Work Roll Cooling System Design Optimisation in Presence of Uncertainty

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Abstract
The paper presents a framework to optimise the design of work roll based on the cooling performance. The framework develops Meta models from a set of Finite Element Analysis (FEA) of the roll cooling. A design of experiment technique is used to identify the FEA runs. The research also identifies sources of uncertainties in the design process. A robust evolutionary multi-objective algorithm is applied to the design optimisation in order to identify a set of good solutions in the presence of uncertainties both in the decision and objective spaces.

Keywords:
Roll cooling design, Uncertainty, Design optimisation, Multi-objective optimisation

1 INTRODUCTION
Roll cooling optimisation can be considered as a process of finding the best set of manufacturing parameters which guarantees an efficient use of water cooling and application. The optimisation of the rolling system is crucial for improving process time, reducing cost as well as increasing product quality. To obtain the desired cooling conditions, it is essential to know and to control as accurately as possible the relevant process parameters. The hot rolling process takes place in a relatively harsh environment with safety implications due to high temperature, machinery, moving stock and overall conditions (space, etc.). Following a detailed mapping of the main input factors (dependent, independent) affecting roll cooling and, hence, roll life, factors such as roll temperature, stock temperature, roll speed, roll stock contact length, cooling heat transfer coefficient, delay time, roll stock contact length and roll gap heat transfer coefficient have been considered as the main factors influencing cooling conditions. All these factors have inherent uncertainty, i.e. they follow some statistical distribution, which is in general a priori not known. Previous studies identified that temperature difference in the roll as well as the developed stresses (thermal and mechanical) are key roll cooling design quality factors or responses [1, 2]. Both measures can contradict each other, depending on the amount of under or overcooling, i.e. the improvement in one quality factor comes with a decrease in the other factor. Therefore, no single optimum solution exists but a set of best possible compromise solutions can be found from which experts can choose depending on their preference [3]. Stress and temperature in the roll cannot be assumed to be perfectly constant. In practice they fluctuate slightly. To address the issues of roll cooling design as described above, this paper presents the application of robust evolutionary multi-objective optimisation using a new dominance-criteria technique. This technique is designed to identify Pareto fronts in noisy environments. A predecessor of this algorithm with noisy fitness functions is described in [4]. In this paper the new technique is adopted and applied in the case of noisy decision space as well as noisy fitness functions. More work on the technique also can be found in [5]. The paper also presents the underlying roll cooling model and experimental application of the new optimisation technique on the model. Also result analysis and concluding remarks are presented.

Figure 1: Principle of rolling system.

2 DEVELOPMENT OF ROLL COOLING MODEL
The section develops a mathematical model of a roll cooling system design. The model is to represent a complex behaviour of a real life rolling process in a simplified and controllable manner. Designs of Experiment (DoE) methods are used to develop the surrogate model. Alternative/surrogate model represents the underlying characteristics of the issues being investigated, such as: rolling process factors and parameters, as well as the influence of those factors on the thermal behaviours of rolls during rolling [6]. The proposed meta-modelling framework was introduced to carry out computation of intensive design simulations. Since the framework is based on a response surface methodology it inherits the following advantages: providing insights into the relationship between output responses y and the input design variables x which can be used to evaluate design process parameters uncertainty. Nevertheless, there is also uncertainty in the meta-model. Uncertainty in the meta-model is due to the fact that it is an approximate representation of the real world rolling practice, where there is inevitable forced accuracy compromise and losses of information during
design of experiment. In the next section the model building methodology and evaluation of the uncertainty will be discussed.

2.1 Model building methodology

Experimental Procedures

Problem Definition: The purpose is to identify the main factors influencing effective roll cooling and therefore minimizing effect of thermal fatigue whilst increasing roll life. The problem definition leads to the identification of change of characteristics and behaviours of rolls which occur during cooling. The change in characteristics and behaviours of rolls are later used as a measure in determining solution for optimum roll cooling. Here, also roll wear and influencing factors, as well as the dependency, if any, between factors, are investigated.

2.2 Identifying optimum rolls cooling measure

Change in roll surface temperature (\(\Delta T\)) is an important roll cooling design objective that expresses the effect of roll cooling during hot rolling. It is a suitable measurement since it displays rolls thermal behaviour (i.e., how well the current cooling design meets the requirements). The change in temperature is measured as the difference between maximum and minimum values over a cycle in quasi steady state heat exchange rolling conditions: \(\Delta T = T_2 - T_1\), measured in Kelvin [K].

Roll Stress [MPa]: Another equally important measure/objective in optimising roll cooling is keeping the roll maximum principal stress (MPS) at the roll surface as low as possible. Behaviour of stress on the roll is a useful objective to consider since it has a proportional effect with change in temperature in rolls and hence on thermal fatigue. Roll surface passing under the water jets undergoes a cyclic state of tensile stresses due to the roll cooling being applied after that surface has been in contact with the hot stock where the stress is compressive in nature. This tensile stress is a contributory factor to thermal crack growth.

Identifying Contributing Factors: The aim is to understand the issues concerning the roll cooling problems and identify specific contributing factors to the identified. This step also lists the most important design variables from a large number of potentially important factors. Defining regions of interest was according to roll cooling experts. The choice of design variables were driven by the need to mimic the real design problem experienced in the plant. Seven variables were identified and their operating range specified. Table 1 below shows the factors identified and factor levels recommended. HTC 1 and HTC 2 are the specified. Table 1 below shows the factors identified and factor levels used in the simulations. Seven variables were identified and their operating range large number of potentially important factors. Defining the boundaries are therefore model accuracy is expected from higher number of levels in the design space [7, 8]. Therefore a 3-level is allocated for each of the seven identified main factors.

Table 1: Factors and factor levels used in the simulations.

<table>
<thead>
<tr>
<th>Levels</th>
<th>Roll entry temperature (°C)</th>
<th>Roll exit temperature (°C)</th>
<th>Roll speed (m/s)</th>
<th>HTC 1 (W/m².K)</th>
<th>HTC 2 (W/m².K)</th>
<th>Delay time (sec)</th>
<th>BUS section length (cm)</th>
<th>HIC 1 (°F/°C)</th>
<th>HIC 2 (°F/°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>40</td>
<td>980</td>
<td>0.14</td>
<td>1.5</td>
<td>20</td>
<td>10</td>
<td>15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>60</td>
<td>1100</td>
<td>0.698</td>
<td>52.5</td>
<td>20</td>
<td>10</td>
<td>15</td>
<td></td>
<td></td>
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<tr>
<td>C</td>
<td>80</td>
<td>1250</td>
<td>1.256</td>
<td>51</td>
<td>30</td>
<td>10</td>
<td>15</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Experiment: The finite element runs were performed using Abaqus Standard version 6.2.2. Due to its high thermal resistance characteristics high chromium steel material has been selected for the work roll. The same loading and boundary conditions were applied in the simulations so that the responses are measured under similar conditions. For each run, values of the two response variables are recorded. Response values for \(\Delta T\) are collected from the roll at a depth, calculated based on speed of rolls and roll /stock contact length at time when the temperature reaches the end of steady state [9]. The depth indicates the maximum heat penetration in the roll when in contact with the stock at a given roll speed. Heat penetration depth can be expressed mathematically as:

\[
P = \sqrt{\frac{\alpha}{C_p \rho}}
\]

Where \(\alpha\) is thermal diffusivity of the roll material and expressed as a function of thermal conductivity (\(K\)) and the product of density \(x\) specific heat capacity (\(p C_p\)), mathematically it can be expressed as:

\[
\alpha = \frac{K}{(p C_p)}
\]

\(p\) and \(C_p\) represent roll material density and specific heat capacity respectively. The parameter \(t\) is the stock and rolls contact time and is expressed as a function of roll /stock projected contact length (L) divided by roll rotational speed (\(\Omega\)) and roll radius (r). Mathematically it is expressed as the following:

\[
t = \frac{L}{\Omega r}
\]

Based on the roll material considered for the simulation, high chromium steel, the following values are allocated to calculate the roll heat penetrating depth: \(K = 48\) w/mk, \(p = 7833\)kg/m³, \(C_p = 478\)Jkg⁻¹.K⁻¹.

2.3 Finite element analysis and data extraction

The finite element runs were performed using Abacus version 6.2.2. Change in temperature of roll and stress, as a response, is the target to be collected and analysed. The sample result below shows the effect of combination of variables and their contribution for the variation of temperature in roll during rolling. X-Y plot on field output (ODB) of the simulation result used to analysis and determined the trend and exact value of the responses from the roll as shown in Figure 4. Each run responses recorded from simulation result are later used to develop the meta-model using a statistical tool. Sample results of the recorded responses are shown below. The response from the FEA shows that how design variable parameter variation and scheduling design set can have effect on the thermal behaviour (temperature) and mechanical property (stress) of rolls, and the cooling system reaction in normalizing to that effect during hot rolling process. Temperature is calculated as the difference between temperatures of the roll after simulation taken at a depth and the roll initial temperature, temperature before simulation (Figure 4). While stress is represented by the value directly measured from the roll surface after simulation (Figure 5). Samples (Table 2) below shows example of input parameters set and the data /response extracted from finite element simulation output. Considering two steps of the process, rolling and delay time and the responses data have been collected at the begging and end of each step. Therefore a total of four data values, have been collected. The delay time during rolling is a time when no stock pass in the roll gap. Delay time can occur at any time in the process and resulted due to uncontrollable or uncontrollable activities. Generally unsolicited delay time, too short and too long, considered
uncertainty in the rolling process since it has a direct effect on the roll temperature distribution.

Table 2: Input parameters and responses used in the modelling.

<table>
<thead>
<tr>
<th>Responses</th>
<th>Process parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roll temp</td>
<td>Max. stress on roll surface</td>
</tr>
<tr>
<td>Roll temper</td>
<td>Factor</td>
</tr>
<tr>
<td>Roll speed</td>
<td>Factor</td>
</tr>
<tr>
<td>Stock temper</td>
<td>Factor</td>
</tr>
<tr>
<td>At the end</td>
<td>At the end of steady state</td>
</tr>
<tr>
<td>At the end of</td>
<td>At the end of steady state</td>
</tr>
<tr>
<td>Acceleration</td>
<td>Factor</td>
</tr>
<tr>
<td>Delay time</td>
<td>Factor</td>
</tr>
<tr>
<td>Contact length</td>
<td>Factor</td>
</tr>
<tr>
<td>Heat at roll &amp; mill</td>
<td>Factor</td>
</tr>
</tbody>
</table>

Fitting the model: The models for the temperature and stress response surfaces were generated by fitting a second order polynomial to the results from the FEM simulations, the fit was carried out using Statistica® software package resulted in the following two models:

Temperature: 
\[ \text{Temperature} = -0.7915 \times (x_1 + \epsilon_1') - 0.0014 \times (x_1 + \epsilon_1')^2 - 0.0488 \times (x_2 + \epsilon_2') + 2.851 \times 10^{-5} \times (x_2 + \epsilon_2')^3 + 30.6809 \times (x_3 + \epsilon_3') - 1.7359 \times (x_4 + \epsilon_4') + 0.0200 \times (x_4 + \epsilon_4')^2 - 0.0038 \times (x_5 + \epsilon_5') + 6.9791 \times 10^{-4} \times (x_5 + \epsilon_5')^2 - 2.4565 \times (x_6 + \epsilon_6') + 0.0721 \times (x_6 + \epsilon_6')^2 - 1.4177 \times (x_7 + \epsilon_7') + 0.1333 \times (x_7 + \epsilon_7')^2 + 83.5805 + \epsilon_1 \]  

Stress: 
\[ \text{Stress} = -1.9147 \times (x_1 + \epsilon_1') + 0.0272 \times (x_1 + \epsilon_1')^2 + 0.0489 \times (x_2 + \epsilon_2') - 6.7312 \times 10^{-5} \times (x_2 + \epsilon_2')^2 - 1.4309 \times 10^{-2} \times (x_3 + \epsilon_3') + 70.5653 \times (x_3 + \epsilon_3')^2 + 1.3606 \times 10^{-2} \times (x_4 + \epsilon_4') - 0.0096 \times (x_4 + \epsilon_4')^2 - 0.5041 \times (x_5 + \epsilon_5') + 0.0067 \times (x_5 + \epsilon_5')^2 + 4.0248 \times (x_6 + \epsilon_6') - 0.1024 \times (x_6 + \epsilon_6')^2 - 16.5929 \times (x_7 + \epsilon_7') + 0.8525 \times (x_7 + \epsilon_7')^2 + 1.3624 \times 10^{-2} + \epsilon_2 \]  

Validation of the Model: The section gives justification for the acceptability of the meta-model by analysing post processing statistical features from the regression. The features helps to determine the relevance of the independent input factors in the model building process as well as measuring the ability of the model to predict the system response over the search space. The criteria of the performance are based on two measures: \( R^2 \) and \( R^2_{adj} \). \( R^2 \) and \( R^2_{adj} \) measure the amount of variation explained by the model. When \( R^2 \) equals 1 (perfect fit) i.e. all N model outputs equal their corresponding simulation outputs. Higher \( R^2 \) implies lower variation between observed and predicted values, therefore a better model. The respective basic quality of the fit of the deterministic FEM data, \( R^2 \) and \( R^2_{adj} \), are 0.91 and 0.89 respectively for change in temperature, 0.95 and 0.88 for stress.

3 REVIEW OF ROLL THERMAL MODELLING

Cooling of rolls is a critical concern in the rolling system design particularly, in the operation of hot mills. Untimely loss of rolls is a common occurrence during hot rolling process. Main sources of these phenomena are the
severe temperature variations and the resulting thermal stresses during work-rolls contact in the process. To control roll thermal stresses and roll life, it is necessary to know the temperature variations in the work-roll during the hot rolling process. There are a number of published studies that have focused on determining the temperature field in the work-roll and how it affects the roll life during rolling. Parke and Baker [10] used a computational method for determining the temperature field in the finishing stand work-roll. The results from their model were then used to design the optimum water spray condition. A two-dimensional finite element method was used by Seluzalec [11] to predict the temperature distribution within the work-rolls in a roll forging process. Devadass and Samarasekara [12] utilized a one-dimensional heat transfer model that was based on the finite difference method. The model was coupled with the assumption of homogeneous work to estimate the steady state temperature distributions in the work-rolls and the rolled metal during the finishing stage. Teseng et al [13] combined experimental and numerical methods to predict temperature distributions in work-rolls and to evaluate roll life. Another research work, Teseng et al [14] used an analytical method to solve the heat transfer, partial differential equations and thus determine the temperature field in a work-roll for a single pass hot strip rolling process. The cooling of both the work-rolls and the product was simulated with the aid of a mathematical model and the results are presented in [15]. In that paper the temperature fields in the work-roll and the rolled metal are predicted and the effects of various cooling conditions on work-roll temperature variations are determined. In all of these works however, the result shows only the work-roll temperature transfer prediction, estimate how it affects the roll life during rolling. Most of these research papers also lack looking in to the uncertain and fuzzy issues in rolling system affecting the cooling process. To fill these gaps therefore, the paper will focus on modelling of a rolling system design for optimum cooling of rolls, by integrating uncertainties in rolling process effecting cooling, deterministic parameters as well as the qualitative nature of rolling causing unexpected temperature variation in rolls leading to untimely roll ware. Therefore tackling cooling problems requires a multi dimensional approach. Hence, a multiple model approach would be most appropriate to consider for achieving a better cooling solution. This will require multiple representations and multiple models for each form of information (subjective and deterministic variable representation). The challenge is however in building a system with a selection of representations to integrate as one model. Today, due to its accuracy, thermal modelling is one of the most important ways used to model quantitative representation of data in metal forming. The most common technique and the most used is the Finite Element Method (FEM). This method is the only one which can give the behaviour of the temperature during metal forming with acceptable results. Because of the development of the numerical analysis, the FEM is no longer "nice to have", but rapidly becomes a cost effective way of representing/modelling the real life forming problems. The finite element method is a technique based on discretisation. A number of finite points called nodes are identified. So, the work piece is divided into an assemblage of elements connected together. Once the boundaries known, the flow equation can be resolved. This is the best technique to analyse temperature in metal forming process, since the method gives the temperature distribution on the roll at any points/nodes required. C. J. Walters gives an example of an application of finite element method in forging [16]. The first is the capability of obtaining detailed solutions of stress, strains and temperature. And the second one gives a detail analysis of the fact that a computer code can be used several times and for different kinds of problems.

4 UNCERTAINTY AND SOURCES OF UNCERTAINTY IN THE PROBLEM

Design uncertainty is comprised of design imprecision, uncertainty in choosing among alternatives, and stochastic uncertainty, usually associated to measurement limitations [18, 19, 20]. This summarise the inevitability of uncertainty in engineering design optimisations. If reliable optimal solution is to be found this inevitability must be considered in an optimisation task. There are several general sources contribute to the uncertainties in simulation predictions. These contributors can be categorized as follows:

- Variability of input values x (including both design parameters and design variables), called "input parameter uncertainty"
- Uncertainty due to limited information in estimating the characteristics of model parameters p, called "model parameter uncertainty" and
- Uncertainty in the model structure F (·) itself (including uncertainty in the validity of the assumptions underlying the model), referred to as "model structure uncertainty".

The robust non dominated technique considered here to deal with the optimisation aimed to address all or most of those uncertainty issues to achieve the best optimised solution. Once the uncertainties and the sources of uncertainty related to the problem has identified, the next step is representation of the uncertainty mathematically in the model. The integrated model is then introduced in the optimisation using the robust non dominance criterion genetic algorithm technique.

5 FEATURE OF THE ROBUST NON DOMINANCE CRITERION

The robust non dominance criterion is a GA based multi-objective optimisation method designed to reflect the general situation of real world applications, such as rolling system, where high disturbance process environment and inevitable uncertainty in input variables occur. The technique also designed to find a solution for problems with uncertainty by introducing a non dominance criterion between design points in the solution space that are created as a result of presence of noise in the problem. The idea here is based on standard approach to evaluate the objective functions a fixed number of times k for a given decision vector. The problem at hand is then to estimate the true Pareto front PFtrue from a set of k noisy samples. The criterion for dominance between points is determined by computing the median values of the objective functions for all points of the Pareto set. Here the initial step is computing the k design solution points, realizing of two objective functions and for seven different points in the decision space and the related median. Afterwards the points are connected to one another so that convex hulls can be formed. Then required average distances are computed. Here a measure of uncertainty of a solution in m-dimensional objective space can be introduced as average deviation of a sample set to the estimate of the solution in each coordinate direction. Taking P: = med (Rk), as a robust estimate of a solution, and the convex hull of all k sample points around P describes a worst case representative of solution P.
containing all k samples. The absolute distances in each dimension of all points in the convex hull to P can be used to define the uncertainty vector.

A robust dominance criterion then determined given the uncertainty bounds around a solution P all points within the box formed by the bounds are represented by P. This implies that the conventional Pareto-dominance definition may not hold any more if any two points P and Q are inside the uncertainty vicinity of each other. Although these points may dominate each other in a noise-free case, in the case with noise it is impossible to tell which point dominates the other. For the analysis in this paper a real-coded Matlab version of NSGA-II was chosen so that is able to provide a source of comparison of the technique used to deal with problem with uncertainty in the paper.

More details of the technique can be found in [4, and 5].

6 Optimisation experiment and result

Here presented the engineering design optimisation with uncertainty non-dominance criteria technique. The optimisation carried out in problem with uncertainty in the decision (Parameter) space and objective space. The technique also applied to the design optimisation problem where uncertainty presence in both spaces. The experimental results are presented below.

6.1 Experimental details and discussion of results

The experiment is intended to prove a case study on real life engineering design optimisation problems with uncertainty. The experiments are conducted on the mathematical model developed in the previous section for real life roll cooling design problem. Robust non dominance optimisation approach was applied to the mathematical model and a minimisation of change of temperature (∆T) and work rolls maximum principal stress (MPS) sought.

6.2 Experimental details

The section investigates various levels of uncertainty and their effect on the cooling of rolls. For comparison initially a deterministic search was carried out, i.e. search with no noise. Then the search problem with noise in the problem using the new Pareto-dominance technique was experimented. A total of 6 experiments were carried out based on the set shown in table 3 below. In the experiment a standard NSGA-II setting has been applied, with crossover probability \( p_c = 0.9 \) and mutation probability is \( p_m = 1/n \) where \( n \) is the number of decision variables. The distribution indices for cross over and mutation operators are \( v_c = 20 \) and \( v_m = 20 \) respectively. A population size of \( pop = 200 \) resulted in sufficient spread of the solutions along the Pareto front and all the experiments have been performed with \( gen=200 \) generations.

<table>
<thead>
<tr>
<th>DS</th>
<th>1</th>
<th>2</th>
<th>1</th>
<th>3</th>
<th>1</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>FF</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

Optimisation with and without uncertainty in the DS and FF

Table 3: Experimental Set.

Descriptions

FF = Fitness Function
DS = Decision Space
1 = Deterministic search (no uncertainty in the problem)
2 = Uncertainty with lower sigma (5% DS & 6.25% FF)
3 = Uncertainty with higher sigma (10% DS & 12.5% FF)

The experimental results are presented in the next section. The first section is the grid search on the 7 dimensional decision spaces. Here no uncertainty is considered. Thus the result highlights the (estimated) true Pareto front. In the second section the results of experiments taking into account uncertainty. Based on information from real world rolling practice, two levels of uncertainty, lower to higher sigma values, are considered. The levels are 5% and 10% of the decision space and uncertainty of 6.25% and 12.5% for the fitness function.

The two levels are used to represent commonly noticed degree of uncertainty and worst case scenario in rolling practice. The performances of the solution are based on these uncertainty values introduced in the problem in the form of perturbation where the perturbation represents a normal distribution with sigma (σ) values. The sigma is the value in the design space calculated as a percentage of decision space of each decision variables listed in table [1] for example \[ σ = 5\%, 10\% (x_{max} - x_{min}) \] where \( i = 1,…,7 \).

Experimental result

Below the result from grid mapping (point cloud) and the Pareto search (the thick line under the cloud). Here no uncertainty is applied. The results are used to illustrate and provide a comparison between the true Pareto problem without uncertainty from grid search and the impact of the new Pareto-dominance criteria. As shown in the result gives a Pareto front with the same convex shape as the grid search from standard NSGA-II. This means that in a deterministic environment the new Pareto-dominance criterion behave like the conventional dominance criterion as expected.

![Figure 7: ∆f / s map generated by exhaustive grid search of the decision space.](image)

Uncertainty in the Decision Space: here presented results of the problem with uncertainty in the decision space. Two experiments carried out Sigma σ = 5% and 10% was used. The two values are used in the experiment so that uncertainty of margins 5% as well as the worst case scenario margins 10% can be tested in the optimisation. The robust non dominance optimisation technique applied and result observed. The results show that the spread of the Pareto is clustered and scattered. (Figures 8.1, 8.2). This is in fact an expected feature of solutions according to the Pareto dominance in uncertain environment. However, unlike results of other experiments presented in the next sections for uncertainty in the fitness function this property is uniquely observed more in the case of problems in the decision space. Nevertheless the all over spread of the solution lies around behind the true Pareto front. From the result it has been learnt that in this particular case more investigation required to study the scattering behaviour and improve the solution. Which is the beyond the scope of this paper. Additional work dealing with optimisation problem with uncertainty in the decision space and solution proposed for improved result are presented in [5].
Uncertainty in the fitness function: here experiment conducted design optimisation problem with uncertainty in the model i.e. fitness functions $\Delta f$ (change in temperature) and MPS (maximum principal stress). Unlike the result observed uncertainty in the decision variables, here the robust non dominance technique applied find solutions that are evenly spread on the true Pareto front (Figure 9.1). The same problem, with worst case scenario uncertainty level, considered very strong in real life rolling practise experimented. The result although, it shows a slight increase in scattering and a shift away from the true Pareto front nevertheless the solution remains close to the true Pareto front. The result here means that uncertainty in the model even in worst case scenario can be dealt with in the optimisation using robust non dominated criterion technique presented in section 5.

Uncertainty in the decision space and objective space: here the experiment carried out to observe the optimisation of problems with uncertainty in the decision space and in the fitness functions. As presented above the two cases have been experimented separately and each resulted with Pareto of unique characteristics. Here result shows that the Pareto dominance criteria technique find optimal solution but with few design solution points in comparison with result presented in the previous sections. This is may be due to higher overall noise level and particularly the presence of uncertain decision space in the problem so that not many non dominated points detected in the convex hull (Figures 10.1 and 10.2). However the results suggest that the uncertainty in decision space and the fitness function can be dealt with in the optimisation using the robust non dominated criterion technique. As presented below the algorithm find Pareto front that is very close to the true Pareto. For comparison, three random samples of design solution for problem without uncertainty and with uncertainty in the decision space and fitness function (Figure 10.2) presented in Table 4 and 5.

Table 4: Design solution at three random points along the true Pareto (Figure 7).

<table>
<thead>
<tr>
<th>High $\Delta T$</th>
<th>Low $\Delta T$</th>
<th>$\Delta T$ &amp; $S$ close to 0</th>
<th>Low $\Delta T$</th>
<th>High $S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.4145759*10^+900</td>
<td>5.674349*10^-03</td>
<td>x1</td>
<td>7.1389919*10^-01</td>
<td>x1</td>
</tr>
<tr>
<td>1.2469034*10^+903</td>
<td>1.2454972*10^-03</td>
<td>x2</td>
<td>1.2500000*10^-03</td>
<td>x2</td>
</tr>
<tr>
<td>6.9746913*10^-001</td>
<td>2.0034947*10^-03</td>
<td>x3</td>
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</tr>
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<td>1.7674115*10^-001</td>
<td>3.3330631*10^-01</td>
<td>x4</td>
<td>3.6230130*10^-01</td>
<td>x4</td>
</tr>
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<td>3.2041030*10^-01</td>
<td>x5</td>
<td>3.6913030*10^-01</td>
<td>x5</td>
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<td>1.0153120*10^-01</td>
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<td>1.3912930*10^-01</td>
<td>x6</td>
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<tr>
<td>9.3795008*10^-001</td>
<td>9.4454267*10^-03</td>
<td>x7</td>
<td>9.3915562*10^-03</td>
<td>x7</td>
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<td>2.3327717*10^-01</td>
<td>x1</td>
<td>6.1183017*10^-03</td>
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<td>2.0709041*10^-01</td>
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</tr>
</tbody>
</table>
Table 5: Design solution at three random points along the Pareto of the problem with uncertainty (Figure 10.2).

<table>
<thead>
<tr>
<th>High ΔT</th>
<th>Low S</th>
<th>ΔT &amp; S close to 0</th>
<th>Low ΔT</th>
<th>High S</th>
</tr>
</thead>
<tbody>
<tr>
<td>x1</td>
<td>y1</td>
<td>z1</td>
<td>x0</td>
<td>y0</td>
</tr>
<tr>
<td>x2</td>
<td>y2</td>
<td>z2</td>
<td>x1</td>
<td>y1</td>
</tr>
<tr>
<td>x3</td>
<td>y3</td>
<td>z3</td>
<td>x2</td>
<td>y2</td>
</tr>
<tr>
<td>x4</td>
<td>y4</td>
<td>z4</td>
<td>x3</td>
<td>y3</td>
</tr>
<tr>
<td>x5</td>
<td>y5</td>
<td>z5</td>
<td>x4</td>
<td>y4</td>
</tr>
<tr>
<td>x6</td>
<td>y6</td>
<td>z6</td>
<td>x5</td>
<td>y5</td>
</tr>
<tr>
<td>x7</td>
<td>y7</td>
<td>z7</td>
<td>x6</td>
<td>y6</td>
</tr>
<tr>
<td>x8</td>
<td>y8</td>
<td>z8</td>
<td>x7</td>
<td>y7</td>
</tr>
</tbody>
</table>

ΔT = Change in temperature
S = Stress (principal)
\( f_1 \) = fitness function 1 (maximum principal stress)
\( f_2 \) = fitness function 2 (Change in temperature)
\( x_1, x_2, x_3, x_4, x_5, x_6, x_7 \) are input variables/design points

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REFERENCES
