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Complexity and Cost Effectiveness Measures for Systems Design

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ABSTRACT

This paper proposes two measures intended to aid high level decision makers in comparing alternatives during pre-competitive studies or during the architectural design process of composite systems. The first measure is a complexity estimate and is based on Boltzmann's entropy concept. It measures the distribution of functional couplings in the system's decomposition. The second measure is intended to estimate the costs and benefits of a functional coupling related to system's performance.

1. INTRODUCTION

It has become a conventional wisdom that the decisions taken during the first 20 percent of the design stage commit approximately 80 percent of the total product costs. In recognition of this fact, major industrial sectors such as defense and aerospace, shipbuilding, and construction are currently trying to reduce risks by "front loading" their engineering programs, so that more (at least 15%) of the total resources are spent during the early stages of the product development process. One particular need in this respect has been to provide tools which could aid high-level decision makers such as systems architects, integrators and programme managers for comparison of alternatives on the basis of cost, value, performance (effectiveness) and technical risk. This paper reports on the initial stages of our research efforts to derive suitable metrics for cost-effectiveness and complexity (as a major contributor to technical risk).

The following section introduces briefly the concepts of Architectural Design, Axiomatic Design and Entropy. In Section 3 two estimates for complexity and cost effectiveness are proposed. In Section 4 our approach is compared to existing work in the field. Finally conclusions are drawn and future work is outlined.

2. TERMINOLOGY AND DEFINITONS

2.1 Architectural design

Architectural design is the process of transforming the technical requirements into a desired design solution. The first step is the *Logical Design* which converts the set of technical requirements into a set of logical representations or models of the solution. Example models include functional flow diagrams, object oriented models, algorithms derived from contextual diagrams and so forth. The second step of the Architectural Design process is the *Physical Design,* which generates alternative physical solutions. (More detailed description of the process can be found in Guide for ISO/IEC 15288 – System Life Cycle Processes.)

2.2 Axiomatic design

The underlying hypothesis of the axiomatic design (AD) theory is that there exist fundamental principles that govern good design practice. The main distinguishable components of the AD are domains, hierarchies, and design axioms. The foundation axioms are:

Axiom 1. Maintain the independence of the functional requirements.

Axiom 2. Minimise the information content of the design.

According to the AD theory (Suh, 1990), the design process takes place in four domains (Fig.1): Customer, Functional, Physical and Process. Through a series of iterations, the design process converts customer's needs (CNs) into Functional Requirements (FRs) and constraints (Cs), which in turn are embodied into Design Parameters (DPs). DPs determine (but also can be affected by) the manufacturing or Process Variables (PVs). The decomposition process starts with the decomposition of the overall functional requirement – in practice this should correspond to the top system requirement. Before decomposing to a lower level, the DPs must be determined for that level in the physical domain. This iterative process is called zigzagging. Zigzagging also involves the other domains since manufacturing considerations may constrain design decisions, while "over-specified" requirements could virtually prohibit the discovery of feasible design solutions. As a result of this process, it is not unusual to ask the customer to relax or re-formulate some of the requirements.

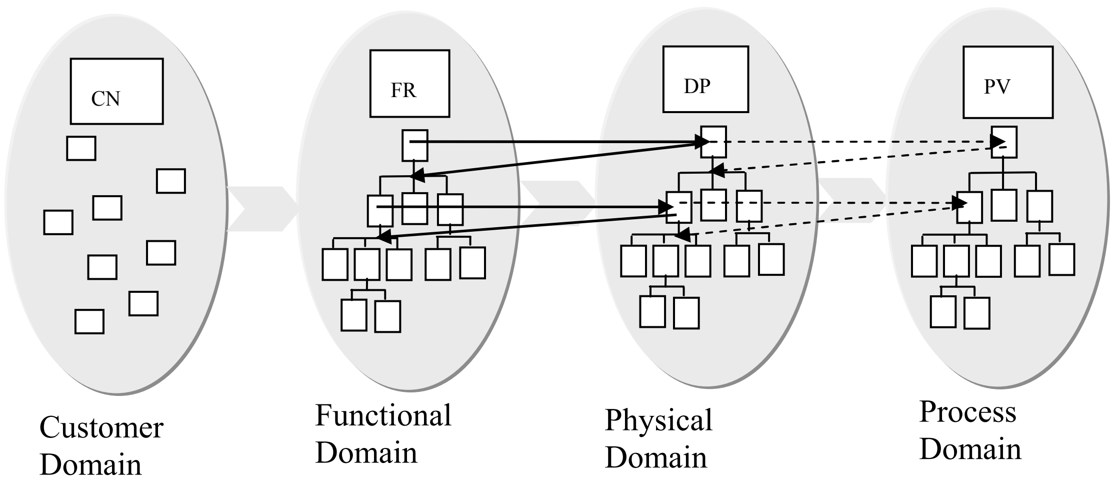

Fig.1. Decomposition by zigzagging.

At each level of the design hierarchy, the relations (the dependencies) between the FRs and the DPs can be represented in an equation of the form:

$$F\vec{R} = [A]\, D\vec{P} \qquad (1)$$

where each element of the *design matrix* [A] can be expressed as $A_{ij} = \partial FR_i/\partial DP_j$ (i = 1,…, m and j = 1,…, n). Eq.1 is called the *design equation* and can be interpreted as "choosing the right set of DPs to satisfy given FRs". Each element A_{ij} is represented as a partial derivative to indicate dependency of a FR_i on a DP_j. For simplicity the value of an element A_{ij} can be expressed as 0 (i.e. the functional requirement does not depend on the particular design parameter), or otherwise X. Depending on the type of the resulting design matrix [A], three types of designs exist: uncoupled, decoupled and coupled (Fig.2).

$$\begin{bmatrix} FR1 \\ FR2 \\ \vdots \\ FRn \end{bmatrix} \quad \begin{bmatrix} X & 0 & \dots & 0 \\ 0 & X & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & X \end{bmatrix} \begin{bmatrix} DP1 \\ DP2 \\ \vdots \\ DPn \end{bmatrix}$$

$$\begin{bmatrix} FR1 \\ FR2 \\ \vdots \\ FRn \end{bmatrix} \quad \begin{bmatrix} X & 0 & \dots & 0 \\ X & X & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ X & 0 & \dots & X \end{bmatrix} \begin{bmatrix} DP1 \\ DP2 \\ \vdots \\ DPn \end{bmatrix}$$

$$\begin{bmatrix} FR1 \\ FR2 \\ \vdots \\ FRn \end{bmatrix} \quad \begin{bmatrix} X & 0 & \dots & X \\ X & X & \dots & X \\ \vdots & \vdots & \ddots & \vdots \\ X & X & \dots & X \end{bmatrix} \begin{bmatrix} DP1 \\ DP2 \\ \vdots \\ DPn \end{bmatrix}$$

Fig. 2. Examples of design types. From top to bottom: uncoupled, decoupled and coupled design. (Adapted from Tate, 1999)

Uncoupled design occurs when each FR is satisfied by exactly one DP. The resulting matrix is diagonal and the design equation has an exact solution, i.e. Axiom 1 is satisfied. Unfortunately this rarely happens in practice. When the design matrix is lower triangular the resulting design is decoupled, which means that a sequence exists, where by adjusting DPs in a certain order, the FRs can be satisfied. This is a very important finding, as the design process is determined to a great extent by this sequence. The design matrix of a coupled design contains mostly non-zero elements and thus the FRs cannot be satisfied independently. It follows that by avoiding coupled designs one can avoid excessive iterations and thus shorten the product development cycle. A coupled design can be decoupled, for example, by adding components to carry out specific functions – however this comes at a price.

One additional factor that affects coupling is the number of FRs, m, relative to the number of DPs, n. If $m>n$ then the design is either coupled or the FRs cannot be satisfied. If $m<n$ then the design is redundant. (Note that in both cases the design matrix [A] is not squared)

The complexity of the problem increases tremendously when the process factors are being considered simultaneously with the design ones. By analogy to (Eq.1) the design parameters at a certain level of the design hierarchy can be considered as requirements for the manufacturing process. Thus the design equation for the manufacturing process (or design for manufacturability) is similarly given as:

$$\vec{DP} = [B]\vec{PV} \qquad (2)$$

By substituting (Eq. 2) in (Eq. 1), the two matrix equations can be combined into a single relation, linking the requirements with the manufacturing process:

$$\vec{FR} = [A][B]\vec{PV} = [C]\vec{PV} \qquad (3)$$

where [C] = [A]x[B]. The multiplication order reflects the chronological order of the design and manufacturing processes. In theory, if the resulting matrix [C] is diagonal, then the design is uncoupled and all design and manufacturing parameters satisfy the functional requirements. In practice this is extremely rare and either [A], i.e. the design, or [B], i.e. the manufacturing process has to be modified during the product realisation process.

The Second Design Axiom states that minimising the *information content* of a DP increases the probability of success of satisfying a function (Suh, 1990). The information content is defined by the equation (see also Fig.3):

$$I = \log\left(\frac{\text{system range}}{\text{common range}}\right) \qquad (4)$$

In Fig. 3 *design range* is the tolerance within which the DP can vary; *system range* is the capability of the manufacturing system. The information content of a system can be computed by summation of the individual information contents of all DPs only if the latter are probabilistically independent.

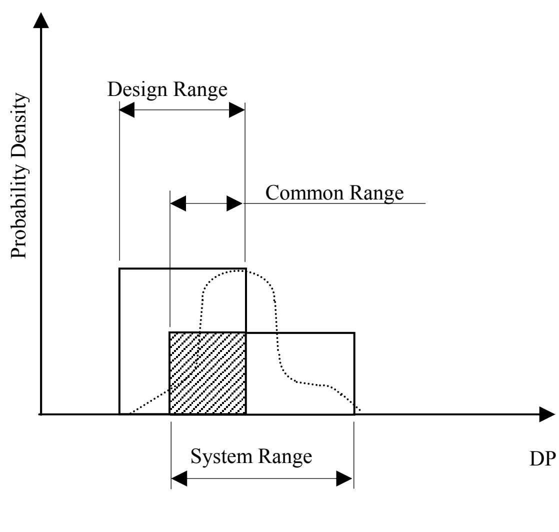

Fig. 3. Probability distribution of a design parameter. The solid line represents uniform distribution, the dotted line - a non-uniform distribution. (Adapted from Suh, 1990).

Frey et al (2000) proved that simple summation cannot be performed for decoupled designs and offered a method for computing information content. Currently there is no method for computing the information content of coupled designs, which is a serious problem, given that most complex systems are inherently coupled.

2.3 Entropy and disorder

Let N elements be characterised by qualitative properties Q_j (j=1,…, K), which form a classification or a division, i.e. each element has one and only one of the K properties. The properties Q_j are also called *cells* of the classification. A complete description specifies for each element the cell to which it belongs. Thus the possible number of (individual) descriptions is:

$$Z = K^N \tag{5}$$

If for example we try to put N_1 (out of N in total) molecules into a cell, we have a description represented by a variation:

$$V_N^{N_1} = N.(N-1).(N-2).....(N-N_1+1) = \frac{N!}{(N-N_1)!}$$

Ignoring variants such as AB, BA, ABC, CAB, etc., then the description becomes a combination:

$$C_N^{N_1} = \frac{V_N^{N_1}}{N_1!} = \frac{N!}{(N-N_1)!\,N_1!}$$

The general case, where N_j is the number of elements (molecules) belonging to a sell Q_j (j = 1,…,K), is a multiplicity, as shown in Fig. 4 and Eq.6.

$$\Omega = C_N^{N_1}.C_{N-N_1}^{N_2}.C_{N-N_1-N_2}^{N_3}....C_{N-N_1-...-N_{K-1}}^{N_K} = \frac{N!}{N_1!N_2!...N_K!} \tag{6}$$

The multiplicity Ω in Eq. 6 is often called *degree of disorder*.

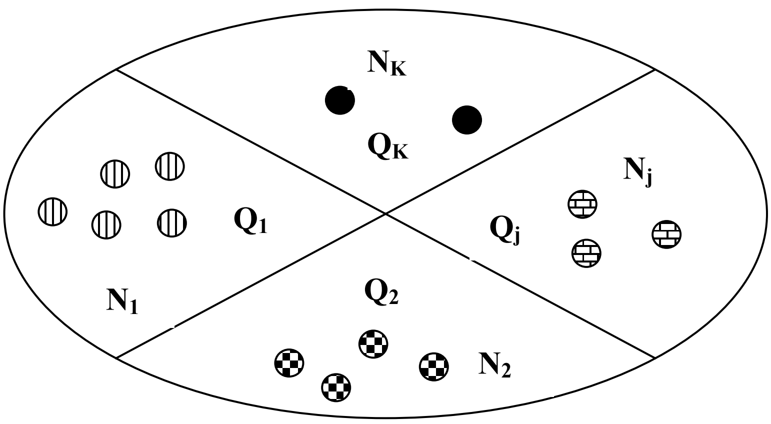

Fig. 4. Example of statistical description.

Boltzmann considers the state of gas body *g* at a given time *t* where the gas body consists of N molecules, each characterised by n magnitudes ϕ_j. For each magnitude ϕ_j its interval of admitted values is divided into small intervals of equal length Δ_j. Hereby the n-dimensional space, called also μ-space or module space, is divided into a system of cells of equal volume: $\upsilon^{\mu} = \Delta_i, \ldots, \Delta_n$. Let K be the number of these cells within the total range of admitted values, R^{μ}; then:

$$\upsilon^{\mu} = V^{\mu}/K, \tag{7}$$

where V^{μ} is the volume of R^{μ}. The μ- cells are analogous to the cells Q_j (j = 1,…, K) in the classification system described above[1].

Let f_j be the density in Q_j, i.e. the number of molecules per unit of μ-volume:

$$f_j = N_j / \upsilon^{\mu} \tag{8}$$

Boltzmann defines a function for a statistical description as follows:

$$H = \sum_{j=1}^{K} [\, f_j \ln f_j \,] \upsilon^{\mu} \tag{9}$$

Taking into consideration equations 5,6 and 9, Boltzmann's entropy can be expressed as (see for example Carnap, 1977):

$$S = -kH \quad \cong \quad k \left\langle \ln \frac{\Omega}{Z} + N \ln \frac{V^{\mu}}{N} \right\rangle \tag{10}$$

where k is the Boltzmann constant; k = $1.38.10^{-16}$ erg/C.

3. ESTIMATES FOR ARCHITECTURAL DESIGN

In this section I introduce two measures for architectural design, which need to fulfill two main requirements - they should be relatively simple and easy to apply, and be sufficiently accurate for the early stages of complex systems design.

3.1 Complexity measure

When two completed design solutions are compared, one can decide which one is more complex on the basis of measurable physical quantities such as number of parts, connections, supply chain size, and so forth. At an early design stage many of those quantities are not necessarily known and should be inferred. Our hypothesis is that the distribution of FR-DP couplings gives a good idea of complexity. This assumption stems from a corollary of the first design axiom which prescribes that coupled designs should be decoupled if possible. The question then is: Given two coupled or two decoupled designs, which one is more complex? Obviously the number of couplings plays a role, but their distribution is equally if not more important, since it will affect the time and the probability of the iterative design process converging on a solution. Thus the estimate has to take into account both the size of the problem and the couplings distribution. The distribution aspect has a certain analogy with *rank (degrees of freedom)* and *sparsity* of a coefficient matrix and their effect on finding a solution of systems of linear equations. The difference is that the design equations are not necessarily linear and also many of the

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[1] μ is a superscript label in the equations in this section.

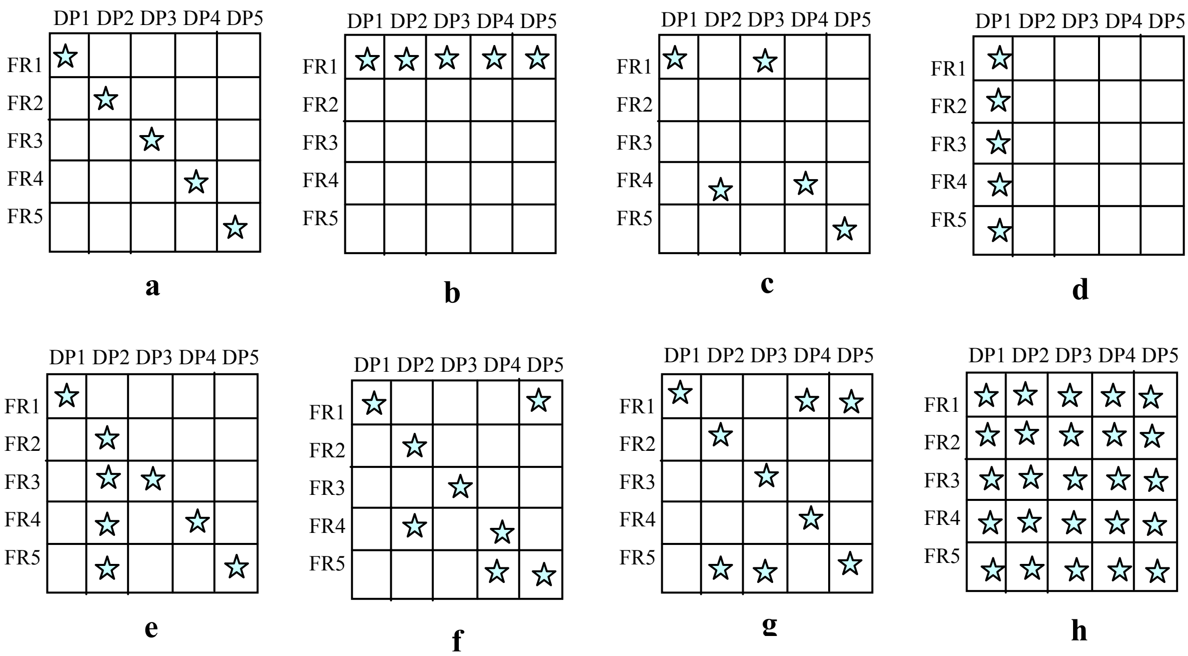
entries in the design matrix do not have numerical values, especially during the conceptual design stage. Thus, at that stage, the matrix is most useful in directing the design process rather than producing numbers.

The candidate estimates for complexity measures, which have been considered so far are:

- $\Omega = N!/(N_1!\ldots N_K!)$ - following directly from Eq. 6,
- $\mathrm{Ln}[\Omega]$ – intended to produce more tangible numbers,
- $\Sigma\, N_j \ln N_j$ - from Eq. 8 and Eq. 9, where the volume is assumed equal to unity.

A relatively simple example demonstrating the similarities and the differences between the measures is depicted in Fig.5 and Table 1.

Fig. 5. Design matrices and coupling distributions. The star symbol denotes a



coupling.

	Fig 5-a	Fig 5-b	Fig 5-c	Fig 5-d	Fig 5-e	Fig 5-f	Fig 5-g	Fig 5-h

N	5	5	5	5	8	8	9	25
K	5	5	5	5	5	5	5	5
$\Omega = N!/(N_1!\ldots N_K!)$	120	120	120	1	1680	5040	22680	$6.22.10^{14}$
$Ln[\Omega]$	4.79	4.79	4.79	0	7.43	8.53	10.03	34.06
$\Sigma N_j \ln N_j$	0	0	0	8.05	5.55	4.16	5.55	40.24

Table 1. Complexity measures.

In Table 1, N is the total number of couplings (the total number of "star" symbols) in each case in Fig. 5. K is the number of design parameters (number of columns). For each case in Fig. 5, K=5. N_j is the number of couplings per design parameter (i.e. per column), j = 1,…, K. Consider the case of Fig. 5-e: N=8; K=5. Then $\Omega = 8!/(1!4!1!1!1!) = 1680$; $Ln\Omega = \ln 1680 \approx 7.43$; $\Sigma N_j \ln N_j = 1\ln1+4\ln4+1\ln1+1\ln1+1\ln1 = 4\ln4 \approx 5.55$.

It can bee seen from Table 1 that Ω and $Ln[\Omega]$ produce values which agree better with the statistical concept of entropy. The estimate $\Sigma N_j \ln N_j$ produces results, which seem to convey better the meaning of Axiom 1. Thus for the completely coupled design in Fig. 5-d, the estimate correctly produces the second largest complexity value, while $Ln[\Omega] = 0$. The explanation for this is that a classification cell, in the axiomatic design sense, is represented by a single design parameter, $DP_{j,}$ rather than by an individual element of the design matrix (A_{ij}). At this stage the function $\Sigma N_j \ln N_j$ is seen as the preferred estimate for complexity that will be applied to industrial case studies for test and validation.

3.2 Cost effectiveness index

Axiomatic design favours uncoupled designs, but practice has shown that coupling of functions needs to be considered in the context of a trade-off between cost, performance and other business criteria (e.g. revenues, market share, brand image, etc., see Gonzalez-Zugasti, et. al., 2001, p.31). As a simple example consider the heating system of a (budget) car. The heat is taken more or less directly from the engine and there is very little one can do in a cold winter morning, but to wait for the engine to warm up. The example shows that the designers, certainly aware of the problem, have weighed the requirements ("propel car" and "heat cockpit") and have compromised the heating in order to keep the costs down. Because of the need to combine and trade-off additional multiple objectives the dimensionless *Index of Value and Cost Effectiveness (I_V)* is proposed[2]:

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[2] For brevity *(I_V)* is referred to occasionally as a *cost effectiveness index*.

$$I_V = \sum_{i=1}^{n} \sum_{j=1}^{K} f_{ij} c_{ij} , \qquad (11)$$

where:

i = [1,…, n] is the number of functional requirements (FR).

j = [1,…, K] is the number of design parameters (DP).

f_{ij} is the degree to which a coupling at design parameter DP_j fulfils the functional requirement FR_i , $0 <= f_{ij} <= 1$, and also, $0 < \Sigma f_j <= 1$. This parameter is related to performance. The rationale is that a functional requirement can be allocated over a number of design parameters, equipment, or subsystems respectively. (For example, Birkler et. al. 2001, p.83, report on a certain allocation of functions in the integrated mission equipment package of the Joint Strike Fighter, which will affect the entry of a new vendor in a future competition). While it is desirable that the functional requirement is fully satisfied (i.e. $\Sigma f_j = 1$), in practice this may not be the case due to the trade-offs involved in systems design.

c_{ij} is the cost-benefit ratio of coupling the i^{th} functional requirement with the j^{th} design parameter. The cost-benefit ratio can be determined from Eq. 12 and Table 2:

$$c_{ij} = \sum_{l=1}^{w} \frac{s_{ij}^{l}}{g_{ij}^{l}} \qquad (12)$$

s^l_{ij} - Estimated benefits due to the coupling of FR_i with DP_j	g^l_{ij} – Estimated costs incurred due to the coupling of FR_i with DP_j
Shared resources, e.g. fewer components, common components, reused components, etc. Availability of COTS (commercial-of-the-shelf) products	Cost of design efforts to extract additional functionality from existing components or accommodate COTS constraints.
Transaction Costs (Make vs Buy); Workshare (jobshare); Savings due to outsourcing	Lost revenue due to delays (time to market), loss of company knowledge. Costs due to work-share: transactions, logistics, etc.
Capability of the (sub)system to evolve (technology refresh and technology insert).	The investment needed to create variants based on an inflexible platform will increase to cover the adjustments needed to create the family (see Gonzalez-Zugasti, et. al., 2001, p.32.)
Savings due to competition (e.g. does the coupling allow introduction of subsystem vendors in a competition)	Cost of introducing competition (e.g. Birkler et. al. 2001)
Savings and competitive edge due to innovation	Cost/risk of introducing innovation

Table2. An example of estimated savings (benefits) and value versus estimated incurred costs due to a coupling. The list is not complete.

The entries in Table 2 are speculative and are based on previous literature search. The next stage of the research will include field studies where the cost/benefit factors will be specified more precisely after interviews with a cross section of company executives and senior engineers.

It is often difficult to express quantitatively the costs and benefits in absolute numbers. It may be easier then to use a ratio:

$1/p <= c_{ij} <= p \qquad (p > 1)$,

rather than in absolute numbers. The ratio may be subjective, but will allow an experienced analyst to take into consideration intangible costs/benefits[3]. Furthermore functional requirements are usually prioritised during the design process, thus Eq.12 can be modified to accommodate preferential treatment at each coupling:

$$I_V = \frac{\sum_{i=1}^{n} r_i \sum_{j=1}^{K} f_{ij} c_{ij}}{\sum_{i=1}^{n} r_i}, \tag{13}$$

where r_i is the rank (weight) of the requirement.

The I_V is a compound measure, but its components can be used separately as checks. For example, when a particular functionality is distributed over a few design parameters it should be no less than a predetermined value, α ($0 < \alpha <= 1$).

4. DISCUSSION

Entropy has been so far our first and only choice in deriving a measure for complexity. Entropic measures have already been proposed for a class of manufacturing operations (Frizelle and Suhov, 2001). El-Haik and Yang (1999) have applied entropy concepts to derive elaborate formulae for design complexity. These seem more applicable to cases where there are already analytical relationships between the FRs and DPs. El-Haik and Yang view complexity as a compound measure consisting of variability, correlation and vulnerability. Variability accounts for the variability of DPs due to factors such as manufacturing. Correlation is due to statistical correlation amongst two or more DPs. Vulnerability accounts for the size and the topological structure of the design matrix, namely the position of the nonzero sensitivity coefficients. Vulnerability depends also on the size of the nonzero coefficients. In this respect our entropy measure is more akin to vulnerability. It may appear less comprehensive, but should be easier to apply during the early stages of the design of complex systems, with less information available.

[3] For example *Transaction Costs Economics* concentrates on “make or buy” decisions based on costs alone, but does not take into account important functions of the firm, such as protection against speculative and volatile markets, the preservation of culture, tradition, and “crown jewels”.

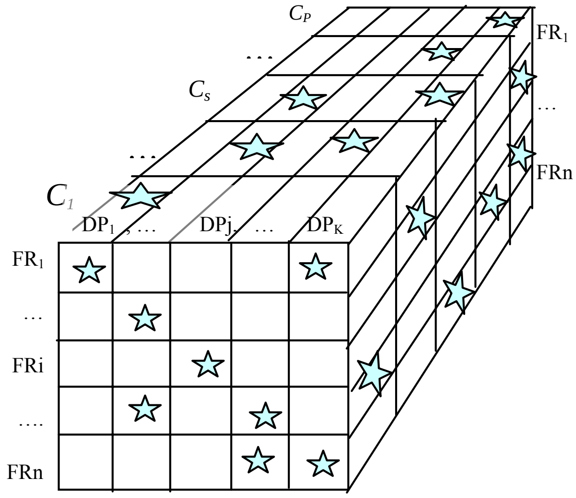

Fig.6. The *design cube*. Constraints (C) affect functional requirements (FR) and the choice and tolerances of the design parameters (DP).

Constraints play an important role in the choice of design parameters and can even require the reformulation of functional requirements. Currently our complexity measure does not account for constraints, including performance characteristics (nor does El-Haik and Yang's formula). These may be included into a similar framework, which for simplicity I shall call the *Design Cube* (although in general it is a parallelepiped), as shown in Fig. 6. It is an outstanding question whether the entropy measure can be applied over and across the different dimensions of the design cube to produce a better description of complexity. In this respect, cases such as those shown in Fig. 5-a,b, c, are not identical in terms of technical or design complexity, as one might infer, should the formula $\Sigma\ N_j \ln N_j$ is applied along the columns only (see section 3.1).

The entropy measure $\Sigma\ N_j \ln N_j$ and the cost effectiveness index (I_V) can be applied independently. The two measures are not completely independent however, since complexity is related to both cost and effectiveness. The relationship is not necessarily a causal one, but rather a correlation. The sign and shape of the correlation will depend on the particular case. In general one should aim for low entropy and high I_V.

5. CONCLUSIONS AND FUTURE WORK

Two estimates, intended to be used by systems architects, integrators, and programme managers for decision support during the early stages of complex systems design, are proposed in this paper. The first estimate, based on Boltzmann's statistical concept of entropy, measures complexity in terms of the size and the distribution of couplings in the design matrix. As such it is akin to El-Haik and Yang (1999) vulnerability measure, but is intended to be simpler and easily applicable during the early design stages.

The second measure is the cost, value, and performance effectiveness index. It is designed to take into consideration not only cost and performance issues, but also intangible or difficult to quantify costs and benefits.

The two measures are complementary and one should aim for low entropy and high cost effectiveness index. Both measures can be used for non-square design matrices, which is an advantage considering that this occurs often in practice.

The findings reported in this paper represent initial work done for a larger project tackling the decomposition of complex systems *(COPE)*. The aim of the project is to develop a decomposition strategy, which takes into consideration not only technical but also business factors. The next stage is to formalise the factors affecting the decomposition, after which the two proposed estimates will be tested and validated in industrial case studies.

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