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GRINDING THEORY

by

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SUMMARY

This report presents a review and extension of current grinding theory. Grinding operations are classified on the basis of (a) the cutting action of the grit, (b) the geometry of wheel and work, (c) the type of chip produced, (d) the existence or otherwise of a continuous lateral traverse of the wheel.

The theory is developed using the above method of classification. Recent experimental work by Purcell(12) on the wear of the wheel in surface grinding is reinterpreted in the light of the theory. It is shown to be consistent with the postulate of a constant task per grit. A method of calculation is suggested by which a satisfactory surface grinding technique may be applied to the form grinding of gears, where the task per grit is not independent of the imposed grinding conditions.

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NOTATION

b	width of statistical mean chip
C	number of active grits per unit surface area
D	wheel diameter
D_w	work diameter
d	nominal wheel depth of cut
d_e	effective wheel depth of cut
f	cross feed/rev of dressing tool
	cross feed/traverse in reciprocating surface grinding
	feed/grit in cylindrical grinding
k	number of active grits per unit length
ℓ	undeformed chip length
m	mean spacing between grits
N	angular speed of wheel (rev/unit time)
n	number of fresh active grits per unit length of path of dressing tool
p	pitch of grits relative to workpiece
q	distance moved by work during time of contact of a grit
r	ratio of chip width to thickness
s	feed rate per unit time
t_M	maximum undeformed chip thickness
T	time taken for work to move a distance p
T_c	time of contact of grit with work
V	surface speed of wheel
v	work speed
w'	effective width of wheel
x	depth of cut in form grinding, measured normal to surface

1. Introduction

Grinding is essentially a cutting process. The action of an individual cutting element, or grit, is roughly similar to that of the tooth of a milling cutter. A chip is formed in a similar manner. The theory of grinding is concerned, therefore, with the geometry of chip formation. This is of practical interest since (a) the maximum force on a grit is a function of the chip thickness, (b) the total volume of a chip must be accommodated by the pores in the bond material holding the grits. The force on a grit governs the tendency to lose grits, and hence the rate of wheel wear. Similarly, if a chip cannot be accommodated by the available voids in the wheel it will be dragged across the ground surface and so damage it.

In a theoretical analysis of cylindrical grinding Guest[1] distinguished between (a) the depth of cut as measured by the total thickness of metal removed from the surface per pass (the infeed, wheel or radial depth of cut) and, (b) the thickness of a chip as seen by a grit (the chip or grit depth of cut). As in milling, the grit depth of cut, i.e. the chip thickness, is limited by the number of cutting elements in simultaneous contact with the workpiece in the plane of rotation. Guest further assumed that each chip was of triangular cross section; the chip cross sectional area, and therefore the force per grit, then varies as the square of the chip thickness.

In a similar analysis Alden[2] obtained an expression for the maximum chip thickness which reduces to Guest's formula when the length of the chip is small compared with both wheel and work diameter. The force on a grit was assumed to vary directly as the chip thickness.

Hutchinson[3] differentiated between upcut and downcut grinding and calculated the length of the chip and the chip thickness. The latter was calculated, however, assuming that the average grit spacing was equal to the length of the chip. This implied that only one grit was in contact with the work at any instant. If this assumption is corrected Guest's formula is obtained. Hutchinson also considered problems of work heating, chip formation and wheel dressing.

More recent theories of grinding are due to, or derive from, the work of Shaw and his associates[5, 6, 7, 8, 9, 10]. These theories are concerned with the definition of the mean grit, the mean grit spacing, the mean chip and the width of a statistical mean chip. Grinding theory has also been extended to cover extreme conditions peculiar to internal grinding, very light depths of cut and high work speeds[8]. The reference cited suggests a classification of types of chip based on the geometry and shape of the chip. Finally, the theory has been used by Shaw[10] to indicate how grinding burn, residual stresses due to grinding, and surface cracking may be minimised.

Experimental work by Purcell[12] on the wear of the wheel in surface grinding has shown that the profile of the wheel periphery tends to take up a characteristic profile, depending upon the imposed grinding conditions, so as to impose a constant task per grit.

The present report attempts to classify grinding processes and presents a theoretical analysis based on this classification. The suggested classification is based on considerations of: the cutting action of the grit, the geometrical configuration of the wheel and work, and the type of chip produced. It will be shown how the postulate of a constant task per grit may be incorporated in current theory. A method of applying satisfactory surface grinding techniques to the form grinding of gears is outlined. Finally, the classification of types of chip presented in reference (8) is extended and the definition made more precise; and the theory of grinding at high work speeds is extended to cover a practical case not considered in the literature.

2. The Grinding Wheel

The grinding wheel is a disc or cylinder composed of abrasive particles embedded in a porous matrix. The abrasive particles, known as grits or grains, present sharp edges to the work. The action is a true cutting process, but more complex than is the case with a lathe tool due to the irregular shape of the grits. Similarly the size of the grits is not uniform but is controlled within limits. The porous matrix is composed of the bond material and pores or voids. The bond serves to provide tool posts (bond posts) to hold the abrasive grits. The pores are spaces not occupied by either grits or bond material and they provide accommodation for the chips removed by the cut. Ideally each cutting grit is accompanied by a void sufficient to carry the cut chip. This is then washed away by the cutting fluid as the grit clears the workpiece.

The inherent properties of a wheel depend on the type of abrasive, the size and distribution of the grits, the amount and type of bond material and the volume of the pores relative to that of grits and bond. The inherent wear properties of a wheel are determined by the resistance to abrasion of the grits, their strength, and the strength of the bond posts. The resistance to abrasion is a function of the type of abrasive and may be modified by the nature of the material of the workpiece and the chip-grit interface temperature. The strength of a grit is the resistance offered to fracture during cutting. It is a function of the type of abrasive and of the grit size. The strength of the bond is the resistance offered to breakdown of the bond posts and consequent loss of grits. It is a function of the type and relative amount of bond material.

Typical abrasives are:

aluminium oxide (alundum), denoted by the letter - A

silicon carbide (carborundum) - S

diamond - D

The term grit size relates to the fineness of the mesh used in grading the abrasive grits. the larger the 'size' the finer the grit. The range of grit sizes is as follows:

coarse	6 - 24
medium	30 - 60
fine	70 - 600

The grit size (S) is related to the grit diameter (g in.) by the approximate formula[9]

$$S.g = 0.6$$

Thus a grit size of 30 corresponds to a grit diameter of 0.02 in.

The wheel grade is a measure of the inherent strength or hardness of the wheel, i.e. the resistance to breakdown and wear. Since under correct cutting conditions wheel wear takes place by abrasion and bond post fracture the grade or hardness is a function of the relative amount of bond post material. The resistance to abrasion of the grits is not usually considered in the classification of a wheel. The wheel grades are:

very soft	C - G
soft	H - K
medium	L - O
hard	P - S
very hard	T - Z

The term structure refers to the porosity of the wheel. Alternatively it may refer to the grit spacing. The range of porosity is from dense (O) to porous (15).

The following bond materials are generally used:

vitreous or vitrified bond (baked clay) - V

silicate - S

resinoid - B

rubber - R

shellac - S

oxychloride - O

Using the above notation a typical wheel might be classified as follows:

A 36 L 8 V

This represents a wheel having an aluminium oxide abrasive (A), a grit size of 36 mesh, a medium strength bond, i.e. a hardness or wheel grade L, a fairly high grit spacing or porosity (8), and a vitrified bond (V).

3. Grinding as a Cutting Process

Grinding is a true cutting process. Microscopic examination of a taper section of a ground surface reveals minute Vee shaped scratches. Microscopic examination of grinding swarf reveals very minute chips formed by the cutting action of individual grits. The cutting process at an individual grit takes place however at a higher cutting speed and with a smaller depth of cut as compared with cutting with a single point tool. Typical values are as follows:

	Cutting	Grinding
cutting speed (ft/min)	40 - 400	3,000 - 6,500
chip thickness (in.)	0.005 - 0.020	0.00003 - 0.00010

Later it will be found convenient to consider grinding as a cutting process geometrically akin to milling. The following features are peculiar to grinding however: The size and shape of the grits varies in a random manner within the limits of a specified grit size. The grits are set in the wheel in a random manner, and not all grits visible on the surface of the wheel take part in cutting. Those that do are termed active grits. Finally, the deeper the cut the greater the number of active grits that may be expected to be in contact with the work surface. Current grinding theory attempts to account for these conditions excepting the last. It is assumed that the number of active cutting grits per unit area of the wheel surface is independent of the depth of cut.

Cutting action of a grit

In orthogonal cutting with a single point tool the chip width is uniform and equal to the width of the tool. Consider, however, orthogonal cutting with the Vee shaped tool shown, Fig. 1. A simple check, using plasticene as a workpiece, will show that if a chip is formed on the flat face of the tool its shape will be roughly as indicated. The ratio r of chip breadth b' to depth of cut t is a constant for a given tool; thus

$$\frac{b'}{t} = r$$

A real grinding grit, even when sharp, does not have such a simple shape. However observation of the Vee shaped profile of the scratches formed on the surface of a ground workpiece suggests that the above relation holds approximately, at least for surface grinding at the periphery of the wheel. Note that for this type of chip the cross sectional area varies as the square of the depth of cut.

The grit depth of cut is the undeformed chip thickness seen by an individual grit. According to the type of grinding involved the chip thickness may vary along the length of the chip. The chip width varies accordingly. Moreover, a grit might cut into a previously uncut surface, or its path might overlap the cut made by a preceding grit. A precise specification of actual chip cross section is therefore difficult. On the other hand a knowledge of the actual variation in chip cross section along the chip, or from chip to chip, is not usually necessary. Since the uncut surface of the workpiece may be formed from a sufficient number of undeformed chips of uniform cross section so as to leave no voids it is convenient to define a statistical mean chip of uniform width which is assumed to be the same for all chips. The cross sectional area of such a mean chip varies as the chip thickness and not as the square of the thickness.

From the point of view of the cutting action of the grit it is convenient to classify grinding processes as:

1. (a) surface grinding
   (b) plunge grinding

2. (a) peripheral grinding
   (b) side-wheel grinding

In surface grinding the plane of motion of the workpiece is parallel to the plane of motion of the cutting grits. In plunge grinding, on the other hand, the motion of the work is normal to the direction of motion of the active wheel surface.

Peripheral grinding refers to a cutting process which takes place at the periphery of the wheel. Side-wheel grinding refers to a process in which the active grits are at the side face of the wheel. In practice, due to non-uniform wheel wear it may happen that the cutting action may be intermediate between these two extremes.

In peripheral surface grinding the width b of the statistical mean chip is defined in terms of the mean chip thickness. In surface grinding, as in cylindrical or slab milling, the undeformed chip thickness varies from zero at the start of the cut to a maximum, t_M, near the end of the cut, the mean chip thickness is therefore $\frac{1}{2}.t_M$ to a very good approximation, and hence

$$b = \tfrac{1}{2}.r.t_M$$

The constant r is found experimentally from measurements on a taper section of the surface grinding scratches, and is assumed to be the same for all chips. In practice the value of r lies between 10 and 30 and depends on the shape and condition of the grits.

In surface grinding at the side face of the wheel, Fig. 2(b), the width of the chip is equal to the depth of cut, or infeed per pass, of the wheel and independent of chip thickness.

For plunge grinding at the periphery of the wheel, Fig. 2(a), the chip thickness may be shown to be constant over the length of the chip. The width of the mean chip is defined as

$$b = r.t$$

where the chip thickness t is equal to the infeed per grit. Thus, if v is the velocity of the work normal to the periphery of the wheel and V the surface speed of the wheel,

$$t = \frac{v}{V} . m$$

When plunge grinding at the side face of a wheel the width of the mean chip is again a constant, equal to the infeed of the work per grit. Thus

$$b = \frac{v}{V} . m$$

where v is the speed of the work normal to the side face of the wheel.

4. Grinding as a Milling Process

The geometry of grinding will be treated in each case as for an analogous milling process. Before doing so, however, it is necessary to define the number of active grits or teeth in a plane of rotation. In practice it is usually possible to determine by experiment the number of active grits C per unit area of the wheel surface. The number of active grits k per unit length of the surface, measured in a plane of rotation, is the number of grits contained in the strip AB, Fig. 3 , cut out of unit area of the cutting surface of the wheel.

In peripheral grinding the width of the strip is equal to the width of the mean chip. In side wheel grinding the strip AB becomes a curved strip of unit length. Its width is measured in a plane of rotation and it is logical therefore to take the defining width as equal to the mean chip thickness. It is assumed that the number surface density C of active grits is uniform. The mean grit or tooth is now defined as a tooth having a width equal to the width of the defining strip AB. The mean distance between successive grits is of course 1/k.

In peripheral grinding the width of the strip AB is equal to b. Then

$$\begin{aligned} k = 1/m &= C. \text{(area of strip)} \\ &= C.b.1 \\ &= C.b \end{aligned}$$

For surface grinding $\quad b = \frac{1}{2}.r.t_M$

Hence $\quad m = \dfrac{2}{C.r.t_M}$

Similarly, for plunge grinding $b = rt$

and $$m = \frac{1}{C.r.t}$$

In side-wheel surface grinding the width of the defining strip is equal to the mean chip thickness $\frac{1}{2}.t_M$. Thus

$$m = \frac{2}{C.t_M}$$

Similarly, for side-wheel plunge grinding,

$$m = \frac{1}{C.t}$$

Note that in general the value of m so defined is less for peripheral grinding than for side-wheel grinding by a factor r.

The ideal wheel is now defined as a milling cutter having teeth uniformly spaced a distance m at the cutting radius, each of which cuts out the statistical mean chip.

5. Determination of Grit Surface Density

The number of active grits per unit area (C) may be found experimentally as follows:[10]

A thin layer of soot is spread evenly on a ground metal surface and the wheel allowed to roll over the surface under its own weight. If the wheel be now allowed to roll over a flat sheet of white paper a soot replica of the surface grits is obtained. The number per unit area may then be estimated. It is assumed that the value of C found by this method is a fair estimate of the active grit surface density. The effect, if any, of the depth of cut is not allowed for. The value of C obtained is slightly dependent on the load on the wheel.

6. Effect of Dressing Technique on Grit Density

The dressing of a grinding wheel consists in traversing a diamond point or other tool across the face of the rotating wheel. The effect is to break out worn grits and loaded pores and present a set of fresh, sharp, grits. The surface density C, and hence the number of teeth presented to the work, is a function of the speed of traverse of the dressing tool. This may be shown as follows:

Assume that the dressing tool produces n fresh grits per unit length of its path relative to the wheel face. Let the feed of the tool across the wheel for each revolution be f. The developed length of the wheel circumference is $\pi.D$, Fig. 4. It is convenient to consider a width b of the wheel face equal to the mean chip width in the subsequent grinding operation. In the figure AB is a path of the tool relative to the wheel. It makes an angle θ with the end face of the wheel, where

$$\tan\theta = \frac{f}{\pi.D}$$

and θ = helix angle of the path.

The length of such a path is

$$AB = \frac{f}{\sin\theta}$$

if f is small compared with D.

The number of fresh grits produced on the path AB

$$= n\,(AB)$$

$$= \frac{n.f}{\sin\theta}$$

The average number of such complete paths (including the sum of incomplete paths) is

$$= \frac{b}{f}$$

Hence, the number of grits in the area considered is

$$K = \frac{n.f}{\sin\theta} \cdot \frac{b}{f}$$

and, since $\sin\theta \triangleq \tan\theta = f/\pi.D$ if θ is small, the number of grits per unit length of circumference is

$$k = \frac{K}{\pi.D}$$

$$= \frac{n.f}{f/\pi.D} \cdot \frac{b}{f} \cdot \frac{1}{\pi.D}$$

$$= \frac{n}{(f/b)}$$

The number of 'teeth' on the wheel face therefore depends strongly on the ratio of the feed per revolution during dressing to the mean chip thickness width in the subsequent grinding process. Halving the relative feed (f/b) tends to increase k by a factor of four. Similarly, since an increase in the depth of cut of the dressing tool tends to increase the number of fresh grits per unit length (n) it will also tend to increase k. However, if a sufficiently large number of passes of the dressing tool be made the total number of fresh grits will tend to a limiting value independent of the method of dressing. Finally, it should be noted that the same dressing technique may produce a different value for k (or C) according to the nature of the subsequent grinding operation (different b).

7. Classification of Grinding Operations

The essential similarities and differences as between one grinding process and another is brought out by a proper classification of operations. In the study of the cutting action of individual grits it has already been found convenient to distinguish between surface grinding and plunge grinding, and again between peripheral and side-wheel grinding. In surface grinding the active grits move parallel to the direction of motion of the work surface at the start of the cut. In plunge grinding the grit motion tends to be normal to the work velocity vector. Peripheral and side-wheel grinding are distinguished according as the active grits are on the periphery or the side face of the wheel. In each case the chip tends to flow radially inwards towards the wheel centre, but in peripheral grinding the chip is accommodated by the voids in the bond material and in cutting on the side face of the wheel the chip tends to move into the spaces between the grits. Further, the depth of cut normal to the side face of the wheel is always limited by the size of the grits, whereas in peripheral grinding the depth of cut seen by the wheel, as distinct from the grit, is not so limited.

A further classification of grinding operations is possible according to the geometric configuration of wheel and work. Fig. 5 illustrates the more usual processes. Each case, plunge grinding excepted, may be further subdivided into upcut or downcut grinding. In upcut grinding the wheel surface moves in the sense opposite to that of the work, in downcut grinding the active grits move in the same sense as the work.

Peripheral surface grinding, case (b) Fig. 5, is roughly analogous to cylindrical, barrel or slab milling. Cases (e) and (f) are termed peripheral side-wheel grinding since, although the side face of the wheel is presented to the work surface, cutting may take place at the periphery of the wheel rather than at the side face. Geometrically these processes are roughly similar to face milling. In practice wheel wear at the active corner of the wheel may modify the assumed action. When the wheel is shaped to grind a definite formed surface, as in case (j), the cutting action in a plane of rotation is essentially peripheral grinding. True side-wheel grinding is illustrated by the process of gear generation, case (h), in which the wheel is so controlled as to present the side face tangent to the surface to be ground.

Finally, peripheral surface grinding in which the wheel has a lateral feed or traverse parallel to its axis of rotation, may be further subdivided. In reciprocating surface grinding of a flat workpiece the traverse is intermittent and occurs at the start of each longitudinal traverse or cut. No lateral motion takes place during the cut and the forces on the wheel are restricted to the plane of motion. In cylindrical surface grinding on the other hand the lateral traverse is continuous, the wheel moves parallel to its axis during the cut and a transverse force is exerted on the wheel in the sense opposite to that of the motion. Although the lateral traverse rate is small relative to the longitudinal work speed the transverse force is not negligible and modifies the forces on the active grits.

8. Calculation of Chip Thickness - Plunge Grinding

In plunge grinding, Fig. 6, the work is fed directly into the wheel without cross feed. In calculating the grit depth of cut, i.e. the maximum chip thickness, the following assumptions are made:

1. The rate of feed relative to the wheel is v, normal to the wheel surface, and its magnitude is small compared with the surface speed of the wheel.

2. The chips are of uniform thickness t.

3. The length of a chip is equal to the mean width of the work in the direction of grinding.

4. The length of the path of contact between the wheel and the work is greater than the distance between successive grits. The chip thickness is then governed by the distance which the wheel advances between the engagement of successive grits.

5. The wheel is an ideal wheel.

The path of a grit relative to the work is AD, shown exaggerated in Fig. 6. The distance travelled by the work during the time of contact of a grit with the work is

$$q = DC = v . T_c$$

where T_c is the time of contact and is equal to ℓ / V.

The thickness of a chip is equal to the distance travelled by the work in the time T taken for a grit to move a distance equal to the grit spacing m. Thus

chip thickness $t = AB = CD$

$$= v.T$$

where $T = \frac{m}{V}$

Hence $t = \frac{v}{V}.\ m$

Since the chip is of uniform thickness

$$m = \frac{1}{C.r.t} \quad \text{for peripheral grinding}$$

$$= \frac{1}{C.t} \quad \text{for side-wheel grinding}$$

Therefore the chip thickness is given by

$$t = \frac{v}{V} \cdot \frac{1}{C.r.t.}$$

i.e. $$t = \sqrt{\frac{1}{C.r} \cdot \frac{v}{V}} \qquad (1)$$

where, for side wheel grinding, r is unity.

Since the surface speed of the wheel is V, = π.D.N, it is seen that the chip thickness is proportional to the square root of the infeed rate, and inversely proportional to the square root of the wheel diameter or the speed of rotation.

9. Calculation of Chip Thickness - Surface Grinding

The following analysis is valid for surface grinding at either the periphery or the side face of the wheel, cases (b) and (e), Fig. 5.

In Fig. 7 the surface speed of the wheel is V. The work surface BCD moves from left to right with speed v. It is convenient, however, to consider the motion of the wheel relative to the work. Thus in the figure O_1 and O_2 are successive positions of the wheel relative to the workpiece.

The wheel depth of cut d is the height of the uncut surface above the ground surface, i.e.

$$d = FA_2$$

For most practical cases it is possible to make the following assumptions:

1. The work speed (4 to 60 ft/min) is small compared with the surface speed (3,000 to 6,000 ft/min) of the wheel, i.e.

$$v \ll V \text{ or } \frac{v}{V} \ll 1.$$

2. The length ℓ of the path of contact of a grit with the work is greater than the distance m between successive grits of the ideal wheel. This implies that more than one grit is in

simultaneous contact with the work in a given plane of rotation. The chip thickness is therefore governed by the distance p moved by the work in the time taken for a grit to move a distance m round the circumference of the wheel, Fig. 8.

In upgrinding a given grit starts to cut at A_1, Fig. 7, almost vertically below the wheel centre O_1 and leaves the workpiece at C. The length ℓ of the chip formed by the mean grit is therefore equal to the length of the arc A_1C, i.e.

$$\ell = A_1 C$$
$$= A_1 A_2 + A_2 C$$
$$= BC + A_2 C, \text{ since } A_1 A_2 = BC.$$

If the time taken for a grit to travel the distance A_2C is T_c, the work moves a distance q, = CB, in this time, where

$$A_2 C = V.T_c$$

and $$BC = v.T_c$$

i.e. $$BC = \frac{v}{V} . A_2 C$$

Hence $$\ell = A_2 C (1 + \frac{BC}{A_2C})$$
$$= A_2 C (1 + \frac{v}{V})$$

Now $$A_2 C = \tfrac{1}{2}.D.\theta$$

where θ is the angle $A_2\hat{O}_2C$ (radian) subtended at the wheel centre by the arc A_2C. From the geometry of the figure

$$\cos\theta = 1 - \tfrac{1}{2}. \frac{d}{D}$$

Hence the length of a chip is

$$\ell = \tfrac{1}{2} (1 + \frac{v}{V}).D. \text{ arc } \left[\cos (1 - \tfrac{1}{2}. \frac{d}{D})\right] \qquad (2)$$

For downcut grinding the same expression is valid if the work speed v is taken negative.

The time interval between the engagement of successive grits is T, = m/V. The distance travelled by the work in this time is

$$p = v.T$$
$$= m. \frac{v}{V}$$

This distance is termed the pitch of the grits relative to the work, or the feed per grit.

If, in Fig. 7, CB is now interpreted as the pitch p, (compare C'B Fig. 8) then the maximum chip thickness is

$$
\begin{aligned}
t_M &= \text{EB, approximately, since CB is small} \\
&= \text{CB} \sin \theta, \quad \text{since } E\hat{C}B = \theta \\
&= p \sin \theta \\
&= m.\ \frac{v}{V}\ .\ \sin \theta \\
&= m.\ \frac{v}{V}.\ \sin\left[\arc\cos\left(1 - \tfrac{1}{2}.\ \frac{d}{D}\right)\right]
\end{aligned}
$$

Now, for surface grinding

$$
\begin{aligned}
m &= \frac{2}{C.r.t_M} \quad \text{in peripheral grinding} \\
&= \frac{2}{C.t_M} \quad \text{in side-wheel grinding}
\end{aligned}
$$

Hence it follows that the chip thickness is given by

$$
\begin{aligned}
t_M &= \sqrt{\frac{2}{C.r}\ .\ \frac{v}{V}\ .\ \sin\left[\arc\cos\left(1 - \tfrac{1}{2}.\ \frac{d}{D}\right)\right]} \\
&= \sqrt{\frac{4}{C.r}\ .\ \frac{v}{V}\sqrt{\frac{d}{D} + \frac{d^2}{D^2}}} \qquad (3)
\end{aligned}
$$

where r is unity for side-wheel grinding.

For surface grinding, case (b) Fig. 5, it is usually possible to assume that the wheel depth of cut (0.001 in. say) is small compared with the wheel diameter (8 to 24 in. say).

i.e. $\quad d \ll D, \quad \text{or} \quad \frac{d}{D} \ll 1$

It follows that the angle θ subtended by the arc of contact of the wheel with the work is small (about one degree). Hence making use of this assumption

$$
\begin{aligned}
A_2C &= \text{FC, approximately, Fig. 7} \\
&= \sqrt{(O_2C)^2 - (O_2F)^2} \\
&= \sqrt{(\tfrac{1}{2}.D)^2 - (\tfrac{1}{2}.D - d)^2} \\
&= \sqrt{D.d - d^2} \\
&= \sqrt{D.d} \quad \text{approximately, since d is relatively small.}
\end{aligned}
$$

Hence the length of the mean chip is now

$$
\ell = \left(1 + \frac{v}{V}\right)\sqrt{D.d} \qquad (4)
$$

Similarly, the maximum chip thickness is now

$$t_M = m.\frac{v}{V}.\sin\theta$$

$$= m.\frac{v}{V}.\theta \text{ , since } \theta \text{ is small}$$

$$= m.\frac{v}{V}.\frac{\text{arc } A_2C}{O_2C}$$

$$= m.\frac{v}{V}.\frac{\sqrt{D.d}}{\frac{1}{2}.D}$$

$$= 2.m.\frac{v}{V}\sqrt{\frac{d}{D}}$$

It follows from the definition of m that

$$t_M = \sqrt{\frac{4}{C.r}.\frac{v}{V}\sqrt{\frac{d}{D}}} \quad (5)$$

when d/D is small compared with unity. Again r is unity for side-wheel grinding. Since $V = \pi.D.N$ it follows that the maximum chip thickness is proportional to

$$r^{-\frac{1}{2}}.\ C^{-\frac{1}{2}}.\ v^{\frac{1}{2}}.\ d^{\frac{1}{4}}.\ D^{-\frac{3}{4}}.\ N^{-\frac{1}{2}}$$

For example to increase the chip thickness by a factor of two it is necessary to increase the wheel depth of cut by a factor of eight.

10. Peripheral Side Wheel Grinding, Case (f)

Peripheral side-wheel grinding is a cutting process roughly similar to face milling, Fig. 10a. In the Figure the workpiece moves from left to right and the grinding wheel rotates about an axis normal to the surface of the work. It is assumed that the wheel overlaps the work and that the wheel centre is offset from the centre line of the work by an amount e. The cutting action is assumed to take place at the periphery of the wheel, i.e. on $\overline{AB}$, Fig. 10(b).

It is convenient to consider the motion of the wheel relative to the work, i.e. as if the work were at rest and the wheel axis moving from right to left. Then O_1, O_2 O_3 and O_4 are successive positions of the wheel axis relative to the work. Similarly P_1, ..., P_4 are the corresponding positions of a mean grit. When the wheel centre is at O_1 the grit considered is at P_1 and the grit immediately preceding it is at P where it has just started to cut.

The distance between successive grits, measured on the circumference of the wheel, is m, $= 1/k = P_1P$. After a time T, $= m/V$, the grit will have moved from P_1 to P_2, where it starts to cut. In the same time the wheel centre will have advanced a distance p relative to the work, where

$$p = O_1 O_2$$
$$= P_1 P_2$$
$$= v.T$$
$$= \frac{m.v}{V}$$

The initial thickness of the chip at the start of the cut is

$$t_2 = P_2 P'_2$$
$$= p.\cos\phi_2 \text{ approximately.}$$

After a further time interval T_c the grit starts to leave the work at P_4, where

$$T_c = \frac{\ell}{V}$$
$$= \text{time of contact of grit with the chip.}$$

Here ℓ is the length of the path ($P_2\ P_3\ P_4$) of the grit. If the table speed is small compared with the peripheral speed of the wheel ($v \ll V$) then ℓ is approximately equal to the arc $P_2\ P'_3\ P'_4$ of the wheel circumference. Thus

$$\text{length of chip,} \quad \ell = \tfrac{1}{2} D (\phi_2 + \phi_4)$$

where ϕ_2 and ϕ_4 are in radian.

At P_4 the maximum chip thickness is

$$t_M = t_2 + t_4$$

where

$$t_4 = p.\cos\phi_4$$

Thus

$$t_M = p.(\cos\phi_2 + \cos\phi_4)$$
$$= \frac{v.m}{V}(\cos\phi_2 + \cos\phi_4)$$

From the definition of the mean grit

$$m = \frac{2}{C.r.t_M}$$

Hence

$$t_M = \frac{2.v}{C.r.V.t_M}(\cos\phi_2 + \cos\phi_4)$$

or

$$t_M = \left\{ \frac{2.v}{C.r.V}(\cos\phi_2 + \cos\phi_4) \right\}^{\frac{1}{2}}$$

The total chip volume is

$$= \tfrac{1}{2}.t_M.\ell$$

The angles ϕ_2, ϕ_4 may be calculated from the geometry of Fig. 10, thus

$$\sin\phi_2 = \frac{\tfrac{1}{2}.w - e}{\tfrac{1}{2}.D} = \frac{w - 2.e}{D}$$

and, similarly

$$\sin\phi_4 = \frac{w + 2.e}{D}$$

11. Cylinder Grinding

The geometry of cylindrical grinding is essentially the same as for peripheral grinding of a flat surface provided that allowance is made for the curvature of the workpiece. Figs. 9a and 9b illustrate internal and external grinding respectively. The work diameter is D_w. The wheel depth of cut is d, $= A_2F'$.

Now A_2F, Fig. 9(a), is the wheel depth of cut d' in an equivalent plane surface grinding operation which would give the same chip length A_2C. The formulae already found for surface grinding may therefore be applied to the present case provided that the wheel depth of cut is small compared with the wheel diameter and the work diameter and that the wheel depth of cut is replaced by d'. It is necessary, therefore, to calculate d' in terms of the actual depth of cut d and the curvature of the workpiece.

From the figure

$$CF = \tfrac{1}{2}.D\sin\theta = \tfrac{1}{2}.D_w.\sin\phi$$

or

$$\frac{\sin\theta}{\sin\phi} = \frac{D_w}{D}$$

Also

$$\cos\theta = \frac{O_2F}{O_2A_2} = 1 - \frac{2.d'}{D}$$

and

$$\cos\phi = \frac{OF}{OF'} = 1 - \frac{2(d - d')}{D_w}$$

Since

$$\frac{\sin^2\theta}{\sin^2\phi} = \frac{1 - \cos^2\theta}{1 - \cos^2\phi}$$

it follows that

$$\frac{D_w^2}{D^2} = \frac{d'/D - d'^2/D^2}{(d - d')/D_w - (d - d')^2/D_w^2}$$

Solving for d', and remembering that d'^2 and $(d - d')^2$ may be neglected in comparison with the remaining terms, gives

$$d' = \frac{d}{(1 + \frac{D}{D_w})}$$

It may be shown that a similar expression may be obtained for internal grinding if D_w is taken to be negative.

Hence the length of a chip is

$$\ell = (1 + \frac{v}{V}) \sqrt{\frac{D.d}{(1 + \frac{D}{D_w})}}$$

and the maximum chip thickness is given by

$$t_M = \sqrt{\frac{4}{C.r} \cdot \frac{v}{V} \sqrt{\frac{d}{D} \frac{1}{(1 + \frac{D}{D_w})}}} \qquad (6)$$

where D_w is positive for external grinding, negative for internal grinding.

12. Form Grinding

The principle of form grinding is illustrated in Fig. 11. The surface to be ground is inclined at an angle α to the wheel axis and the periphery of the wheel is shaped correspondingly. The thickness of material to be removed in one traverse of the wheel is x, measured normal to the ground surface. The cutting action is peripheral grinding. An ideal grit, of width b, cutting in the plane of rotation of the wheel, commences to cut at A, starts to leave the work surface at P, and finishes the cut at C. The width of the ideal chip varies as shown in the figure. The thickness of the mean chip increases along its length essentially as in surface grinding. From the figure

$$d_1 = d_2 - b.\cot \alpha$$

where the total wheel depth of cut is

$$d_2 = x.\sec \alpha$$

Numerically, since the thickness x is small, and when the angle α is not too nearly a right angle, the arc PC may be neglected by comparison with the arc AC. Similarly, d_2 is usually small compared with the effective wheel diameter in the plane of rotation of the grit considered. The chip length and thickness may therefore be calculated as for surface grinding with a wheel depth of cut d_2, = x. sec α. Evidently the wheel depth of cut increases as the angle α is increased.

When grinding a curved surface the above analysis remains true to a first approximation. Angle α is now the slope of the tangent to the formed surface at the point considered. However, when α approaches a right angle the tangent to the surface tends to be normal to the wheel axis. The wheel depth of cut is no longer small compared with the wheel diameter. Equations (2) and (3) now apply. Finally, when α is very nearly a right angle the wheel depth of cut is limited by the curvature of the profile to be ground. This case will be illustrated by reference to the form grinding of gears.

13. Form Grinding of Gears

Fig. 12(a) illustrates the form grinding of a gear tooth of involute form. OA is the radius of the base circle, diameter D_b, centre O. The thickness of material to be removed in one pass of a wheel is x, measured normal to the gear surface and shown exaggerated in the diagram. A grit starting to cut at B on the base circle would leave the gear surface at C, since each grit cuts in the vertical plane of rotation of the wheel. In general the depth of cut varies over the flank of the tooth and is greatest on that part of the flank nearest the base circle. Attention will be confined to this case. Also, in practice the gear profile may be undercut below the base circle by an amount d_o as shown. The total maximum wheel depth of cut is therefore $d + d_o$.

The calculation of d may be carried out as follows:

It may be assumed that both x and d are small compared with the base circle diameter. Similarly the angles θ, ψ are small also, Fig. 12(a).

The radius of curvature r of an element of arc ds_o of the involute is given by

$$ds_o = r\, d\theta_o$$

Also $$r = \tfrac{1}{2} D_b\, \theta_o$$ by the property of the involute curve.

i.e. $$ds_o = \tfrac{1}{2} D_b\, \theta_o\, d\theta_o \tag{7}$$

Also $\psi_o = \frac{\pi}{N}$, where N is the number of teeth on the wheel.

For the triangular element shown

$$\frac{dx}{ds} = \tan(\psi + \theta) \simeq \psi_o + \theta_o$$

$$\frac{dz}{ds} = \sec(\psi + \theta) \simeq \sec(\psi_o + \theta_o)$$

and $ds \simeq ds_o$

Hence, using equation (7)

$$\frac{x}{\frac{1}{2} D_b} = \int_o^{\theta^o} \theta_o \left(\frac{\pi}{N} + \theta_o\right) d\theta_o \tag{8a}$$

$$\frac{d}{\frac{1}{2} D_b} = \int_o^{\theta^o} \theta_o \sec\left(\frac{\pi}{N} + \theta_o\right) d\theta_o \tag{8b}$$

Equations (8a), (8b) have been solved numerically to give Fig. 12(b) which plots $d/(\frac{1}{2} D_b)$ as a function of $x/(\frac{1}{2} D_b)$ for a range of values of N. For example, if x = 0.001 in., D_b = 4 in; the wheel depth of cut is 0.05 in.

14. Chip Classification

In certain extreme cases, for example internal cylindrical surface grinding at very light depths of cut and high work speeds, the shape and type of chip formed may differ from the more usual cases already considered. A possible classification of chip types has been suggested[8]. The present report follows the classification offered in the reference cited, excepting that a more precise definition of the types of chip will be attempted. The method of definition requires a different notation and reveals certain differences in the method of classification, as will now be shown.

It will be recalled that the time of contact of a grit with the work is given by ℓ / V and the distance travelled by the work in this time is q, $= \ell \cdot \frac{v}{V}$. Similarly, the time interval between the engagement of successive grits is m/V, and the corresponding distance travelled by the work in this time is p, $= m \cdot \frac{v}{V}$. The method of classification is based on the relative magnitudes of the quantities p and q, as indicated in the following table :

Chip type	Definition		
A	$p < q$	or	$\frac{m}{\ell} < 1$
B	$p = q$		$\frac{m}{\ell} = 1$
C	$m > p > q$		$\frac{m}{\ell} > 1 > \frac{v}{V}$
D	$p = m > q$		$\frac{m}{\ell} > \frac{v}{V} = 1$
E	$p > m > q$		$\frac{m}{\ell} > \frac{v}{V} = 1$

In general the chip length ℓ is a function of the depth of cut, wheel diameter and the configuration of wheel and work. It is a function also of the ratio v/V. The spacing m between successive grits is a function of wheel structure and the method of dressing. Also, from the definition of the mean grit, it is also a function of the maximum chip thickness and therefore to some extent of the grinding geometry.

The theoretical analysis of the grinding process so far considered has been restricted to the type A chip. For this chip m is less than ℓ and there are a number of grits simultaneously in contact with the work. The maximum chip thickness is determined by the pitch per grit p, and is always less than the wheel depth of cut.

For the remaining types of chip only one grit is in contact with the work at any instant; or none. The chip thickness is now determined by the magnitude of the quantity q rather than p. Hence, at the relatively low work speeds already considered ($v/V \ll 1$) q is small compared with ℓ (type B or type C chip). Also q is equal to or less than p. The maximum chip thickness is now given by

$$t_M = q. \sin \theta , \qquad \text{approximately,}$$

where θ is the angle turned through by the wheel in time T_c. Hence it may be shown that

$$\begin{aligned} t_M &= \frac{v}{V}. \ell. \sin \theta \\ &\doteq 2. \ell. \frac{v}{V}. \sqrt{\frac{d}{D}} \\ &= 2.d. \frac{v}{V} \qquad \text{since} \qquad \ell \doteq \sqrt{D.d} \end{aligned}$$

Fig. 13 illustrates the formation of the chips of type A and C. The type A chip is the type I chip of the reference cited[8]. The types III and v chips of that reference are both type C in the present classification. The type III chip is formed when a grit is always in contact with the work. The type v chip is formed when intermittent cutting occurs as in the figure. In both cases it is evident that a type C chip is formed when the wheel depth of cut is very small; the figure shows the depth of cut exaggerated relative to the wheel diameter.

The effect of very high work speeds on A and C chips leaves the geometry essentially unaltered. However it is no longer possible to assume that p and q are small compared with ℓ, and a modification to the theory is required when calculating the chip thickness. The length of the chip is no longer calculable by an elementary formula; however, for the type v chip it is evident that the length is very nearly twice that given by the formula $\ell = (1 \pm \frac{v}{V}) \sqrt{D.d}$.

Finally, chips D and E occur only when the work speed is equal to or greater than the wheel speed. For most practical purposes these conditions need not be considered.

15. Grinding at high work speeds

When the work speed is not small compared with the surface speed of the wheel the feed per grit is no longer small compared with the arc of contact between wheel and work. The following modified analysis for this case[8] refers to a type I chip, Fig. 7, $(p < q,\ m < \ell)$. The maximum chip thickness is

$$\begin{aligned} t_M &= BE \\ &= O_2E - O_2B \\ &= O_2E - \sqrt{(O_2F)^2 + (FB)^2} \end{aligned}$$

Let CF = x and CB = p, then FB = x - p. Hence

$$\begin{aligned} t_M &= \tfrac{1}{2}.D - \sqrt{(x-p)^2 + (\tfrac{1}{2}.D - d)^2} \\ &= \tfrac{1}{2}.D\left\{ 1 - \sqrt{1 + \left[\frac{d^2 + x^2 + p^2 - 2.x.p - D.d}{(\tfrac{1}{2}.D)^2} \right]} \right\} \\ &= \tfrac{1}{2}.D.(1 - \sqrt{1+z}) \\ &= \tfrac{1}{2}.D\left[1 - (1 + \tfrac{1}{2}.z - \ldots)\right] \\ &= -(\tfrac{1}{2}.D)^2 . \frac{z}{D} \end{aligned}$$

Therefore $$t_M = 1 + \frac{1}{D.d}\left[2.x.p - (x^2 + p^2 + d^2)\right]$$

Now d^2 may be neglected in comparison with the remaining terms; also $x^2 = (cF)^2 = D.d$. Hence,

$$\frac{t_M}{d} = \frac{2.p}{\sqrt{D.d}} - \frac{p^2}{D.d} \qquad (9a)$$

and since

$$p = m.\frac{v}{V} = \frac{2}{C.r.t_M} . \frac{v}{V}$$

$$\frac{t_M}{d} = \frac{2.B}{(t_M/d)} - \frac{B^2}{(t_M/d)^2} \qquad (9b)$$

where $$B = \frac{2}{C.r.d.\sqrt{D.d}} . \frac{v}{V}$$

Equation (9b) for t_M/d has three real roots for B less than 32/27. For each positive value of B there are two possible positive values of t_M/d, as shown in Fig. 15. The reference cited[8] chooses the greater of the two positive values. However, since $p < q$ it is unlikely that the undeformed chip thickness will approach the wheel depth of cut for this type of chip, since there are a number of grits in simultaneous contact. Hence it is suggested that the smaller value of t_M/d is the correct one. The modification of theory required for cylindrical grinding is given in Ref. 8.

When v/V is small B^2 can be neglected and the following solution is obtained, viz.,

$$\frac{t_M}{d} = \sqrt{2.B}$$

i.e.

$$t_M = \sqrt{\frac{4}{C.r} \cdot \frac{v}{V} \sqrt{\frac{d}{D}}}$$

The case $p > q$ (i.e. $m > \ell$, type III chip) has already been considered in section 14 for low work speeds. It was found that the chip thickness is now governed by the magnitude of q, rather than the feed per grit. Fig. 14 illustrates this case for high work speeds. The chip thickness is BE, where CB = q. It may be shown that equation (9a) remains valid for this case if p is replaced by q in that equation.

Since $q = \ell . v/V$ it follows that t_M/d is now given by

$$\frac{t_M}{d} = \frac{2.\ell}{\sqrt{D.d}} \cdot \frac{v}{V} - \frac{\ell^2}{D.d} \cdot \frac{v^2}{V^2} \qquad 9(c)$$

The l ength of a chip is

$$\ell = A_1 B' (1 + \frac{v}{V})$$

where $A_1B' = \frac{1}{2}.D(\theta + \theta_1)$

and $\theta \doteq \sin\theta$, if θ is small

$= 2\sqrt{\frac{d}{D}}$, if d/D is small.

$$\theta_1 = \frac{\text{arc } A'A}{\frac{1}{2}.D}$$

$\doteq p/D$, approximately

Hence $\ell = \frac{1}{2}.(2\sqrt{D.d} + p)(1 + \frac{v}{V})$

i.e.

$$\frac{\ell}{\sqrt{D.d}} = (1 + \frac{v}{V})(1 + \frac{\frac{1}{2}.B}{(t_M/d)}) \qquad (10)$$

Eliminating ℓ from equations 9(c) and (10) gives

$$\frac{t_M}{d} = 2.A - A^2 \qquad (11a)$$

where

$$A = \frac{v}{V}(1 + \frac{v}{V})(1 + \frac{\frac{1}{2}.B}{(t_M/d)}) \qquad (11b)$$

This solution yields two real and positive values of t_M/d for each value of B, when v/V is less than $\frac{1}{2}.(\sqrt{5} - 1)$, as shown in Fig. 16. Again it is suggested that the smaller of the two solutions is the correct one for this problem.

When v/V is small A^2 can be neglected and hence

$$\frac{t_M}{d} = 2.A$$

$$= \frac{2.\ell}{D.d} \cdot \frac{v}{V}$$

$$= 2d. \frac{v}{V}, \quad \text{as in section 14.}$$

Finally, it may be remarked that the above analysis, for the case $m > \ell$, is applicable to the case of a very light depth of cut where the length of contact between wheel and work is very small.

16. Specific Energy

Fig. 17 shows the forces acting on a grinding wheel. The horizontal cutting force is F_H and the vertical normal force is F_V. The transverse force F_T opposes the sense of the continuous cross traverse. It may be neglected when the cross traverse is zero or when the traverse is intermittent, as in reciprocating surface grinding.

In surface grinding the work done per unit time is

$$F_H (V \pm v)$$

The volume of metal removed per unit time is

$$v.w.d$$

where w is the effective width of the wheel (Section 23).

The specific energy (u') is defined as the work done per unit volume of metal removed, thus

$$u' = \frac{F_H(V \pm v)}{v.w.d}$$

$$= \frac{F_H}{w.d} \left(\frac{V}{v} \pm 1\right)$$

The energy required per unit cross section area of metal removed is

$$U = \frac{F_H(V \pm v)}{w.v}$$

$$= u'.d$$

For a given material the specific energy is found experimentally to be a function of the chip depth of cut.(7, 8, 9) Fig. 18 shows a typical plot of u' against the maximum undeformed chip thickness t_M. The specific energy increases with decreasing chip thickness, as in metal cutting, up to the point A where, for dry surface grinding, it tends to remain constant. At A, in a typical case of grinding a hardened steel(8)

$$u' = u_c = 10.10^6 \text{ in.-lb/in}^3.$$

$$t_M = t_c = 40.10^{-6} \text{ in.}$$

The following table compares the specific energy for grinding, fracture of a mild steel specimen and cutting with a single point tool.

fracture of M.S.	$u' = 10^5$ in.-lb/in^3.
cutting	$= 5.10^5$
grinding	$= 10.10^6$

If u is defined as the specific energy required to cut a given material with a 15° rake single point tool and a feed rate (t) of 0.01 in./rev., the following approximate relations have been suggested:[9]

$$u' = 30.u \qquad t_M < t_c \tag{13a}$$

$$u' = \frac{0.0092.u}{t_M^n} \qquad t_M > t_c \tag{13b}$$

where $n \triangleq 0.8$.

The increase in specific energy with a decrease in undeformed chip thickness has been ascribed to the following effects:[15]

1. A size effect; the strength of a material increases with decreasing grain size when the cross section is of the same order of magnitude as the grain size.

2. The dulled edge of the cutting grit has an increasing relative effect on the cutting force per grit when the chip is small.

3. The total energy U is made up of two parts - a shear energy U_s and a friction energy U_f, i.e.

   $$U = U_s + U_f$$

   The shear energy decreases as the chip thickness is reduced. The friction energy remains sensibly constant and relatively independent of the chip thickness. Hence a decrease in chip thickness results in U_f being a greater proportion of the total energy. The specific energy therefore tends to increase as the chip thickness is reduced.

For small thicknesses of chip the specific energy is sensibly constant. This appears to indicate a cutting process of a different nature. It is suggested that the removal of material may now take place by abrasion rather than by true cutting. The swarf appears as an ash rather than as chips.

In surface grinding a type I chip is normally formed. On the other hand it has been pointed out that, for internal cylinder grinding, the flat portion of the $u' - t_M$

curve is associated with a type III chip[8], Fig. 19. As the chip thickness is decreased further the specific energy tends to increase again. This portion of the curve is associated with a type v chip. The increase in energy is said to be due to the greater relative effect of chip-grit interface friction, due to the greater length of the chip[8].

17. Grit Forces

The work done in a surface grinding process may be written

$$u'.(b.d).v \qquad \text{per unit time}$$

The number of chips produced per unit time is

$$V.\ k$$

$$= V.b.C$$

The work done per chip is, therefore

$$\frac{u'.b.d.v}{V.b.C}$$

$$= \frac{u'}{C} \cdot \frac{v}{V} \cdot d$$

The mean force $\overline{F}$ on a grit is $= \dfrac{\text{work done/chip}}{\text{undeformed chip length}}$

$$= \frac{u'.v}{C.V} \cdot \frac{d}{\ell}$$

Also, since

$$t_M = \sqrt{\frac{4}{C.r} \cdot \frac{v}{V} \cdot \frac{d}{\ell}}$$

it follows that[9]

$$\overline{F} = u'.r.t_M^2$$

The maximum force per grit may be assumed to be twice the mean force, and is therefore

$$F_M = 2.\overline{F}$$

$$= 2.u'.r.t_M^2$$

The specific cutting pressure, $F' = \dfrac{\text{force per grit}}{\text{area of chip}}$

$$= \frac{F_M}{b.t_M}$$

$$= \frac{2.u'.r.t_M^2}{\frac{1}{2}.r.t_M^2}$$

$$= 4.u'$$

For very fine grinding ($t_M < t_c$) u' is independent of t_M. Thus

$$F_M = (\text{constant}).\ t_M^2$$

For thicker chips ($t_M > t_c$)

$$u' = \frac{\text{constant}}{t_M^n}$$

where $0.6 \leqslant n \leqslant 0.9$.

Hence $F_M = (\text{constant}).\ t_M^{(2-n)}$

18. Cutting properties of a wheel

The cutting properties of a wheel are characterised by:-

the rate of metal removal

the rate of wear

the surface finish produced

the tendency to "load"

the tendency to "burn" .

Loading of a wheel is the collection of metallic chips in the voids which are not dislodged when the grits clear the work. Burning of the work surfaces arises from excessive temperature rise at the work surface.

Each of the cutting properties are a function of:

(a) the inherent properties of the wheel, according to its grade, structure and so on.

(b) The apparent properties of the wheel as modified by the task set in the particular grinding operation.

The task set may be simply described in terms of the load per grit and the volume of the mean chip relative to the void volume available.

19. Intrinsic Hardness

Consider unit volume of a wheel, taken near the surface. Let the volumes of grit, bond and voids be V_g, V_b and V_p respectively. The following relation is satisfied

$$V_g + V_b + V_p = 1$$

If the grit volume is made up of n grits per unit of total volume, each of mean volume v_g then $V_g = n.v_g$, and

$$n.v_g + V_b + V_p = 1$$

The relative volumes of grit, bond and pores are not independent. For example, an increase in bond volume, i.e. in strength of bond post, is balanced by a loss of porosity and/or grit volume. On the other hand it might be possible to decrease the number of grits n while increasing the grit size so as to leave V_g, and therefore V_b and V_p, unaltered.

Suppose that for a particular grinding process it is possible to specify the length, thickness and volume of the mean chip. The load on a mean grit is then determined in principle. Then, for a given abrasive the tendency for the grit to fracture is some function of the mean volume per grit, v_g. The tendency for a bond post to fracture is a function of V_b, being greater as the bond post volume is decreased.

The intrinsic hardness of a wheel is connected with its strength, i.e. the converse of its tendency to lose grits. Thus when confronted with a standard task the strength might be increased by increasing v_g and V_b. If v_g is increased relative to V_b the tendency to bond post fracture, rather than grit fracture, is increased.

20. Wheel Loading

As already seen, an increase in v_g or V_b will be accompanied by a decrease in the relative void fraction. This might be acceptable so long as the voids remaining are, on average, sufficient to accommodate the volume of the mean grit chip. Moreover, a decrease in the space available for voids may be accompanied by a decrease in the volume of the mean void. There will be a limiting state in which the void space is no longer able to cope with the volume of the mean chip. At this point V_p, or the volume of the mean void, v_p, is too small relative to the chip volume. The chip tends to score and rub on the work, producing a defective surface and a tendency to grinding burn. The chip may become embedded in the pore and is not released when the cut of a grit is completed. This state of affairs will occur when the decrease in void volume is accompanied by an increase in bond post strength. On the other hand it may happen that the increased bond post strength is now insufficient for the imposed task. The overloaded bond posts tend to fracture and release the chips. Although chips may no longer be left embedded in the wheel there may still be a tendency for the wheel to score and burn.

The elimination of wheel loading may be obtained

(a) at the expense of bond post volume and strength

(b) by reducing grit diameter, which is accompanied by a reduction in grit strength and an increase in the tendency towards grit fracture. This is not normally desirable, for grit fracture tends to produce two or more smaller grits without additional void space for the extra grits.

(c) by reducing the number of grits per unit volume. In practice this will alter both the size of chip produced and the surface finish (C is a function of n).

The table below illustrates the probable effects on wheel characteristics of changes in inherent properties of the wheel. A positive sign refers to an increase in the quantity considered, a negative sign to a decrease. The table assumes a given task, i.e. chip thickness and volume, and neglects the additional effect of changes in C on the task.

n	v_g	V_g	V_p	V_b	C	bond post fracture	grit fracture	loading
-	-	.	.	.	+	.	+	.
.	+	+	-	.	.	+	-	+
.	+	+	.	-	.	+	-	-
+	.	+	.	-	+	+	.	-
+	.	+	-	.	+	+	.	+
.	.	.	+	-	.	+	.	-

21. Relative Hardness

It may be shown that the relative hardness of a wheel is altered by the grinding conditions.

The mean force on a grit is

$$\bar{F}_g = u' . t_M^2 . r$$

and $u' = 30.u \qquad t < t_c$

$$u' = \frac{0.0092.u}{t_M^n} \qquad t > t_c$$

It follows that, for very small chips, the force per grit varies as t_M^2. For t greater than the critical value t_c the force per grit varies as $t_M^{(2-n)}$; i.e. as $t_M^{1.2}$ if $n \triangleq 0.8$.

Since
$$t_M = \left[\frac{4}{C.r} \cdot \frac{v}{V} \cdot \sqrt{\frac{d}{D} (1 \pm \frac{D}{D_w})^{-1}} \right]^{\frac{1}{2}}$$

it follows that t_M and $\bar{F}_g$ are both increased, and the effective hardness is decreased, by:(9)

1. decreasing work speed v,
2. increasing wheel speed N, or wheel diameter D
3. increasing the number of active grits C (i.e. by dressing technique, change of grain size or spacing)
4. decreasing the wheel depth of cut d
5. increasing the work diameter D_w in external grinding, or decreasing the work diameter in internal grinding.

22. Wheel Wear

Fig. 20 illustrates the progress of total wheel wear, as measured by the total volume of wheel lost, plotted on a base of total volume of metal removed. After initial dressing the rate of wear of the fresh wheel is high (OA in Fig. 20). Over the normal operating range AB the rate of wear is sensibly constant. The grinding ratio is the ratio

$$\frac{\text{rate of metal removal}}{\text{rate of wheel wear}}$$

$$= \frac{x}{y}$$

and is measured on the straight portion of the wear curve.

It has been suggested that the nature of wheel wear may be by attrition, fracture or by chemical action[13].

Attritious wear is the wearing away of the sharp cutting edges of the grits. This is said to correspond to the flat portion of the wear curve[13]. Eventually a flat surface is produced on the grit and the load on the grit tends to increase. The grit load may then be sufficient to rupture either grit or bond post and fracture wear is said to result. The wear rate then rises sharply (curve BC, Fig. 20). If the bond posts or grits do not fracture the dull grits remain and the wheel tends to become glazed. It is suggested in the reference cited that if wear is purely mechanical in nature, and if the forces on the grits are insufficient to cause fracture wear, then neither the wheel grade nor mechanical variables (wheel settings) should affect the grinding ratio. This has been borne out by experiments on plunge grinding[13]. On the other hand, where fracture wear does occur wheel wear is dependent on the mechanical variables (wheel and work speed and so on), wheel grade and the total metal removed.

Under certain conditions chemical wear may occur. This arises from chemical reactions between grit and work. The tendency towards chemical wear is greatest at high grit face - chip interface temperatures. Apparently chemical wear is not normally important when grinding steel with an aluminium oxide wheel. Chemical reactions between grit and work are important when grinding titanium for example.

The foregoing is the conventional interpretation of the process of grinding wheel wear. On the other hand it has been shown[12] that in surface grinding the wear at the periphery of the wheel is non uniform. When this is taken into account an alternative interpretation of the wear process is possible. This will be discussed in the following section.

23. Wear Profile in Surface Grinding

It has been shown experimentally[12] that, for surface grinding, the wear at the wheel periphery is non-uniform. Fig. 21 illustrates the characteristic profile obtained under normal cutting conditions. Initially, after dressing, the leading edge A of the wheel is square. Grits at this edge are relatively unsupported and weakly bonded to the wheel. When cutting starts the wheel crumbled away at the leading edge and the wear is comparatively rapid, as is found on the initial portion of a wear curve. Eventually the stable profile BCD is established, in which each grit has roughly the same support. Further wear takes place at the active cutting face CD. The result is that the face CD retreats uniformly across the periphery towards the trailing edge E. That is, the distance BC, = X, increases, leaving the shape of the profile unaltered. This regime of stable wear rate corresponds to the linear portion of the wear curve. Finally, when the trailing edge D of the profile reaches the face E of the wheel, this edge, being relatively unsupported, crumbles away. This regime corresponds with the final, relatively steep, portion of the wear curve.

In contrast with the wear theory advanced in the previous section normal wear on the linear portion of the wear curve takes place by both attrition and fracture on the active face C^D.

In Fig. 21 the cross feed per traverse of the wheel is CC' = DD' = f. For surface grinding of flat surfaces the cross feed takes place intermittently at the end of each traverse, before the wheel recommences its cut. The wheel profile shown in the figure was obtained under these conditions. In cylindrical grinding, on the other hand, the cross feed is continuous and f may be interpreted as the cross feed per grit. That is

$$f = s. \frac{m}{V}$$

where s is the cross feed rate in ft/min., say.

In general, the cross feed may be less than, equal to, or greater than the effective width w of the wheel, Fig. 21. The condition f < w will be considered first. In this case the effective wheel depth of cut is less than the nominal geometric depth of cut AB, d. For the profile shown the maximum effective depth of cut is

$$\begin{aligned} C'C'' &= D'D'' \\ &= d_e \\ &= f. \tan \alpha \end{aligned}$$

and is independent of the nominal depth of cut.

The wear profile shown is fixed by the nominal depth of cut, the angle α of the wear land CD, and the distance X representing the current amount of total uniform wear. If it is assumed that the porosity of the wheel is sufficient to accommodate the chip formed then it may be shown that the angle of the wear land is a function of the task set the wheel. The following argument is due in part to the reference cited and is developed here on the basis of the theory given in the present report.

Consider therefore the effect of a change in cutting conditions. This change is experienced by the wheel as a change in the task per grit. The task per grit may be defined as the maximum grit force. For surface grinding this is a function of the specific energy and the maximum chip thickness. For most chip thicknesses the maximum force per grit varies as $t_M^{1.2}$. For cylindrical grinding, in which there is a continuous cross feed, the effective hardness of the wheel is modified by virtue of the lateral force associated with the cross feed. This is some function of the cross feed and may be assumed to increase with the cross feed. For the moment only surface grinding with intermittent cross feed will be considered. The task set the mean grit then reduces to some function of the maximum chip thickness.

The following postulate will now be formulated:(12)

The wheel profile is self regulating and adjusts itself so that the task experienced by the mean grit tends to be equal to some critical value.

In effect, each grit tends to cut at a fixed force per grit F^* (or a fixed maximum chip thickness t^*).

Evidently, for a given wheel and given machine settings (d, v, V, ... etc.) the only self adjustment allowed to the wheel is an alteration of the effective wheel depth of cut by a change of slope of the wear land. Since $d_e = f.\tan\alpha$, the effective depth of cut (and therefore chip thickness and task) can be increased by an increase in the angle of the wear land. Thus an increase in the severity of the cutting conditions can be compensated for by a change in α so as to maintain the chip thickness unaltered.

Since

$$t_M = \sqrt{\frac{4}{C.r}\ \frac{v}{V}\ \sqrt{\frac{d_e}{D}}}$$

where d_e has been substituted for d, the following changes in slope of the wear profile may be predicted (a positive sign denotes an increase, a negative sign a decrease in the magnitude of the quantity considered).

d	D	v	V	C	α
+					.
	+				+
		+			-
			+		+
				+	+

The effect of a reduction in α for a constant nominal depth of cut is to produce a longer, flatter, wear profile CD.

Since the effective depth of cut, and therefore the chip thickness, is independent of the nominal depth of cut, changes in d leave the land angle unaltered.

In general the flatter the wear profile the greater will be both the wear rate and the rate of metal removal, for the effective cutting width of the wheel is increased.

From the formula for d_e it is seen that an increase in the intermittent cross feed requires a reduction in the angle α in order to maintain the same effective depth of cut. This is true only so long as the feed is less than the effective width of the wheel.

If, on the other hand, the intermittent cross feed is greater than the width w of the profile ($f > w$) it follows that the maximum effective wheel depth of cut is equal to the nominal depth of cut, Fig. 22. The self regulating action of the wheel is lost. If, however, the grits are still cutting within their capacity the profile of Fig. 22 is a possible one (but the postulate of a constant task per grit no longer holds).

Changes in cutting conditions which increase the task beyond some critical value cannot be accommodated by a wheel having this latter profile, for the effective depth of cut is now fixed. It may be expected that the wheel profile would crumble in order to establish another profile such that f is again less than w and the self-regulating characteristics of the wheel are regained.

If the cross feed is continuous, as in cylindrical grinding, f is now the cross feed per grit. The foregoing arguments still hold, except that the critical value of the task which is accepted by the mean grit is lower due to the transverse force and the land angle is correspondingly smaller. An increase in continuous cross feed rate will increase the lateral force on a grit and hence the wear land angle will be reduced still further to maintain the same task.

It may be concluded that a fundamental parameter is the working value of the critical task per grit. This is a function of the inherent properties of the wheel and is independent of the machine settings, (except that cross feed in continuous cross traverse may affect this value). For surface grinding the critical task is measured by a critical chip thickness given by

$$t^{+} = \sqrt{\frac{4}{C.r}\ \frac{v}{V}\ \sqrt{\frac{d_e}{D}}}$$

where $d_e = f.\tan\alpha \qquad (f < w)$

$= d \qquad (f > w)$

In peripheral side wheel grinding it is possible that non uniform wheel wear occurs at the leading edge of the wheel. However it has not been found possible to predict a simple wear profile able to provide self regulation of the effective depth of cut. Similarly, there is no theoretical reason for a characteristic wear profile to occur in gear grinding, for example. However these conclusions have not been tested experimentally.

Finally, when calculating the average force on a grit and the rate of metal removal, the term effective wheel width (w') has been used. For surface grinding this is defined as follows:

The rate of metal removal per unit (R) $= w'.d.v$

But $R = (\text{area } CC'DD').v$ (Fig. 21)

$= f.d.v$

i.e. $R = w'.d.v = f.d.v$

$w' = f$

$=$ feed per traverse

This width is not to be confused with the width w of the wear land.

24. Grinding Temperatures

The temperature changes arising from the grinding process which will be considered here relate to:

1. the mean chip temperature θ_t,

2. the mean temperature at, or just below, the ground surface, θ_s.

The former is of interest in connection with wheel wear, the latter is associated with surface damage, grinding burn and residual stresses in the workpiece.

It has been suggested(16) that θ_t is primarily a function of the specific energy of cutting u'. On the other hand a dimensional analysis(9) indicates that θ_t is proportional to the expression

$$u' \sqrt{\frac{V.\,t_M}{K.\,\rho\,.\,c}}$$

where K = thermal conductivity, ρ = density and c = specific heat of the workpiece material. From the relations between u' and chip thickness (equations 13a, b) it may be shown that, for a given workpiece,

θ_t is proportional to $\sqrt{V.\,t_M}$, $\qquad t_M \leqslant t_c$

or θ_t is proportional to $\sqrt{V.\,t_M^{-0.3}}$ $\qquad t_M > t_c$

When fine grinding, therefore, chip temperatures may be reduced by reducing the chip thickness. When the chips are relatively large ($t_M > t_c$) a reduction in θ_t may be obtained by an increase in chip thickness.

The surface temperature θ_s, together with the time at this temperature, governs the tendency for austenite to be formed at the surface of a steel workpiece. This may be accompanied by an overtempered martensite layer beneath the surface of a hardened steel workpiece(14).

Austenite is formed when the following conditions are satisfied:

(a) the temperature is above the critical value for the steel
(b) the temperature is maintained above the critical temperature for a sufficiently long time
(c) the surface is subsequently quenched.

The temperatures produced by grinding are very high. However published experimental results indicate widely varying values according to the point at which the temperature is measured. The time at temperature is very short. The greatest temperatures are reached during the time of contact of the grit with the work. This may be of the order of 2.10^{-8} seconds. Quenching may be effected by air or by the coolant. It may also be assisted by the freshly cut nature of the ground surface and the relatively large area presented by the grinding scratches, together with heat flow to the main body of the metal.

If formed, the surface layer of austenite appears to be very thin, having a thickness less than the depth of the scratches formed by the grits[14]. The transformed layer is relatively soft and affects the wear properties of the surface. Further, when put to use, the austenitic surface layer may suffer a reverse transformation due to load or abrasion. The associated dimensional change gives rise to residual surface stresses and surface cracks. Under good grinding conditions the formation of austenite need not occur[10].

The severe temperature gradients existing in the work during grinding tend to produce severe residual stresses. The temperature rise during grinding tends to produce tensile residual stress. The effect of any subsequent surface quenching is to produce compressive residual stresses. It has also been suggested that the effect of mechanical working of the surface by the wheel also tends to produce compressive residual stress at the surface[18].

Experimentally it has been found that the pattern of residual stress due to grinding gives rise to the greatest tensile stress at a layer about 0.001 in. below the surface[17]. The stress at the surface tends to be compressive. In dry surface grinding the quenching effect is due to air only, the surface stress may then be tensile. The effect of a grinding fluid is primarily a lubrication effect, thus reducing the mechanical working of the material and also the temperatures attained. The quenching effect of the coolant is probably to make the surface stress more compressive.

At a surface speed v the rate at which work is done during grinding is

$$E = w'.d.v.u'$$

The energy per unit volume is u'. The energy per unit surface area is

$$U = \frac{E}{w'.v}$$

$$= u'.d$$

It has been suggested[16] that the surface temperature θ_s is a function of U.

i.e. $$\theta_s = \text{function } (u'.d)$$

Hence, from equations (13a) and (13b)

$$\theta_s = \text{function } (d) \qquad t_M \leqslant t_c$$

$$= \text{function } \left(\frac{d}{t_M^{0.8}}\right) \qquad t_M > t_c$$

The tendency towards surface damage of the work is greatest in fine grinding and is decreased as the chip thickness is increased. Surface damage is most effectively minimised, however, by limiting the wheel depth of cut.

Even if the grinding temperature is high, phase changes or overtempering can be minimised by reducing the time at temperature, i.e. by reducing the time of contact between grit and work.

The time of contact is

$$T_c = \frac{1}{v} = \frac{\sqrt{D.d}}{v}$$

and is reduced by an increase in work speed, or a reduction in wheel depth of cut or wheel diameter. This argument does not apply when considering the formation of residual stresses due to purely thermal effects. The time of contact is not important in this case. These residual stresses are minimised largely by reducing θ_s, i.e. by reducing the wheel depth of cut.

To sum up, the technique of reducing or minimising surface damage and residual stresses is as follows:(16)

1. To reduce u' by
   - (a) use of sharp cutting grits
   - (b) use of a good lubricant
   - (c) increase in t_M by a decrease in wheel speed or diameter, an increase in work speed or a decrease in C by suitable dressing technique.

2. Reduce the wheel depth of cut

3. Reduce the time of contact (surface damage only) by
   - (a) a small depth of cut
   - (b) a small diameter wheel
   - (c) an increase in work speed

It has been shown (section 25) that in <u>surface grinding</u> it may not always be possible to control the effective depth of cut or chip thickness by changes in machine settings. The wheel may tend to impose its own cutting conditions as far as individual grits are concerned. The self regulating property of wheel wear tends to impose a constant task on the grit. The above arguments relating changes in θ_s to the machine settings require modification in this case.

Under these conditions surface grinding with intermittent cross feed is regulated by the condition that the task per grit is sensibly constant ($t_M = t^*$). The relation between chip thickness and effective depth of cut d_e is

$$t_M = t^* = \sqrt{\frac{4}{C.r} \; \frac{v}{V} \sqrt{\frac{d_e}{D}}}$$

It follows that changes in v, V, D and C respectively result in changes in the effective depth of cut such that, for a given wheel,

$$d_e.v^2 = \text{constant}$$

$$\left.\begin{array}{l} d_e/V = \text{constant} \\ d_e/D = \text{constant} \end{array}\right\} \text{or, } \frac{d_e}{ND^2} = \text{constant}$$

$$d_e/C^2 = \text{constant}$$

If it is now assumed that

$$\theta_s = \text{function } (u'.d_e)$$

where u' is now a constant (constant task) then a reduction in θ_s results from a reduction in d_e. That is, the severity of workpiece surface damage is reduced by:

1. an increase in v

2. a decrease in V

3. a decrease in D

4. a decrease in C

The above conclusions are essentially the same as already found.

A change in intermittent cross feed on the other hand has no effect on workpiece damage. For, as in section 25, an increase in cross feed results in a change of wheel profile, which keeps d_e and therefore θ_s. constant.

On the other hand θ_t, which is a function of u' (a constant according to this theory) is unaltered by changes in any of the machine variables. θ_t can be altered only by a change in the characteristic properties of the wheel, i.e. by a change in the task of which the wheel is capable. A reduction in chip temperature can be effected only by a change in wheel grade or hardness.

In <u>cylindrical grinding</u> the cross feed is continuous. Since

$$d_e = f.\tan\alpha$$
$$= s.\frac{m}{V}.\tan\alpha$$

a change in cross feed per grit can be produced by a change in the rate of feed (s) or by a change in V or C (m). Changes in s or in f do not in themselves have any effect on grinding temperature. The effect of changes in m and V has already been discussed.

Finally, it may be remarked that for reciprocating surface grinding the tendency towards grinding burn appears to be much more sensitive to changes in machine settings than is the case for other grinding processes. For example, consider the effect of a change in work speed v. The tendency to burn is a function of the product $u'.d_e$. In surface grinding, assuming a constant task per grit, u' is constant and d_e is inversely proportional to the square of the work speed. For other grinding processes, assuming conventional theory to apply and no self regulating action, d_e is a constant and u' is proportional to $t_M^{-0.8}$. That is

$$u' \propto t_M^{-0.8}$$

or

$$\propto \frac{1}{v^{0.4}} \qquad \text{from equation (5)}$$

as compared with $1/v^2$ for surface grinding, where the modified theory applies. A similar argument applies to changes in V, D and C.

25. Application of Grinding Theory

Let is be assumed that a suitable peripheral surface grinding technique has been developed which gives a satisfactory wear rate or grinding ratio, and in which the tendency towards grinding burn has been satisfactorily minimised. It is required to determine suitable grinding conditions for the form grinding of gears, for example. The problem has been set in this way for two reasons. Firstly, surface grinding techniques are better known and most easily developed. Secondly, experiments on surface grinding reveal certain fundamental properties of the grinding process, as has been shown. The following argument assumes that the postulate of a constant task per grit applies to the surface grinding process studied. The wheel profile then adjusts itself by preferential wheel wear so as to perform a characteristic constant task measured by the constant chip thickness t^*.

Since

$$t^* = \sqrt{\frac{4}{C.r} \cdot \frac{v}{V} \sqrt{\frac{d_e}{D}}}$$

it follows that

$$\frac{d_e}{D} = Z. \frac{V^2}{v^2}$$

where

$$Z = \frac{t^{*4}\, C^2 .\, r^2}{16}$$

and is a constant for a given wheel, which will be termed the grinding constant. The value of Z is found experimentally from the slope α of the wheel profile. For, since the effective wheel depth of cut d_e is given by

$$d_e = f.\ \tan\alpha$$

it follows that, for the surface grinding process,

$$Z = \frac{f.\ \tan\alpha}{D} \cdot \frac{v^2}{V^2}$$

In the form grinding process it will be assumed that the wheel cuts under the conditions, depth of cut and task, imposed upon it. The concept of a constant task per grit does not apply. Ideally, therefore, it is necessary to so adjust the imposed conditions that the chip depth of cut in form grinding is not greater than that which is found to occur in the surface grinding operation. Moreover it may be necessary to limit the grinding temperature θ_s and the time of contact of a grit with the work in order to limit the tendency towards grinding burn to a degree not greater than in the surface grinding process, assuming the latter to be satisfactory.

For the same wear rate, grinding ratio or task per grit the wheel depth of cut is now limited by the condition

$$d_{f1} = Z.\ D_{f2} \cdot \frac{V_f^2}{v_f^2}$$

where the suffix f denotes form grinding.

In order to limit the grinding temperature to the accepted value the depth of cut is found from

$$d_{f2} \leqslant d_e \qquad\qquad t_f \leqslant t_c$$

or

$$d_{f2} \leqslant d_e \left[\frac{t_f}{t^*}\right]^{0.8} \qquad\qquad t_f \geqslant t_c$$

In the latter case, since

$$t^* = \left[\frac{16.Z}{C^2.r^2}\right]^{0.25}$$

and

$$t_f = \left[\frac{16}{C^2.r^2} \cdot \frac{d_{f2}}{D_f} \cdot \frac{v_f^2}{V_f^2}\right]^{0.25}$$

it follows that, solving for d_{f2},

$$d_{f2} \leqslant d_e^{1.25}\left[\frac{v_f^2}{Z.\ D_f.\ V_f^2}\right]^{0.25}$$

Finally, if the time of contact T_c is to be limited to the accepted value, the criterion is

$$\frac{\sqrt{D_f \cdot d_{f2}}}{v_f} \leqslant T_c = \frac{\sqrt{D.\ d_e}}{v}$$

i.e.

$$d_{f3} \leqslant d_e \cdot \frac{D}{D_f} \cdot \frac{v_f^2}{v^2}$$

Thus, for equally good or better grinding conditions, it is necessary that d_f shall be equal to or less than the least of the three quantities d_{f1}, d_{f2} and d_{f3}. In practice it may not be possible to limit the wheel depth of cut so as to fulfil all three conditions simultaneously. In that case, if d_f is greater than d_{f1}, the grinding ratio will be correspondingly lower than for the standard surface grinding operation. If d_f is greater than d_{f2} the grinding temperature, and hence the tendency towards grinding burn or the residual stresses, will be correspondingly greater. Finally, if d_f is greater than d_{f3}, the tendency towards a phase transformation in the ground surface will be correspondingly greater.

26. Conclusions

Grinding operations have been considered according to the mode of action of the active grit, the geometry of wheel and work, the type of chip produced and the method of lateral cross-traverse. It has been shown that, for most conditions of cutting, the action of wheel wear in peripheral surface grinding is such as to present a constant task to the grit. This is achieved by a self regulating action in which the wheel depth of cut is varied by changes in the angle of the wear land. The wheel depth of cut is always less than the nominal geometrical depth of cut.

Incorporation of the concept of a constant task per grit in to current grinding theory allows of a more satisfactory explanation of the effects of changes in machine variables. The self regulating action peculiar to surface grinding does not appear to occur in other grinding processes. However, the fundamental information which may be obtained from experiments on peripheral surface grinding may be applied to other processes, such as form grinding, when once the grinding constant has been determined for the wheel in question. It has been found theoretically that peripheral surface grinding is more sensitive than other processes to changes in machine settings, at least as far as grinding burn is concerned. Experimental results[4] suggest that the grinding ratio is unaltered by changes in (calculated) chip area or length, or by changes in cutting speed. This is in accordance with the Purcell wear profile theory, as is the fact that doubling the time rate of metal removal tends to double the time rate of wheel wear. The reference cited suggests that the reason for these results is that wheel wear took place by abrasion or chemical wear, rather than by bond post rupture. It may now be seen that these results could well be true also when bond post rupture does occur. On the other hand these conclusions are not necessarily true for processes other than peripheral surface grinding. For these processes there is no longer any reason why the grinding ratio should remain constant with changes in cutting speed; unless bond post rupture does not occur.

In general it may be pointed out that the conclusions reached in this report are primarily theoretical and require further experimental verification.

27. Acknowledgements

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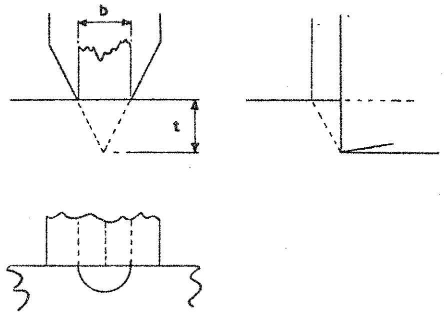

FIG. 1. CUTTING WITH VEE SHAPED TOOL.

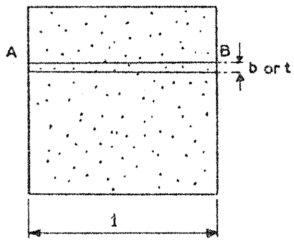

FIG. 3. GRIT SPACING AND SURFACE DENSITY.

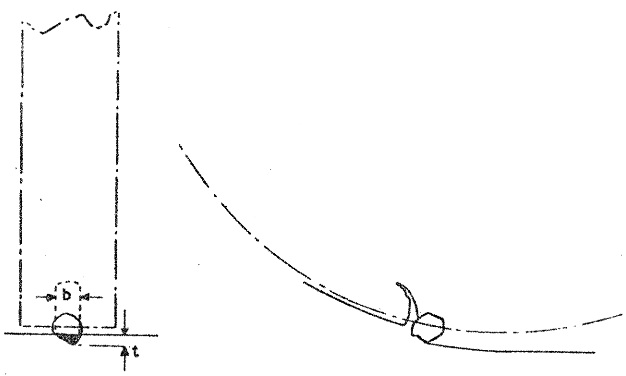

FIG. 2a. PERIPHERAL GRINDING.

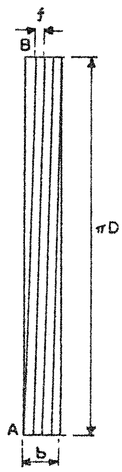

FIG. 4. EFFECT OF FEED / REV ON C DURING DRESSING.

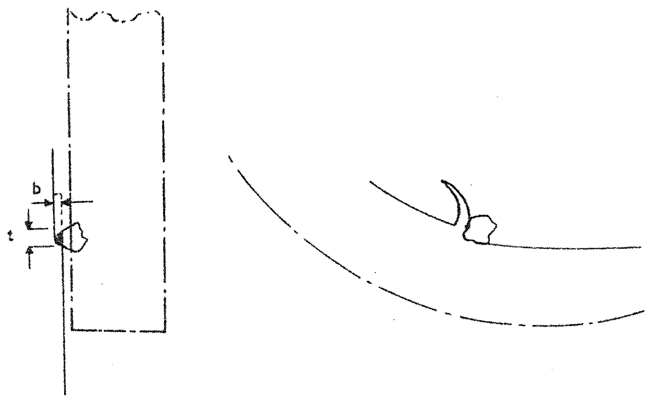

FIG. 2b. TRUE SIDE-WHEEL GRINDING.

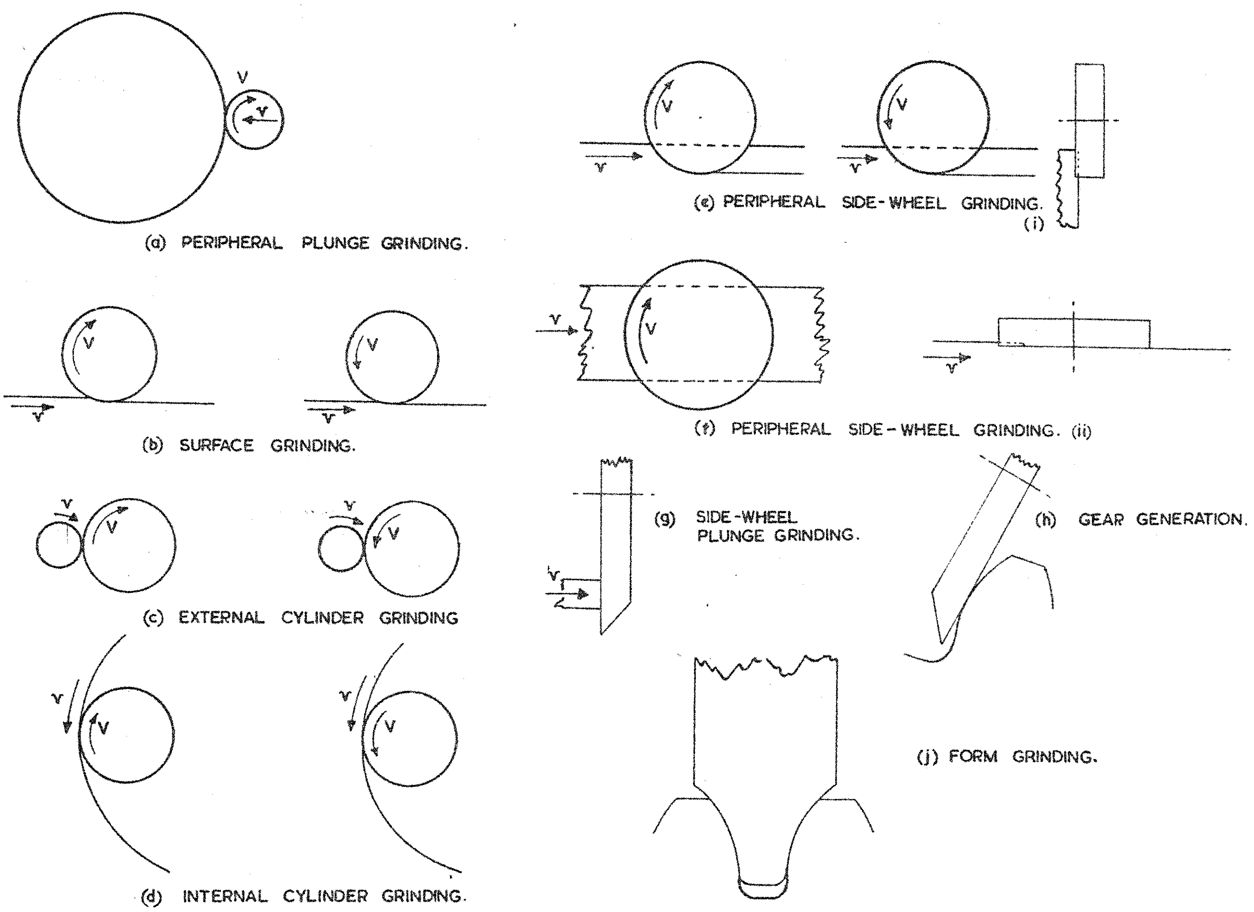

FIG.5. CLASSIFICATION OF GRINDING OPERATIONS.

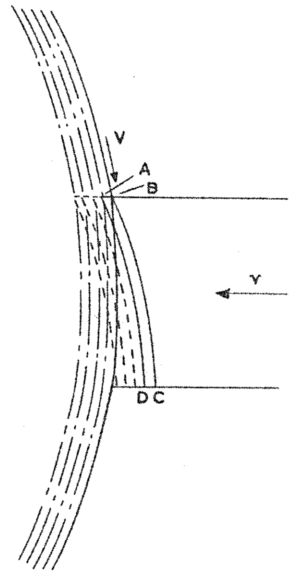

FIG. 6. GEOMETRY OF PLUNGE GRINDING.

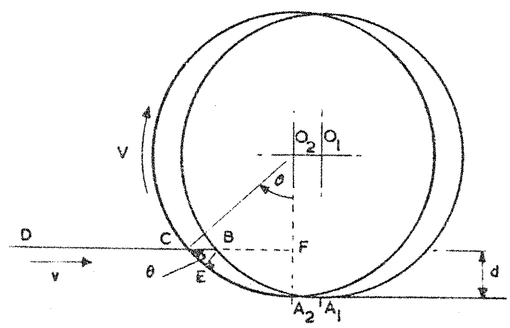

FIG. 7. GEOMETRY OF PERIPHERAL SURFACE GRINDING.

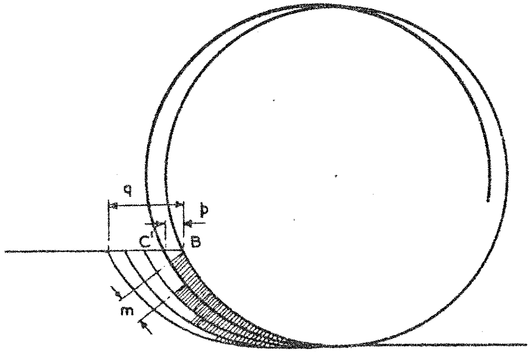

FIG. 8. CHIP THICKNESS IN SURFACE GRINDING.

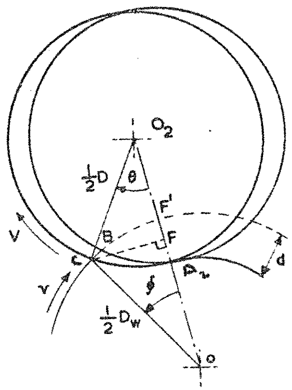

FIG. 9a. GEOMETRY OF EXTERNAL CYLINDER GRINDING.

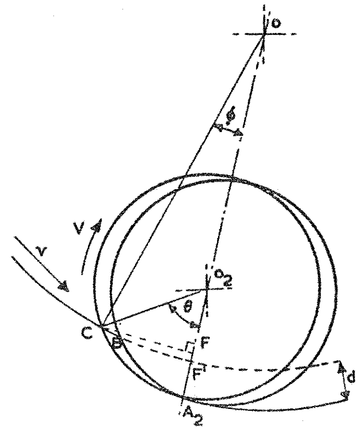

FIG. 9b. GEOMETRY OF INTERNAL CYLINDER GRINDING.

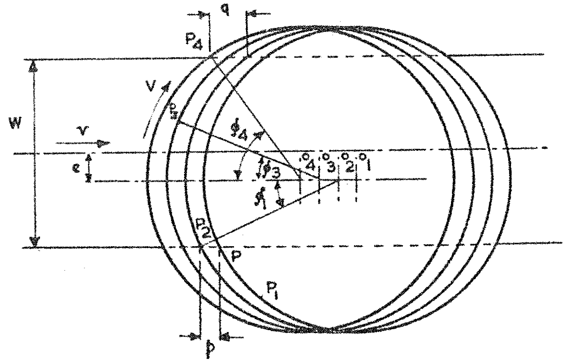

FIG.10. GEOMETRY OF PERIPHERAL SIDE WHEEL GRINDING.

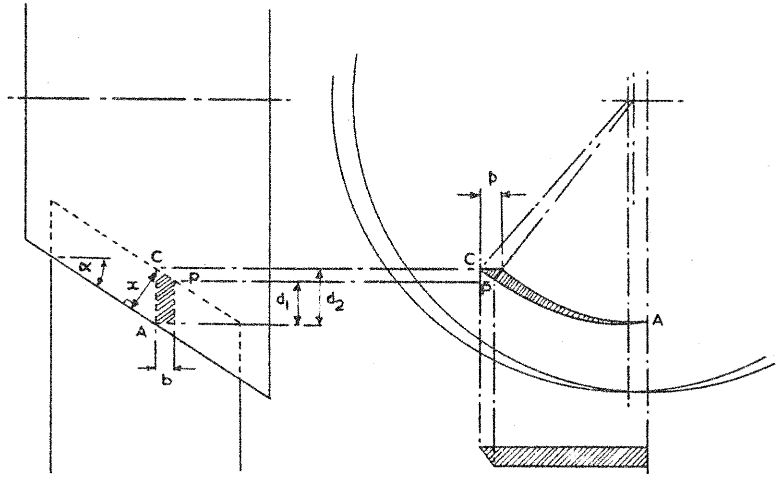

FIG. 11. GEOMETRY OF FORM GRINDING.

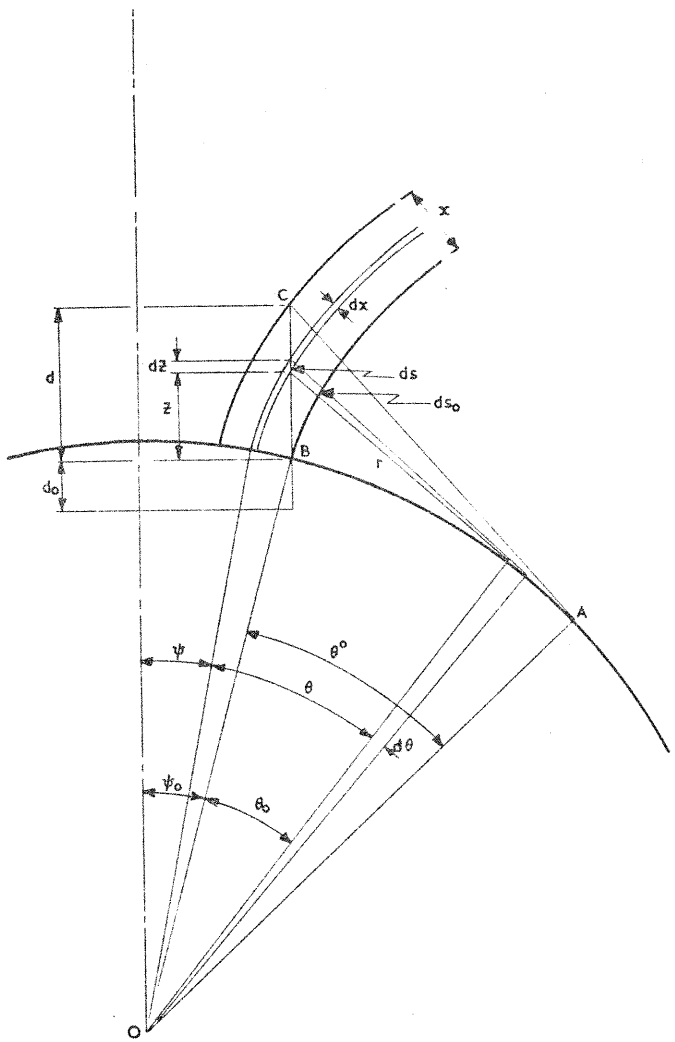

FIG. 12 a FORM GRINDING OF GEARS

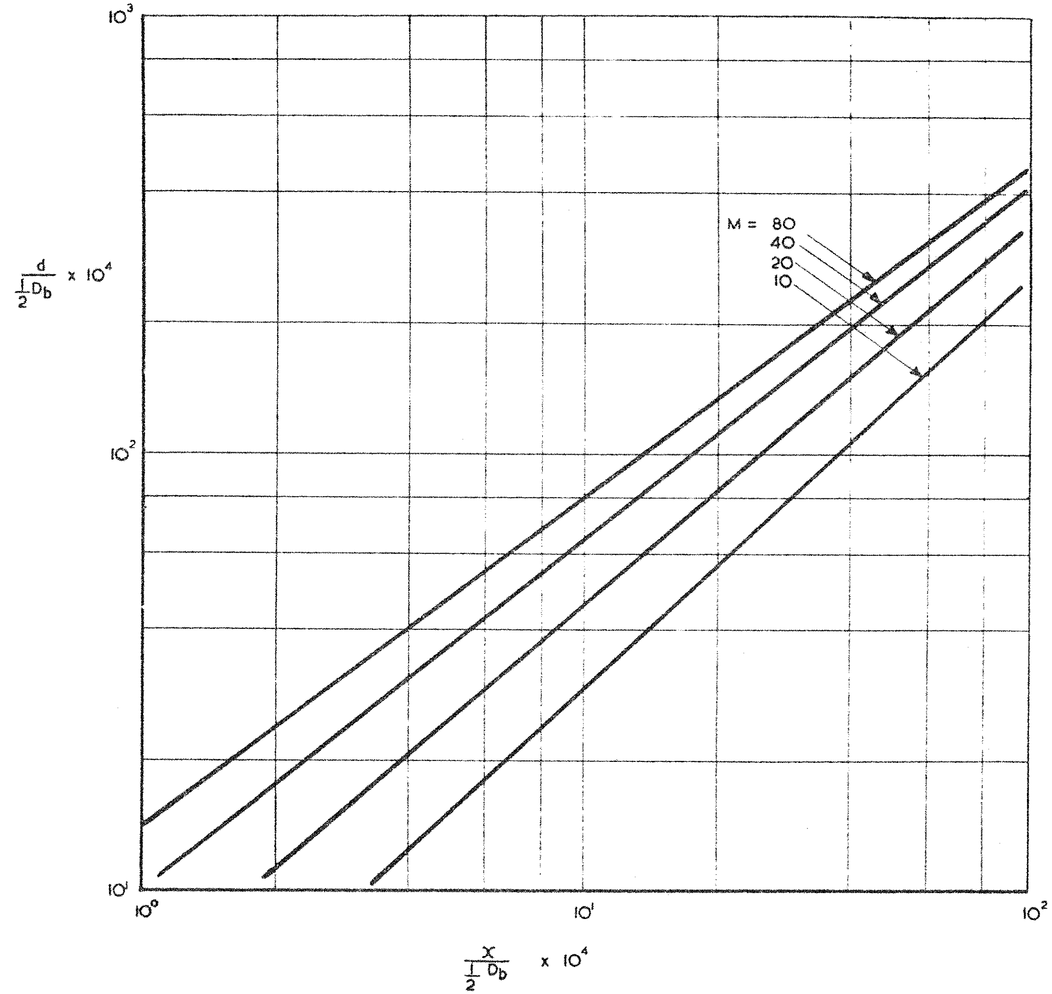

FIG. 12 b FORM GRINDING OF GEARS

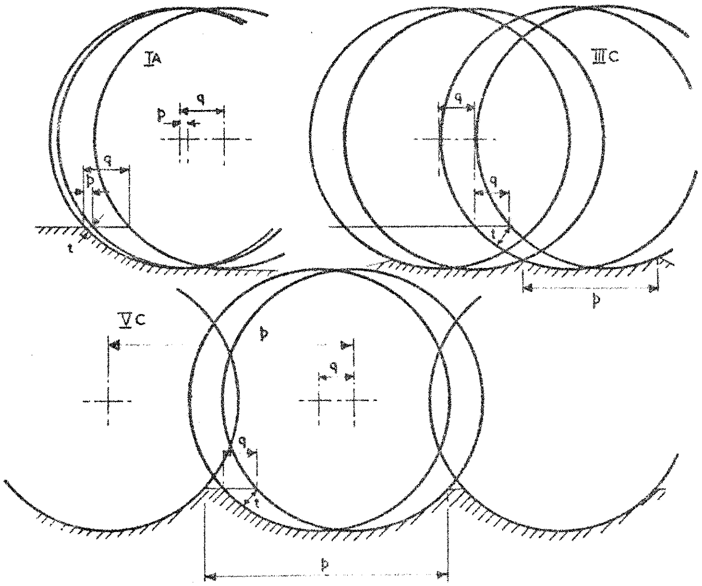

FIG. 13. CHIP CLASSIFICATION.

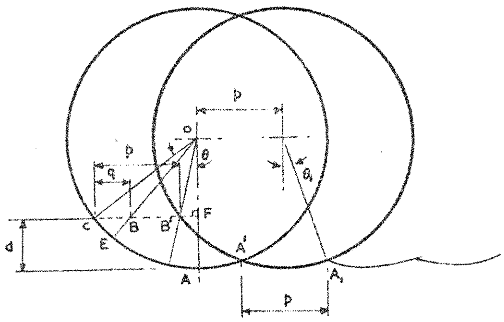

FIG. 14. GEOMETRY OF TYPE III CHIP.

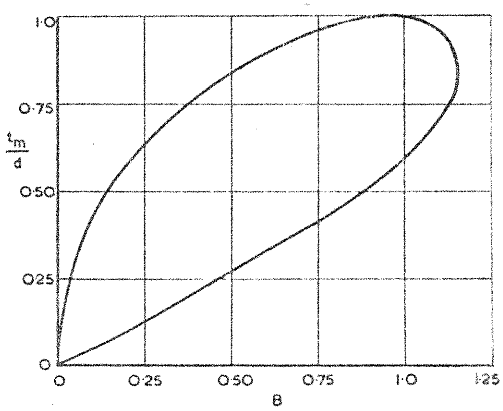

FIG. 15. CHIP THICKNESS AT HIGH WORK SPEEDS, CASE 1.

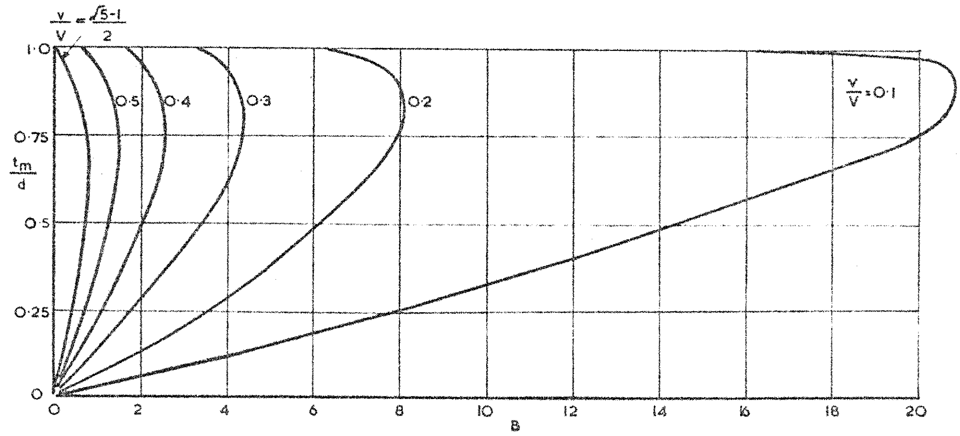

FIG. 16. CHIP THICKNESS AT HIGH WORK SPEEDS, CASE 2.

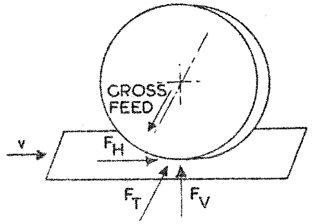

FIG. 17. FORCES ON GRINDING WHEEL.

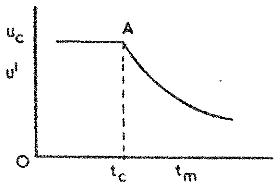

FIG. 18. $u' - t_m$ GRAPH

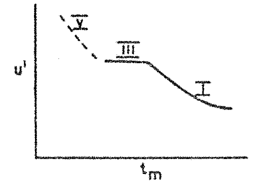

FIG. 19. $u' - t_m$ GRAPH FOR INTERNAL CYLINDER GRINDING.

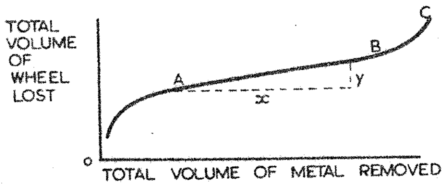

FIG. 20. WEAR CURVE.

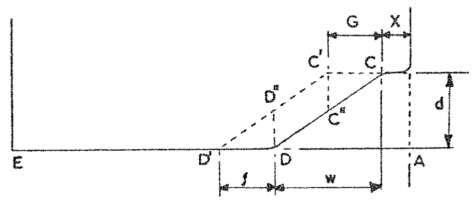

FIG. 21. WEAR PROFILE (f < w).

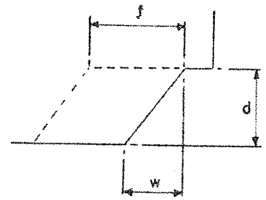

FIG. 22. WEAR PROFILE (f > w).