figures for Total Cost per week as a function of r are also presented in Table 7.2., and plotted in Figure 7.6..

Again, a curved distribution is shown; with Total Cost per week increasing most rapidly when time-windows are narrow and almost constant when time-windows are more than 300 minutes in width. This graph bears a close resemblance to that of Figure 7.1., which illustrates the relationship between fleet-size and r.

Because of the consistently curved nature of the distribution shown in Figure 7.6., it is not difficult to fit a regression-line to these points, although, due to the geometric relationship between Total Cost and time-window width, this may only be achieved after the Total Cost figures contained in Table 7.1. are subjected to a logarithmic transformation. This is the same technique that is used in Chapter 4 for deriving expressions for Stem Distance and Delivery Distance as a function of fleet-size, etc.. Figure 7.7.1. shows the transformed version of Figure 7.6., and confirms that the relationship between the logarithms of TC and r is more or less linear; the regression-line which describes these points may be described by the following expression,

$$\text{Log. } TC = 3{\cdot}7 - 0{\cdot}177 \text{ Log.} r$$

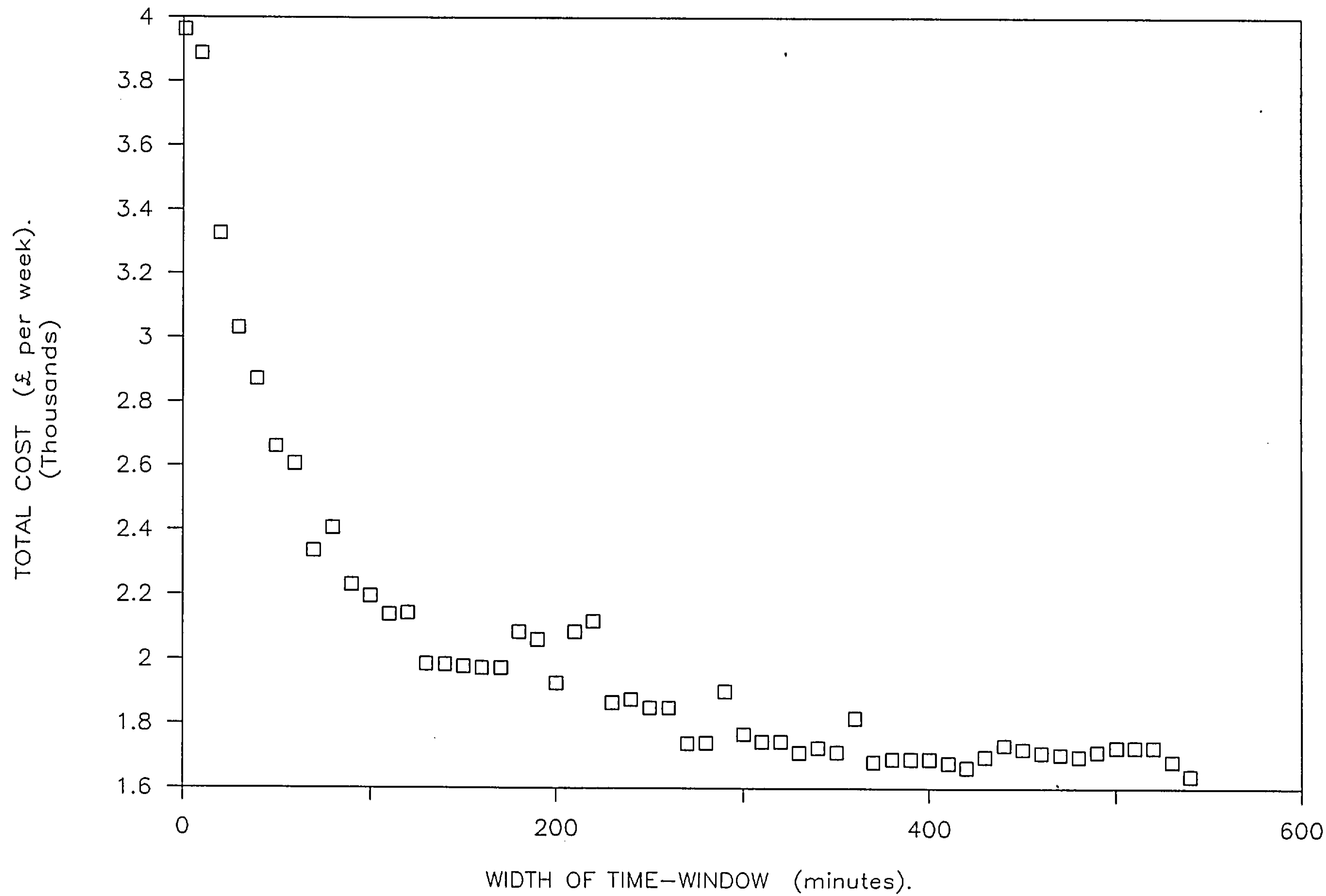
$$(R^2 = 0{\cdot}923)$$

or,

$$TC = 5012.r^{-0{\cdot}177} \qquad \text{(E.7.2.)}$$

Equation E.7.2. may be used as a predictive tool, and Table 7.2. compares the estimates of Total Cost as a function of time-window width, derived from this expression, with the Total Cost figures that are already presented in Table 7.1.. To give an indication of the degree of correlation that exists between observed and predicted figures for Total Cost, Figure 7.7.2. reproduces the distribution of points shown in Figure 7.6., and superimposes the regression-curve that is calculated from Equation E.7.2.; this close correlation is confirmed by the R^2 value of 0·923.

Figure 7.6. The effect of time-windows on Total Cost per week.



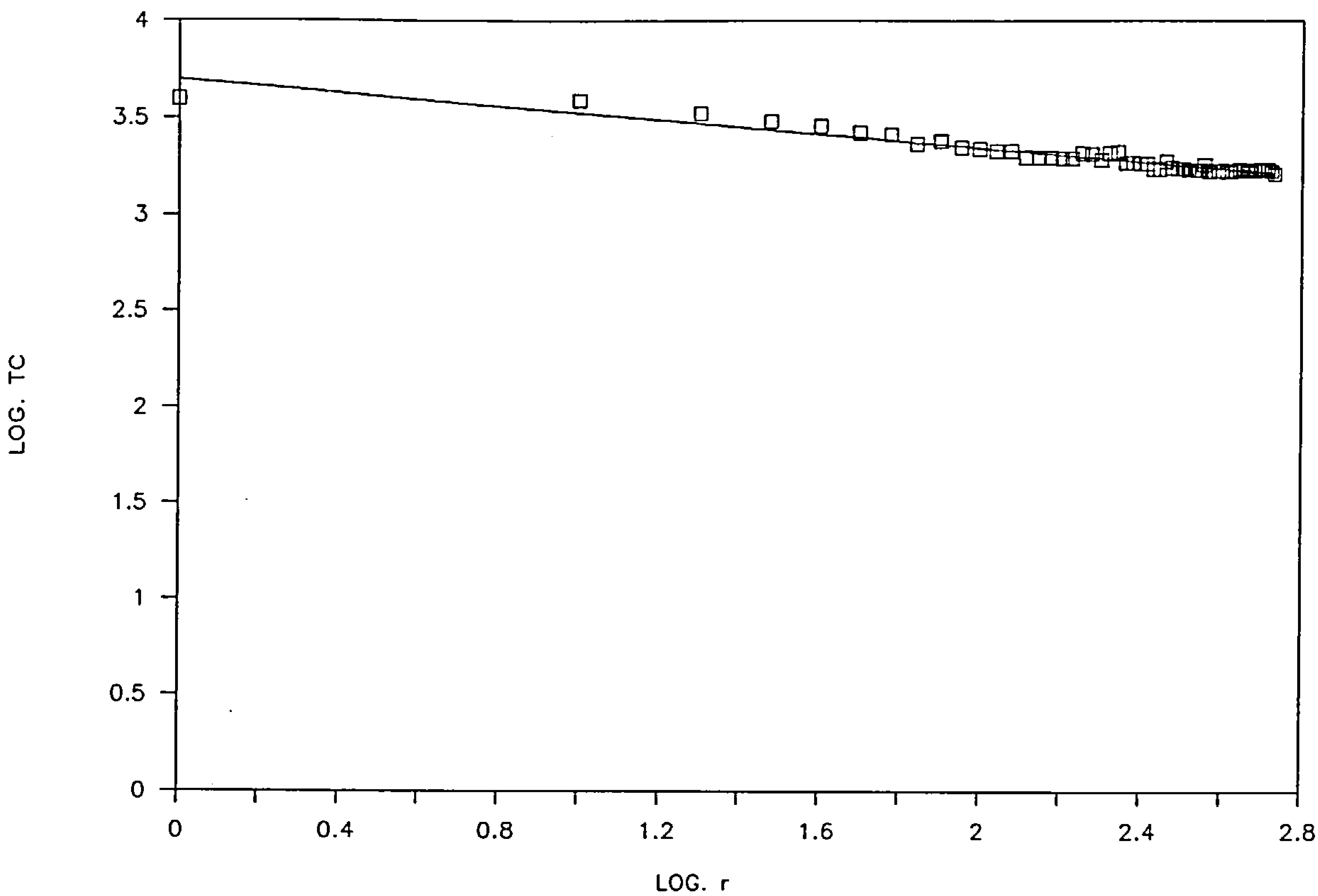


Figure 7.7.1. Logarithmic regression: Total Cost/r.

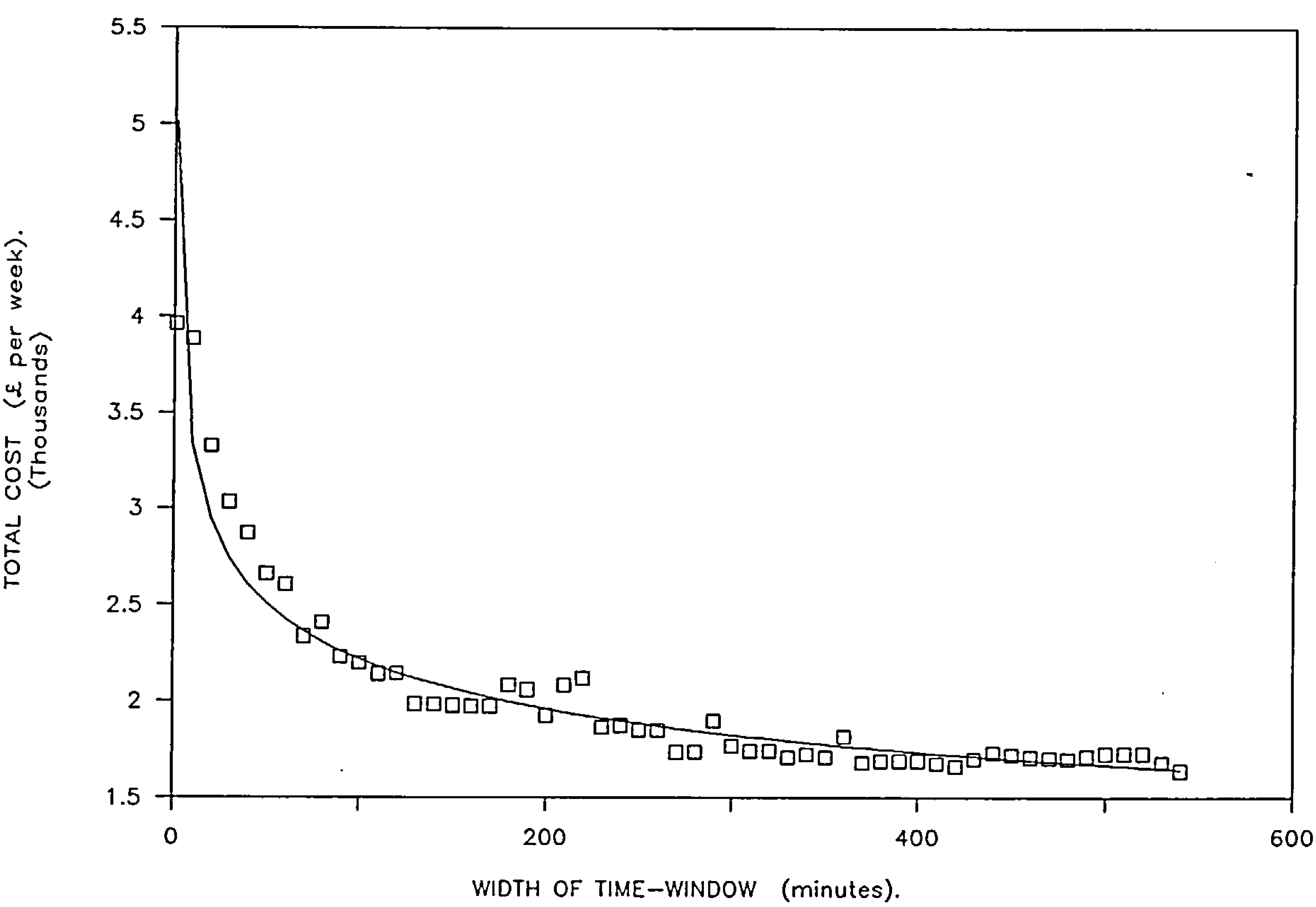


Figure 7.7.2. Observed and predicted Total Cost figures, as r changes.

A similar exercise may readily be carried out in order to quantify the relationship between r and fleet-size, since the curve shown in Figure 7.1. is very like that of Figure 7.6., although the numerical details of regression equation that might be associated with this graph are of limited importance here; what is of value, though, is the form of the distributions produced as a result of these simulations. Although the figures contained in Tables 7.1. and 7.2. are greatly influenced by the assumptions that have been made, and by the problem formulation that has been adopted for the purposes of the current analysis, there would seem to be no reason to believe that a geometric relationship between the severity of time-window constraints, (represented by r), and distribution costs, similar to that shown in Figure 7.6., would not be found under similar conditions. It should be noted, however, that the shape of the Total Fleet Mileage curve, shown in Figure 7.2., cannot be taken to represent the way in which increasingly stringent time-constraints cause Travelling-Salesman solutions to deteriorate from the optimum; this is because the simulation procedure summarised in Figure 6.2. is founded upon a tour-building algorithm, (SEE Figure 6.1.), which is not guaranteed to produce shortest-path solutions in the absence of time-windows. It is more appropriate, therefore, to state that the relationships described by Figures 7.1. to 7.5. may be partially a reflection of the way in which the performance of the algorithm used to generate vehicle-tours is affected as timing constraints become more restrictive. Because of the acknowledged importance of the algorithm used for the results of the entire simulation exercise, the following section describes two alternative techniques that could have been used in this analysis, and compares the output produced from them with the results described above. Meanwhile, Sub-section 7.1.1. discusses the results of sensitivity analyses carried out on the simulation model.

7.1.1. Sensitivity analysis

There are two reasons for carrying out such an analysis on the simulation model's output: to test the sensitivity of the results produced to changes in the model's in-built assumptions, and to assess the extent to which there is variance between simulation runs.

In order to meet both objectives, completely new data were generated. Although there is no reason why these figures should differ from those already presented in this Chapter, it should be noted that subsequent graphs and tables in this section are not derived from the same set of results as Figures 7.1. to 7.6..

In the first of these sensitivity analyses, the effect of changing the time that must be spent at each customer location, (l), on both the number of vehicles and mileage required in the model's solutions, was examined. Having generated a fresh set of control data, (ie. with no changes to the simulation model's assumptions, so that l=30), for values of r from 1 to 540, (on average, 5 iterations for each r-value were made), the run was repeated with l fixed at 15 minutes and then 45 minutes. Obviously, an increase in the amount of time that has to be spent at each customer's premises will increase the number of vehicles that are required to perform the delivery task, and thus increase total fleet mileage; the objective of the sensitivity analysis is to ascertain the extent to which this is the case. Similarly, runs were made, (with l fixed at 30 minutes), with the length of the working day changed from 9 hours to 10 hours, and then to 8 hours. The effect of changing average vehicle speed or the size of the delivery-area would have virtually the same effect as altering the length of the working day, as such changes would merely reduce the number of locations that can be visited by each vehicle before having to return to the depot, and so the effect of changing these variables was not considered.

The results of the analysis are shown in Figure 7.8. and Figure 7.9., and indicate that neither altering the value of l nor the length of the working day by the amount mentioned above has very much effect on the total fleet mileage required, and the effect of extending or contracting the working day by one hour typically makes a difference of one vehicle either side of the figure produced by the "control run". Changing the value of l by 15 minutes does, however, have a far greater impact on the number of vehicles required for the delivery task, as Figure 7.8.1. clearly shows. The significance of these differences in fleet-size lies in the effect that the number of vehicles used has on the total cost of an operation; this is illustrated in Figure 7.10.1.. As with Figure 7.6., these data refer to the total weekly cost of 0.75-ton vans operating in a square delivery-area of 50 miles x 50 miles. Comparison with Figure 7.10.2 indicates that the effect, on the model's results, of altering drop-time by 15 minutes has a noticeably greater effect on the cost of the distribution task than changing the length of the working day by one hour.

The second reason for a sensitivity analysis on the model's results being necessary is the fact that there is variance between runs in the simulation process. The issue of the presence of this variance is discussed in full in Section 4.2.. To investigate the extent to which this might be a factor in relation to the results presented in this Chapter, the simulation program was run 50 times with r fixed at 1 minute, and then 50 times with r fixed at 10 minutes, and then 400 minutes. The dispersion statistics associated with this analysis are shown in Table 7.3., and the conclusion from these figures is that, after 50 runs, there is little variance from the mean. When r=1, for example, there is a 68.2% probability of the Total Fleet Mileage estimate generated by the model being within 2.481

Figure 7.8.1. The effect of changing drop-time on fleet-size

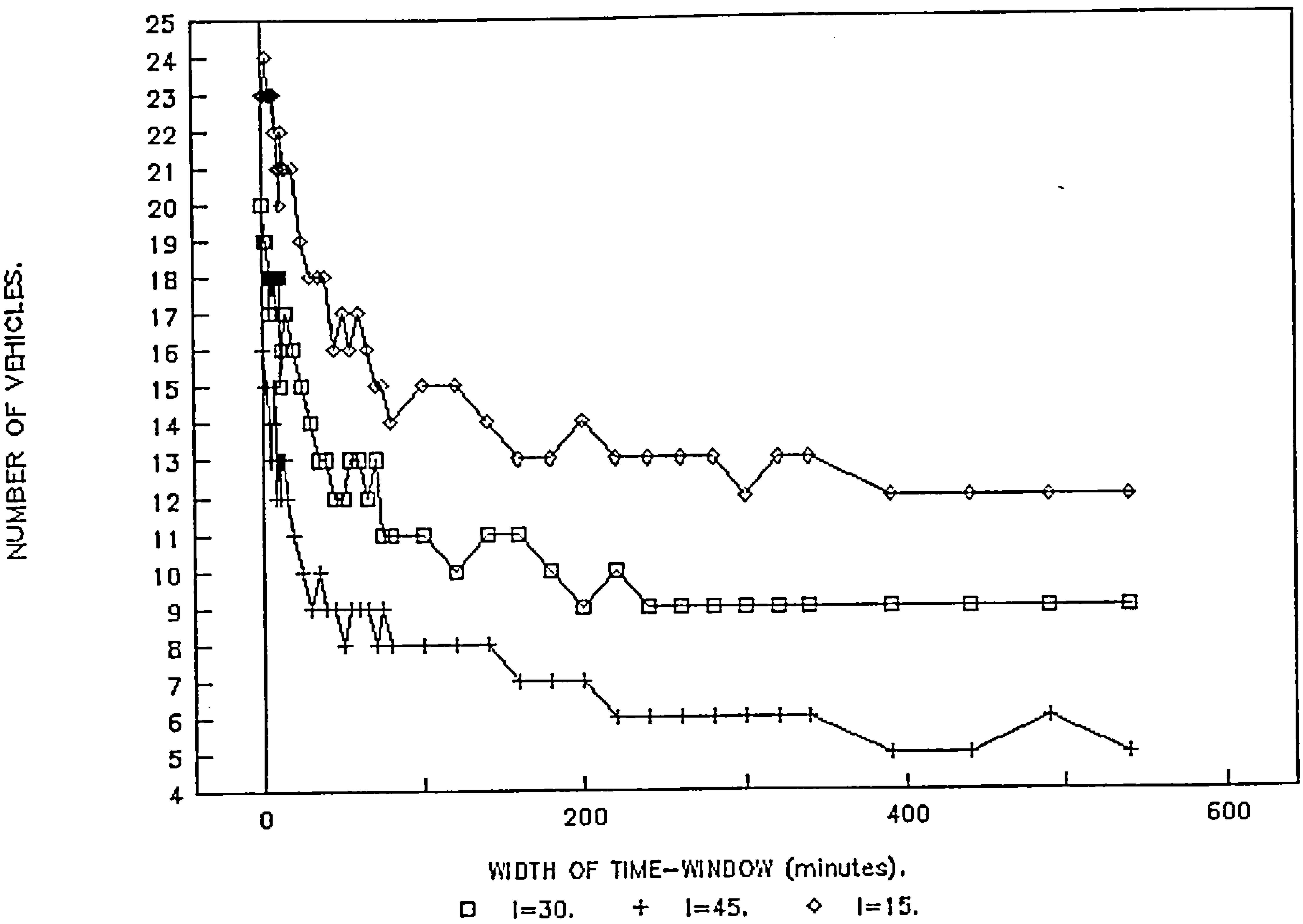


Figure 7.8.2. The effect of changing drop-time on Total Fleet Mileage

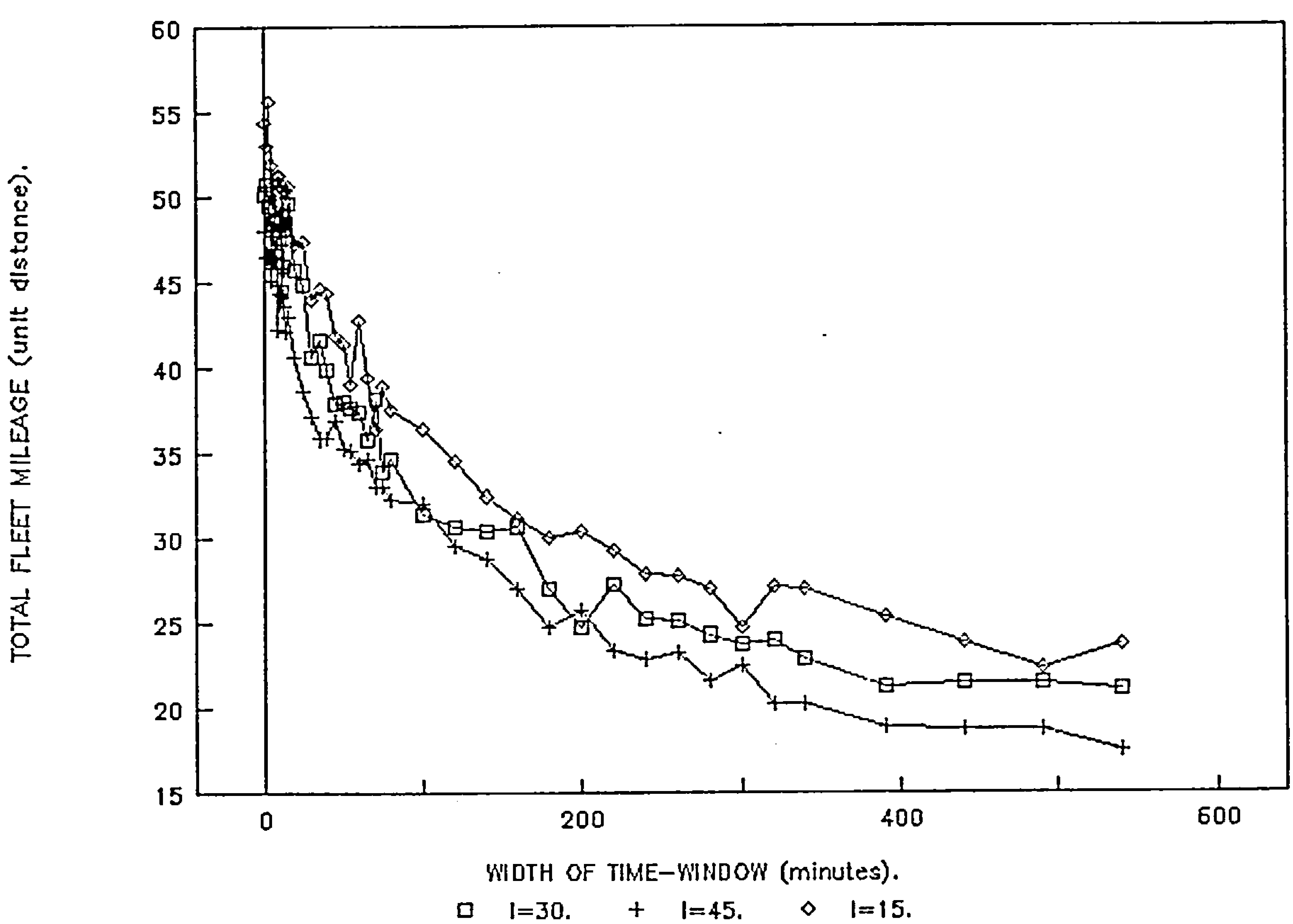


Figure 7.9.1. The effect of changing the length of the working day on fleet-size

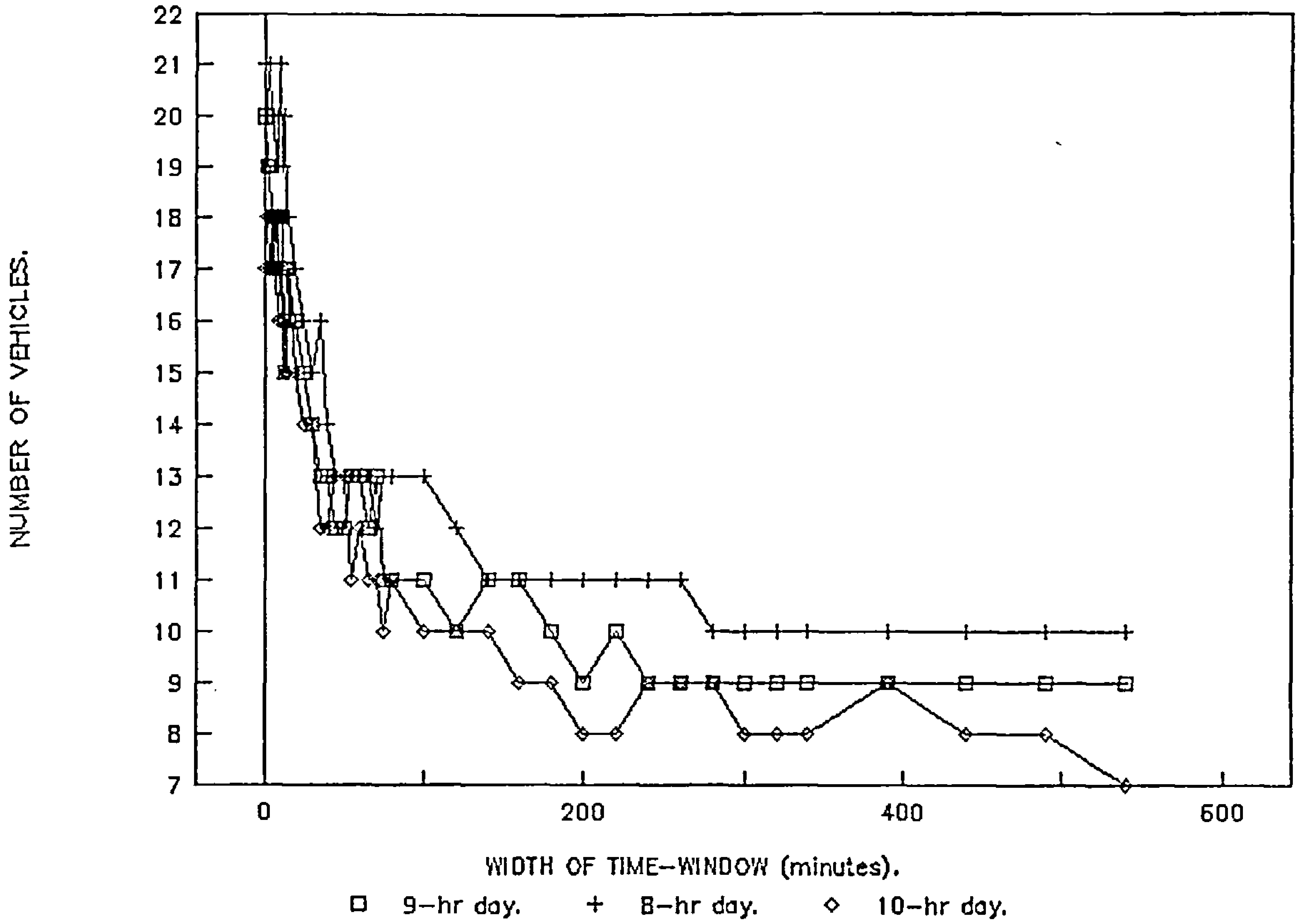


Figure 7.9.2. The effect of changing the length of the working day on Total Fleet Mileage

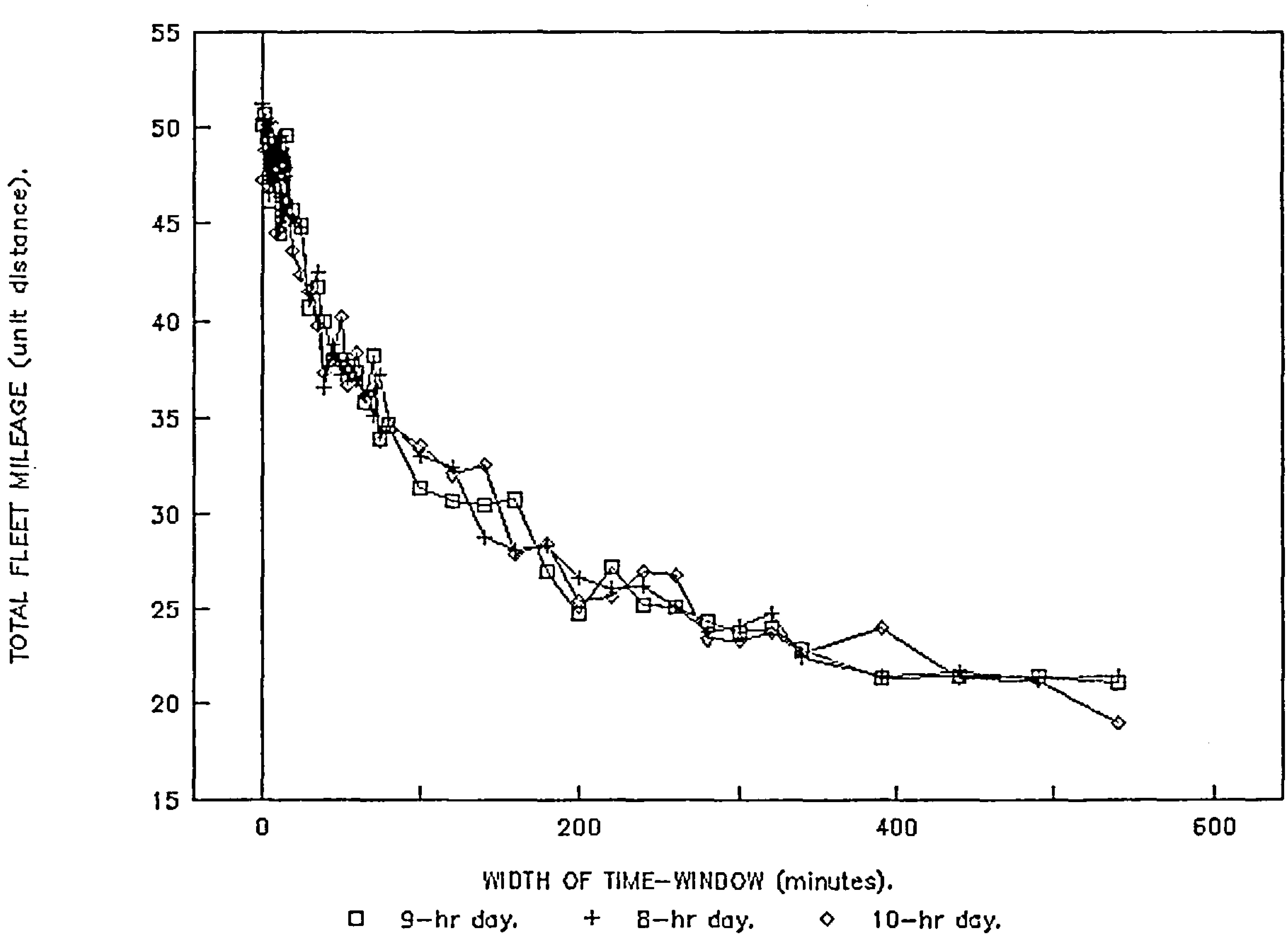


Figure 7.10.1. The effect of changing drop-time on Total Cost per week

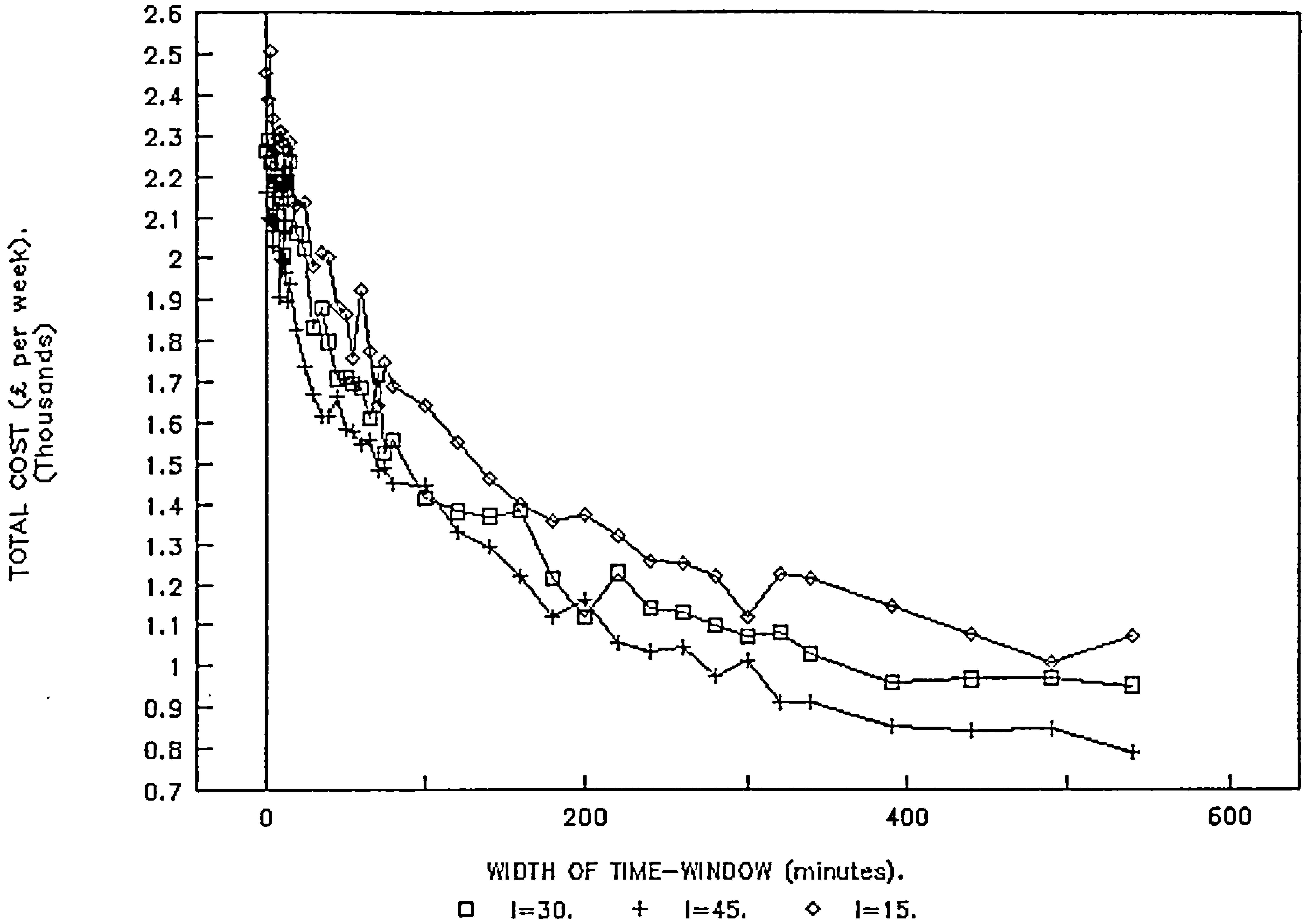


Figure 7.10.2. The effect of changing the length of the working day on Total Cost per week

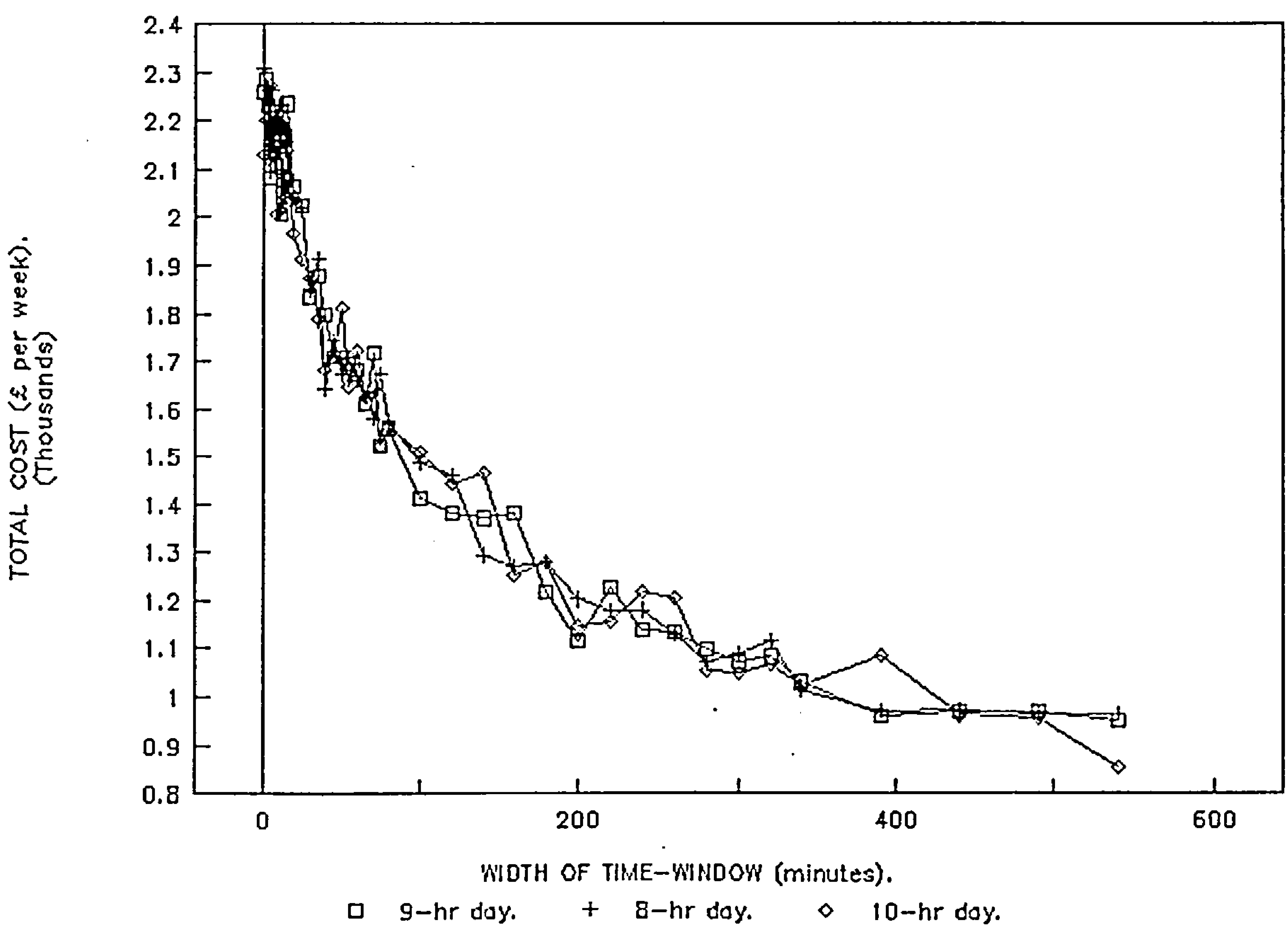
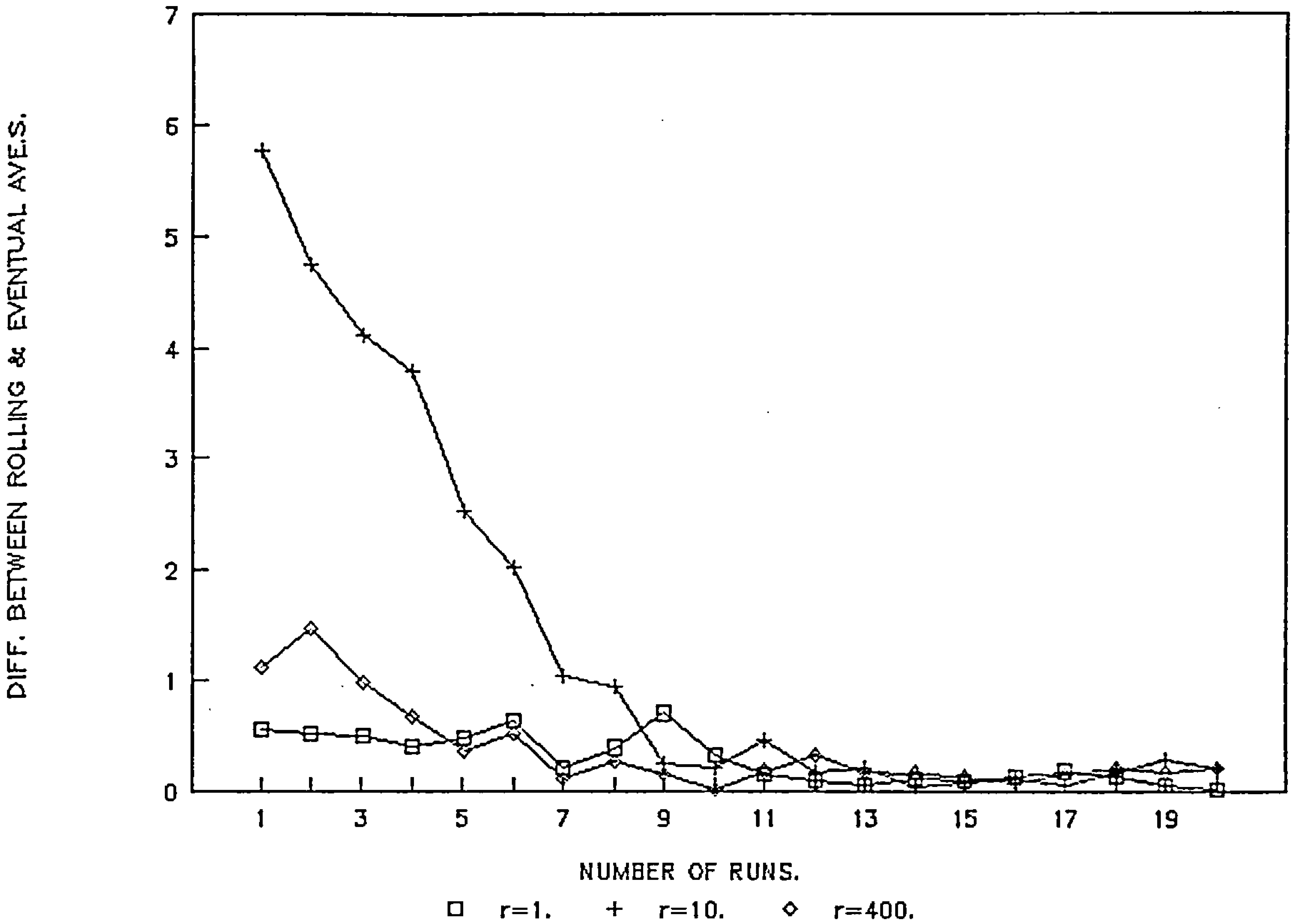


Table 7.3. The variance in the Total Fleet Mileage estimates of 50 simulation runs, (unit distance).

	r=1	r=10	r=400
Mean Value	50.176*	47.628*	22.089*
Min. Value	44.122	42.274	19.292
Max. Value	55.334	53.404	26.096
Standard Deviations	2.481	2.569	1.381
Number of Iterations	50	50	50

distance units of the mean, 50.156. In practice, however, it is not feasible to make 50 iterations of the simulation program for each value of r, and so the important question to be asked is: after how many iterations of the program will the average value of, say, Total Fleet Mileage, not vary significantly with the generation of more readings? In other words, after how many iterations does the average value level out? To answer this question, a rolling average of all Total Fleet Mileage estimates was recorded for the three sets of 50 runs, and the results of all three are shown in Figure 7.11.. Clearly, there is a great deal of variation in the difference between the rolling average and the eventual average after 50 runs for the first few iterations of the program but this is purely a function of how "erratic" the initial estimate is; for example, the first estimate when r=1 differs from the overall mean by only 0.56 distance units, and by 5.81 distance units when r=10. The disparity between the value generated at the first iteration, and the eventual mean value is of little significance, however; what is of importance is the point at which the three curves in Figure 7.11. level out, and for values of r of 1,10 and 400, this appears to be after 11, 9 and 7 iterations, respectively. It is interesting to note that, on the evidence of these three curves, the number of runs required to achieve this levelling out tends to increase as time-window width decreases. Generally, it may be concluded that, if the average of, say, 10 runs were taken as the estimate of Total Fleet Mileage for each value of r, then it may reasonably be assumed, on the evidence of this sensitivity analysis, that this figure would not change substantially if more runs were made. In practice, all of the figures included in this chapter are the result of at least 5 iterations of the simulation program.

Figure 7.11. The difference between "rolling average" of Total Fleet Mileage estimates, and the average Figure after 50 runs



7.2. The Results Obtained Using Alternative Simulation Techniques

In the course of the development of the simulation program that is described above, two alternative tour-building strategies were considered. The first of these involves the use of a different Shortest-Path algorithm for constructing tours in Generalised Cost-space, whilst the second drops the notion of Generalised Cost, and instead defines the relative location of each customer in terms of time.

7.2.1. The use of the Savings Method in the presence of time-windows

The route-building algorithm used to derive the results that have been discussed in this chapter is based on the Nearest Neighbour criterion, since a vehicle's next destination on its schedule is simply the nearest available location that requires a visit. A similar tour-construction procedure may be employed using the concept of "Savings", so that the "distance" from the vehicle's location, i, at any time, to each of the other locations, j, is equal to the saving that would be made as a result of linking i and j directly, in a tour.

The use of the Savings Method, however, does nothing to overcome the problems, outlined in Section 6.3., concerning the initial formation of a matrix of values, and so it is still necessary to adopt a myopic approach for the process of linking together locations to form a tour. The difference between the context in which the Savings calculation is used in Chapter 4 and its utilisation in the current chapter, is that distances are now measured in terms of Generalised Cost. It is the consideration of waiting time as well as actual travelling time, along with the fact that a myopic tour-building strategy is used, that makes a Savings-based method used in the presence of time-windows rather different in nature to one which is employed in a non-time-constrained context; this is true from the point of view of both the way in which the tour-building procedure is initiated, and the manner in which the process continues.

To reiterate what has already been explained in Chapter 4, the Savings Method normally begins by assuming that all customer-locations are served individually by one vehicle which travels from the depot to one location, and back to the depot again; in this situation, the number of vehicle-trips is equal to P, and C=1. The procedure commences by calculating the distance that is saved by linking together a given pair of points, so reducing the number of vehicles used by 1. The calculation of this distance-saving may be illustrated with the aid of Figure 7.12. In this situation, the saving made from the inclusion of points A and B in the same trip, instead of serving them in separate vehicle-trips from the depot, "X", is, in algebraic terms, equal to,

$$(2a + 2b) - (a + b + x)$$

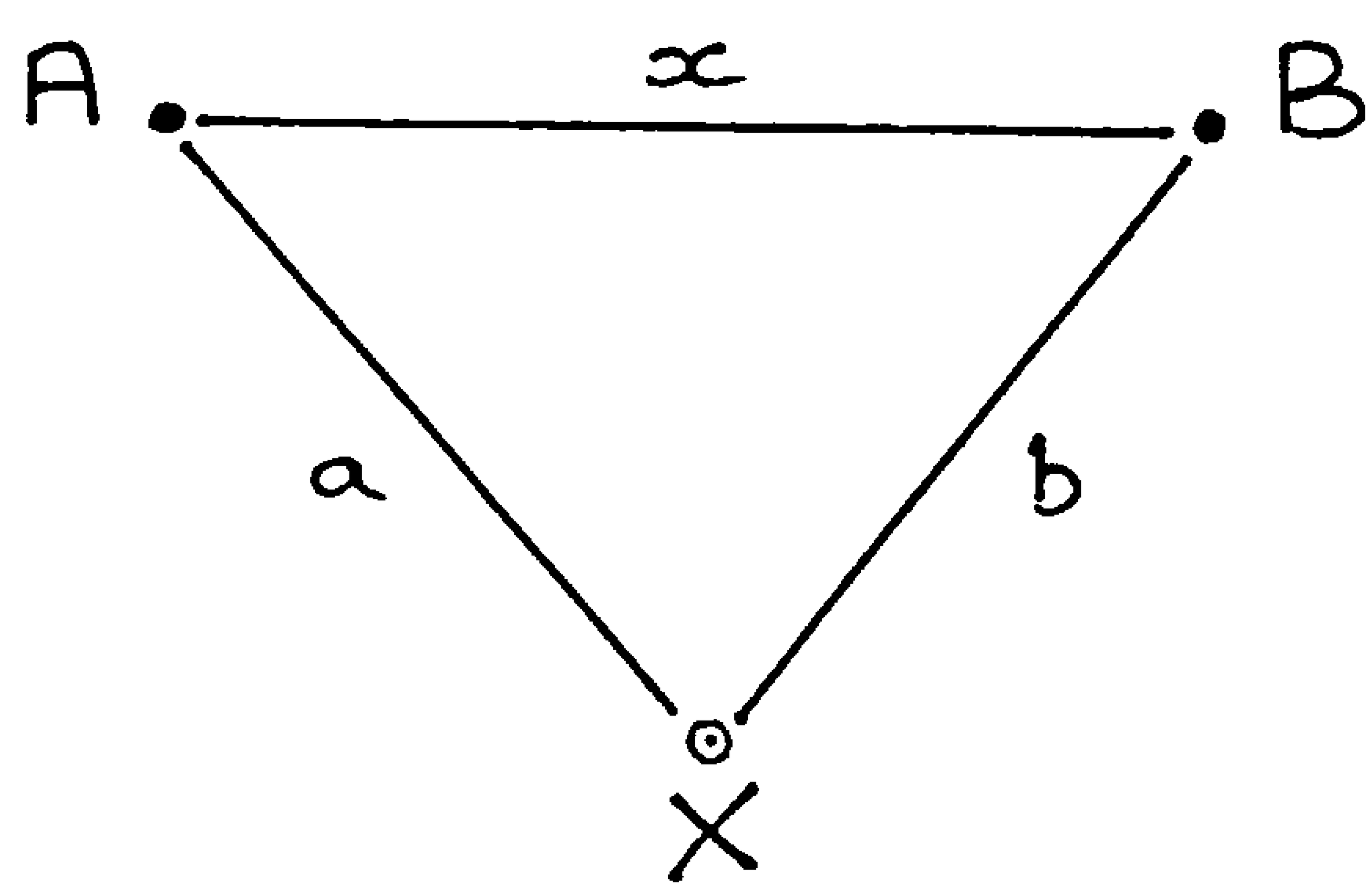
A simplified version of this expression is one which has been reproduced earlier in this thesis as Equation E.4.6.,

$$S = a + b - x$$

where, S = the "Savings Value" of linking a pair of points,
x = the distance separating the two points in question,
and, a & b are the respective distances of points A and B from the depot.

In the non-time-constrained situation, the first step to be executed in the process of constructing a tour would be to link the pair of locations having the highest S-value; since

Figure 7.12, Illustration of Savings Method



it is necessary here to proceed in a myopic manner, however, the first link to be made must originate from the depot. The first step in this case, therefoe, is for the vehicle to be directed to the nearest customer in Generalised Cost-space, which may be assumed to be point A, for the purposes of Figure 7.12. Once the vehicle arrives at A, the saving made by travelling directly to point B is, purely in terms of distance, still (a + b - x), but the complication, here is the importance of the direction of travel when waiting-time is considered. The fact that the Generalised Cost of travelling between a pair of points is not the same in both directions has already been discussed in Seciton 6.3.2.; in terms of Equation E.4.6., the two journeys which need to be costed are from A to the depot, X, and from the depot to B. As these Generalised Cost measurements are being made purely for the purposes of comparison, the cost of travelling from A to X may be omitted, since this journey is common to each calculation. The difference between a Savings-based method, in this particular context, and the Nearest Neighbour equivalent, therefore, is that the "distance" from the vehicle's location to each of the customers requiring a visit is calculated as (b - x), whilst the latter technique simply uses the measure "x".

In practice, the Savings criterion will tend to favour the linking of those locations that are further away from the depot, and those that specify times for delivery that are later in the day; when r is so large as to make time-windows relatively insignificant, the bias will just be towards the more remote locations. The main advantage of using the (b - x) calculation is that customers that are most distant from the depot, and/or specify late delivery-times, are more likely to be included in a vehicle-tour early on in the tour-building process; this contrasts with the problem of dealing with such locations at the end of the Routing & Scheduling process, associated with the Nearest Neighbour approach, which may lead to the use of one vehicle to serve relatively few customers.

A possible disadvantage of the Savings-based approach, however, is that, when a remote or "late" location is linked early on in the tour-building process, there is little time left for the vehicle to travel to other customer-locations before having to return to the depot. This is likely to reduce the number of customers that may be served in a day, and thus cause the number of vehicles used to be unnecessarily high.

Such hypotheses about the relative strengths and weaknesses of these two route-building strategies were tested by once again running the simulation program many times, and deriving figures for parameters such as fleet-size and Total Fleet Mileage, for each value of r. The results obtained using the Savings formula are presented in Table 7.4., and are compared with those derived using the Nearest Neighbour approach in Figures 7.13 to 7.16..

These graphs provide a very clear indication of the difference between the two methods. Figure 7.13 indicates that the Savings Method tends, for most values of r, to produce solutions which require substantially more vehicles, but which, as Figure 7.14. shows, involve less Total Fleet Mileage. This mileage reduction supports remarks made, in Section 7.1., to the effect that the route-building algorithm based on the Nearest Neighbour criterion is not guaranteed to produce a minimum-distance solution in the absence of time-windows.

The consistent difference in the mileage per vehicle-trip required using the Savings Method, as indicated by Figure 7.15., confirms that solutions based on this technique are characterised by generally larger fleets making shorter round-trips, which results in lower Total Mileage figures, in comparison with equivalent solutions obtained using the Nearest Neighbour approach. The fact that a larger fleet of vehicles can cover fewer miles may seem, at first, to contradict the findings of Chapter 4, which demonstrates that there is a positive relationship between TFM and n. The explanation for this apparent anomaly is that it is assumed, in Chapter 4, that vehicle-routes are non-overlapping; with time-windows, of course, this is no longer a realistic assumption, and so this earlier finding does not apply in the present context.

Another difference between the two graphs of Figures 7.13. and 7.14. is that, whereas both curves derived using the Nearest Neighbour approach tend to level out as time-windows become wider, both fleet-size and Total Fleet Mileage continue to decline when the Savings Method is used. In other words, neither n nor TFM appear to have a geometric relationship with r, using the latter approach. The effect of this is that, for larger time-windows the difference in Total Fleet Mileage is quite considerable, and, when the value of r exceeds 500 minutes, the Savings Method even produces solutions which also involve fewer vehicles.

A more important issue, though, in this section, is the comparison between the two techniques when timing constraints are more stringent. Although Figure 7.15. shows that the value of d is lower, using the Savings criterion, for all values of r, Figures 7.13 and 7.14. indicate conclusively that the Nearest Neighbour approach is superior when time-windows are narrower. The conclusion is supported by Figure 7.16., which compares the two techniques in terms of Total Cost per week. Again, the curve obtained using the Savings Method continues to fall as the value of r increases, whilst the corresponding line for the Nearest Neighbour technique levels out, but it is not until the value of r rises to around 350 minutes that the two curves intersect. This graph clearly shows that the latter approach produces the lower-cost solutions when timing constraints are effective, a tendency that is increasingly accentuated as the width of time-windows is reduced.

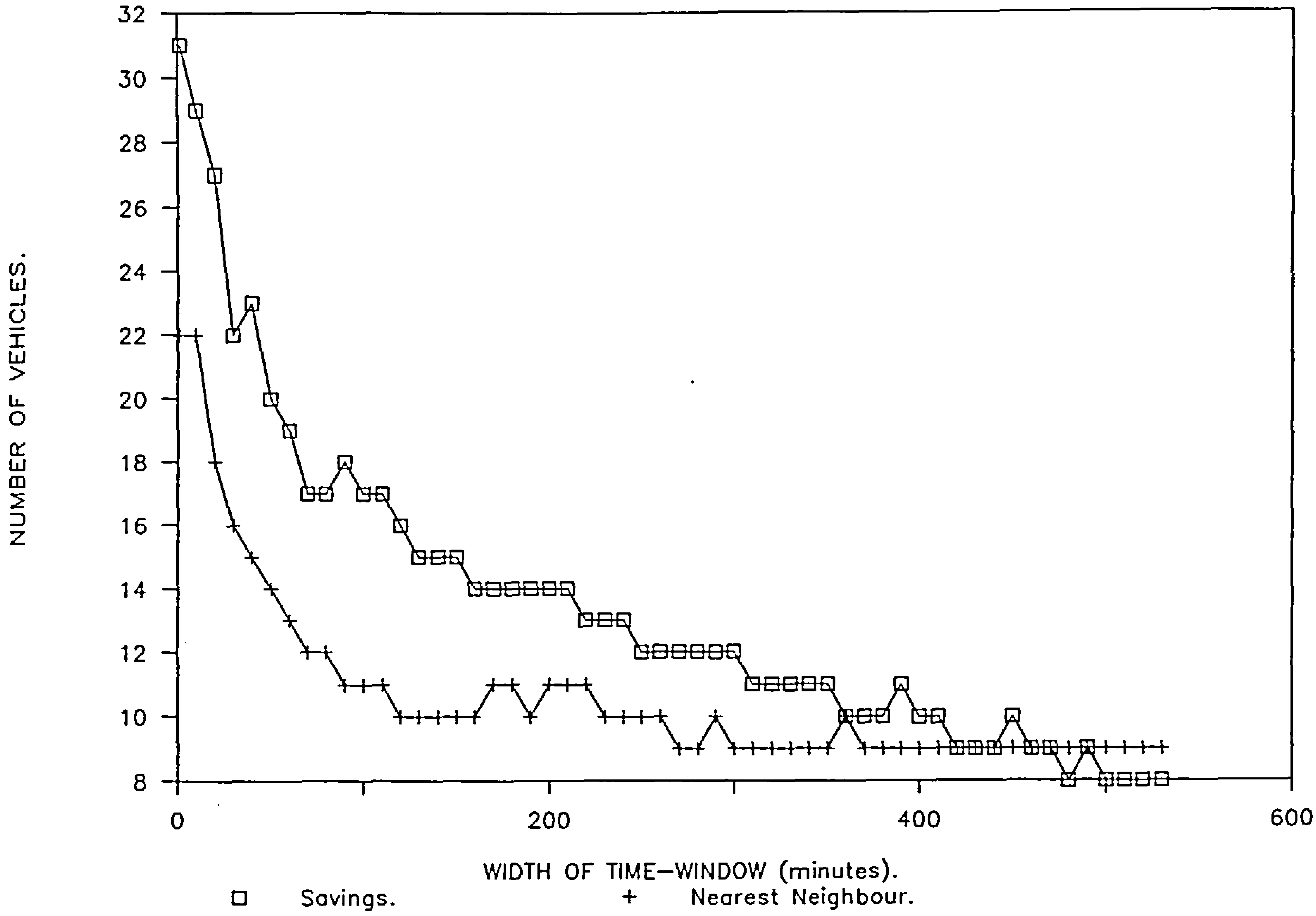


Figure 7.13 Comparison of results generated using Savings and Nearest Neighbour criteria: r/n.

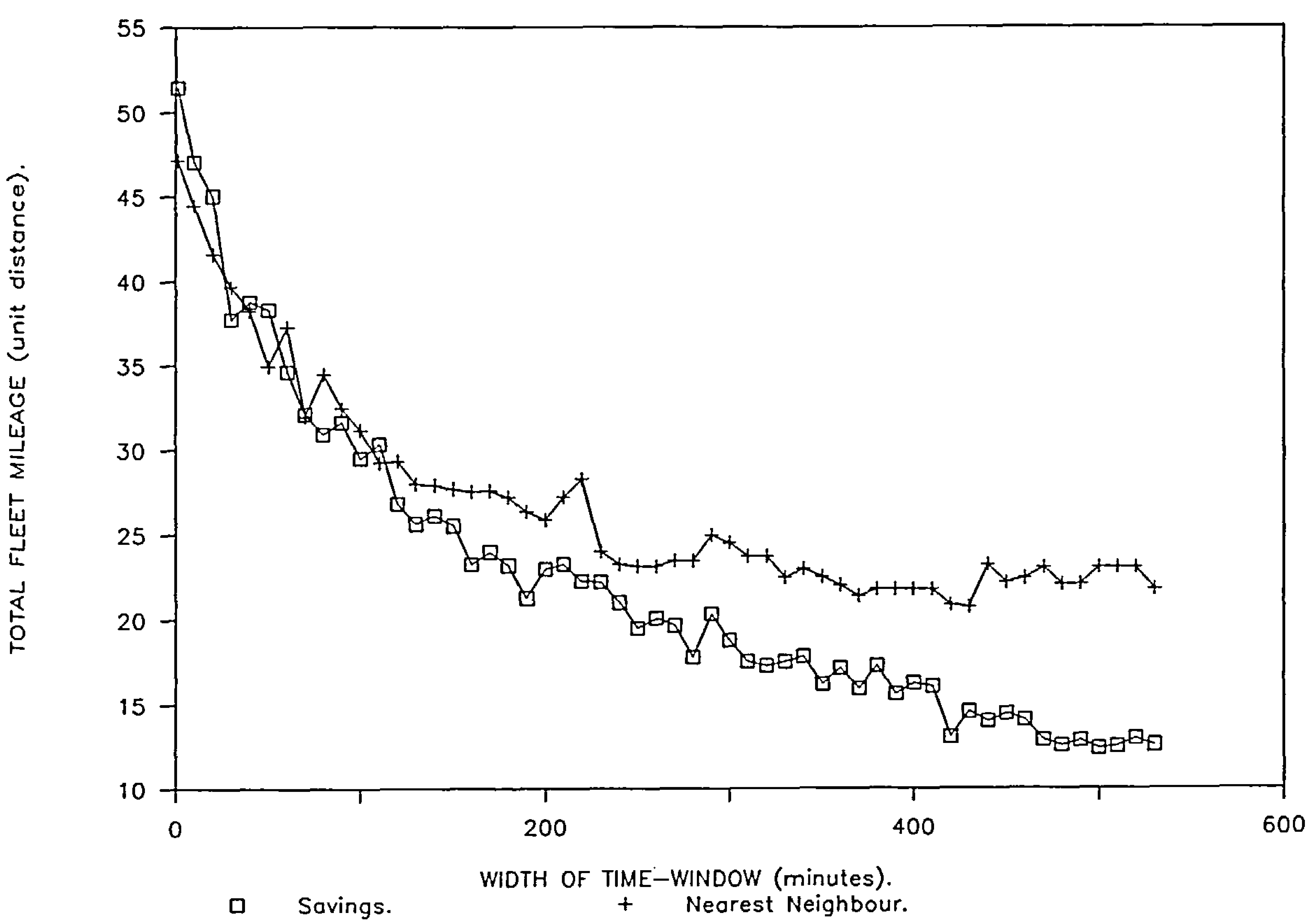


Figure 7.14. Comparison of results generated using Savings and Nearest Neighbour criteria: r/TFM.

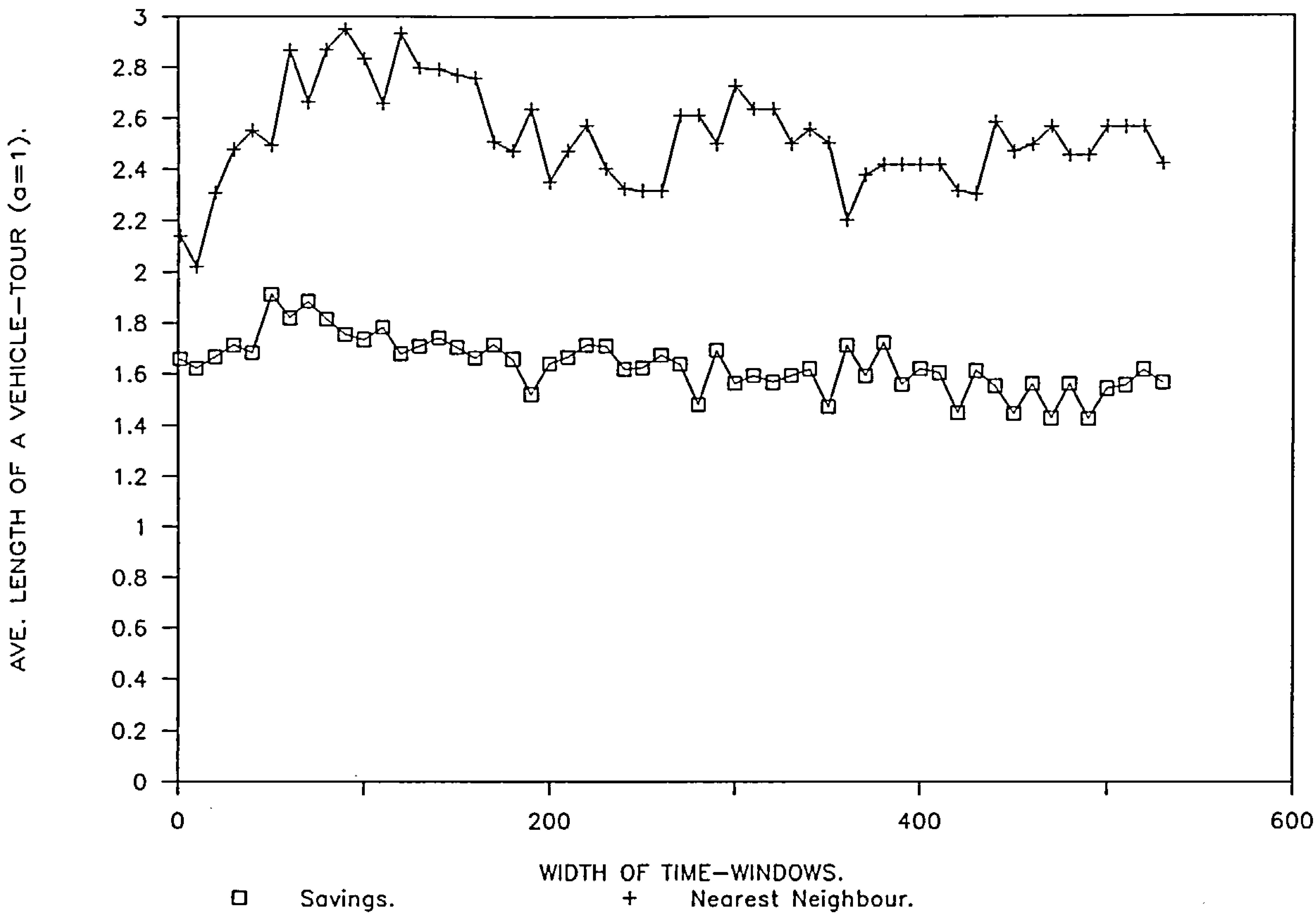


Figure 7.15.. Comparison of results generated using Savings and Nearest Neighbour criteria: r/d

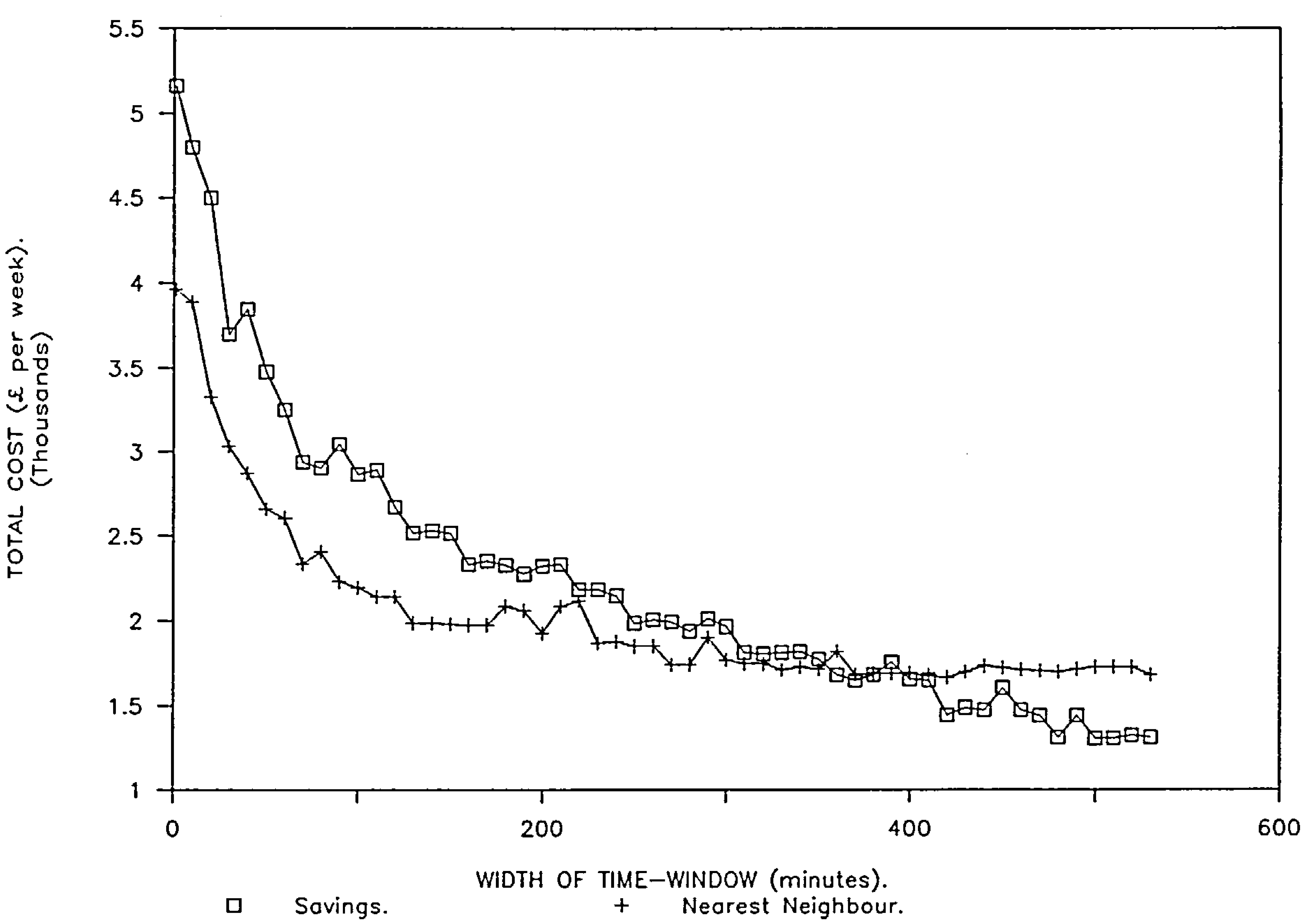


Figure 7.16. Comparison of results generated using Savings and Nearest Neighbour criteria: r/Total Cost.

7.2.2. The use of time as an alternative to Generalised Cost

Having presented an alternative set of results, derived using a different Shortest-Path algorithm, in the previous sub-section, attention is now turned to the results produced from a series of simulations which employ the same myopic, Nearest Neighbour-based routing strategy; the difference, in this case, is that time is the sole measure that is used to estimate the distance between pairs of locations. To be more precise, tours are built by successively directing a vehicle to the location at which it can arrive, AND START UNLOADING, the soonest. In this way, account is still being taken of both the spatial location and the w-value of each location, although there is no monetary weighting attached to times in order to distinguish between waiting-time and in-transit time, as there was when Generalised Cost was used.

The results obtained using this time-based technique are presented in Table 7.5., and are illustrated graphically by Figures 7.17. to 7.20.. Clearly, these diagrams bear a very close resemblance to those produced using the Generalised Cost concept, (ie. Figures 7.1. to 7.4.), except that the latter technique performs rather better in terms of Total Fleet Mileage, and thus Total Cost per week, when time-windows are narrower than about 60 minutes. Figure 7.1. and 7.17., which show the relationship between time-window width and fleet-size, are almost identical, suggesting that the observed differences in Total Fleet Mileage figures when r is small are a result of differences in the average length of a round-trip; this may be confirmed by comparison of Figures 7.3. and 7.19..

The variance that exists between the two sets of results are, admittedly, very slight, although the differences in the figures for Total Cost when r is small, (SEE Figures 7.6. to 7.20.), do provide some justification for the Generalised Cost approach's emphasis on cost-minimisation.

Passing reference is made in Section 7.2.1. to the findings, on the relationship between Total Fleet Mileage and fleet-size in the absence of time-windows, that are discussed in Chapter 4. Although the relationship between these two variables under conditions of time-constraints is not one of the primary aims of this part of the thesis, the data required for exploring this relationship is, nevertheless, available as a result of the simulation exercises that have already been performed. The following section therefore uses this information to examine the effect of time-windows on this relationship.

7.3. The Relationship Between Total Fleet Mileage and Fleet Size in the Presence of Time-Windows

Although it is not valid to compare the absolute figures for Total Fleet Mileage with and without time-window constraints,

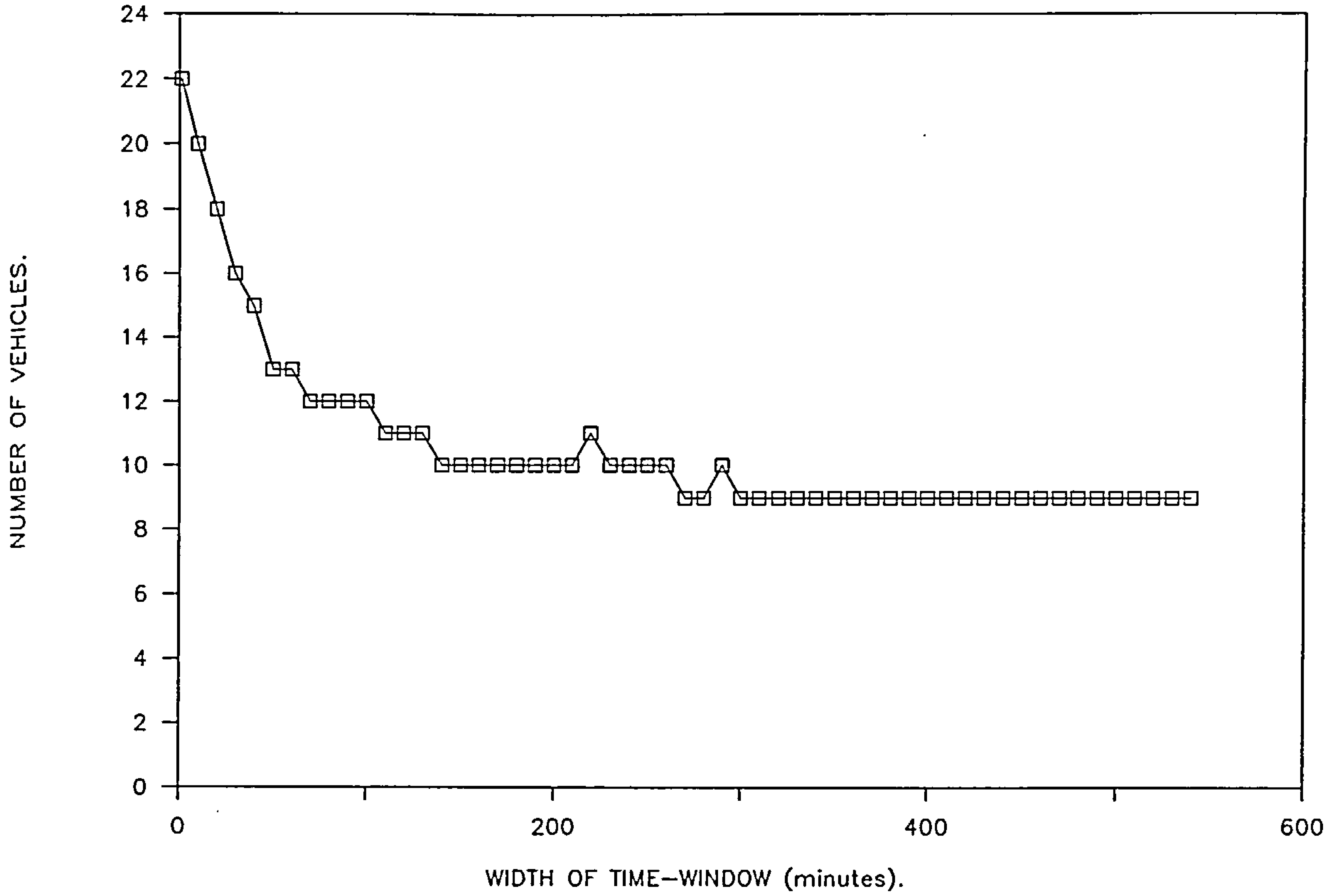


Figure 7.17 . The effect of time-windows on n, using time-minimisation as an alternative to Generalised Cost

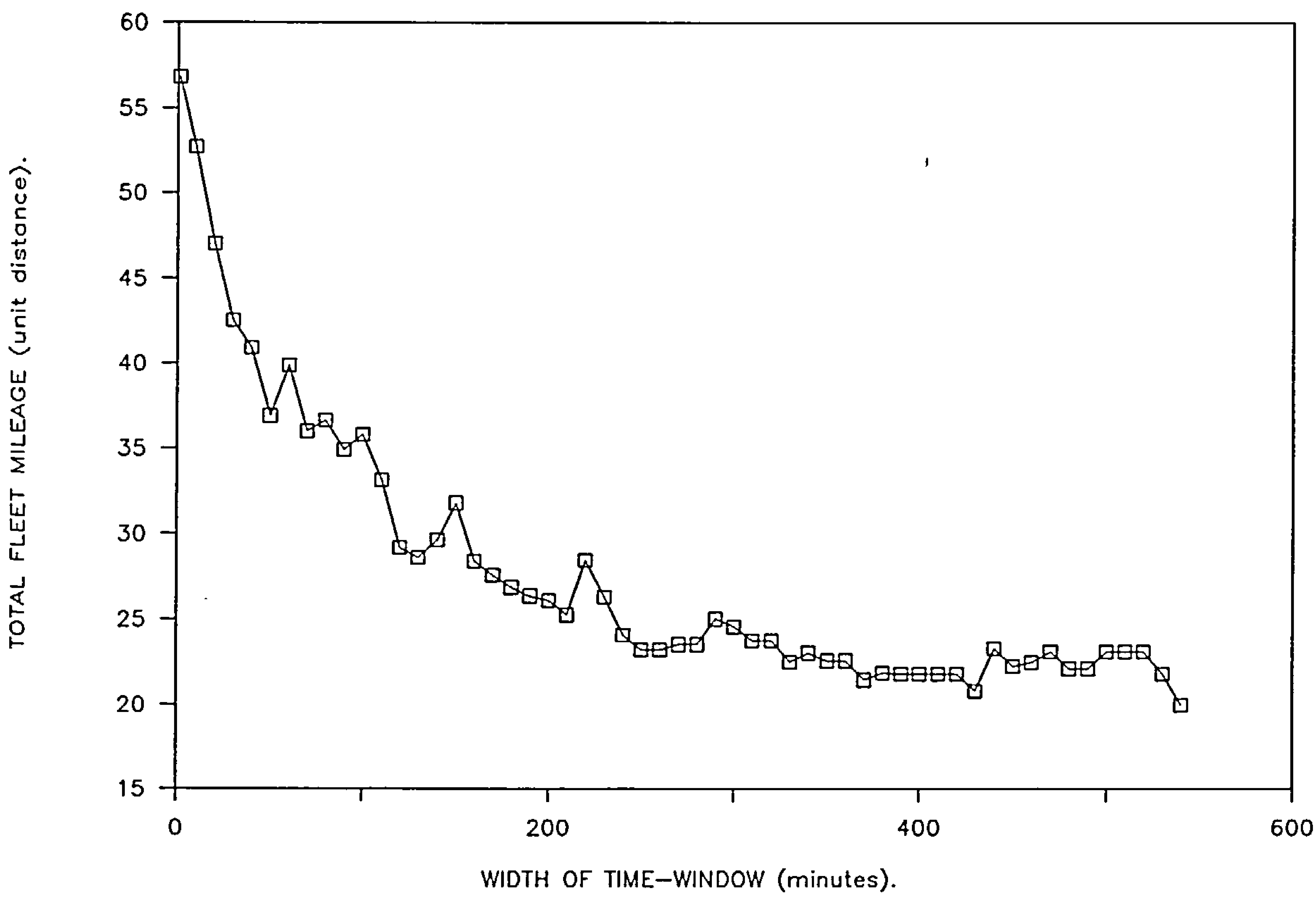


Figure 7.1 8. The effect of time-windows on TFM, using time-minimisation as an alternative to Generalised Cost

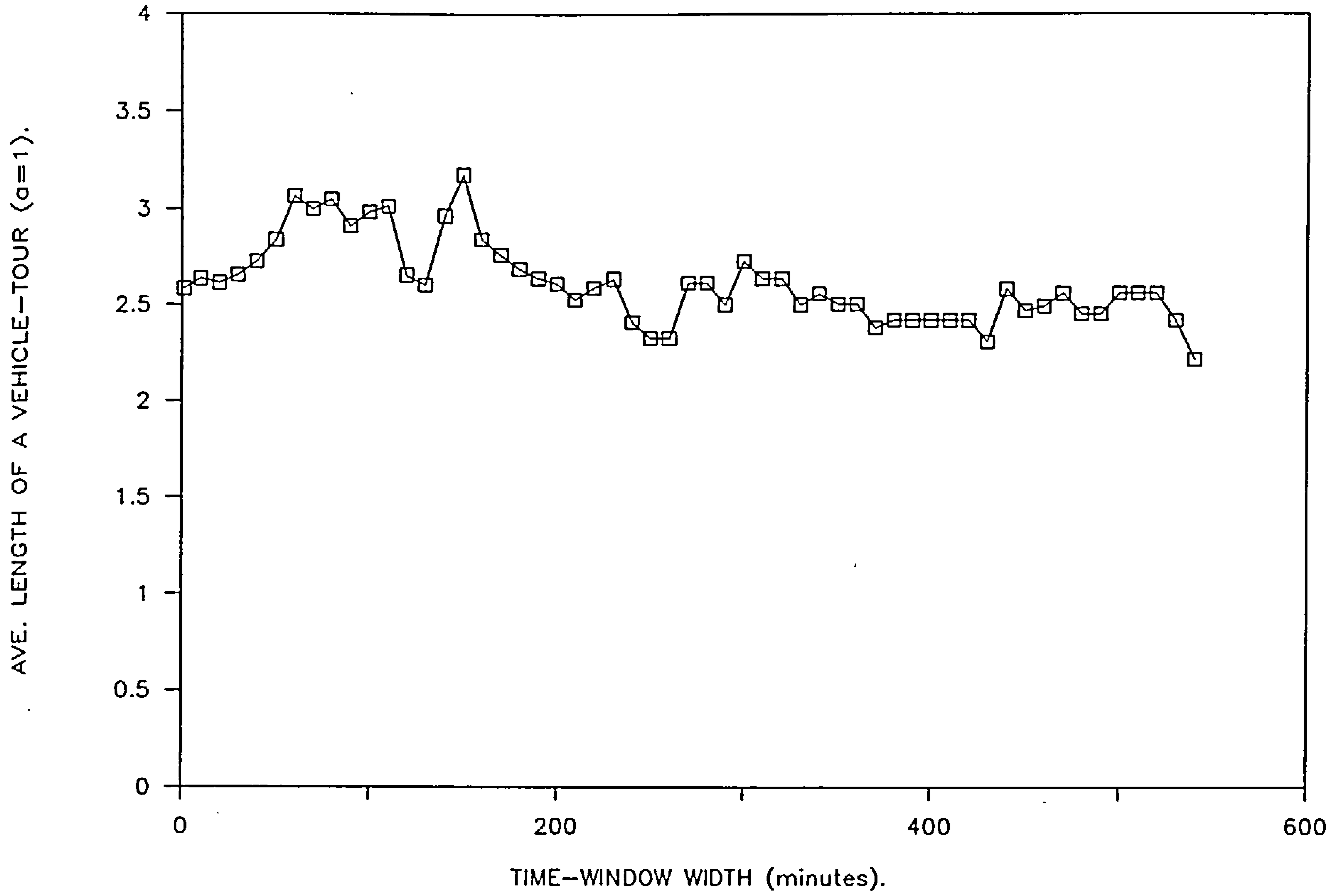


Figure 7.19.. The effect of time-windows on d, using time-minimisation as an alternative to Generalised Cost

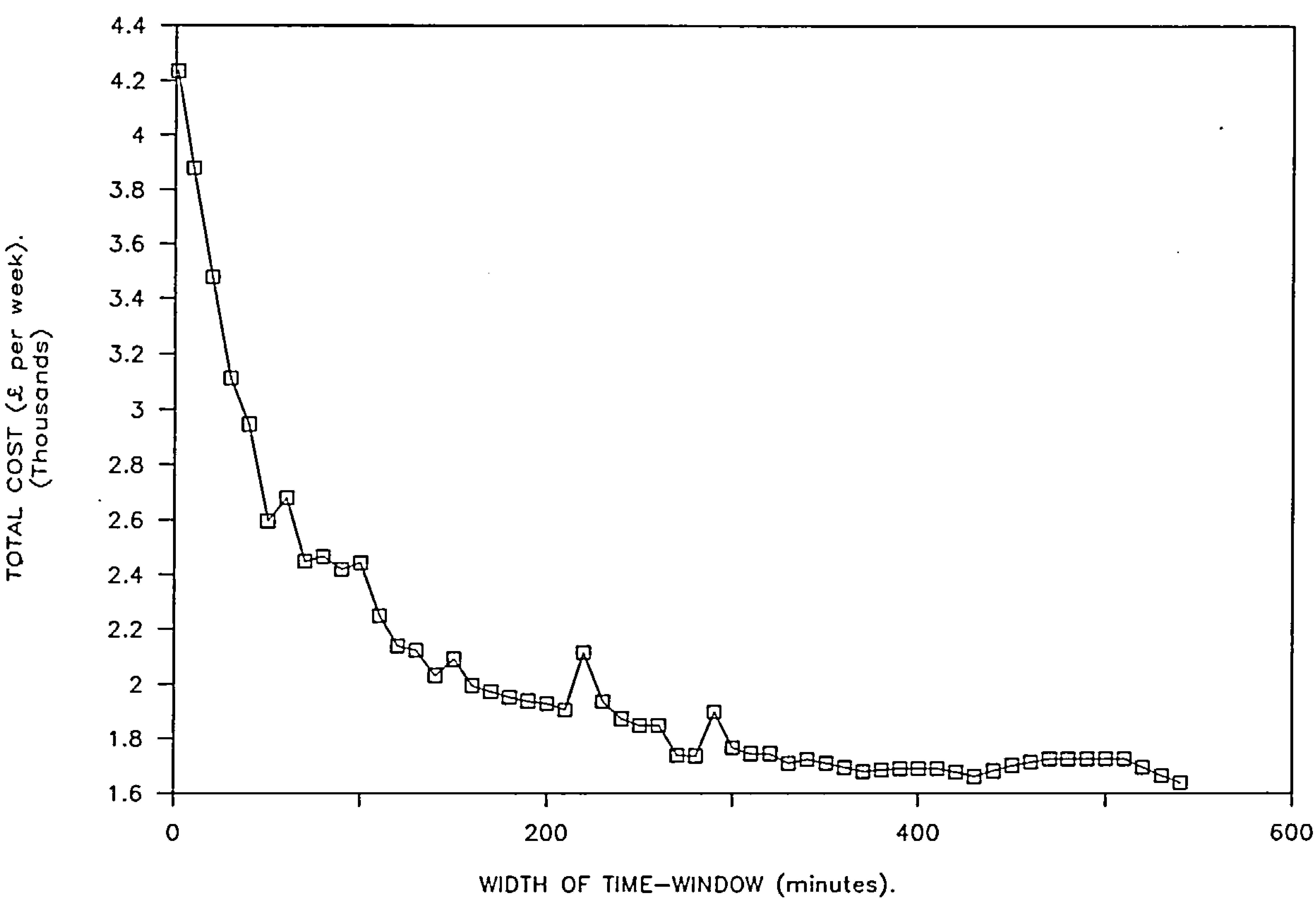


Figure 7.20. The effect of time-windows on Total Cost using time-minimisation as an alternative to Generalised Cost

owing to the fact that the sets of results presented in Chapters 4 and 7 are obtained using different algorithms, it is still interesting to compare the shape of the respective relationships.

The relevant graphs in Chapter 4 are Figure 4.9., particularly the curve for Total Distance as a function of fleet-size, and Figure 4.11., which shows the relationship between fleet-size and the average length of a vehicle-trip. The equivalent data for the time-constrained context has already been presented, in Table 7.1.. As there are sometimes several different mileage figures for each value of n in this table, average values for TFM and d were calculated in each case; these averages are contained in Table 7.6.. Figures 7.21. and 7.22. combine both these averages, and the raw data of Table 7.1., for TFM and d, respectively. (N.B. The average figures for when n=21 and 22, appearing in Table 7.6., are derived from simulations in which the value of r ranges from 1 to 15 minutes; these detailed results have been omitted from Table 7.1., for the sake of brevity).

Another factor that inhibits the making of comparisons between the results obtained with and without the imposition of time-windows is the fact that no data is available in Table 7.1. for n-values of less than 9. This applies particularly to the figures on average tour-length, as the steep portion of the d-curve of Figure 4.11. corresponds mainly to small fleets. Considering the scale of the y-axis in Figure 7.22., the average length of a vehicle-trip is more or less constant, with only a slight decline with increasing vehicle-size in evidence; the same observation might also be made about the corresponding portion of the line shown in Figure 4.11.. The lack of data for Total Fleet Mileage when n is small, in the presence of time-windows, should not present the same problem for making comparisons, since the line drawn in Figure 4.9. is virtually linear. Although the same cannot quite be said about the relationship shown in Figure 7.21., the general tendency for Total Fleet Mileage to increase at a more or less constant rate as fleet size increases is largely repeated. In fact, although there is a limited extent to which firm conclusions may be drawn from this very rough comparison of the output produced by two rather different simulation programs, it would appear that the general nature of the relationship between Total Fleet Mileage and fleet-size varies little according to whether or not time-window constraints are imposed.

7.4. Summary of the Findings on the Effect of Time-Windows on Distribution Costs

Chaper 6 begins by outlining the difficulties that are presented by the introduction of time-windows, making particular reference to the ways in which such constraints inhibit the use of existing algorithms for solving Travelling-Salesman-type problems. The major difficulty involved here, is the fact that time-windows hinder the initial setting up

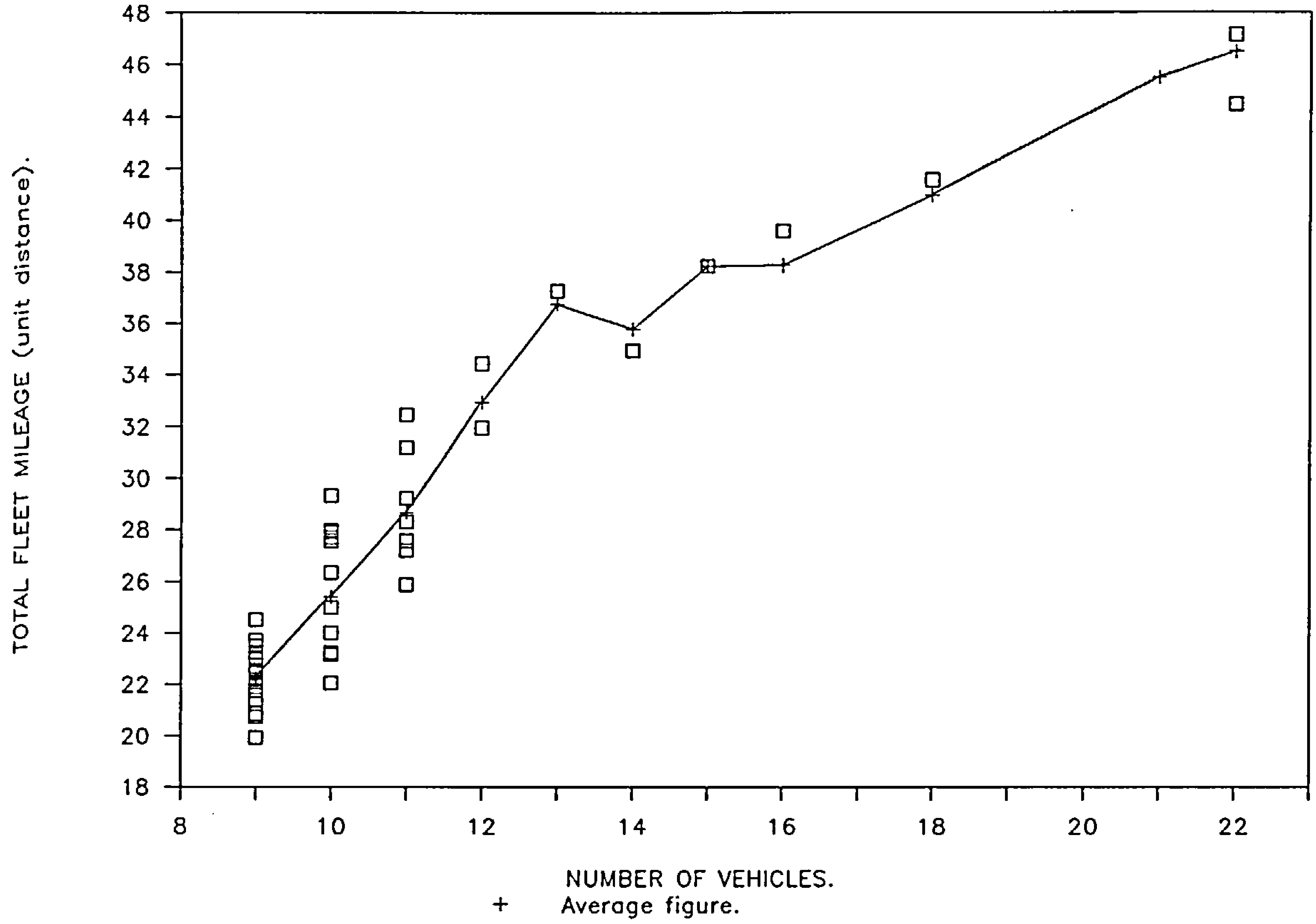


Figure 7.21. The relationship between Total Fleet Mileage and n in the presence of time-windows

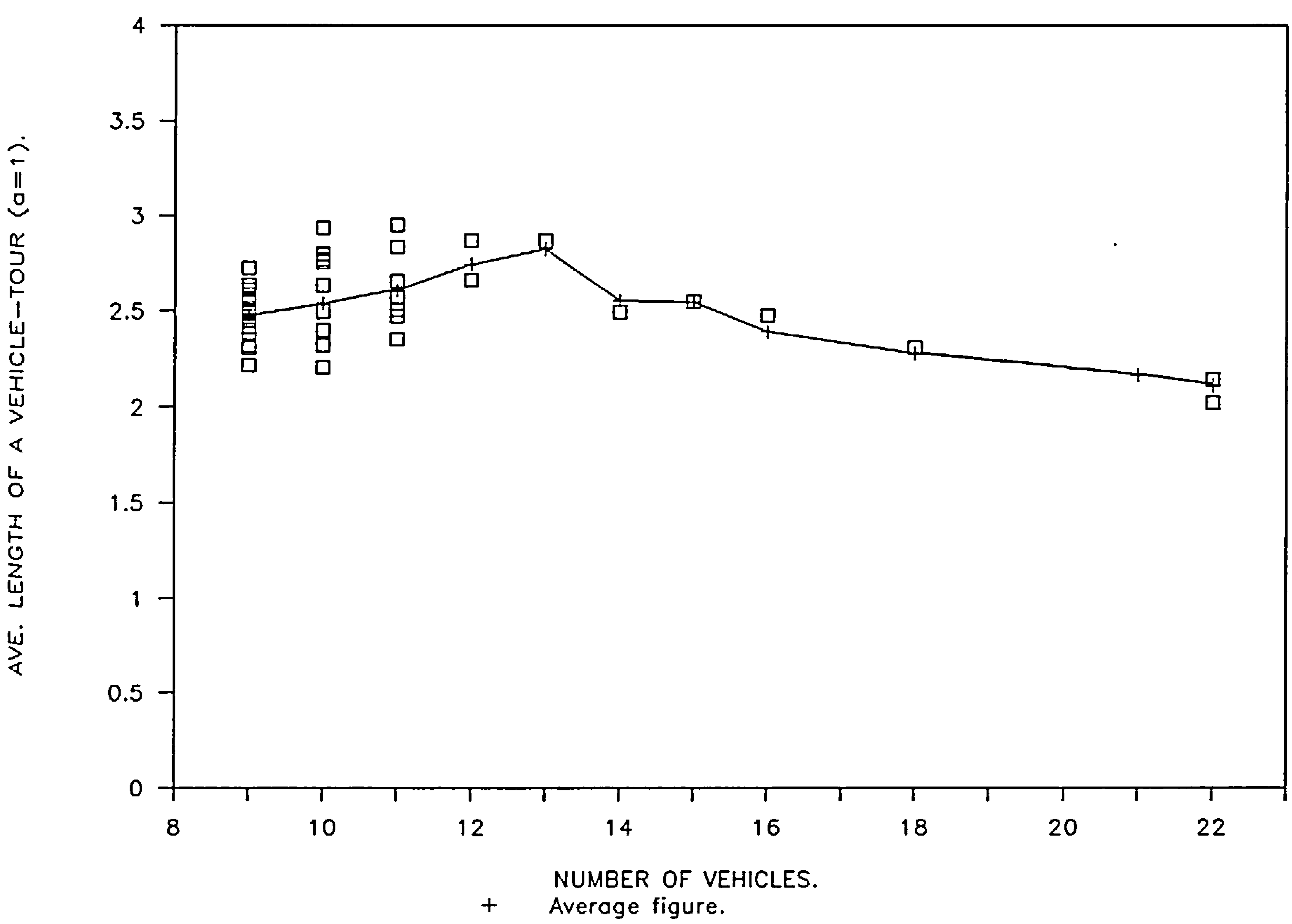


Figure 7.22. The relationship between Total Fleet Mileage and d in the presence of time-windows

of a distance-matrix to describe the relative locations of the customers that are to be served. In order to overcome this, an alternative, myopic Shortest-Path algorithm is proposed; prior to this procedure being put into operation, and to reflect the fact that the saving that is made in a fleet's travelling-time is not the same as that of a saving in waiting-time, all customer-locations are defined in terms of the Generalised Cost of travel. The formula for Generalised Cost appears as Equation E.6.5., and the program used to generate Travelling-Salesman solutions is summarised by the flow-diagram of Figure 6.2..

The results of the simulation exercises carried out using the procedure developed in Chapter 6, have now been presented in Chapter 7. The most important finding, here, is the nature of the relationship between the Total Cost of an operation and r, the width of the time-windows that are imposed, (SEE Figure 7.6.). This graph upholds the hypothesis that distribution costs will increase as timing constraints become more stringent, and, after transformation of the data using logarithms, a regression-line is fitted to this geometric distribution of points, (SEE Figure 7.7.2.). Despite the close correlation that exists between this regression-line and figures produced by simulation, (the R^2 - value associated with the regression is 0·923), there is a limit to the usefulness of the expression that describes this relationship, Equation E.7.2., as a tool for estimating Total Cost per week. This is because the numerical details of the estimates derived from this equation are very dependent upon the particular assumptions that are built into the problem formulation that is involved here. Furthermore, the demonstrable increase in simulated Total Cost figures as time-windows narrow cannot be said to take place from a base-line level that represents the weekly cost of a distribution operation using an optimum routing strategy; this is due to the fact that the tour-building algorithm used, that has been developed for the specific purpose of overcoming the problems that are posed by time-windows, is by no means guaranteed to produce optimum solutions in the absence of such constraints. The results presented in Chapter 7 do, nevertheless, provide vivid insights into the way in which solutions to Travelling-Salesman Problems deteriorate as timing constraints become more severe, and there seems to be no reason to doubt that the geometric relationship shown in Figure 7.6. is not a true reflection of the way in which time-windows affect distribution costs in the real world.

The analysis goes on to examine the behaviour of the major parameters of a distribution operation - namely Total Fleet Mileage, fleet-size and the average length of a vehicle-trip - as r changes, and compares the resulting graphs with Figure 7.6.. As Figure 7.1. and 7.2. indicate, the curves for n and TFM show the same geometric relationship with time-window width as that for Total Cost. The fact that the value of d is virtually constant as r changes, (SEE Figure

7.3.), suggests that it is the increase in the number of vehicles required that is the main influence on the behaviour of the Total Fleet Mileage curve of Figure 7.2.. Figure 7.4. shows how the average distance between consecutive stops in a vehicle-tour, i_d, increases as time-windows narrow. This graph confirms that the overall effect of timing constraints becoming increasingly stringent is that vehicles are able to visit fewer more widely dispersed customer-locations in each working-day; this reduction in the value of C directly increases fleet size. Using the findings of Chapter 4, which explores the relationship between Total Fleet Mileage and fleet-size in the absence of time-window constraints, it may be concluded that it is the increase in the number of vehicle-trips, and therefore stem-journeys, that are made that causes TFM to behave in the manner indicated in Figure 7.2..

Because the above conclusions were drawn on the basis of many iterations of a computerised simulation model, sensitivity analyses were carried out to test the validity of the conclusions. These analyses showed that changing neither the model's assumption about average drop-times by 15 minutes, nor the assumed length of a working day by 1 hour had any significant effect on Total Fleet Mileage estimates. It was shown, however, that changing the length of the working day by this amount changed the fleet size required, typically by one vehicle, whilst a 15-minute change in drop-time changed the required size of the fleet substantially more, (typically by 3 or 4 vehicles). To assess the extent to which there may be a problem of variance between successive Total Fleet Mileage estimates, the model was run 50 times with no changes made to the value of any of its parameters. An analysis of the difference between the rolling average of the estimates and the eventual average figure was carried out. This exercise suggested that, after about 10 iterations of the program, there was very little discrepancy between these two averages.

It should be emphasised, of course, that the introduction of time-windows into the problem formulation limits the extent to which analogies may be drawn between the results of simulations carried out with and without the imposition of time-windows. Figures 7.21. and 7.22., however, indicate that the relationship of n to both TFM and d, with timing constraints, is probably not very different to the corresponding relationships in the non-time-constrained context, as shown in Figures 4.9. and 4.11..

4. SUMMARY AND CONCLUSION

CHAPTER 8

SUMMARY

The role of Part 4 of the thesis is to summarise the major findings of the research, and to outline the conclusions that may be drawn from the results that are described in foregoing chapters. Whilst the current chapter provides a resume of the main end-products of this research, Chapter 9 discusses the extent to which the objectives outlined in Chapter 2 have been met, and goes on to evaluate the degree to which the methods used here have been successful, and the way in which the thesis relates to other work done in this field. Chapter 9 closes with a section that makes suggestions for further research which might be carried out as an extension of the work that has been reported in the current thesis.

The main focus of Chapter 3 is the relationship between two important decision-variables - fleet-size, (n), and vehicle size, (x), and the basic dilemma involved in making a trade-off between the two when deciding on the composition of a vehicle fleet. This relationship is examined using the concept of Economies of Scale in Transport, which utilises the fact that the cost per ton (carrying-capacity) per mile of operating a goods vehicle decreases geometrically as vehicle carrying-capacity increases. This phenomenon is demonstrated by means of a deliberately simplistic hypothetical situation of an operation which involves delivering a fixed weekly consignment of goods from a single depot to a single destination using a uniform fleet of vehicles - using cost-data published by Commercial Motor magazine, the presence of Economies of Scale of the type described is demonstrated both graphically, (SEE Figure 3.7.), and algebraically. The important policy implication of Economies of Scale in this context is that, given the constraints and assumptions associated with the particular problem formulation dealt with in this chapter, it is more cost-effective to employ a fleet, for the delivery of a fixed weekly consignment of goods, (t), which consists of the smallest number of vehicles possible, than to use a larger fleet made up of smaller vehicles. The basis of Section 3.5.'s algebraic proof of this finding is an expression, (Equation E.3.11.), which expresses Total Cost per week as a function of the parameters t and x, (with the length of the fixed round-trip, d, given).

The removal of the assumption of a fixed vehicle trip, so that d is influenced by x, by the latter's effect on n, leads to the discovery that the curve describing the fleet's Running Cost per week is "U"-shaped when plotted against the number of vehicles in the fleet, (SEE Figure 3.22.). This is because Running Cost is a function of both vehicle-cost per mile, which increases with vehicle size, and the number of miles travelled per week, which decreases as vehicle size increases due to the associated reduction in fleet size. The implication of this "U"-shaped Running Cost curve is that, when d is variable, it is possible that the Total Cost curve may also be "U"-shaped, depending on the relative influence of Standing Costs and Running Costs, (SEE Figure 3.23.). The main variables that determine whether the Total Cost curve will be "U"-shaped are therefore parameters such as area-size and fuel costs per mile; the possibility does, however, mean that, in the absence of the assumption that vehicles travel on a route of fixed length, it is feasible, under certain conditions, for Total Cost to actually decline as n increases and x is reduced. In such circumstances, it is therefore possible to use this Total Cost curve to establish an optimum, least-cost fleet-size, with its associated value of x, to perform a given delivery-task.

The theme of the influence of fleet-size on Total Distribution Cost, although not in the context of the way in which it affects x, is continued in Chapter 4, in which the spatial implications of the number of vehicles employed, which are introduced in Chapter 3, now become the central issue. In fact, in the absence of any consideration of the size of the weekly consignment of goods to be delivered, or vehicle size, Chapter 4 concentrates on the way in which fleet-size affects Total Cost purely as a result of increasing the number of vehicle trips that are made, and thus Total Fleet Mileage. More precisely, with the total number of outlets to be served, (P), fixed, fleet-size is shown to affect Fleet Mileage by influencing the number of "stem journeys" that must be made to and from the depot, the average number of customer-locations that are served in each round-trip, and the average size of each vehicle's delivery-zone, (retaining the assumption of non-overlapping vehicle tours).

Using stochastic simulation once again, by means of a route-building computer program based on the Savings Method, a formula for Total Fleet Mileage as a function of fleet-size, (n), and the average number of stops per tour, (C), is developed, (C being a function of P and n). The novelty of the approach here lies in the way in which Stem Distance, (the total distance that is travelled by a fleet between the depot and the first and last customer-locations of each vehicle tour), and Delivery Distance, (the total distance travelled between consecutive customers), are estimated separately, and, for the latter estimate, Delivery Distance is further disaggregated to the level of the term "i", which represents the average distance between consecutive stops. From an initial equation for Total Fleet Mileage as a function of n and C, (SEE Equation E.4.20.), after algebraic manipulation, an expression for Total Fleet Mileage as a function of the "external" constraints P and k is developed, (SEE Equation E.4.21.), where P is the total population of customers requiring a delivery, and k represents the maximum number of deliveries that may be made each day by a single vehicle. The introduction of the parameter k, in this chapter, may be seen as the reappearance of vehicle carrying-capacity as a constraint, although this limit could equally be a result of temporal constaints.

The graphs drawn from Equation E.4.20., (SEE Figure 4.31.), show that both Stem Distance and Total Fleet Mileage appear to increase linearly with n, whilst Delivery Distance is roughly constant - this underlines the assertion that it is the increase in Stem Distance that is the major factor in causing Total Fleet Mileage to rise as the fleet grows, in terms of the number of vehicles used. Despite this apparent linearity, however, the simulation analysis also provides data which reveals that the lines of Figure 4.31. are, in fact, slightly curved; Figure 4.33.2., based on figures generated from Equation E.4.20., graphs the Marginal Cost, in terms of the extra distance travelled by the fleet, of each vehicle that is added, for both a 50-customer and a 100-customer situation. The distinctly "U"-shaped, but asymmetrical, nature of these curves is then accounted for by Figure 4.34., which disaggregates the Marginal Cost curve for the 100-customer instance into the curves for Marginal Cost in terms of both Stem Distance and Delivery Distance - this graph shows that both of these lines are geometric in form, but that the Marginal Cost of Stem Distance <u>increases</u> with n, whilst that of Delivery Distance <u>decreases</u>. Similar graphs are also drawn to illustrate the figures generated from E.4.21., (SEE Figures 4.32. and 4.33.1.), the latter Figure demonstrating the increasingly gradual decline of Marginal Cost in terms of distance, of adding an extra customer-location to the fleet's delivery-schedule. The use of Marginal Cost figures in this way is itself an illustration of how the nature of vehicle tours is examined in Chapter 4 using the detailed data that is provided by the tour-building simulation program.

As well as providing a great deal of information about the various components of vehicle-tours, mentioned above, and the relationships that exist between key parameters, Equations E.4.20. and E.4.21. also serve as useful predictive tools for estimating the length of vehicle tours and Total Fleet Mileage, given certain conditions and constraints. Because all such predictions are the result of a number of iterations of a simulation program, however, each predicted value is a mean figure, and therefore has a measurable variance and Standard Deviation with which it is associated. Section 4.5. provides an illustration of how this variance may be used to place confidence limits around these estimates, and suggests how statistical indices such as skewness and kurtosis might be employed in a similar way for qualifying the predictions that are derived from these expressions.

An example of the usefulness of distinguishing between Stem Distance and Delivery Distance is presented in the following chapter, Chapter 5, which deals with the impact of driver's hours limitations on Total Cost. The main focus of the first part of this chapter is the effect of the variable "H", the total amount of time that is available for each daily vehicle-trip, on Fleet Mileage estimates, that are disaggregated into Stem Distance and Delivery Distance; this set of estimates is derived from an equation that has been adapted from E.4.21., which expresses Total Fleet Mileage as a function of P and k. The resulting graph , shown by Figure 5.2.2., reveals the geometric shape of the Total Cost curve, and clearly shows that Total Cost rises rapidly as the value of H declines to below about 6 hours, in this particular case. The same graph indicates, with equal clarity, that it is mainly due to the increase in Stem Distance that this sharp increase in Total Cost takes place, (Delivery Distance is seen to be almost constant), and Table 5.2. confirms that it is the limitation of the number of outlets that may be visited in each vehicle trip that causes this rise in the number of stem-journeys that are made.

The analysis of the relationship between the parameters H and k then leads on to a discussion of the effect on Total Cost of allowing drivers to stay away from the operating centre overnight - the major advantage of an overnight stay is that the number of customer-locations that may be visited in a round-trip, k, is increased, and the number of stem journeys that must be made is reduced, which in turn, reduces the Total Mileage of the Fleet. The associated decrease in cost must however, be traded off against both the increased vehicle-cost per mile that results from having to use larger vehicles, and the overnight expenses that must be paid to each driver. Again, a simulation program was developed to compare the cost of delivering a weekly consignment of goods both using a system of overnight stays, and imposing the constraint that each vehicle must return to the depot at the end of each working-day; this program is based on the equation for Total Fleet Mileage as a function of n and C, derived in Chapter 4, (ie. Equation E.4.20.), and Chapter 3's E.3.6., which enables Total Cost to be calculated as a function of fleet-size, Total Fleet Mileage and vehicle carrying-capacity. Using the size of the whole delivery-area as the main independent variable in this section of the analysis, output from the program shows the value of "a" at which it becomes cheaper to allow drivers to make overnight stays, (SEE Figure 5.7.). This simulation data also shows, once again, the extent of the influence of Stem Distance on Total Fleet Mileage, through Figure 5.10., which illustrates the way in which the percentage of the latter that is accounted for by Stem Distance increases with increasing area-size; clearly, this percentage growth is far more pronounced in this graph for the Daily Round-Trips option than for a system that allows overnight stays.

The curves for Total Cost per week for both situations, (SEE Figure 5.15.), shows the same geometric shape as the Total Cost curve of Figure 5.7., which is described in an earlier section of Chapter 5, although, in Figure 5.15., it is the increasing size of the delivery-area, and not the diminishing amount of time required in which to make deliveries, that causes the sharp increase in Total Cost after a critical value of a, (which will vary according to the assumptions that are made). In both cases, though, it is shown that it is the limitation of k, the number of customers that may be served in a single delivery-round, that actually causes this cost increase. Although the cost curve for Overnight Stays in Figure 5.15. has the same geometric form as that for Daily Round Trips, the advantage of being able to effectively increase the value of the constraint k means that this rapid cost increase with area size is delayed until a reaches a considerably larger value. Therefore, for the same portion of this graph, the Overnight Stays curve appears to rise linearly whilst the corresponding curve for Daily Round Trips increases geometrically.

The main findings of Part 3 - which is concerned with the external influences on distribution operations, which are normally beyond the control of the operator - centre around the effect that time-windows have on estimates of both Fleet Mileage and Total Cost. Throughout this section of the analysis time constraints are represented by the independent variable "r", which defines the "width" of a time-window, in minutes. Although it is acknowledged that time-windows may take on various forms in practice, just one type is used here as a general representation of time-constraints that are imposed on a fleet's schedules by the customer. In fact, it is assumed that <u>every</u> customer specifies <u>one</u> time-window of the same width, a time-window whose mid-point may take on any random value in time, provided that it is feasible for a vehicle to make a delivery to the relevant location from the depot, given the constraints on the length of the working day.

The presence of such time-windows greatly complicates the building of vehicle-tours using the savings-based method used in Chapter 4, for reasons that are specified in Section 6.3., and so, in order to test the effect of time-windows on estimates of Total Fleet Mileage, it has been necessary to design a heuristic method for constructing Travelling-Salesman tours under such conditions. The development of this algorithm provides the main subject-matter for Chapter 6.

The fundamental trade-off involved with this heuristic is that between minimising the aggregate distance that is travelled by the fleet and minimising the total amount of time that all vehicles spend waiting at an outlet for a time-window to "open". It has been necessary, therefore, to develop a means of estimating the Value of Time, in order that the relative location of each customer may be defined in terms of Generalised Cost, (SEE Equation E. 6. 5. for a detailed definition of this measure). Using the "myopic" Shortest-Path algorithm described by Figure 6.1., a simulation program was used to construct and measure Travelling-Salesman tours in the presence of time-window constraints - although this program was not designed to produce optimum tours, it has been successful in providing output to enable the effect of time-window constraints on Total Fleet Mileage and Total Cost to be examined.

It is not surprising that the results of these simulations support the hypothesis that the narrowing of time-window widths will increase fleet size, Total Fleet Mileage and Total Cost, although it is interesting to note the markedly geometric nature of the Total Cost curve of Figure 7. 6., as well as that of the curves for n and Total Mileage shown in Figures 7. 1. and 7. 2., respectively. Again, it is demonstrated how such relationships may be quantified, and, after logarithmic transformation of the data and a regression analysis, an expression E.7.2., is derived for Total Cost per week, as a function or r,

$$\underset{(\text{£ per wk})}{TC} = 5011 \cdot 87 . r^{-0 \cdot 177}$$

Although the numerical details of this equation are unimportant, in view of the fact that such details are very much dependent on the assumptions made for the purposes of the analysis, the regression that produced this equation returned an R^2-value of 0·923, indicating that Equation E.7.2. gives quite an accurate description of the relationship between Total Cost and time-window width. The advantage of simulation providing a great deal of disaggregated data, enabling a detailed analysis of the relationships that are discussed in Chapter 7, is again in evidence, and, comparing Figures 7.6 and 7.1., there is the strong suggestion that it is the enforced increase in fleet size that causes the observed geometric rise in Total Cost with decreasing r. Figure 7.3. indicates that the average length of a vehicle-trip remains more or less constant as time-window width changes, confirming that increases in Total Fleet Mileage, and thus Total Cost, must therefore be due to increases in the number of round-trips made by the fleet. The explanation of the actual effect of increasingly stringent time-constraints on the spatial behaviour of a vehicle-fleet is completed by Figure 7.4., which graphs the average distance between consecutive stops against r. It may be deduced from these Figures that, as time-windows become narrower, vehicles are able to visit fewer, but more widely-spaced, locations in each working-day; in algebraic terms, therefore, the reduction in C causes n to increase, which in turn, causes an increase in Stem Distance, Total Fleet Mileage and, consequently, Total Cost per week.

To complement the research done on the relationship between fleet size and Total Mileage, that's described in Chapter 4, the output produced for the time-constrained situation of Chapter 7 was examined to see if a similar relationship between these two variables is in evidence. This analysis was carried out on what is only a relatively small body of data in comparison with that used in Chapter 4, and, in any case, the significance of comparisons that might be made is limited, due to the fact that the results generated by the two simulation programs involved utilise very different route-building algorithms, and, in the non-time-constrained context, it is assumed that vehicle-tours never overlap. Figure 7.21., which graphs Total Fleet Mileage against fleet size, does, however, show a roughly linear relationship which is not unlike that of Figure 4.9., albeit far less marked in the former case. The relationship between the average length of a vehicle-tour and fleet size, on the other hand, (shown in Figure 7.22.), fails to show the decline in d with increasing n that's demonstrated by Figure 4.11., and this almost certainly because of the absence of the non-overlapping tours assumption.

As well as providing a workable heuristic for constructing Travelling-Salesman tours in the presence of timing constraints, and allowing inferences to be made about the way in which time-windows affect distribution operations, this section of the thesis also provides evidence of how the performance of route-building algorithms deteriorated under such conditions. Apart from that of the procedure that was used to generate the data on which the results discussed so far are based, the performance of similar techniques, notably one employing the Savings criterion, (SEE Section 7.2.1.), was also analysed, and a similar set of graphs was produced, (SEE Figures 7.13 to 7.16., inclusive). Comparison of the relevant graphs reveals that the technique based on the Savings criterion for choosing the next location in a tour generates solutions which generally require more vehicles, require less Total Fleet Mileage and, on average, shorter vehicle-tours than the heuristic that is actually used. More importantly, as Figure 7.16. illustrates, the latter technique produces solutions of less Total Cost per week when time-windows are narrow enough to be effective, (ie. when r is less than about 350 minutes), underlining once again the important influence of fleet size on Total Cost figures.

CHAPTER 9

CONCLUSION AND DISCUSSION OF AREAS REQUIRING FURTHER RESEARCH

Having summarised the major findings of the thesis in the previous chapter, it is now necessary to review the research objectives that have been set down in Chapter 2, and to evaluate the extent to which they have been satisfied. The current chapter also discusses how far the research has been successful in providing additional useful information concerning the relationships between key variables of distribution operations, and both considers the advantages and disadvantages of the general methodology and particular methods that are used, and makes suggestions as to the scope that exists for further research in this area.

9.1. The Achievements of the Thesis, with Particular Reference to the Research Objectives

The central objective of the research, as stated in Section 2.1., is the development of analytical expressions to describe the effect that both fleet size and time-window constraints have on the Total Cost of a distribution operation; the chapters that present the relevant analytical results, here, are Chapters 4 and 7.

In both of these chapters, the development of a precise expression is described - Equation E.4.21. estimates Total Fleet Mileage as a function of the total number of customers to be served and the constraint on the number that may be visited within the limits of a working-day, whilst E.7.2. quantifies the effect of time-window constraints, represented here by the width of each time-window, r, on Total Cost per week, (given basic information such as area-size, size of weekly consignment, etc.). Because of the nature of their development, however, these two expressions vary in the extent to which they may be used as predictive tools.

Equation E.4.21., first of all, can certainly be used for predicting Total Fleet Mileage in situations that use similar constraints and assumptions to those outlined in the problem formulation at the beginning of Chapter 4. It is true that this formulation describes a rather simplified abstraction of a distribution system, using basic concepts such as a continuous, homogeneous delivery-area, a uniform fleet of vehicles and non-overlapping vehicle-tours, etc., but this scenario conforms quite closely to the type used by many studies of the classical Travelling Salesman Problem, particularly those claiming to use the technique of Continuous Space Modelling, (SEE Section 1.2.2. for a description of this approach). As a means of predicting aggregate tour length within a delivery-zone of known dimensions, which contains a population of customers of known size requiring delivery of a given consignment of goods, therefore, Equation E.4.21. is applicable to a wide range of hypothetical problems of this type. As with all research activities in which a simplified , computerised model is used as a representation of complex system, there is a problem as to the extent to which the findings of such a model are applicable to the relevant real-life situation. However, one of the advantages of using a simulation-based methodology, among others which will be outlined in Section 9.3., is that the program employed may be readily elaborated in order to more closely imitate an actual distribution system, and the constraints and assumptions involved may be changed to enable the model to deal with a range of problem formulations.

The expression for Total Cost as a function of time-window width presented in Chapter 7, (Equation E.7.2.), provides an accurate means of estimating the cost of performing a given delivery-task within the confines of the problem formulation described in this chapter, and given the type of time-constraint that is assumed here. Although it is very unlikely that a distribution system would, in practice, be constrained by a time-window that is uniform for every customer in the delivery-schedule, the results produced by the simulation program demonstrate the behaviour of Total Cost estimates in response to increasingly stringent restrictions of this type. In other words, changes in the value of the precise independent variable, r, are used here as a representation of the varying degrees of severity of a set of, often diverse, constraints that inhibit the optimum scheduling of a fleet. The advantage of using r in this way lies in its flexibility, as it may take on a value of 1, to represent a scenario in which timing requirements are absolutely precise, and may take on a value that is equal to the length of a working-day, to represent a situation in which timing constraints are not significant. What is of interest, of course, is the effect that the parameter r has on Total Cost estimates as its value varies between these two extremes.

Because of the complexity of time-windows in practice, (Section 6.1. provides examples of some types of time-windows), there is a danger that the use of one particular type in a model might be an over-simplification of the phenomenon. It should be pointed out, however, that the program described in Chapter 6 is flexible enough to cater for the characteristics of many more types of time constraint. There would be no problem, for example, in allowing the variable r to vary randomly from customer to customer, or to manipulate the simulation process to ensure any degree of "clustering" of time-windows in time, (so that x% of all time-windows must open before 11am, for example); it is also possible for only a specified percentage of customers to specify timing constraints, a percentage which can, of course, be changed at each iteration. Despite these possibilites for increasing the variety of constraints that are imposed on a hypothetical system, the problem still remains as that of generalising the results of a simulation procedure to be able to draw general conclusions as to the effect that timing constraints have on Total Distribution Costs. This is to say that, whether the parameter to be varied in the simulation process is the percentage of customers imposing a constraint or the severity of the constraint at each location, its role as the independent variable in the analysis is exactly the same as that of r in the research that is described in Chapters 6 and 7.

The question still remains , though, as to whether the particular type of time-constraint described by r, and its effect of Distribution Cost, is really representative of both the constraints that are encountered in real systems, and the other types of constraint that may be applied to the same hypothetical situation. Clearly, this question can only be satisfactorily answered by more, similar research in this area, (SEE Section 9.3.). Nevertheless, the results gained from simulation so far do provide detailed information on the mechanisms that lead to the Total Cost curve of Figure 7.6. and to the form of the expression that describes it, indicating quite clearly that it is the increase in the number of vehicles that are required as time-constraints become more stringent that has the strongest influence on the observed relationship between r and Total Cost. Because it would seem logical to expect the increasing severity of any type of time-window to affect vehicle-tours by the same fundamental process of reducing the number of locations that may be visited in one day, thus increasing the number of round-trips that are required, there is every reason to also expect the cost curve so-produced to closely resemble that of Figure 7.6.. What is important, though, is the generality of the coefficient -0·177, of Equation E.7.2., which describes the slope of this curve; to obtain such quantitative information clearly requires further detailed analysis. Nevertheless, the conclusions that may be drawn from the current research, featuring just one type of time-window, still provide important insights into the nature of the effect of time-constraints on the costs of distribution, since no attempt has yet been made elsewhere to quantify this particular relationship.

The latter point refers to one of the secondary objectives described in Chapter 2, that of making a contribution to the work that has already been done in developing analytical expressions which may be used to make estimates of, say, the Total Mileage of a fleet, in a situation where there is no detailed data available at a customer-specific level. The context of the current research has already been brought into sharper focus by Figure 1.2., which pin-points the Travelling-Salesman Problem with Time-Windows as being the specific subject area to which the thesis seeks to contribute. In this respect, the current research has been successful in filling the under-researched niche that is described by the symbol "X" in this diagram. Although it should be pointed out that there is no novelty in considering the time-constrained Travelling-Salesman Problem itself - the work of both Baker, (1), and Savelsberg, (2), has already been described briefly in Chapter 1 - most research on time-windows has concentrated on the development of a convenient route-building procedure, and there is nothing in the literature that is comparable to the current thesis, considering the emphasis that is placed here on the relationship between the severity of time constraints and Distribution Costs.

Whereas a detailed model of the relationship between Total Fleet Mileage and the number of vehicles used is developed in the non-time-constrained context described in Chapter 4, it has not been possible to derive reliable equivalent expressions which take account of the effect of time-restrictions, given the information that has been generated so far. Whilst it is true that the general form of the relationship graphed in Figure 7. 21. is very reminiscent of the corresponding curves of Chapter 4, (eg. Figure 4.31.3.), it should be noted that the former graph is based on a far more limited body of data. Furthermore, comparison of output from these two chapters is hindered by the fact that the relevant simulation programs, on which all the reported results are based, utilise two very different routing algorithms - the program described in Chapter 4 is designed to generate optimum tours, based on the Savings Method, whilst the results of Chapter 7, because of the algorithmic difficulties caused by time-windows, rely on a myopic heuristic that is not guaranteed to produce optimum solutions when timing restrictions are ineffective. The extent to which analogies may be made between these two fundamentally separate pieces of research is therefore limited.

In addition to addressing the major research-objectives that are outlined in Chapter 2, the thesis has also produced a great deal of other information on the relationships that exist between the main variables of distribution operations - the following section discusses these findings, and highlights the other important contributions of the current research.

9.2. Other Important Contributions Resulting from the Research

A theme that is maintained throughout the thesis, as far as concerns the results that are presented, is the geometric nature of the relationships described; this is

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(1) BAKER, E.K., (op cit)

(2) SAVELSBERG, M.W.P., (op cit)

illustrated by graphs such as Figures 4.11., 4.24. and 7.6., which are all very similar in shape. Although the inverse relationship shown here between each pair of variables is not at all surprising - in the case of Figure 7.6., for instance, it is to be expected that Total Cost will decrease as time-window constraints are relaxed - it is nevertheless informative to note that the relationships plotted here are clearly geometric, as opposed to being linear. This is important as it indicates that, as the variable represented by the x-axis approaches a critical point, the value of the dependent variable will be expected to increase rapidly.

In the case of Figure 7.6., as the similarity with Figure 7.1. suggests, it is the rapid increase in the value of n - or, more precisely, in the number of separate vehicle-tours required, given a limit on the length of a working-day - that is mainly responsible for the geometric relationship between Distribution Cost and the severity of time constraints shown in this graph. A similar point might be made in relation to the relationship between Total Cost per week and delivery-area size that is shown in Figure 5.15.. In this situation, it is the fact that the average spacing between customers increases with the value of "a" that increases the time that it takes to visit a fixed number of customers. Because fewer locations may be served within the time allowed for each day, the number of vehicle-trips required rises, which, primarily by increasing the number of "stem-journeys" made to and from the operating-centre, increases Total Fleet Mileage.

The importance of fleet size to Distribution Cost is re-emphasised in Chapters 3 and 4, which demonstrate n's indirect influence on Total Cost per week through its effect on x, and the direct relationship between fleet size and Total Fleet Mileage, respectively.

It is Chapter 4's disaggregation of mileage figures into Delivery Distance and Stem Distance, and then down to "i", the average distance between consecutive stops in a tour, that provides interesting insights into the behaviour of the component parts of Travelling-Salesman tours as fleet size changes. Detailed information on the relationship between i and n, for example, (SEE Figures 4.22., 4.24., etc.), and disaggregated Marginal Cost (in terms of distance) data such as that illustrated by Figure 4.34., may be used as an aid to the building of more accurate models of distribution systems.

A useful product of the investigation of the effect of time-windows on Distribution Costs, because of the algorithmic difficulties that these restrictions create, is the development of an original procedure for constructing vehicle routes under such conditions, (SEE Chapter 6). Using a "myopic" Shortest Path technique, and being founded upon a concept of Generalised Cost that trades off time and distance that is saved against savings in "waiting time" at customer-locations, this algorithm is not designed with the aim of generating optimum solutions to Travelling-Salesman Problems. It does, however, have the flexibility to construct tours in the context of a variety of time-constrained formulations; although the current thesis assumes a relatively simple situation of a set of customers all specifying a time-window of the same width, there is no problem in adapting the program used, to build tours in a situation where only a certain percentage of customers state time restrictions, or where the time-windows in force are of more than one type. This procedure, described by Figure 7.8., is also flexible in that the Value of Time component, that provides a basis for the aforementioned trade-off between waiting-time and travelling-time, may be readily adjusted.

In order to evaluate the accuracy of this algorithm, of course, it would be necessary to compare its solutions with those of other algorithms using the same numerical examples; such an exercise would also have the utility of giving a better indication of how close Chapter 6's algorithm comes to producing optimum tours.

Also of interest in connection with the discussion, in Part 3, of the Time Constrained Travelling-Salesman Problem, is the demonstration of how the performance of an algorithm based on the Savings formula deteriorates as time-windows become narrower, (SEE Figure 7.16.). This is important, since the Savings principle is commonly used in route-construction algorithms. Again, though, these findings are based on one variety of time-window constraint, and so further analysis is required before general statements may be made as to the decreasing effectiveness of this widely-used criterion. This is one example of how the results of the current research have left room for such further analysis - the following section takes a broader view of the areas for further research that are suggested by the thesis as a whole.

9.3. Discussion of Areas Requiring Further Research

The key finding in the investigation into the effect of time-constraints on Distribution Cost is the form of Equation E.7.2., which describes the observed relationship between Total Cost per week estimates and the width of the time-window imposed at each customer-location. This equation has already been reproduced in Chapter 8; here, it is stressed that it is the extent to which the coefficient, "-0·177," can be taken to be a generalisation of the relationship described, that determines the usefulness of this finding, and thus of this section of the analysis. By the same token, it is the testing of the value of this coefficient in similar situations, and in relation to different problem-formulations, that should form the basis of further research emanating from the current thesis. For example, because the analysis has concentrated on one type of time-window, it would be informative to explore the value of this coefficient, and thus the precise form of the Total Cost/time-constraint relationship, when the time-windows imposed by customers are of a mixed nature, (ie. with a non-uniform value of r). It is an advantage of the simulation-based methodology that is used here that the program constructed for the generation of Travelling-Salesman tours, described by Figure 7.8., is flexible enough to enable this additional data to be produced, merely by introducing an extra random-element into this procedure. An alternative means of representing the severity of time-restrictions on the routing of a fleet is to hold r constant whilst varying the percentage of locations at which a time-window is imposed, which is the approach adopted by Savelsberg, (op cit). This scenario, along with the situation in which customers' time-windows are "clustered" in time, so as to create peaks and troughs of demand for a visit throughout the day, or the concession of having more than one time-window to aim at for each location, could also readily be simulated by making minor adjustments to the same program.

There are a number of other ways in which the nature of the time constraints imposed could be altered in order to make the process of stochastic simulation more closely approximate an actual distribution operation. For example, the assumption that there is an absolute requirement for every time-window to be "hit" could be replaced by means of a problem formulation which states that lateness is permitted, but that there is a cost associated with this lateness. This would require a "penalty" to be built in to the route-building algorithm for the late arrival at a customer's premises, to reflect the inconvenience that this customer is caused. A generalised Cost measure would have to be established for this loss of "customer-utility", in much the same way that a Generalised Cost index is calculated in Chapter 6 to assign a monetary value to the opportunity cost of waiting-time; this is arguably a more realistic formulation of the problem than the no-lateness condition currently imposed. There are, of course, a number of possibilities for elaborating on the basic situation of a set of customers all imposing a uniform time-window, although it should be re-emphasised that, for any attempt to study the effect of the increasing stringency of these constraints on the cost of an operation, it is the value of the equivalent coefficient to that shown in Equation E.7.2. that is of prime importance.

Given the detailed analysis of the relationship between Fleet Size and Total Fleet Mileage in Chapter 4, it would be useful for further research to establish a similar relationship in the presence of time-windows, as, given the fact that the algorithms used to generate vehicle-tours in Parts 2 and 3 are fundamentally different, it is not possible to do this satisfactorily on the basis of the results described in foregoing chapters. To a certain extent, this could be achieved by using the program described in Figure 7.8. to generate similar data for a wider range of n-values to those represented in Figure 7.21.. The main way in which Travelling-

Salesman tours constucted in a time-constrained and non-time-constrained situation differ is in the absence of the assumptions of non-overlapping routes in the former case, and it is this factor that might be expected to make the major difference in the Fleet Mileage/Fleet Size relationships observed. The extent to which conclusions can be drawn from the data on which Figure 7.21. is based is also limited by the fact that the independent variable in this instance, n, is itself a function of time-window width, and so the value of r is certainly not the same for each value of n.

As far as concerns the scope for further research beyond that which is presented in Chapter 4, the main areas would appear to involve sensitivity analyses. Again, as is the case with the research carried out on time-windows, because of the flexibility of the simulation technique adopted here, this may be readily achieved by changing the values of certain parameters that are built into the program; among the variables that could be altered in this way are the time that is spent at each outlet, l, and average road-speed, S. As well as performing sensitivity analyses on the program's parameters, a similar exercise may be carried out on the assumptions and constraints that form the basis of the problem formulation described in Chapter 2. For instance, the constaint that vehicle-tours must be non-overlapping may be relaxed, and tours may be built on the basis of different zoning strategies, such as those discussed in Section 4.1. and illustrated by Figure 4.3.. Similarly, the assumption that all customers are located randomly throughout the delivery-area may be abandoned, and varying degrees of spatial clustering of customers represented by making minor alterations to the program described in Figure 4.8..

In other words, having developed a simulation-type procedure to generate random customer-coordinates, Travelling-Salesman tours, and the Fleet Mileage and Total Cost data associated with them, a framework exists within which any number of sensitivity analyses may be performed, and problem formulations tested; for each formulation, as has been demonstrated here, it is possible to develop a set of analytical expressions such as those presented in Chapter 4.

The overall formulation of the problem considered here could be widened further to include factors such as variable demand, (both between customers and over time), multi-depot networks and non-uniform vehicle-fleets, although to do so would be to move away from the basic Travelling-Salesman Problem and address related subject-areas such as the Vehicle Loading Problem, the Depot Location Problem and the Fleet Mix Problem, respectively, (SEE Chapter 1 for a discussion of these formulations). This section has therefore concentrated mainly on the scope that exists for further research within the problem formulation that is outlined in Chapter 2, and is therefore confined to the area of interest that is defined by Figure 1.2..

A P P E N D I X   A

COST TABLES PUBLISHED BY COMMERCIAL MOTOR 1982

APPENDIX A. Cost tables published by Commercial Motor, 1982

GOODS VEHICLES RIGIDS 15cwt-2 ton Loads Diesel TABLE 2

Carrying capacity	*15cwt*	*20cwt*	*30cwt*	*40cwt*	*Carrying capacity*
Unladen weight	*21cwt*	*22cwt*	*30cwt*	*35cwt*	*Unladen weight*

STANDING COSTS

(per week)	£	£	£	£	(per week)
Licences	2.00	2.00	2.22	3.86	Licences
Wages	92.15	92.15	92.15	92 15	Wages
Rent and rates	7.68	8.02	8.45	9.06	Rent and rates
Insurance	3.84	3.84	3.84	3.84	Insurance
Interest	14.13	19.28	22.03	21.65	Interest
Total (per week)	119.80	125.29	128.69	130.56	Total (per week)
(pence per hour)	299.55	313.29	321.78	326.47	(pence per hour)

RUNNING COSTS

(pence per mile)	p	p	p	p	(pence per mile)
Fuel	4.29	5.00	6.00	6.00	Fuel
Lubricants	0.40	0.40	0.40	0.41	Lubricants
Tyres	1.26	1.33	1.41	3.41	Tyres
Maintenance	6.41	7.48	9.26	9.83	Maintenance
Depreciation	5.65	7.71	8.81	7.22	Depreciation
Total	18.01	21.92	25.88	26.87	Total

TOTAL OPERATING COST—per mile

Miles per week	p	p	p	p	Miles per week
100	137.83	147.24	154.59	157.46	100
200	77.92	84.58	90.24	92.16	200
300	57.95	63.69	68 78	70.40	300
400	47.96	53.25	58 06	59.52	400
500	41.97	46.98	51.62	52.99	500

TOTAL OPERATING COST—per week

Miles per week	£	£	£	£	Miles per week
100	137.83	147.24	154.59	157.46	100
200	155.84	169.16	180.47	184.33	200
300	173.85	191.08	206.35	211.20	300
400	191.86	213.00	232.23	238.07	400
500	209 87	234.92	258.11	264.94	500

MINIMUM CHARGE—per mile

Miles per week	p	p	p	p	Miles per week
100	192.96	206.14	216.43	220.44	100
200	109 09	118.41	126.34	129.02	200
300	81.13	89.17	96.29	98.56	300
400	67.14	74.55	81.28	83.33	400
500	58.76	65.77	72.77	74.19	500

MINIMUM CHARGE—per week

Miles per week	£	£	£	£	Miles per week
100	192.96	206.14	216.43	220.44	100
200	218.18	236 82	252.66	256.06	200
300	243.39	267.51	288.89	295.68	300
400	268 60	298 20	325.12	333.30	400
500	293.82	328.89	361.35	370.92	500

MINIMUM CHARGE—time plus mileage

	p	p	p	p	
Per hour	419.37	438.61	450.49	457.06	Per hour
Per mile	25.21	30.69	36.23	37.62	Per mile

SUPPORTING DATA ON WHICH COSTS ARE BASED

Cost of fuel (pence per gallon)	150	150	150	150	Cost of fuel (pence per gallon)
Fuel consumption (mpg)	35	30	25	25	Fuel consumption (mpg)
Cost of tyres (not spare)	£252	£331	£352	£852	Cost of tyres (not spare)
Mileage life of tyres (in thousands)	20	25	25	25	Mileage life of tyres (in thousands)
Mileage life of vehicle (in thousands)	75	75	75	90	Mileage life of vehicle (in thousands)
Cost of vehicle (less cost of tyres and residual value)	£4,240	£5,785	£6,610	£6,495	Cost of vehicle (less cost of tyres and residual value)

GOODS VEHICLES RIGIDS 3 ton-8 ton Loads Diesel TABLE 3

Carrying capacity / Unladen weight	3 tons / 2 tons	4 tons / 2½ tons	5 tons / 2¾ tons	6 tons / 3 tons	7 tons / 3½ tons	8 tons / 3¾ tons	5-ton van / 3 tons	Carrying capacity / Unladen weight
STANDING COSTS								
(per week)	£	£	£	£	£	£	£	(per week)
Licences	3.86	4.84	5.33	5.82	6.88	7.40	5.82	Licences
Wages	151.53	151.53	151.53	181.59	181.59	181.59	151.53	Wages
Rent and rates	9.46	10.40	10.80	11.30	11.46	12.24	10.69	Rent and rates
Insurance	11.97	13.11	13.64	16.51	17.93	18.82	14.60	Insurance
Interest	24.49	28.78	31.79	34.97	37.17	39.60	32.65	Interest
Total (per week)	201.31	208.66	213.09	250.19	255.03	255.65	215.29	Total (per week)
(pence per hour)	503.35	521.69	532.79	625.53	637.64	649.15	538.28	(pence per hour)
RUNNING COSTS								
(pence per mile)	p	p	p	p	p	p	p	(pence per mile)
Fuel	7.50	8.33	9.38	10.71	10.71	10.71	9.38	Fuel
Lubricants	0.45	0.45	0.45	0.50	0.50	0.50	0.45	Lubricants
Tyres	2.16	1.80	1.86	2.38	2.58	4.62	1.86	Tyres
Maintenance	8.27	8.87	10.49	11.70	13.11	14.79	12.43	Maintenance
Depreciation	8.16	9.59	10.60	11.66	12.39	13.20	10.89	Depreciation
Total	26.54	29.04	32.78	36.95	39.29	43.82	35.01	Total
TOTAL OPERATING COST—per mile								
Miles per week	p	p	p	p	p	p	p	Miles per week
200	127.21	133.38	139.34	162.06	166.82	173.65	142.67	200
400	76.87	81.21	86.06	99.50	103.05	108.74	88.84	400
600	60.10	63.82	68.30	78.65	81.80	87.10	70.90	600
800	51.71	55.12	59.42	68.23	71.17	76.28	61.92	800
1,000	46.67	49.91	54.09	61.97	64.80	69.79	56.54	1,000
TOTAL OPERATING COST—per week								
Miles per week	£	£	£	£	£	£	£	Miles per week
200	254.42	266.76	278.67	324.11	333.63	347.30	285.33	200
400	307.50	324.84	344.23	398.01	412.21	434.94	355.35	400
600	360.58	382.92	409.79	471.91	490.79	522.58	425.37	600
800	413.66	441.00	475.35	545.81	569.37	610.22	495.39	800
1,000	466.74	499.08	540.91	619.71	647.95	697.86	565.41	1,000
MINIMUM CHARGE—per mile								
Miles per week	p	p	p	p	p	p	p	Miles per week
200	178.09	186.73	195.08	226.88	233.55	243.11	199.74	200
400	107.62	113.69	120.48	139.30	144.27	152.24	124.38	400
600	84.14	89.35	95.62	110.11	114.52	121.94	99.26	600
800	72.39	77.17	83.19	95.52	99.64	106.79	86.69	800
1,000	65.34	69.87	75.73	86.76	90.72	97.71	79.16	1,000
MINIMUM CHARGE—per week								
Miles per week	£	£	£	£	£	£	£	Miles per week
200	356.19	373.46	390.14	453.75	467.08	486.22	399.46	200
400	430.50	454.78	481.92	557.21	577.09	608.92	497.49	400
600	504.81	536.09	573.71	660.67	687.11	731.61	595.52	600
800	579.12	617.40	665.49	764.13	797.12	854.31	693.55	800
1,000	653.44	698.71	757.27	867.59	907.13	977.00	791.57	1,000
MINIMUM CHARGE—time plus mileage								
	p	p	p	p	p	p	p	
Per hour	704.69	730.37	745.91	879.74	892.70	908.81	753.55	Per hour
Per mile	37.16	40.66	45.89	51.73	55.01	61.35	49.01	Per mile
SUPPORTING DATA ON WHICH COSTS ARE BASED								
Cost of fuel (pence per gallon)	150	150	150	150	150	150	150	Cost of fuel (pence per gallon)
Fuel consumption (mpg)	20	18	16	14	14	14	16	Fuel consumption (mpg)
Cost of tyres (not spare)	£540	£540	£652	£833	£903	£1,618	£652	Cost of tyres (not spare)
Mileage life of tyres (in thousands)	25	30	35	35	35	35	35	Mileage life of tyres (in thousands)
Mileage life of vehicle (in thousands)	90	90	90	90	90	90	90	Mileage life of vehicle (in thousands)
Cost of vehicle (less cost of tyres and residual value)	£7,347	£8,635	£9,539	£10,492	£11,151	£11,881	£9,797	Cost of vehicle (less cost of tyres and residual value)

GOODS VEHICLES RIGIDS 10 ton-20 ton Loads

TABLE 5

Carrying capacity / Unladen weight	10 tons / 5½ tons (6-wheeler)	17 tons / 7 tons (8-wheeler)	20 tons / 8 tons (Artic)	Carrying capacity / Unladen weight

STANDING COSTS

(per week)	£	£	£	(per week)
Licences	12.88	18.22	21.77	Licences
Wages	196.48	196.48	224.71	Wages
Rent and rates	12.21	12.21	13.12	Rent and rates
Insurance	23.60	32.86	35.91	Insurance
Interest	52.23	58.41	110.23	Interest
Total (per week)	279.40	318.18	405.74	Total (per week)
(pence per hour)	743.55	195.49	1,014.40	(pence per hour)

RUNNING COSTS

(pence per mile)	p	p	p	(pence per mile)
Fuel	12.50	12.50	15.00	Fuel
Lubricants	0.50	0.51	0.54	Lubricants
Tyres	3.59	5.80	6.51	Tyres
Maintenance	13.00	13.27	14.52	Maintenance
Depreciation	7.84	8.76	13.23	Depreciation
Total	37.43	40.84	49.80	Total

TOTAL OPERATING COST—per mile

Miles per week	p	p	p	Miles per week
400	111.79	120.39	151.24	400
600	87.00	93.87	117.43	600
800	74.61	80.61	100.52	800
1,000	67.17	72.66	90.38	1,000
2,000	52.30	56.75	70.09	2,000

TOTAL OPERATING COST—per week

Miles per week	£	£	£	Miles per week
400	447.14	481.56	604.96	400
600	522.00	563.24	704.56	600
800	596.86	644.92	804.16	800
1,000	671.72	726.60	903.76	1,000
2,000	1,046.02	1,135.00	1,401.76	2,000

MINIMUM CHARGE—per mile

Miles per week	p	p	p	Miles per week
400	156.51	168.55	211.74	400
600	121.80	131.42	164.40	600
800	104.45	112.85	140.73	800
1,000	94.04	101.72	126.53	1,000
2,000	73.22	79.45	98.13	2,000

MINIMUM CHARGE—per week

Miles per week	£	£	£	Miles per week
400	626.00	674.18	846.94	400
600	730.80	788.54	986.38	600
800	835.60	902.89	1,125.82	800
1,000	940.41	1,017.24	1,265.26	1,000
2,000	1,464.43	1,589.00	1,962.46	2,000

MINIMUM CHARGE—time plus mileage

	p	p	p	
Per hour	1,040.97	1,113.69	1,420.16	Per hour
Per mile	52.40	57.18	69.72	Per mile

SUPPORTING DATA ON WHICH COSTS ARE BASED

Cost of fuel (pence per gallon)	150	150	150	Cost of fuel (pence per gallon)
Fuel consumption (mpg)	12	12	10	Fuel consumption (mpg)
Cost of tyres (not spare)	£1,617	£2,608	£3,256	Cost of tyres (not spare)
Mileage life of tyres (in thousands)	45	45	50	Mileage life of tyres (in thousands)
Mileage life of vehicle (in thousands)	200	200	200	Mileage life of vehicle (in thousands)
Cost of vehicle (less cost of tyres and residual value)	£15,671	£17,524	£33,070	Cost of vehicle (less cost of tyres and residual value)

GOODS VEHICLES ARTICS 10 ton-22 ton Loads TABLE 6

Carrying capacity	10 tons	12 tons	14 tons	16 tons	18 tons	22 tons	21-ton van	Carrying capacity
Unladen weight	4 tons	4½ tons	5 tons	6½ tons	8 tons	10 tons	11 tons	Unladen weight

STANDING COSTS

(per week)	£	£	£	£	£	£	£	(per week)
Licences	9.95	12.42	14.20	20.28	25.62	38.95	43.68	Licences
Wages	196.48	196.48	196.48	196.48	196.48	224.71	224.71	Wages
Rent and rates	12.48	13.86	14.58	15.20	15.78	16.37	16.37	Rent and rates
Insurance	23.32	31.22	32.60	32.60	33.47	61.65	61.65	Insurance
Interest	56.78	60.22	67.09	77.39	77.63	108.92	121.21	Interest
Total (per week)	299.01	314.20	324.95	341.95	348.98	450.60	467.62	Total (per week)
(pence per hour)	747.58	785.54	812.40	854.92	872.50	1,126.54	1,169.61	(pence per hour)

RUNNING COSTS

(pence per mile)	p	p	p	p	p	p	p	(pence per mile)
Fuel	12.50	12.50	15.00	16.67	18.75	21.43	21.43	Fuel
Lubricants	0.50	0.50	0.50	0.51	0.51	0.51	0.51	Lubricants
Tyres	5.96	5.96	5.36	5.36	4.47	6.25	6.25	Tyres
Maintenance	11.31	11.77	13.96	14.92	15.49	16.29	22.55	Maintenance
Depreciation	8.52	9.03	10.06	9.67	9.32	13.07	14.55	Depreciation
Total	38.79	39.76	44.88	47.13	48.54	57.55	65.29	Total

TOTAL OPERATING COST—per mile

Miles per week	p	p	p	p	p	p	p	Miles per week
100	337.82	353.98	369.84	389.10	397.54	508.17	533.13	100
200	188.31	196.87	207.36	218.11	223.04	282.86	299.21	200
300	138.47	144.50	153.20	161.12	164.87	207.76	221.24	300
400	113.55	118.31	126.12	132.62	135.79	170.20	182.25	400
500	98.60	102.60	109.87	115.52	118.34	147.67	158.86	500
1,000	68.69	71.18	77.38	81.33	83.44	102.61	112.07	1,000

TOTAL OPERATING COST—per week

Miles per week	£	£	£	£	£	£	£	Miles per week
100	337.82	353.98	369.84	389.10	397.54	508.17	533.13	100
200	376.61	393.74	414.72	436.23	446.08	565.72	598.42	200
300	415.40	433.50	459.60	483.36	494.62	623.27	663.71	300
400	454.19	473.26	504.48	530.49	543.16	680.82	729.00	400
500	492.98	513.02	549.36	577.62	591.70	738.37	794.29	500
1,000	686.93	711.82	773.76	813.27	834.40	1,026.12	1,120.74	1,000

MINIMUM CHARGE—per mile

Miles per week	p	p	p	p	p	p	p	Miles per week
100	472.95	495.57	517.78	544.74	556.56	711.44	746.38	100
200	263.63	275.62	290.30	305.35	312.26	396.00	418.89	200
300	193.86	202.30	214.48	225.57	230.82	290.86	309.74	300
400	158.97	165.63	176.57	185.67	190.11	238.28	255.15	400
500	138.04	143.64	153.82	161.73	165.68	206.74	222.40	500
1,000	96.17	99.65	108.33	113.86	116.82	143.65	156.90	1,000

MINIMUM CHARGE—per week

Miles per week	£	£	£	£	£	£	£	Miles per week
100	472.95	495.57	517.78	544.74	556.56	711.44	746.38	100
200	527.25	551.24	580.61	610.72	624.51	792.01	837.79	200
300	581.56	606.90	643.44	676.70	692.47	872.56	929.19	300
400	635.67	662.56	706.27	742.69	828.38	953.15	1,020.60	400
500	690.17	718.23	769.10	808.67	828.38	1,033.72	1,112.01	500
1,000	961.70	996.55	1,083.26	1,138.58	1,168.16	1,436.57	1,569.04	1,000

MINIMUM CHARGE—time plus mileage

	p	p	p	p	p	p	p	
Per hour	1,046.61	1,099.76	1,137.36	1,196.89	1,221.50	1,577.16	1,637.45	Per hour
Per mile	54.31	55.66	62.83	65.98	67.96	80.57	91.41	Per mile

SUPPORTING DATE ON WHICH COSTS ARE BASED

Cost of fuel (pence per gallon)	150	150	150	150	150	150	150	Cost of fuel (pence per gallon)
Fuel consumption (mpg)	12	12	10	9	8	7	7	Fuel consumption (mpg)
Cost of tyres (not spare)	£2,680	£2,680	£2,680	£2,680	£2,680	£3,749	£3,749	Cost of tyres (not spare)
Mileage life of tyres (in thousands)	45	45	50	50	60	60	60	Mileage life of tyres (in thousands)
Mileage life of vehicle (in thousands)	200	200	200	240	250	250	250	Mileage life of vehicle (in thousands)
Cost of vehicle (less cost of tyres and residual value)	£17,036	£18,066	£20,127	£23,218	£23,289	£32,676	£36,364	Cost of vehicle (less cost of tyres and residual value)

Conversion of 1982 Figures to 1989 levels

According to the corresponding figures published by Commercial Motor magazine for 1989, (1), Running Costs per mile have changed only negligibly since 1982 for the whole range of vehicle-sizes displayed in this Appendix, (with several figures showing a slight decrease).

Standing Cost per week figures for rigid-chassis vehicles, however, show an increase of between 30·8% and 32·7% over this 7-year period, whilst the same figures for an 18-ton articulated vehicle, the only vehicle-category of this type that appears in both sets of figures, indicate an increase of 49·5%.

(1) Commercial Motor Buyers' Guide, 1989. (Reed Business Publishing Ltd., 1989).

APPENDIX B

TABLES

Table 3.1. Expressions describing transport costs in pence per ton per mile

	Running Cost	Standing Cost
Rigid-Chassis Vehicles	$21{\cdot}3796\ x^{-0{\cdot}727}$	$12.106\ x^{-0{\cdot}629}$
Articulated Vehicles	$11{\cdot}885\ x^{-0{\cdot}5}$	$9{\cdot}8175\ x^{-0{\cdot}535}$

Table 3.2. Cost figures published by the Freight Transport Association

tonnes gvw	£ per year			TOTAL STANDING COST	p per mile			TOTAL RUNNING COST	TOTAL COST
	V.E.D.	Insurance	Depreciation		Fuel	Tyres	Maintenance		
≤ 3·5 (petrol)	106	142	1161	1409	10·58	0·63	7·66	18·87	5 499
≤ 3·5 (diesel)	105	168	1638	1911	6·58	0·76	4·84	12·18	5 858
7 - 5	130	172	2498	2800	9·52	1·07	13·02	23·61	19087
9 - 13	370	169	2500	3039	10·86	1·32	13·36	25·54	22 656
13 - 16	992	207	3449	4648	12·53	1·6	9·06	23·19	27 269
≤ 24 (artic)	972	257	3650	4879	16·04	1·21	10·93	28·18	31 143
32·52 (artic)	2450	224	4689	7363	18·56	2·87	11·34	32·77	41583
38 (artic)	3018	282	5494	8794	19·86	5·03	9·23	34·12	47 351

Table 3.3. The percentage of Total Cost attributable to each cost component

Vehicle Size (tons)	Wages	Insurance	Rent	Licences	Interest	Tyres	Maintenance	Fuel	Depreciation
0·75	30·7	1·28	2·56	0·67	4·71	4·2	21·4	14·3	18·8
1	26·75	1·11	2·33	0·58	5·60	3·9	21·7	14·5	22·4
1·5	23·78	0·99	2·18	0·57	5·69	3·6	23·9	15·5	22·7
2	23·08	0·96	2·27	0·97	5·42	8·5	24·6	15·0	18·1
3	32·47	2·56	2·03	0·83	5·25	4·6	17·7	16·1	17·5
4	30·36	2·63	2·08	0·97	5·77	3·6	17·8	16·7	19·2
5	28·01	2·52	2·00	0·99	5·88	3·4	19·4	17·3	19·6
6	29·33	2·67	1·82	0·94	5·65	3·8	18·9	17·3	18·8
7	28·03	2·77	1·77	1·06	5·74	4·0	20·2	16·5	19·1
8	26·17	2·71	1·76	1·07	5·71	6·7	21·3	15·4	19·0
10	30·06	3·61	1·87	1·97	7·99	5·5	19·9	19·1	12·0
17	27·03	4·52	1·68	2·31	8·04	7·9	18·3	17·2	12·0
20	24·87	3·97	1·45	2·41	12·20	7·2	16·1	16·6	14·6

Table 3.7. Estimates of Total Cost per week as a function of td

t	x	vehicle type	n	(£ per wk) Standing Cost	(£ per wk) Running Cost	(£ per wk) Total Cost	(p per ton per mile) Average Cost
5	1	rigid	1	121	106	227	46·0
10	2	rigid	1	156	129	286	28·6
15	3	rigid	1	182	144	326	21·7
20	4	rigid	1	202	156	359	18·0
25	5	rigid	1	220	166	386	15·4
50	10	artic	1	286	188	474	9·5
75	17	rigid	1	306	204	510	6·8
100	20	rigid	1	368	242	610	6·1
125	25	artic	1	439	297	736	5·9
150	30	artic	1	477	325	803	5·3
175	38	artic	1	491	337	828	4·7
200	20	rigid	2	736	484	1220	6·1
225	25	artic	2	789	535	1324	5·9
250	25	artic	2	877	594	1471	5·9
300	30	artic	2	955	651	1606	5·3
350	35	artic	2	1026	703	1729	4·9
360	38	artic	2	1010	694	1704	4·7
400	30	artic	3	1273	868	2141	5·3
425	30	artic	3	1353	922	2275	5·3
450	30	artic	3	1432	976	2409	5·3
500	35	artic	3	1465	1004	2470	4·9
550	38	artic	3	1542	1060	2603	4·7

Table 3.8. Estimates of cost as a function of vehicle size, using Equations E.3.9., E.3.10. and E.3.11

x	artic/rigid	n	(£/wk) Standing Cost	(£/wk) Running Cost	(£/wk) Total Cost
0·75	r	51	5 513	5 007	10 520
1	r	38	4 600	4 062	8 662
1·5	r	26	3 565	3 025	6 590
2	r	19	2 975	2 454	5 429
3	r	13	2 305	1 828	4 133
4	r	10	1 923	1 483	3 406
5	r	8	1 672	1 261	2 932
6	r	7	1 490	1 104	2 595
8	r	5	1 244	896	2 140
10	r	4	1 081	762	1 843
12	a	4	987	652	1 639
14	a	3	909	604	1 513
16	a	3	846	565	1 411
17	r	3	774	518	1 292
18	a	3	795	532	1 327
20	r	2	699	460	1 159
22	a	2	714	481	1 195
25	a	2	667	452	1 118
30	a	2	605	412	1 017
35	a	2	557	382	939
38	a	1	533	366	899

Table 3.9. Estimates of cost as a function of fleet size, using Equation E.3.10., (a=50)

n	X	Daily Fleet Mileage	(£/wk) Standing Cost	(£/wk) Running Cost	(£/wk) Total Cost
1	20	293·7	395·35	780·53	1175·88
2	10	322·0	572·84	605·10	1177·94
4	5	378·6	830·01	503·08	1333·08
5	4	406·9	935·25	483·60	1418·85
10	2	548·4	1355·13	460·87	1816·00
20	1	831·4	1963·50	494·06	2457·56
40	½	1397·4	2845·00	587·18	3432·18

Table 3.10 Estimates of cost as a function of fleet size, using Equation E.3.10., (a=150)

n	x	TFM	td	(£/wk) Standing Cost	(£/wk) Running Cost	(£/wk) Total Cost
1	20	881·1	88110	395	2342	2737
2	10	966·0	48300	573	1815	2388
4	5	1135·8	28395	830	1509	2339
5	4	1220·7	24414	935	1451	2386
10	2	1645·2	16452	1355	1383	2738
20	1	2494·2	12471	1963	1482	3446
40	½	4192·2	10480	2845	1761	4607

Table 4.1. Results of manual tour-building exercise. (p=50 a=1 (square))

Nº of vehicles	Ave. Total Fleet Mileage
1	6.004
2	6.684
4	7.251
6	8.695
9	10.533
10	10.412
11	11.768
12	12.408
13	11.754
14	13.793
15	13.971

Table 4.2. Estimates of Total Fleet Mileage. (P=50, a=1 (square))

Nº of vehicles	Using E.4.2. based on manually-constructed tours	Using computerised Savings Method	Using Eilon et al's formula
1	5.874	—	5.55
2	6.440	6.617	6.24
3	7.006	7.015	6.94
4	7.572	7.680	7.64
5	8.138	7.920	8.34
6	8.704	8.855	9.04
7	9.270	9.130	9.75
8	9.836	9.668	10.44
9	10.402	—	11.20
10	10.968	10.830	11.83
11	11.534	—	12.61
12	12.100	12.232	13.32
13	12.666	—	13.93
14	13.232	—	14.87
15	13.798	14.274	15.33

Table 4.4. Disaggregated distance figures. (P=100, a=1 (square))

n	C	Stem Distance	Delivery Distance	Total Distance
3	33	0·621	8·595	9·217
4	25	1·090	8·446	9·536
5	20	1·799	7·792	9·591
6	17	2·106	8·391	10·497
7	14	2·810	8·768	11·578
8	13	2·855	9·388	12·243
10	10	4·134	9·160	13·294
12	8	4·867	9·340	14·206
14	7	7·022	8·567	15·589
16	6	7·480	9·135	16·615
18	6	8·888	9·653	18·541
20	5	10·890	9·168	20·058
22	5	11·555	9·885	21·440
25	4	13·967	9·293	23·260
35	3	19·330	9·579	28·909

Table 4.5. Stem Distance as a percentage of Total Distance (P=100, a=1 (square))

n	C	%
3	33	6·73
4	25	11·51
5	20	18·70
6	17	19·99
7	14	24·41
8	13	23·32
10	10	31·01
12	8	34·26

n	C	%
14	7	45·05
16	6	44·83
18	6	48·11
20	5	54·49
22	5	53·66
25	4	58·91
35	3	65·98

Table 4.6. Disaggregated distances per vehicle. (P=100, a=1 (square))

n	C	Stem Distance	Delivery Distance	Total Distance
3	33	0·207	2·865	3·072
4	25	0·272	2·112	2·384
5	20	0·360	1·559	1·918
8	13	0·357	1·173	1·530
10	10	0·413	0·916	1·329
12	8	0·406	0·778	1·183
16	6	0·468	0·571	1·038
20	5	0·545	0·458	1·003
25	4	0·559	0·372	0·930
35	3	0·552	0·274	0·826

Table 4.7. Stem Distance per vehicle, with C fixed. (C=20, a=1 (circle))

n	Stem Dist.	n	Stem Dist.
1	0·265	10	0·219
2	0·282	12	0·232
3	0·269	15	0·233
4	0·291	20	0·250
5	0·280	25	0·259
6	0·214	35	0·274
8	0·269		

Table 4.8. The relationship between D_r and n, with square and circular delivery-areas.

n	(a x a) Square area Dr	(diameter = a) Circular area Dr
1	0·413	0·310
2	0·388	0·335
3	0·370	0·333
4	0·380	0·336
5	0·407	0·337
6	0·402	0·319
7	0·395	
8	0·383	0·338
10	0·382	0·328
12	0·323	0·334
14	0·377	
15	0·347	0·333
16	0·343	
18	0·350	
20	0·365	0·343
22	0·339	
25	0·336	0·335
35	0·299	0·328
40	0·346	
50	0·325	
100	0·326	

Table 4.9. Stem Distance per vehicle with n fixed. (n=20, a=1 (circle))

C	Stem Dist.	C	Stem Dist.
1	0.663	10	0.399
3	0.593	12	0.317
4	0.609	15	0.299
5	0.473	20	0.283
6	0.535	25	0.251
8	0.379	35	0.198

Table 4.10. Partially-transformed data from Table 4.9..

Log.C	Stem Dist. per vh	Log.C	Stem Dist. per vh
0	0.663	1	0.399
0.477	0.593	1.0792	0.317
0.602	0.607	1.1761	0.299
0.699	0.473	1.301	0.283
0.778	0.535	1.398	0.251
0.903	0.379	1.544	0.198

Table 4.11. Comparison of observed and predicted figures for Stem Distance per vehicle

C	Observed Stem	Predicted Stem	C	Observed Stem	Predicted Stem
1	0.663	0.7285	10	0.399	0.3867
3	0.593	0.565	12	0.317	0.3596
4	0.609	0.5227	15	0.299	0.3265
5	0.473	0.4896	20	0.283	0.2838
6	0.535	0.4625	25	0.251	0.2507
8	0.379	0.4198	35	0.198	0.2007

Table 4.12. Variation in the number of inter-nodal links, as n and C change. (P=100)

n	C	Inter-nodal links	n	C	Inter-nodal links
1	100	99	12	8	84
2	50	98	14	7	84
3	33	96	16	6	80
4	25	96	18	6	90
5	20	95	20	5	80
6	17	96	25	4	75
7	14	91	35	3	70
8	12	88	50	2	50
10	10	90			

(N.B. Number of inter-nodal links = n (C-1))

Table 4.13. The relationship between i and n. (P=100, a=1 (circle))

n	C	i	n	C	i
4	25	0·0808	13	8	0·0999
5	20	0·073	14	7	0·0973
6	17	0·0714	15	7	0·0948
7	14	0·076	16	6	0·1083
8	13	0·076	18	6	0·1068
9	11	0·091	20	5	0·1081
10	10	0·089	25	4	0·1069
11	9	0·094	35	3	0·1177
12	8	0·0984			

Table 4.14. The relationship between i and n with C fixed, (C=20, a=1 (circle))

n	observed i	predicted i	residuals
1	0·171	0·153	+0·018
2	0·127	0·118	+0·009
3	0·101	0·101	—
4	0·089	0·090	−0·001
5	0·071	0·083	−0·012
6	0·071	0·077	−0·006
8	0·062	0·069	−0·007
10	0·062	0·064	−0·002
12	0·060	0·059	+0·001
15	0·055	0·054	+0·001
20	0·051	0·049	+0·002
25	0·047	0·045	+0·002
35	0·042	0·039	+0·003

Table 4.15. The relationship between i and n, with C fixed (C=15, a=1 (circle))

n	Observed i	Predicted i	Residuals
1	0·183	0·169	+0·014
2	0·142	0·132	+0·010
3	0·122	0·114	+0·008
4	0·099	0·102	−0·003
5	0·083	0·094	−0·011
6	0·080	0·088	−0·008
7	0·082	0·084	−0·002
8	0·077	0·080	−0·003
10	0·072	0·074	−0·002
12	0·071	0·069	+0·002
14	0·056	0·065	−0·009
18	0·061	0·059	+0·002
25	0·058	0·053	+0·005
35	0·052	0·047	+0·005

Table 4.16. The relationship between i and n, with C fixed. (C=10,a=1 (circle))

n	Observed i	Predicted i	Residuals
1	0·242	0·210	+0·032
2	0·182	0·164	+0·018
3	0·142	0·142	—
4	0·125	0·128	−0·003
5	0·105	0·118	−0·013
6	0·107	0·111	−0·004
7	0·087	0·105	−0·018
8	0·093	0·100	−0·007
10	0·083	0·092	−0·009
12	0·085	0·086	−0·001
14	0·082	0·082	—
16	0·080	0·078	+0·002
18	0·069	0·075	−0·006
20	0·079	0·072	+0·007
25	0·072	0·066	+0·006
35	0·069	0·059	+0·010

Table 4.17. The relationship between i and C, with n fixed (n=20, a=1 (circle))

C	Observed i	Predicted i	Residuals
3	0·141	0·146	−0·005
4	0·110	0·124	−0·014
5	0·129	0·109	+0·020
6	0·091	0·098	−0·007
8	0·087	0·083	+0·004
10	0·076	0·074	+0·002
12	0·070	0·066	+0·004
15	0·060	0·058	+0·002
20	0·049	0·050	−0·001
25	0·043	0·044	−0·001
35	0·035	0·036	−0·001

Table 4.18. Observed and predicted values for i, with both n and C variable. (P=100, a=1 (circle))

n	C	Observed i	Predicted i	Residuals
1	100	—	0·062	—
2	50	—	0·070	—
3	33	—	0·075	—
4	25	0·081	0·079	+0·002
5	20	0·073	0·082	−0·009
6	17	0·071	0·083	−0·012
7	14	0·076	0·088	−0·012
8	13	0·076	0·087	−0·011
9	11	0·091	0·091	—
10	10	0·089	0·092	−0·003
11	9	0·094	0·094	—
12	8	0·098	0·097	+0·001
13	8	0·100	0·095	+0·005
14	7	0·097	0·099	−0·002
15	7	0·095	0·096	−0·001
16	6	0·108	0·104	+0·004
18	6	0·106	0·098	+0·008
20	5	0·108	0·104	+0·004
25	4	0·106	0·108	−0·002
35	3	0·117	0·112	+0·005
50	2	—	0·122	—

Table 4.19. Observed and predicted values for Total Distance, Stem Distance and Delivery Distance, with both n and C variable. (P=100, a=1 (circle))

n	C	Observed Stem Distance	Predicted Stem Distance	Residual	Observed Delivery Distance	Predicted Delivery Distance	Residual	OBSERVED TOTAL FLEET MILEAGE	PREDICTED TOTAL FLEET MILEAGE	RESIDUAL
3	35	—	0·500	—	—	7·416	—	—	7·916	—
4	25	0·880	0·907	-0·027	7·758	7·537	+0·221	8·639	8·444	+0·195
5	20	1·510	1·334	+0·176	6·931	7·756	-0·825	8·442	9·090	-0·648
6	17	2·080	1·775	+0·305	6·859	8·007	-0·148	8·939	9·782	-0·843
7	14	2·505	2·314	+0·191	6·912	7·968	-1·056	9·418	10·282	-0·864
8	13	2·428	2·750	-0·322	—	8·334	—	10·506	11·084	-0·578
9	11	2·874	3·363	-0·489	—	8·191	—	11·060	11·554	-0·494
10	10	3·881	3·907	-0·026	7·970	8·299	-0·329	11·852	12·206	-0·354
11	9	4·178	4·504	-0·326	8·272	8·297	-0·025	12·450	12·801	-0·351
12	8	4·738	5·166	-0·428	8·270	8·176	+0·094	13·008	13·342	-0·334
13	8	5·407	5·597	-0·190	9·086	8·603	+0·483	14·494	14·200	+0·294
14	7	6·557	6·362	+0·195	8·172	8·307	-0·135	14·729	14·669	+0·060
15	7	6·803	6·816	-0·013	—	8·680	—	15·333	15·496	-0·163
16	6	7·670	7·711	-0·041	8·666	8·190	+0·476	16·336	15·901	+0·435
18	6	7·821	8·675	-0·854	—	8·827	—	17·431	17·502	-0·071
20	5	10·369	10·290	+0·079	8·647	8·332	+0·315	19·016	18·622	+0·394
25	4	14·800	13·860	+0·940	8·016	8·123	-0·007	22·816	21·983	+0·833
35	3	22·515	21·204	+1·311	8·291	7·834	+0·457	30·806	29·038	+1·768

Table 4.20. Estimates of Stem Distance, Delivery Distance and Total Distance, using Equation E.4.21., with k fixed. (k=10, a=1 (circle))

P	DELIVERY	STEM	TOTAL	n	C	DELIVERY	STEM	TOTAL
1	0.443241	0.039064	0.482305	1	1	0	0.802	0.802
2	0.688898	0.078129	0.767027					
3	0.891629	0.117194	1.008823					
4	1.070706	0.156259	1.226965					
5	1.234026	0.195323	1.429350	1	5	1.239	0.515	1.753
6	1.385797	0.234388	1.620185					
7	1.528591	0.273452	1.802043					
8	1.664124	0.312517	1.976641					
9	1.793614	0.351582	2.145196					
10	1.917961	0.390647	2.308608	1	10	1.918	0.391	2.309
11	2.037858	0.429711	2.467569					
12	2.153847	0.468776	2.622624					
13	2.266369	0.507841	2.774210					
14	2.375781	0.546905	2.922687					
15	2.482385	0.585970	3.068355	2	7.5	2.514	0.884	3.398
16	2.586431	0.625035	3.211467					
17	2.688137	0.664099	3.352237					
18	2.787688	0.703164	3.490853					
19	2.885246	0.742229	3.627476					
20	2.980953	0.781294	3.762247	2	10	2.981	0.781	3.762
21	3.074934	0.820358	3.895293					
22	3.167300	0.859423	4.026723					
23	3.258150	0.898488	4.156638					
24	3.347575	0.937552	4.285127					
25	3.435653	0.976617	4.412271					

P	DELIVERY	STEM	TOTAL	n	C	DELIVERY	STEM	TOTAL
26	3.522459	1.015682	4.538141	4	8.25	4.14	1.7	5.84
27	3.608059	1.054746	4.662805					
28	3.692511	1.093811	4.786323					
29	3.775874	1.132876	4.908750					
30	3.858197	1.171941	5.030138					
31	3.939528	1.211005	5.150534					
32	4.019910	1.250070	5.269980					
33	4.099382	1.289135	5.388518					
34	4.177984	1.328199	5.506184					
35	4.255749	1.367264	5.623013					
36	4.332709	1.406329	5.739038					
37	4.408896	1.445393	5.854289					
38	4.484337	1.484458	5.968795					
39	4.559059	1.523523	6.082582					
40	4.633087	1.562588	6.195675	4	10	4.633	1.563	6.196
41	4.706445	1.601652	6.308098					
42	4.779155	1.640717	6.419872					
43	4.851237	1.679782	6.531019					
44	4.922713	1.718846	6.641559					
45	4.993599	1.757911	6.751511					
46	5.063915	1.796976	6.860891					
47	5.133677	1.836040	6.969718					
48	5.202901	1.875105	7.078007					
49	5.271602	1.914170	7.185773					
50	5.339795	1.953235	7.293030	5	10	5.34	1.953	7.293

P	DELIVERY	STEM	TOTAL	n	C	DELIVERY	STEM	TOTAL
51	5.407494	1.992299	7.399794					
52	5.474711	2.031364	7.506076					
53	5.541460	2.070429	7.611889					
54	5.607752	2.109493	7.717246					
55	5.673599	2.148558	7.822157					
56	5.739012	2.187623	7.926635					
57	5.804001	2.226687	8.030689					
58	5.868577	2.265752	8.134329					
59	5.932749	2.304817	8.237566					
60	5.996526	2.343882	8.340408	6	10	5.997	2.344	8.34
61	6.059918	2.382946	8.442864					
62	6.122933	2.422011	8.544944					
63	6.185579	2.461076	8.646655					
64	6.247864	2.500140	8.748005					
65	6.309797	2.539205	8.849002					
66	6.371383	2.578270	8.949654	7	9.43	6.394	2.808	9.202
67	6.432632	2.617334	9.049967					
68	6.493548	2.656399	9.149948					
69	6.554140	2.695464	9.249604					
70	6.614412	2.734529	9.348941					
71	6.674373	2.773593	9.447966					
72	6.734027	2.812658	9.546685					
73	6.793380	2.851723	9.645103					
74	6.852438	2.890787	9.743226					
75	6.911207	2.929852	9.841059					

P	DELIVERY	STEM	TOTAL	n	C	DELIVERY	STEM	TOTAL
76	6.969691	2.968917	9.938608					
77	7.027895	3.007981	10.03587					
78	7.085826	3.047046	10.13287					
79	7.143487	3.086111	10.22959					
80	7.200883	3.125176	10.32605	8	10	7.201	3.125	10.326
81	7.258018	3.164240	10.42225					
82	7.314898	3.203305	10.51820					
83	7.371526	3.242370	10.61389					
84	7.427906	3.281434	10.70934					
85	7.484042	3.320499	10.80454					
86	7.539939	3.359564	10.89950					
87	7.595599	3.398628	10.99422					
88	7.651028	3.437693	11.08872					
89	7.706227	3.476758	11.18298					
90	7.761202	3.515823	11.27702	9	10	7.761	3.516	11.277
91	7.815955	3.554887	11.37084					
92	7.870489	3.593952	11.46444					
93	7.924808	3.633017	11.55782					
94	7.978915	3.672081	11.65099					
95	8.032813	3.711146	11.74395					
96	8.086505	3.750211	11.83671					
97	8.139994	3.789275	11.92927					
98	8.193282	3.828340	12.02162					
99	8.246374	3.867405	12.11377					
100	8.299270	3.90647	12.20574	10	10	8.299	3.906	12.205

Table 4.21.1. Marginal Cost, in terms of distance travelled as a function of P, with k fixed, using Equation E.4.21. (a=1 (circle))

P	k=10	k=5
1	0.234722	0.349529
2	0.241795	0.306434
3	0.218141	0.282687
4	0.202385	0.266868
5	0.190835	0.255272
6	0.181858	0.246260
7	0.174598	0.238971
8	0.168554	0.232904
9	0.163412	0.227741
10	0.158961	0.223273
11	0.155054	0.219351
12	0.151586	0.215869
13	0.148477	0.212748
14	0.145667	0.209927
15	0.143111	0.207360
16	0.140770	0.205010
17	0.138615	0.202847
18	0.136622	0.200846
19	0.134771	0.198988
20	0.133045	0.197255
21	0.131430	0.195634
22	0.129915	0.194112
23	0.128438	0.192680
24	0.127143	0.191329
25	0.125870	0.190051
26	0.124663	0.188840
27	0.123517	0.187690
28	0.122427	0.186595
29	0.121387	0.185551
30	0.120395	0.184555
31	0.119446	0.183602
32	0.118537	0.182690
33	0.117666	0.181815
34	0.116829	0.180975
35	0.116025	0.180167
36	0.115251	0.179390
37	0.114505	0.178642
38	0.113786	0.177920
39	0.113092	0.177224
40	0.112422	0.176551
41	0.111774	0.175900
42	0.111147	0.175270
43	0.110539	0.174661
44	0.109951	0.174070
45	0.109380	0.173497
46	0.108826	0.172941
47	0.108298	0.172400
48	0.107765	0.171876
49	0.107257	0.171365
50	0.106763	0.170869
51	0.106282	0.170386
52	0.105813	0.169915
53	0.105356	0.169457
54	0.104911	0.169010
55	0.104477	0.168574
56	0.104053	0.168149
57	0.103640	0.167734
58	0.103236	0.167328
59	0.102842	0.166932
60	0.102456	0.166545
61	0.102079	0.166167
62	0.101710	0.165797
63	0.101350	0.165435
64	0.100997	0.165080
65	0.100651	0.164733
66	0.100312	0.164393
67	0.099981	0.164060
68	0.099656	0.163734
69	0.099337	0.163414
70	0.099025	0.163100
71	0.098718	0.162793
72	0.098417	0.162491
73	0.098122	0.162195
74	0.097833	0.161904
75	0.097548	0.161618
76	0.097269	0.161338
77	0.096995	0.161062
78	0.096725	0.160792
79	0.096460	0.160526
80	0.096200	0.160264
81	0.095944	0.160007
82	0.095692	0.159755
83	0.095444	0.159506
84	0.095201	0.159261
85	0.094961	0.159021
86	0.094725	0.155784
87	0.094492	0.158551
88	0.094264	0.158321
89	0.094039	0.158095
90	0.093817	0.157872
91	0.093598	0.157653
92	0.093383	0.157437
93	0.093171	0.157224
94	0.092962	0.157014
95	0.092756	0.156807
96	0.092553	0.156603
97	0.092353	0.156402
98	0.092155	0.156204
99	0.091961	0.156009
100		

Table 4.21.2. Marginal Cost, in terms of distance travelled as a function of n, with P fixed, using Equation E.4.20.. (a=1 (circle))

n	P=100	P=50
1	0.946027	0.318858
2	0.748471	0.695152
3	0.677772	0.656360
4	0.645827	0.642343
5	0.630313	0.638298
6	0.623015	0.638938
7	0.620256	0.641969
8	0.620168	0.646269
9	0.621721	0.651243
10	0.624309	0.656558
11	0.627558	0.662018
12	0.631228	0.667507
13	0.635162	0.672954
14	0.639253	0.678317
15	0.643429	0.683571
16	0.647641	0.688700
17	0.651852	0.693698
18	0.656038	0.698561
19	0.660182	0.703290
20	0.664273	0.707888
21	0.668302	0.712356
22	0.672263	0.716700
23	0.676154	0.720923
24	0.679973	0.725030
25	0.683718	0.729026
26	0.687390	0.732916
27	0.690990	0.736703
28	0.694518	0.740393
29	0.697976	0.743989
30	0.701364	0.747496
31	0.704686	0.750917
32	0.707942	0.754256
33	0.711135	0.757517
34	0.714265	0.760702
35	0.717336	0.763815
36	0.720348	0.766859
37	0.723304	0.769837
38	0.726205	0.772751
39	0.729053	0.775603
40	0.731850	0.778397
41	0.734596	0.781133
42	0.737295	0.783816
43	0.739946	0.786446
44	0.742552	0.789025
45	0.745115	0.791555
46	0.747634	0.794038
47	0.750112	0.796476
48	0.752550	0.798870
49	0.754948	0.801221

Table 4.22. Dispersion statistics of Stem Distance per vehicle as a function of C

	C=5	C=10	C=20
Mean	561·5	423·4	322·8
Standard Deviation	165·44	153·25	120·26
Skewness	0·018	0·515	0·691
Kurtosis	2·384	3·592	3·289
Number of observations	93	223	67

Table 4.23. Stem Distance as a percentage of Total Fleet Mileage, as estimated from Equation E.4.21.. (k=10, a=1)

P	%	P	%	P	%	P	%	P	%	P	%
1	8.99572	19	20.45131	37	24.68948	55	27.46759	73	29.56653	91	31.26318
2	10.13599	20	20.76668	38	24.87032	56	27.59838	74	29.66971	92	31.34869
3	11.51690	21	21.06025	39	25.04731	57	27.72723	75	29.77171	93	31.43339
4	12.73538	22	21.34299	40	25.22062	58	27.85420	76	29.87256	94	31.51731
5	13.66519	23	21.61573	41	25.39042	59	27.97934	77	29.97228	95	31.60046
6	14.46674	24	21.87922	42	25.55685	60	28.10272	78	30.07090	96	31.68256
7	15.17459	25	22.13412	43	25.72005	61	28.22438	79	30.16844	97	31.76452
8	15.81053	26	22.38101	44	25.88016	62	28.34437	80	30.26494	98	31.84545
9	16.38928	27	22.62043	45	26.03730	63	28.46275	81	30.36040	99	31.92567
10	16.92131	28	22.85285	46	26.19158	64	28.57955	82	30.45487	100	32.00518
11	17.41436	29	23.07870	47	26.34311	65	28.69482	83	30.54834		
12	17.87432	30	23.29838	48	26.49199	66	28.80860	84	30.64086		
13	18.30578	31	23.51223	49	26.63833	67	28.92093	85	30.73244		
14	18.71242	32	23.72058	50	26.78221	68	29.03185	86	30.82309		
15	19.09721	33	23.92374	51	26.92371	69	29.14140	87	30.91284		
16	19.46260	34	24.12196	52	27.06293	70	29.24960	88	31.00171		
17	19.81064	35	24.31551	53	27.19993	71	29.35651	89	31.08971		
18	20.14305	36	24.50461	54	27.33479	72	29.46214	90	31.17686		

Table 4.24. Comparison between Stem Distance per vehicle figures as calculated from Equation E.4.20., and $(1\cdot 8.D_r)$. (P=100, a=1 (circle))

n	C	Stem Distance per Vehicle E.4.20.	Stem Distance per Vehicle (1·8.Dr)	n	C	Stem Distance per Vehicle E.4.20.	Stem Distance per Vehicle (1·8.Dr)
35	3	0·606	0·5968	9	11	0·374	0·5968
25	4	0·554		8	13	0·344	
20	5	0·514		7	14	0·331	
16	6	0·482		6	17	0·3	
14	7	0·454		5	20	0·267	
12	8	0·431		4	25	0·227	
11	9	0·409		3	33	0·167	
10	10	0·391					

Table 4.25. Distance estimates using Equation E.4.23.. (P=100, a=1 (circle))

n	C	Total Stem Distance	Total Delivery Distance	Total Fleet Mileage
1	100	0·597	5·968	6·565
2	50	1·194	↓	7·162
3	33	1·808	↓	7·776
4	25	2·387	↓	8·355
5	20	2·984	↓	8·952
6	17	3·511	↓	9·479
7	14	4·263	↓	10·231
8	13	4·591	↓	10·559

n	C	Total Stem Distance	Total Delivery Distance	Total Fleet Mileage
10	10	5·968	5·968	11·936
12	8	7·46	↓	13·428
14	7	8·526	↓	14·494
16	6	9·947	↓	15·915
18	6	9·947	↓	15·915
20	5	11·936	↓	17·904
25	4	14·92	↓	20·888
35	3	19·89	↓	25·861

Table 4.26. Estimates of Total Fleet Mileage as a Function of P, with k fixed, using Equation E.4.24.. (k=10, a=1 (square))

P	STEM	DELIVERY	TOTAL
1	0.059679	0.59679	0.656469
2	0.119358	0.843988	0.963346
3	0.179037	1.033670	1.212707
4	0.238716	1.19358	1.432296
5	0.298395	1.334463	1.632858
6	0.358074	1.461830	1.819904
7	0.417753	1.578957	1.996710
8	0.477432	1.687977	2.165409
9	0.537111	1.79037	2.327481
10	0.59679	1.887215	2.484005
11	0.656469	1.979328	2.635797
12	0.716148	2.067341	2.783489
13	0.775827	2.151756	2.927583
14	0.835506	2.232983	3.068489
15	0.895185	2.311357	3.206542
16	0.954864	2.38716	3.342024
17	1.014543	2.460628	3.475171
18	1.074222	2.531965	3.606187
19	1.133901	2.601347	3.735248
20	1.19358	2.668926	3.862506
21	1.253259	2.734835	3.988094
22	1.312938	2.799193	4.112131
23	1.372617	2.862104	4.234721
24	1.432296	2.923661	4.355957
25	1.491975	2.98395	4.475925
26	1.551654	3.043043	4.594697
27	1.611333	3.101011	4.712344
28	1.671012	3.157915	4.828927
29	1.730691	3.213812	4.944503
30	1.79037	3.268753	5.059123
31	1.850049	3.322786	5.172835
32	1.909728	3.375954	5.285682
33	1.969407	3.428297	5.397704

P	STEM	DELIVERY	TOTAL
34	2.029086	3.479853	5.508939
35	2.088765	3.530657	5.619422
36	2.148444	3.58074	5.729184
37	2.208123	3.630131	5.838254
38	2.267802	3.678860	5.946662
39	2.327481	3.726952	6.054433
40	2.38716	3.774431	6.161591
41	2.446839	3.821320	6.268159
42	2.506518	3.867641	6.374159
43	2.566197	3.913413	6.479610
44	2.625876	3.958657	6.584533
45	2.685555	4.003389	6.688944
46	2.745234	4.047626	6.792860
47	2.804913	4.091386	6.896299
48	2.864592	4.134682	6.999274
49	2.924271	4.17753	7.101801
50	2.98395	4.219942	7.203892
51	3.043629	4.261933	7.305562
52	3.103308	4.303513	7.406821
53	3.162987	4.344696	7.507683
54	3.222666	4.385492	7.608158
55	3.282345	4.425913	7.708258
56	3.342024	4.465967	7.807991
57	3.401703	4.505665	7.907368
58	3.461382	4.545017	8.006399
59	3.521061	4.584030	8.105091
60	3.58074	4.622715	8.203455
61	3.640419	4.661078	8.301497
62	3.700098	4.699129	8.399227
63	3.759777	4.736873	8.496650
64	3.819456	4.77432	8.593776
65	3.879135	4.811474	8.690609
66	3.938814	4.848344	8.787158

P	STEM	DELIVERY	TOTAL
67	3.998493	4.884936	8.88342
68	4.058172	4.921256	8.97942
69	4.117851	4.957310	9.07516
70	4.17753	4.993103	9.17063
71	4.237209	5.028641	9.26585
72	4.296888	5.063931	9.36081
73	4.356567	5.098975	9.45554
74	4.416246	5.133781	9.55002
75	4.475925	5.168353	9.64427
76	4.535604	5.202694	9.73829
77	4.595283	5.236810	9.83209
78	4.654962	5.270706	9.92566
79	4.714641	5.304385	10.01902
80	4.77432	5.337852	10.11217
81	4.833999	5.37111	10.2051
82	4.893678	5.404163	10.29784
83	4.953357	5.437015	10.39037
84	5.013036	5.469670	10.48270
85	5.072715	5.502131	10.57484
86	5.132394	5.534402	10.66679
87	5.192073	5.566486	10.75855
88	5.251752	5.598386	10.85013
89	5.311431	5.630105	10.94153
90	5.37111	5.661647	11.03275
91	5.430789	5.693013	11.12380
92	5.490468	5.724208	11.21467
93	5.550147	5.755234	11.30538
94	5.609826	5.786093	11.39591
95	5.669505	5.816789	11.48629
96	5.729184	5.847323	11.57650
97	5.788863	5.877695	11.66656
98	5.845542	5.907919	11.75646
99	5.908221	5.937925	11.84620
100	5.9679	5.9679	11.9358

Table 4.27. Marginal Cost as a function of P, with k fixed using Equation E.4.24.. (a=1 (square))

P	k=10	k=5	P	k=10	k=5	P	k=10	k=5	P	k=10	k=5
1	0.306877	0.366556	26	0.117646	0.177325	51	0.101259	0.160938	76	0.093795	0.153474
2	0.249361	0.309040	27	0.116583	0.176262	52	0.100861	0.160540	77	0.093574	0.153253
3	0.219583	0.279267	28	0.115575	0.175254	53	0.100475	0.160154	78	0.093357	0.153036
4	0.200562	0.260241	29	0.114619	0.174298	54	0.100099	0.159778	79	0.093145	0.152824
5	0.187046	0.246725	30	0.113711	0.173390	55	0.099733	0.159412	80	0.092936	0.152615
6	0.176805	0.236484	31	0.112846	0.172525	56	0.099377	0.159056	81	0.092732	0.152411
7	0.168698	0.228377	32	0.112022	0.171701	57	0.099030	0.158709	82	0.092531	0.152210
8	0.162071	0.221750	33	0.111235	0.170914	58	0.098692	0.158371	83	0.092334	0.152013
9	0.156524	0.216203	34	0.110482	0.170161	59	0.098363	0.158042	84	0.092140	0.151819
10	0.151791	0.211470	35	0.109761	0.169440	60	0.098042	0.157721	85	0.091949	0.151628
11	0.147691	0.207370	36	0.109070	0.168749	61	0.097729	0.157408	86	0.091762	0.151441
12	0.144094	0.203773	37	0.108407	0.168086	62	0.097423	0.157102	87	0.091578	0.151257
13	0.140905	0.200584	38	0.107770	0.167449	63	0.097125	0.156804	88	0.091398	0.151077
14	0.138053	0.197732	39	0.107158	0.166837	64	0.096833	0.156512	89	0.091220	0.150899
15	0.135481	0.195160	40	0.106568	0.166247	65	0.096549	0.156228	90	0.091045	0.150724
16	0.133147	0.192826	41	0.105999	0.165678	66	0.096270	0.155949	91	0.090873	0.150552
17	0.131016	0.190695	42	0.105451	0.165130	67	0.095998	0.155677	92	0.090704	0.150383
18	0.129060	0.188739	43	0.104922	0.164601	68	0.095732	0.155411	93	0.090538	0.150217
19	0.127257	0.186936	44	0.104411	0.164090	69	0.095472	0.155151	94	0.090374	0.150053
20	0.125588	0.185267	45	0.103916	0.163595	70	0.095217	0.154896	95	0.090213	0.149892
21	0.124036	0.183715	46	0.103438	0.163117	71	0.094968	0.154647	96	0.090054	0.149733
22	0.122590	0.182269	47	0.102975	0.162654	72	0.094723	0.154402	97	0.089898	0.149577
23	0.121236	0.180915	48	0.102526	0.162205	73	0.094484	0.154163	98	0.089744	0.149423
24	0.119967	0.179646	49	0.102091	0.161770	74	0.094250	0.153929	99	0.089593	0.149272
25	0.118772	0.178451	50	0.101669	0.161348	75	0.094020	0.153699	100		

Table 4.28. Estimates of Total Fleet Mileage as a function of n and C, with P fixed, using Equation E.4.27.. (P=100, a=1 (circle))

n	STEM	DELIVERY	TOTAL	n	STEM	DELIVERY	TOTAL
1	0.6631	5.052	5.7151	26	17.2406	5.052	22.2926
2	1.3262	5.052	6.3782	27	17.9037	5.052	22.9557
3	1.9893	5.052	7.0413	28	18.5668	5.052	23.6188
4	2.6524	5.052	7.7044	29	19.2299	5.052	24.2819
5	3.3155	5.052	8.3675	30	19.893	5.052	24.945
6	3.9786	5.052	9.0306	31	20.5561	5.052	25.6081
7	4.6417	5.052	9.6937	32	21.2192	5.052	26.2712
8	5.3048	5.052	10.3568	33	21.8823	5.052	26.9343
9	5.9679	5.052	11.0199	34	22.5454	5.052	27.5974
10	6.631	5.052	11.683	35	23.2085	5.052	28.2605
11	7.2941	5.052	12.3461	36	23.8716	5.052	28.9236
12	7.9572	5.052	13.0092	37	24.5347	5.052	29.5867
13	8.6203	5.052	13.6723	38	25.1978	5.052	30.2498
14	9.2834	5.052	14.3354	39	25.8609	5.052	30.9129
15	9.9465	5.052	14.9985	40	26.524	5.052	31.576
16	10.6096	5.052	15.6616	41	27.1871	5.052	32.2391
17	11.2727	5.052	16.3247	42	27.8502	5.052	32.9022
18	11.9358	5.052	16.9878	43	28.5133	5.052	33.5653
19	12.5989	5.052	17.6509	44	29.1764	5.052	34.2284
20	13.262	5.052	18.314	45	29.8395	5.052	34.8915
21	13.9251	5.052	18.9771	46	30.5026	5.052	35.5546
22	14.5882	5.052	19.6402	47	31.1657	5.052	36.2177
23	15.2513	5.052	20.3033	48	31.8288	5.052	36.8808
24	15.9144	5.052	20.9664	49	32.4919	5.052	37.5439
25	16.5775	5.052	21.6295	50	33.155	5.052	38.207

Table 4·29· Estimates of Stem Distance, Delivery Distance and Total Fleet Mileage as a function of P, with k fixed, using Equation E.4.27.. (k=10, a=1 (circle))

P	DELIVERY	STEM	TOTAL
1	0.5052	0.06631	0.57151
2	0.714460	0.13262	0.847080
3	0.875032	0.19893	1.073962
4	1.0104	0.26524	1.27564
5	1.129661	0.33155	1.461211
6	1.237482	0.39786	1.635342
7	1.336633	0.46417	1.800803
8	1.428921	0.53048	1.959401
9	1.5156	0.59679	2.11239
10	1.597582	0.6631	2.260682
11	1.675558	0.72941	2.404968
12	1.750064	0.79572	2.545784
13	1.821524	0.86203	2.683554
14	1.890285	0.92834	2.818625
15	1.956631	0.99465	2.951281
16	2.0208	1.06096	3.08176
17	2.082992	1.12727	3.210262
18	2.143382	1.19358	3.336962
19	2.202115	1.25989	3.462005
20	2.259323	1.3262	3.585523
21	2.315117	1.39251	3.707627
22	2.369598	1.45882	3.828418
23	2.422854	1.52513	3.947984
24	2.474964	1.59144	4.066404
25	2.526	1.65775	4.18375
26	2.576024	1.72406	4.300084
27	2.625096	1.79037	4.415466
28	2.673267	1.85668	4.529947
29	2.720585	1.92299	4.643575
30	2.767094	1.9893	4.756394
31	2.812834	2.05561	4.868444
32	2.857842	2.12192	4.979762
33	2.902153	2.18823	5.090383
34	2.945796	2.25454	5.200336
35	2.988803	2.32085	5.309653
36	3.0312	2.38716	5.41836
37	3.073011	2.45347	5.526481
38	3.114261	2.51978	5.634041
39	3.154972	2.58609	5.741062
40	3.195165	2.6524	5.847565
41	3.234858	2.71871	5.953568
42	3.274070	2.78502	6.059090
43	3.312817	2.85133	6.164147
44	3.351117	2.91764	6.268757
45	3.388984	2.98395	6.372934
46	3.426433	3.05026	6.476693
47	3.463476	3.11657	6.580046
48	3.500128	3.18288	6.683008
49	3.5364	3.24919	6.78559
50	3.572303	3.3155	6.887803
51	3.607849	3.38181	6.989659
52	3.643049	3.44812	7.091169
53	3.677911	3.51443	7.192341
54	3.712446	3.58074	7.293186
55	3.746663	3.64705	7.393713
56	3.780570	3.71336	7.493930
57	3.814176	3.77967	7.593846
58	3.847488	3.84598	7.693468
59	3.880514	3.91229	7.792804
60	3.913262	3.9786	7.891862
61	3.945738	4.04491	7.990648
62	3.977948	4.11122	8.089168
63	4.009900	4.17753	8.187430
64	4.0416	4.24384	8.28544
65	4.073052	4.31015	8.383202
66	4.104264	4.37646	8.480724
67	4.135240	4.44277	8.578010
68	4.165985	4.50908	8.675065
69	4.196506	4.57539	8.771896
70	4.226806	4.6417	8.868506
71	4.256890	4.70801	8.964900
72	4.286764	4.77432	9.061084
73	4.316430	4.84063	9.157060
74	4.345894	4.90694	9.252834
75	4.375160	4.97325	9.348410
76	4.404231	5.03956	9.443791
77	4.433112	5.10587	9.538982
78	4.461805	5.17218	9.633985
79	4.490315	5.23849	9.728805
80	4.518646	5.3048	9.823446
81	4.5468	5.37111	9.91791
82	4.574780	5.43742	10.01220
83	4.602591	5.50373	10.10632
84	4.630234	5.57004	10.20027
85	4.657713	5.63635	10.29406
86	4.685032	5.70266	10.38769
87	4.712191	5.76897	10.48116
88	4.739196	5.83528	10.57447
89	4.766047	5.90159	10.66763
90	4.792748	5.9679	10.76064
91	4.819300	6.03421	10.85351
92	4.845708	6.10052	10.94622
93	4.871972	6.16683	11.03880
94	4.898095	6.23314	11.13123
95	4.924080	6.29945	11.22353
96	4.949928	6.36576	11.31568
97	4.975642	6.43207	11.40771
98	5.001224	6.49838	11.49960
99	5.026676	6.56469	11.59136
100	5.052	6.631	11.683

Table 5.1. The relationship between H and k, using Equation E.5.5.. (P=200, a=100 (circle))

H(hrs)	k	INT(k)
3.2500	1.2609	1.0000
3.5000	2.4458	2.0000
3.7500	3.6306	3.0000
4.0000	4.8155	4.0000
4.2500	6.0003	6.0000
4.5000	7.1852	7.0000
4.7500	8.3700	8.0000
5.0000	9.5549	9.0000
5.2500	10.7397	10.0000
5.5000	11.9246	11.0000
5.7500	13.1094	13.0000
6.0000	14.2943	14.0000
6.2500	15.4791	15.0000
6.5000	16.6640	16.0000
6.7500	17.8488	17.0000
7.0000	19.0337	19.0000
7.2500	20.2185	20.0000
7.5000	21.4034	21.0000
7.7500	22.5882	22.0000
8.0000	23.7731	23.0000
8.2500	24.9579	24.0000
8.5000	26.1428	26.0000
8.7500	27.3276	27.0000
9.0000	28.5125	28.0000
9.2500	29.6973	29.0000
9.5000	30.8822	30.0000
9.7500	32.0670	32.0000
10.0000	33.2519	33.0000
10.2500	34.4367	34.0000
10.5000	35.6216	35.0000
10.7500	36.8064	36.0000
11.0000	37.9913	37.0000
11.2500	39.1761	39.0000
11.5000	40.3610	40.0000
11.7500	41.5458	41.0000
12.0000	42.7307	42.0000
12.2500	43.9155	43.0000
12.5000	45.1004	45.0000
12.7500	46.2852	46.0000
13.0000	47.4701	47.0000
13.2500	48.6550	48.0000
13.5000	49.8398	49.0000
13.7500	51.0247	51.0000
14.0000	52.2095	52.0000
14.2500	53.3944	53.0000
14.5000	54.5792	54.0000
14.7500	55.7641	55.0000
15.0000	56.9489	56.0000
15.0000	56.9489	56.0000

Table 5.2. The effect of H on distance estimates. (P=200, a=100 (circle))

H	INT(k)	DELIVERY DISTANCE	STEM DISTANCE	TOTAL FLEET MILEAGE
3.25	1	0	8021.47	8021.47
3.5	2	611.6550	3391.365	4003.020
3.75	3	759.4928	2019.372	2778.864
4	4	812.3383	1385.998	2198.336
4.25	6	840.5679	803.2296	1643.797
4.5	7	841.4947	649.1273	1490.622
4.75	8	839.1176	538.1568	1377.274
5	9	834.9873	454.9735	1289.960
5.25	10	829.9270	390.647	1220.574
5.5	11	824.3965	339.6490	1164.045
5.75	13	812.8807	264.4302	1077.311
6	14	807.1470	236.0823	1043.229
6.25	15	801.5166	212.1236	1013.640
6.5	16	796.0222	191.6572	987.6795
6.75	17	790.6816	174.0101	964.6917
7	19	780.4897	145.2315	925.7212
7.25	20	775.6399	133.3865	909.0264
7.5	21	770.9503	122.8827	893.8330
7.75	22	766.4160	113.5182	879.9342
8	23	762.0313	105.1286	867.1600
8.25	24	757.7900	97.57919	855.3692
8.5	26	749.7125	84.57133	834.2838
8.75	27	745.8637	78.94104	824.8048
9	28	742.1337	73.80052	815.9342
9.25	29	738.5167	69.09318	807.6099
9.5	30	735.0073	64.77053	799.7778
9.75	32	728.2907	57.11805	785.4088
10	33	725.0741	53.72076	778.7948
10.25	34	721.9460	50.57159	772.5176
10.5	35	718.9024	47.64657	766.5489
10.75	36	715.9393	44.92458	760.8639
11	37	713.0532	42.38702	755.4402
11.25	39	707.4983	37.80100	745.2993
11.5	40	704.8233	35.72482	740.5481
11.75	41	702.2127	33.77718	735.9899
12	42	699.6637	31.94759	731.6113
12.25	43	697.1740	30.22668	727.4006
12.5	45	692.3623	27.07778	719.4401
12.75	46	690.0361	25.63525	715.6713
13	47	687.7601	24.27207	712.0322
13.25	48	685.5325	22.98254	708.5150
13.5	49	683.3514	21.76149	705.1129
13.75	51	679.1221	19.50624	698.6284
14	52	677.0707	18.46377	695.5345
14.25	53	675.0594	17.47310	692.5325
14.5	54	673.0869	16.53091	689.6178
14.75	55	671.1518	15.63413	686.7859
15	56	669.2528	14.77993	684.0328

Table 5.3. The relationship between Total Cost per week and the size of the delivery-area

a	Daily Round-Trips TFM	Daily Round-Trips TC (£ per wk)	Overnight Stays TFM	Overnight Stays TC (£ per wk)
250.00	1325.19	370.73	678.90	414.39
300.00	1590.23	423.11	814.68	456.77
350.00	1855.27	475.49	950.46	499.16
400.00	2120.30	527.88	1086.24	541.54
450.00	2385.34	580.26	1222.02	583.92
500.00	2650.38	632.65	1357.80	626.31
550.00	2915.42	685.03	1493.58	668.69
600.00	3180.46	737.41	1629.36	711.08
650.00	3445.49	789.80	1765.14	753.46
700.00	3710.53	842.18	1900.92	795.84
750.00	3975.57	894.57	2036.70	838.23

Table 5.4.1. Disaggregation of weekly costs using daily round-trips, when t=3·75 tons

a	n^v	x	C	Stem Cost	Running Cost	Total Cost	Stem Cost as a percentage of Total Cost
50	3	0.75	13.33	50.29	174.85	501.26	10.03
75	3	0.75	13.33	75.43	262.27	588.68	12.81
100	3	0.75	13.33	100.57	349.69	676.11	14.88
125	3	0.75	13.33	125.72	437.12	763.53	16.47
150	4	0.75	10.00	231.63	614.05	1049.27	22.08
175	4	0.75	10.00	270.24	716.39	1151.61	23.47
200	4	0.75	10.00	308.84	818.73	1253.95	24.63
225	4	0.75	10.00	347.45	921.07	1356.29	25.62
250	5	0.75	8.00	531.83	1176.25	1720.27	30.92
275	5	0.75	8.00	585.01	1293.87	1837.90	31.83
300	5	0.75	8.00	638.19	1411.50	1955.52	32.64
325	6	0.75	6.67	892.44	1732.73	2385.56	37.41
350	6	0.75	6.67	961.09	1866.01	2518.84	38.16
375	7	0.75	5.71	1272.83	2239.69	3001.33	42.41
400	7	0.75	5.71	1357.68	2389.01	3150.64	43.09
425	8	0.75	5.00	1728.79	2816.58	3687.02	46.89
450	9	0.75	4.44	2143.55	3282.64	4261.89	50.30
475	10	0.75	4.00	2602.42	3787.75	4875.79	53.37
500	11	0.75	3.64	3105.91	4332.39	5529.24	56.17
525	14	0.75	2.86	4463.67	5668.20	7191.47	62.07
550	16	0.75	2.50	5551.79	6744.24	8485.12	65.43
575	20	0.75	2.00	7708.39	8788.78	10964.88	70.30

Table 5.4.2. Disaggregation of weekly costs using overnight stays, when t=3·75

a	n^v	x	C	Stem Cost	Running Cost	Total Cost	Stem Cost as a percentage of Total Cost
50	3	1.5	66.67	1.85	122.66	544.80	0.34
75	3	1.5	66.67	2.77	183.99	606.13	0.46
100	3	1.5	66.67	3.70	245.32	667.46	0.55
125	3	1.5	66.67	4.62	306.65	728.79	0.63
150	3	1.5	66.67	5.55	367.98	790.12	0.70
175	3	1.5	66.67	6.47	429.32	851.45	0.76
200	3	1.5	66.67	7.40	490.65	912.78	0.81
225	3	1.5	66.67	8.32	551.98	974.11	0.85
250	4	1	50.00	22.03	587.92	1072.16	2.05
275	4	1	50.00	24.23	646.72	1130.96	2.14
300	4	1	50.00	26.43	705.51	1189.75	2.22
325	4	1	50.00	28.63	764.30	1248.54	2.29
350	4	1	50.00	30.84	823.09	1307.33	2.36
375	4	1	50.00	33.04	881.89	1366.13	2.42
400	4	1	50.00	35.24	940.68	1424.92	2.47
425	4	1	50.00	37.44	999.47	1483.71	2.52
450	4	1	50.00	39.65	1058.26	1542.50	2.57
475	5	0.75	40.00	67.08	1095.52	1639.55	4.09
500	5	0.75	40.00	70.61	1153.18	1697.21	4.16
525	5	0.75	40.00	74.14	1210.84	1754.87	4.22
550	5	0.75	40.00	77.67	1268.50	1812.52	4.29
575	5	0.75	40.00	81.20	1326.16	1870.18	4.34

Table 5.5.1. Disaggregation of weekly costs using daily round-trips, when t=200 tons

a	n^v	x	C	Stem Cost	Running cost	Total Cost	Stem Cost as a percentage of Total Cost
50	3	14	13.33	113.14	393.39	1398.18	8.09
75	3	14	13.33	169.71	590.09	1594.87	10.64
100	3	14	13.33	226.28	786.79	1791.57	12.63
125	3	14	13.33	282.86	983.49	1988.27	14.23
150	4	10	10.00	469.79	1245.41	2383.19	8.09
175	4	10	10.00	548.09	1452.97	2590.76	10.64
200	4	10	10.00	626.39	1660.54	2798.33	12.63
225	4	10	10.00	704.69	1868.11	3005.90	14.23
250	5	8	8.00	1014.90	2244.66	3553.89	19.71
275	5	8	8.00	1116.39	2469.13	3778.36	21.16
300	5	8	8.00	1217.88	2693.59	4002.83	22.38
325	6	7	6.67	1642.10	3188.23	4683.38	23.44
350	6	7	6.67	1768.41	3433.48	4928.63	28.56
375	7	6	5.71	2245.50	3951.23	5598.60	29.55
400	7	6	5.71	2395.20	4214.64	5862.02	30.43
425	8	5	5.00	2901.82	4727.69	6487.27	35.06
450	9	5	4.44	3597.99	5509.99	7489.52	35.88
475	10	4	4.00	4110.06	5982.07	8006.79	40.11
500	11	4	3.64	4905.23	6842.23	9069.42	40.86
525	14	3	2.86	6517.11	8275.75	10823.40	44.73
550	16	3	2.50	8105.80	9846.81	12758.41	48.04
575	20	2	2.00	10075.19	11487.31	14618.52	51.33

Table 5.5.2. Disaggregation of weekly costs using overnight stays, when t=200 tons

a	n^{γ}	x	C	Stem cost	Running Cost	Total Cost	Stem Cost as a percentage of Total Cost
50	7	30	28.57	46.26	420.63	3762.28	1.23
75	7	30	28.57	69.39	630.95	3972.60	1.75
100	7	30	28.57	92.52	841.27	4182.91	2.21
125	7	30	28.57	115.65	1051.59	4393.23	2.63
150	7	30	28.57	138.78	1261.90	4603.55	3.01
175	7	30	28.57	161.90	1472.22	4813.87	3.36
200	7	30	28.57	185.03	1682.54	5024.18	3.68
225	7	30	28.57	208.16	1892.86	5234.50	3.98
250	7	30	28.57	231.29	2103.17	5444.82	4.25
275	7	30	28.57	254.42	2313.49	5655.14	4.50
300	7	30	28.57	277.55	2523.81	5865.45	4.73
325	7	30	28.57	300.68	2734.12	6075.77	4.95
350	7	30	28.57	323.81	2944.44	6286.09	5.15
375	7	30	28.57	346.94	3154.76	6496.40	5.34
400	7	30	28.57	370.07	3365.08	6706.72	5.52
425	7	30	28.57	393.20	3575.39	6917.04	5.68
450	7	30	28.57	416.33	3785.71	7127.36	5.84
475	7	30	28.57	439.45	3996.03	7337.67	5.99
500	7	30	28.57	462.58	4206.35	7547.99	6.13
525	7	30	28.57	485.71	4416.66	7758.31	6.26
550	7	30	28.57	508.84	4626.98	7968.63	6.39
575	7	30	28.57	531.97	4837.30	8178.94	6.50

Table 5.6. Total Cost per week of alternative systems for values of a from 50 to 3000 miles. (t=3·75 tons)

	Daily Round-trips			Overnight stays		
a	n^v	X	Total Cost	n^v	X	Total Cost
50	3	0.75	501.26	3	1.5	544.80
75	3	0.75	588.68	3	1.5	606.13
100	3	0.75	676.11	3	1.5	667.46
125	3	0.75	763.53	3	1.5	728.79
150	4	0.75	1049.27	3	1.5	790.12
175	4	0.75	1151.61	3	1.5	851.45
200	4	0.75	1253.95	3	1.5	912.78
225	4	0.75	1356.29	3	1.5	974.11
250	5	0.75	1720.27	4	1	1072.16
275	5	0.75	1837.90	4	1	1130.96
300	5	0.75	1955.52	4	1	1189.75
325	6	0.75	2385.56	4	1	1248.54
350	6	0.75	2518.84	4	1	1307.33
375	7	0.75	3001.33	4	1	1366.13
400	7	0.75	3150.64	4	1	1424.92
425	8	0.75	3687.02	4	1	1483.71
450	9	0.75	4261.89	4	1	1542.50
475	10	0.75	4875.79	5	0.75	1639.55
500	11	0.75	5529.24	5	0.75	1697.21
525	14	0.75	7191.47	5	0.75	1754.87
550	16	0.75	8485.12	5	0.75	1812.52
575	20	0.75	10964.88	5	0.75	1870.18
1000				8	0.75	4340.00
1500				12	0.75	6022.00
2000				18	0.75	9665.00
2500				28	0.75	15741.00
3000				53	0.75	31339.00

Table 7.1. The effect of time-windows on n, TFM, d, i_d and TC. (P=100, a=1 (square))

r	n	TFM	d	i_d	TC
10	22	44.484	2.022	0.365	3687
20	18	41.600	2.311	0.353	3327
30	16	39.646	2.478	0.342	3032
40	15	38.242	2.549	0.333	2873
50	14	34.954	2.497	0.307	2661
60	13	37.286	2.868	0.330	2607
70	12	31.958	2.663	0.285	2337
80	12	34.442	2.870	0.308	2407
90	11	32.460	2.951	0.292	2231
100	11	31.192	2.836	0.281	2196
110	11	29.228	2.657	0.263	2140
120	10	29.340	2.934	0.267	2143
130	10	27.996	2.800	0.255	1986
140	10	27.944	2.794	0.254	1984
150	10	27.706	2.771	0.252	1978
160	10	27.582	2.758	0.251	1974
170	11	27.594	2.509	0.249	1974
180	11	27.218	2.474	0.245	2084
190	10	26.366	2.637	0.240	2060
200	11	25.880	2.353	0.233	1926
210	11	27.224	2.475	0.245	2084
220	11	28.308	2.573	0.255	2118
230	10	24.030	2.403	0.218	1866
240	10	23.266	2.327	0.212	1875
250	10	23.180	2.318	0.211	1850
260	10	23.180	2.318	0.211	1850
270	9	23.494	2.610	0.216	1739
280	9	23.502	2.611	0.216	1740
290	10	24.998	2.500	0.227	1901
300	9	24.542	2.727	0.225	1769
310	9	23.720	2.636	0.218	1746
320	9	23.720	2.636	0.218	1746
330	9	22.490	2.499	0.206	1711
340	9	22.988	2.554	0.211	1725
350	9	22.534	2.504	0.207	1712
360	10	22.060	2.206	0.201	1819
370	9	21.412	2.379	0.196	1681
380	9	21.770	2.419	0.200	1691
390	9	21.770	2.419	0.200	1691
400	9	21.770	2.419	0.200	1691
410	9	21.770	2.419	0.200	1678
420	9	20.866	2.318	0.191	1665
430	9	20.754	2.306	0.190	1699
440	9	23.256	2.584	0.213	1733
450	9	22.220	2.469	0.204	1722
460	9	22.444	2.494	0.206	1710
470	9	23.064	2.563	0.212	1705
480	9	22.090	2.454	0.203	1699
490	9	22.090	2.454	0.203	1713
500	9	23.068	2.563	0.212	1727
510	9	23.068	2.563	0.212	1727
520	9	23.068	2.563	0.212	1727
530	9	21.778	2.420	0.200	1683
540	9	19.936	2.215	0.183	1639

Table 7.2. Observed and predicted Total Cost figures, as r changes

r	Observed TC	Predicted TC
1	3962	5012
10	3887	3334
20	3327	2949
30	3032	2745
40	2873	2609
50	2661	2508
60	2607	2428
70	2337	2363
80	2407	2308
90	2231	2260
100	2196	2218
110	2140	2181
120	2143	2148
130	1986	2118
140	1984	2090
150	1978	2065
160	1974	2041
170	1974	2019
180	2084	1999
190	2060	1980
200	1926	1962
210	2084	1945
220	2118	1929
230	1866	1914
240	1875	1900
250	1850	1886
260	1850	1873
270	1739	1861
280	1740	1849
290	1901	1837
300	1769	1826
310	1746	1816
320	1746	1805
330	1711	1796
340	1725	1786
350	1712	1777
360	1819	1768
370	1681	1760
380	1691	1751
390	1691	1743
400	1691	1736
410	1678	1728
420	1665	1721
430	1699	1713
440	1733	1707
450	1722	1700
460	1710	1693
470	1705	1687
480	1699	1680
490	1713	1674
500	1727	1668
510	1727	1662
520	1727	1657
530	1683	1651
540	1639	1646

Table 7.4. The effect of time-windows on n, TFM, d and TC, using the savings formula

r	n	TFM	d	TC
1	31	51.450	1.660	5162
10	29	47.066	1.623	4799
20	27	45.038	1.668	4502
30	22	37.708	1.714	3697
40	23	38.758	1.685	3846
50	20	36.302	1.915	3474
60	19	34.622	1.822	3250
70	17	32.102	1.888	2940
80	17	30.908	1.818	2906
90	18	31.606	1.756	3046
100	17	29.506	1.738	2867
110	17	30.326	1.784	2890
120	16	26.872	1.680	2673
130	15	25.672	1.711	2519
140	15	26.152	1.743	2533
150	15	25.554	1.704	2516
160	14	23.296	1.664	2333
170	14	24.010	1.715	2353
180	14	23.216	1.658	2330
190	14	21.278	1.520	2276
200	14	22.992	1.642	2324
210	14	23.310	1.665	2333
220	13	22.274	1.713	2184
230	13	22.230	1.710	2183
240	13	21.030	1.618	2149
250	12	19.504	1.625	1986
260	12	20.076	1.673	2003
270	12	19.654	1.638	1991
280	12	17.766	1.480	1938
290	12	20.332	1.694	2010
300	12	18.752	1.563	1965
310	11	17.510	1.592	1811
320	11	17.244	1.568	1803
330	11	17.518	1.593	1811
340	11	17.812	1.619	1819
350	11	16.188	1.472	1773
360	10	17.124	1.712	1680
370	10	15.936	1.594	1646
380	10	17.248	1.725	1683
390	11	15.612	1.561	1757
400	10	16.210	1.621	1654
410	10	16.060	1.606	1650
420	9	13.050	1.450	1445
430	9	14.538	1.615	1487
440	9	13.982	1.554	1472
450	10	14.448	1.447	1605
460	9	14.068	1.563	1474
470	9	12.870	1.430	1440
480	8	12.516	1.564	1311
490	9	12.842	1.427	1440
500	8	12.368	1.546	1306
510	8	12.468	1.559	1309
520	8	12.948	1.619	1323
530	8	12.548	1.569	1312

Table 7.5. The effect of time-windows on n, TFM, d and TC, using time-minimisation as an alternative to Genralised Cost

r	n	TFM	d	TC
1	22	56.830	2.583	4235
10	20	52.724	2.636	3880
20	18	47.038	2.613	3460
30	16	42.510	2.657	3113
40	15	40.880	2.725	2947
50	13	36.890	2.838	2595
60	13	39.860	3.066	2679
70	12	35.976	2.998	2450
80	12	36.614	3.051	2468
90	12	34.932	2.911	2421
100	12	35.798	2.983	2445
110	11	33.148	3.013	2251
120	11	29.182	2.653	2139
130	11	28.638	2.603	2124
140	10	29.636	2.964	2032
150	10	31.784	3.178	2092
160	10	28.402	2.840	1997
170	10	27.594	2.759	1974
180	10	26.848	2.685	1954
190	10	26.356	2.636	1940
200	10	26.064	2.606	1931
210	10	25.230	2.523	1908
220	11	28.452	2.587	2118
230	10	26.304	2.630	1938
240	10	24.054	2.405	1875
250	10	23.180	2.318	1850
260	10	23.180	2.318	1850
270	9	23.502	2.611	1740
280	9	23.494	2.610	1739
290	10	24.998	2.500	1901
300	9	24.542	2.727	1769
310	9	23.720	2.636	1746
320	9	23.720	2.636	1746
330	9	22.490	2.499	1711
340	9	22.988	2.554	1725
350	9	22.534	2.504	1712
360	9	22.534	2.504	1696
370	9	21.412	2.379	1681
380	9	21.788	2.421	1686
390	9	21.770	2.419	1691
400	9	21.770	2.419	1691
410	9	21.770	2.419	1691
420	9	21.770	2.419	1677
430	9	20.754	2.306	1662
440	9	23.256	2.584	1682
450	9	22.220	2.469	1703
460	9	22.444	2.494	1715
470	9	23.064	2.563	1727
480	9	22.090	2.454	1727
490	9	22.090	2.454	1727
500	9	23.068	2.563	1727
510	9	23.068	2.563	1727
520	9	23.068	2.563	1696
530	9	21.778	2.420	1665
540	9	19.936	2.215	1639

Table 7.6. The relationship between Total Fleet Mileage and Fleet Size in the presence of time-windows, (average Figures)

n	TFM	d
9	22.306	2.478
10	25.420	2.542
11	28.680	2.607
12	32.939	2.745
13	36.765	2.828
14	35.797	2.557
15	38.242	2.549
16	38.287	2.393
18	41.028	2.279
21	45.514	2.167
22	46.480	2.113

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