THE COLLEGE OF AERONAUTICS

CRANFIELD

SOME VALUES OF THE REDUCTION FACTOR FOR PLASTIC BUCKLING OF PLATES IN SHEAR

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Some Values of the Reduction Factor for Plastic Buckling of Plates in Shear

- by -

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SUMMARY

In this note values are obtained for the plasticity reduction factor for plates in shear, based on incremental plasticity theory. These values are compared with other theoretical results and with some experimental results.

CONTENTS

NOTATION

x_1, x_2, x_3	Rectangular cartesian coordinates. Ox_1 and Ox_2 are axes in the middle surface of the plate and Ox_3 is normal to the plate.
V_3	Displacement in the x_3 direction.
S	Stress resultants applied to the edges of the plate.
D	Flexural rigidity of the plate.
b	Width of long plate.
a	Side of square plate.
s	Length of half waves of the buckled plate.
α	Slope of the nodal lines.
E	Young's Modulus
G	Shear Modulus
G_T	Tangent Shear Modulus
G_s	Secant Shear Modulus
ν	Poisson's Ratio
μ	$\dfrac{2\,G_T\,(1+\nu)}{E}$
m	Material characteristic in the expression $\epsilon = \frac{f}{E} + \epsilon_R \left(\frac{f}{f_R}\right)^m$ (See R.Ae.S Data Sheet 00.02.04)
η	Plasticity reduction factor $= \dfrac{\text{actual buckling stress}}{\text{elastic buckling stress}}$
τ_b	Buckling shear stress lb/in^2

Introduction

The plastic buckling stress of a plate is usually obtained by multiplying the elastic buckling stress by a factor η, the plasticity reduction factor.

D. H. Smith, Ref. 3, obtained some experimental values of η for plates in shear and the aim of this note is to calculate theoretical values of η, based on incremental theory, for comparison. Other theoretical values of η, based on total strain theory and some suggested approximations for η, G_s/G and G_T/G are also shown in Figures 3 and 4.

1. The Approximate Solution of the Equation for Plastic Shear Buckling of an Infinite Plate

Consider equation (14), Ref.1,

$$S = -\tfrac{1}{2}D \frac{\int_0^a\int_0^b\left\{\left(\frac{\partial^2 V_3}{\partial x_1^2}\right)^2 + \left(\frac{\partial^2 V_3}{\partial x_2^2}\right)^2 + 2\nu\frac{\partial^2 V_3}{\partial x_1^2}\frac{\partial^2 V_3}{\partial x_2^2} + \frac{4G_T(1-\nu^2)}{E}\left(\frac{\partial^2 V_3}{\partial x_1 \partial x_2}\right)^2\right\} dx_1\, dx_2}{\int_0^a\int_0^b\left(\frac{\partial V_3}{\partial x_1} \quad \frac{\partial V_3}{\partial x_2}\right) dx_1\, dx_2}. \qquad \ldots(1.1)$$

Following Timoshenko, let the deflection be

$$V_3 = A \sin \frac{\pi x_2}{b} \sin \frac{\pi}{s}(x_1 - \alpha x_2). \qquad \ldots(1.2)$$

With

$$\left.\begin{aligned} \xi_1 &= \frac{x_1}{b} \\ \xi_2 &= \frac{x_2}{b} \\ \lambda &= \frac{s}{b} \end{aligned}\right\}, \qquad \ldots(1.3)$$

the deflection (1.2) becomes

$$V_3 = A \sin \pi\, \xi_2 \sin \frac{\pi}{\lambda}(\xi_1 - \alpha \xi_2) \qquad \ldots(1.4)$$

and equation (1.1) becomes

$$S = \frac{-D}{2b^2} \frac{\int_0^{2\lambda}\int_0^1 \left[\left(\frac{\partial^2 V_3}{\partial \xi_1^2}\right)^2 + \left(\frac{\partial^2 V_3}{\partial \xi_2^2}\right)^2 + 2\nu \frac{\partial^2 V_3}{\partial \xi_1^2}\frac{\partial^2 V_3}{\partial \xi_2^2} + 2\mu(1-\nu)\left(\frac{\partial^2 V_3}{\partial \xi_1 \partial \xi_2}\right)^2\right] d\xi_1\, d\xi_2}{\int_0^{2\lambda}\int_0^1 \left(\frac{\partial V_3}{\partial \xi_1} \frac{\partial V_3}{\partial \xi_2}\right) d\xi_1\, d\xi_2}, \quad \ldots(1.5)$$

where $\mu = \frac{2\, G_T(1+\nu)}{E}$.

Substituting (1.4) in equation (1.5),

$$S = \frac{D\pi^2}{2b^2}\left\{ \left(\frac{1}{\alpha} + \alpha^3\right)\frac{1}{\lambda^2} + 6\alpha + \frac{\lambda^2}{\alpha} + \left(\frac{\alpha}{\lambda^2} + \frac{1}{\alpha}\right)(2\nu + 2\mu\ (1-\nu)) \right\} \ldots(1.6)$$

By putting $\frac{\partial S}{\partial \lambda} = 0 = \frac{\partial S}{\partial \alpha}$ for stationary values of S, the following two equations are obtained,

$$\left.\begin{aligned} -\left(\frac{1}{\alpha} + \alpha^3\right)\frac{2}{\lambda^3} + \frac{2}{\alpha}\lambda &= \frac{2\alpha}{\lambda^3}\left\{2\nu + 2\mu\ (1-\nu)\right\} \\ \left(3\alpha^2 - \frac{1}{\alpha^2}\right)\frac{1}{\lambda^2} + 6 - \frac{\lambda^2}{\alpha^2} &= \left(\frac{1}{\alpha^2} - \frac{1}{\lambda^2}\right)\left\{2\nu + 2\mu\ (1-\nu)\right\} \end{aligned}\right\} \ldots(1.7)$$

The solutions of equations (1.7) and (1.6) are tabulated in Table 1 and plotted in Figure 1.

2. The Approximate Solution of the Equation for Plastic Shear Buckling of a Square Plate

For the square plate, following Timoshenko, let the deflection be

$$V_3 = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} \sin \frac{m \pi x_1}{a} \sin \frac{n \pi x_2}{a} . \qquad \ldots(2.1)$$

Substituting (2.1) in equation (1.1),

$$S = -\frac{D \pi^4}{64 a^2} \quad \frac{\sum_m \sum_n a_{mn}^2 \left[m^4 + n^4 + m^2 n^2 \left\{ 2\nu + 2\mu \ (1 - \nu) \right\} \right]}{\sum_m \sum_n \sum_p \sum_q a_{mn} a_{pq} \dfrac{m\, n\, p\, q}{(p^2 - m^2)(n^2 - q^2)}} . \qquad \ldots(2.2)$$

The system of constants a_{mn}, a_{pq}, to make S a minimum, are selected as in Ref. 2. This process leads to the following set of equations,

$$\begin{bmatrix} (2+F)y & 4/9 & 0 & 0 & 0 \\ 4/9 & 16(2+F)y & -4/5 & -4/5 & 36/25 \\ 0 & -4/5 & (82+9F)y & 0 & 0 \\ 0 & -4/5 & 0 & (82+9F)y & 0 \\ 0 & 36/25 & 0 & 0 & 81(2+F)y \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{22} \\ a_{13} \\ a_{31} \\ a_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} , \qquad \ldots(2.3)$$

where $y = -\dfrac{\pi^2}{32} \cdot \dfrac{\pi^2 D}{a^2 S}$

and $F = 2\nu + 2\mu \ (1 - \nu)$.

For the set of equations (2.3) to have a solution other than $a_{11} = a_{22} = \ldots = a_{33} = 0$, the determinant of the coefficients must vanish; and hence the critical values of S can be found.

These results are tabulated in Table 2 and plotted in Figure 2.

3. Comparison of Some Plasticity Reduction Factors

In Figures 3 and 4 the values of the plasticity reduction factor obtained in paragraphs 1 and 2 are compared with :-

(i) Bijlaard's theoretical factors based on total strain plasticity theory.

(ii) Values of G_s/G and G_T/G based on the Mises-Hencky distortion energy hypothesis.

(iii) Experimental values obtained by D.H.Smith.

All the plasticity reduction factors have been calculated for aluminium alloy DTD.603 (.2% proof stress in the 48,000 lb/in^2 region; m = 16 in the stress-strain expression), the material used by Smith in his tests.

4. Discussion of Results

The calculations in this note for aluminium alloy DTD.603 show as usual, see Benthem Ref. 4, that incremental theory predicts values of η that are too high. Although Smith's test results are not conclusive, the general trend is more in agreement with theoretical values of η based on total strain theory.

5. References

1.	Hemp, W.S.	"Plastic Buckling of a Plate in Shear", College of Aeronautics Note No. 64, May 1957.
2.	Timoshenko, S.	"Theory of Elastic Stability", 1936.
3.	Smith, D.H.	"Investigation into the Plastic Buckling of Plates in Shear", College of Aeronautics Thesis, June 1957.
4.	Benthem, J.P.	"On the Buckling of Bars and Plates in the Plastic Range". Pt. II N.A.C.A. T.M.1392 March 1956.
5.	Krivetsky, A .	"Plasticity Coefficients for the Plastic Buckling of Plates and Shells". Journal of the Aeronautical Sciences, June 1955.
6.	Barrett, A.J.	"The Estimation of Shear Web Buckling Stresses", Royal Aeronautical Society Draft Data Sheet 02.03.S.11.
7.	Barrett, A.J.	"Note on Non-Elastic Shear Buckling", Royal Aeronautical Society Draft Data Sheet 02.03.S.12.

TABLE 1

μ	α	λ	$\frac{b^2 S}{D\pi^2}$	$\left(\frac{b^2 S}{D\pi^2}\right) / \left(\frac{b^2 S}{D\pi^2}\right)_{\mu=1.0}$
1.0	0.707	1.23	5.67	1.0
0.8	0.686	1.20	5.40	0.955
0.6	0.665	1.17	5.14	0.909
0.4	0.643	1.14	4.87	0.861
0.2	0.622	1.11	4.59	0.812
0	0.600	1.08	4.31	0.762

TABLE 2

μ	y	$\frac{a^2S}{D\pi^2}$	$\left(\frac{a^2S}{D\pi^2}\right) / \left(\frac{a^2S}{D\pi^2}\right)_{\mu = 1.0}$
1.0	3.274×10^{-2}	9.42	1.0
0.8	3.493×10^{-2}	8.83	0.937
0.6	3.745×10^{-2}	8.24	0.874
0.4	4.039×10^{-2}	7.64	0.810
0.2	4.385×10^{-2}	7.03	0.747
0	4.798×10^{-2}	6.43	0.682

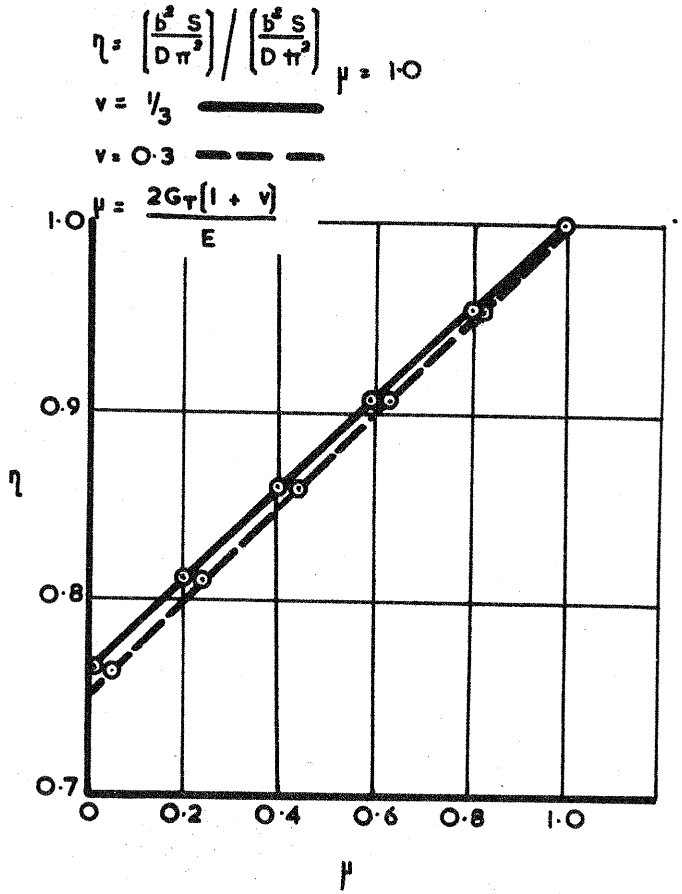

FIG. 1. INFINITE PLATE.

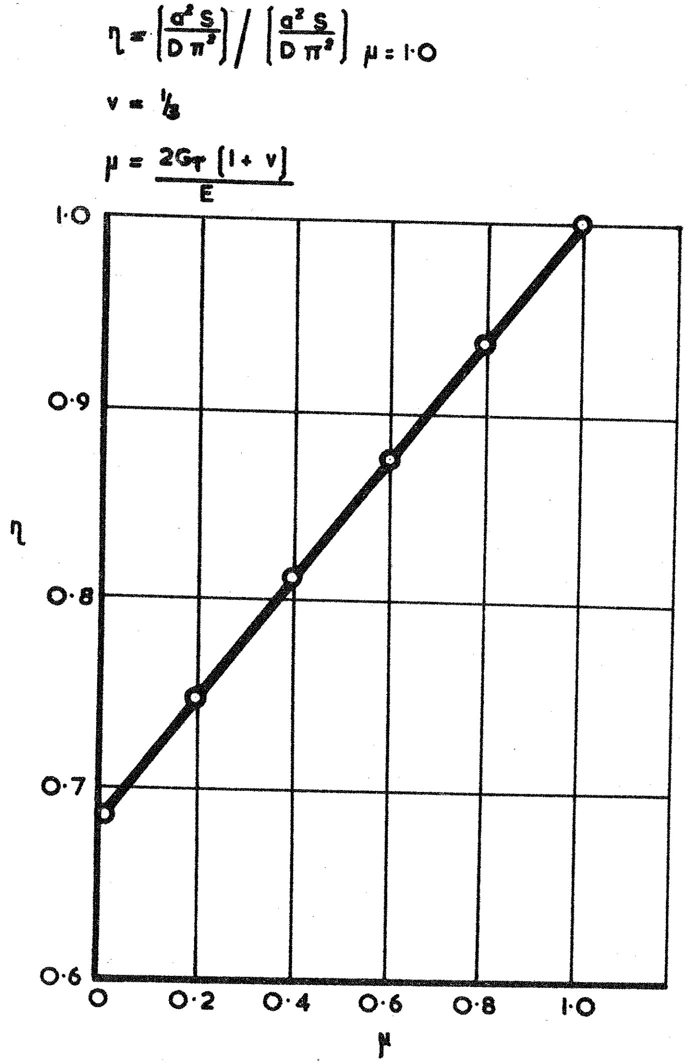

FIG. 2. SQUARE PLATE

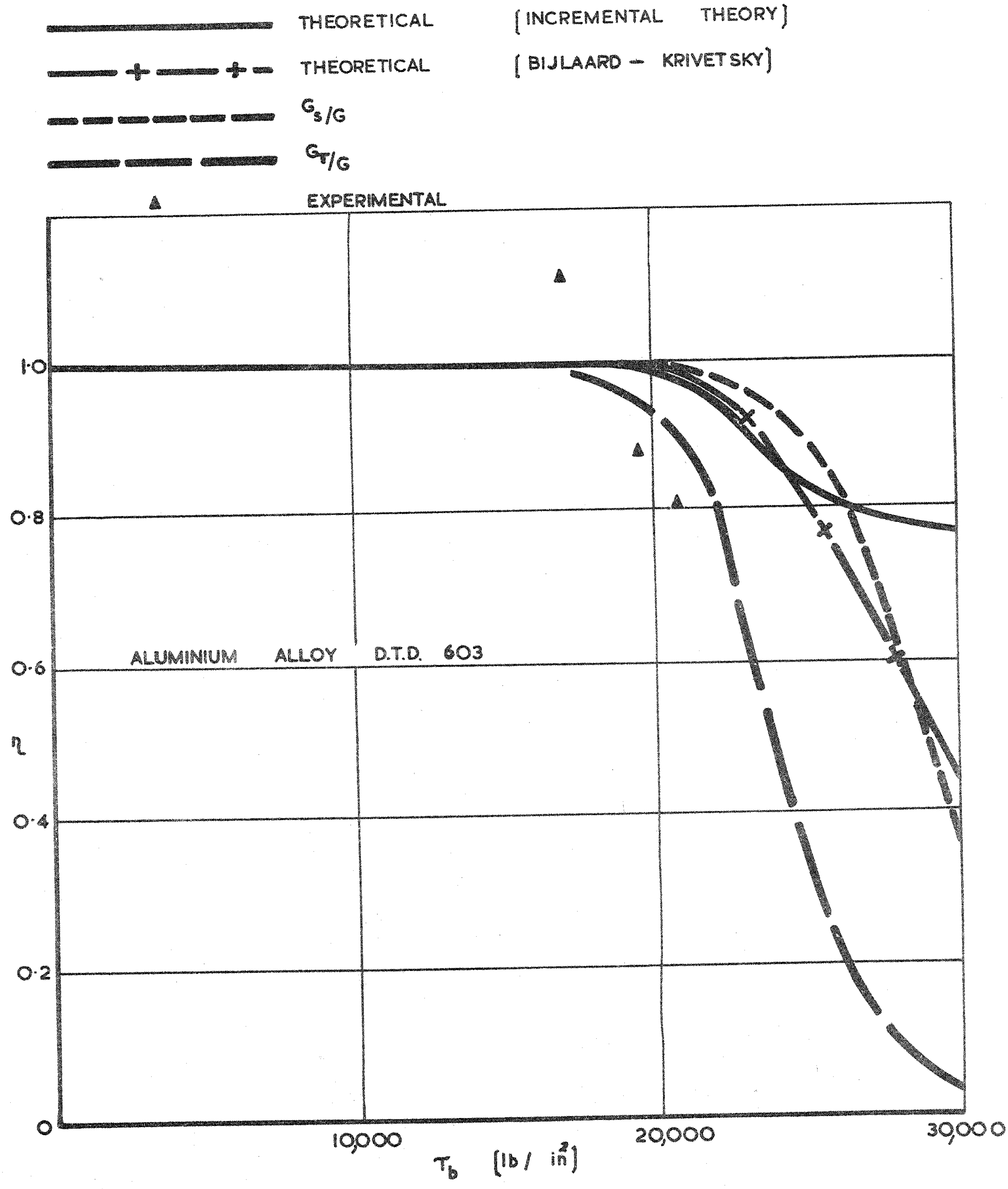

FIG. 3. INFINITE PLATE.

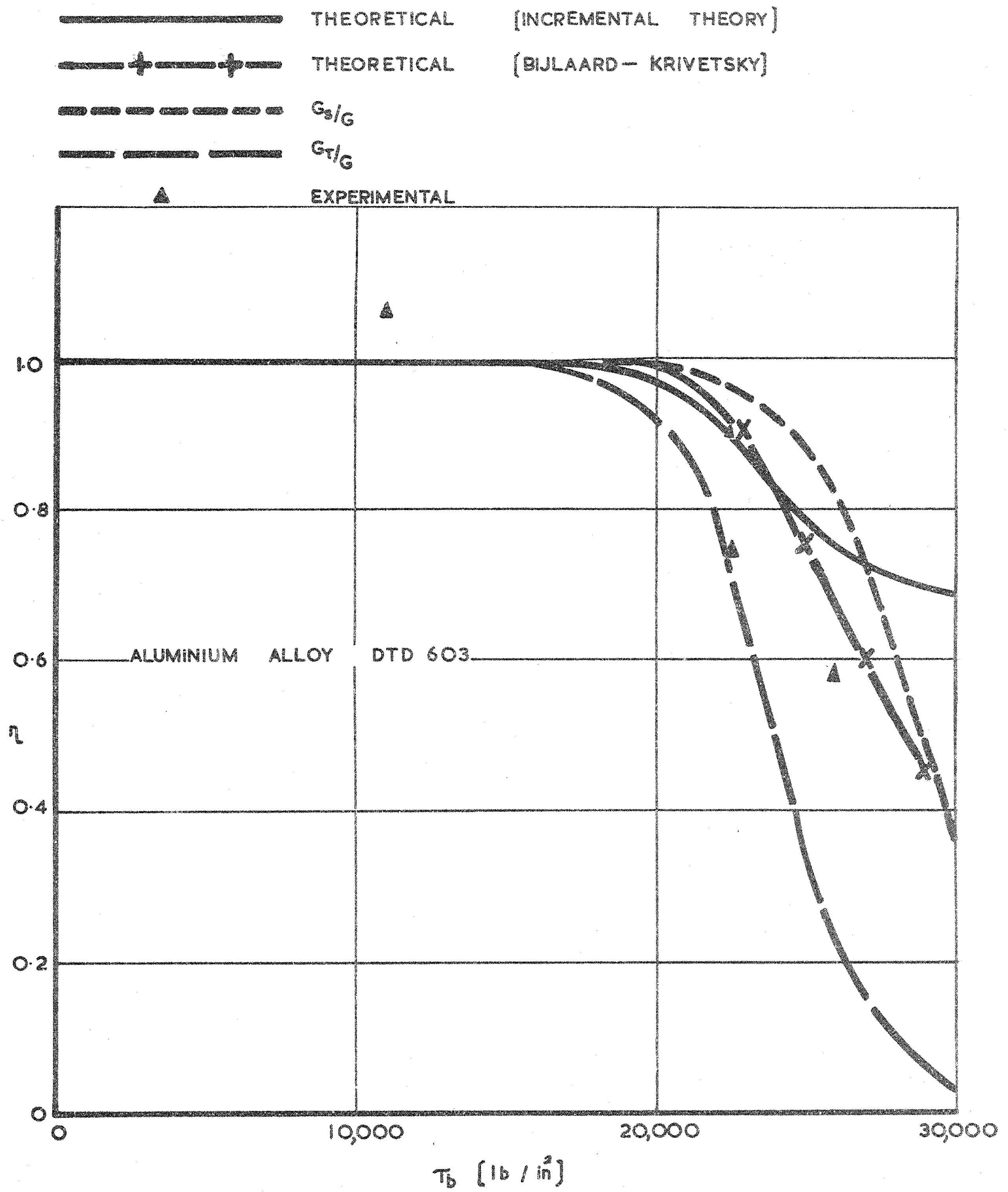

FIG. 4. SQUARE PLATE.