Cranfield UNIVERSITY

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Abstract

This paper describes the development of a simple theory of the longitudinal controls fixed static stability of tailless aeroplanes. The classical theory, as developed for the conventional aircraft, is modified to accommodate the particular features of the tailless aeroplanes. The theory was then applied to a particular blended-wing-body tailless civil transport aircraft, BWB-98.

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Contents

Nomenclature

$a_1 = \dfrac{\partial C_L}{\partial \alpha} = C_{L_\alpha}$	Lift curve slope
$a_2 = \dfrac{\partial C_L}{\partial \eta} = C_{L_\eta}$	Elevator lift curve slope
$a_3 = \dfrac{\partial C_L}{\partial \beta} = C_{L_\beta}$	Elevator tab lift curve slope
ac	Aerodynamic center
AoA	Angle of attack
b_o	Basic elevator hinge moment
$b_1 = \dfrac{\partial C_{h_e}}{\partial \alpha}$	Elevator hinge moment derivative with respect to α
$b_2 = \dfrac{\partial C_{h_e}}{\partial \eta}$	Elevator hinge moment derivative with respect to η
$b_3 = \dfrac{\partial C_{h_e}}{\partial \beta}$	Elevator hinge moment derivative with respect to β
BWB-98	Blended-Wing-Body 98
$\bar{\bar{c}}$	Aerodynamic chord
C_D	Drag coefficient
$(C_D)_\alpha$	Drag coefficient due to angle of attack
$(C_D)_c$	Drag coefficient due to camber
C_{D_o}	Drag coefficient for zero lift
CG	Center of Gravity
C_{h_e}	Hinge moment coefficient
C_L	Lift coefficient
$(C_L)_\alpha$	Lift coefficient due to angle of attack
$(C_L)_c$	Lift coefficient due to camber
$(C_L)_{wb}$	Lift coefficient for the wing-body-nacelles configuration
C_{L_o}	Lift coefficient for zero angle of attack

$C_{L_q} = \dfrac{\partial C_L}{\partial \hat{q}}$	Lift coefficient derivative wrt to the no-dimensional pitch rate
$C_{L_{trim}}$	Lift coefficient to trim
C_m, $C_{m_{CG}}$	Pitch moment coefficient at the center of gravity
C_{m_o}	Pitch moment coefficient at the ac
$\left(C_{m_o}\right)_{wb}$	Pitch moment coefficient at the ac for the wing-body-nacelles configuration
$\overline{C}_{m_o}$	Basic pitch moment for $\alpha = \eta = \beta = 0$
C_{m_p}	Pitch moment due to the power units
$C_{m_q} = \dfrac{\partial C_m}{\partial \hat{q}}$	Pitch moment derivative wrt the no-dimensional pitch rate
$C_{m_\alpha} = \dfrac{\partial C_m}{\partial \alpha}$	Angle of attack pitch moment curve slope
$C_{m_\beta} = \dfrac{\partial C_m}{\partial \beta}$	Elevator tab pitch moment curve slope
$C_{m_\eta} = \dfrac{\partial C_m}{\partial \eta}$	Elevator pitch moment curve slope
C_W	Weight coefficient
D	Drag
D_α	Drag due to angle of attack
D_c	Drag due to camber
g	Acceleration due to gravity
h	CG position on $\overline{\overline{c}}$
h_c	Location of the aerodynamic forces due to camber on $\overline{\overline{c}}$
H_e	Hinge moment
h_m	Controls fixed maneuver point on $\overline{\overline{c}}$
H_m	Controls fixed maneuver margin
h_n	Controls fixed neutral point on $\overline{\overline{c}}$

$h_n^{'}$	Controls free neutral point on $\bar{\bar{c}}$
h_o	Aerodynamic center location on $\bar{\bar{c}}$
$(h_o)_{wb}$	Aerodynamic center location on $\bar{\bar{c}}$ for the wing-body-nacelles configuration
k	Drag polar constant
K_n	Controls fixed static stability margin
$K_n^{'}$	Controls free static stability margin
L	Lift
L_α	Lift due to angle of attack
L_c	Lift due to camber
m	Mass
M_{ac}	Pitch moment about ac
M_{CG}	Pitch moment about CG
M_o	Basic pitch moment about CG
$\hat{q}$	No-dimensional pitch moment rate
S	Wing area
V	Airspeed
z	CG position perpendicular to $\bar{\bar{c}}$, along the z axis
α	Angle of attack
α_{trim}	Angle of attack to trim
β	Elevator tab angle
β_{trim}	Elevator tab angle to trim
η	Elevator angle
η_{free}	Elevator angle for controls free
η_{trim}	Elevator angle to trim
μ	Mass ratio or longitudinal relative density factor

1. INTRODUCTION

Aircraft development over the last sixty years, or so, has focused on improving the performance and utility of conventional configurations comprising wing, fuselage and tail. Moreover, all design, aerodynamic and flight dynamic tools have been developed to apply primarily to this class of aircraft configuration. However, today, new configurations continue to evolve and the concept of a large tailless, or flying wing, passenger carrying transport aircraft seems to be a possible successor to the conventional aeroplane. Although during the past 60 years, or so, a variety of tailless aircraft have been constructed and flown in the world, it seems that many notable designs were radical departures from the normal and were experimental. The exception, of course, is the large variety of low aspect ratio tailless delta wing aircraft, which have seen operational service over the years.

The omission of the horizontal tail is the principal physical difference between the conventional aircraft and a tailless configuration. Its omission introduces more differences in the flight characteristics, sufficient to warrant deeper research into the development of the equations of motion directly relevant to the high aspect ratio civil transport configuration.

Thus, in a tailless configuration all aerodynamic controls are situated in the wing and the usual assumptions regarding aerodynamic forces and zero-lift pitch moment no longer apply. An elevon is a control surface, which functions as both as an elevator and an aileron, and in the tailless configuration it is universally applied in the wing trailing edge. However, the elevon is no more than a flap, which when deflected changes the effective wing camber and hence changes the lift, drag and pitch moment due to camber. Properties that are usually regarded as "constant" in a conventional aeroplane.

2. AERODYNAMIC MODEL

2.1 Aerodynamic forces and moment

The aircraft configuration in its simplest form may be represented by an airfoil as shown in Fig. 1. The aerodynamic forces are split into two components, Fig. 1, corresponding to,

i) Aerodynamic force due to the angle of attack

ii) Aerodynamic force due to camber

The former is assumed to act at the aerodynamic center and the latter at a point half of the mean aerodynamic chord.

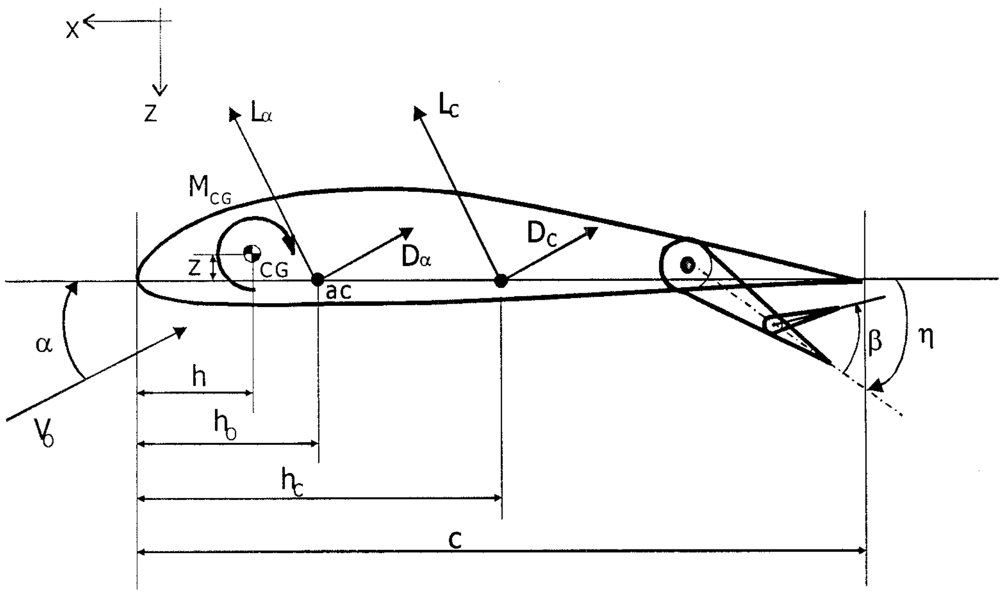

Fig 1 – Section of the wing

The subscript α and c meaning the terms are due either to AoA or due to the camber. With reference to Fig. 1, the expressions for lift and drag coefficients are as follows;

Lift due to AoA,

$$\left(C_L\right)_\alpha = \frac{\partial C_L}{\partial \alpha}\alpha = a_1\alpha \tag{1}$$

Lift due to camber,

$$\left(C_L\right)_c = C_{L_o} + \frac{\partial C_L}{\partial \eta}\eta + \frac{\partial C_L}{\partial \beta}\beta = C_{L_o} + a_2\eta + a_3\beta \tag{2}$$

Total lift,

$$C_L = \left(C_L\right)_\alpha + \left(C_L\right)_c \tag{3}$$

$$C_L = C_{L_o} + a_1\alpha + a_2\eta + a_3\beta \tag{4}$$

Total drag may be expressed,

$$C_D = \left(C_D\right)_\alpha + \left(C_D\right)_c \tag{5}$$

The total drag can also be represented as a function of C_L, usually known as "drag polar". The drag polar usually is parabolic and can be approximated by one of the following equations,

$$C_D = C_{D_o} + kC_L^{\ 2} \tag{6}$$

$$C_D = C_{D_{ref}} + \beta\left(C_L - C_{L_{ref}}\right)^2 \tag{7}$$

$$C_D = C_{D_o} + k_1 C_L + k_2 C_L^{\ 2} \tag{8}$$

The lift and drag due to camber are independent of AoA, and often they are moved to the aerodynamic center, being summed up with the lift and drag due solely to AoA, generating the total lift and drag, L and D, and a pitch moment, M_{ac}. From Fig.1 follows,

$$M_{ac} = (L_c \cos\alpha + D_c \sin\alpha)(h_o - h_c)\bar{c} \tag{9}$$

Dividing the forces this way was made clear where the aerodynamic pitching moment around the aerodynamic center comes from. Now it is clear that the pitching moment around the ac is independent of the CG position, dependent of the AoA and dependent of the camber through the values of lift and drag (and therefore dependent on the same variables as the later).

The total aerodynamic lift and drag forces, the pitching moment around the ac, together with the CG position and the total pitching moment around the CG are presented in Fig. 2.

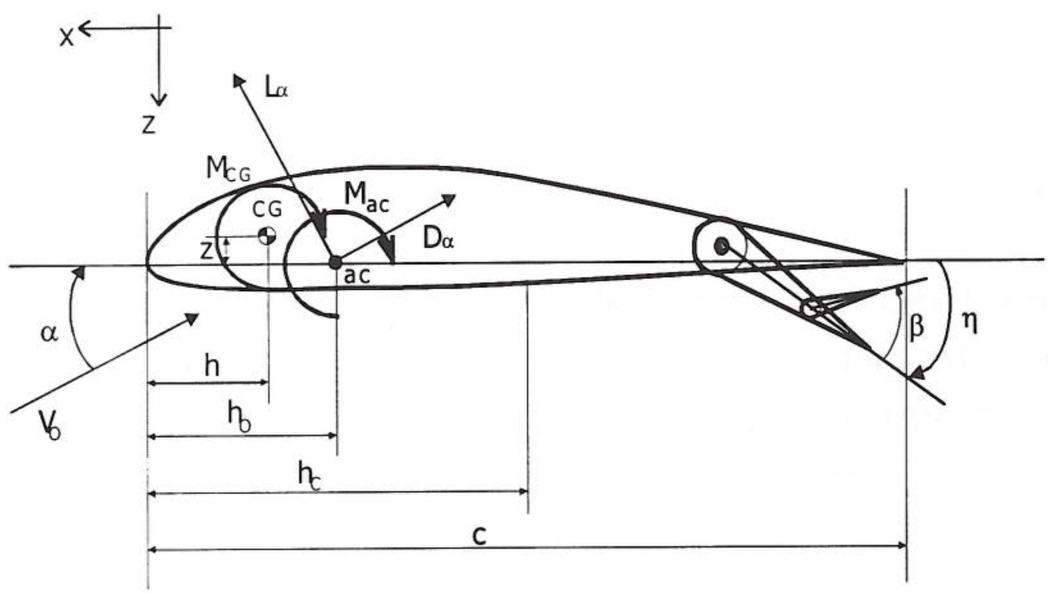

Fig. 2 – Section of the wing

Bearing in mind that the pitch moment equation is the basic equation for the static stability analysis, with reference to Fig. 2, and supposing the airfoil is subjected to an angle of attack, α, follows,

$$M_{CG} = M_{ac} - (L\cos\alpha + D\sin\alpha)(h_o - h)\overline{\overline{c}} - (L\sin\alpha - D\cos\alpha)z \tag{10}$$

Usually the CG position does not lay in the chord, therefore the appearance of the term in z. In terms of aerodynamic coefficients equation (10) reduces to,

$$C_{m_{CG}} = C_{m_o} - (C_L\cos\alpha + C_D\sin\alpha)(h_o - h) - (C_L\sin\alpha - C_D\cos\alpha)\frac{z}{\overline{\overline{c}}} \tag{11}$$

where, $C_{m_o} \equiv C_{m_{ac}}$.

$$C_{m_o} = ((C_L)_c\cos\alpha + (C_D)_c\sin\alpha)(h_o - h_c) \tag{12}$$

When pitching moment has no indices to indicate the application point, this is assumed to be the CG.

Equation (11) takes into account only the effects of the wing. However, it can also include the effects of body and nacelles, in the case when a body and/or nacelles are present, as long as the values C_{m_o}, h_o and C_L are interpreted as those for the wing-body-nacelles configuration, $(C_{m_o})_{wb}$, $(h_o)_{wb}$ and $(C_L)_{wb}$. The indices "wb" will be used to differentiate between the values for the complete aircraft and those for the wing alone, when both values are used.
Usually $(C_{m_o})_{wb}$ is more negative than C_{m_o}, the wing-body-nacelles aerodynamic center, $(h_o)_{wb}$, is forward of h_o, and, the total lift coefficient for the total aircraft is higher than for a wing alone.

Finally, another important term is the contribution of the propulsive system, which can be split in two parts:

i) the pitch moment coming from the interaction of the propulsive slipstream with the others parts of the airplane, and

ii) that coming from the forces acting on the unit itself, for example, the plane of the acting thrust. The former is assumed to be included in the moments already given for the wing-body-nacelles. The remaining moment from the propulsion units are supposed to be included in a term denoted by C_{m_p}.

Thus, the equation for the total pitching moment for tailless aircraft is given by,

$$C_m = C_{m_o} - (C_L \cos\alpha + C_D \sin\alpha)(h_o - h) - (C_L \sin\alpha - C_D \cos\alpha)\frac{z}{\bar{\bar{c}}} + C_{m_p} \qquad (13)$$

2.2 Assumptions

It should be noted that so far, no assumptions have been made about thrust, compressibility or aeroelastic effects. However, for a simple analysis some assumptions need to be made. Thus, for the remainder of this report, the following simplifications are assumed.

(i) Trimmed equilibrium flight

(ii) Constant mass

(iii) Quasi-steady flight

(iv) Normal atmosphere

(v) The structure does not distort

(vi) Compressibility effects can be ignored

(vii) The wing aerodynamic coefficients a_1, a_2, a_3, b_0, b_1, b_2 and b_3 are constants independent of forward speed

Additionally, for small angles of attack follows that $\cos\alpha \approx 1$ and $\sin\alpha \approx \alpha$. Substituting these values in equation (13), it reduces to,

$$C_m = C_{m_o} - C_L(h_o - h) - (C_L\alpha - C_D)\frac{z}{\bar{\bar{c}}} + C_{m_p} \tag{14}$$

Moreover, the term $(C_L\alpha - C_D)\frac{z}{\bar{\bar{c}}}$ may be considered small comparatively to the others, therefore, a further simplification follows,

$$C_m = C_{m_o} - C_L(h_o - h) + C_{m_p} \tag{15}$$

Finally, in case the thrust moment is also negligible a linear equation in C_L is found,

$$C_m = C_{m_o} - C_L(h_o - h) \tag{16}$$

Equation (16) is much simpler and may represent well the reality in some situations. However, even in cruise condition, where commercial aircrafts are most of their time, the effects of thrust may not be negligible. Moreover, in the case of tailless aircraft where the CG travel is smaller than for conventional aircrafts, $\frac{z}{\bar{\bar{c}}}$ may not be negligible when compared to h. Nevertheless, equation (14) resulting just from the assumption of small angle of attack is very simple and is valid in many situations.

3. STATIC STABILITY

3.1 General

Why do a static stability analysis?

Usually when designing a new aircraft some or all of the following questions may arise:

i) Will it be possible to trim the aircraft?

ii) What is the aircraft behavior when externally upset from its trim state?

iii) What is the aircraft behavior when the pilot changes the trim through the controls?

iv) How much control deflection will be necessary to change the trim condition or to correct a gust upset?

Sufficient insight may be gained when these questions are answered with simple theory, without needing large calculations. This may be achieved through a static stability analysis. A static stability theory is mainly applicable to cruise conditions or to quasi steady maneuvers, where the main concern is the longitudinal motion. Thus, this report is concerned only with the longitudinal static stability analysis.

An aircraft, or any other body, will be in equilibrium, or trim, when the resultant forces and moments acting on it are zero. In a cruise condition the thrust force will balance the drag force along the axial axis. In the normal axis, the lift will balance the weight force. The only variable to be checked is the pitch moment around the CG, which has to be zero. Thus, the condition for trim ability will be dictated by the equation $C_m = 0$.

Supposing now that, it is possible to trim the aircraft and it is flying in a cruise condition when it is upset by a vertical gust. The wind velocity will have a normal component, which is seen by the wing as a change in α.

This change will modify the lift and drag forces as well as the pitch moment. Over the action of this pitch moment the aircraft will rotate. If this rotation will be in the direction to oppose the upsetting gust the aircraft is said to be statically stable, otherwise it will be unstable. Therefore, a way to know whether an aircraft possesses stability or not, is through the change of pitch moment with C_L. And for a stable aircraft this derivative has to be negative, $\frac{dC_m}{dC_L} < 0$.

The change of pitch moment is made relatively to C_L because it is more general. The disturbance could also be in the forward speed not affecting the AoA, but only C_L. Especially when C_L has a dependency on Mach number or velocity. However in some cases, and under the assumptions stated above, using the derivative in α, $\frac{\partial C_m}{\partial \alpha}$, will give the same results. $\frac{\partial C_m}{\partial \alpha}$ can be used when C_L vs α are linearly related with no velocity dependency, i.e., C_L is given by equation (1) and the lift curve slope, a_1, can be assumed constant.

To change an established equilibrium or to restore an upset equilibrium, the pilot or the automatic control system may adjust thrust, change the aerodynamic configuration, by operating controls, such as elevators, wing flaps or spoilers, or by moving the CG position.

Now, the questions to be answered are, whether the control source is sufficient to correct the aircraft, and how the aircraft will respond to control deflection. Whether the control power is sufficient or not, will be determined by the change of pitch moment with control deflection, actually $\frac{\partial C_m}{\partial \delta}$.

From this brief discussion, one can see that the pitch moment is the key to the static stability analysis, equation (10) in the previous paragraph.

A measure of the static stability of an aircraft, the static margin, is given by the pitch moment derivative with respect to lift coefficient. Thus, differentiating equation (13) with respect to C_L,

$$\frac{dC_m}{dC_L}=\frac{dC_{m_o}}{dC_L}-\left(\cos\alpha-C_L\frac{d\alpha}{dC_L}\sin\alpha+\frac{dC_D}{dC_L}\sin\alpha+C_D\frac{d\alpha}{dC_L}\cos\alpha\right)(h_o-h)-$$

$$-\left(\sin\alpha+C_L\frac{d\alpha}{dC_L}\cos\alpha-\frac{dC_D}{dC_L}\cos\alpha+C_D\frac{d\alpha}{dC_L}\sin\alpha\right)\frac{z}{\overline{\overline{c}}}+\frac{dC_{m_p}}{dC_L} \quad (17)$$

Using the equality $\frac{\partial\alpha}{\partial C_L}=\frac{1}{a_1}$, equation (17) becomes,

$$\frac{dC_m}{dC_L}=\frac{dC_{m_o}}{dC_L}-\left[\left(1+\frac{C_D}{a_1}\right)\cos\alpha+\left(\frac{dC_D}{dC_L}-\frac{C_L}{a_1}\right)\sin\alpha\right](h_o-h)-$$

$$-\left[\left(1+\frac{C_D}{a_1}\right)\sin\alpha+\left(\frac{C_L}{a_1}-\frac{dC_D}{dC_L}\right)\cos\alpha\right]\frac{z}{\overline{\overline{c}}}+\frac{dC_{m_p}}{dC_L} \quad (18)$$

Assuming constant AoA, $\alpha=\alpha_e$, equation (17) reduces simply to,

$$\frac{dC_m}{dC_L}=\frac{dC_{m_o}}{dC_L}-\left(\cos\alpha_e+\frac{dC_D}{dC_L}\sin\alpha_e\right)(h_o-h)-\left(\sin\alpha_e-\frac{dC_D}{dC_L}\cos\alpha_e\right)\frac{z}{\overline{\overline{c}}}+\frac{dC_{m_p}}{dC_L} \quad (19)$$

As said before, in static stability analysis just trim conditions and quasi-steady maneuvers are of interest, thus, equation (19) holds a good approximation and it will be used in the remaining derivation.

In classical static stability analysis, to proceed further it is necessary to distinguish between controls fixed, or controls free. In this report the same approach will apply, in the case of military or large commercial aircraft today, it does not make literal sense to talk about controls free conditions, as all of them possess irreversible powered flight controls.

3.2 Controls fixed static stability

When the stick is fixed the pilot "holds" the elevator and tab angles constant. Thus $\eta = const$ and $\beta = const$ or,

$$\frac{\partial \eta}{\partial C_L} = 0 \tag{20}$$

$$\frac{\partial \beta}{\partial C_L} = 0 \tag{21}$$

As the elevator is fixed, the camber of the wing will not change, therefore, C_{m_o}, which is due to the lift due to camber, will not change.

Then equation (17) reduces to,

$$\frac{dC_m}{dC_L} = -\left(\cos\alpha_e + \frac{\partial C_D}{\partial C_L}\sin\alpha_e\right)(h_o - h) - \left(\sin\alpha_e - \frac{\partial C_D}{\partial C_L}\cos\alpha_e\right)\frac{z}{\bar{\bar{c}}} + \frac{\partial C_{m_p}}{\partial C_L} \tag{22}$$

By definition, the controls fixed static margin is given by,

$$K_n = -\frac{dC_m}{dC_L} \tag{23}$$

Thus, it follows that,

$$K_n = \left(\cos\alpha_e + \frac{\partial C_D}{\partial C_L}\sin\alpha_e\right)(h_o - h) + \left(\sin\alpha_e - \frac{\partial C_D}{\partial C_L}\cos\alpha_e\right)\frac{z}{\bar{\bar{c}}} - \frac{\partial C_{m_p}}{\partial C_L} \tag{24}$$

If equation (14) were used instead of equation (13) in the differentiation, or assuming the small angles approximation of equation (24), the controls-fixed static margin is as follow,

$$K_n = \left(1 + \frac{\partial C_D}{\partial C_L}\alpha_e\right)(h_o - h) + \left(\alpha_e - \frac{\partial C_D}{\partial C_L}\right)\frac{z}{\overline{\overline{c}}} - \frac{\partial C_{m_p}}{\partial C_L} \tag{25}$$

If all simplifications were considered and equation (16) used instead, the following result would appear,

$$K_n = (h_o - h) \tag{26}$$

Equation (26) is a known result for tailless aeroplanes, and enables to conclude that for a stable aeroplane the CG position has to be forward of the aerodynamic center, as the static margin for a stable aeroplane has to be positive.

$$K_n > 0 \Leftrightarrow (h_o - h) > 0 \Leftrightarrow h_o > h \tag{27}$$

However, as suggested before, the assumed simplifications may not always apply, principally with respect to the thrust moments. Therefore, if the term $\frac{\partial C_{m_p}}{\partial C_L}$ is negative, the power units will contribute to a more stable aircraft. Moreover, in the case when the CG is very near the aerodynamic center, the term $\frac{z}{\overline{\overline{c}}}$ may not be negligible. In that case, and as $\left(\frac{\partial C_D}{\partial C_L}\right)$ is always positive, if z is made negative this term will also contribute to a more stable aeroplane.

In these two cases, it would be possible to have a stable aeroplane with the CG aft of the aerodynamic center.

3.2.1 Controls fixed neutral point, h_n

The CG location on the aerodynamic chord where, for controls fixed, the aircraft is neutrally stable, and aft of which it is unstable, is called the controls fixed neutral point, $h_n\bar{\bar{c}}$. It is the CG position where the controls fixed static margin is null. The controls fixed neutral point is related to the controls fixed static margin through the following equation,

When $K_n = 0$, $h = h_n$ or,

$$K_n = h_n - h = -\frac{dC_m}{dC_L} \tag{28}$$

Thus, from equation (22) or equation (24) it follows,

$$0 = \left(\cos\alpha_e + \frac{\partial C_D}{\partial C_L}\sin\alpha_e\right)(h_o - h_n) - \left(\sin\alpha_e + \frac{\partial C_D}{\partial C_L}\cos\alpha_e\right)\frac{z}{\bar{\bar{c}}} - \frac{\partial C_{m_p}}{\partial C_L}$$

$$h_n = h_o + \frac{1}{\left(\cos\alpha_e + \dfrac{\partial C_D}{\partial C_L}\sin\alpha_e\right)}\left\{\left(\sin\alpha_e - \frac{\partial C_D}{\partial C_L}\cos\alpha_e\right)\frac{z}{\bar{\bar{c}}} - \frac{\partial C_{m_p}}{\partial C_L}\right\} \tag{29}$$

where h_n defines the neutral point controls fixed.

Thus, for a stable aircraft the CG has to be forward of the controls fixed neutral point, h_n, or the controls fixed static margin has to be positive,

$$K_n = h_n - h > 0$$

$$h < h_o + \frac{1}{\left(\cos\alpha_e + \frac{\partial C_D}{\partial C_L}\sin\alpha_e\right)}\left\{\left(\sin\alpha_e - \frac{\partial C_D}{\partial C_L}\cos\alpha_e\right)\frac{z}{\bar{\bar{c}}} - \frac{\partial C_{m_p}}{\partial C_L}\right\} \tag{30}$$

For small angles of attack and considering that $\frac{\partial C_D}{\partial C_L}\alpha_e << 1$, equation (30) reduces to,

$$h < h_o + \left(\alpha_e - \frac{\partial C_D}{\partial C_L}\right)\frac{z}{\bar{\bar{c}}} - \frac{\partial C_{m_p}}{\partial C_L} \tag{31}$$

If equation (15) had been used instead, and differentiated with respect to α, a rather simpler but equally valid within the assumptions made, result would appear as follow,

$$\frac{\partial C_m}{\partial \alpha} = -\frac{\partial C_L}{\partial \alpha}(h_o - h) + \frac{\partial C_{m_p}}{\partial \alpha} \tag{32}$$

Now, to find the neutral point the derivative is made null, $\frac{\partial C_m}{\partial \alpha} = 0$, and it comes,

$$h_n = h_o - \frac{1}{C_{L_\alpha}}\left(\frac{\partial C_{m_p}}{\partial C_L}\frac{\partial C_L}{\partial \alpha}\right) \Leftrightarrow h_n = h_o - \frac{\partial C_{m_p}}{\partial C_L} \tag{33}$$

Substituting equation (33) again in equation (32), it follows,

$$\frac{\partial C_m}{\partial \alpha} = \frac{\partial C_L}{\partial \alpha}(h - h_n) \Leftrightarrow C_{m_\alpha} = C_{L_\alpha}(h - h_n) \tag{34}$$

Equation (34) is a classical result, which is valid whether C_{m_o} and C_{m_p} vary with α or not. This equation makes it possible to find h_n from flight tests, by measuring C_L and C_m with α changes. Moreover, integrating the above equation,

$$C_m = C_{m_o} + C_L(h - h_n) \tag{35}$$

Which shows a linear C_m vs. C_L relation dependent on the static margin and CG position. This equation shows that it is possible to change the equilibrium by changing the CG position. Comparing equation (35) with equation (16) follows that the neutral point only coincide with the aerodynamic center when considering all assumptions as for derivation of equation (16).

3.2.2 Elevator angle to trim

The elevator angle to trim is the elevator angle deflection that makes $C_m = 0$. Thus, from equation (13), together with equation (4) and letting $C_m = 0$, the elevator angle to trim may be calculated.

$$0 = C_{m_o} - (C_L \cos\alpha_{trim} + C_D \sin\alpha_{trim})(h_o - h) - (C_L \sin\alpha_{trim} - C_D \cos\alpha_{trim})\frac{\overline{z}}{\overline{c}} + C_{m_p} \tag{36}$$

and,

$$C_L = C_{L_O} + a_1\alpha_{trim} + a_2\eta_{trim} + a_3\beta \tag{37}$$

These two equations show non-linearity in α_{trim}, and to calculate the elevator angle to trim, η_{trim}, a numerical solution is necessary. If equation (14) is used instead, together with equation (37), the elevator angle to trim is given by,

$$\eta_{trim} = \frac{1}{a_2}\left\{\frac{C_{m_o} + C_D \frac{z}{\overline{\overline{c}}} + C_{m_p}}{(h_o - h)} - \left(C_{L_o} + a_1 \alpha_{trim} + a_3 \beta\right)\right\} \tag{38}$$

However, equation (38) is dependent on α_{trim}. Substituting this result again in equation (37) leads to,

$$C_L = \frac{C_{m_o} + C_D \frac{z}{\overline{\overline{c}}} + C_{m_p}}{(h_o - h)} \tag{39}$$

which is independent of α_{trim}. Thus, to calculate α_{trim} and η_{trim} both equation (13) and equation (37) have to be calculated numerically and simultaneously.

An alternative method can be used to calculate α_{trim} and η_{trim}, by using a different definition of C_m. Thus, if C_m is defined using stability derivatives as follow,

$$C_m = \overline{C}_{m_o} + \frac{\partial C_m}{\partial \alpha}\alpha + \frac{\partial C_m}{\partial \eta}\eta + \frac{\partial C_m}{\partial \beta}\beta \tag{40}$$

where $\overline{C}_{m_o}$ is the basic pitch moment for $\alpha = \eta = \beta = 0$, the derivatives are called stability derivatives and may be written as,

$$\frac{\partial C_m}{\partial \alpha} = C_{m_\alpha}, \quad \frac{\partial C_m}{\partial \eta} = C_{m_\eta}, \quad \frac{\partial C_m}{\partial \beta} = C_{m_\beta} \tag{41}$$

The convention is that a positive surface control deflection is when deflecting it down. Thus, usually C_{L_η} is positive, as a down deflection increases the lift coefficient, and C_{m_η} is negative, as that increase in lift induces a negative pitch moment.

The lift coefficient equation is given by equation (37) in terms of stability derivatives.

Rewriting equation (37) and equation (40), isolating the angle of attack and elevator terms in the first member, it follows that,

$$C_{m_\alpha}\alpha + C_{m_\eta}\eta = C_m - \overline{C}_{m_o} - C_{m_\beta}\beta \tag{42}$$

$$C_{L_\alpha}\alpha + C_{L_\eta}\eta = C_L - C_{L_o} - C_{L_\beta}\beta \tag{43}$$

For equilibrium, $C_m = 0$, and in matrix format,

$$\begin{bmatrix} C_{m_\alpha} & C_{m_\eta} \\ C_{L_\alpha} & C_{L_\eta} \end{bmatrix}\begin{bmatrix} \alpha \\ \eta \end{bmatrix} = \begin{bmatrix} -\overline{C}_{m_o} - C_{m_\beta}\beta \\ C_L - C_{L_o} - C_{L_\beta}\beta \end{bmatrix} \tag{44}$$

Solving the system of equations (44), the angle of attack and control surface deflection to trim is given as follow,

$$\alpha_{trim} = \frac{\hat{C}_m C_{L_\eta} - \hat{C}_L C_{m_\eta}}{\det} \tag{45}$$

$$\eta_{trim} = \frac{\hat{C}_L C_{m_\alpha} - C_{L_\alpha}\hat{C}_m}{\det} \tag{46}$$

where

$$\hat{C}_m = -\overline{C}_{m_o} - C_{m_\beta}\beta \tag{47}$$

$$\hat{C}_L = C_{L_{trim}} - C_{L_o} - C_{L_\beta}\beta \tag{48}$$

$$\det = C_{L_\eta}C_{m_\alpha} - C_{L_\alpha}C_{m_\eta} \tag{49}$$

Equation (49) may be simplified by calculating the value of C_{m_η} from equation (35),

$$C_{m_\eta} = \frac{\partial C_{m_o}}{\partial \eta} + C_{L_\eta}(h - h_n) \qquad (50)$$

Substituting equation (50) and equation (34) in the expression for the denominator, equation (49), it follows that,

$$\det = C_{L_\eta} C_{m_\alpha} - C_{L_\alpha} C_{m_\eta} = -C_{L_\alpha} \frac{\partial C_{m_o}}{\partial \eta} \qquad (51)$$

The new trimmed lift equation may be obtained from equations (45), (48) and (49) as follows,

$$C_{L_{trim}} = C_{L_o} + C_{L_\beta}\beta - \frac{C_{m_o} C_{L_\eta}}{C_{m_\eta}} + \left(C_{L_\alpha} - \frac{C_{L_\eta}}{C_{m_\eta}} C_{m_\alpha} \right) \alpha_{trim} \qquad (52)$$

Using equation (34) the trimmed lift slope follows as,

$$\left(\frac{dC_L}{d\alpha} \right)_{trim} = C_{L_\alpha} \left(1 - \frac{C_{L_\eta}}{C_{m_\eta}} (h - h_n) \right) = C_{L_\alpha} \left(1 + K_n \frac{C_{L_\eta}}{C_{m_\eta}} \right) \qquad (53)$$

From equation (53) it is seen that the lift curve slope for trimmed flight is different from just C_{L_α} for normal flight, and it is dependent on the static margin. Moreover, for an aircraft usually C_{L_η} is positive and C_{m_η} is negative, thus,

i) for a stable aircraft, $K_n > 0$ and C_{L_α} is decreased

ii) for an unstable aircraft, $K_n < 0$ and C_{L_α} is increased

3.2.3 Variation of elevator angle to trim with lift coefficient

Assuming a stable aircraft in trim, the independent variable in controls fixed static stability analysis is the elevator angle to trim. Allowing elevator angle to vary with trim, it may be shown, that the variation of elevator angle to trim with C_L is given by,

$$\frac{d\eta_{trim}}{dC_L} = -\frac{(h-h_n)}{\frac{\partial C_{m_{ac}}}{\partial \eta}} \tag{54}$$

As it was expected, the change of angle to trim with C_L is a function of the static margin and varies with the CG location. This means that, the larger the static margin , the larger the deflection of elevator angle to trim for a given C_L.

3.2.4 Variation of elevator angle to trim with speed

In the absence of compressibility, aeroelastic effects, and propulsive system effects, the aerodynamic coefficients of equation (46) are constant and the variation of η_{trim} with speed is simple. As η_{trim} is a unique function of $C_{L_{trim}}$ for each CG position, and $C_{L_{trim}}$ is in turn fixed by the speed for horizontal flight,

$$C_{L_{trim}} = \frac{mg}{\frac{1}{2}\rho V^2 S} \tag{55}$$

then, η_{trim} becomes a unique function of V.

$$\eta_{trim} = -\frac{\left(\frac{2mg}{\rho V^2 S} - C_{L_o} - C_{L_\beta}\beta\right)C_{m_\alpha} - C_{L_\alpha}\hat{C}_m}{C_{L_\alpha}C_{m_\eta} - C_{L_\eta}C_{m_\alpha}} \tag{56}$$

The desirable variation of η_{trim} with $C_{L_{trim}}$, or speed is, for any CG position, an increase in downward deflection of the elevator with increasing speed. The "gradient" of the movement $\frac{\partial \eta}{\partial V}$ is seen to decrease with rearward movement of the CG until it vanishes altogether at the neutral point. In this condition the pilot in effect has no control over the trim speed, and control of the vehicle becomes very difficult. For even more rearward positions of the CG the gradient reverses, and the controllability deteriorates still further.

When the aerodynamic coefficients vary with speed, which is the case for high Mach numbers, this simple analysis is not possible and an account for the propulsive effects should also be taken into account.

3.3 Controls free static stability

3.3.1 Hinge moment

A control force has to be applied to overcome the aerodynamic load that resists the rotation of the aerodynamic control surface about its hinge. This force may be supplied entirely by a human pilot, partly by a mechanical device, or the pilot may be altogether mechanically disconnected from the control surface. In any case, the force that has to be applied to the control surface must be known with precision to design the control system and surface actuator that connects the primary control in the cockpit to the aerodynamic surface.

However, first of all it is necessary to define the aerodynamic hinge moment. Therefore, the elevator hinge moment, C_{h_η} is defined by,

$$C_{h_\eta} = \frac{H_\eta}{\frac{1}{2}\rho V^2 S_\eta \bar{c}_\eta} \tag{57}$$

where S_η is the elevator area aft of the hinge line, and $\bar{c}_\eta$ is the mean chord of the surface. In many practical cases it is sufficient to assume that for finite surfaces, C_{h_η} is a linear function of α, η and β as follow,

$$C_{h_\eta} = b_o + b_1\alpha + b_2\eta + b_3\beta \tag{58}$$

where b_o is the basic hinge moment, for zero α, η and β, $b_1 = \dfrac{\partial C_{h_\eta}}{\partial \alpha}$, $b_2 = \dfrac{\partial C_{h_\eta}}{\partial \eta}$ and

$b_3 = \dfrac{\partial C_{h_\eta}}{\partial \beta}$.

The force that the control system must exert to hold the elevator at the desired angle is in direct proportion to the hinge moment. When the control is free, then $C_{h_\eta} = 0$, and the elevator angle is given by,

$$\eta_{free} = -\frac{1}{b_2}(b_o + b_1\alpha + b_3\beta) \tag{59}$$

And the corresponding lift and moment are,

$$C_{L_{free}} = C_{L_o} + C_{L_\alpha}\alpha + C_{L_\eta}\eta_{free} + C_{L_\beta}\beta \tag{60}$$

$$C_{m_{free}} = \overline{C}_{m_o} + C_{m_\alpha}\alpha + C_{m_\eta}\eta_{free} + C_{m_\beta}\beta \tag{61}$$

Substituting equation (59) into equations (60) and (61) it follows,

$$C_{L_{free}} = C_{L_o}' + C_{L_\alpha}'\alpha \tag{62}$$

$$C_{m_{free}} = C_{m_o}' + C_{m_\alpha}'\alpha \tag{63}$$

where

$$C_{L_o}' = C_{L_o} - \frac{b_o}{b_2}C_{L_\eta} - \frac{b_3}{b_2}C_{L_\eta}\beta \tag{64}$$

$$C_{L_\alpha}' = C_{L_\alpha} - \frac{b_1}{b_2}C_{L_\eta} \tag{65}$$

$$C_{m_o}' = C_{m_o} - \frac{b_o}{b_2}C_{m_\eta} - \frac{b_3}{b_2}C_{m_\eta}\beta \tag{66}$$

$$C_{m_\alpha}' = C_{m_\alpha} - \frac{b_1}{b_2}C_{m_\eta} \tag{67}$$

Rearranging equation (65) as follows,

$$C_{L_\alpha}' = C_{L_\alpha}\left(1 - \frac{b_1}{b_2}\frac{C_{L_\eta}}{C_{L_\alpha}}\right) \tag{68}$$

the free elevator factor is found, the value in parentheses which usually has a value less than unity.

3.3.2 Controls-free neutral point, h_n'

As for controls-fixed the following equation is also valid for controls free,

$$C_{m_\alpha}' = C_{L_\alpha}'(h - h_n') \tag{69}$$

Substituting equation (69) into equation (67) and rearranging,

$$(h - h_n') = \frac{1}{C_{L_\alpha}'}\left(C_{m_\alpha} - \frac{b_1}{b_2}C_{m_\eta}\right) \tag{70}$$

Now, substituting equation (50) and equation (34) into equation (70), after some rearranging and then substituting equation (65), it follows that,

$$(h - h_n') = (h - h_n) - \frac{b_1}{C_{L_\alpha}' b_2}\frac{\partial C_{m_o}}{\partial \eta} \tag{71}$$

$$h_n' = h_n + \frac{b_1}{C_{L_\alpha}' b_2}\frac{\partial C_{m_o}}{\partial \eta} \tag{72}$$

Rewriting equation (71) in terms of the controls fixed static margin, K_n, and defining $K_n' = h_n' - h$ as the controls free static margin,

$$K_n' = K_n + \frac{b_1}{C_{L_\alpha}' b_2} \frac{\partial C_{m_o}}{\partial \eta} \tag{73}$$

3.3.3 Trim tabs

In an aircraft where the pilot has to fly for long periods at a constant speed or C_L, it is very fatiguing to maintain the force to hold the elevator, which happens when the angle η_{trim} differs from the angle η_{free}.

The trim tab angle required is calculated for C_{h_η} and C_m both zero. Thus, from equation (58) follows,

$$\beta_{trim} = -\frac{1}{b_3}\{b_o + b_1 \alpha_{trim} + b_2 \eta_{trim}\} \tag{74}$$

Substituting the values of α_{trim}, equation (45), and η_{trim}, equation (46), into equation (74) and rearranging follows,

$$\beta_{trim} = -\frac{1}{b_3}\left\{b_o + \frac{\hat{C}_m}{\det}\left(b_1 C_{L_\eta} - b_2 C_{L_\alpha}\right) + \frac{\hat{C}_L}{\det}\left(b_1 C_{m_\eta} - b_2 C_{m_\alpha}\right)\right\} \tag{75}$$

Equation (75) shows a linear dependence on C_L. Not so visible is a linear dependence with the CG position, which is found from rearranging equation (70) as shown below,

$$\left(b_1 C_{m_\eta} - b_2 C_{m_\alpha}\right) = -C_{L_\alpha}' b_2 \left(h - h_n'\right) \tag{76}$$

Substituting equation (76) into equation (75) follows,

$$\beta_{trim} = -\frac{1}{b_3}\left\{b_o + \frac{\hat{C}_m}{\det}\left(b_1 C_{L_\eta} - b_2 C_{L_\alpha}\right) - \frac{C_{L_\alpha}{}' b_2}{\det}\left(h - h_n{}'\right)\hat{C}_L\right\} \tag{77}$$

where $\hat{C}_L$ is given by equation (47), $\hat{C}_m$ by equation (48) and det by equation (49) or equation (51).

3.3.4 Control force to trim

The forces that exert up the control surfaces are important whether the aircraft has reversible or irreversible controls, as for reversible controls they need to be within certain limits given by the authorities and for irreversible controls they will be related with the synthetic feel. In the latter case, as now a days it is possible to design any desired synthetic feel, it might seem that the control forces would not be important. However, the controls surfaces have structural limits and the bigger the forces will be, the bigger the requirements for power to move the surfaces, and thus increase in weight, even if the pilot will have the correct feeling through a synthesized feel system.
Thus, the control force is related with the hinge moment through the expression,

$$P = GH_\eta = G\frac{1}{2}\rho V^2 S_\eta \bar{c}_\eta C_{h_\eta} \tag{78}$$

where G is the gearing function, dependent on the kind of control power used, S_η and $\bar{c}_\eta$ are the control surface area and chord respectively. At trim the value of the hinge moment when the tab angle is at an arbitrary position follows from equation (58) and equation (74),

$$C_{h_\eta} = b_o + b_1\alpha_{trim} + b_2\eta_{trim} + b_3\beta = b_3\left(\beta - \beta_{trim}\right) \tag{79}$$

and, as said before, the hinge moment is zero when $\beta = \beta_{trim}$. Thus, substituting equation (77) in equation (79) follows,

$$C_{h_\eta} = b_3\beta + b_o + \frac{\hat{C}_m}{\det}\left(b_1 C_{L_\eta} - b_2 C_{L_\alpha}\right) - \frac{C_{L_\alpha}{}' b_2}{\det}(h - h_n{}')\hat{C}_L \tag{80}$$

For level flight the lift coefficient in trim equals the weight, $C_L = \dfrac{2mg}{\rho S V^2}$, and substituting this result in equation (80), the hinge moment follows then,

$$C_{h_\eta} = b_3\beta + b_o + \frac{\hat{C}_m}{\det}\left(b_1 C_{L_\eta} - b_2 C_{L_\alpha}\right) + \frac{C_{L_\alpha}{}' b_2}{\det}(h - h_n{}')\left(C_{L_o} + C_{L_\beta}\beta\right) - \frac{C_{L_\alpha}{}' b_2}{\det}(h - h_n{}')\frac{2mg}{\rho S V^2} \tag{81}$$

The control force follows from substituting equation (81) in equation (78).

$$P = -GS_\eta \bar{c}_\eta \frac{C_{L_\alpha}{}' b_2}{\det}(h - h_n{}')\frac{mg}{S} +$$
$$+\frac{1}{2}G\rho S_\eta \bar{c}_\eta V^2\left[b_3\beta + b_o + \frac{\hat{C}_m}{\det}\left(b_1 C_{L_\eta} - b_2 C_{L_\alpha}\right) + \frac{C_{L_\alpha}{}' b_2}{\det}(h - h_n{}')\left(C_{L_o} + C_{L_\beta}\beta\right)\right] \tag{82}$$

or,

$$P = A + B\frac{1}{2}\rho V^2 \tag{83}$$

where

$$A = -GS_\eta \bar{c}_\eta \frac{C_{L_\alpha}{}' b_2}{\det}(h - h_n{}')\frac{mg}{S} \tag{84}$$

$$B = GS_\eta \bar{c}_\eta\left[b_3\beta + b_o + \frac{\hat{C}_m}{\det}\left(b_1 C_{L_\eta} - b_2 C_{L_\alpha}\right) + \frac{C_{L_\alpha}{}' b_2}{\det}(h - h_n{}')\left(C_{L_o} + C_{L_\beta}\beta\right)\right] \tag{85}$$

If the set tab angle was initially zero, $\beta = 0$, expression for B will simplify as follow,

$$B = GS_\eta \bar{c}_\eta\left[b_o + \frac{C_{m_o}}{\det}\left(b_1 C_{L_\eta} - b_2 C_{L_\alpha}\right) + \frac{C_{L_\alpha}{}' b_2}{\det}(h - h_n{}')C_{L_o}\right] \tag{86}$$

4. MANEUVERABILITY

4.1 General

When the aircraft is brought out of trim by the pilot, or by the flight control system, the disturbance may be not small and may be for a prolonged time. This condition is regarded as manoeuvring flight and a different approach has to be made to analyse static stability. In analytical terms, the manoeuvre is regarded as an increment in steady motion over and above the initial trim state, in response to an increment in control angle.

The wing is the main device which produces the main aerodynamic force. In rotating the airframe in roll, pitch and yaw, it will make the lift vector, which acts normal to the direction of flight in the plane of symmetry, rotate to the desired direction of motion to produce the acceleration to manoeuvre.

Therefore, manoeuvring flight is sometimes called "accelerated flight" and is defined as the condition when the airframe is subject to temporary, or transient, out-of-trim linear and angular accelerations, resulting from the displacement of controls relative to their trim settings.

Moreover, manoeuvrability is mainly concerned with the ability to rotate about the aircraft axes; the modulation of the normal, or lift force, and the modulation of the axial, or thrust, force. However, it is essential that the changes do not impair the stability of the aeroplane, or in other words, no tendency to diverge in manoeuvring flight.

The manoeuvrability of an airframe is a critical factor in its overall flying and handling qualities. An aircraft should not be too manoeuvre stable, nor too few manoeuvre stable. There should be the right balance between control power, manoeuvre stability, static stability and dynamic stability over the entire flight envelope of the aeroplane.

4.2 Controls fixed

4.2.1 Elevator angle per "g"

A pull-up manoeuvre will be analysed here, however the same results would be reached using another steady manoeuvre. Thus, in a pull-up manoeuvre the normal acceleration in the lower part of the path, while still in symmetric flight, following an elevator deflection $\delta\eta$ relative to η_{trim} will be proportional to the difference between the generated lift and weight, and may be expressed as follow,

$$L - W = ma_n \tag{87}$$

$$a_n = (n-1)g \tag{88}$$

where g is the gravity acceleration, $L = nW$ and n is the load factor. The angular velocity responsible for the circular path is given by,

$$w = \frac{a_n}{V} = \frac{(n-1)g}{V} \tag{89}$$

In reality this angular velocity for a longitudinal manoeuvre in the symmetrical plane is the pitch velocity, $w = q$. Due to this angular velocity, the airflow field around the aircraft will change, and the following increments appear,

$$\delta C_L = C_{L_\alpha}\delta\alpha + C_{L_q}\hat{q} + C_{L_\eta}\delta\eta \tag{90}$$

$$\delta C_m = C_{m_\alpha}\delta\alpha + C_{m_q}\hat{q} + C_{m_\eta}\delta\eta \tag{91}$$

Where $\hat{q} = \frac{q\bar{\bar{c}}}{2V}$, $C_{L_q} = \frac{\partial C_L}{\partial \hat{q}}$ and $C_{m_q} = \frac{\partial C_m}{\partial \hat{q}}$. From equation (89) $\hat{q}$ may be rewritten as,

$$\hat{q} = (n-1)\frac{C_W}{2\mu} \tag{92}$$

$$C_W = \frac{mg}{\frac{1}{2}\rho V^2 S} \tag{93}$$

$$\mu = \frac{2m}{\rho S \bar{\bar{c}}} \tag{94}$$

where C_W is the weight coefficient and μ the mass ratio and are defined as above. The increment in lift coefficient, δC_L, may be related to the load factor, n, as follow,

$$\delta C_L = (n-1)C_W \tag{95}$$

Assuming that the curved flight path is steady, that is, without angular acceleration, then $\delta C_m = 0$. Therefore, making equation (91) equal to zero and equating equation (95) to (90) follow, in matrix format,

$$\begin{bmatrix} C_{L_\alpha} & C_{L_\eta} \\ C_{m_\alpha} & C_{m_\eta} \end{bmatrix} \begin{bmatrix} \delta\alpha \\ \delta\eta \end{bmatrix} = \begin{bmatrix} \left(1 - \frac{C_{L_q}}{2\mu}\right)(n-1)C_W \\ -C_{m_q}(n-1)\frac{C_W}{2\mu} \end{bmatrix} \tag{96}$$

Solving for $\delta\alpha$ and $\delta\eta$,

$$\frac{\delta\alpha}{(n-1)} = \frac{1}{C_{L_\alpha}}\left(C_W - C_{L_q}\frac{C_W}{2\mu} - C_{L_\eta}\frac{\delta\eta}{(n-1)}\right) \tag{97}$$

$$\frac{\delta\eta}{(n-1)} = -\frac{\left(1 - \frac{C_{L_q}}{2\mu}\right)C_W C_{m_\alpha} + C_{m_q}\frac{C_W}{2\mu}C_{L_\alpha}}{C_{L_\alpha}C_{m_\eta} - C_{L_\eta}C_{m_\alpha}} \tag{98}$$

The ratio $\dfrac{\delta\eta}{(n-1)}$ is called the elevator angle per g. The denominator is equal to equation (49) and can be simplified, as shown before, to equation (51). Substituting equation (34) in equation (98) and rearranging follows,

$$\frac{\delta\eta}{(n-1)} = -\frac{C_W C_{L_\alpha}\left(2\mu - C_{L_q}\right)}{\det 2\mu}\left(h - h_n + \frac{C_{m_q}}{2\mu - C_{L_q}}\right) \tag{99}$$

or,

$$\frac{\delta\eta}{(n-1)} = \frac{C_W\left(2\mu - C_{L_q}\right)}{2\mu\dfrac{\partial C_{m_o}}{\partial\eta}}\left(h - h_n + \frac{C_{m_q}}{2\mu - C_{L_q}}\right) \tag{100}$$

From both equations above it seems that the elevator angle per g is linear with h. However, in reality, C_{L_q} and C_{m_q} are also dependent on the CG position, and this dependence is even more important for tailless aeroplanes.

4.2.2 Controls-fixed manoeuvre point, h_m

The controls-fixed manoeuvre point, h_m, is the point where the elevator angle per "g" is null, $\dfrac{\delta\eta}{(n-1)} = 0$. Thus, from equation (99) follows,

$$h_m = h_n - \frac{C_{m_q}(h_m)}{2\mu - C_{L_q}(h_m)} \tag{101}$$

To find the controls-fixed manoeuvre point, the derivatives C_{L_q} and C_{m_q} have to be evaluated for $h = h_m$. If these two derivatives can be assumed independent of the CG position, then the controls-fixed manoeuvre point may be given by,

$$\frac{\delta\eta}{(n-1)} = -\frac{C_W C_{L_\alpha}(2\mu - C_{L_q})}{2\mu \det}(h - h_m) \tag{102}$$

Where

$$H_m = h_m - h \tag{103}$$

is known as the controls-fixed manoeuvre margin.

$$\frac{\delta\eta}{(n-1)} = \frac{C_W C_{L_\alpha}(2\mu - C_{L_q})}{2\mu \det} H_m \tag{104}$$

Also important is the relation between the controls-fixed manoeuvre margin and the controls-fixed static margin. Introducing equation (101) into equation (103) and taking account equation (28) follows that,

$$H_m = K_n - \frac{C_{m_q}(h_m)}{2\mu - C_{L_q}(h_m)} \tag{105}$$

As usually $C_{m_q}(h_m)$ is negative and the denominator is always positive, then $H_m > K_n$, as it would be expected.

4.3 Controls free

4.3.1 Controls free manoeuvre point, h_m'

In a manoeuvre the hinge moment will be also influenced by the pitch rate, thus the incremental hinge moment will be given by,

$$\delta C_{h_\eta} = b_1 \delta\alpha + C_{h\eta_q} \hat{q} + b_2 \delta\eta \tag{106}$$

Dividing equation (106) by $(n-1)$, and introducing equation (92) and equation (97), after some rearranging it follows that,

$$\frac{\delta C_{h_\eta}}{(n-1)} = \frac{C_W}{2\mu C_{L_\alpha}}\left[\left(2\mu - C_{L_q}\right)b_1 + C_{h_{\eta q}} C_{L_\alpha}\right] + \left(b_2 - b_1\frac{C_{L_\eta}}{C_{L_\alpha}}\right)\frac{\delta\eta}{(n-1)} \tag{107}$$

In equation (107) the last bracket is equal to $b_2 \frac{C_{L_\alpha}'}{C_{L_\alpha}}$, from comparing to equation (68).

Substituting the elevator angle per g, given by equation (101), in equation (107), follows that,

$$\frac{\delta C_{h_\eta}}{(n-1)} = \frac{C_W}{2\mu C_{L_\alpha}}\left[\left(2\mu - C_{L_q}\right)b_1 + C_{h_{\eta q}} C_{L_\alpha}\right] - b_2 C_{L_\alpha}'\frac{\left(2\mu - C_{L_q}\right)}{2\mu \det}\left(h - h_m\right) \tag{108}$$

The controls free manoeuvre point, h_m', is the CG position when the hinge moment is equal to zero. Therefore, making equation (108) equal to zero and $h = h_m'$, after some rearrangement follows that,

$$h_m' = h_m + \frac{\det}{b_2 C_{L_\alpha}'}\left[\frac{b_1}{C_{L_\alpha}} + \frac{C_{h_{\eta q}}}{\left(2\mu - C_{L_q}\right)}\right] \tag{109}$$

Introducing equation (109) again in equation (108) follows that,

$$\frac{\delta C_{h_\eta}}{(n-1)} = -b_2 C_{L_\alpha}' C_W \frac{\left(2\mu - C_{L_q}\right)}{2\mu \det}\left(h - h_m'\right) \tag{110}$$

or, in terms of the controls free manoeuvre margin, $H_m' = h_m' - h$,

$$\frac{\delta C_{h_\eta}}{(n-1)} = b_2 C_{L_\alpha}{}' C_W \frac{(2\mu - C_{L_q})}{2\mu \det} H_m{}' \tag{111}$$

4.3.2 Control force per 'g'

Similarly to equation (78), the incremental control force is given by,

$$\delta P = \frac{1}{2} G \rho S_\eta \bar{c}_\eta V^2 \delta C_{h_\eta} \tag{112}$$

Finally, substituting equation (111) into equation (112), the control force per g is given by,

$$Q = \frac{\Delta P}{(n-1)} = \frac{1}{2} G \rho S_\eta \bar{c}_\eta V^2 \frac{b_2 C_{L_\alpha}{}' C_W (2\mu - C_{L_q})}{2\mu \det} H_m{}' \tag{113}$$

or

$$Q = \frac{\Delta P}{(n-1)} = -\frac{1}{2} G \rho S_\eta \bar{c}_\eta V^2 \frac{b_2 C_{L_\alpha}{}' C_W (2\mu - C_{L_q})}{2\mu \det} (h - h_m{}') \tag{114}$$

5. APPLICATION TO THE COLLEGE OF AERONAUTICS BWB-98 TAILLESS CIVIL TRANSPORT CONCEPT

5.1 General information

BWB-98 is a blended-wing-body tailless civil transport aircraft designed by the College of Aeronautics, Cranfield, to meet the same specifications as that for Airbus A-3XX [2]. As the name says, the body and wing are blended in a way that the aircraft is classified as a high aspect ratio flying wing. General data for two typical flight conditions cruise and approach are presented in Table 1.

Flight Condition	**Typical cruise**	**Approach**
Altitude, h (m)	10059	0
Density, ρ $\left({Kg}/{m^3}\right)$	0.3921	1.225
Sound speed, a $\left({m}/{s}\right)$	305.8	340.3
Mach number, Ma	0.85	0.23
Speed, V_o $\left({m}/{s}\right)$	260.0	77.
Mass, m (Kg)	443680	322599
Moment of inertia, I_{yy} $\left(Kg.m^2\right)$	42.702x10^6	-
Area, S $\left(m^2\right)$	1390.6	1390.6
Mean aerodynamic chord, $\bar{\bar{c}}$ (m)	27.28	27.28
CG position, X_{CG} (m)	31.9	31.23
Aerodynamic center, X_o (m)	32.42	31.638
Static margin, K_n	1.9%	1.5%

Table 1 – General characteristics of the BWB-98

Aerodynamic Characteristics	**Cruise condition**	**Approach condition**
Lift coefficient, C_L	0.236	1.05
Lift curve slope, a_1 $\left(rad^{-1}\right)$	5.382	3.327
Drag coefficient for zero lift, C_{D_o}	0.04163	0.013908
Drag polar constant, k	0.059153	0.056592
Basic pitch moment coefficient, C_{m_o}	0.004403	0.004747

Table 2 – Aerodynamic data of the BWB-98 for a Cruise and Approach condition

In Table 2 and Table 3 are presented some aerodynamic data for both flight conditions. The lift coefficient due to camber, C_{L_0}, is not given in the data and it is assumed zero. Moreover, the BWB-98 has several trailing-edge surfaces [2], and the chosen pitch control elevon, control surface or flap number 6, possesses the largest pitch moment increment per degree of deflection.

	Cruise (Ma = 0.85)		Approach (Ma = 0.23)	
# Flaps	C_{m_η} (rad^{-1})	a_2 (rad^{-1})	C_{m_η} (rad^{-1})	a_2 (rad^{-1})
1 (in)	-0.0728	0.2518	-0.0469	0.1797
2	-0.1058	0.3606	-0.0745	0.2754
3	-0.0911	0.3225	-0.0682	0.2595
4	-0.0733	0.3189	-0.0544	0.2447
5	-0.1030	0.4835	-0.0746	0.3369
6	-0.1394	0.4726	-0.1097	0.3405
7 (out)	-0.0678	0.1528	-0.0581	0.1224

Table 3 – Elevator pitch moment slope, C_{m_η}, elevator lift curve slope, a_2, for two different flight conditions.

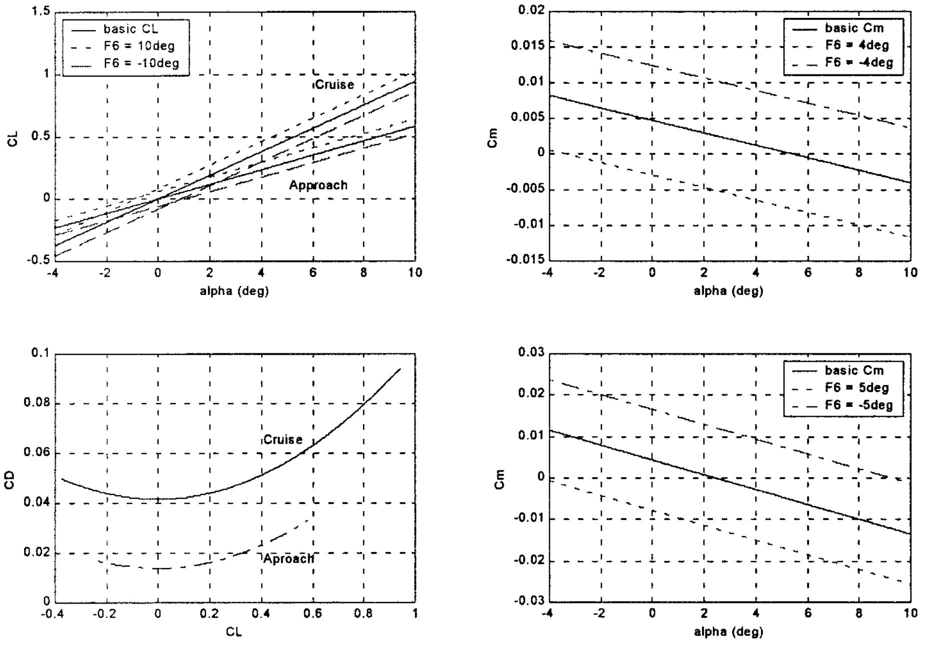

Fig. 1 – Aerodynamic characteristics of the BWB-98 in cruise and approach conditions

In Fig. 1 the aerodynamic data is plotted against AoA and lift coefficient. To plot the lift coefficient it was used equation (4). The drag polar was plotted using equation (6). For the pitch moment plots equation (40) was used.

5.2 Trim

To fly in a steady condition it is necessary to trim the aircraft. The AoA and elevator angle to trim were calculated using equation (45) and equation (46) respectively, supposing that no tab device is fitted, or $\beta = 0$. Results are presented in Table 4. Moreover, due to the rather large AoA in the approach condition the AoA and elevator angle to trim were calculated considering an equal deflection of all control surfaces. The elevator pitch moment coefficient slope and the elevator lift curve slope were calculated as the sum of each individual control surface as follows,

$$C_{m_{all}} = \left(C_{m_{\eta 1}} + C_{m_{\eta 2}} + C_{m_{\eta 3}} + \ldots\right) \tag{115}$$

$$C_{L_{all}} = \left(C_{L_{\eta 1}} + C_{L_{\eta 2}} + C_{L_{\eta 3}} + \ldots\right) \tag{116}$$

	α_e (deg)		η_e (deg)	
	Cruise	Approach	Cruise	Approach
Flap 6	2.52	18.57	-0.04	-6.0
All flaps	2.52	18.8	-0.008	-1.4

Table 4 – AoA and elevator angles to trim for cruise and approach conditions.

As it can be seen in Table 4, deflecting all control surfaces the elevator angle necessary to trim is significantly reduced, but the AoA necessary to generate the trim lift is still rather large. This is probably due to the low value of the lift slope curve, a_1. Hence, making a_1 for the approach condition of the same value as for cruise condition, the necessary AoA to trim is now $\alpha_{ap} = 11.63\,\text{deg}$ for just one surface deflected, and, $\alpha_{apt} = 11.56\,\text{deg}$ for all surfaces deflected the same angle.

5.3 Controls fixed static stability

5.3.1 Static margin

In Table 1 the static margin is already given for controls fixed. From Table 5 it is found that this data was obtained simply by using equation (26).

	$h_o = \frac{X_o}{\bar{\bar{c}}}$	$h = \frac{X_{CG}}{\bar{\bar{c}}}$	$K_n = h_o - h$
Cruise	1.188	1.169	1.9%
Approach	1.1598	1.1448	1.5%

Table 5 – Controls fixed static margin calculation

Using equation (25) it was calculated the influence of the CG normal position on the static margin. This influence can be seen in Fig. 2. In this figure is also plotted the results using equation (24).

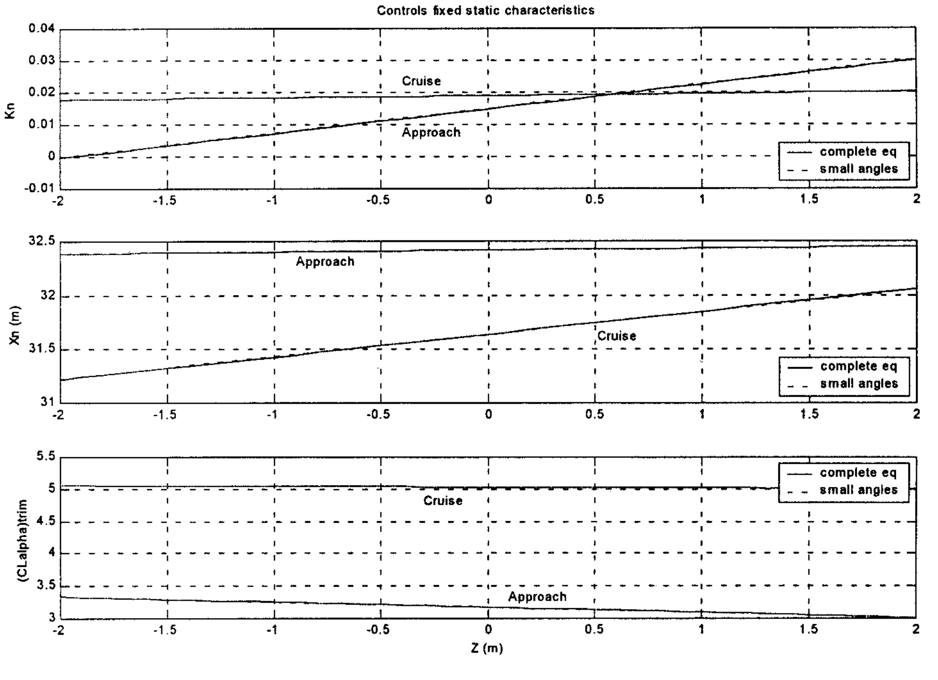

Fig. 2 – Static margin, neutral point position and lift curve slope at trim for cruise and approach conditions

From the first plot of Fig. 2 two conclusions may be drawn,

i) Equation (24) for small angles is a rather good approximation and may be used instead of equation (25).

ii) In the approach condition a shift of CG about 2m in the positive side of the normal axis increase the static margin of about 2%.

5.3.2 Neutral point

In the second plot of Fig. 2 the influence of the CG normal position, Z, in the neutral point position, X_n is shown. The complete equation used is equation (29), while for small angles approximation was used equation (31). As before, the latter equation held very good results and may be used instead of the complete equation.

5.3.3 Trimmed lift curve slope

In the last plot of Fig. 2 it is shown the decreasing in the lift curve slope, given by equation (53), that it would be expected when increasing the static margin, due to the aircraft being stable.

In case an unstable configuration is used the values expected are those shown in the first plot of Fig. 3.

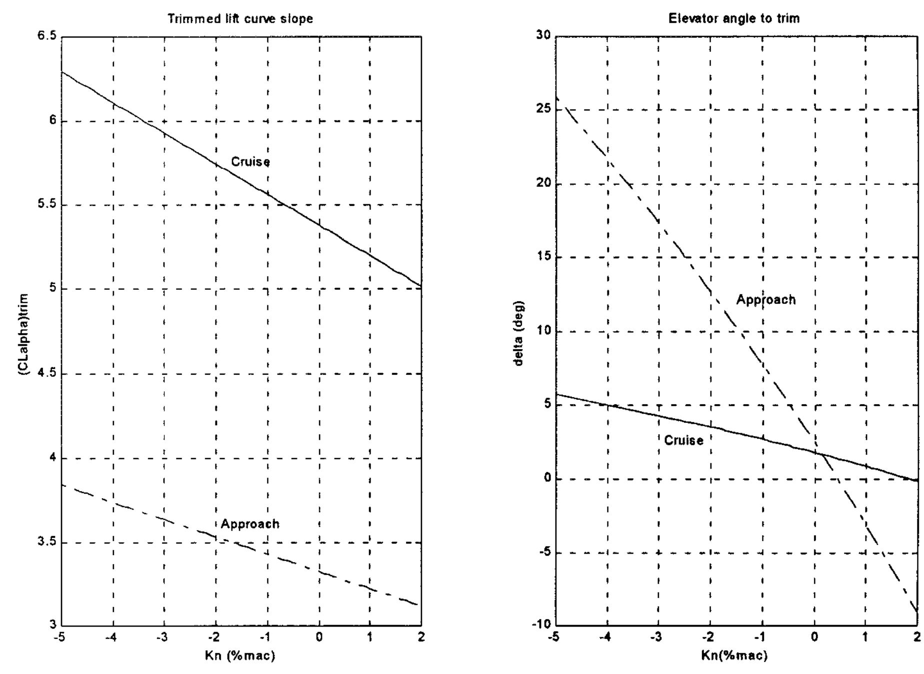

Fig. 3 – Trimmed lift curve slope and elevator angle to trim versus static margin.

The curve slopes in the first plot of Fig. 3, follow from equation (53),

$$\frac{\partial\left(\frac{dC_L}{d\alpha}\right)_{trim}}{\partial K_n} = C_{L_\alpha}\frac{C_{L_\eta}}{C_{m_\eta}} \tag{117}$$

In reality C_{m_η} is also a function of the CG axial position. However, in case this variation may be neglected, the elevator angle to trim as a function of static margin follows from equation (46),

$$\eta_{trim} = \frac{-\hat{C}_L K_n + \hat{C}_m}{-C_{L_\eta} K_n - C_{m_\eta}} \tag{118}$$

The results from this equation are presented in the second plot of Fig 3. Although the elevator deflections for an unstable aircraft, or negative static margin, are now positive, the elevator angle necessary to trim in the approach condition becomes rather large and it may happen the control power not being enough.

6. Conclusions

Firstly, it is assumed that the data for the approach condition is for a "clean" configuration (without flaps and landing gear) due to the low values of lift and drag curves comparative to the cruise condition.
Moreover, and as stated before, attention has to be paid to the validity of the theory applied. Thus, the conclusions given take into account that, using data from Fig. 1, in the cruise condition the maximum value for lift coefficient will be 1 or 1.2, while for the approach condition the maximum value will be 0.8. This limitation avoids the usual zone of non-linearity at angles of attack greater than 12-15 degrees.
Now, bearing in mind the limitations of the theory applied the following conclusions can be made:

- Some modifications should be done in the approach condition to achieve the lift coefficient required without having so large AoA.
- The approximation of small angles of attack holds good results and may be used instead of the full equations.
- Some modifications should be done in the approach condition to achieve the lift coefficient required without having so large AoA.
- The use of negative static margin, unstable aircraft, may be a way to improve the trimmed lift curve slope, however, care has to be taken to be able to trim the aircraft in all situations.

As the data used is for a large transport aircraft there is no need to analyze the controls free stability as it is supposed to be controls fixed all the time by means of a full flight control system. Moreover, at this stage of the design there was available no pitch rate stability derivatives, C_{L_q} and C_{m_q}, thus, it was not possible to analyze the maneuver case as well. Nevertheless, the theory was developed and may be used for future applications.

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