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Chaos Theory: Implications for Supply Chain Management.

By

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Before joining Cranfield, Richard was a Principal Teaching Fellow at the Warwick Manufacturing Group, University of Warwick, U.K. In this position he was responsible for course management and development in the area of logistics and supply chain management and collaborating on industrial projects on the subject. He works with leading European and international companies from a variety of Industries on logistics and supply chain projects. He has undertaken lecture tours of Hong Kong & India at the invitation of local Universities & Confederations of Industry. He has published widely in the area of logistics and is Editorial Advisor to a number of leading journals in the area. His Doctoral research, undertaken at the University of Warwick, has applied chaos and complexity science to logistics and supply chain management.

Richard is both a European and Chartered Engineer; he is a member of the Institute of Logistics and Institute of Electrical Engineers (Manufacturing Division).

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ABSTRACT

Since the late 1950's it has been recognised that the systems used internally within supply chains can lead to oscillations in demand and inventory as orders pass through the system. The uncertainty generated can result in late deliveries, order cancellations and an increased reliance on inventory to buffer these effects. Despite the best efforts of organisations to stabilise the dynamics generated, industry still experiences a high degree of uncertainty. The failure to significantly reduce uncertainty through traditional approaches may in part be explained by chaos theory.

This paper defines deterministic chaos and demonstrates that supply chains can display some of the key characteristics of chaotic systems, namely: Chaos exhibits sensitivity to initial conditions, it has "Islands of Stability", generates patterns, invalidates the reductionist view and undermines computer accuracy.

The implications for the management and design of supply chains are briefly discussed.

INTRODUCTION

The term chaos is currently much used within the management literature. The term chaos has been used to describe the seemingly random disorder of customer demands for products (e.g. Womack and Jones [1 p.81]) and by Tom Peters [2] in his book "Thriving on chaos" to describe disorganised yet responsive business structures that rapidly adapt and gain competitive advantage. Chaos is also used as a metaphor to describe how a small change can be amplified to have a large effect on the system. This has resulted from the popularisation of the "butterfly effect" where a butterfly flapping its wings creates tiny changes in the atmosphere that result in the creation of a tornado a few weeks later [3]. Authors [1 p.87, 4]) describing amplification within the supply chain have used the term chaos in this context. Using chaos as a metaphor for amplification within systems is an over simplification and can lead to misunderstanding. It has been shown that chaos and amplification are linked yet distinctly different types of behaviour [5].

Chaos theory has been applied and found in a variety of systems including cardiac systems [6], stock market data [7] and the management of digital telephone exchanges [8]. Chaos is even being exploited in advanced washing machines to improve cleaning [9] and in the manufacturing quality control of springs [10].

CHAOS

The Collins English dictionary describes chaos as meaning "complete disorder and confusion". However, within this paper the term chaos describes deterministic chaos. The definition used in this work is adapted from that proposed by Kaplan and Glass [11 p.27] and Abarbanel [12 p.15]:

Chaos is defined as aperiodic, bounded dynamics in a deterministic system with sensitivity dependence on initial conditions, and has structure in phase space.

The key terms can be defined as follows:

- **Aperiodic;** the same state in never repeated twice.
- **Bounded;** on successive iterations the state stays in a finite range and does not approach plus or minus infinity.
- **Deterministic;** there is a definite rule with no random terms governing the dynamics.
- **Sensitivity to initial conditions;** two points that are initially close will drift apart as time proceeds.
- **Structure in Phase Space;** Nonlinear systems are described by multidimensional vectors. The space in which these vectors lie is called phase space (or state space). The dimension of phase space is an integer [12]. Chaotic systems display discernible patterns when viewed. Stacey [13 p.228] emphasises this by defining chaos as;

  "order (a pattern) within disorder (random behaviour)".

Professor Ian Stewart proposes the following simplified definition [14 p.17]:

"Stochastic behaviour occurring in a deterministic system".

Stochastic means random or lawless, deterministic systems are governed by exact unbreakable laws or rules. So chaos is "Random (or Lawless) behaviour governed entirely by laws!"

Chaos is deterministic, generated by fixed rules that in themselves involve no element of chance (hence the term deterministic chaos). In theory, therefore, the system is predictable, but in practice the non-linear effects of many causes make the system less predictable. The system is also extremely sensitive to the initial conditions, so an infinitesimal change to a system variable's initial condition may result in a completely different response.

This presents us with a good news / bad news scenario. The good news is that apparently random behaviour may be more predictable than was first thought, so information collected in the past, and subsequently filed as being too complicated, may now be explained in terms of simple rules. The bad

news is that due to the nature of the system there are fundamental limits to the horizon and accuracy of prediction. Past patterns of system behaviour are never repeated exactly but may reoccur within certain limits. For example the weather, a true chaotic system, has limits to the forecast horizon. Even if every variable was known exactly the theoretical maximum forecast is 2 to 3 weeks [14].

For non-linear dynamic systems the assumption of one-to-one, cause and effect relationships implicit in most human logic do not hold. For chaotic systems, a tiny change in conditions may result in an enormous change in system output, whereas a substantial change in conditions may be absorbed without significant effect to the system's output.

NON-LINEAR SYSTEMS AND CHAOS

Mathematicians have discovered that non-linear feedback systems, are particularly prone to chaos. Information is fed back thus impacting on the outcome in the next period of time.

Jay Forrester proposes the following definition for a feedback system [15]:

> *" An information-feedback system exists when the environment leads to a decision that results in action which affects the environment and thereby influences future decisions"*

The process is continuous and new results lead to new decisions that keep the system in continuous motion.

All logistics and supply chain management systems are made up of a series of feedback control loops. This is the way the majority of business systems operate. Within logistics and supply chain management a large proportion of the feedback loops are non-linear.

For example, the availability of inventory affects the shipment rate from a warehouse. When inventory is near the desired level, shipment rate can equal order rate but as inventory reduces the shipment rate can become halted or checked by the amount of available inventory. This in turn leads to the issue of "service level".

The relationship between service level and cost to an organisation is depicted as a steeply rising curve. This results from the high costs of carrying additional safety stock to cover those times of unexpectedly high demand.

It is therefore possible that the systems of control developed for managing the supply chain exhibit chaotic behaviour.

CHARACTERISTICS OF CHAOTIC SYSTEMS

There are five key characteristics of chaotic systems that have implications for supply chain management. These are:

Chaos exhibits sensitivity to initial conditions

The characteristic of sensitivity is a central concept of chaos theory. However it should be emphasised that sensitivity does not automatically imply chaos [16 p.512]. This misunderstanding is prevalent in much of the management literature and is linked to the popularisation of the "butterfly effect".

Sensitivity to initial conditions within chaotic systems is more distinct. Given a small deviation in initial conditions, this small difference or error becomes amplified until it is the same order of magnitude as the correct value. The amplitude of the error is magnified exponentially until there is no means of differentiating the actual signal from the signal generated by the error. This results in two systems with starting conditions varied by a fraction of one percent producing outcomes over time that are totally different. The error propagation of the system results in the system being inherently unpredictable and therefore long term forecasting of such systems is generally impossible. This error propagation can be quantified by the use of Lyapunov Exponents.

Lyapunov exponents have proved to be one of the most useful diagnostic tools for detecting chaos. The exponent is a measurement of sensitivity to initial conditions. The maximum Lyapunov exponent can be described as the maximal average factor by which an error is amplified within a system. A system can be defined as chaotic if at least one positive Lyapunov exponent is present. If the maximum exponent is negative the system is stable or periodic [17].

The magnitude of the exponent gives a reflection of the time scale over which the dynamics of the system are predictable, so the exponent can be used to approximate the average prediction horizon of a system [17, 18]. After this prediction horizon has been reached the future dynamics of the system become unforecastable. This occurs because any small error is amplified so that the magnitude of the dynamics generated by the error exceeds the original dynamics being measured. This results in cause and effect relationship between current data and previous data becoming increasingly blurred and eventually lost. The concept of prediction horizons can enable managers to differentiate between short and long term strategic decisions and also define a limit to which any forecasting is effective [19].

The sensitivity to initial conditions also results in dramatic changes occurring unexpectedly. Stable behaviour can be followed by rapid change or a system behaving in a seemingly random manner may change to a stable form of behaviour without any warning.

There are many sensitive systems that do not behave chaotically. A simple example of this is the equation:

$X_{t+1} = CX_t$ where C is a parameter much greater than 1.

Any small error is magnified by a factor of C during each iteration. This system is sensitive to initial conditions but in no way can be defined as chaotic. The error will remain proportionally identical as the system is iterated

Chaos has "Islands of Stability".

Chaotic Systems are often related to aperiodic behaviour. Aperiodic behaviour is characterised by irregular oscillations that neither exponentially grow nor decay nor move to steady state [11 p.11]. These oscillations never repeat the same state twice. It is a behaviour that is neither periodic nor stochastic [12].

Chaotic systems typically can exhibit other domains of behaviour that may include stable convergent behaviour, oscillating periodic behaviour, and unstable behaviour [20]. A system may be operating in a stable manner but when a parameter is changed periodic or chaotic behaviour may be witnessed. Also, it is also not uncommon for chaotic systems to spontaneously switch between different modes of behaviour as the system evolves with time. Systems have been observed that will produce aperiodic behaviour for a long period of time and then spontaneously "lock" on to a stable periodic solution [21]. It is therefore possible for "Islands of stability" to be present in between areas of chaotic behaviour. This characteristic has been harnessed for some chaotic systems, by changing a parameter so that the chaotic system can be controlled to produce more regular behaviour.

Chaos generates patterns.

Despite the generation of apparently random data, chaotic systems produce patterns in the data. These patterns never repeat exactly but have characteristic properties. An example of this is a snowflake. The snowflake is generated by deterministic relationships within the environment but tiny changes become amplified. This results in every snowflake being different, but when observed it clearly belongs to the category of snow flakes. The patterns generated by systems are referred to as "attractors".

An attractor can be defined as [12 p.199]:

> *"The set of points in phase space visited by the solution to an evolution equation long after (initial) transients have died out"*

Attractors are geometric forms that characterise the long-term behaviour of a system in phase space [22]. Systems have a stable state, the state to which all initial conditions tend to gravitate; this state serves as an attractor. In classical systems, if the system gravitates towards a single point it is said to have a point attractor, if it gravitates towards a stable cyclic response it has a periodic attractor, if the attractor results from a combination of 2 or more periods the system can be defined as a quasi-periodic attractor. The term "strange attractor" is used to describe the shape of the chaotic patterns generated. These strange attractors are a classic feature of chaotic systems [23].

To understand the nature of chaotic attractors one needs to understand a simple stretching and folding operation. This results in a shuffling process being undertaken on the chaotic attractor, akin to a dealer shuffling a deck of cards. The randomness of the chaotic orbits is the result of this shuffling process [22]. The stretching and folding process can create patterns that reveal more detail as they are increasingly magnified, these are referred to as fractals.

We cannot predict exactly the behaviour of chaotic systems, but by understanding the patterns generated some degree of prediction is possible. Chaos generates endless individual variety, which is recognisably similar. As systems evolve with time recognisable patterns are generated. This property enables analysts to make some predictions about the system. One can predict the qualitative nature of the patterns generated and the quantitative limits within which the pattern will move [13].

Chaos invalidates the reductionist view.

One consequence of chaotic systems is that in general the reductionist view becomes invalid. The reductionist view argues that a complex system or problem can be reduced into simple forms for the purpose of analysis [24]. It is then believed that the analysis of the individual parts gives an accurate insight into the working of the whole system. This methodology of reductionism is also often applied to improvement within supply chain systems. The optimisation of the individual units, for example manufacturing, purchasing, and distribution, is believed to result in the optimisation of the global system. Goldratt and Cox [25] demonstrated that in manufacturing environments this is often not the case. One of the rules for manufacturing developed by Goldratt and Cox state:

> *"The sum of the local optimums is not equal to the global optimum"*

Chaos theory states that a small change to an individual unit within a system may result in dramatic effects on the global system. These effects may not in all cases be beneficial to the operation of the global system.

Chaos undermines computer accuracy.

Even simple equations can behave chaotically and these can have a dramatic effect on perceived computer accuracy. To demonstrate this phenomena a simple example will be described.

The example demonstrates chaos by iterating a simple equation using a standard spreadsheet package. The simple equation to be iterated is as follows:

$$X_t = KX_{t-1}^2 - 1$$

This simple equation for certain values of K is chaotic. The use of spreadsheets to demonstrate chaos enables an accessible method to demonstrate the nature of such systems [26].

When K is 1 then a stable periodic orbit occurs, the system is attracted to a cycle of 0, -1, 0, -1. The equation is relatively stable up until K = 1.5 producing periodic behaviour or quasi-periodic behaviour. At K = 1.5 chaos occurs and the dynamics become increasingly complicated as K increases. However, at K = 1.74 the system behaves chaotically but at K = 1.76 stable behaviour occurs. This stable behaviour continues until K = 1.81 and then chaotic behaviour reoccurs. Therefore an "island of stability" is present between K = 1.76 to 1.80.

PLACE TABLE 1 HERE

Table 1 shows the iteration of this equation when K = 2 on two different spreadsheet and hardware platforms and two starting conditions differing by a tiny amount. It can be seen that after forty iterations the results start to diverge rapidly. If K is greater than 2 the equation becomes unstable and the solution approaches infinity.

This raises a fundamental issue about the impact of chaos on computer systems. An identical program run on two different makes of computer, or different standard software packages doing the same calculations can produce significantly different results [14 p.21].

Peitgen [16 p.40] further emphasises this point by stating:

> *"More and more massive computations are being performed now using black box software packages developed by sometimes very well known and distinguished centers. These packages, therefore, seem to be very trustworthy, and indeed they are. But this does not exclude the fact that the finest software sometimes produces total garbage, and it is an art in itself to understand and predict when and why this happens. ...More decisions in the development of science and technology, but also in economy and politics are based on large-scale computations and simulations. Unfortunately, we can not take for granted that an honest error propagation analysis has been carried out to evaluate the results."*

In the remainder of this paper each of these characteristics will be related to the supply chain. To demonstrate some of these effects a number of supply chain simulations were used. These will now be described.

METHODOLOGY

The research described below was undertaken by the author to gain an insight into the potential generation of chaos within warehouse supply chains.

Chaos in warehouse supply chains.

The supply chains are characterised by automatic inventory control algorithms and EDI (electronic data interchange) between the echelons. The supply chain structures simulated for the following example can be seen in Figure 1. It is accepted that warehouse supply chains are in reality more complex but the model does capture the main components of such a system.

PLACE FIGURE 1 HERE

The simulation approach used to investigate the non-linear dynamic behaviour of supply chains was a development of that created by Mike Wilson at Logistics Simulation Ltd. This software and approach to the simulation was chosen as it is used commercially as a training and strategic development tool by a number of blue chip companies including ICI and Black & Decker [27]. It has been tested widely in industry and is shown to mimic with good accuracy the characteristics of actual warehouse supply chains. The simulation has been subjected to rigorous validation by the author, engineers and scientists within the University of Warwick and also external practitioners and academics. The simulation enabled the generation of large quantities of "clean" data that could be used in any further analysis [5].

The investigations outlined in this paper were conducted using an automatic re-order algorithm within the warehouse, which forecasts demand, calculates the optimum inventory cover level and places an order to account for expected demand for a given period. This is a widely used re-order and stock control algorithm used in industry [28] [29 pp.105-107]. The simulation can monitor inventory levels on a daily basis at all points of the chain. It also monitors the supply chain's ability to satisfy end customer demand.

The data used for analysis was the time series output of the apparent inventory level within warehouses in the simulated supply chain. This is the quantity of stock on hand minus any backorder not yet received from suppliers. This was used as it reflects well the inventory dynamics and has a direct relationship to the order pattern generated by the warehouse.

Searching for chaos

The tools for analysis of non-linear time series data are reasonably advanced and have been applied to a variety of systems. The analysis techniques can be classified as either geometric or algorithm based. Geometric techniques include return maps and phase plots, algorithm based techniques include the calculation of Lyapunov exponents and correlation dimensions that can be used to quantify the degree of chaos.

For the data to be characterised as chaotic it must be shown that the data satisfies the definition of chaos described above. A system is deterministic when future events are causally set by past events. Ensuring the data are bounded, aperiodic and sensitive to initial conditions can be quantified by using a number of standard tools. Standard statistical techniques can be used for checking that the data is

bounded. The calculation of Lyapunov exponents is a key technique for characterising chaotic behaviour and quantifying sensitivity to initial conditions this also detects periodic behaviour. Structure in phase space can be distinguished by the use of geometric techniques.

The non-linear analysis methodology used in this work has been developed from methodologies proposed by Abarbanel, Kaplan and Glass, and Sprott and Rowlands [11, 12, 30]. A detailed description of the methodology is documented by Wilding [5].

RESULTS

The results described below demonstrate that supply chains can display the characteristics of chaotic systems that were described earlier in this paper.

Chaos exhibits sensitivity to initial conditions

Rigourous analysis of the time-series data generated by the supply chain simulation has found positive positive lyapunov exponents are present, thus indicating sensitivity to initial conditions [Wilding, 1997b]. Figure 2 demonstrates the effect of sensitivity to initial conditions within warehouse 2 in a two warehouse supply chain. This is a somewhat crude experiment; but it gives a useful insight into the dynamics. It can be seen that slightly differing initial stock levels over time results in the graphs diverging. In the case where a small random input is placed over the initial demand the random input seems to temporarily stabilise the dynamics. However after a certain period a series of "chaotic spikes" can be observed.

PLACE FIGURE 2 HERE

Chaos has "Islands of Stability".

Figure 3 demonstrates the impact of changing the service level setting for warehouse 1 in a one-warehouse supply chain at a daily demand level of 10. It can be seen that islands of stability occur at approximately 98.7% and 99.3% resulting in a prediction horizon of 10,000 days. At other service level settings such as 99.8% the prediction horizon is reduced to 100 days due to chaos being generated. This demonstrates that "Islands of Stability" can be found for different parameter settings.

PLACE FIGURE 3 HERE

Chaos generates patterns.

Figures 4 and 5 show the relationship between the inventory level in warehouse 1 and warehouse 2 and 3 respectively. Each figure shows a distinctive pattern that never repeats precisely. This is typical of a chaotic system. A limited number of points has been plotted because as the number of points plotted increases the pattern starts to fill up the phase space.

PLACE FIGURES 4 & 5 HERE

Chaos invalidates the reductionist view.

Figures 6 and 7 also demonstrate how changing the supplier lead-time impacts on the prediction horizon of the data series from the warehouses. These results reinforce the indications that a change in a parameter within the supply chain impacts on all echelons in the chain. A particularly interesting finding is that for the shortest supplier lead-time (3 days) the prediction horizon in warehouse 1 is significantly lower than that of the other lead-times. However in warehouse 2 no significant difference is seen. A possible explanation for this is that warehouse 2 carries less cover stock as the supplier lead-time is shorter i.e. the algorithm calculates less stock is required as the supplier can deliver quickly. This then results in warehouse 1 having less inventory buffer in the warehouse supplying it. Therefore if any uncertainty due to chaos occurs warehouse 1 is more liable to be impacted upon, resulting in increased chaos and thus a reduced prediction horizon. This is an example of where the reduction of a lead-time (time compression) does not always result in improved dynamic behaviour for all players in the supply chain. The change of a variable in one echelon of the supply chain may result in detrimental performance in another echelon.

PLACE FIGURES 6 & 7 HERE

Chaos undermines computer accuracy.

As demonstrated above deterministic chaos can be generated by simple equations in standard spreadsheets. The results of iterative calculations can be sensitive to both the hardware and software used to undertake the calculations. Increasingly, industry is investing in faster communication systems and is purchasing "black box" packages for the management of key functions within the supply chain. The human decision making process is increasingly being automated. The interaction of such systems has to date seen little research. The dynamics of such systems are usually explored over a relatively short period of time with performance monitored in detail for, say, 6 months after installation. This is an inadequate time frame to assess such systems. The potential for the system to produce chaos requires the system to be monitored with care for much longer time periods. In theory, uncertainty within the supply chain could be caused by the algorithms being used operating under conditions that generate chaos.

MANAGERIAL IMPLICATIONS.

The implications of this work are that a system which is meant to control and level fluctuations, and consequently buffer the system from instability, can create dynamics which turn a stable predictable, demand pattern into one which is unpredictable with occasional explosive changes in demand, so further destabilising the system. Thus a system designed to optimise stock holding and order management can actually increase unpredictability and costs incurred across the total supply chain.

Deterministic chaos can be generated within supply chains. The increasing use of enterprise planning systems which are often little understood by the managers using them leaves supply chains susceptible to uncertainty generated by chaos.

Systems that generate chaos can also generate stable behaviour. A small change of a variable can cause the system to act in chaotic manner. This has been witnessed in the authors' investigations. For certain levels of demand the model behaves in a stable manner but as demand is raised the system becomes chaotic. By a further increase in demand the system becomes stable again. By searching for these "Islands of stability" it may be possible to optimise batch sizes which reduce chaos. It may also explain why a system that has been performing well for years becomes chaotic under new market conditions.

Removal of chaos

The key to the removal of chaos is the use of systems that do not have direct feedback loops. Simulations using simple re-order point systems do not produce chaotic behaviour as no feedback loops are present, however demand amplification has been shown to be a major drawback with this type of system. Many lean approaches to manufacturing do not rely on complex feedback systems. Focusing on the uninterrupted flow and matching the pull from the customer, which is the basis of such techniques, can be seen to eliminate feedback and consequently the conditions required producing further chaos. However, the misapplication of lean manufacturing, such as wholesale reduction of inventory and lead-times, can result in the system exhibiting increased chaos. One characteristic of chaotic systems is that they can perform in a counter intuitive manner. The study outlined above demonstrates that the reduction of a supplier lead-time which is known to reduce demand amplification resulted in an increase in chaos [5].

Management within a chaotic system

Key implications for management operating within a chaotic system are [19]: -

- Dramatic change can occur unexpectedly. Chaotic spikes in demand can occur which are generated by the system and not as the result of external events.

- Long term planning is very difficult. If long-term plans are made they need to be reviewed on a regular basis.

- Supply chains do not reach stable equilibrium. Small perturbations will always prevent equilibrium being achieved.

- Short-term forecasts and prediction of patterns can be made. It is better to allocate resource to the development of effective short-term decision making processes rather than long term.

- Treat the supply chain as a complete system. Small changes made to optimise one echelon of the supply chain can result in massive changes in other parts of the supply chain. Driving down inventory and lead-times may not always improve performance; it could result in the system slipping into chaos.

- Remove chaos by focusing on the customer; communicate demand information as far upstream as possible, use simple lean approaches.

- When changing hardware or software platforms, which are critical to your organisations operation, undertake detailed validation. Computers are prone to chaos.

- Simulation of systems and non-linear dynamic analysis of key outputs should be a mandatory part of any supply chain re-engineering proposal. Search for "Islands of stability". Remember that if a simplified model of the system generates chaos the real system with increased complexity will also.

CONCLUSION.

This paper has demonstrated that a simple supply chain can exhibit the characteristics of a chaotic system. Chaos generation contributes to the uncertainty experienced within the supply chain. It should be recognised that this is one of a number of sources of uncertainty that contribute to the total dynamics experienced by managers within supply chains. Wilding [5] describes the trade-off between deterministic chaos, demand amplification and what are described as "Parallel Interactions", which occur between suppliers in the same tier in the supply chain. All three effects combine to create the total uncertainty experienced. In some supply chains the contribution of deterministic chaos to uncertainty may be small, while in others it may make a significant contribution to the total uncertainty experienced.

Quantitative techniques are going to become increasingly important for logisticians to master. Chaos theory has already created a revolution in many areas of science. Traditional viewpoints have been challenged and a greater understanding gained. The application of chaos to supply chains will undoubtedly generate a similar response in the field of supply chain management.

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Number of Iterations	IBM 486 using Excel Spread sheet		PSION 3a using Psion Spread sheet	
Start Value	0.54321	0.54321000000001	0.54321	0.54321000000001
5	0.890035	0.890035	0.890035	0.890035
10	-0.84727	-0.84727	-0.84727	-0.84727
20	-0.07355	-0.07355	-0.07355	-0.07355
40	0.625099	0.614856	0.62497	0.614805
60	0.455086	-0.97999	-0.4463	-0.30702
80	-0.9822	-0.098716	0.306851	-0.80001
100	0.05050847	0.0349483	0.322846	-0.58814

Table 1 - The Iteration of $X_t = 2X_{t-1}^2 - 1$ using Excel and Psion, Using the same start conditions and a small error introduced.

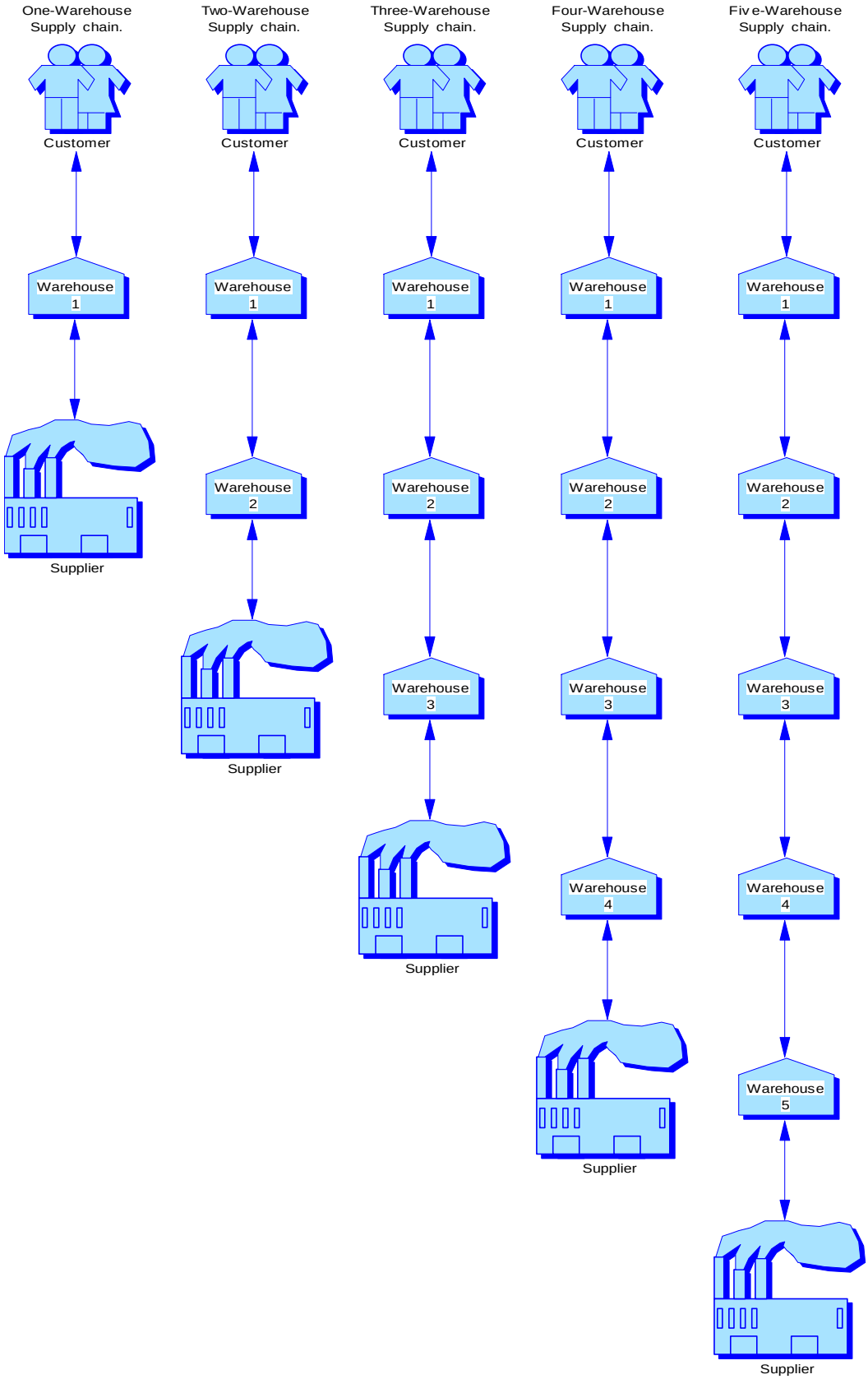

Figure 1 – Supply chain structures of increasing complexity.

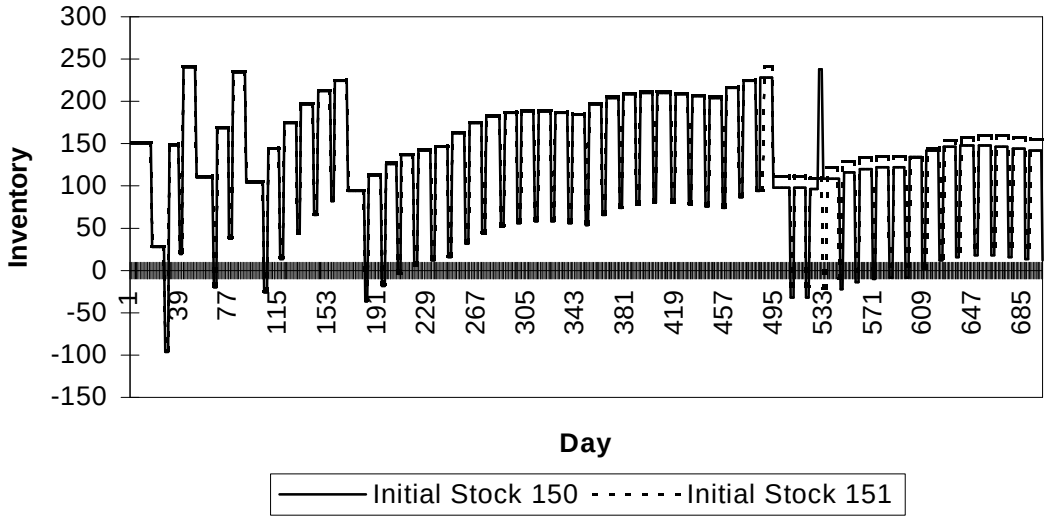

Figure 2a – Inventory in Warehouse 2 for slightly different initial stock levels.

Note the divergence of the data series at Day 457 and "Chaotic spike" at approximately Day 533.

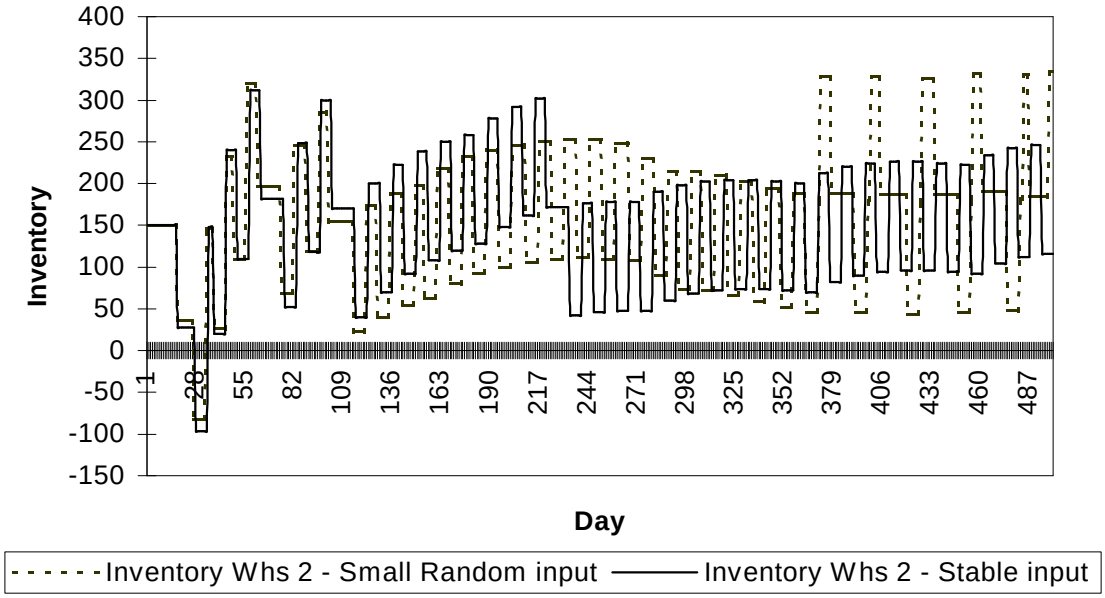

Figure 2b – Inventory in warehouse 2 for same starting conditions but using stable demand and demand with small random input.

Note that data series with small random input seems to stabilise between Day 136 and 179. This is then followed by a series of "chaotic spikes".

Figure 2 – Demonstrations of sensitivity to initial conditions within Warehouse 2.

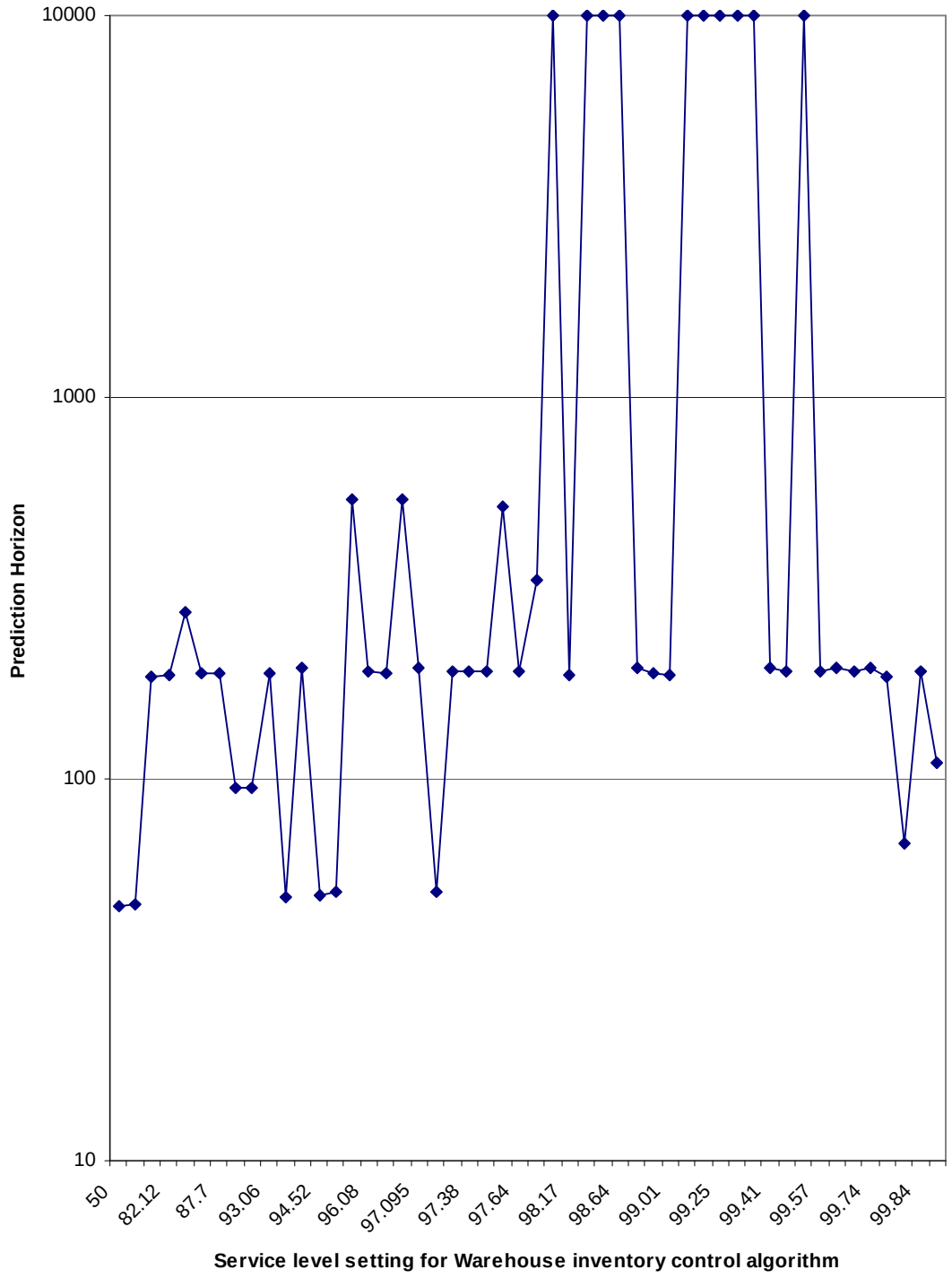

Figure 3 – Prediction horizon for different service level settings of warehouse algorithm in one warehouse supply chain.

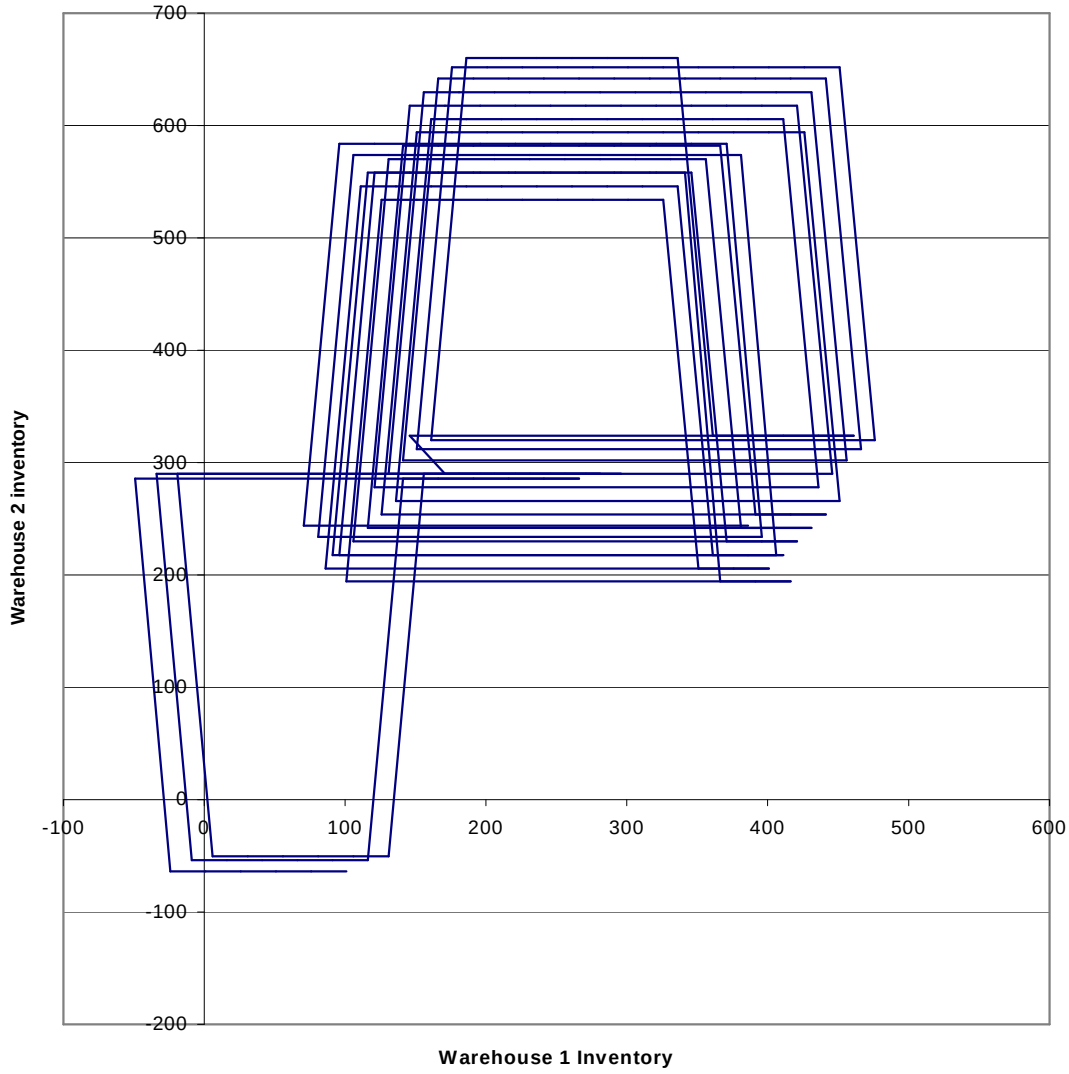

Figure 4 – Relationship between Warehouse 1 inventory level and Warehouse 2 inventory level in 5 warehouse supply chain, Demand = 25.

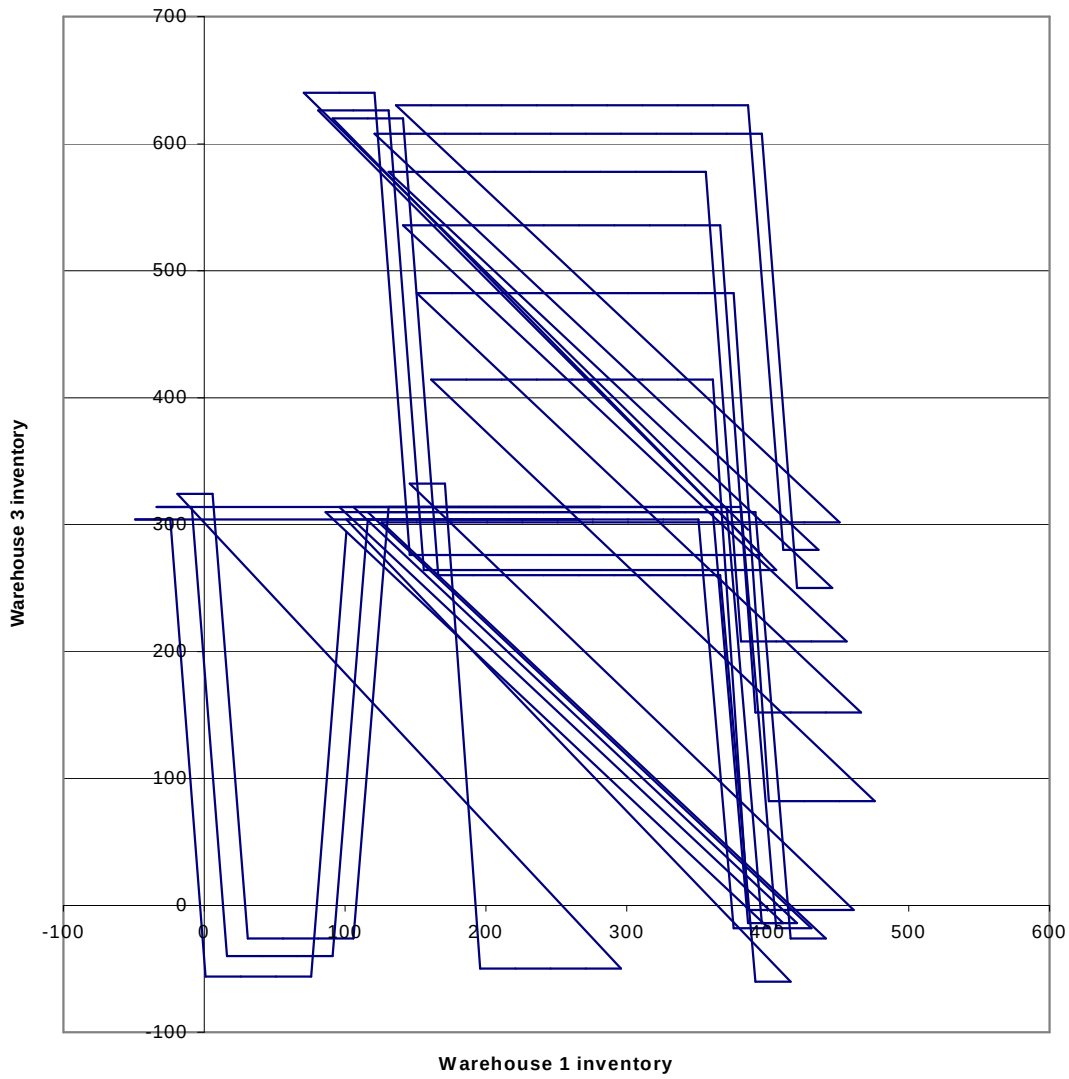

Figure 5 – Relationship between Warehouse 1 inventory level and Warehouse 3 inventory level in 5 warehouse supply chain, Demand = 25.

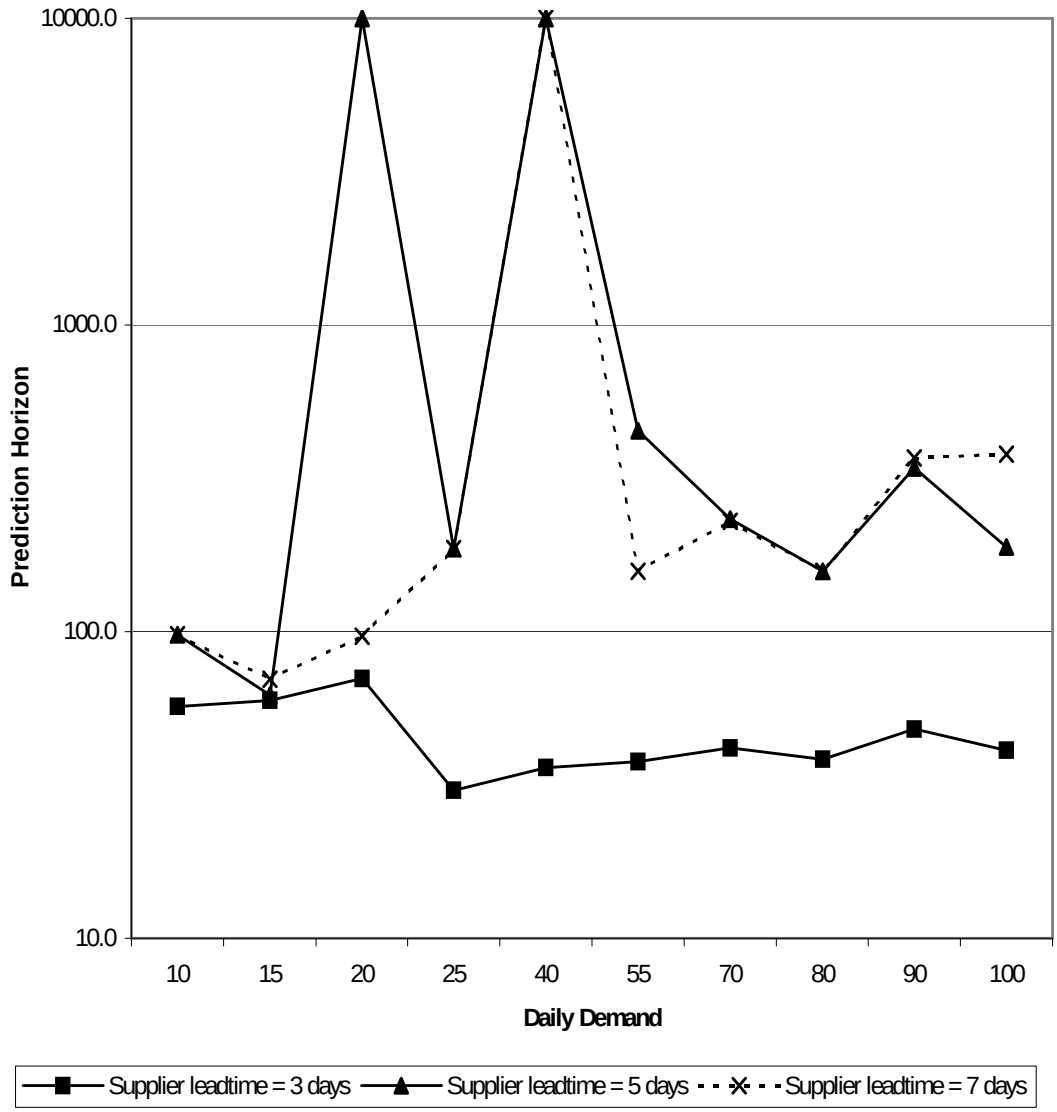

Figure 6 – Prediction horizon for Warehouse 1 in two warehouse supply chain with increasing daily demand level and different supplier lead-time.

It is interesting to note that for the shortest lead-time the degree of chaos has increased and subsequently the prediction horizon has reduced. This is the result of the automatic reorder system in the warehouse reducing the amount of safety stock (cover) and this therefore results in a reduced inventory buffer within the system. This can be seen as an example where time compression is not always beneficial.

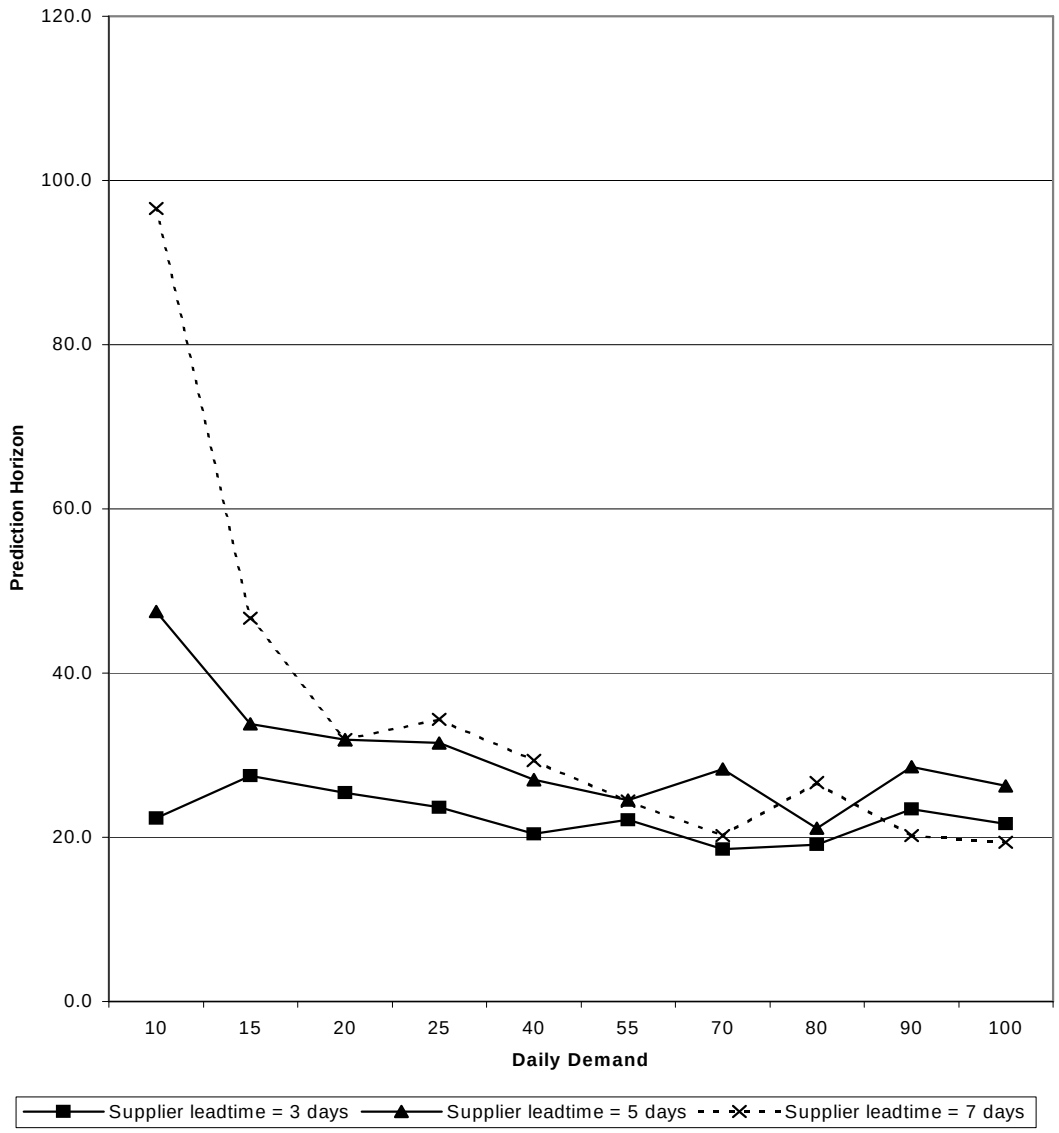

Figure 7 – Prediction horizon for Warehouse 2 in two warehouse supply chain with increasing daily demand level and different supplier lead-time.

It is interesting to note that for the shortest lead-time the degree of chaos is not significantly different in contrast to Figure 6.

Chaos Theory: Implications for Supply Chain Management.

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