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INITIAL EVALUATION OF THE MODIFIED STEPWISE REGRESSION PROCEDURE TO ESTIMATE AIRCRAFT STABILITY AND CONTROL PARAMETERS FROM FLIGHT TEST DATA

by

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NOTATION

C_x	Drag coefficient
C_y	Sideforce coefficient
C_z	Lift coefficient
C_l	Rolling moment coefficient
C_m	Pitching moment coefficient
C_n	Yawing moment coefficient
g	Gravity acceleration
I_x, I_y, I_z	Aircraft moment of inertia - axix x, y, z respectively
I_{xz}	Aircraft product of inertia
$\mathring{L}_y, \mathring{L}_p, \mathring{L}_r$	Dimensional rolling moment derivative due to sideslip, roll, yaw.
$\mathring{L}_\xi, \mathring{L}_\zeta$	Dimensional rolling moment derivative due to aileron, rudder.
$\mathring{M}_u, \mathring{M}_w, \mathring{M}_{\dot{w}}, \mathring{M}_p, \mathring{M}_\eta$	Dimensional pitching moment derivatives due to u,w,p, etc.
m	Aircraft mass
$\mathring{N}_v, \mathring{N}_p, \mathring{N}_r, \mathring{N}_\xi, \mathring{N}_\zeta$	Dimensional yawing moment derivatives due to sideslip, roll, etc
p, q, r	Rate of roll, pitch and yaw, respectively.
u, v, w	Velocity components in axis x, y and z respectively
U_o, V_o, W_o	Steady state velocities x, y and z axis.
$\mathring{X}_u, \mathring{X}_w, \mathring{X}_{\dot{w}}, \mathring{X}_q, \mathring{X}_\eta$	Dimensional drag force derivative due to u,w, etc.
$\mathring{Y}_v, \mathring{Y}_p, \mathring{Y}_r, \mathring{Y}_\xi, \mathring{Y}_\zeta$	Dimensional sideforce derivative due to sideslip, roll, yaw, etc.
$\mathring{Z}_u, \mathring{Z}_w, \mathring{Z}_{\dot{w}}, \mathring{Z}_q, \mathring{Z}_\eta$	Dimensional lift force derivative due to u,w, etc.
α	Angle of attack
β	Angle of sideslip
δa, δr	Aileron and rudder deflections, respectively
θ, ϕ, ψ	Attitude pitch, roll, yaw, respectively
η, ξ, ζ	Elevator, aileron and rudder deflections, respectively.
λ	Confidence level in F distribution calculation

1. INTRODUCTION

This report presents the initial results of research in progress in the College of Aeronautics at the Cranfield Institute of Technology. The object of the research is to investigate the use of the Modified Stepwise Regression procedure to estimate aircraft stability and control parameters from flight test data.

The Stepwise Regression Method for parameter identification is a procedure that inserts independent variables into a regression model. Each time a new variable is added to the model a regression is performed and the fit is examined when the new variable, or any other previously included variable, may be rejected from the model if non-significant to the fit. The iterative process continues until a satisfactory fit (or model) is obtained. When the procedure is applied to a conveniently chosen set of aircraft variables it may produce a model representative of aircraft dynamics (or performance) and the coefficients of the model contain the aircraft dynamic and control parameters.

The research will be developed in the next two years and, together with an insight into the Modified Stepwise Regression (MSR), will lead to a facility for parameter estimation, teaching and research studies. The research programme will include initial instrumentation analysis, investigation of methods, software development for simulation, signal conditioning, parameter estimation, and analysis of flight test experiments. The flight tests will be carried out with one of the College of Aeronautic's Handley Page Jetstream 200 aircraft, a light twin turboprop aircraft powered by Astazou MK16 engines. The aircraft is fully instrumented and equipped with an electronic data acquisition system for use as an airborne laboratory.

The main programme milestones are:

- Definition of MSR algorithm and MSR Fortran program.
- Definition of a simulation program to generate data to use in the initial runs of the MSR Fortran program as well as simulate airplane maneuvers to be used in planning future flights.
- Analysis of flight test instrumentation.
- Development of an initial software package to support initial flight test.
- Analysis of initial flights and feedback to flight maneuvers, methodology for improving test methods and software development.
- Final software development/integration.
- Report.

2. BACKGROUND

In the last 30 years, parameter estimation and system identification have been developed as powerful techniques and strategy for determining the properties of a system by the measurement of its relevant input/output parameters. During this period several different approaches have been proposed and tested and, in particular, the application to the determination of airplane stability and control derivatives from flight data has been developed. In this area previous approaches were based mainly on time consuming steady-state measurements and on the measurement of free oscillations. The identification of airplane parameters using modern control theory encouraged the development of new methods of flight testing and data analysis. Today, from one test, it is possible to determine all the stability and control parameters of an airplane. There are several methods for the estimation of airplane parameters. Their basic differences are due to assumptions regarding an optimal criterion, a criterion to establish a gradient and search direction and a criterion about signal noise. However, under certain assumptions all the methods can be seen as equals (Ref 1). The methods most commonly encountered in the bibliography are the *Equation Error*, the *Output Error*, the *Kalman Filter* and *Extended Kalman Filter* and the *Maximum Likelihood Method.*

The simplest technique for airplane parameter estimation is the *Equation Error Method* which is based on the principle of Least Square fitting. It represents the application of linear regression to each equation of motion separately. It gives biased estimation because of the measurement noise in the state and input variables and requires the measurement of all state variables involved in the process.

The *Output Error Method* minimizes the error between the model output and the actual output for the same input. The process assumes that only the measured output is corrupted by noise and there are no other disturbances on the airplane. The optimization is nonlinear and the modified Newton-Raphson technique is usually applied in the iterative solution. The output error method is also called *Maximum Likelihood* method when this criterion is used in the cost function. It is susceptible to results degradation when process noise exists.

The *Extended Kalman Filter (EKF)* is an approximate filter for nonlinear systems based on first-order linearization. It estimates simultaneously the airplane state variables and the other unknown parameters. To do this, it is first necessary to include the unknown parameters in the state vector. Once this is done a standard *Kalman* filter can be applied for the estimation, resulting in a not very complicated algorithm.Its main disadvantage is that it requires *a priori* knowledge of the covariances which are normally unknown. If the *a priori* values for the parameters are poor, the method exhibits poor convergence.

The generalized *Maximum Likelihood* estimation method consists of a combination of the *Kalman Filter* (or EKF depending on linear or nonlinear system, respectively) for estimating the state and a modified Newton-Raphson iterative procedure for estimating parameters. In general the unknown parameters can include stability and control derivatives, bias terms in the state and output equations, initial conditions for state variables and measurement and process noise covariances. When it is assumed that no process noise exists, the method reduces to the output error method and the estimates are obtained by integration of the equations of motion only.

Irrespective of the method used, it is necessary to formulate a mathematical model of the airplane under test. The problem of modelling an airplane raises, therefore, the question of how complex the model should be. A more complex model can be justified for the correct description of airplane motion. However, in the case of parameter estimation, it is not clear what should be the best relationship between model complexity, measurement information and results quality. In general, linear models can produce significant results, however, the interest in poststall and spin flights has created a need to extend parameter estimation to flight regimes where nonlinear aerodynamic effects could become pronounced.

The *stepwise regression* (SR) was introduced to airplane parameter estimation as an efficient alternative for the determination of model structure from flight data. It is seen as a good alternative to the application where non-linear terms are required in the analysis since it does not require complex mathematical solution as would be required by other methods in such cases. The first work in this area using the stepwise regression was presented in the U.S.A in 1974 (Ref 2) and by Klein in the Cranfield College of Aeronautics in 1975 (Ref 3). The use of the stepwise regression has also been proposed in The Netherlands by The Technical University of Delft and the National Aerospace Laboratory (NLR) for aerodynamic model identification (Ref 4) and previously by Gerlach for flight test instrumentation calibration (Ref 5). Further developments of the SR method was carried out in the U.S.A by Klein, Batterson and Murphy (Ref 6). In this work the airplane equations of motion are in general form with aerodynamic force and moment coefficients expressed in terms of multivariable polynomials in input and output variables. The stepwise regression has been modified by the addition of a constraint were the regression starts with the introduction of linear terms. After all linear terms have been included then non-linear terms are analyzed. This is called the *Modified Stepwise Regression - (MSR).* More recently, Hess, and Ly (Ref 7), proposed the use of the Stepwise Regression in the determination of model structure for airplane simulation. Recently, new research has been carried out at Cranfield when the MSR method was used to estimate the stability and control parameters of a small B.Ae. Hawk aircraft model flown in a dynamic wind tunnel facility (Ref 8).

The present research programme is concerned with the use of a MSR procedure to estimate stability and control parameters of a H.P. Jetstream airplane from flight test data. The main objective of the research is to demonstrate that even using the very simple regression method, the flexibility to adjust the model in the stepwise regression, allows very acceptable results to be obtained.

3. STEPWISE REGRESSION

Linear regression is used to define a functional relationship between a dependent variable and a set of independent variables. It is assumed that the dependent variable can be approximated by a linear combination of the independent variables. For the identification of airplane state and control parameters, the mathematical model of aerodynamic forces and moments may be expressed as:

$$y(t) = \beta_0 + \beta_1 x_1 + \cdots + \beta_{n-1} x_{n-1} \qquad 3.1$$

Here the independent variable *y(t)* represents the resultant coefficient of aerodynamic force or moment C_x, C_y, C_z, C_m, C_l, C_n and $\beta_0, \beta_1, \cdots, \beta_n$ are the stability and control derivatives where β_0 is the value of any particular coefficient corresponding to the initial steady-flight conditions. $x_1, x_2, \ldots, x_{n-1}$ are the aircraft state and control variables, or any combination of variables, at a given instant in time.

For a sequence of observations (N measurements) of y and x the resultant set of equation may be expressed in matrix form and since equation (1) is only an approximation of the actual aerodynamic relationship a term ε is included in the equation right side in order to reflect the error (equation error).

$$\begin{bmatrix} y(1) \\ y(2) \\ y(3) \\ \vdots \\ y(N) \end{bmatrix} = \begin{bmatrix} 1 & x_{1(1)} & x_{2(1)} & \cdots & x_{n-1(1)} \\ 1 & x_{1(2)} & x_{2(2)} & \cdots & x_{n-1(2)} \\ 1 & x_{1(3)} & x_{2(3)} & \cdots & x_{n-1(3)} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & x_{1(N)} & x_{2(N)} & \cdots & x_{n-1(N)} \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \vdots \\ \beta_N \end{bmatrix} + \begin{bmatrix} \varepsilon_{(1)} \\ \varepsilon_{(2)} \\ \varepsilon_{(3)} \\ \vdots \\ \varepsilon_{(N)} \end{bmatrix} \qquad 3.2$$

For N>n the unknown parameters (β_i) may be estimated from the measurements by a least-square technique in which the square of the equation error is minimized. For the system identification of aircraft one can determine the stability and control derivatives by a simple regression process when modelling the aircraft equations of motion.

The Stepwise Regression is a procedure that inserts independent variables into a regression model. Each time a new variable is added to the model, a regression is performed and the fit is examined in order to identify whether the new variable improves the fit, when compared with the previous one (Ref 9). The set of variables which produce the best fit will define the model (or best model) and the resulting coefficients will be the final coefficients of the regression.

The insertion order of the independent variables is determined by the use of the partial correlation coefficient, as a measure of the importance of the variables in the overall regression. At every regression step the variables incorporated into the model in the previous stages as well as the variable entering in the present stage are examined using the F statistic. A variable may be retained or rejected from the model depending on the value the partial F_p. Where,

$$F_P = \hat{\beta}_j^2 / s^2(\hat{\beta}_j) \qquad 3.3$$

β_j is the estimate of the parameter j, and **s** is its variance

By this means a variable may be included or rejected and even a variable entered in the model in a previous stage may be rejected if it results in a poor fit. The process of selecting/checking continues until no more variables can be introduced and no more can be rejected. However, there are some additional criteria that may be used in the determination of the best model fit as explained below.

3.1 MODIFIED STEPWISE REGRESSION

The mathematical representation of airplane forces and moments is normally presented as a set of linearized equations whose coefficients are the aerodynamic derivatives and the independent variables are the airplane states. This representation is good for some applications. However, the interest in post stall and spin flights has created the need to extend parameter estimation into flight areas where non-linear aerodynamic effects become more pronounced. The stepwise regression method seems to be an ideal tool for analysing data of flights in non-linear aerodynamic regions of flight. The concept of *Modified Stepwise Regression* has been introduced (Ref.6) meaning that the linear terms are introduced and may not be removed from the aircraft/regression model. In other words the modification introduces a constraint preventing the removal of linear terms regardless of their partial correlation coefficients F_P, All the linear terms are entered and examined first in an order according to their partial correlation coefficients. After all linear terms are included then non-linear terms are analyzed, being included or rejected according to their significance and the significance of all the terms already included in the model.

3.2 MODIFIED STEPWISE REGRESSION PROCEDURE

The MSR procedure is implemented by disassembling the equations of motion to a set of equations, each one representing one of the state or control variables. The regression is performed for each state represented by its respective model. Hence the procedure is repeated until models have been estimated for all states.

The MSR procedure may be summarized as follows:

STEP 1 Formulate a mathematical model, for example the linearizes equation in the derivative of u is

$\dot{u} = X_u.u + X_w.w + X_q.q - g.\theta + X_\eta.\eta$

This equation may be represented by $y = \beta_0 + \beta_1 x_1 + \ldots + \beta_{n-1} x_{n-1}$ in which β_0 may be equal to zero.

STEP 2 For N>>n the estimate of the aerodynamic derivatives can be calculated by:

$$\beta = \left[X^T X\right]^{-1} X^T Y \qquad 3.4$$

when $[x^T x]$ is positive definite.
This is the solution throught the *Normal Equations* method.

STEP 3 Calculate:

(i) The residual sum of squares:

$$RSS = \sum_{i=1}^{N}\left[y(i) - \hat{y}(i)\right]^2 \qquad 3.5$$

(ii) The residual variance:

$$s^2(\varepsilon) = RSS \,/\, (N - n) \qquad 3.6$$

It is equivalent to attributing all the errors to model errors rather than measurement error, which is a good approach since it will be used in judging model adequacy.

(iii) The covariance matrix of parameter error is

$$E\{(\beta - b)(\beta - b)^T\} = \sigma^2 [X^T X]^{-1}$$

where,

$$\left[X^T X\right]^{-1} = \begin{bmatrix} c_{00} & c_{01} & \cdots & c_{0n-1} \\ c_{10} & c_{11} & \cdots & c_{1n-1} \\ \vdots & \vdots & \cdots & \vdots \\ c_{n-10} & c_{n-11} & \cdots & c_{n-1n-1} \end{bmatrix} \qquad 3.7$$

σ^2 is replaced by its estimate s^2 calculated by 3.6 .
The estimated standard error for each parameter estimate is given by:

$$s_{\beta 0} = s\sqrt{c_{00}} \;;\quad s_{\beta 1} = s\sqrt{c_{11}} \;;\quad \cdots \;;\; s_{\beta n-1} = s\sqrt{c_{n-1}} \qquad 3.8$$

where $s = \sqrt{s^2(\varepsilon)}$ is obtained from (ii) above.

STEP 4 In this step the overall regression is examined for the possibility that all of estimates are zero.

The null hypothesis is rejected if $F > F(\nu_1, \nu_2, \lambda)$ where:

$$F = \frac{\beta^T X^T Y - N \bar{y}^2}{(n-1)s^2(\varepsilon)} \quad ; \quad \bar{y} = \frac{1}{N}\Sigma Y(i) \qquad 3.9$$

F is a random variable having an F-distribution with $\nu_1 = n\text{-}1$ and $\nu_2 = N\text{-}n$ degrees of freedom and significance level λ.
Values of F-distribution for various significance levels λ may be found in statistical reference tables (see, for example, Ref 10).

STEP 5 The significance of each individual term of the regression is analyzed through the partial F-test. For each independent variable the value of F_P is calculated.

$$F_p = \frac{[\beta_j]^2}{s^2_{\beta j}} \qquad 3.10$$

In the expression $s^2_{\beta j}$ is the estimated variance of β calculated in step 3. If $F_p > F(1, N-n, \lambda)$ the parameter under test is significant and may be kept in the regression equation. If $F_P < F$ it is possible that β_j is equal to zero and that variable should be removed from the model. If one or more parameter is non-significant, the one with lowest value of F_P is rejected from the regression model.

STEP 6 The correlation coefficient R^2 is calculated and it gives an indication of the goodness of the fit to the equation with the measured data.

$$R^2 = \frac{\beta^T X^T Y - N(\bar{y})^2}{Y^T Y - N(\bar{y})^2} = \frac{F}{(N-n)/(n-1)+F} \qquad 3.11$$

STEP 7 Choice of new variables to enter in the model.
The variables not included in the current mathematical model are examined to identify how well they correlate with the y, given the variables already included in the regression.

Consider a model where only x_1 has been included. A new independent variable z_2 is constructed by finding the residuals of x_2 after regressing it on x_1.

The residual from fitting the model is:

$$x_2 = a_0 + a_1 x_1 + \varepsilon$$

z_2 is given by $z_2 = x_2 - \hat{a}_0 - \hat{a}_1 x_1$

In the same way the variables $z_3, z_4, \ldots, z_{n-1}$ are formed by regressing the variables x_3 on x_1, x_4 on x_1, and so on. A new dependent variable y^* is formed by the residuals of y regressed on x_1 using the model given by:

$$y^* = y - \hat{\beta}_0 - \hat{\beta}_1 x_1$$

In the next step, a new correlation which involves the variables y^*, z_2, $z_3, \ldots, z_{n-1}$ is formulated. This partial correlation may be written as $r_{ij.1}$ meaning the correlation of z_j and y^* related to the model containing the variable x_1.

The correlation coefficient is calculated by the following expression:

$$r_{jy} = \frac{S_{jy}}{\left(S_{jj} S_{yy}\right)^{1/2}} \qquad 3.12$$

where

$$S_{jy} = \sum_N \left[x_j(i) - \bar{y} \right] \left[y(i) - \bar{y} \right]$$

$$S_{jj} = \sum_N \left[x_j(i) - \bar{x}_j \right]^2 \qquad 3.13$$

$$S_{yy} = \sum_N \left[y(i) - \bar{y}_j \right]^2$$

$$\bar{x}_j = \frac{1}{N} \sum_N x_j(i) \quad , \quad \bar{y} = \frac{1}{N} \sum_N y(i)$$

If there are no further variables to be rejected or no further variables to be included during the iteration then the procedure stops, otherwise the steps 1 to 8 are repeated.

3.3 CHOICE OF 'F' VALUES

The tabulated values of $F(1,N\text{-}n,\lambda)$ are dependent on the number of samples, the number of parameters in the model and the chosen risk level. For N > 100, the effect of n on F is small ; therefore, $F(1,N\text{-}n,0.01)$ is taken as 7, regardless of N and n. The tabulated values of $F(n\text{-}1,N\text{-}n,\lambda)$ for N > 100 and $\lambda = 0.01$ vary approximately from 3.0 to 2.3. It is indicated in reference (Ref 3) that the observed F-values should not only exceed the percentage point of the F-distribution but should be about four times the selected percentage point. Reference 3 defines 12 as an acceptable value for F in aircraft parameter estimation while (Ref 8) defines 4 and (Ref 7) defines 5.

Experience with test computation showed that a model based only on the statistical significance of individual parameters in the regression equation can have too many parameters (Ref 4). Then, more quantities and their values should be analyzed as possible criteria for the selection of an adequate model. Of the quantities that could be examined the following may be considered (Ref 3):

(a) The computed value of F_P for each parameter considered in the model.. Since F_P is the inverse of parameter variance, it should have the maximum value for an adequate model.

(b) The computed value of F. The model that has the maximum F-value may be the best because F is given as the ratio of regression mean square to the residual mean square.

(c) R^2, the correlation coefficient, is no longer the most commonly used parameter as the measure of the fit correctness (1 = perfect fit).

(d) The value of the residual sum of the squares (RSS) is defined:

$$RSS = \sum_{i=1}^{N}\left[y(i) - \bar{y}(i) \right]^2 \qquad 3.14$$

(e) The residual variance $s^2(\varepsilon)=RSS/(N-n)$ which should be compared with an unbiased estimate of the variance $\sigma^2(\varepsilon)$, if available.

(f) In an adequate model, the time history of the residuals should be close to a random sequence, uncorrelated and Gaussian.

(g) The predicted sum of squares is also a good criterion to use

$$PRESS = \sum_{i=1}^{N} \frac{\left[y(i) - \hat{y}(i) \right]^2}{1 - \frac{Var\{y(i)\}}{\sigma^2}} \qquad 3.15$$

3.4 MSR FORTRAN PROGRAM

The Modified Stepwise Procedure presented in the previous paragraph has been programmed in FORTRAN language. The program is running in both a 286 IBM PC and a 486 Viglen PC using MS Fortran Versions 5.0 and 5.1, in the 286 and 486 PC respectively. In addition to the main program three routines have been writen to manipulate matrices transposition, multiplication and inversion, and one has been writen that reads run time instructions. The routine that does matrix inversion uses the Gaussian Elimination Method (Ref.11).

The MSR program strictly follows the steps defined in paragraph 3.2 and it interacts with the user via screen/keyboard, i.e., Run Time Input (RTI) is requested on the screen (see Appendix A, the program listing). The minimum value of F and F_P to reject a variable has been set equal to 5.0. A new value may be adopted in the future when analyzing actual flight data depending on the quality of the results. At the beginning of a program run, the user specifies the independent variables to be included in the initial model. After initialization, the program asks only if the user wants to reprocess the regression when a variable is rejected. The data file name is the first run time information to be requested. Any name may be used, however the .DAT extension is assumed. The data file consists of a sequence of x_i, the state and control parameters (as in the model), the last determining the value of y_i. The first line of the file must present, as integers, the total number of samples and the number of independent variables (x) in the data file (Example: 200 8 meaning that the file contains 200 samples and 8 independent variables). Appendix B shows an example of a data file. It is assumed that the data file contains the required data to analyze one model regression, i.e., one equation only. At the present stage of program development the program is unable to combine independent variables read in the data file in order to generate new terms in the regression. The maximum number of samples that can be used in a regression is 300, which allows for aproximately 9 seconds of flight at a sample rate of 32 samples/second. Results of a program run are discussed in section 6.

4. AIRCRAFT EQUATIONS OF MOTION

The MSR procedure does not necessarily require the aircraft equations of motion since it does not integrate equations. However, a model must be chosen to physically represent the aircraft motion and it may be a difficult objective to achieve. The model does not need to be in state space form and the regression may be done over the state variables, their derivatives and any combination of the state variables.

In order to validate the MSR program a simple computer simulation of an aircraft was used. The simulation will also be useful for designing flight test procedures at a later stage in the project. Consequently, the equations of motion for an aircraft are required for the simulation program rather than for the MSR program. Hence, the equations of motion are proposed below, refered to body axes and based on the following assumptions:

(i) the airplane is a rigid body
(ii) small disturbance.

The linearized aircraft equations of motion for small perturbations referred to wind axis may be written (Ref 15):

$$M\dot{x}(t) = A'x(t) + B'x(t)$$

where $x(t)^T = [u\ w\ q\ \theta\ h\ v\ p\ r\ \phi\ \psi]$ and $u(t)^T = [\eta\ \tau\ \xi\ \zeta]$

M, A' and B' are defined as follows:

$$
M' = \begin{bmatrix}
m & 0 & 0 & 0 & 0 & & & & & \\
0 & (m-\mathring{Z}_{\dot{w}}) & -\mathring{Z}_{\dot{q}} & 0 & 0 & & & & & \\
0 & -\mathring{M}_{\dot{w}} & (I_y-\mathring{M}_{\dot{q}}) & 0 & 0 & & & 0 & & \\
0 & 0 & 0 & 1 & 0 & & & & & \\
0 & 0 & 0 & 0 & 1 & & & & & \\
 & & & & & m & 0 & 0 & 0 & 0 \\
 & & & & & 0 & (I_x-\mathring{L}_{\dot{p}}) & -(I_{xz}+\mathring{L}_{\dot{r}}) & 0 & 0 \\
 & & 0 & & & 0 & -(I_{xz}+\mathring{N}_{\dot{p}}) & (I_z-\mathring{N}_{\dot{r}}) & 0 & 0 \\
 & & & & & 0 & 0 & 0 & 1 & 0 \\
 & & & & & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

$$
A' = \begin{bmatrix}
\mathring{X}_u & \mathring{X}_w & \mathring{X}_q - mW_0 & -mg\cos\alpha & 0 & & & & & \\
\mathring{Z}_u & \mathring{Z}_w & \mathring{Z}_q + mU_0 & -mg\sin\alpha & 0 & & & & & \\
\mathring{M}_u & \mathring{M}_w & \mathring{M}_q & 0 & 0 & & & 0 & & \\
0 & 0 & 1 & 0 & 0 & & & & & \\
0 & -1 & 0 & U_0 & 0 & & & & & \\
 & & & & & \mathring{Y}_v & \mathring{Y}_p + mW_0 & \mathring{Y}_r - mU_0 & mg\cos\alpha & -mg\sin\alpha \\
 & & & & & \mathring{L}_v & \mathring{L}_p & \mathring{L}_r & 0 & 0 \\
 & & 0 & & & \mathring{N}_v & \mathring{N}_p & \mathring{N}_r & 0 & 0 \\
 & & & & & 0 & 1 & 0 & 0 & 0 \\
 & & & & & 0 & 0 & 1 & 0 & 0
\end{bmatrix}
$$

$$
B' = \begin{bmatrix}
\mathring{X}_\eta & \mathring{X}_\tau & & \\
\mathring{Z}_\eta & \mathring{Z}_\tau & & \\
\mathring{M}_\eta & \mathring{M}_\tau & 0 & \\
0 & 0 & & \\
0 & 0 & & \\
 & & \mathring{Y}_\xi & \mathring{Y}_\zeta \\
 & & \mathring{L}_\xi & \mathring{L}_\zeta \\
 & 0 & \mathring{N}_\xi & \mathring{N}_\zeta \\
 & & 0 & 0 \\
 & & 0 & 0
\end{bmatrix}
$$

where

$U_0 = V\cos\alpha$, $W_0 = V\sin\alpha$ and V is the flight path speed.

In general, the longitudinal-lateral coupling derivatives are very small and can be neglected. The longitudinal and lateral groups of equations may be separated for small perturbations. The lateral thrust derivatives (denoted by τ) may be assumed null because the thrust vector lies in the airplane plane of symmetry.

4.1 LONGITUDINAL EQUATIONS OF MOTION

The upper left hand portion of matrix A' always has an inverse, hence it can be separated from the matrix A' generating the equations for the longitudinal modes alone. The engine dynamics do not influence the small perturbation stability and the height does not influence any other state, this allows simplifications in the equations. Multiplying by the inverse of the mass matrix M, results in the following longitudinal equations of motion referred to body axes (Ref 12),

$$\dot{u} = X_u u + X_{\dot{u}} \dot{u} + X_w w + X_{\dot{w}} \dot{w} + X_q q + X_{\dot{q}} \dot{q} - W_0 q - g\cos\alpha\theta + X_\eta \eta + X_{\dot{\eta}} \dot{\eta}$$

$$\dot{w} = Z_u u + Z_{\dot{u}} \dot{u} + Z_w w + Z_{\dot{w}} \dot{w} + Z_q q + Z_{\dot{q}} \dot{q} + U_0 q - g\sin\alpha\theta + Z_\eta \eta + Z_{\dot{\eta}} \dot{\eta}$$

4.1

$$\dot{q} = M_u u + M_{\dot{u}} \dot{u} + M_w w + M_{\dot{w}} \dot{w} + M_q q + M_{\dot{q}} \dot{q} + M_\eta \eta + M_{\dot{\eta}} \dot{\eta}$$

$$\dot{\theta} = q$$

where $X_x, \ldots, Z_z$ are normalized, calculated as below:

$$X_u = \frac{\mathring{X}_u}{m} \quad , \quad X_w = \frac{\mathring{X}_w}{m} \quad , \quad Z_u = \frac{\mathring{Z}_u}{m} \quad , \quad Z_w = \frac{\mathring{Z}_w}{m} \quad , \quad Z_q = \frac{\mathring{Z}_q}{m}$$

$$M_u = \frac{\mathring{Z}_u \mathring{M}_{\dot{w}}}{m I_y} + \frac{\mathring{M}_u}{I_y} \quad , \quad M_w = \frac{\mathring{Z}_w \mathring{M}_{\dot{w}}}{m I_y} + \frac{\mathring{M}_w}{I_y} \quad , \quad M_q = \frac{\mathring{Z}_q \mathring{M}_{\dot{w}}}{m I_y} + \frac{\mathring{M}_{\dot{w}} U_0}{I_y} + \frac{\mathring{M}_q}{I_y}$$

$$X_\eta = \frac{\mathring{X}_\eta}{m} \quad , \quad Z_\eta = \frac{\mathring{Z}_\eta}{m}$$

$$M_\eta = \frac{\mathring{Z}_\eta \mathring{M}_{\dot{w}}}{m I_y} + \frac{\mathring{M}_\eta}{I_y}$$

The equations above are the classical longitudinal equations which will constitute the basic equations to be considered in the regression process, i.e., will constitute the basic model, to which nonlinear terms may be added.

4.2 LATERAL EQUATIONS OF MOTION

As for the longitudinal case, the lower right hand side of the matrix A' always has an inverse allowing for separation of the lateral mode. Because yaw attitude does not influence any other state it can be omitted too, resulting, after multiplication by the inverse of the resulting mass matrix M, in the following set of equations, refered to body axes (Ref 12):

$$\dot{v} = Y_v v + Y_{\dot{v}} \dot{v} + Y_r r + Y_{\dot{r}} \dot{r} + Y_p p + Y_{\dot{p}} \dot{p} + Y_\xi \xi + Y_\zeta \zeta - U_0 r + W_0 p + g \cos\alpha\phi$$

$$\dot{p} = \frac{I_{xz}}{I_{xx}} \dot{r} + L_v v + L_{\dot{v}} \dot{v} + L_r r + L_p p + L_{\dot{p}} \dot{p} + L_\xi \xi + L_\zeta \zeta \qquad 4.2$$

$$\dot{r} = \frac{I_{xz}}{I_{xx}} \dot{p} + N_v v + N_{\dot{v}} \dot{v} + N_r r + N_{\dot{r}} \dot{r} + N_p p + N_{\dot{p}} \dot{p} + N_\xi \xi + N_\zeta \zeta$$

$$\dot{\phi} = p$$

where

$$Y_v = \frac{\mathring{Y}_v}{m}, \quad Y_p = \frac{\mathring{Y}_p}{m}, \quad Y_r = \frac{\mathring{Y}_r}{m}, \quad L_v = \frac{\mathring{L}_v}{I_x}, \quad L_p = \frac{\mathring{L}_p}{I_x}, \quad L_r = \frac{\mathring{L}_r}{I_x}$$

$$N_v = \frac{\mathring{N}_v}{I_z}, \quad N_p = \frac{\mathring{N}_p}{I_z}, \quad N_r = \frac{\mathring{N}_r}{I_z}$$

$$Y_\xi = \frac{\mathring{Y}_\xi}{m}, \quad Y_\zeta = \frac{\mathring{Y}_\zeta}{m}, \quad L_\xi = \frac{\mathring{L}_\zeta}{I_x}, \quad L_\zeta = \frac{\mathring{L}_\zeta}{I_x}, \quad N_\xi = \frac{\mathring{N}_\xi}{I_z}, \quad N_\zeta = \frac{\mathring{N}_\zeta}{I_z}$$

4.3 GENERAL MODEL

As stated above, the MSR Procedure does not require an equation to integrate and the proposed regression model, which is an objective to be achieved, need not be in state space form. Hence the regression may be carried out on models similar to the ones presented above (equations 4.1 1nd 4.2) or by a model such as the following, (Ref 18), which is equivalent:

$$C_L = C_{Lo} + C_{L\alpha}\alpha + C_{L\eta}\eta + C_{L\dot{\alpha}}\frac{l\mu}{U_0}\dot{\alpha} + C_{Lq}\frac{l\mu}{U_0}q$$

$$C_D = C_{D0} + C_{D\alpha}\alpha + C_{D\eta}\eta$$

$$C_m = C_{m0} + C_{m\alpha}\alpha + C_{m\eta}\eta + C_{m\dot{\alpha}}\frac{l\mu}{U_0}\dot{\alpha} + C_{mq}\frac{l\mu}{U_0}q$$

$$C_Y = C_{Y0} + C_{Y\beta}\beta + C_{Y\delta a}\delta a + C_{Y\delta r}\delta r + C_{Yp}\frac{s}{U_0}p + C_{Yr}\frac{s}{U_0}r \qquad 4.3$$

$$C_l = C_{l0} + C_{l\beta}\beta + C_{l\delta a}\delta a + C_{l\delta r}\delta r + C_{lp}\frac{s}{U_0}p + C_{lr}\frac{s}{U_0}r$$

$$C_n = C_{n0} + C_{n\beta}\beta + C_{n\delta a}\delta a + C_{n\delta r}\delta r + C_{np}\frac{s}{U_0}p + C_{nr}\frac{s}{U_0}r$$

An attractive alternative equation model which appears in the literature (Ref 1 and 3), is the representation of the aerodynamic coefficients as multivariable polynomials in response and control variables (valid for subsonic conditions, i.e., assumed continuity). The parameters are the coefficients of the Taylor series expansion around the values corresponding to initial steady state flight. In particular, (Ref 3) uses the following representation:

(i) The longitudinal coefficients C_x , C_y and C_m represented as functions of α, β, η, α^2, $q\alpha$, $\eta\alpha$, $\alpha\beta^2$, β^2, α^n (n=3,...8).

(ii) The lateral coefficients C_y, C_l and C_n represented as functions of β, p, r, δa, δr, α, $\alpha\beta$, $p\alpha$, $r\alpha$, $\delta a\alpha$, $\delta r\alpha$, $\beta\alpha^2$, $p\alpha^2$, $r\alpha^2$, $\delta a\alpha^2$, $\delta r\alpha^2$, β^2, β^3, β^4,...., $\beta^3\alpha^2$, α^2 , α^3
C_x, C_y ,C_z , C_l, C_m and C_m are assumed to be known, i.e., are calculated from the measurements of a_x , a_y, a_z, Thrust, p,q,r (and derivatives).

(iii) (Ref 1) defines a more general expression mainly directed to the application for nonlinear conditions:

$$C = C_0(\bar{\alpha},\bar{\beta}) + \sum_i C_{\alpha^{(i)}}\alpha^i + \sum_i C_{\beta^{(i)}}\beta^i + \sum_i \sum_j C_{\alpha^{(i)}\beta^{(j)}}\alpha^i\beta^j$$

which is analogous to the ones presented under (i) and (ii), above.
It is important to note that equation 4.1, 4.2 and 4.3 are derived from the equilibrium of forces and moments and the last equation 4.3, differs from the former only by the US notation. However, the equation format suggested in (i), (ii) and (iii) does not follow a logical approach unless a model representing a particular maneuver is required, based on a combination of sensible parameters.

5. AIRCRAFT SIMULATION

In order to do an initial evaluation of the MSR Method as well as of the MSR Fortran program, an aircraft simulation program has been written to generate data to run the MSR FORTRAN. Otherwise, the simulation program will be useful to model the basic airplane equations of motion for small perturbations so that any input can be modelled and the real airplane response can be analyzed prior to any flight test.
The simulation has been programmed using the Advanced Continuous Simulation Language- ACSL (Ref.14) and initially used the full mathematical model and data for the B-747, as presented in (Ref 13). In a second stage the model of the B-747 will be replaced by the model of the H.P Jetstream, as set out in (Ref 16) or (Ref 17).

Two simulation programs have been developed, one for decoupled longitudinal motion and the other for decoupled lateral motion. It was not considered necessary, at this time, to consider coupled responses. However, this may be taken into account later.

5.1 LONGITUDINAL MODEL SIMULATION

The model used for the B-747 is described in (Ref 13). However, it is almost the same as the model presented above in paragraph 4.1.

The equations were presented in state space form $\dot{x} = Ax + Bu$ and they appear in the simulation program in the following form:

```
   ud = a11*u + a12*w + a13*q + a14*theta + b11*eta
   wd = a21*u + a21*w + a23*q + a24*theta + b21*eta
   qd = a31*u + a31*w + a33*q + a34*theta + b31*eta
tetad = a41*u + a42*w + a43*q + a44*theta + b41*eta
```

corresponding to the following equations

$$\dot{u} = X_u u + X_w w + X_q q - g\cos\alpha\theta + X_\eta \eta$$

$$\dot{w} = Z_u u + Z_w w + Z_q q - g\sin\alpha\theta + Z_\eta \eta$$

$$\dot{q} = M_u u + M_w w + M_q q + M_\eta \eta$$

$$\dot{\theta} = q$$

The step input to the elevator (eta) is modelled using the STEP input function begining at the first integration step and finishing at any desired instant. STEP input or DOUBLET input to the elevator may be programmed. Integration may be performed by several alternative algorithms, however, Runge-Khutta 2nd order was used. The program uses the following structure, as defined in Ref 14, and presented below:

```
PROGRAM TITLE

   INITIAL REGION
       Specify Constants ( Speed, Values of Stability and Control
        Parameters)
       Set Control Surface Deflection and Duration.
   END OF INITIAL

   DYNAMIC REGION
       Specify Time Interval
       Specify Data Save Interval

        DERIVATIVE REGION
            Define Equation of Motion
             Integrate States
            Perform Additional Calculations/Conversions
         END OF DERIVATIVE

   END OF DYNAMIC
       Output Responses
   END OF PROGRAM
```

5.2 LATERAL MODEL SIMULATION

Analogous to the longitudinal simulation program, a program has been written for the lateral model representing the B-747 airplane, as presented in (Ref13). The equations (linear) are presented in state space form $\dot{x} = Ax + Bu$, appearing in the program as a set of equations represented by:

pd = a11*p + a12*r + a13*phi + a14*beta + b11*rud + b12*ail
rd = a21*p + a21*r + a23*phi + a24*beta + b21*rud + b22*ail
phid = a31*p + a31*r + a33*phi + a34*beta + b31*rud + b31*ail
betad = a41*p + a42*r + a43*phi + a44*beta + b41*rud + b41*ail

where 'rud' represents the rudder input while 'ail' represents aileron input.

The above set of linear equations represents the lateral equations of motion,

$$\dot{v} = Y_v v - U_0 r + W_0 p + g \cos\alpha\phi + Y_\zeta \zeta + Y_\xi \xi$$

$$\dot{p} = L_v v + L_p p + L_r r + L_\zeta \zeta + L_\xi \xi$$

$$\dot{r} = N_v v + N_p p + N_r r + N_\zeta \zeta + N_\xi \xi$$

$$\dot{\phi} = p$$

The inputs to aileron or rudder are modelled by a sum or difference of STEP functions to represent an impulse or a doublet. The program structure is similar to that shown for the longitudinal simulation.

6. INITIAL RESULTS

The simulation programs reproduce reasonably the airplane motion in both response to longitudinal and lateral inputs. Values of airplane state response were recorded for several inputs generating data to be used in the MSR Fortran program. Using these data, the MSR program reconstructed accurately the initial coefficients of the equations of motion, i.e., the aerodynamic derivatives, in both longitudinal and lateral modes.

Appendix B presents a printout of the MSR Program data file, output from the simulation program, corresponding to a 5 degrees elevator step of 3 seconds duration for the airplane flying at 20000 ft pressure altitude and 0.5 Mach. Appendix C presents the MSR printout showing the iterative process of model estimation and the final values achieved for the model coefficients. The objective of the exercise was to determine the coefficients, in this case the dimensional coefficients, by the regression of a set of 59 samples of *u*, *w*, *q*, *theta*, *eta* and *udot* (derivative of *u*) generated by the simulation program. This corresponds to the adjustustment of a model of, for example, the following form

udot = Bu * *u* + Bw * *w* + Bq * *q* + Btheta * *theta* + Beta * *eta*

or equivalently

y = B0 + B1*X1 + B2*X2 + B3*X3 + B4*X4 + B5*X5

For the initial application of the MSR program the model was reduced to the minimum form

y = B1*X1 + B2*X2 + B3*X3

where, X1 = *u*, X2 = *w* and X3 = *q*

After the first regression no variable was rejected and the variable X5 was chosen as the best to be included in the model. A second regression was performed, now with variables X1, X2, X3 and X5, and again no variables were rejected and the variable X4 was chosen to be included. A third regression was performed, now with X4 included in the previous model, and no rejection was decided. A fourth regression with X0 included (the last available) was performed. Since X0 does not reach the minimum F_P to remain in the model, X0 was rejected.
The best model was therefore found to be

y = B1*X1 + B2*X2 + B3*X3 + B4*X4 + B5*X5

The estimated coefficients are very close to the expected ones (used in the simulation). Table 1, next page, presents the estimated coefficients and their actual values.

Table 1					
	B1	B2	B3	B4	B5
Actual	-0.00161	0.080078	-61.36810	-31.97350	2.01637
Estimated	-0.00163	0.08008	-61.36828	-31.97527	2.01638

The same estimation has been repeated with 100, 200 and 300 samples, corresponding to 5, 10 and 15 seconds of simulated flight, respectively, reproducing similar results for all cases.

For the other states the following results were obtained

$$dw = -0.73495u - 0.440289w + 517.325q - 3.87340\theta - 17.1794\eta$$
$$\hat{dw} = -0.7369u - 0.44027w + 517.3231q - 3.890720\theta - 17.17962\eta$$

$$dq = 0.000298u - 0.001619w - 0.481833q + 0.000483\theta - 1.07813\eta$$
$$\hat{dq} = 0.00030u - 0.00162w - 0.48182q + 0.00055\theta - 1.07813\eta$$

The symbol ^ denotes estimated values while *dw*, *dq* denote derivatives of *w* and *q*, respectively.

For the lateral motion the following results were obtained for 5 degrees of rudder doublet with 2 seconds duration.

$$d\beta = -0.0821423\beta + 0.118404p - 0.992965r + 0.061689\phi + 0.012777\zeta$$
$$\hat{d\beta} = -0.08214\beta + 0.11840p - 0.99297r + 0.061669\phi + .01278\zeta$$

$$dp = -2.00247\beta - 0.654732p + 0.443283r + 0.111186\zeta$$
$$\hat{dp} = -2.00248\beta - 0.65474p + 0.44328r + 0.11119\zeta$$

$$dr = 1.1999\beta - 0.159638p - 0.477847r - 1.06915\zeta$$
$$\hat{dr} = 1.17992\beta - 0.15962p - 0.47784r - 1.06915\zeta$$

Again the symbol ^ denotes estimated values while $d\beta$, *dp* and *dr* denote derivative.

7. CONCLUSIONS AND SHORT TERM OBJECTIVES

The results obtained from the simulations are promising and experiments with actual flight test data will be commenced in the near future. There are, however, some objectives to be achieved before test flights can commence.

(i) Develop a signal conditioning package to filter the data and detect spurious points. The package must have a reasonable plotting capability.

(ii) Replace the B-747 model used in the simulation by the model of the H.P Jetstream as presented on (Ref 16) or (Ref 17) since the Jetstream will be used for flight tests.

(iii) Analysis of inputs (elevator , rudder or aileron deflections) required to excite the aircraft in the required modes.

(iii) Replace the matrix inversion in the MSR FORTRAN program by a Householder transformation in order to increase the robustness of the program in the event of an ill conditioned matrix, computer roundoff problems, etc.

iv) Looking into the possibility of moving the MSR Program from the DOS environment to the Windows environment in order to take advantage of the extended PC memory (if supported by Microsoft FORTRAN) in order to increase the length of the vectors. An alternative solution might be to transform the regression process into a recursive process, thereby allowing the of use any number of samples.

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APPENDIX A - MSR Program Listing

```
C*** PROGRAM MOD. STEPWISE REGRESSION ***************
C
C    NAT = NUMBER OF SAMPLES OF EACH VARIABLE
C    NV  = MAXIMUM NUMBER OF INDEPENDENT VARIABLES
C    IVV = ACTUAL NUMBER OF VARIABLES IN THE MODEL
C    NN  = ACTUAL NUMBER OF VARIABLES IN THE DATA ARCHIVE
C    ISTAT(IV) = DEFINE STATUS OF THE VARIABLE IN THE MODEL
C            IF.EQ.-1 IS NEGLETED.
C    ISTATU(I) = VARIABLE NUMBER
C    X(I,J) = INDEPENDENT VARIABLES READ FROM FLIGHT DATA
C    XWORK(I,J) = THE X(s) ACTUALY USED BY THE MODEL
C    XWORK1 = NEW IND. VARIABLE OF REGRESSION.
C    Y(I,J) = DEPENDENT VARIABLE - FROM FLIGHT DATA
C
      REAL*8 X(300,11),Y(300,1),SB(11),Z(11),FP(11),DY(300),
     +     YHAT(300,1),XHAT(300,1),XWORK1(300,1),BTXTRY(1,1)

      REAL*8 XWORK[ALLOCATABLE] (:,:),    XTR[ALLOCATABLE] (:,:),
     +       XTRX[ALLOCATABLE] (:,:), XTRXI[ALLOCATABLE] (:,:),
     +       XTRY[ALLOCATABLE] (:,:),     B[ALLOCATABLE] (:,:),
     +        BTR[ALLOCATABLE] (:,:),    BN[ALLOCATABLE] (:,:),
     +       XWTY[ALLOCATABLE] (:,:), IDENT[ALLOCATABLE] (:,:),
     +       WORK[ALLOCATABLE] (:,:)

      REAL*8 SYY,SJY,SJJ,YAVER,MODRJY,RJY,ZAV,RMAX,DYAV,
     +     FMIN,FPMIN,R2,F,VAR,RESS
      INTEGER*2 ISTAT(11),ISTATU(12),I,J,K,L,M,N,IN,NAT,NV,NN,IV,
     +       IVV,ITER,NEWVAR,IT
      CHARACTER*8 INAME
      CHARACTER*1 ICHAR
      CHARACTER*6 IMOD(12)/
     *'  Y =',' B0 + ','B1*X1+','B2*X2+','B3*X3+',
     * 'B4*X4+','B5*X5+','B6*X6+','B7*X7+','B8*X8+','B9*X9+',
     * 'B10X10'/
      LOGICAL PEND
      FMIN=5.
      IN=1
      IOLD=12
      PEND=.FALSE.
C
C*** DATA READING ***
C
      PRINT *,'NAME OF THE ARCHIVE OF FLIGHT DATA'
      READ *,INAME
      OPEN(UNIT=6,FILE='DATO.DAT',STATUS='OLD')
      OPEN(UNIT=8,FILE='MSROUT')
      READ(6,*)NAT,NN
      NV=NN+1
      DO I=1,NAT
       X(I,1)=1.
```

```
      READ(6,*) LINE,(X(I,J+1),J=1,5)
     ENDDO
     DO I=1,NAT
      READ(6,*) LINE,Y(I,1)
     ENDDO
     CLOSE(UNIT=6)
C
C*** READING THE MODEL ***
C
     DO I=1,NV
      ISTAT(I)=-1
     ENDDO
     PRINT *,'ENTER THE VARIABLES TO BE INCLUDED IN THE MODEL'
     PRINT *,'TYPE 1ST VARIABLE NUMBER, I2,'
     READ *,IV
     ISTAT(IV+1)=0
   10 PRINT *,'ENTER NEXT VAR. NUMBER, I2, - TO STOP ENTER 99'
     READ *,IV
     IF(IV.EQ.12) THEN
      PRINT *,'MAX. No. OF VARIABLES EXCEEDED'
      IV=99
     ENDIF
     IF(IV.NE.99) THEN
      ISTAT(IV+1)=0
      GO TO 10
     ENDIF
     PRINT *,' OK - ALL VARIABLES NOW ENTERED'
C
C*** Y AVERAGE
C
     YAVER=0.0
     DO M=1,NAT
      YAVER=YAVER+Y(M,1)
     ENDDO
     YAVER=YAVER/FLOAT(NAT)
C
C<<<<<< REGRESSION PROCESS >>>>>>>>>>>>>>>>>>>>>>>>
C
     NEWVAR=-3
     ITER=0

  999 CONTINUE
     IVV=0
     DO M=1,NV
      IF(ISTAT(M).EQ.0) THEN
       IVV=IVV+1
       ISTATU(IVV)=M-1
      ENDIF
     ENDDO

C*** PRINTING THE MODEL

     WRITE(8,200)
     WRITE(*,201)
```

```
 200 FORMAT(/T05,'REGRESSION MODEL:'//)
 201 FORMAT(/T05,'REGRESSION MODEL:')
   WRITE(8,205) IMOD(1),(IMOD(ISTATU(L)+2),L=1,IVV)
   WRITE(*,205) IMOD(1),(IMOD(ISTATU(L)+2),L=1,IVV)
 205 FORMAT(T02,11A6)

   IF(PEND) GO TO 1111
   ITER=ITER+1
   IF(ITER.GT.20) GO TO 1111
    ALLOCATE (XWORK(NAT,IVV))
   DO N=1,NAT
    DO L=1,IVV
     XWORK(N,L)=X(N,ISTATU(L)+1)
    ENDDO
   ENDDO

    ALLOCATE (XTR(IVV,NAT))
   CALL TRANSP(XWORK,NAT,IVV,XTR)        ! X TRANSPOSE
C
    ALLOCATE (XTRX(IVV,IVV))
   CALL MPROD(XTR,XWORK,IVV,NAT,NAT,IVV,XTRX) ! XTR*X=XTRX
C
    ALLOCATE (WORK(IVV,2*IVV))
    ALLOCATE (IDENT(IVV,IVV))
    ALLOCATE (XTRXI(IVV,IVV))
   CALL INVMAT(XTRX,IVV,IVV,XTRXI,WORK,IDENT) ! INVERSE XTRX
C
    ALLOCATE (XTRY(IVV,1))
   CALL MPROD(XTR,Y,IVV,NAT,NAT,IN,XTRY)   ! XTRANSP*Y
C
    ALLOCATE (B(IVV,1))
   CALL MPROD(XTRXI,XTRY,IVV,IVV,IVV,IN,B)  ! REGRES.COEF.
C
C*** STATISTICS ***
C
   CALL MPROD(XWORK,B,NAT,IVV,IVV,IN,YHAT) ! YHAT = Y ESTIMATED

    ALLOCATE (BTR(1,IVV))
   CALL TRANSP(B,IVV,IN,BTR)          ! B TRANSPOSE
C
   CALL MPROD(BTR,XTRY,IN,IVV,IVV,IN,BTXTRY)
C
   DYAV=0.0
   RESS=0.0
   DO L=1,NAT
    DY(L)=Y(L,1)-YHAT(L,1)            ! RESIDUE
    RESS = RESS + DY(L)*DY(L)           ! RESIDUAL SUM SQUARES
    DYAV=DYAV + DY(L)
   ENDDO
    DYAV=DYAV/NAT
    SYY=0.0
   DO L=1,NAT
    SYY=SYY+(DY(L)-DYAV)**2
   ENDDO
```

```
C
   VAR=RESS/(NAT-IVV)                ! RESIDUAL VARIANCE

C
   DO K=1,IVV
    SB(K)=SQRT(VAR*XTRXI(K,K))        ! ESTIMATED STD ERROR
   ENDDO

   F=(BTXTRY(1,1)-NAT*(YAVER**2))/(VAR*(IVV-1)) ! F VALUE

   R2=F/((NAT-IVV)/(IVV-1)+F)         ! CORRELATION COEF.

C
C *** PRINTING THE SIGNIFICANT PARAMETERS
C
   WRITE(8,*) ' '
   WRITE(8,*) ' '
   DO K=1,IVV
    WRITE(8,210) ISTATU(K),B(K,1),SB(K)
 210 FORMAT(T05,'VARIABLE  X',I2,'  COEF.Bj = ',F10.5,'  STD ERROR',
  * E12.6)
   ENDDO
   WRITE(8,215) R2,F,RESS,VAR
 215 FORMAT(//T05,'CORRELATION COEF. "R2".... = ',F10.6/
  *      T05,'"F" COEFFICIENT.......... = ',E12.6/
  *      T05,'RESIDUAL SUM OF SQUARES... = ',E12.6/
  *      T05,'RESIDUAL VARIANCE......... = ',E12.6//)

C
C<<<<<<  VARIABLE TO BE REJECTED  >>>>>>>>>>>>>>>>>>>>>>>>
C   THE NULL CASE :

   IF(ITER.EQ.1.AND.F.LT.FMIN) THEN
    PRINT *,' ALL B(J)=0 - REGRESSION ABORTED'
    WRITE(8,220)
 220 FORMAT(/T05,'* REGRESSION ABORTED: F LOWER THAN Fmin *')
    GO TO 1111
   ENDIF
   WRITE(8,222)
 222 FORMAT(/T05,'PARTIAL CORRELATION COEFFICIENTS'/)
C
C***  PARTIAL TEST - FP -- VARIABLE TO BE REJECTED
C
   IT=0
   FPMIN=FMIN
   DO J=1,IVV
    FP(J)=(B(J,1)**2)/(SB(J)**2)
     IF(FP(J).LT.FPMIN) THEN
      FPMIN=FP(J)
      IT=ISTATU(J)+1          ! THE LAST WILL BE THE REJECTED
     ENDIF
     WRITE(8,224) ISTATU(J),FP(J)
 224    FORMAT(T05,'FP(',I2,') ........ = ',E10.4)
   ENDDO
```

```
      IF(IT.NE.0) THEN
       IF(IT.EQ.NEWVAR-1) THEN
        PRINT *,' * LAST INTRODUCED VARIABLE WAS REJECTED *'
        WRITE(8,226) IT-1
 226    FORMAT(/T05,'LAST INTRODUCED VARIABLE WAS REJECTED..X',I2/)
        ISTAT(IT)=-2
        GO TO 230
       ELSE
        PRINT *,' ONE VARIABLE REJECTED ',IT-1
        WRITE(8,228) IT-1
 228    FORMAT(//T05,'ONE VARIABLE REJECTED....X',I2/)
        PRINT *,' REPROCESS WITHOUT THE REJECTED VARIABLE ? '
        CALL SREAD(ICHAR)
        ISTAT(IT)=-2            ! RESET STATUS VARIABLE TO REJECT
         IF(ICHAR.EQ.'Y') THEN
          DEALLOCATE(XWORK,XTR,XTRX,XTRXI,XTRY,B,BTR,WORK,IDENT)
          WRITE(8,*) '  NEW REGRESSION WITHOUT REJEC. VARIABLE'
          WRITE(8,*) ' '
          GO TO 999
         ENDIF
       ENDIF
      ELSE
       PRINT *,' NO VARIABLE REJECTED'
       WRITE(8,229)
 229  FORMAT(/T05,'NO VARIABLE REJECTED')
      ENDIF
C
C<<<<< IDENTIFICATION NEW VARIABLE TO INCLUDE IN THE MODEL >>>>>>
C
 230  WRITE(8,231)
 231  FORMAT(//T05,'ANALYSIS OF NEW VARIABLES '/)
      NEWVAR=-3
      RMAX=0.0

      ALLOCATE (XWTY(IVV,1))
      ALLOCATE (BN(IVV,1))
C
      DO L=1,NV
       IF(ISTAT(L).EQ.0.OR.ISTAT(L).EQ.-2) GO TO 1000
       DO J=1,NAT
        XWORK1(J,1)=X(J,L)     ! NEW INDEPENDENT VARIABLE
       ENDDO

C     REGRESSION

      CALL MPROD(XTR,XWORK1,IVV,NAT,NAT,IN,XWTY)
      CALL MPROD(XTRXI,XWTY,IVV,IVV,IVV,IN,BN)  ! NEW COEF.
      CALL MPROD(XWORK,BN,NAT,IVV,IVV,IN,XHAT)  ! NEW ESTIMATE

      ZAV=0.0
      DO I=1,NAT
       Z(I)=XWORK1(I,1)-XHAT(I,1)           ! RESIDUE
       ZAV=Z(I)+ZAV                    ! AVERAGE RESIDUE
      ENDDO
```

```
      ZAV=ZAV/FLOAT(NAT)
      SJY=0.0
      SJJ=0.0
      DO I=1,NAT
       SJY=SJY+(Z(I)-ZAV)*((Y(I,1)-YHAT(I,1))-DYAV)
       SJJ=SJJ+(Z(I)-ZAV)**2
      ENDDO
      RJY=0.0
      IF((SYY*SJJ).NE.0.0) RJY=SJY/SQRT(SYY*SJJ)
      MODRJY=DABS(RJY)
      WRITE(8,232) L-1,MODRJY
  232 FORMAT(T05,'VARIABLE  X',I2,'.... RJY=',E12.6)
      IF(MODRJY.GT.RMAX) THEN
       RMAX=MODRJY
       NEWVAR=L-1              ! IDENTIFY THE CHOSEN VARIABLE
      ENDIF
 1000 CONTINUE
      ENDDO  ! END OF NEW VARIABLE CHOICE - RETURN TO THE LOOP

      IF(NEWVAR.EQ.-3) THEN
       WRITE(8,235)
  235 FORMAT(/T05,'NO MORE VARIABLES TO BE INCLUDED - PROGRAM END'//
     +        T05,'FINAL MODEL:')
       PEND=.TRUE.
       GO TO 999
      ENDIF

      DEALLOCATE(XWORK,XTR,XTRX,XTRXI,XTRY,B,BTR,XWTY,BN,WORK,IDENT)

      IF(ISTAT(IT).EQ.-2)ISTAT(IT)=-1  ! RESET STAT. REJEC. VAR
      ISTAT(NEWVAR+1)=0               ! RESET STATUS NEW VARIABLE
      WRITE(8,240) NEWVAR
  240 FORMAT(/T05,'THE NEW BEST VARIABLE IS: X',I2/)
      DO J=1,NV
       IF(ISTAT(J).EQ.-2) ISTAT(J)=-1
      ENDDO
      GO TO 999            ! TRY A NEW REGRESSION
 1111 CONTINUE
      CLOSE(UNIT=8)
      STOP
      END
C
C>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>
C     Matrices Transposition................................

      SUBROUTINE TRANSP(AM,IR,IC,AMT [REFERENCE])
C
      INTEGER*2 IR,IC
      REAL*8 AM(IR,IC),AMT(IC,IR)

       DO I=1,IC
        DO J=1,IR
         AMT(I,J)=AM(J,I)
        ENDDO
```

```
      ENDDO
      RETURN
      END
C
C>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>
C     Matrices Multiplication..................................

      SUBROUTINE MPROD(A,B,IRA,ICA,IRB,ICB,AB [REFERENCE])

      INTEGER*2 IRA,ICA,IRB,ICB
      REAL*8 A(IRA,ICA),B(IRB,ICB),AB(IRA,ICB)
C
      DO J=1,ICB
       DO I=1,IRA
        AB(I,J)=0.0
         DO K=1,ICA
          AB(I,J)=AB(I,J)+A(I,K)*B(K,J)
         ENDDO
        ENDDO
       ENDDO
       RETURN
       END
C
C>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>
C
      SUBROUTINE INVMAT(A,IAR,IAC,AINV,WORK,IDENT [REFERENCE])
C
C     Matrix inversion - max xx*xx matrix
C
      INTEGER*2 IAR,IAC
      REAL*8 A(IAR,IAC),WORK(IAR,2*IAC),AINV(IAR,IAC),IDENT(IAR,IAC)
      REAL*8 WKDIV,WKMULT
C
C ... N = NUMBER OF ROWS    (I)
C ... M = NUMBER OF COLUMNS (J)
C ... N = M OR CANNOT INVERT THE MATRIX A
C
      N = IAR
      M = IAC
C
C     TO CREATE THE APPROPRIATE IDENTITY MATRIX In=IDENT(N,M)

      DO 20 I=1,N
        DO 10 J=1,M
           IDENT(I,J)=0.0
   10   CONTINUE
           IDENT(I,I)=1.0
   20 CONTINUE

C ... TO ADJOIN THE A AND IDENT MATRICES

      MDASH=2*M
      DO 40 I=1,N
       DO 30 J=1,M
```

```
        WORK(I,J)=A(I,J)
        WORK(I,M+J)=IDENT(I,J)
30    CONTINUE
40  CONTINUE

C ... TO MAKE WORK(1,1)=1.0

    WKDIV=WORK(1,1)

    DO 50 J=1,MDASH
      WORK(1,J)=WORK(1,J)/WKDIV
50  CONTINUE

C ...  TO MAKE ZEROS BELOW DIAGONAL OF LHS OF MATRIX WORK

    DO 90 I=2,N
      DO 70 K=I,N
        WKMULT=WORK(K,I-1)
        DO 60 J=1,MDASH
          WORK(K,J)=WORK(K,J)-(WKMULT*WORK((I-1),J))
60      CONTINUE
70    CONTINUE
        WKDIV=WORK(I,I)
        DO 80 J=I,MDASH
          WORK(I,J)=WORK(I,J)/WKDIV
80      CONTINUE
90  CONTINUE

C ... TO GET THE UPPER LHS TO ZEROS

    DO 130 K=N,2,-1

      DO 120 I=1,K-1
        WKMULT=WORK(I,K)

        DO 110 J=1,MDASH
          WORK(I,J)=WORK(I,J)-(WKMULT*WORK(K,J))
110     CONTINUE
120   CONTINUE
130 CONTINUE

C ... TO EXTRACT INVERSE MATRIX ON RHS AND
C ... MULTIPLY BY ORIGINAL TO SEE ACCURACY OF IDENTITY

    DO 150 I=1,N
      DO 140 J=M+1,MDASH
        AINV(I,J-M)=WORK(I,J)
140   CONTINUE
150 CONTINUE
    RETURN
    END
C>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>
    SUBROUTINE SREAD(CHAR)
    CHARACTER*1 CHAR
```

```
CHAR=' '
DO WHILE ((CHAR.NE.'N').AND.(CHAR.NE.'Y'))
  WRITE(*,'(A)') ' ENTER Y OR N :'
  READ(*,'(A)') CHAR
ENDDO
RETURN
END
```

APPENDIX B - MSR Program Data File

Data generated by ACSL simulation program. The first line indicate the number of sample and the number of independent variables in the file. First column of the second and following lines indicates line, remaining columns are u, w, q, theta, eta and ud (derivative of u).

59 5

1	-0.0157866	0.1340430	0.0046433	1.166E-04	-0.0872665	-0.4538830
2	-0.0452299	0.3823930	0.0091604	4.623E-04	-0.0872665	-0.7222070
3	-0.0878470	0.7392550	0.0135452	0.0010305	-0.0872665	-0.9808160
4	-0.1431500	1.1988200	0.0177922	0.0018145	-0.0872665	-1.2296200
5	-0.2106470	1.7552600	0.0218965	0.0028073	-0.0872665	-1.4685800
6	-0.2898440	2.4027600	0.0258539	0.0040017	-0.0872665	-1.6976400
7	-0.3802470	3.1355400	0.0296605	0.0053902	-0.0872665	-1.9116810
8	-0.4813630	3.9478500	0.0333132	0.0069652	-0.0872665	-2.1261200
9	-0.5926980	4.8339900	0.0368095	0.0087190	-0.0872665	-2.3256200
10	-0.7137640	5.7883100	0.0401472	0.0106436	-0.0872665	-2.5153700
11	-0.8440750	6.8052700	0.0433247	0.0127311	-0.0872665	-2.6954600
12	-0.9831530	7.8793800	0.0463409	0.0149734	-0.0872665	-2.8660200
13	-1.1305200	9.0052600	0.0491952	0.0173625	-0.0872665	-3.0271700
14	-1.2857200	10.177700	0.0518874	0.0198902	-0.0872665	-3.1790800
15	-1.4482800	11.391400	0.0544177	0.0225485	-0.0872665	-3.3219000
16	-1.6177600	12.641500	0.0567867	0.0253293	-0.0872665	-3.4558200
17	-1.7937200	13.922900	0.0589956	0.0282246	-0.0872665	-3.5810400
18	-1.9757200	15.231000	0.0610456	0.0312263	-0.0872665	-3.6977800
19	-2.1633600	16.561200	0.0629386	0.0343265	-0.0872665	-3.8062600
20	-2.3562200	17.908900	0.0646767	0.0375176	-0.0872665	-3.9067100
21	-2.5539000	19.269900	0.0662621	0.0407917	-0.0872665	-3.9993900
22	-2.7560300	20.639900	0.0676976	0.0441413	-0.0872665	-4.0845500
23	-2.9622400	22.015100	0.0689862	0.0475590	-0.0872665	-4.1624500
24	-3.1721600	23.391500	0.0701310	0.0510376	-0.0872665	-4.2333600
25	-3.3854600	24.765500	0.0711354	0.0545698	-0.0872665	-4.2975700
26	-3.6018100	26.133600	0.0720030	0.0581488	-0.0872665	-4.3553400
27	-3.8209000	27.492600	0.0727378	0.0617679	-0.0872665	-4.4069800
28	-4.0424100	28.839100	0.0733436	0.0654205	-0.0872665	-4.4527500
29	-4.2660800	30.170400	0.0738247	0.0691002	-0.0872665	-4.4929600
30	-4.4916200	31.483500	0.0741852	0.0728010	-0.0872665	-4.5279000
31	-4.7187900	32.775900	0.0744297	0.0765168	-0.0872665	-4.5578600
32	-4.9473300	34.045000	0.0745626	0.0802421	-0.0872665	-4.5831200
33	-5.1770300	35.288700	0.0745884	0.0839713	-0.0872665	-4.6039800
34	-5.4076600	36.504700	0.0745119	0.0876992	-0.0872665	-4.6207300
35	-5.6390400	37.691200	0.0743378	0.0914209	-0.0872665	-4.6336600
36	-5.8709700	38.846300	0.0740708	0.0951315	-0.0872665	-4.6430400
37	-6.1032900	39.968400	0.0737156	0.0988265	-0.0872665	-4.6491600
38	-6.3358400	41.056100	0.0732772	0.1025020	-0.0872665	-4.6522900
39	-6.5684700	42.107900	0.0727603	0.1061530	-0.0872665	-4.6527000
40	-6.8010700	43.122900	0.0721696	0.1097760	-0.0872665	-4.6506600
41	-7.0335000	44.099900	0.0715099	0.1133690	-0.0872665	-4.6464300
42	-7.2656800	45.038000	0.0707860	0.1169260	-0.0872665	-4.6402500

43	-7.4975000	45.936600	0.0700024	0.1204460	-0.0872665	-4.6323800
44	-7.7288900	46.795100	0.0691639	0.1239260	-0.0872665	-4.6230600
45	-7.9597800	47.612900	0.0682750	0.1273620	-0.0872665	-4.6125100
46	-8.1901200	48.389800	0.0673400	0.1307530	-0.0872665	-4.6009600
47	-8.4198700	49.125400	0.0663636	0.1340950	-0.0872665	-4.5886300
48	-8.6489800	49.819800	0.0653498	0.1373880	-0.0872665	-4.5757400
49	-8.8774300	50.472900	0.0643029	0.1406300	-0.0872665	-4.5624700
50	-9.1052200	51.084800	0.0632271	0.1438180	-0.0872665	-4.5490200
51	-9.3323400	51.655800	0.0621263	0.1469520	-0.0872665	-4.5355800
52	-9.5587800	52.186100	0.0610043	0.1500300	-0.0872665	-4.5223200
53	-9.7845700	52.676200	0.0598650	0.1530520	-0.0872665	-4.5094100
54	-10.009700	53.126600	0.0587120	0.1560170	-0.0872665	-4.4970100
55	-10.234300	53.537700	0.0575486	0.1589230	-0.0872665	-4.4852700
56	-10.458300	53.910300	0.0563784	0.1617710	-0.0872665	-4.4743200
57	-10.681700	54.245000	0.0552045	0.1645610	-0.0872665	-4.4643100
58	-10.904700	54.542700	0.0540300	0.1672920	-0.0872665	-4.4553500
59	-11.127300	54.804200	0.0528579	0.1699640	-1.650E-05	-4.2716300

APPENDIX C - MSR Program Printout

Regression of the B-747data generated by simulation in a ACSL simulation program. The simulation corresponds to airplane response to a 3 seconds elevator step of 5 degrees, at 20000 ft pressure altitude and Mach 0.5

REGRESSION MODEL:

Y = B1*X1+B2*X2+B3*X3

VARIABLE X 1 COEF.Bj = .43134 STD ERROR .190278E-01
VARIABLE X 2 COEF.Bj = .06765 STD ERROR .398107E-02
VARIABLE X 3 COEF.Bj = -63.96062 STD ERROR .536269E+00

CORRELATION COEF. "R2".... = .997818
"F" COEFFICIENT........ = .128035E+05
RESIDUAL SUM OF SQUARES... = .172136E+00
RESIDUAL VARIANCE......... = .307385E-02

PARTIAL CORRELATION COEFFICIENTS

FP(1) .... = .5139E+03
FP(2) .... = .2888E+03
FP(3) .... = .1423E+05

NO VARIABLE REJECTED

ANALYSIS OF NEW VARIABLES

VARIABLE X 0.... RJY= .893785E+00
VARIABLE X 4.... RJY= .781431E+00
VARIABLE X 5.... RJY= .992648E+00

THE NEW BEST VARIABLE IS: X 5

REGRESSION MODEL:

Y = B1*X1+B2*X2+B3*X3+B5*X5

VARIABLE X 1 COEF.Bj = .35810 STD ERROR .256260E-02
VARIABLE X 2 COEF.Bj = .05120 STD ERROR .544707E-03
VARIABLE X 3 COEF.Bj = -58.47990 STD ERROR .109054E+00
VARIABLE X 5 COEF.Bj = 2.29053 STD ERROR .368632E-01

CORRELATION COEF. "R2".... = .999970
"F" COEFFICIENT........ = .598157E+06
RESIDUAL SUM OF SQUARES... = .241771E-02
RESIDUAL VARIANCE......... = .439583E-04

PARTIAL CORRELATION COEFFICIENTS

FP(1) .... = .1953E+05
FP(2) .... = .8837E+04
FP(3) .... = .2876E+06
FP(5) .... = .3861E+04

NO VARIABLE REJECTED

ANALYSIS OF NEW VARIABLES

VARIABLE X 0.... RJY= .526633E-01
VARIABLE X 4.... RJY= .100000E+01

THE NEW BEST VARIABLE IS: X 4

REGRESSION MODEL:

Y = B1*X1+B2*X2+B3*X3+B4*X4+B5*X5

VARIABLE X 1 COEF.Bj = -.00163 STD ERROR .443470E-04
VARIABLE X 2 COEF.Bj = .08008 STD ERROR .358933E-05
VARIABLE X 3 COEF.Bj = -61.36828 STD ERROR .369251E-03
VARIABLE X 4 COEF.Bj = -31.97526 STD ERROR .393633E-02
VARIABLE X 5 COEF.Bj = 2.01638 STD ERROR .476624E-04

CORRELATION COEF. "R2".... = 1.000000
"F" COEFFICIENT........ = .538236E+12
RESIDUAL SUM OF SQUARES... = .197857E-08
RESIDUAL VARIANCE......... = .366402E-10

PARTIAL CORRELATION COEFFICIENTS

FP(1) .... = .1350E+04
FP(2) .... = .4978E+09
FP(3) .... = .2762E+11
FP(4) .... = .6599E+08
FP(5) .... = .1790E+10

NO VARIABLE REJECTED

ANALYSIS OF NEW VARIABLES

VARIABLE X 0.... RJY= .797162E-01

THE NEW BEST VARIABLE IS: X 0

REGRESSION MODEL:

Y = B0 + B1*X1+B2*X2+B3*X3+B4*X4+B5*X5

VARIABLE X 0 COEF.Bj = .00000 STD ERROR .813167E-05
VARIABLE X 1 COEF.Bj = -.00163 STD ERROR .446248E-04
VARIABLE X 2 COEF.Bj = .08008 STD ERROR .365038E-05
VARIABLE X 3 COEF.Bj = -61.36833 STD ERROR .381736E-03
VARIABLE X 4 COEF.Bj = -31.97538 STD ERROR .396610E-02
VARIABLE X 5 COEF.Bj = 2.01642 STD ERROR .840890E-04

CORRELATION COEF. "R2".... = 1.000000
"F" COEFFICIENT........ = .425344E+12
RESIDUAL SUM OF SQUARES... = .196588E-08
RESIDUAL VARIANCE......... = .370920E-10

PARTIAL CORRELATION COEFFICIENTS

FP(0) .... = .3422E+00
FP(1) .... = .1334E+04
FP(2) .... = .4813E+09
FP(3) .... = .2584E+11
FP(4) .... = .6500E+08
FP(5) .... = .5750E+09

ONE VARIABLE REJECTED....X 0

NEW REGRESSION WITHOUT REJEC. VARIABLE

REGRESSION MODEL:

Y = B1*X1+B2*X2+B3*X3+B4*X4+B5*X5

VARIABLE X 1 COEF.Bj = -.00163 STD ERROR .443470E-04
VARIABLE X 2 COEF.Bj = .08008 STD ERROR .358933E-05
VARIABLE X 3 COEF.Bj = -61.36828 STD ERROR .369251E-03
VARIABLE X 4 COEF.Bj = -31.97526 STD ERROR .393633E-02
VARIABLE X 5 COEF.Bj = 2.01638 STD ERROR .476624E-04

CORRELATION COEF. "R2".... = 1.000000
"F" COEFFICIENT........ = .538236E+12
RESIDUAL SUM OF SQUARES... = .197857E-08
RESIDUAL VARIANCE......... = .366402E-10

PARTIAL CORRELATION COEFFICIENTS

FP(1) .... = .1350E+04
FP(2) .... = .4978E+09
FP(3) .... = .2762E+11
FP(4) .... = .6599E+08
FP(5) .... = .1790E+10

NO VARIABLE REJECTED

ANALYSIS OF NEW VARIABLES

NO MORE VARIABLES TO BE INCLUDED - PROGRAM END

FINAL REGRESSION MODEL:

Y = B1*X1+B2*X2+B3*X3+B4*X4+B5*X5