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The Longitudinal Condtrols Fixed Static Stability of Tailless Aircraft

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THE LONGITUDINAL CONTROLS FIXED STATIC STABILITY OF TAILLESS AIRCRAFT

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Abstract

This paper describes the development of a simple theory of the longitudinal controls fixed static stability of tailless aeroplanes. The classical theory, as developed for the conventional aircraft, is modified to accommodate the particular features of the tailless aeroplanes. The theory was then applied to a particular blended-wing-body tailless civil transport aircraft, BWB-98.

Acknowledgements

Contents

Nomenclature

$a_1 = \dfrac{\partial C_L}{\partial \alpha} = C_{L_\alpha}$ Lift curve slope

$a_2 = \dfrac{\partial C_L}{\partial \eta} = C_{L_\eta}$ Elevator lift curve slope

$a_3 = \dfrac{\partial C_L}{\partial \beta} = C_{L_\beta}$ Elevator tab lift curve slope

AoA	Angle of attack
BWB-98	Blended-Wing-Body 98
$\bar{\bar{c}}$	Aerodynamic chord
C_D	Drag coefficient
$(C_D)_\alpha$	Drag coefficient due to angle of attack
$(C_D)_c$	Drag coefficient due to camber
C_{D_o}	Drag coefficient for zero lift
CG	Center of Gravity
C_L	Lift coefficient
$(C_L)_\alpha$	Lift coefficient due to angle of attack
$(C_L)_c$	Lift coefficient due to camber
C_{L_o}	Lift coefficient for zero angle of attack
C_m	Pitch moment coefficient at the center of gravity
C_{m_o}	Basic pitch moment coefficient

$C_{m_\eta} = \dfrac{\partial C_m}{\partial \eta}$ Elevator pitch moment curve slope

D	Drag
D_α	Drag due to angle of attack
D_c	Drag due to camber
g	Acceleration due to gravity
h	CG position on $\bar{\bar{c}}$

h_c	Location of the aerodynamic forces due to camber on $\bar{\bar{c}}$
h_n	Controls fixed neutral point on $\bar{\bar{c}}$
h_n'	Controls free neutral point on $\bar{\bar{c}}$
h_o	Aerodynamic center location on $\bar{\bar{c}}$
k	Drag polar constant
K_n	Controls fixed static stability margin
K_n'	Controls free static stability margin
L	Lift
L_α	Lift due to angle of attack
L_c	Lift due to camber
m	Mass
M_{CG}	Pitch moment about CG
M_o	Basic pitch moment about CG
S	Wing area
V	Airspeed
α	Angle of attack
β	Tab angle
η	Elevator angle
η_{trim}	Elevator angle to trim
ρ	Air density

1. Introduction

Aircraft development over the last sixty years, or so, has focused on improving the performance and utility of conventional configurations comprising wing, fuselage and tail. Moreover, all design, aerodynamic and flight dynamic tools have been developed to apply primarily to this class of aircraft configuration.

However, today, new configurations continue to evolve and the concept of a large tailless, or flying wing, passenger carrying transport aircraft seems to be a possible successor to the conventional aeroplane.

Although during the past 60 years, or so, a variety of tailless aircraft have been constructed and flown in the world, it seems that many notable designs were radical departures from the normal and were experimental. The exception, of course, is the large variety of low aspect ratio tailless delta wing aircraft, which have seen operational service over the years.

The lack of the horizontal tail is the principal physical difference between the conventional aircraft and a tailless configuration. Its omission introduces more differences in the flight characteristics, sufficient to warrant deeper research into the development of the equations of motion directly relevant to the high aspect ratio civil transport configuration.

Thus, in a tailless configuration all aerodynamic controls are situated in the wing and the usual assumptions regarding aerodynamic forces and zero-lift pitch moment no longer apply. An elevon is a control surface, which functions as both as an elevator and an aileron, and in the tailless configuration it is universally applied in the wing trailing edge. However, the elevon is no more than a flap, which when deflected changes the effective wing camber and hence changes the lift, drag and pitch moment due to camber. Properties that usually remains "constant" in a conventional aeroplane.

2. Aerodynamic Model

2.1 Assumptions

(i)   Trimmed equilibrium flight

(ii)   Constant mass

(iii)   Quasi-steady flight

(iv)   Normal atmosphere

(v)   Thrust in equilibrium acting at the CG (thrust moment negligible)

(vi)   Propeller and slipstream effects can be neglected

(vii)   The structure does not distort

(viii) Compressibility effects can be ignored

(ix)   The wing aerodynamic coefficients a_1, a_2, a_3, b_0, b_1, b_2 and b_3 are constants independent of forward speed

2.2   Aerodynamic forces and moment

The aircraft configuration in its simplest form may be represented by an airfoil as shown in Fig. 1. The aerodynamic forces are split into two components, Fig. 1, corresponding to,

i)   Aerodynamic force due to the angle of attack

ii)   Aerodynamic force due to camber

The former is assumed to act at the aerodynamic center and the latter at a point half of the mean aerodynamic chord.
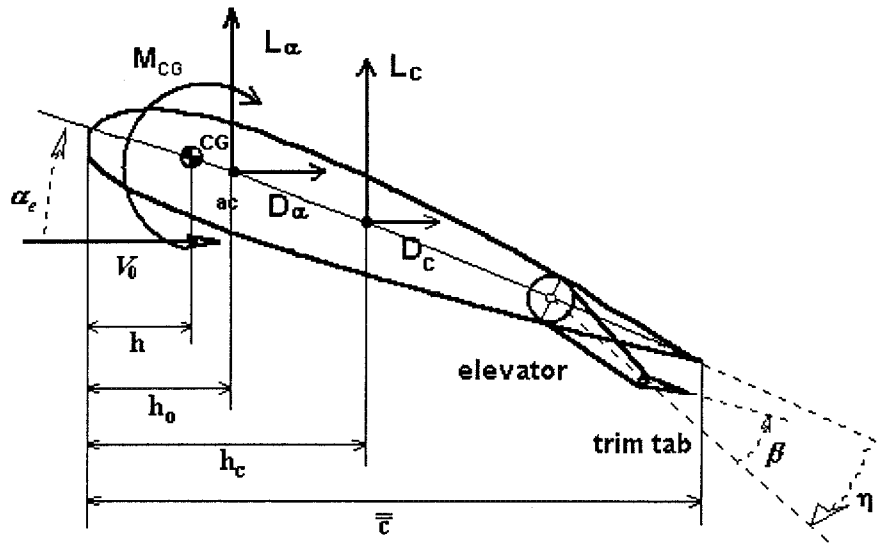
Fig 1 – Section of the wing

The subscript α and c meaning the terms are due either to AoA or due to the camber. With reference to Fig. 1, the expressions for lift and drag coefficients are as follows;

Lift due to AoA,

$$\left(C_L\right)_\alpha = \frac{\partial C_L}{\partial \alpha}\alpha = a_1\alpha \tag{1}$$

Lift due to camber,

$$\left(C_L\right)_c = C_{L_o} + \frac{\partial C_L}{\partial \eta}\eta + \frac{\partial C_L}{\partial \beta}\beta = C_{L_o} + a_2\eta + a_3\beta \tag{2}$$

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Total lift,

$$C_L = (C_L)_\alpha + (C_L)_c \tag{3}$$

Total drag may be expressed,

$$C_D = C_{D_o} + (C_D)_\alpha + (C_D)_c \tag{4}$$

or

$$C_D = C_{D_o} + kC_L^2, \tag{5}$$

The lift coefficient for null angle of attack is included in equation (2) and not in equation (1) because, in reality, it depends on the aerofoil camber. The zero lift drag coefficient C_{D_o} also includes parasite drag effects.

The pitch moment equation is the basic equation for the static stability analysis. In Fig. 1 the airfoil is subjected to an angle of attack, α. Thus, assuming that the CG and the aerodynamic center lie on the mean aerodynamic chord, and that the zero lift drag is negligible, the pitching moment about the CG follows,

$$M_{CG} = -(L_\alpha \cos\alpha + D_\alpha \sin\alpha)(h_o - h)\overline{\overline{c}} - (L_c \cos\alpha + D_c \sin\alpha)(h_c - h)\overline{\overline{c}} \tag{6}$$

or, in terms of aerodynamic coefficients,

$$C_m = -((C_L)_\alpha \cos\alpha + (C_D)_\alpha \sin\alpha)(h_o - h) - ((C_L)_c \cos\alpha + (C_D)_c \sin\alpha)(h_c - h) \tag{7}$$

or, rearranging equation (7)

$$C_m = C_{m_o} - (C_L \cos\alpha + C_D \sin\alpha)(h_o - h)$$ (8)

Where,

$$C_{m_o} = -((C_L)_c \cos\alpha + (C_D)_c \sin\alpha)(h_c - h_o)$$ (9)

Equation (6), (7) and (8) are different to the equation describing pitch moment for a conventional aircraft. This is due to the fact that it is a flying wing, thus, there is no contribution from the fuselage and the wing contribution is all included in the camber terms.

In the case of small perturbations, α is small and it follows that $\cos\alpha \approx 1$ and $\sin\alpha \approx 0$. Then, equation (7) simplifies to,

$$C_m \approx -C_L(h_o - h) - (C_L)_c(h_c - h_o) \approx C_{m_o} - C_L(h_o - h)$$ (10)

3. Static Stability

A measure of the static stability of an aircraft, the static margin, is given by the pitch moment derivative with respect to lift coefficient. Thus, differentiating equation (7) with respect to C_L,

$$\frac{\partial C_m}{\partial C_L} = -\left(\frac{\partial (C_L)_\alpha}{\partial C_L} \cos \alpha + \frac{\partial (C_D)_\alpha}{\partial C_L} \sin \alpha \right)(h_o - h) -$$

$$-\left[\left(a_2 \frac{\partial \eta}{\partial C_L} + a_3 \frac{\partial \beta}{\partial C_L} \right) \cos \alpha + \frac{\partial (C_D)_c}{\partial C_L} \sin \alpha \right](h_c - h) \qquad (11)$$

In a classical static stability analysis to proceed further it is necessary to distinguish between controls fixed, or controls free. Although in this report the same will be done too, in the case of military or large commercial aircraft, today, it does not make literal sense to talk about controls free, as all of them possess irreversible powered flight controls.

3.1 Controls fixed static stability

When the stick is fixed the pilot "holds" the elevator and tab angles constant. Thus $\eta = const$ and $\beta = const$ or,

$$\frac{\partial \eta}{\partial C_L} = 0 \qquad (12)$$

$$\frac{\partial \beta}{\partial C_L} = 0 \qquad (13)$$

Then equation (11) reduces to,

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$$\frac{\partial C_m}{\partial C_L} = -\left(\frac{\partial (C_L)_\alpha}{\partial C_L}\cos\alpha + \frac{\partial (C_D)_\alpha}{\partial C_L}\sin\alpha\right)(h_o - h) - \left[\frac{\partial (C_D)_c}{\partial C_L}\sin\alpha\right](h_c - h) \tag{14}$$

or

$$\frac{\partial C_m}{\partial C_L} = -\left(\frac{\partial (C_L)_\alpha}{\partial C_L}\cos\alpha + \frac{\partial C_D}{\partial C_L}\sin\alpha\right)(h_o - h) - \frac{\partial (C_D)_c}{\partial C_L}\sin\alpha(h_c - h_o) \tag{15}$$

Since $(C_L)_\alpha = C_{L_o} + a_1\alpha$, differentiating this expression with respect to C_L follows,

$$\frac{\partial (C_L)_\alpha}{\partial C_L} = a_1\frac{\partial \alpha}{\partial C_L} = 1 \tag{16}$$

since, by definition, $\partial \alpha/\partial C_L = 1/a_1$.

On the other hand, the derivative of drag coefficient with respect to lift coefficient, from equation (5) follows,

$$\frac{\partial C_D}{\partial C_L} = 2kC_L \tag{17}$$

Substituting equation (16) and equation (17) into equation (15),

$$\frac{\partial C_m}{\partial C_L} = -(\cos\alpha + 2kC_L\sin\alpha)(h_o - h) - \frac{\partial (C_D)_c}{\partial C_L}\sin\alpha(h_c - h_o) \tag{18}$$

By definition, the controls fixed static margin is given by,

$$K_n = -\frac{\partial C_m}{\partial C_L} \tag{19}$$

Thus, it follows that,

$$K_n = (\cos\alpha + 2kC_L \sin\alpha)(h_o - h) + \frac{\partial(C_D)_c}{\partial C_L}\sin\alpha(h_c - h_o) \tag{20}$$

Again, for small perturbations, equation (20) is approximately,

$$K_n \cong (h_o - h) \tag{21}$$

Which is a classical result for tailless aeroplanes. However, if the angle of attack is large the static margin will not only be dependent on the center of gravity position, but also on the other α dependent terms appearing in equation (20). One of the parameters that static margin is dependent on is the lift coefficient. For trimmed flight it is possible to write the lift coefficient as a function of the airspeed. Whence,

$$L = W \Leftrightarrow C_L = \frac{2mg}{\rho S V^2} \tag{22}$$

Substituting equation (22) in equation (20) it follows that,

$$K_n = \left(\cos\alpha + 4k\frac{mg}{\rho S V^2}\sin\alpha\right)(h_o - h) + \frac{\partial(C_D)_c}{\partial C_L}\sin\alpha(h_c - h_o) \tag{23}$$

which is equivalent to equation (20).

3.2 Controls fixed neutral point, h_n

The CG location on the aerodynamic chord where, for controls fixed, the aircraft is neutrally stable, and aft of which it is unstable, is called the controls fixed neutral point. It is the CG position where the controls fixed static margin is null. Thus, from equation (20),

$$0 = -\left(\cos\alpha + 2kC_L \sin\alpha\right)\left(h_o - h\right) - \frac{\partial\left(C_D\right)_c}{\partial C_L}\sin\alpha\left(h_c - h_o\right)$$

$$h_n = h_o + \frac{\dfrac{\partial\left(C_D\right)_c}{\partial C_L}\sin\alpha\left(h_c - h_o\right)}{\left(\cos\alpha + 2kC_L \sin\alpha\right)} \qquad (24)$$

where h_n defines the neutral point controls fixed.

Thus, for stable aircraft the CG has to be forward of the controls fixed neutral point, h_n , or the controls fixed static margin has to be positive,

$$K_n = h_o - h_n > 0$$

$$h_n < h_o + \frac{\dfrac{\partial\left(C_D\right)_c}{\partial C_L}\sin\alpha\left(h_c - h_o\right)}{\left(\cos\alpha + 2kC_L \sin\alpha\right)} \qquad (25)$$

From equation (21), for small angles of attack, it is seen that the neutral point is in reality the aerodynamic center, h_o. Then, from equation (25) it follows that the center of gravity

can be behind of the aerodynamic center in the case of large angles of attack provided that $\dfrac{\partial(C_D)_c}{\partial C_L}$ is positive.

3.3 Elevator angle to trim

Substituting equation (2) into equation (7) then,

$$C_m = (C_L \cos\alpha + C_D \sin\alpha)(h_o - h) - ((C_{L_o} + a_2\eta + a_3\beta)\cos\alpha + (C_D)_c \sin\alpha)(h_c - h_o) \quad (26)$$

Thus, the elevator angle to trim follows from equation (26) when $C_m = 0$,

$$\eta_{trim} = -\frac{1}{a_2}\left\{\frac{1}{\cos\alpha}\left[\frac{[C_L \cos\alpha + (C_D)\sin\alpha](h_o - h)}{(h_c - h_o)} - (C_D)_c \sin\alpha\right] + C_{L_o} + a_3\beta\right\} \quad (27)$$

Substituting equation (5) into equation (27) then,

$$\eta_{trim} = -\frac{1}{a_2}\left\{\frac{1}{\cos\alpha}\left[\frac{[C_L \cos\alpha + (C_{D_o} + kC_L{}^2)\sin\alpha](h_o - h)}{(h_c - h_o)} - (C_D)_c \sin\alpha\right] + C_{L_o} + a_3\beta\right\} \quad (28)$$

In a first approximation some more simplifications can be made to expression (28), thus

    (i) β is considered null, tab set to datum (or no tab)

    (ii) $(C_D)_c$ is considered small and multiplied by $\tan\alpha$ will be neglected

    (iii) $h_o \approx 0.25$

    (iv) $h_c \approx 0.5$

Then, the elevator angle to trim is approximated by,

$$\eta_{trim} \approx -\frac{1}{a_2}\left\{ \frac{\left[C_L\cos\alpha + \left(C_{D_o} + kC_L{}^2\right)\sin\alpha\right]\!\left(h_o - h\right)}{0.25\cos\alpha} + C_{L_o} \right\} \qquad (29)$$

For small perturbations equation (29) reduces to,

$$\eta_{trim} \approx -\frac{1}{a_2}\left\{ \frac{C_L\left(h_o - h\right)}{0.25} + C_{L_o} \right\} \qquad (30)$$

3.4 Variation of elevator angle to trim with lift coefficient

Assuming a stable aircraft in trim, the independent variable in controls fixed static stability analysis is the elevator angle to trim. Then, equating the pitch moment equation, (7), to zero and differentiating it with respect to C_L and allowing elevator angle to vary with trim, it may be shown, after some rearrangement, that

$$\frac{\partial\eta}{\partial C_L} = -\frac{\left(\cos\alpha + 2kC_L\sin\alpha\right)\!\left(h_o - h\right)}{a_2\left(h_c - h_o\right)\cos\alpha} - \frac{1}{a_2}\frac{\partial\left(C_D\right)_c}{\partial C_L}\tan\alpha \qquad (31)$$

Comparing equation (20) to equation (31), it is seen that the variation of elevator angle to trim with lift coefficient is a function of the static margin, K_n. Thus rearranging equation (31) it follows that,

$$\frac{\partial\eta}{\partial C_L} = -\frac{K_n}{a_2\left(h_c - h_o\right)\cos\alpha} \qquad (32)$$

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Thus, for small angles of attack it follows that approximately,

$$\frac{\partial \eta}{\partial C_L} = -\frac{\left(h_o - h\right)}{a_2\left(h_c - h_o\right)} \equiv -\frac{K_n}{a_2\left(h_c - h_o\right)} \approx -\frac{4K_n}{a_2} \tag{33}$$

As in conventional aeroplanes the variation of elevator angle to trim with lift coefficient is proportional to the static margin. However, in this case there is also a non-linear dependency on the lift coefficient.

5. Application to the College of Aeronautics BWB-98 tailless civil transport aircraft concept

5.1 General Information

BWB-98 is a blended-wing-body tailless civil transport aircraft designed by the College of Aeronautics, Cranfield, to meet the same specifications as that for Airbus A-3XX [2]. As the name says, the body and wing are blended in a way that the aircraft is classified as a high aspect ratio flying wing.

The general aerodynamic characteristics of this large aircraft are presented in Table 1. Some aerodynamic characteristics for typical cruise and approach conditions are presented in Table 2.

General characteristics	Values
Span, b	80 m
Wing area, S	1390.6 m^2
Root cord, c_r	46.2 m
Tip cord, c_t	4 m
Leading edge sweptback, Λ_{LE}	38.3°
Mean aerodynamic chord, $\bar{\bar{c}}$	27.28 m
Aspect ratio	7.14
Design normal take-off mass	480072Kg
Design maximum landing mass	322599Kg
Basic empty mass	210659Kg

Table 1 – General characteristics of the BWB-98

The lift coefficient due to camber, C_{L_o}, is not given in the data and it is assumed zero. Moreover, the BWB-98 has several trailing-edge surfaces, see Table 3, and the chosen pitch control elevon possesses the largest pitch moment increment per degree of deflection. Thus, from the data, flap number 6 was chosen, as the primary pitch control surface.

Aerodynamic Characteristics	Cruise condition	Approach condition
Height, h	10059 m	0 m
Temp, T	232.8K	288.15K
Sound speed, a	305.8 m/s	324.2 m/s
Density, ρ	0.3921 Kg/m^3	1.225 Kg/m^3
True airspeed, V_t	260 m/s	75 m/s
Mach number, M	0.85	0.23
Static margin, K_n	1.9%	1.5%
Lift coefficient, C_L	0.236	1.05
Lift curve slope, a_1	5.382 rad^{-1}	3.327 rad^{-1}
Elevator lift curve slope, a_2	0.008248 rad^{-1}	0.005944 $\deg^{-1}$
Drag coefficient for zero lift, C_{D_o}	0.04163	0.013908
Drag polar constant, k	0.059153	0.056592
Basic pitch moment coefficient, C_{m_o}	0.004403	0.004747

Table 2 – Aerodynamic data of the BWB-98 for a Cruise and Approach condition

δ	$a_2 \left(1/\deg ree \right)$ (M = 0.85)	$C_{m_\eta} \left(1/\deg ree \right)$ (M = 0.85)	$a_2 \left(1/\deg ree \right)$ (M = 0.23)	$C_{m_\eta} \left(1/\deg ree \right)$ (M = 0.23)
1 inboard	0.004394117	-0.000818507	0.003136305	-0.000818507
2	0.006294388	-0.001271234	0.004806767	-0.001299979
3	0.005628103	-0.001846142	0.004528947	-0.001190006
4	0.005565415	-0.001589909	0.004271003	-0.000949145
5	0.008438098	-0.001797411	0.005879657	-0.001302011
6	0.008248227	-0.002433309	0.005943705	-0.001914565
7 outboard	0.002666738	-0.001183576	0.002137155	-0.001013961

Table 3 – Elevator lift and pitch moment curve slopes for M = 0.85 and M = 0.23
(Cruise and Approach condition respectively)

Using the data given in the tables plots of lift, drag and pitch moment coefficient against angle of attack can be drawn for both approach and cruise conditions.
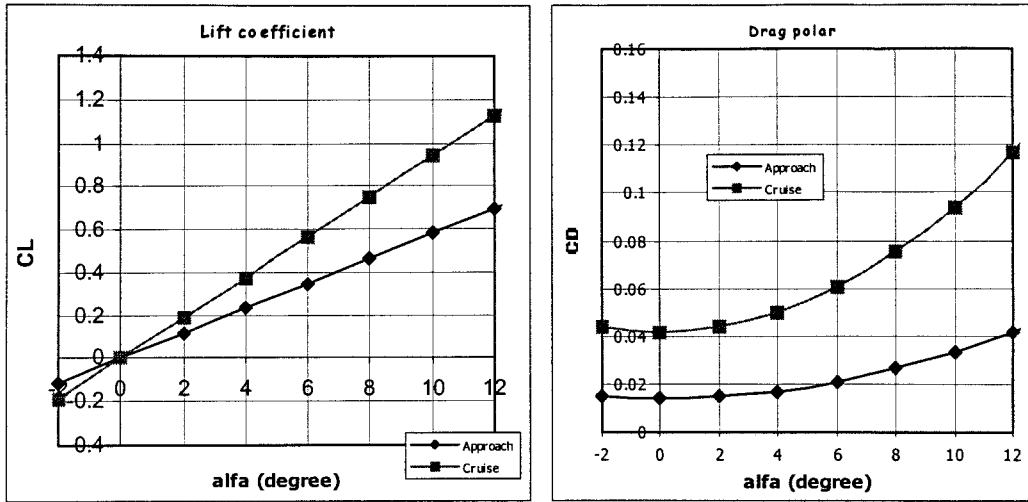
Fig. 2 – Lift and drag coefficient versus angle of attack for two different flight conditions.
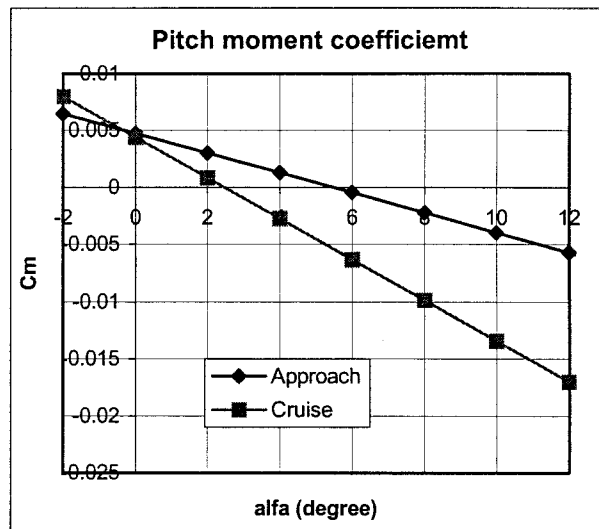

Fig. 3 – Pitch moment coefficient versus angle of attack for two different flight conditions.

Fig. 3 was drawn using equation (10) where the basic pitch moment, C_{m_o}, used is that given in Table 2, which excludes elevator and tab effects. The variation is then due only to angle of attack.

5.2 Theory application

Using equation (30),

$$\eta_{trim} \approx -\frac{1}{a_2}\left\{\frac{C_L(h_o - h)}{0.25} + C_{L_o}\right\} \tag{30}$$

the elevator angle to trim, η_{trim}, versus the lift coefficient, C_L, is calculated for different values of static margin, K_n.
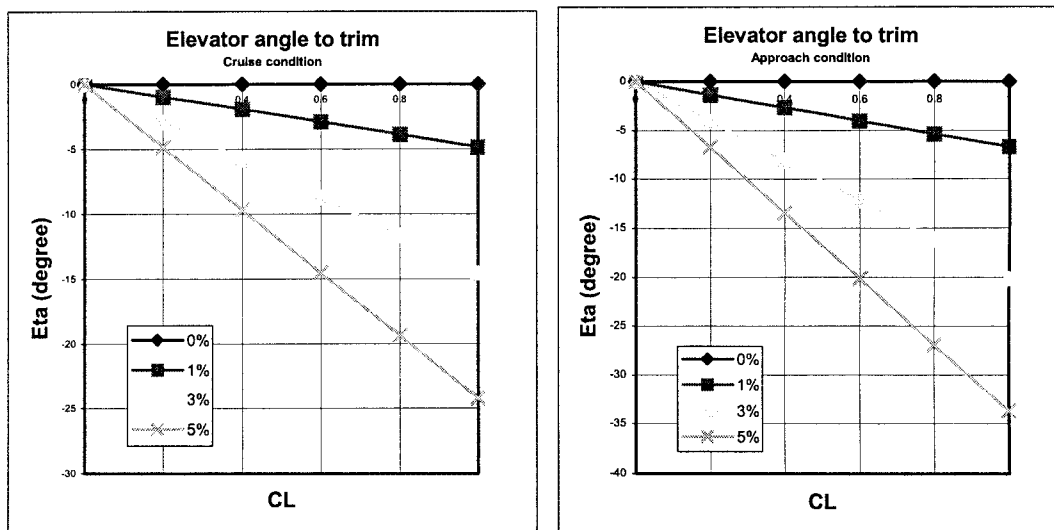

Fig. 4 – Elevator angle to trim for two different flight conditions: cruise and approach, for different CG positions.

It is possible to use the full equation for elevator angle to trim, equation (27) or equation (28), but it is necessary that the equilibrium angle of attack be known. For that, equation (28) has to be solved simultaneously with equation (3). As equation (28) is non-linear, the solution of this system is greatly facilitated with the aid of a MathCad computer program. The results are presented in Fig. 5.
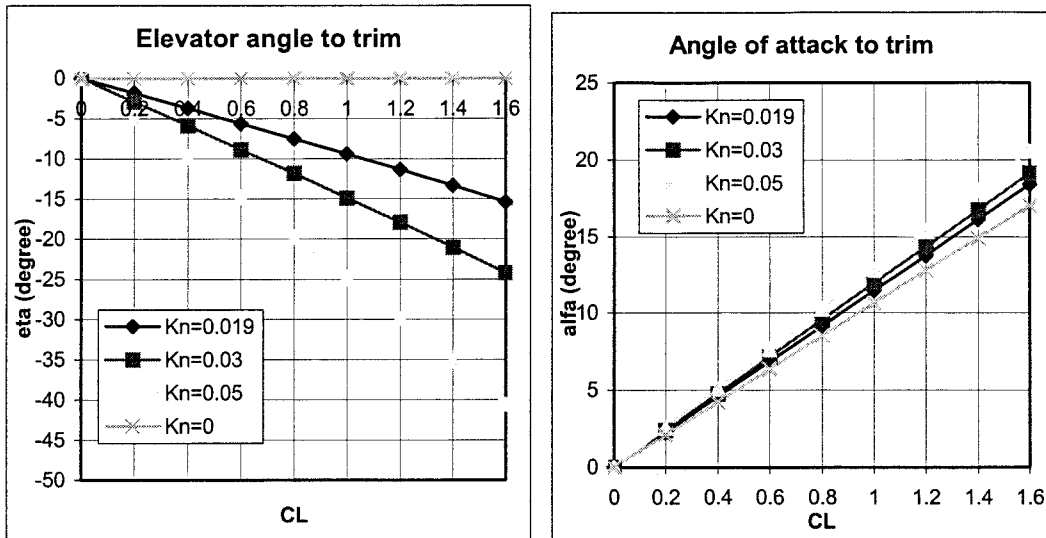
Fig. 5 – Elevator angle to trim and Angle of attack to trim versus lift coefficient for several values of static margin in a general cruise condition.

In Fig. 5 the lift coefficient range may be too large, it may be out of the permitted range of application, because a linear variation of lift with angle of attack and elevator angle is assumed.

Although, the large values were used to observe the effects of non-linearity of equation (28), comparing both Fig. 4 and Fig. 5 it can be concluded that equation (28) and equation (30) give the same results. Thus, this demonstrates that the simplified equations are adequate for initial stability and control estimates.
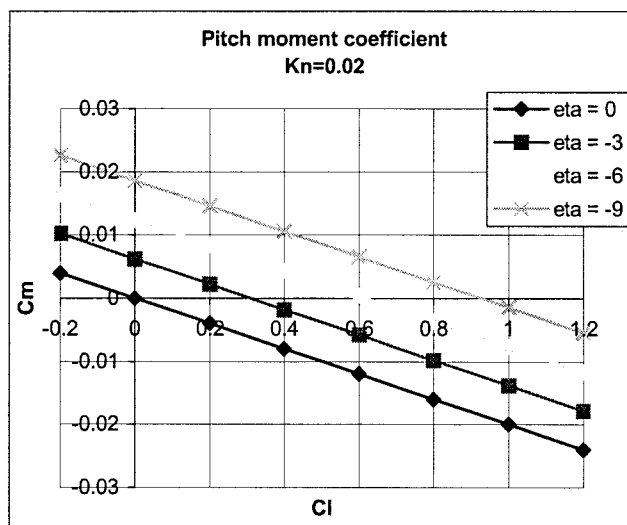

Fig. 6 – Pitch moment coefficient versus lift coefficient for several values of elevator deflection (in degrees) in a general cruise condition.

The plot in Fig. 6 was obtained using equation (10), where this time the basic pitch moment coefficient, C_{m_o}, is calculated using equation (9), rather than using the values given in Table 2.

The variation of the elevator angle to trim with lift coefficient, as can be seen from equation (32), is constant for different lift coefficient values. Fig. 7 shows instead the dependence of this parameter on static margin.
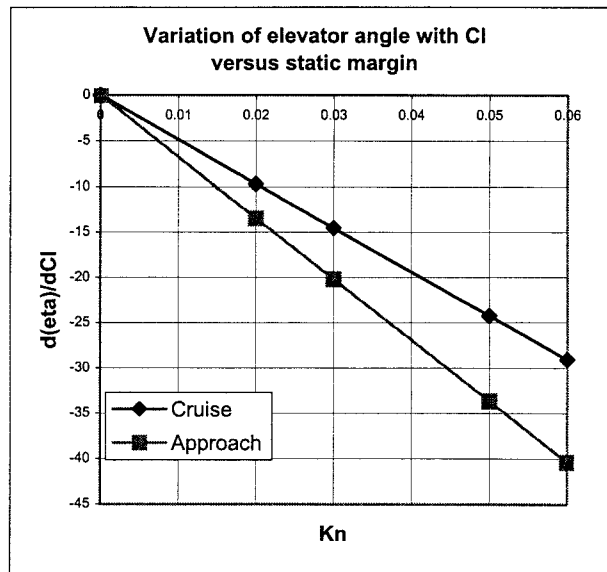

Fig. 7 – Variation of elevator angle to trim with lift coefficient with static margin, K_n.

6. Conclusions

Firstly, concerning the data for the approach condition I assumed this data is for a "clean" configuration (without flaps and landing gear) due to the low values of lift and drag curves comparative to the cruise condition.

Moreover, and as stated before, attention has to be paid to the validity of the theory applied. Thus, the conclusions given take into account that, using data from Fig. 2, in the cruise condition the maximum value for lift coefficient will be 1 or 1.2, while for the approach condition the maximum value will be 0.8. This limitation avoids the usual zone of non-linearity at angles of attack greater than 12-15 degrees.

Now, bearing in mind the limitations of the theory applied the following conclusions can be made:

- Fig. 2 and 3 show the general trend, which is usual in conventional statically stable configurations.

- Considering a maximum elevator deflection of $\eta = -25°$, from Fig. 4 and 5, the maximum static margin is limited to approximately 3%. This value complies with the minimum requirements[2].

- From Fig. 6 the large change of variation of elevator angle to trim with respect to lift coefficient for a small change of static margin is clearly evident.

- From equation (32), increasing the value of a_2 suggests that it would be possible to improve the variation of elevator angle to trim with respect to lift coefficient for increased static margin.

Concluding, the aircraft has static stability and fulfils the requirement for at least 2% of static margin. However, such a small static margin is not adequate for a civil transport aircraft. The requirement of 2% static margin ensures a stable airframe that will almost certainly require artificial stability augmentation as well.

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