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CRANFIELD

COMPOUNDED NORMAL MODES OF FREE VIBRATION OF CANTILEVER PLATES

by

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Compounded normal modes of free vibration of cantilever plates

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S U M M A R Y

Experimental determination of the natural frequencies of a stiffened cantilever plate reveals the simultaneous excitation of two normal modes of vibration.

Comparison is made between the experimental nodal pattern and the nodal pattern calculated by assuming characteristic beam functions for the vibration form of the plate.

Contents

Notation

a	length of plate parallel to the x-axis.
b	length of plate parallel to the y-axis.
m,n	number of nodal lines parallel to the x and y directions respectively.
x,y	co-ordinate distances in plane of plate.
$W(x,y)$	deflection of plate normal to xy plane.
$\theta(x)$, $\phi(y)$	characteristic beam functions in the x and y directions respectively.

Introduction

In the vibration of isotropic rectangular plates, the phenomenon of the simultaneous excitation of two separate modes having equal frequencies is well known. Waller[1] observed complex nodal patterns on isotropic rectangular plates, and she stated that such combinations were possible because of the internal damping of the plates, which reduced the sharpness of resonance.

Warburton[2] considered the existence of modes $m/n \pm n/m$ for a clamped plate and showed that when a plate is square, for the mode $4/2 \pm 2/4$, the nodal lines do not lie parallel to the edges of the plate. Furthermore, it was proved that for a square clamped plate, the modes $m/n \pm n/m$ have discrete frequencies.

Hoppmann[3], investigating the vibration characteristics of an orthogonally stiffened square plate with simply-supported edges, observed two normal modes having almost equal frequencies. He stated that the reason for this was that the nodal patterns of the stiffened plate were the same as those of the equivalent isotropic plate, but that the different elastic compliances, D_x, D_y and H enabled the stiffened plate to have two modes with almost identical frequencies.

It is important in the resonance testing of stiffened plate structures to recognise the existence of compounded normal modes and to be able to determine their component modes.

In the theoretical determination of the natural frequencies of isotropic rectangular plates and stiffened rectangular plates, the use of assumed vibration forms in the energy solutions leads to approximate frequency expressions[2][4]. One check on the validity of such vibration forms is a comparison of experimental nodal patterns with those nodal patterns calculated from the assumed vibration form.

Compounded mode of vibration of a stiffened cantilever plate

During an experimental investigation of the natural frequencies of the stiffened cantilever plate illustrated in figure 1, it was observed that the mode $m/n = 1/4$ was being excited at 144 c.p.s. and the mode $m/n = 2/0$ was being excited at 148 c.p.s. The latter mode was excited by reducing the exciting force to a small value. At 148 c.p.s., when the exciting force was increased the two modes were then excited simultaneously. The stiffeners were attached to the plate by small nuts and bolts which permitted sliding to occur between the stiffeners and the plate, thus creating damping and reducing the sharpness of resonance.

Careful adjustment of the frequency and the magnitude of the exciting force was required to produce the combined nodal pattern. Figures (2A), (2B) and (2C) illustrate the separate and combined modes respectively.

It must be stated that the three modes were excited with the plate clamped at $x = 0$ but with the stiffeners unclamped at $x = 0$. When the stiffeners were clamped at $x = 0$, the compounded nodal pattern could not be produced. This does not however preclude the possibility of exciting such nodes in stiffened plates when both the stiffeners and the plate are clamped.

Figure 2B indicates that the mean nodal line is approximately 0.26 (length of plate) from $x = a$, which is in about the same position as the node for a hinged-free beam in the second mode of vibration. This indicates that the unclamped stiffeners are exerting some 'hinged-free beam' action on the cantilever plate. This becomes further apparent when it is realised that the nodal line for the second bending mode of an isotropic cantilever plate is a curve as shown in figure 3.

Calculation of compounded nodal pattern

The approximate deflection form used in reference (4), to obtain the natural frequencies of a stiffened cantilever plate is

$$W(x,y) = \theta(x)\phi(y), \text{ for stiffeners parallel to the x-axis,}$$

where $\theta(x)$ and $\phi(y)$ are characteristic beam functions in the x and y directions respectively. The characteristic beam functionsare given in references (2) and (5).

a) Mode $m = 2,\ n = 0$

$$W(x,y) = \theta_2(x)\phi_0(y),$$

where $\phi_0(y) = 1$ for the rigid body mode of the free-free beam, and for the second cantilever mode

$$\theta_2(x) = (\cos\gamma\frac{x}{a} - \cosh\gamma\frac{x}{a}) + k_2(\sin\gamma\frac{x}{a} - \sinh\gamma\frac{x}{a}).$$

For $m = 2$, $\gamma = 1.494\pi$ rad. and $k_2 = \dfrac{\sin\gamma - \sinh\gamma}{\cos\gamma + \cosh\gamma} = -1.0184$,

hence

$$\theta_2(x) = \left[\cos 4.7(\frac{x}{a}) - \cosh 4.7(\frac{x}{a})\right] - 1.0184\left[\sin 4.7(\frac{x}{a}) - \sinh 4.7(\frac{x}{a})\right] \quad (1)$$

b) Mode $m = 1,\ n = 4$

$$W(x,y) = \theta_1(x)\phi_4(y),$$

where for the first cantilever mode

$$\theta_1(x) = \left[\cos\left\{1.875(\tfrac{x}{a})\right\} - \cosh\left\{1.875(\tfrac{x}{a})\right\}\right] - 0.734\left[\sin\left\{1.875(\tfrac{x}{a})\right\} - \sinh\left\{1.875(\tfrac{x}{a})\right\}\right] \quad (2)$$

The free-free beam mode is given by

$$\phi(y) = \cos\epsilon(\tfrac{y}{b} - \tfrac{1}{2}) + C\cosh\epsilon(\tfrac{y}{b} - \tfrac{1}{2}) \text{ for } n = 2, 4, 6 \ldots ,$$

where $C = -\frac{\sin\epsilon/2}{\sinh\epsilon/2}$. For $n = 4$, $\epsilon = 3.5\pi$, hence

$$\phi_4(y) = \left[\cos\left\{11.0(\tfrac{y}{b} - \tfrac{1}{2})\right\} - 0.00575\cosh\left\{11.0(\tfrac{y}{b} - \tfrac{1}{2})\right\}\right] \quad (3)$$

The plate was excited by a vibrator connected directly to it and it is assumed that both the modes $m/n = 1/4$ and $m/n = 2/0$ were excited with equal amplitudes at point B in figure 4. Consequently in the following calculations $\phi_4(y)$ is normalised so that it has unit amplitude at $y/b = \frac{1}{2}$.

The equation of the compound nodal curve is

$$W(x,y) = \theta_2(x) + \theta_1(x)\phi_4(y) = 0 \quad (4)$$

or $$\phi_4(y) = -\frac{\theta_2(x)}{\theta_1(x)} \quad (5)$$

Equation (5) may be solved numerically for y in terms of x.

Alternative method

Instead of assuming that $\theta(x)$ is the vibration form of a clamped-free beam, if the hinged-free beam vibration form is assumed, then for $m = 1$, measuring x from the pinned end,

$$\theta_1(x) = (\tfrac{x}{a}) \quad (6)$$

and for $m = 2$

$$\theta_2(x) = \sin(\tfrac{\gamma x}{2a}) + k\sinh(\tfrac{\gamma x}{2a}), \quad (7)$$

where for $m = 2$, $\gamma/2 = 3.927$ rad. and $k = \frac{\sin\gamma/2}{\sinh\gamma/2} = -0.028$.

The maximum value of $\theta_2(x)$ when x = a is, from equation (7),

$$\theta_2(x) = 2\sin\gamma/2 = 1.412.$$

Thus in equation (5), $\theta_1(x)$ must be increased in the ratio 1.412 to 1.00, when using equation (6).

Comparison of compound nodal patterns

Figure 4 illustrates the observed and calculated nodal patterns.

Points A and B do not coincide because the assumption of equal amplitudes at point B is not completely justified. The remaining differences between the nodal patterns are due to (a) lack of uniformity in plate thickness, and (b) variation in $\phi_4(y)$ caused by the two central nodal lines in figure 2A not being parallel to the sides of the plate as assumed in equation (4).

References

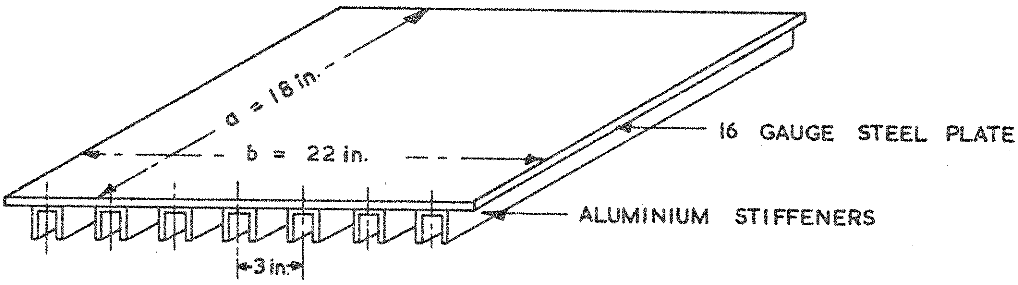
1. Waller, M.D. — Vibration of Free Square Plates. Part II Compounded Normal Modes. Proceedings of the Physical Society of London. Vol. 52, p.452, 1940.

2. Warburton, G.B. — The Vibration of Rectangular Plates. Proceedings of the I.Mech.E. (London) Vol. 168, No. 12. 1954.

3. Hoppmann, W.H., Magness, L.S. — Nodal Patterns of the Free Flexural Vibration of Stiffened Plates, Journal of Applied Mechanics, Vol. 24, pp. 526-530. 1957.

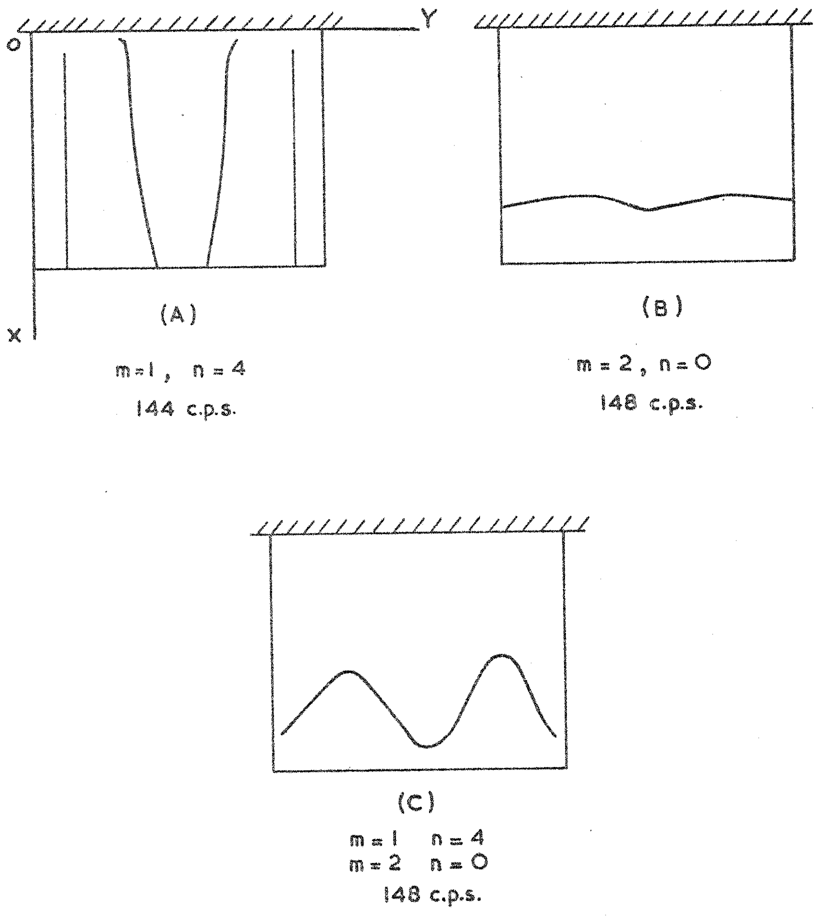
4. Kirk, C.L. — Vibration Characteristics of Stiffened Plates. Ph.D. Thesis, Sheffield University, 1959.

5. Bishop, R.E.D., Johnson, D.C. — The Mechanics of Vibration. Cambridge University Press. 1960.



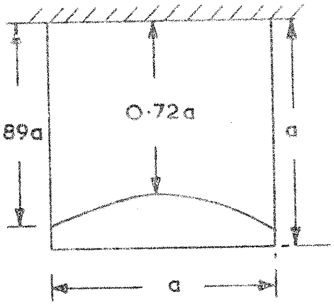
STIFFENED CANTILEVER PLATE

FIG. 1



EXPERIMENTAL NODAL PATTERNS
OF STIFFENED CANTILEVER PLATE.

FIG. 2



THEORETICAL NODAL PATTERN
OF UNSTIFFENED SQUARE CANTI-
LEVER PLATE.

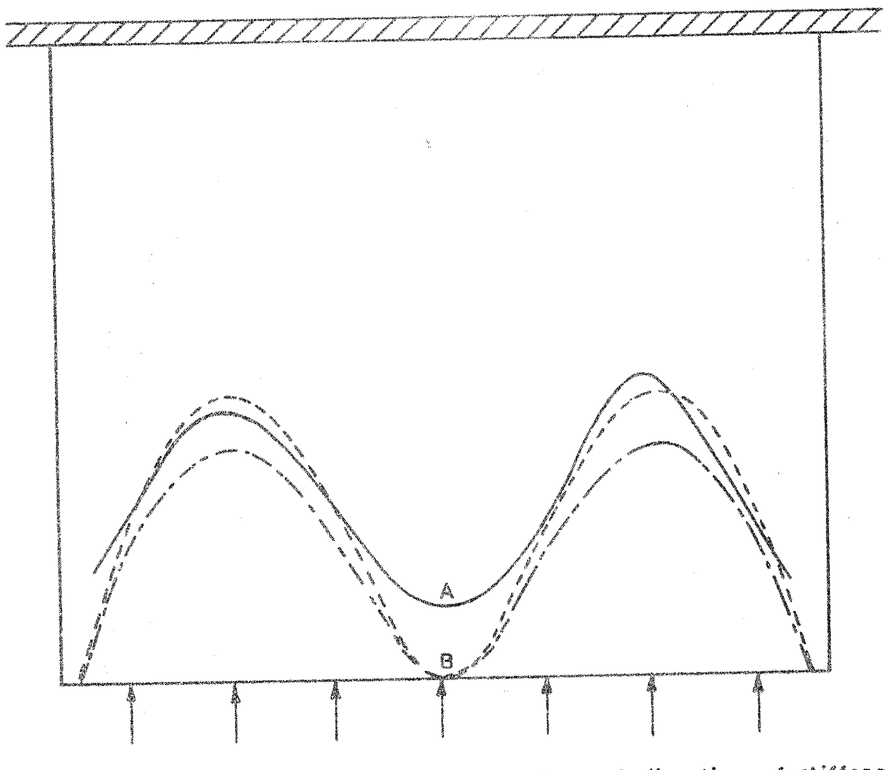
FIG. 3

Reference :

———— Observed nodal pattern for clamped cantilever plate.

- - - - - - - Calculated nodal pattern, assuming cantilever to be hinged at x = o

—·—·— Calculated nodal pattern, assuming cantilever to be clamped at x = o



The seven vertical arrows denote location and direction of stiffeners.

NODAL PATTERN OF TWO DIFFERENT MODES OF
VIBRATION EXCITED SIMULTANEOUSLY.

FIG. 4