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On ice induced instability in free-surface flows

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The problem of stability of a water-coated ice layer is investigated for a free surface flow of a thin water film down an inclined plane. An asymptotic (double-deck) theory is developed for the flow with large Reynolds and Froude numbers which is then used to investigate linear two-dimensional (2D), three-dimensional (3D) and nonlinear 2D stability characteristics. A new mode of upstream-propagating instability arising from the interaction of the ice surface with the flow is discovered and its properties are investigated. In the linear limit, closed-form expressions for the dispersion relation and neutral curves are obtained for the case of $Pr = 1$. For the general case, the linear stability problem is solved numerically and the applicability of the solution with $Pr = 1$ is analysed. Nonlinear double-deck equations are solved with a novel global-marching type scheme and the effects of nonlinearity are investigated. An explanation of the physical mechanism leading to the upstream propagation of instability waves is provided.

1. Introduction

The problem of solidification front stability arises in a number of engineering applications including casting (Glicksman *et al.* (1986)), pipe freezing (Gilpin (1979, 1981); Seki *et al.* (1984)) and airplane icing (Vargas & Reshotko (1998); Rothmayer & Tsao (1998)). In all these applications we encounter an interaction between an unsteady solidification front and the flow of the liquid phase which can lead to instability of the solidification front.

Hydrodynamic and morphological instabilities of the solidification interface in alloy solidification and crystal growth have been a subject of extensive research (see, for example, Glicksman *et al.* (1986); Davis (1990); Batchelor *et al.* (2000)). As a result, the coupling of a forced flow with morphological instabilities is a well-known effect in the models where the interface formation is determined by the solute concentration field near the solid/liquid interface. A characteristic feature of the solidification front instability observed in these problems is the upstream propagation of the solid/liquid interface disturbance (see Brattkus & Davis (1988); Forth & Wheeler (1989)).

The question of stability of the temperature-driven solidification fronts, and icing in particular, received comparatively little attention despite the experimental evidence available. Gilpin (1979) performed a series of experiments in order to study ice growth in a pipe at or near Reynolds numbers typical for laminar-turbulent transition. He found that for the entire range of Reynolds numbers that was obtainable in the experiment (up to 14000), there existed a final steady-state ice profile with a cyclic variation in height along the length of the pipe - an 'ice-band' structure. However another similar experiment by Kikuchi *et al.* (1986) demonstrated a smooth steady-state ice surface development without the ice-band structures. Later Gilpin (1981) extended this study to the range of

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