CoA Note No. 152

THE COLLEGE OF AERONAUTICS

CRANFIELD

THE DESIGN OF A HYDROSTATIC LUBRICATION ARRANGEMENT FOR THE HORIZONTAL AND TRANSVERSE SLIDES OF A No. 3 ARCHDALE VERTICAL MILLING MACHINE

by

J. Loxham and J. Hemp

NOTE NO. 152

May, 1963

THE COLLEGE OF AERONAUTICS

CRANFIELD

The design of a hydrostatic lubrication arrangement for the horizontal and transverse slides of a No. 3 Archdale vertical milling machine.

- by -

J. Loxham, C.G.I.A., M.I.Mech.E., M.I.Prod.E., F.B.I.M.

and

J. Hemp

SUMMARY

The Department of Production and Industrial Administration of The College of Aeronautics has made a detailed analysis into the performance of numerically controlled machine tools, and this shows that when these machines are operating under light cutting loads, the total envelope tolerance is about 0.003 ins. Further investigation into the source of the errors showed that about 80 per cent of this error was due to mechanical mechanisms in the machine tool. The electronic equipment operated inside a total band of 0.0004 ins. over small distances, and 0.0006 ins. over 10 inches. This situation appeared to justify a comprehensive survey into the methods which may be used to improve the mechanical performance of these machines.

After due consideration had been given to a number of possible alternatives, it was decided to fit hydrostatic lubrication to the slides, nut and screw arrangement, and the thrust bearings used on the X and Y axes of an existing No. 3 Archdale Milling Machine. The Z axis was not included in the redesign, because analysis showed that a new head casting would have been necessary. It was thought that this additional expense was not justified and that the data obtained from an examination of the performance given by the X and Y axes would show whether the system could be used with advantage in the design of new machines. The paper gives details of the calculations used to establish the dimensions of the new features fitted to the machine to satisfy the target performance conditions which appeared to be possible and are given in the introduction.

The authors would like to express their appreciation to James Archdale and Co. for making a machine available for this investigation, to the Staveley Research Department, and in particular to Mr. Graham, Mr. Harris, Mr. Huntley, Mr. Cummings and Mr. Hayward.

CONTENTS

Introduction

The following elements of the machine are examined in this memorandum:

(i) The horizontal table slide.
(ii) The transverse table slide.
(iii) The thrust bearings for the lead screws on horizontal and transverse slides.
(iv) The screw and nut arrangement for the lead screws on horizontal and transverse slides.

1. The purpose of the experiment, of which the above forms a part, is to endeavour to provide a machine on which the mechanical performance on two horizontal slides on a programme-controlled vertical milling machine shall be equal to, or better, in response characteristics and accuracy than, the electronic control equipment now available.

If the above can be achieved, it appears that the following performance specification is attainable.

With the knee of the machine and the vertical head clamped and a maximum cutting load of 100 lbs. on an end mill type of cutter of suitable design operating at near optimum conditions for speed, feed, etc., the accuracy of a part machined in low carbon free cutting steel, brass or aluminium alloy shall not depart from the prescribed profile by more than the algebraic sum

(a) plus and minus 0.0001 inch
(b) plus and minus 0.0001 inch for each part of 5 inches moved from datum in a horizontal plane.

2. In this particular investigation the Stavelly three dimensional numerical control system will be used. The hydrostatic lubrication which will be applied to the horizontal and transverse slides of the machine, and the specially designed hydraulic drive with its associated hydrostatically lubricated screw and nut arrangement, are expected to play an important part in attaining the desired result.

3. From a preliminary survey of an existing No. 3 Archdale vertical milling machine, it appeared that, in its modified form, the horizontal and transverse slideways would be as shown in Fig. 1A and Fig. 1B, with hydrostatic lubrication provided at the positions marked thus //////. The maximum loads for which the slides are designed are shown in Figs. 1A, 1B, 2, 3.

4. One of the known, and in some cases serious, disadvantages of hydrostatic lubrication is the heat generated in the slide when the oil gives up its potential energy (as if it fell from a tank to give it pressure) when it passes through the restrictions incorporated in

the hydrostatic slide. In a typical case (and in this actual example), it was decided to use a supply pressure of 500 lbs. per sq. inch and a cell pressure of 250 lbs. per sq. inch when the slides are in their central position. By this arrangement, the oil will give up one half of its potential energy in the form of heat as it passes through the restriction between the main supply pipe and the cell, the remaining half being given up as it passes from the cell to the atmosphere.

In order to keep the distortion due to the changes in temperature of the slides to the lowest possible value, it was decided to

(a) make the total heat input as small as possible
(b) use a heat exchanger designed to keep the temperature of the oil in the cell at ambient temperature.

To satisfy condition (a), it is necessary to consider the quantity of oil which flows through the narrow slot between two parallel plates, which in this instance are the walls of the cell and the flat surface of the slide. This is given by the formula

$$Q = \frac{PLh^3}{12\mu W}$$

where P is the difference in the pressure of the oil at inlet and outlet; h the gap between the pad and slide; μ the viscosity of oil; L the length, and W the width of slot.

One important feature shown by the above equation is that the quantity of oil flowing, and therefore the heat given up by the oil in passing through the bearing, varies as the cube of h. If h = 0.0010" the flow of oil will be in proportion to $10 \times 10 \times 10 = 1000$. If h is reduced to 0.0003", the flow will be in proportion to $3 \times 3 \times 3 = 27$. This much smaller flow transfers a much smaller amount of heat to the machine slides and for this reason it was decided that an attempt should be made to make the mean gap 0.0003". This value was selected because it was thought that by using suitable measuring techniques and skillful workmanship it would be possible to make the mating surfaces of the slides flat and parallel to within a total envelope tolerance of 0.0001".

To satisfy condition (b), it will be necessary to arrange that the oil in the main high pressure supply line be controlled at ambient temperature $-\frac{1}{2}t$ degrees centigrade, and the temperature of the oil in the exhaust pipe line be at ambient temperature $+\frac{1}{2}t$ degrees centigrade, where t is the rise in temperature in degrees centigrade of the oil due to a fall in pressure of 500 lbs. per sq. inch.

5. To enable the slides to carry high loads under operating conditions, it was decided that

(i) when the sliding elements are carrying a vertical load consisting of the weight of the slides plus 30% of the

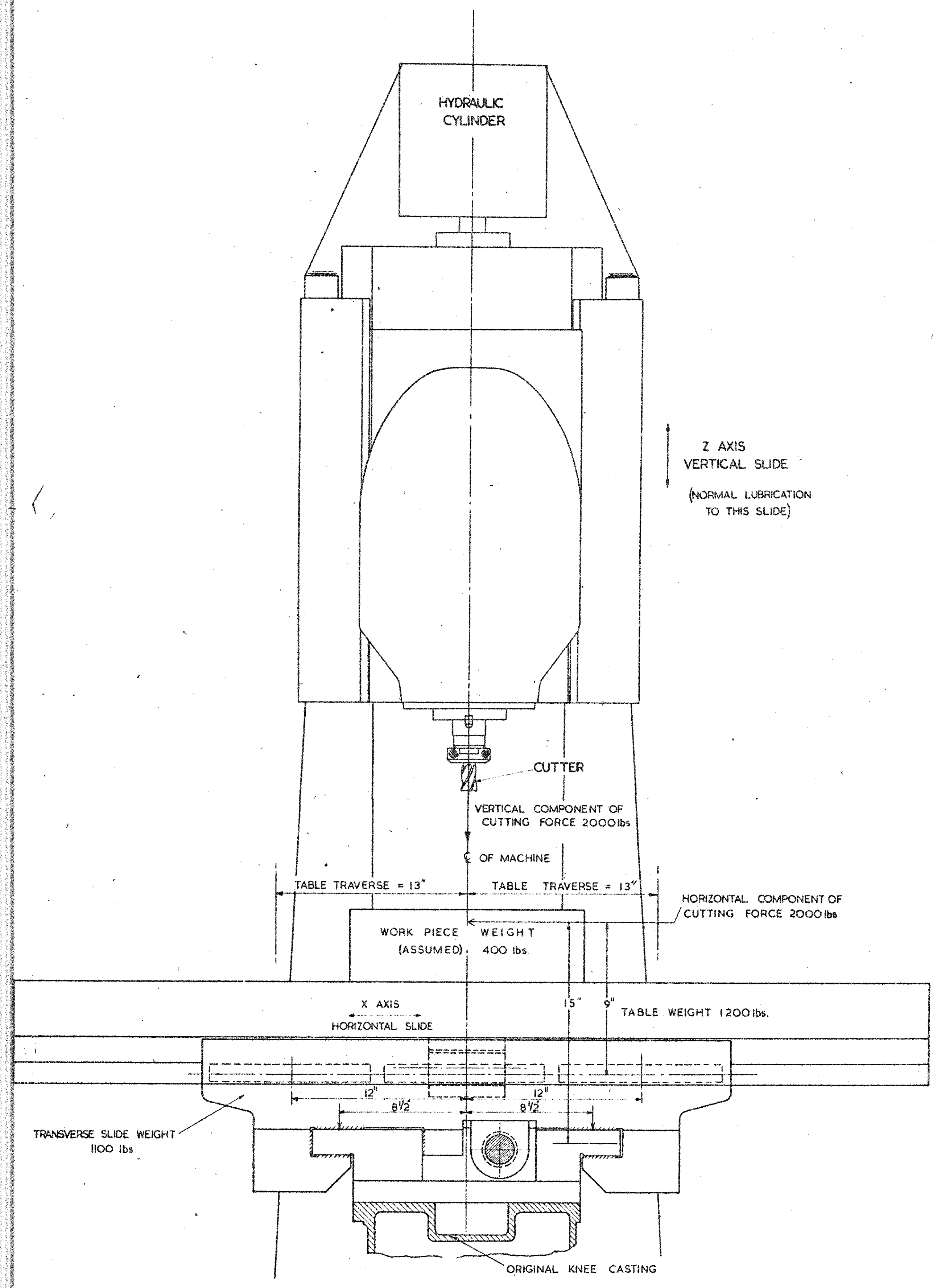


FIG. 1A. FRONT ELEVATION OF ARCHDALE No. 3 VERTICAL MILLING MACHINE MODIFIED BY FITTING HYDROSTATICLY LUBRICATED SLIDES TO X AND Y AXES.

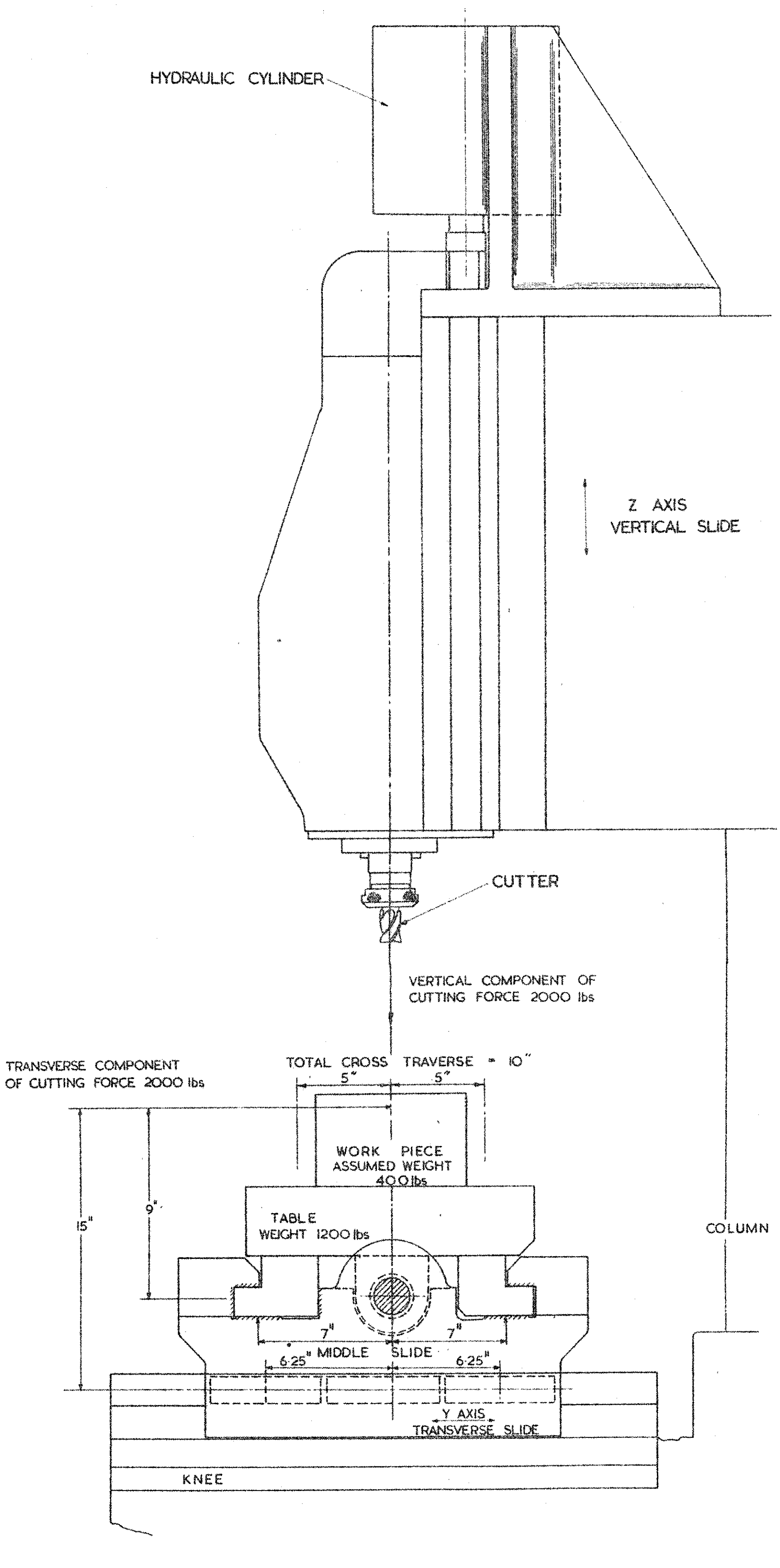


FIG. 1 B. SIDE ELEVATION OF MACHINE MODIFIED FOR HYDROSTATIC SLIDES

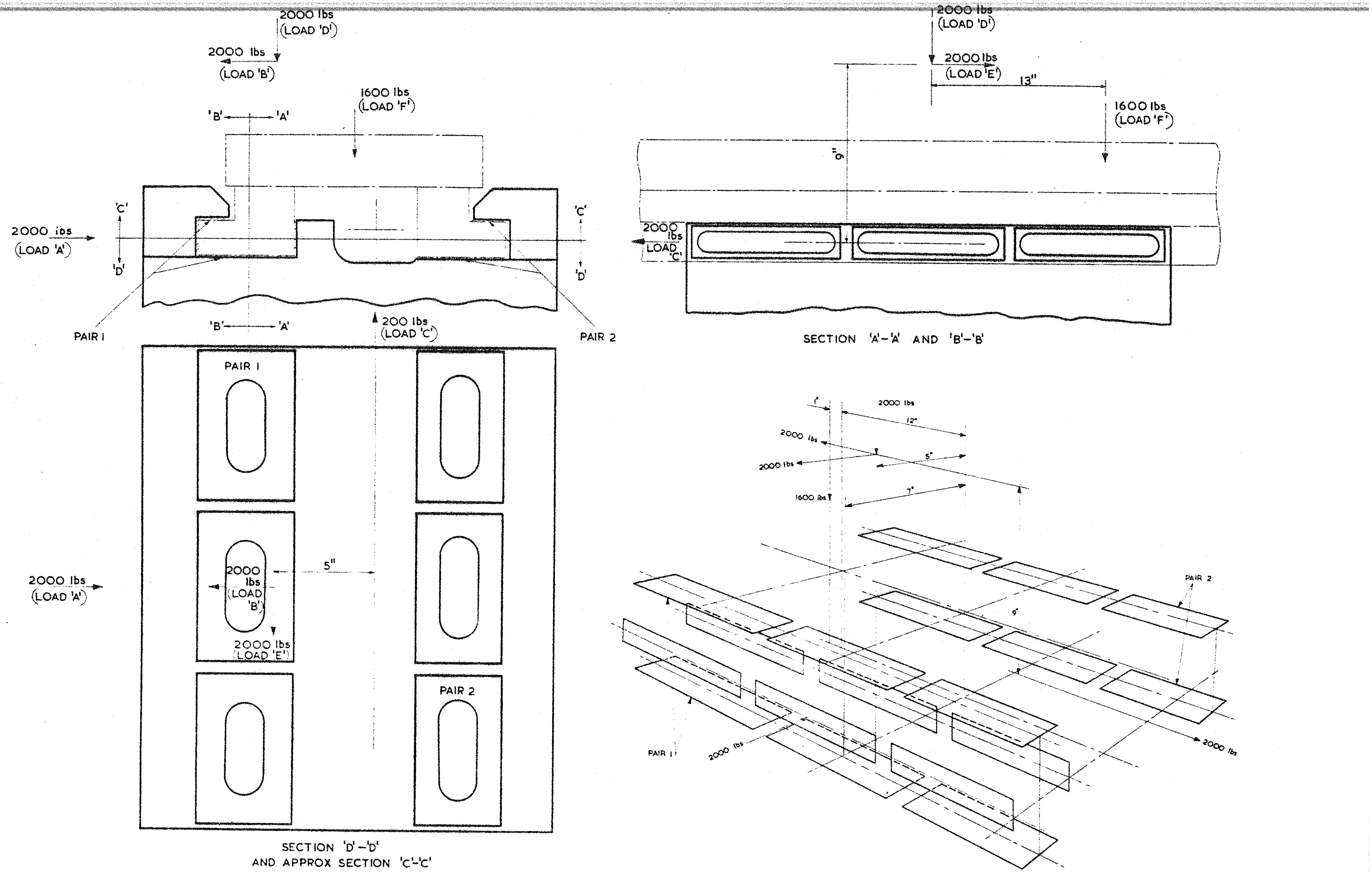


FIG. 2 REPRESENTATION OF THE THRUST EXERTED ON THE HORIZONTAL SLIDE AND DISPOSITION OF THE HORIZONTAL SLIDE PADS.

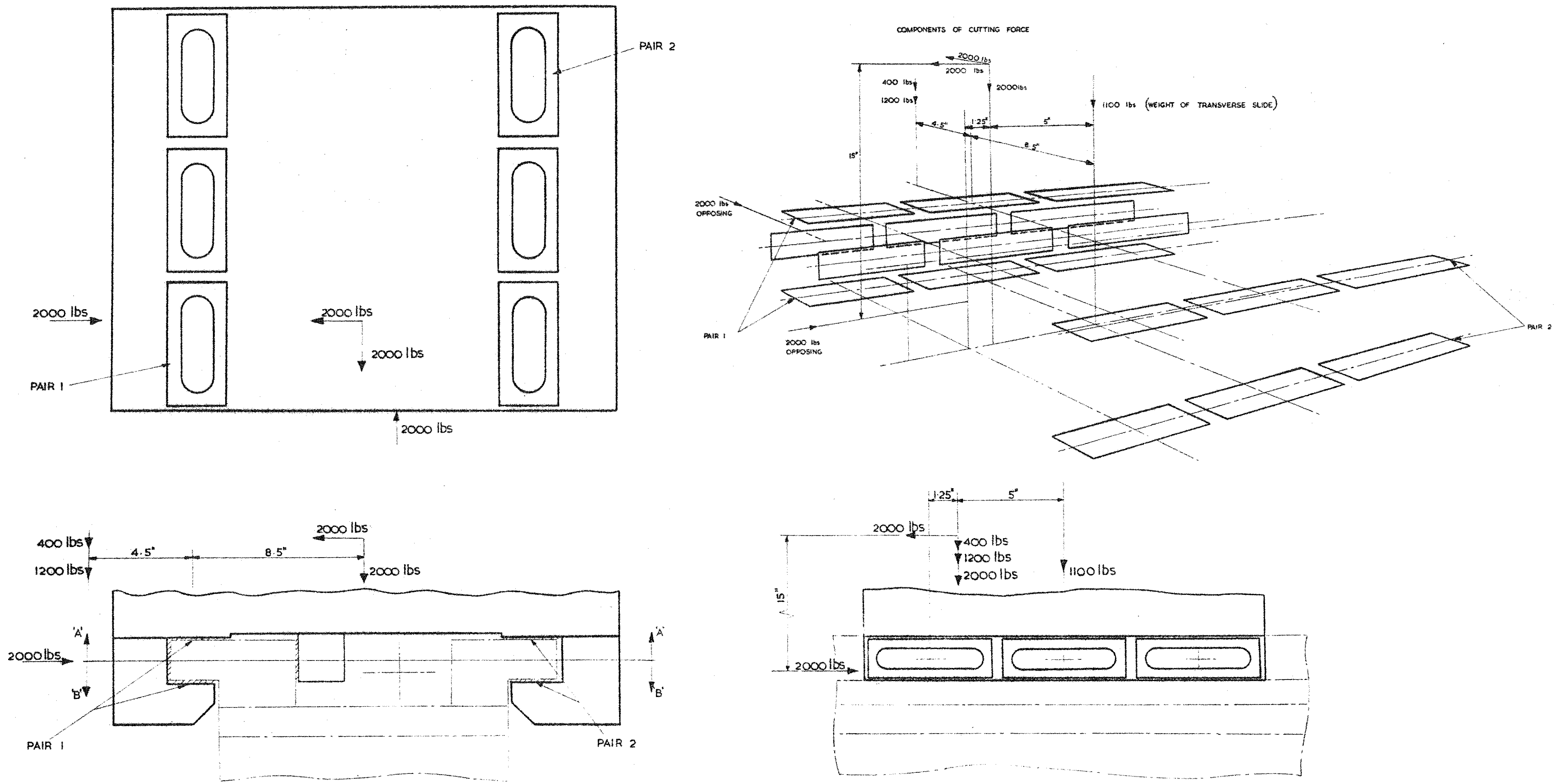


FIG. 3. REPRESENTATION OF THE THRUSTS EXERTED ON THE TRANSVERSE SLIDE AND TRANSVERSE SLIDE PADS.

maximum vertical load for which the slides are designed, the mean gap on each side of the slide shall be 0.0003".

(ii) under the most unfavourable conditions with the maximum load applied and with the maximum possible offset of the horizontal and transverse slides, the mean gap of 0.0003" between the slide and the walls of the cell shall not at any place be reduced to less than 40% of its value at the central position. This is illustrated in Fig.4 and is provided if the displacement from the central position does not exceed 60% of the mean gap.

(iii) twelve pressure cells per slide in four sets of three would be used. In this arrangement, six pressure cells would be placed above the plane members and six below.

6. If, because of exceptional circumstances, it is required to increase the cutting loads to values in excess of those shown in Figs. 1A, 1B, 2, 3, the supply pressure shall be increased from 500 lbs. per sq. inch to a value that will ensure that the displacements quoted in 5 above shall not be exceeded.

7. The symbols and values to which they refer, and units in which they are expressed, are as follows:

Symbol	Meaning of Symbol	Units
A_1	Effective area of pad 1 of a pair	sq. ins.
A_2	Effective area of pad 2 of a pair	sq. ins.
L_1	Length of capillary for pad 1	ins.
L_2	Length of capillary for pad 2	ins.
Q_1	Flow through pad 1 of a pair	cu.ins./sec.
Q_2	Flow through pad 2 of a pair	cu.ins./sec.
R_1	Resistance of pad 1 of a pair	lb.sec./in^5
R_2	Resistance of pad 2 of a pair	lb.sec./in^5
μ	Viscosity of oil	reyns.

Part 1

The design of the horizontal slideways

(i) If the horizontal slide is to move centrally between its supporting pads (shown in Fig.5) during normal working conditions (which are those stated in decision (i) of part 5 of the introduction), those pads must then be providing an upthrust of

$$1,200 + 400 + 30\% \text{ of } 2,000 = 2,200 \text{ lbs.}$$

Since the weight of the horizontal slide is 1,200 lbs., the weight of the workpiece might be 400 lbs. and the greatest vertical component of cutting force is 2,000 lbs.

Now there are six opposing pairs to provide this upthrust, and those pairs are equal to each other. Hence the upthrust required of one pair is one sixth of 2,200 lbs., which is 367 lbs.

(ii) The cell pressures in pads 1 and 2 in Fig. 5 will be 250 p.s.i. (part 3 of the introduction), so to provide an upthrust at the mean gap of 0.0003 we must make the effective area of pad 1 greater than that of pad 2, and if A_1 and A_2 are those effective areas, we must make

$$250\ A_1 - 250\ A_2 = 367 \text{ lbs.}$$

That is the difference in the effective areas must be about 1.5 sq.ins.

(iii) However, four of the six supporting pairs may have to provide much greater upthrusts when the cutting forces are not central. None, however, will have to provide upthrusts greater than that provided by pair 1 in Fig. 2, which must support one half of the weight of the horizontal slide and work piece, nearly one third of the vertical component of the greatest cutting force and (across the width and length of the horizontal slide) the couples arising from the horizontal components and their opposing forces from the guideways and screw. This upthrust is

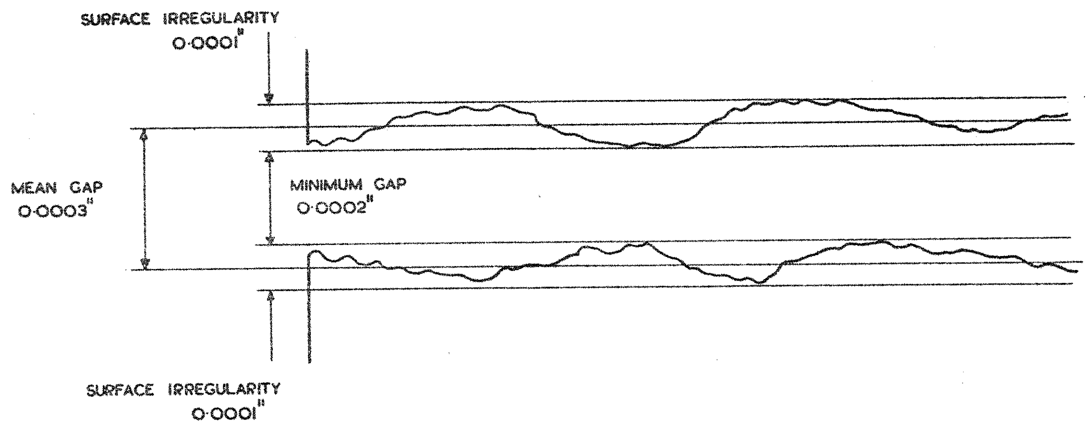
$$\frac{1}{2} \times (1200 + 400) + \frac{1}{3} \times 2000 + \frac{1}{3} \times \frac{9}{14} \times 2000 + \frac{1}{2} \times \frac{9}{24} \times 2000$$

$$= 3,300 \text{ lbs.}$$

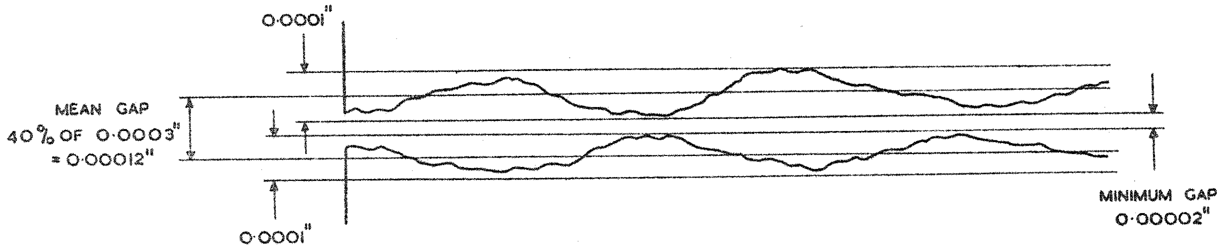
So, by Appendix 3, because more load must be taken with gaps closing as much as 60% of 0.0003 , we must make

$$3,300 < .9398 \times A_1 \times 500 - .1962 \times A_2 \times 500$$

and because $A_2 = A_1 - 1.5$



DIAGRAMMATIC REPRESENTATION OF MEAN GAP OF 0·0003"
WITH SLIDE IN CENTRAL POSITION.



DIAGRAMMATIC REPRESENTATION OF MINIMUM DESIGN GAP WHERE MEAN GAP
OF 0·0003" IS REDUCED TO 40% OF THIS VALUE (60% DISPLACEMENT).

FIG.4. DIAGRAMMATIC REPRESENTATION OF DESIGN GAP IN CENTRAL AND MAXIMUM DISPLACED POSITION WITH MAXIMUM ERROR IN SURFACE OF SLIDE.

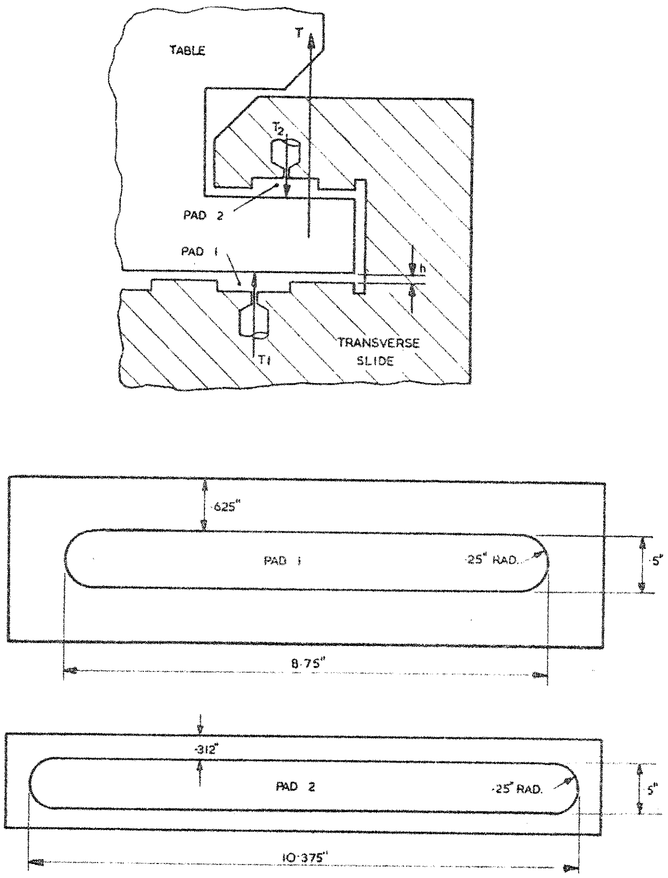


FIG. 5. VERTICAL SECTION THROUGH TABLE BEARING AND DETAILS OF No. 1 AND No. 2 SUPPORTING PADS.

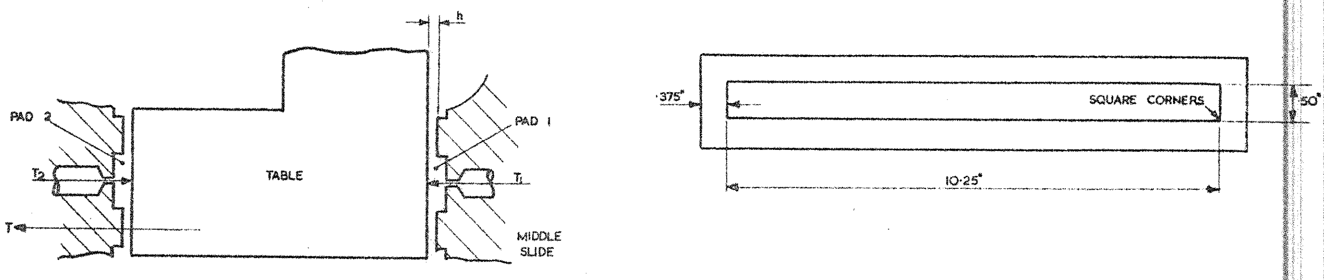


FIG. 6. VERTICAL SECTION THROUGH TABLE SLIDE BEARING SHOWING DETAILS OF VERTICAL PADS.

$$3{,}300 < .9398 \times A_1 \times 500 - .1962 \times (A_1 - 1.5) \times 500$$

$$A_1 > 8.45 \text{ sq.ins.}$$

Any of the same four pairs might have to provide a downthrust equal to, or less than, that provided by pair 2 in Fig. 2, which is

$$\frac{1}{3} \times \frac{9}{14} \times 2000 + \frac{1}{2} \times \frac{9}{24} \times 2000 = 803 \text{ lbs.}$$

So, by Appendix 3, because this downthrust must be provided without gaps closing by more than 60% of 0.0003 , we must make

$$803 < (.9398\ A_2 - .1962\ A_1)\ 500$$

and because $A_1 = A_2 + 1.5$

$$803 < (.9398\ A_2 - .1962\ (A_2 + 1.5)\ 500$$

$$A_2 > 2.55 \text{ sq.ins.}$$

Now the difference $8.45 - 2.55 > 1.5$ (the required difference) therefore we should first choose $A_1 > 8.45$ sq.ins. and then find A_2 knowing the difference.

(iv) The horizontal slide will be urged by horizontal components of cutting force as much in one direction as in any other, so all guiding pads have been made equal. If any of the three pairs have to take as much as 2000 lbs. we should, by Appendix 3, make

$$2000 < (.9398\ A_1 - .1962\ A_2)\ 500$$

or because $A_1 = A_2$, $2000 < .7436 A_1 \times 500$

$$A_1 \text{ and } A_2 > 5.38 \text{ sq.ins.}$$

(v) The effective areas of the pads drawn in Figs. 5 and 6 are now found by Appendix 1 to show that they will be suitable.

Supporting pads:

$$A_1 = (8.75 + 0.5 - 1) \times 0.625 + 8.75 \times 0.5 - 4 \times 0.25^2$$

$$+ \frac{\pi}{2} \; \frac{(0.625 + 2 \times 0.25) \times 0.625}{\log \dfrac{0.625 + 0.25}{0.25}} = 10.18 \text{ sq.ins.} > 8.45 \text{ sq.ins.}$$

$$A_2 = (10.375 + 0.5 - 1) \times 0.312 + 0.5 \times 10.375 - 4 \times 0.25^2$$

$$+ \frac{\pi}{2} \frac{(0.312 + 2 \times 0.25) \times 0.312}{\log \dfrac{0.312 + 0.25}{0.25}}$$

$$= 8.51 \text{ sq.ins.} > 2.55 \text{ sq.ins.}$$

and $A_1 - A_2 = 1.67$ sq. ins. nearly 1.5 sq.ins.

Guiding pads:

$$A_1 = A_2 = 0.5 \times 10.25 + 0.375 \times 10.25 + 0.375 \times 0.5$$

$$= 9.16 \text{ sq.ins.} > 5.38 \text{ sq.ins.}$$

(vi) In order that the characteristics of pads might be easily appreciated by the reader, thrust and stiffness curves are drawn for each pair by means of Appendix 2, and they can be seen in Figs. 7, 8, 9, 10. In the graphs showing stiffness against gap, the stiffnesses of each pad are plotted separately against the gap over pad 1, and the stiffness of the whole bearing is found by addition.

(vii) The resistance to flow of the pads drawn in Figs. 5, 6 is now found so that the resistance of the best capillaries may be known. The method used is that described in Appendix 4.

Supporting pads:

$$R_1 = \frac{6\mu \times 0.625 \times \log \dfrac{0.625 + 0.25}{0.25}}{27 \times 10^{-12} \times (8.75 + 0.5 - 4 \times 0.25) \log \dfrac{0.625 + 0.25}{0.25} + 0.625\pi}$$

$$= 1.42\mu 10^{10} \text{ lb.sec/in}^5$$

Similarly $R_2 = 6.266\ \mu 10^9$

Guiding pads:

$$R_1 = R_2 = \frac{6\mu \times 0.375}{27 \times 10^{-12}(0.5 + 10.25)} = 7.75\ \mu 10^9$$

These values found are also the required capillary resistances, since the cell pressure in each pad must be half the supply pressure (Introduction 3).

(viii) The required lengths of 0.008 bore capillary to have those resistances are found by the method in Appendix 5.

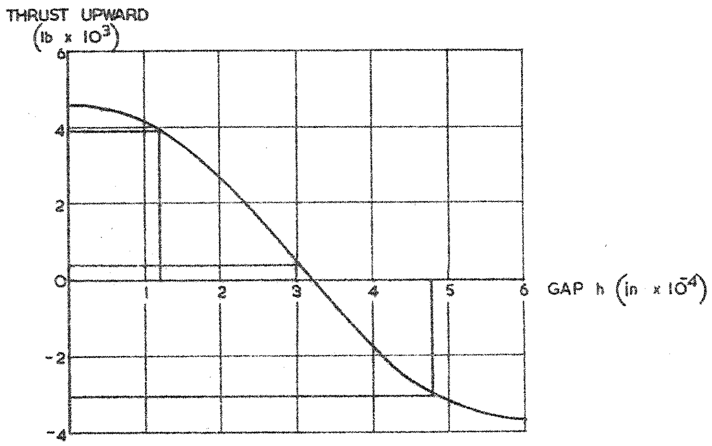


FIG. 7. THRUST GRAPH FOR ONE PAIR OF HORIZONTAL PADS ON HORIZONTAL SLIDE.

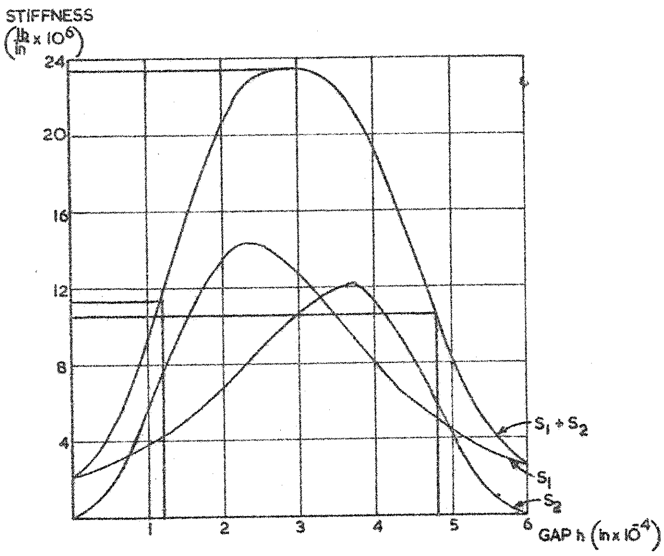


FIG. 8. STIFFNESS CURVES FOR ONE PAIR OF HORIZONTAL PADS ON HORIZONTAL SLIDE.

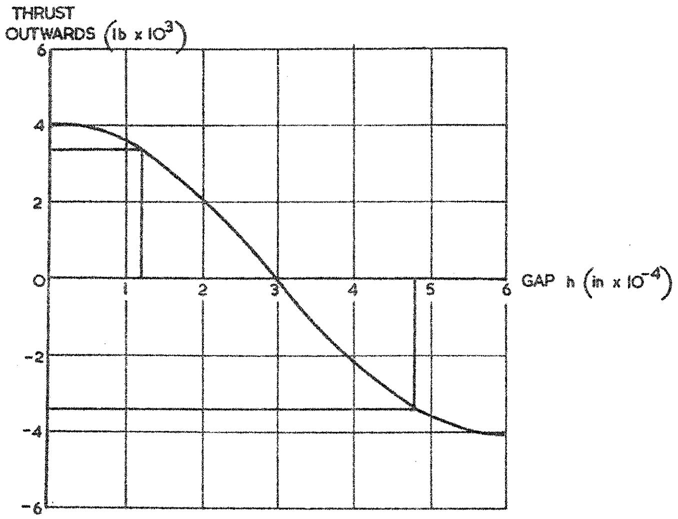


FIG. 9. THRUST GRAPH FOR ONE PAIR OF VERTICAL PADS ON HORIZONTAL SLIDE.

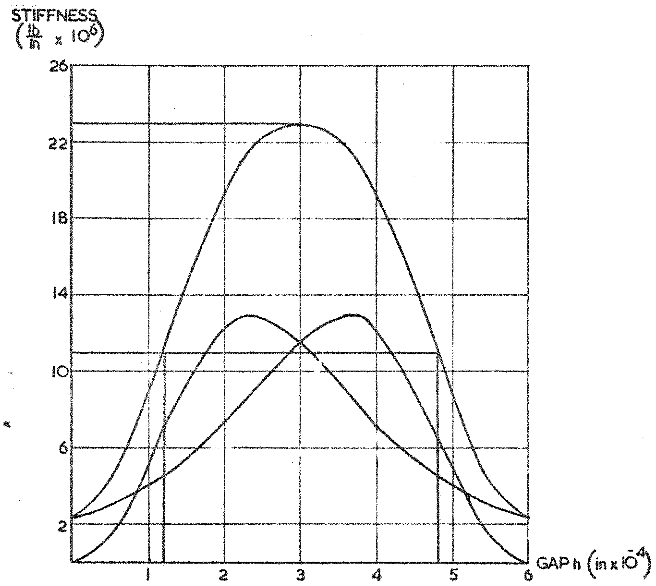


FIG. 10. STIFFNESS CURVES FOR ONE PAIR OF VERTICAL PADS ON HORIZONTAL SLIDE.

Supporting pads:

$$L_1 = 1.006 \; \frac{1.42\mu 10^{10}}{\mu} \; 10^{-10} \; = 1.429 \text{ ins.}$$

Similarly $L_2 = 0.63$ ins.

Guiding pads:

$$L_1 = L_2 = 0.78 \text{ ins.}$$

(ix) The flows through pads at the normal gap induced by pressure drops and retarded by resistance may be found, in terms of the viscosity of oil used, by Appendix 6.

Supporting pads:

$$Q_1 = \frac{500}{2\times 1.42\mu 10^{10}} = \frac{1}{\mu} \times 176 \times 10^{-10} \text{cu.ins/sec } (\mu \text{ in reyns})$$

Similarly $Q_2 = \frac{1}{\mu} \times 39.9 \times 10^{-9}$

Guiding pads: $Q_1 = Q_2 = \frac{1}{\mu} \times 32.2 \times 10^{-9}$

(x) So. using Shell Macron oil 27, the flows are

$$Q_1 = \frac{1}{4.65\times 10^{-6}} \times 176 \times 10^{-10} = 37.8 \times 10^{-4}$$
$$\doteq 0.00378 \text{ cu.ins./sec.}$$

$$Q_2 = 0.008 \text{ cu.ins./sec.}$$

for the supporting pads and for the guiding pads

$$Q_1 = Q_2 = 0.00693 \text{ cu.ins./sec.}$$

(xi) For water (or any fluid whose viscosity is nearly that of water)

Supporting pads:

$$Q_1 = 0.0685 \text{ cu.ins./sec.}$$

$$Q_2 = 0.155 \text{ cu.ins./sec.}$$

Guiding pads:

$$Q_1 = Q_2 = 0.125 \text{ cu.ins./sec.}$$

Part 2

The design of the horizontal transverse slide

(i) If the horizontal transverse slide is to move centrally between its supporting pads (shown in Fig. 11, 12) during normal working conditions (which are those stated in decision (i) of part 5 of the introduction), those supporting pads must then be providing an upthrust of

$$1{,}100 + 1{,}200 + 400 + 30\% \text{ of } 2{,}000 = 3{,}300 \text{ lbs.}$$

Since the weight of the horizontal transverse slide is 1,100 lbs., the weight of the horizontal slide is 1,200 lbs., the weight of a typical workpiece is 400 lbs., and the greatest vertical component of cutting force is 2,000 lbs.

Now there are six opposing pairs to provide this upthrust and these pairs are equal to one another. Hence the upthrust required of one pair is one sixth of 3,300 lbs., which is 550 lbs.

(ii) The cell pressures in pads 1 and 2 in Fig. 11, will be 250 p.s.i. (Introduction 3), so to provide an upthrust at the mean gap of 0.0003 ins., we must make the effect area of pad 1 greater than the effective area of pad 2, and if A_1 and A_2 are those effective areas, we must make

$$250A_1 - 250A_2 = 550 \text{ lbs.}$$

to provide the required upthrust of 550 lbs. That is, the difference in effective areas must be about 2.5 sq.ins.

(iii) However, four of the six supporting pairs may have to provide much greater upthrusts when the cutting forces area not central. None, however, will have to provide upthrusts greater than that provided by pair 1 in Fig. 3, which must support one sixth of the weight of the transverse slide, one third of the weight of the horizontal slide and workpiece, one half of the greatest vertical component of cutting force and (across the width and length of the transverse slide) the couples arising from the horizontal components and their opposing forces from the guideways and screw. This upthrust is

$$\frac{1}{6} \times 1{,}100 + \frac{1}{3} \times 1{,}600 + \frac{1}{2} \times 2000 + \frac{1}{3} \times \frac{5}{17} \times 2000 + \frac{1}{2} \times \frac{15}{12} \times 2000$$

$$= 3{,}400 \text{ lbs.}$$

By Appendix 3, because this load must be taken without gaps closing as much as 60% of 0.0003 ins., we must make

$$3{,}400 < (0.9398A_1 - 0.1962A_2)500$$

and because $A_2 = A_1 - 2.5$

$$3{,}400 < (0.9398A_1 - 0.1962(A_1 - 2.5))500$$

$$A_1 > 8.5 \text{ sq.ins.}$$

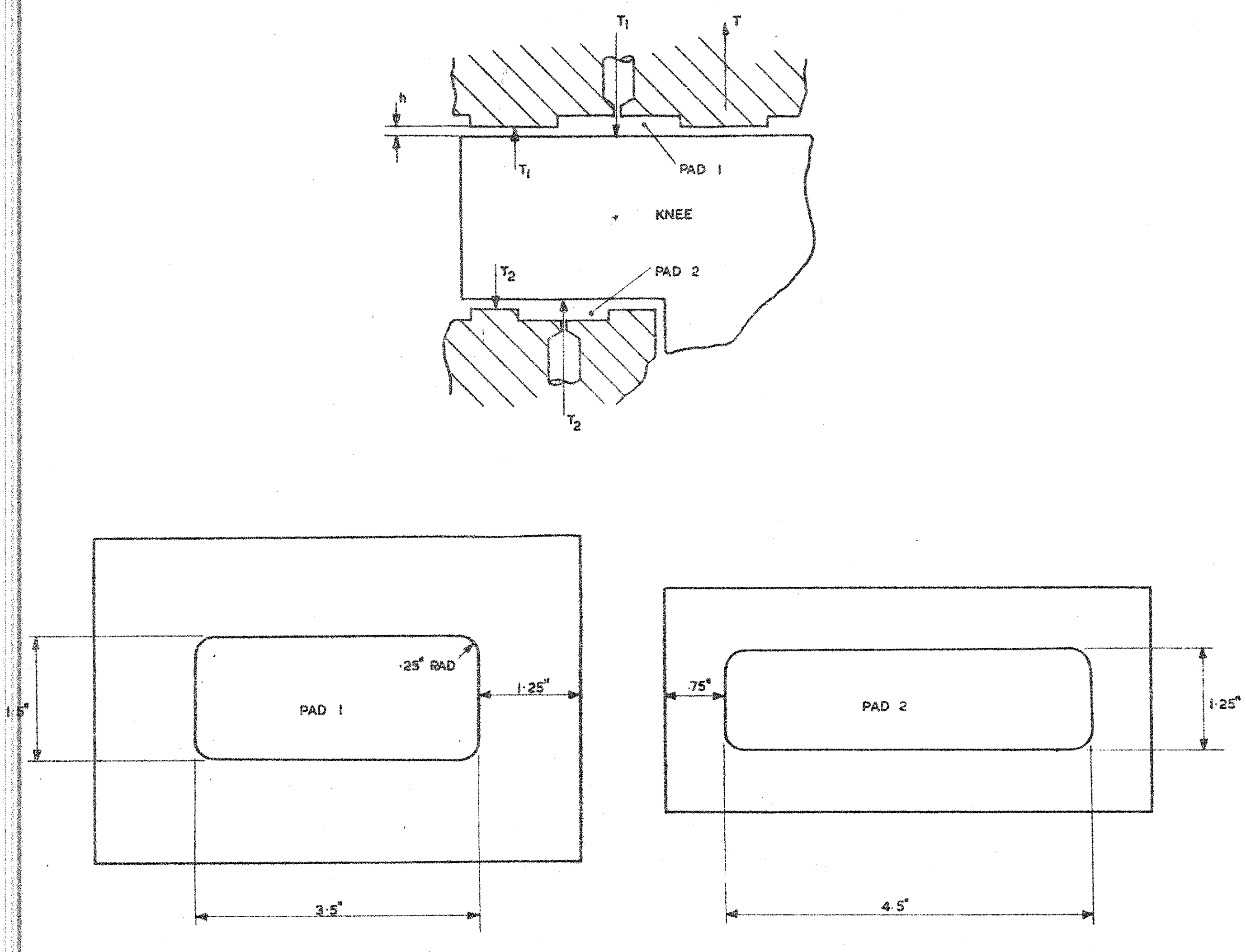


FIG. 11. VERTICAL SECTION THROUGH TRANSVERSE SLIDE BEARING AND DETAILS OF PADS.

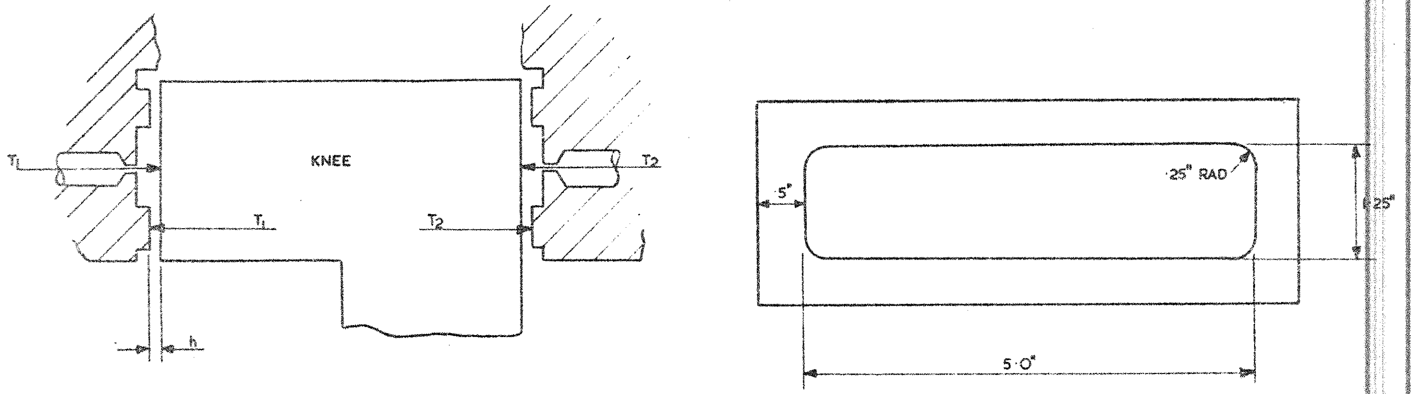


FIG. 12. VERTICAL SECTION THROUGH TRANSVERSE SLIDE GUIDEWAY AND DETAILS OF PAD.

Any of the same four pairs might have to provide a downthrust equal to, or less than, that provided by pair 2 in Fig. 3, which is

$$\frac{1}{3} \times \frac{5}{17} \times 2000 + \frac{1}{2} \times \frac{15}{12} \times 2000 = 1{,}700 \text{ lbs.}$$

By Appendix 3, because this downthrust must be provided without gaps closing by more than 60% of 0.0003 ins., we must make

$$1{,}700 < (0.9398A_2 - 0.1962A_1)500$$

and because $A_1 = A_2 + 2.5$

$$1{,}700 < (0.9398A_2 - 0.1962(A_2 + 2.5))500$$

$$A_2 > 5.23 \text{ sq.ins.}$$

Now the difference $8.5 - 5.23 > 2.5$ (the required difference), therefore we should first choose $A_1 > 8.45$ sq.ins. and then find A_2, knowing the difference.

(iv) The horizontal transverse slide will be urged by horizontal components of cutting force as much in one direction as in any other, so all guiding pads have been made equal. If any of the three pairs have to take as much as 2,000 lbs., we should, by Appendix 3, make

$$2000 < (0.9398A_1 - 0.1962A_2)500$$

or, because $A_1 = A_2$, $2000 < 0.7436A_1 500$

$$A_1 \text{ and } A_2 > 5.38 \text{ sq.ins.}$$

(v) The effective areas of the pads in Figs. 11, 12 are now found by Appendix 1 to show that they are suitable.

Supporting pads:

$$A_1 = (1.5 + 3.5 - 4 \times 0.25) \times 1.25 + 1.5 \times 3.5 - 4 \times 0.25^2$$

$$+ \frac{\pi}{2} \frac{(1.25 + 2 \times 0.25) \times 1.25}{\log \frac{1.25 + 0.25}{0.25}}$$

$$= 11.92 \text{ sq.ins.} > 8.5 \text{ sq.ins.}$$

$$A_2 = (1.25 \times 4.5 - 4 \times 0.25) \times 0.75 + 1.75 \times 4.5 - 4 \times 0.25^2$$

$$+ \frac{\pi}{2} \frac{(0.75 + 2 \times 0.25) \times 0.75}{\log \frac{0.75 + 0.25}{0.25}}$$

$$= 10 \text{ sq.ins.} > 5.22 \text{ sq.ins.}$$

and $A_1 - A_2 = 1.92$ sq.ins. nearly 2.5 sq.ins.

Guiding pads:

$$A_1 = A_2 = (1.25 + 5 - 4 \times 0.25) \times 0.5 + 5 \times 1.25 - 4 \times 0.25^2$$

$$+ \frac{\pi}{2} \frac{(0.5 + 2 \times 0.25) \times 0.5}{\log \dfrac{0.5 + 0.25}{0.25}}$$

$$= 9.34 \text{ sq.ins.} > 5.38 \text{ sq.ins.}$$

(vi) Thrust and stiffness curves, drawn using the equations in Appendix 2, can be found in Figs. 13, 14, 15, 16.

(vii) The resistance to flow of the pads drawn in Figs. 11, 12 is now found by means of Appendix 4.

Supporting pads:

$$R_1 = \frac{6\mu \times 1.25 \times \log \dfrac{1.25 + 0.25}{0.25}}{27 \times 10^{-12} \times (1.5 + 3.5 - 3 \times 0.25) \log \dfrac{1.25 + 0.25}{0.25} + 1.25\pi}$$

$$= 4.485\mu 10^{10} \text{ lb.sec./in}^5$$

Similarly, $R_2 = 2.583\mu 10^{10}$

Guiding pads:

$$R_1 = R_2 = 1.664\mu 10^{10}$$

These values found are also the resistances required of the capillary restrictors for each pad, since the cell pressures are half the supply pressure.

(viii) Hence the required lengths of 0.008 bore capillary may be found by Appendix 5.

Supporting pads:

$$L_1 = 1.006 \frac{4.485\mu 10^{10}}{\mu} 10^{-10} = 4.51 \text{ ins.}$$

Similarly $L_2 = 2.6$ ins.

Guiding pads:

$$L_1 = L_2 = 1.67 \text{ ins.}$$

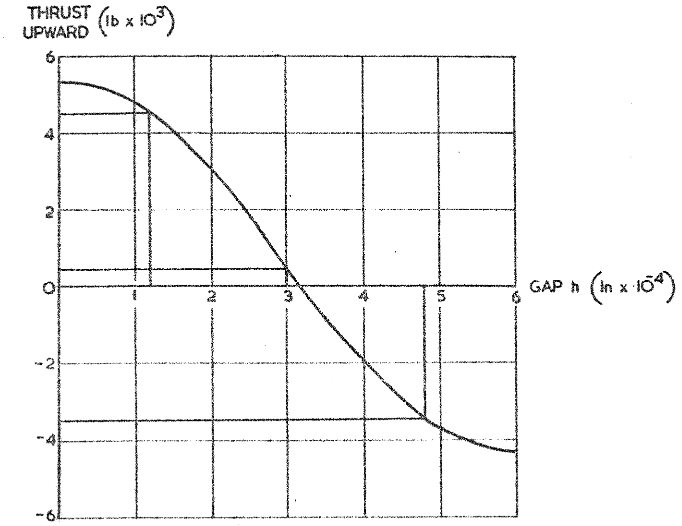


FIG. 13. THRUST GRAPH FOR ONE PAIR OF HORIZONTAL PADS ON TRANSVERSE SLIDE.

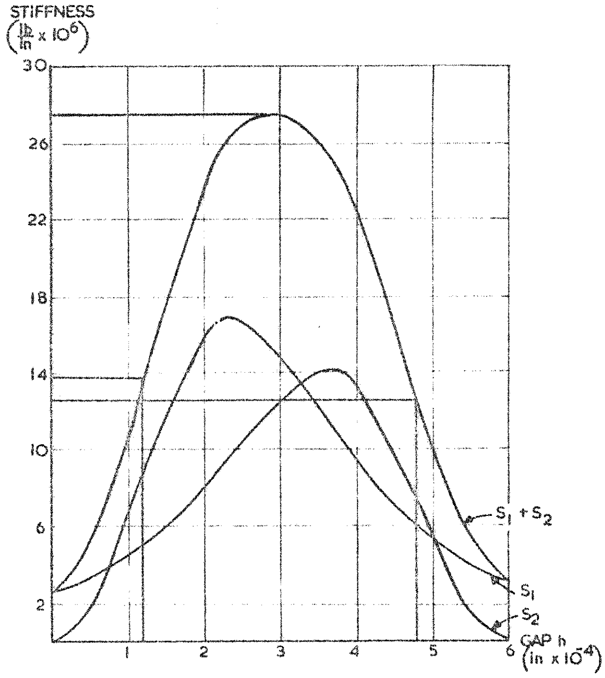


FIG. 14. STIFFNESS CURVES FOR ONE PAIR OF HORIZONTAL PADS ON TRANSVERSE SLIDE.

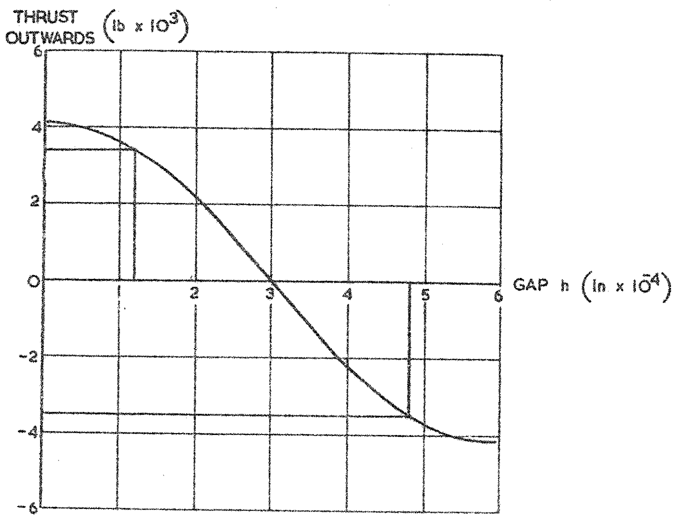


FIG. 15. THRUST GRAPH FOR ONE PAIR OF VERTICAL PADS ON TRANSVERSE SLIDE.

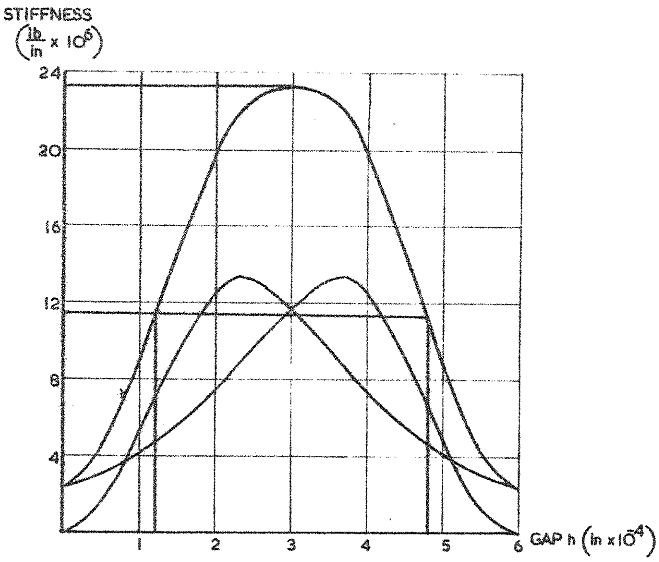


FIG. 16. STIFFNESS CURVES FOR ONE PAIR OF VERTICAL PADS ON TRANSVERSE SLIDE.

(ix) The flows through pads at the mean gap of 0.0003″ can be found, in terms of the viscosity of oil used, by Appendix 6.

Supporting pads:

$$Q_1 = \frac{500}{2 \times 4.485\mu 10^{10}} = \frac{1}{\mu} 55.7 \times 10^{-10} \text{ cu.ins./sec.} (\mu \text{ in reyns})$$

Similarly, $Q_2 = \frac{1}{\mu} 96.8 \times 10^{-10}$

Guiding pads:

$$Q_1 = Q_2 = \frac{1}{\mu} 150 \times 10^{-10}$$

(x) So if Shell Macron 27 oil is used, for the supporting pads:

$$Q_1 = \frac{1}{4.65 \times 10^{-6}} 55.7 \times 10^{-10} = 0.0012 \text{ cu.ins/sec.}$$

$$Q_2 = 0.00208 \text{ cu.ins./sec.}$$

for the guiding pads:

$$Q_1 = Q_2 = 0.00322 \text{ cu.ins./sec.}$$

(xi) Or, if water is used

for the supporting pads:

$$Q_1 = \frac{1}{2.57 \times 10^{-7}} 55.7 \times 10^{-10} = 0.0217 \text{ cu.ins./sec.}$$

$$Q_2 = 0.0376 \text{ cu.ins./sec.}$$

for the guiding pads:

$$Q_1 = Q_2 = 0.0584 \text{ cu.ins./sec.}$$

Part 3

The design of the thrust bearings

(i) The purpose of the thrust bearings is to provide loads on the horizontal and transverse screws to oppose the horizontal components of cutting force which, at greatest, are 2000 lbs.

(ii) The bearings have been designed to make the best use of the space available. Their form is represented in Fig. 17. The thrust bearings, which are equal, consist of two hydrostatically lubricated surfaces, side 1 and side 2, which oppose each other to provide forces in either direction. Four cells in the form of circular arcs are cut in the lands of each side as shown, and escapes are provided both in the centre of the bearings and about their circumferance.

(iii) For the purpose of mathematical treatment, the flow through either side of a thrust bearing shall be regarded as the simple flow through pairs of circular slots shown in Fig. 18.

(iv) The thrust exerted by the land of a circular slot of outer radius R and inner radius r when oil flows from inside to out driven by a pressure drop p is

$$\frac{\pi}{2} p \frac{R^2 - r^2}{\log \frac{R}{r}} - \pi p r^2$$

The thrust exerted by the same slot when oil flows from outisde to inside driven by an equal pressure drop is

$$\pi p R^2 - \frac{\pi}{2} \frac{R^2 - r^2}{\log \frac{R}{r}} p$$

Hence the effective areas of side 1 and side 2 may be found.

$$A_1 = \frac{\pi}{2} \left[\frac{2.483^2 - 1.997^2}{\log \frac{2.483}{1.997}} - \frac{1.741^2 - 1.255^2}{\log \frac{1.741}{1.355}} \right]$$

$$= 8.74 \text{ sq.ins.}$$

Similarly, $A_2 = 8.69$ sq.ins.

(v) Thrust and stiffness curves for a thrust bearing are drawn in Figs. 19, 20, from the equations in Appendix 2.

(vi) The resistance to flow of either side of a bearing at the mean gap of 0.0003" is found by adding the reciprocals of the resistance of the inner and outer lands.

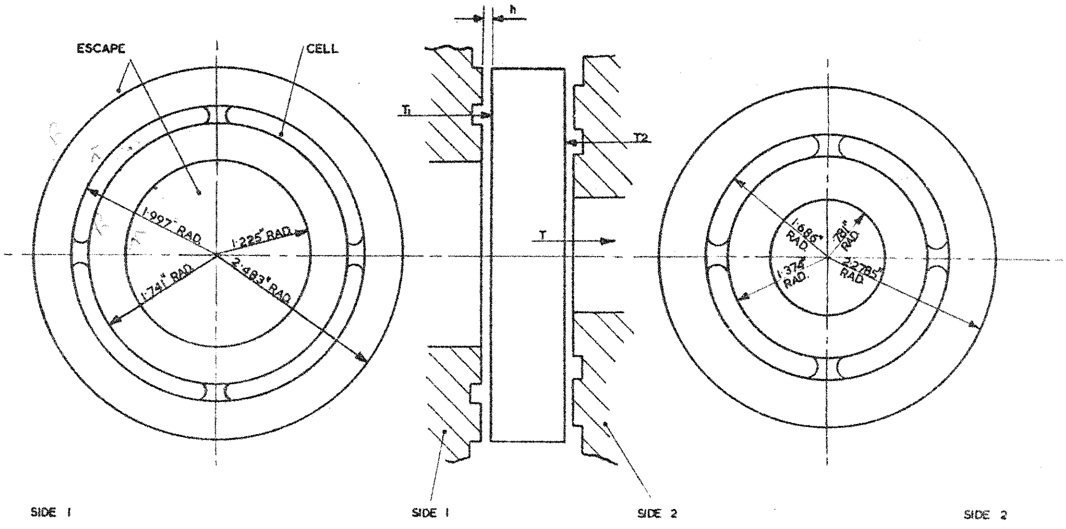


FIG. 17. THRUST BEARING.

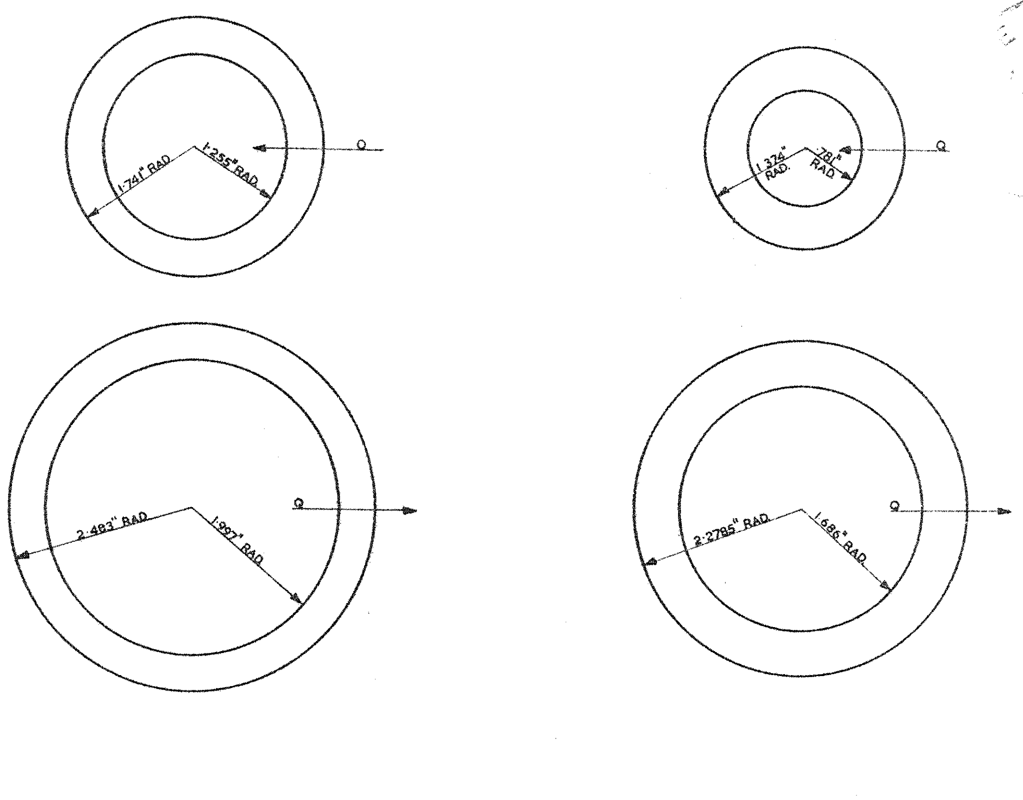


FIG. 18. CIRCULAR SLOTS WITH OIL FLOW Q IN THE DIRECTIONS SHOWN.

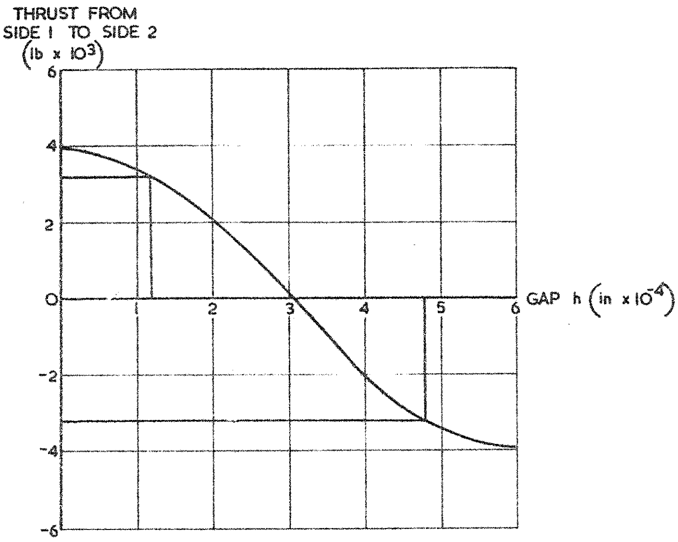


FIG. 19. THRUST GRAPH FOR THE THRUST BEARING.

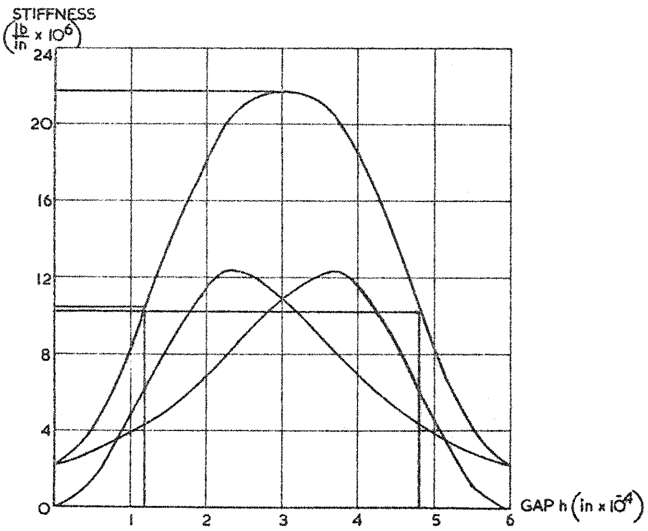


FIG. 20. STIFFNESS CURVES FOR A THRUST BEARING.

The resistance to flow of a circular slot of inner radius r and outer radius R is, at a gap h,

$$\frac{6\mu \log \frac{R}{r}}{\pi h^3}$$

Hence,

$$\frac{1}{R_1} = \frac{\pi 27 \times 10^{-12}}{6\mu \log \frac{2.485}{1.997}} + \frac{\pi 27 \times 10^{-12}}{6\mu \log \frac{1.741}{1.255}}$$

$$R_1 = 9.254\mu 10^{9} \text{ lbs.sec./in}^5$$

Similarly, $R_2 = 1.391\mu 10^{10}$

These values are also the required resistances of the corresponding capillaries, since the cell pressures are half the supply pressure.

(vii) Hence, the required lengths of 0.008 ins. bore capillary may be found for each pad, by Appendix 5.

$$L_1 = 1.006 \times \frac{9.254\mu 10^{9}}{\mu} 10^{-10} = 0.93 \text{ ins.}$$

$$L_2 = 1.4 \text{ ins.}$$

(viii) The flows through either side of a thrust bearing may be found, in terms of the viscosity of oil used, by Appendix 6.

$$Q_1 = \frac{500}{2 \times 9.254\mu 10^{9}} = \frac{1}{\mu} 27.3 \times 10^{-9} \text{ cu.ins./sec. } (\mu \text{ in reyns})$$

$$Q_2 = \frac{1}{\mu} 180 \times 10^{-10}$$

(ix) If Shell Macron 27 oil is used,

$$Q_1 = \frac{1}{4.65 \times 10^{-6}} 27.3 \times 10^{-9} = 0.00588 \text{ cu.ins./sec.}$$

$$Q_2 = 0.00387 \text{ cu.ins./sec.}$$

(x) Or, if water is used,

$$Q_1 = \frac{1}{2.57 \times 10^{-7}} 27.3 \times 10^{-9} = 0.106 \text{ cu.ins./sec.}$$

$$Q_2 = 0.07 \text{ cu.ins./sec.}$$

Part 4

The design of the hydrostatic nuts

(i) Both the horizontal and transverse slides shall be moved by screws turning in hydrostatically lubricated nuts. The surfaces of the threads of the nuts shall be machined very accurately, but only after a continuous cell (.1 ins. wide) has been cut on each side of the thread, a cross-section of which is drawn in Fig. 21. Oil pumped into the cells and forced between the screw and nut, exerting forces on each, may escape at both the outer and inner parts of the nut.

(ii) The axial thrust exerted by one side of the nut on the screw through the oil, equal to that exerted by one side of the screw on the same side of the nut, is very nearly that exerted by nine circular thrust bearings equal to the one drawn in Fig. 22, each at 61° to the axis. Because the axial thrust exerted by a pad in the fustrum on a cone is equal to a fraction of the component of force in the same direction of a circular thrust bearing whose centre is on the axis of the cone, and whose land form exactly coincides with the land form of the fustrum in a straight line which, being produced enough, would cut the axis at the centre of the thrust bearing. That fraction is the ratio of the radii, the lesser to the greater, from any point in the fustrum which is sin61°. This is demonstrated in Appendix 7.

The effective area of the thrust bearing in Fig. 22 is

$$\frac{\pi}{2}\left[\frac{1.429^2 - 1.345^2}{\log \frac{1.429}{1.345}} - \frac{1.245^2 - 1.161^2}{\log \frac{1.245}{1.161}}\right]$$

$$= 1.54 \text{ sq.ins.}$$

Hence the axial thrusts, T_1, T_2, exerted by the screw on the nut, T_1 by one face, T_2 by the other, will be given by

$$T_1 = \frac{9 \times 1.54\sin 61^\circ \times 500\cos 29^\circ}{1 + \dfrac{h^3}{27 \times 10^{-12}}}$$

$$T_2 = \frac{9 \times 1.54\sin 61^\circ \times 500\cos 29^\circ}{1 + \dfrac{(0.0006 - h)^3}{27 \times 10^{-12}}}$$

where h is the gap over side 1 of the screw.

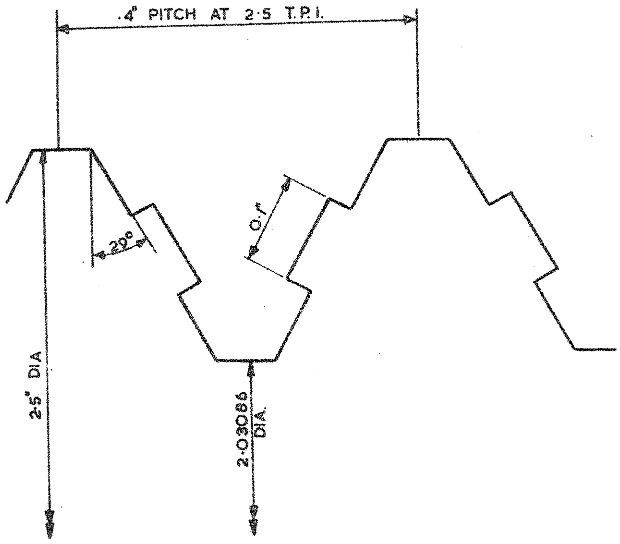


FIG. 21 DETAILS OF THREAD FORM.

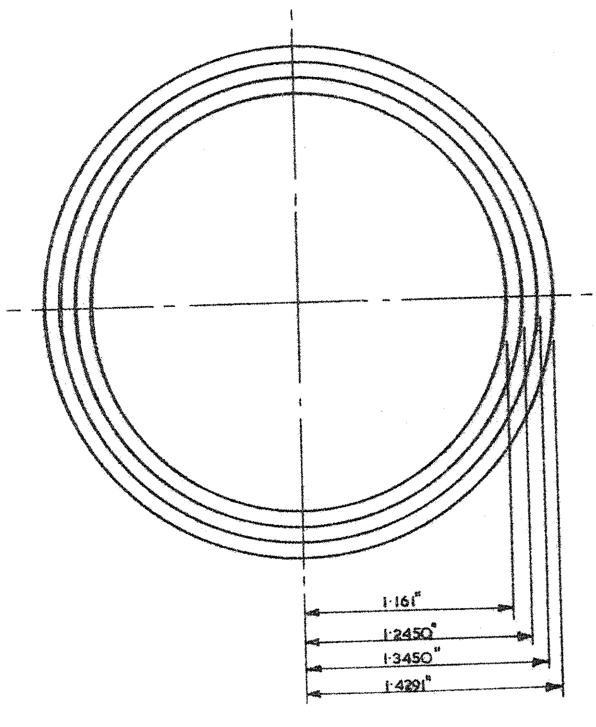


FIG. 22. CIRCULAR SLOT.

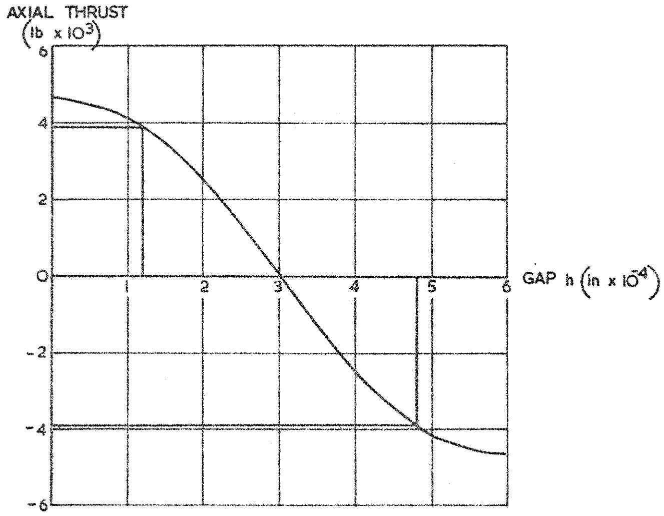


FIG. 23. THRUST GRAPH FOR A NUT AND LEADSCREW.

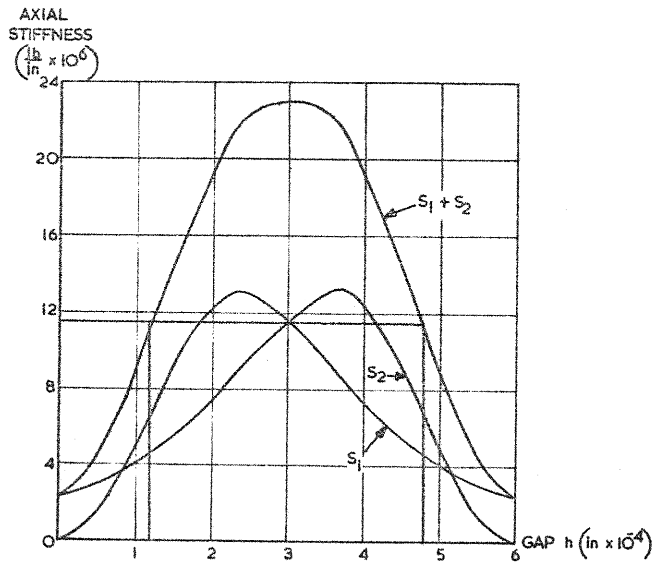


FIG. 24. STIFFNESS CURVES FOR A NUT AND LEADSCREW.

Let x measure the position of the screw in the nut along its axis as the screw moves towards side 1 of the nut making the gap h smaller proportionally by cos29°; then the stiffness of the whole bearing is

$$\frac{d(T_1-T_2)}{dh} \times \frac{dh}{dx} = \frac{d(T_1-T_2)}{dh} \cos 29^\circ$$

In Figs. 23, 24, thrust and stiffness curves are drawn for a screw in the same way that they have been drawn for the other parts of the machine. However, the forces and stiffnesses shown for any gap are not at right angles to the bearing in the nut, but are along the axis of the nut.

(iii) The resistance of one side of the nut thread, whose screw is central, will be very nearly $\frac{1}{\sin 61^\circ}$ times that of nine thrust bearings of the size shown in Fig. 22, arranged in parallel, each at a gap of 0.0003 inches.

Hence,

$$\frac{1}{R_1} = \frac{1}{R_2} = 9\,\frac{\pi 27 \times 10^{-12}}{6\mu \log \frac{1.429}{1.345}} \times \sin 61^\circ + 9\,\frac{\pi 27 \times 10^{-12}}{6\mu \log \frac{1.245}{1.161}}$$

$$R_1 = R_2 = 5.92\mu 10^9 \text{ lb.sec/in}^5$$

This value is the required resistance of capillaries for either part of the bearing in the nut, since the cell pressures are to be half of the supply pressure.

(iv) Now the lengths of 0.008" bore capillary required can be found by Appendix 5.

$$L_1 = L_2 = 1.006\,\frac{5.92\mu 10^9}{\mu}\,10^{-10} = 0.595 \text{ ins.}$$

(v) The flows through either side of the nut may be found, in terms of the viscosity of oil used, by Appendix 6.

$$Q_1 = Q_2 = \frac{500}{2 \times 5.92\mu 10^9} = \frac{1}{\mu}\,40.3 \times 10^{-9} \text{ cu.ins./sec } (\mu \text{ in reyns})$$

(vi) So if Shell Macron oil type 27 is used,

$$Q_1 = Q_2 = \frac{1}{4.65 \times 10^{-6}}\,40.3 \times 10^{-9} = 0.00869 \text{ cu.ins./sec.}$$

(vii) Or, if water is used,

$$Q_1 = Q_2 = \frac{1}{2.57 \times 10^{-7}} \; 40.3 \times 10^{-9} = 0.15 \text{ cu.ins./sec.}$$

Appendix 1

To find the effective area of pads which are of the type shown in Fig. 25, the approximation in sections 1 and 2 of 'The application of hydrostatic bearings to high precision machine tools' by J. Loxham*, has been used, wherefore the effective area of the pad in Fig. 25 is

$$(a + b - 4d)c + ab - 4d^2 + \frac{\pi}{2}\,\frac{(c + 2d)c}{\log e\,\frac{c+d}{d}}$$

However, the effective area of a pad whose pools are cut square in the plane of the lands, like the one drawn in Fig. 26, will be taken to be only

$$(a + b)c + ab$$

Appendix 2

In order that the properties of pads might be well represented, thrust and stiffness curves have been drawn for each pad pair from the equations

$$T = \frac{AP}{1+\left(\frac{h}{ho}\right)^3} \quad \text{and } S = \frac{3AP\,\frac{h^2}{ho^3}}{\left(1+\left(\frac{h}{ho}\right)^3\right)^2}$$

derived in the 'Application of hydrostatic bearings to high precision machine tools'. They give the thrust T and stiffness S of any pad of effective area A, working at a supply pressure P and at a gap h, if h_o is the gap at which the pad has a resistance equal to that of its capillary restrictor. Because the restrictors have been made equal in resistance to the corresponding lands at the mean gap of 0.0003", h_o = 0.0003" for any pad in any part of the machine.

Appendix 3

In Appendix 2 it was stated that the thrust T exerted by a pad of effective area A at a supply pressure P and a gap h was given us by

$$T = \frac{AP}{1+\left(\frac{h}{h_o}\right)^3}$$

and that h_o = 0.0003" for all the pads in the machine. Now the force exerted by a pair of opposing pads on an intermediate slide must be

* To be published.

$$\frac{A_1 P}{1+\left(\frac{h}{0.0003}\right)^3} - \frac{A_2 P}{1+\left(\frac{0.0006-h}{.0003}\right)^3} \quad \text{upwards.}$$

When h is 40% of 0.0003", this force upward, M, is given by

$$M = (0.9398A_1 - 0.1962A_2)P$$

Similarly, if the gap (0.0006-h) over pad 2 is 40% of 0.0003", the downward force N is given by

$$N = (0.9398A_2 - 0.1962A_1)P$$

However, by Introduction 5(ii), we should never have gaps as small as 40% of 0.0003". So in designing a pad pair, we must choose A_1 and A_2 so that the greatest upthrust required is less than M and the greatest downthrust is less than N.

Appendix 4

The resistances of pads, which are of the type drawn in Fig. 25, are nearly,

$$\frac{6\mu c \log \frac{c+d}{d}}{h^3\left[(a + b - 4d)\log \frac{c+d}{d} + \pi c\right]}$$

as shown by section 3 of 'The application of hydrostatic bearings to high precision machine tools'.

However, the resistance of a pad whose cells are cut square at the corners, like the one drawn in Fig. 26, will be taken to be only

$$\frac{6\mu c}{h^3(a+b)}$$

Appendix 5

The restrictors of each pad will be capillaries of 0.008" bore. Their required lengths, which are proportional to their required resistances, are found by means of section 10 of 'The application of hydrostatic bearings to high precision machine tools', which gives the length of a capillary, to have a resistance r, as

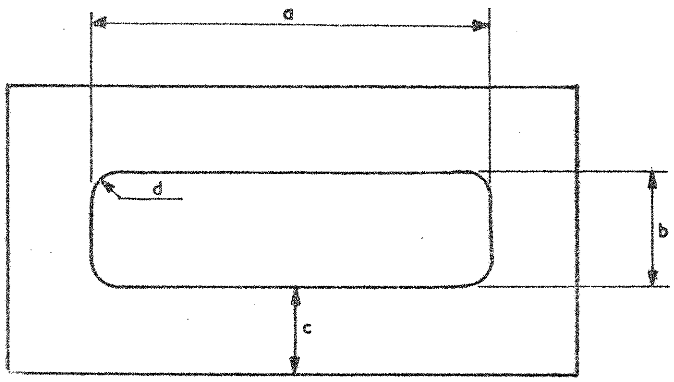


FIG. 25

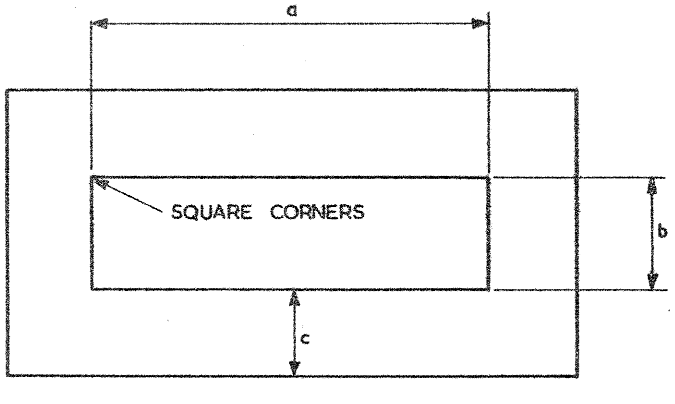


FIG. 26

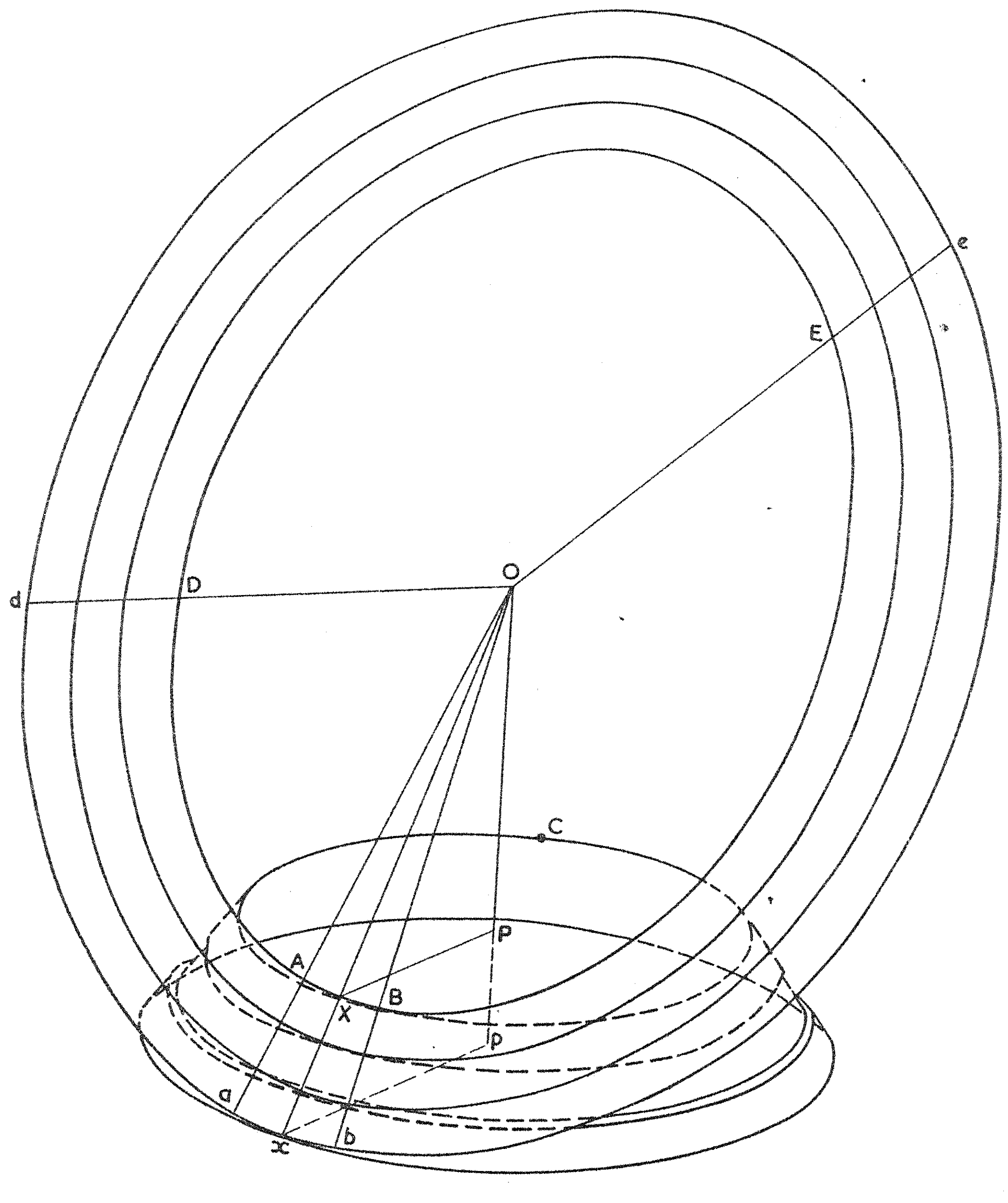


FIG. 27

$$\frac{\pi(0.008)^4}{128} \quad \frac{r}{\mu} = 1.006 \frac{r}{\mu} 10^{-10} \text{ inches.}$$

Appendix 6

The flow through any pad will be $\frac{500}{2R}$ cu.ins./sec., since the supply pressure for any pad is 500 p.s.i. and the resistance R of the lands to flow equals that of the restrictor.

Appendix 7

In Fig. 27, let xXC be the fustrum of a cone vertex O and let xXEeDd be a circular thrust bearing centre O, making an angle XOP with the axis OP of the cone and coinciding with it in Xx produced to meet OP in O. Draw OAa, OBb. Now as the points a, b come together and meet in x, the flow, resistance, thrust, stiffness and every property of equivalent parts of the pad in the fustrum and the circular thrust bearing, in which flows are towards and away from O and ultimately in the same plane, become identical.

So if the cone xXC be rotated about OP, turning the thrust bearing like a bevel and circular rack past the point C, until after one complete turn the thrust bearing has turned through the angle XOD, the properties of the part XxDd of the thrust bearing axially are identical with those of the fustrum, and therefore equal in effective area for axial forces. So if A is the effective area of the thrust bearing, that of the pad in the fustrum is

$$A \times \frac{\text{circ.ABC}}{\text{circ.XED}} = A \times \frac{XP}{XO} = A \times \frac{xp}{xO}$$

And if R is the resistance of the thrust bearing, that of the pad in the fustrum is

$$R \times \frac{\text{circ.XED}}{\text{circ.ABC}} = R \times \frac{XO}{XP} = R \times \frac{xO}{xp}$$