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THE COLLEGE OF AERONAUTICS

CRANFIELD

CLEARANCE FITS AND THE LIMITS ON MATING PARTS

by

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1. Relationship between component and clearance limits

In order that the limits of clearance between two mating parts may be satisfied it is necessary that both the difference between the smaller limit on the one part and the higher limit on the other and the difference between the other two limits on the parts should lie at or between the two clearance limits. Generally it is more economical to allow them to lie at the clearance limits and this assumption will be adopted in this paper.

In order to fix ideas let us consider the two component parts as a shaft and a hole, and let C_1 and C_2 be the clearance limits, S_1 and S_2 the limits on the shaft and H_1 and H_2 the limits on the hole. Now, letting the smaller subscript represent the smaller value in all cases, we have that, (Fig. 1):

$$H_1 - S_2 = C_1 \qquad (a)$$

$$H_2 - S_1 = C_2 \qquad (b)$$

2. Flexibility in the selection of component tolerances

If C_1 and C_2, representing the combined limits, are given we have two equations in four unknowns. They are not simultaneous equations since the two unknowns in one equation do not exist in the other and as a result the equations are independent of each other. Each equation has an infinity of solutions and we can look upon them as representing as it were, two axes of flexibility. Along one we determine the size of the component and along the other we apportion the available tolerance range $(C_2 - C_1)$ between the two components. For instance, we can move along a linear scale and select a value for S_1 thereby solving equation (b) and determining a value for H_2 and we can also choose a value for S_2 within the range from $S_2 = S_1$ to $S_2 = H_2 - C_1$ and obtain a solution for (a).

Such flexibility of choice enables one to take into account the method of manufacture and it may be possible to obtain one or more solutions which represent an optimum. Sometimes flexibility is reduced by one or more restrictions which are themselves intended to contribute towards optimisation. For instance, the approximate size of the components may be stated.

3. The distribution of clearance fits

A knowledge of the distribution of clearances resulting from the assembly of the parts enables one to more satisfactorily define the limits on the parts themselves. This is not so much of an empirical problem as it may at first seem. Mathematical methods can assist in the simulation of industrial processes and we can use these methods to draw conclusions of practical value. If we know the distribution of the sizes of the parts we can determine the distribution of clearances resulting from the assembly of the parts by the use of mathematics, and if we consider two types of distribution, the normal or Gaussian and the rectangular for the parts and treat these at different levels of dispersion, we shall cover much of the range of situations found in practice.

Let us assume that we have a situation where :

$$H_1 = -0.0000 \text{ ins} \qquad S_2 = -0.0008 \text{ ins}$$

$$H_2 = +0.0008 \text{ ins} \qquad S_1 = -0.0013 \text{ ins}$$

these being in fact deviations from a nominal value of one inch. The tolerance for the hole is accordingly 0.0008 ins and for the shaft 0.0005 ins. The resulting tolerance on assembly clearances is 0.0013 ins and the values of C_1 and C_2, the limits on the clearances, are respectively 0.0008 ins and 0.0021 ins.

Normal distribution of component sizes

Suppose that the manufacturing process for components is working under conditions of ideal statistical control and that the size distributions for the parts are normal with 1% of parts being too small and 1% too large in each case. This situation is illustrated in Fig. 2, both distributions are of the same area to represent an equal number of parts. If we randomly assembled all parts, including those beyond limits at the same time, then the distribution of clearance sizes would be normal as illustrated by the continuous line in Fig. 3. The area of this has been made equal to each of those in Fig. 2 to represent the same number of assemblies as there are parts of each type. Now the proportion of assemblies beyond the clearance limits, representing the combination of the limits of Fig. 2, is only about .15%. This represents a reduction in unacceptable parts as such of about 92.5% and means an increase in the number of acceptable parts of nearly 2%.

Suppose now that instead of 2% of parts being beyond limits the figure is 15%. This is illustrated by Fig. 4 where 7.5% of parts are too small and 7.5% of parts too large. The random assembly of all parts, including at the same time those beyond limits leads to the situation shown in Fig. 5 by a continuous line. Here about 4.25% of assemblies are beyond the combined limits and this represents a reduction in unacceptable parts as such of about 70% and a gain in accepted parts of about 12%.

Rectangular distribution of component sizes

Suppose now that the distribution of component sizes is rectangular and not normal. Such a situation would arise in certain circumstances if there was a steady rate of drift from a given setting in the manufacturing process. Once again suppose that 2% of parts are symmetrically beyond limits as illustrated in Fig. 6. The area of these distributions are the same as those already discussed in order to represent the same number of parts and to make comparisons by eye more efficient. The random assembly of parts in this case, including those beyond limits, is shown in Fig. 7 by a continuous line. It is interesting to observe that the clearance distribution for rectangularly distributed part sizes is triangular and not rectangular. There is, as in the earlier situations considered, again the strong tendency to cluster around the mean value for the clearances. In Fig. 7 only .04% of assemblies are beyond the combined component limits and this gives a reduction in the proportion of parts not accepted of 98% and an increase in accepted parts of 2%.

Suppose, once again, that our rectangular component distributions have 15% symmetrically beyond limits as in Fig. 8. Here the proportion of the assembly distribution shown by the continuous line of Fig. 9 that is beyond the combined limits is 2.25%. This represents a decrease in the number of non-accepted parts of 85% and an increase in the number of accepted parts of 15%.

The choice of component limits

We observe then that the proportion of assemblies beyond combined limits is less than that beyond component limits and the difference is greater for rectangular component distributions than for normal ones. This is shown for all proportions in Fig. 10. However the gain in accepted parts is very small for small proportions outside limits though the reduction in the proportion of rejects is high. This is shown in Fig. 11. The gain in acceptable parts and the reduction in rejects both become more significant as the proportion of parts outside limits increases, the values being greater for the rectangular component distributions.

The strong central tendency of assembly clearances shows the importance of care in the choice of component limits. It reveals that component limits should be chosen so that the difference between the central value of each should be, as far as is possible, equal to the ideal clearance.

There is no significant difference between a part or assembly just outside a limit and one just inside, particularly in comparison with the other limit. The fact that the clearance distributions have tails indicates that the component tolerances can be increased without increasing proportionately the number of assemblies beyond the combined tolerances provided all parts are assembled. This advantage could assist in enabling the component limits to be chosen so that the resulting clearances cluster around the ideal clearance.

4. Standard tables

In practice resort is often made to the use of standard tables in selecting component limits. Suppose, for example, it is required that:

(a) The limits of clearance be 0.0005 ins and 0.0020 ins

(b) The parts be nominally of one inch the hole being nearest this size

(c) The hole has slightly more tolerance than the shaft,

then the nearest values we could select from B.S.1916 would be :

Hole H.7	- 0.0000 ins	Shaft F.6	- 0.0008 ins
	+ 0.0008 ins		- 0.0013 ins

and these are in fact the actual values used in the situations treated above.

It will be observed that the standard table values give rise to combined limits 0.0008 and 0.0021 ins that overlap those required, 0.0005 and 0.0020 ins. There is an acceptable range from 0.000685 ins to 0.0005 ins in which no parts are produced for assembly. There is a range beyond the higher limit in which 4% of the assemblies lie. Examining Fig. 10 we see that this value can be obtained with as much as from 15% to 20% of component parts beyond limits. Investigating the tolerance on the clearances we observe that it is less than that allowed on the original requirements. This means smaller tolerances on at least one if not both parts and this would be in the direction of greater costs of manufacture though it may not necessarily be significant.

If we examine the two sets of figures in the light of the central tendency of the clearance distributions we see first of all that the original ideal clearance is 0.00125 ins as against 0.00145 ins of the standard tables. In the case of the rectangular component distribution we can calculate from Fig. 9 that the proportions within ± 0.0002 ins of these values is 38.5% and 45% respectively. The difference in these figures indicates the loss in assemblies that are not around the ideal size. For a narrower range around these values the difference would be greater. An examination of the normal distribution shows that the difference is not so great around the ideal clearance but away from this range it increases very rapidly. Hence we see the importance of choosing component limits with attention not only to the ideal clearance but also with some knowledge of the distribution of sizes that will arise.

5. Clearance distributions with rejection at component stage

In each of the clearance distributions of Figs. 3, 5, 7 and 9 there has been added a broken line. These in each case represent the distribution of clearance sizes for the random assembly of all parts other than those beyond component limits. The area of those in Figs. 3 and 7 are the same and each is 98% of that of the corresponding assembly distribution for all parts. This is to represent a reduction in the number of parts by 2%. For the same reason those in Figs. 5 and 9 have been reduced by 85% of their corresponding all-inclusive assembly distributions. In all cases the distributions show as expected no assemblies beyond the combined tolerances. For the normal component distributions there is revealed to be a greater number of assemblies around the central value than in the situation where all parts were randomly assembled but for rectangular ones the number is less. The rectangular distributions still retain a triangular clearance distribution but the normal ones change the nature of theirs to become more platykurtic. The central clustering tendency still remains indicating a small probability for clearances near the limit. This suggests once again the possibility of wider tolerances for components.

6. Comparison between numerically and mathematically derived clearance distributions.

In addition to making use of mathematical methods to derive clearance distributions a Monte Carlo method was employed in the case of two of them. The component distributions treated were those of Fig. 2 and 4 excluding parts beyond the component limits. There are many ways in which the Monte Carlo procedure can be carried out but in this case the distributions were first of all transformed into histograms. Each distribution was divided into 20 blocks of equal class breadth and the height of each block was determined by the symmetrical intersection of the block with the line of the continuous distribution. The ratio of the height of each block to the total of block heights for each distribution represented the proportion of parts with that block's mid-class size value.

The numbers 1 to 500 were allocated in that order from left to right to each block in proportion to its ratio. Thus if the ratio of the first block was .06 then the numbers 1 to 30 would be allocated to it. If the next block was .07 then it would be allocated the numbers 31 to 65.

Discs identical in all respects and numbered 1 to 500 were made to represent shafts and a similar set made to represent holes. Each set of discs was separately mixed and one disc chosen from the set representing shafts. The number of this was located in the histogram and the mid-class shaft size of the block where it existed represented the shaft size. The disc was then replaced and the set mixed again. A similar procedure was carried out to obtain a hole size. The difference between the shaft and hole size was recorded. The procedure was repeated until 800 differences were obtained. These were then grouped into 20 blocks of equal class breadth and the frequency for each block obtained. The results are shown in Figs. 12 and 13 for the two sets of 800 differences by the continuous line blocks suitably adjusted to the appropriate size. The broken lines represent the mathematical derivation already recorded in Figs. 3 and 5 but with the addition of their derived histograms. A close agreement between the mathematically and the numerically derived results is seen to exist.

7. Economic evaluation

So far as economic aspects of our problem are concerned we may first of all have the possibility of choosing the average size of the components. What would be the most economic average size of the components ? This would involve first of all the costs of manufacture. They depend very much on the type of components and the complete product to which they belong. It may be that a relatively small shaft would mean a saving of material. However it might mean too, greater machining costs. A larger shaft may bring economic advantages but it may at the same time introduce disadvantages. There is accordingly the possibility of an optimum situation which may not necessarily be very uniquely determined. There could be a wide range of average values which are not significantly different from the economic optimum.

However changes in the average size of components could affect the average life of the product or its operating efficiency. These factors need also to be taken into account though we do here meet the problem of evaluating intangibles. To what extent for example will the goodwill of the organisation be affected ?

An interesting problem in minimum cost analysis arises in the allocation of the available clearance fit tolerance between the two components. If we increase the tolerance on the hole we decrease simultaneously the tolerance on the shaft by the same amount. To work with wider limits on the hole would be cheaper per unit but at the same time the costs of the shaft increase. If we think in terms of marginal values, that is the additional gain or cost from increasing or decreasing a tolerance by a small amount, then the point at which the marginal cost of decreasing the tolerance on the shaft is equal to the marginal gain from the resulting increased tolerance on the hole, represents the economic optimum. The optimum need not necessarily be at the point of equal allocation of the available tolerance. It will of course depend on many factors such as the comparative ease of performing the tasks, the rates of production, the type of machines used and their availability.

A similar type of situation exists with the actual choice of limits on the components themselves. The wider the limits the greater the possibility of unacceptable parts but at the same time there may be reduced costs of manufacture.

The statistical analysis has shown that rejection based on the combined tolerances is more efficient in terms of the number of parts actually used in acceptable assemblies than when rejection is based on component limits. There may not necessarily be an economic advantage. The cost of rejection at the assembly stage may be very high relative to the costs of rejection at the component stage. It may in fact need to be destructive so that a hundred per cent inspection at the assembly stage would be in no way economic. The difference otherwise need not arise however from the actual cost of testing but may result from the possible high cost of transport. The parts involved could be manufactured at large distances from the assembly centre. A mathematical model of the considerations of this paragraph is developed in Appendix II and a function derived to enable a decision to be made in a general case. It is simplified since details must vary with the particular application but it indicates how a model can be constructed and evaluated.

8. Extension to other uses and more components

If instead of the difference we had been concerned with the sum of randomly selected samples in the statistical analysis the distributions for assemblies would have been exactly the same. The means of the latter distributions would have been the sum of the means of the component distributions and there would have been the same change by a constant addition or subtraction in the values along the base of the assembly distributions.

We could accordingly use the distributions in a situation where a process is not precise enough to provide sufficient products within a required tolerance range. If the product can be made in two parts then greater precision, though at a greater cost per product manufactured, but not necessarily at a greater cost of production per product accepted, could possibly ensue. Such a method if feasible could possibly save the cost of an additional machine.

Now the more part distributions we sample from, the narrower the dispersion of the final assembly. The normal part size distributions still maintain a normal distribution for assemblies but the rectangular ones approach a normal distribution very rapidly. Thus more accuracy results, if it is feasible, in making the type of product referred to in more than two parts. Another interesting application is in the allocation of a given tolerance in a linkage system. The sum of tolerances on each link need not be equal to the required overall tolerance. If each link is given a little more there would only be a rare chance of the actual total play in the system exceeding the total tolerance specified.

9. Acknowledgements

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10. References

The Control of Quality in Engineering Manufacture, by Professor J. Loxham, Conference on Technology of Engineering Manufacture, March 1958, Page 49.

APPENDIX I

General

(1) Let H represent the distribution of sizes of holes

" S " " " " " " shafts

" C " " " " " " clearances between holes and shafts.

There is no loss of generality if we assume that the mean size of hole is zero and the mean size of shaft is zero.

Let x_1 represent the size of hole

" x_2 " " " " shaft

" x_3 " " " " clearance between a hole and a shaft.

Both x_1 and x_2 are randomly and independently chosen and x_3 is such that:

$$x_3 = x_1 - x_2$$

It follows that if $\bar{x}_i$ represents the means of the distributions then:

$$\bar{x}_1 = \bar{x}_2 = \bar{x}_3 = 0$$

H is accordingly measured along x_1 as deviations from 1.00040 inches, S along x_2 as deviations from 0.99895 inches and C along x_3 as deviations from 0.00145 inches.

(2) If H_1 and H_2 are the actual limits of the hole and S_1 and S_2 those of the shaft, then the combined limits are C_1 and C_2, where:

$$C_1 = H_1 - S_2 \qquad C_2 = H_2 - S_1$$

$$H_2 \geqslant H_1, \qquad S_2 \geqslant S_1, \qquad C_2 \geqslant C_1$$

Along x_1 and x_2 the limits of H and S are respectively $-\xi_1$ and ξ_1 and $-\xi_2$ and ξ_2, where:

$$\xi_1 = (H_2 - H_1)/2 \qquad \xi_2 = (S_2 - S_1)/2$$

and the combined limits along x_3 are $-\xi_3$ and ξ_3, where:

$$\xi_3 = \xi_1 + \xi_2$$

In the numerical examples we have

$$\xi_1 = 0.00040 \text{ inches}$$
$$\xi_2 = 0.00025 \text{ inches}$$
$$\xi_3 = 0.00065 \text{ inches}$$

(3) If the variances of H, S and C are σ_1^2, σ_2^2 and σ_3^2 respectively, then:

$$\sigma_3^2 = \sigma_1^2 + \sigma_2^2$$

While this relationship holds generally it is unnecessary to make use of it in the examples except where H and S are normal as shown in section 5 of this appendix.

(4) If H and S are normal C will be normal but slight divergencies of H and S from normality will not significantly affect C. If H and S are rectangular C will be triangular.

Normal Distributions

(5) The normal distribution is defined by :

$$f(z) = A e^{-z^2/2}$$

$$A = 1/\sqrt{2\pi}$$

$$\int_{-\infty}^{\infty} f(z)\, dz = 1$$

In the examples, where H, S and C are normal

$$z = x_i / \sigma_i \qquad i = 1, 2, 3$$

The value of:

$$\int_{-b}^{b} f(z)\, dz = \text{erf}\ (b)$$

is tabulated in published statistical tables for variations in b.

Let P be the proportion of components beyond limits and P_1 the proportion of assemblies beyond combined limits, then:

$$\text{erf}\ (\xi_1 / \sigma_1) = \text{erf}\ (\xi_2 / \sigma_2) = 1 - P$$
$$\text{erf}\ (\xi_3 / \sigma_3) = 1 - P_1$$

Making use of the statistical tables, we have:

if P = .02

$$b = 2.3$$

$$2.3\,\sigma_1 = \xi_1 = 0.00040 \text{ inches}$$

$$2.3\,\sigma_2 = \xi_2 = 0.00025 \text{ inches.}$$

Therefore $\sigma_1 = 0.0001740$ inches

$\sigma_2 = 0.0001087$ inches

$\sigma_3 = 0.0002052$ inches

$P_1 = 0.0015$

Similarly if P = .15, then b = 1.44 and

$\sigma_1 = 0.0002780$ inches

$\sigma_2 = 0.0001735$ inches

$\sigma_3 = 0.0003277$ inches

$P_1 = 0.0425$

(6) If C excludes those components from H and S where $|x_1| > \xi_1$ and $|x_2| > \xi_2$ respectively, then x_3 will have the distribution:

$$g(x_3) = f(x_3/\sigma_3)\ \phi(x_3)$$

where:

$$\phi(x_3) = \frac{\operatorname{erf}\left\{\min\ \alpha - \gamma,\ \beta + \delta\right\} + \operatorname{erf}\left\{\min\ \alpha + \gamma,\ \beta - \delta\right\}}{2\operatorname{erf}\left\{\xi_1/\sigma_1\right\}\operatorname{erf}\left\{\xi_2/\sigma_2\right\}}$$

$$\alpha = \sigma_3\ \xi_1/\sigma_1\ \sigma_2 \qquad \beta = \sigma_3\ \xi_2/\sigma_1\ \sigma_2$$

$$\gamma = \sigma_1\ x_3/\sigma_2\ \sigma_3 \qquad \delta = \sigma_2\ x_3/\sigma_1\ \sigma_3$$

for $|x_3| < \xi_3$ but vanishes for $|x_3| > \xi_3$

If we apply this to the example where P = .02 and for the value x_3 = 0.00005, we have :

$$\phi(x_3) = \frac{\text{erf}\left\{2.7123 + (3044.4)(0.00005)\right\} + \text{erf}\left\{2.7123 - (3044.4)(0.00005)\right\}}{2\left[\text{erf}\left\{2.3\right\}\right]^2}$$

$$= \frac{.9958 + .9898}{2(.98)^2} = 1.035$$

and $g(x_3) = (.387)(1.035) = .401$

(7) Both f(z) and $g(x_3)$ are probability distributions with base scales in terms of $\sigma_i = 1$ (i = 1, 2, 3).

In order to have a base scale in the units of measurement of x_i we need an ordinate scale for each distribution f(z) which is given by:

$$f(x_i/\sigma_i)/\sigma_i$$

In addition the clearance distributions $g(x_3)$ need to be adjusted to make the total area of each equal to 1 - P and not unity. The ordinates of $g(x_3)$ are therefore calculated as:

$$(1 - P)\left[g(x_3)\right] / \sigma_3$$

Continuing the calculation of section 6, we have:

$$(.98)(.401)/0.0002052 = 1915$$

The ordinate values of the clearance distributions for selected values of x_3 are shown in the table below.

P = .02

x_3	0.00092	0.00065	0.00055	0.00045	0.00025	0.00005	0.00000
x_3/σ_3	4.5	3.16	2.68	2.20	1.22	0.24	0.00
$f(x_3/\sigma_3)/\sigma_3$	0.1	13	54	177	900	1890	1950
$\phi(x_3)$	0.000	0.000	0.385	0.786	1.000	1.035	1.036
$(.98)\ g(x_3)/\sigma_3$	0	0	20	136	885	1915	1980

P = .15

x_3	0.00147	0.00098	0.00065	0.00055	0.00045	0.00025	0.00005	0.0000
x_3/σ_3	4.5	3.0	1.97	1.68	1.38	0.76	0.15	0.00
$f(x_3/\sigma_3)/\sigma_3$	.06	13.4	175	296	469	909	1202	1217
$\phi(x_3)$	0.00	0.00	0.00	0.39	0.69	1.14	1.26	1.26
$(.85)\ g(x_3)/\sigma_3$	0	0	0	98	274	881	1287	1305

TABLE OF ORDINATE VALUES OF CLEARANCE DISTRIBUTIONS AT SELECTED VALUES OF x_3

FOR GAUSSIAN DISTRIBUTIONS OF COMPONENT SIZES.

Rectangular Component Distributions

(8) The rectangular distribution is defined by :

$$l(x_i) = 1/2\, a_i$$

for $|x_i| \leqslant a_i$

and $l(x_i) = 0$

for $|x_i| > a_i$ $\qquad i = 1, 2$

If P is the proportion of components beyond limits then:

$$P = (a_i - \xi_i)/a_i = 1 - (\xi_i/a_i) \qquad i = 1, 2, 3$$

where $a_3 = a_1 + a_2$

Then for example where P = .02

a_1 = 0.000471 inches

a_2 = 0.000294 inches

a_3 = 0.000765 inches

(9) The distribution of x_3 is given by :

$$m(x_3) = (1 + x_3/a_3)/a_3$$

for $-a_3 \leqslant x_3 \leqslant 0$

and $m(x_3) = (1 - x_3/a_3)/a_3$

for $0 \leqslant x_3 \leqslant a_3$

If P_1 is the proportion of assemblies beyond combined limits, then :

$$P_1 = \frac{2}{a_3} \int_{\xi_3}^{a_3} (1 - x_3/a_3)\,dx_3 = P^2$$

Thus in the example for P = .02 we have that P_1 = .0004.

Similarly for P = .15, P_1 = .0225.

(10) If C excludes those components from H and S where $|x_1| > \xi_1$ and $|x_2| > \xi_2$ respectively, then x_3 will have the distribution :

$$n(x_3) = (1 + x_3/\xi_3)/\xi_3$$

for $-\xi_3 \leqslant x_3 \leqslant 0$

and $n(x_3) = (1 - x_3/\xi_3)/\xi_3$

for $0 \leqslant x_3 \leqslant \xi_3$

(11) In order to make the total area of $n(x_3)$ equal to 1 - P the ordinates need to be calculated as:

$$(1 + x_3/\xi_3)/a_3$$

for $-\xi_3 \leqslant x_3 \leqslant 0$

and $(1 - x_3/\xi_3)/a_3$

for $0 \leqslant x_3 \leqslant \xi_3$

APPENDIX II

A mathematical model for determining whether it is more or less economic to reject parts at the component stage or at the assembly stage of manufacture.

Let us assume that costs behave in two types of ways; there are costs that are fixed irrespective of the batch size and there are costs that are constant per unit of the product.

Let P = proportion of parts beyond component limits

" P_1 = proportion of parts beyond combined limits

" A = batch size of number of acceptable assemblies required

" Q = cost of A if rejection is at the component stage only

" Q_1 = cost of A if rejection is at the assembly stage only

" w = cost of raw material and manufacture

" x = cost of inspection at component stage

" y = cost of transportation from assembly stage and cost of assembly

" z = cost of assembly inspection

" subscript 1 = fixed costs per batch

" subscript 2 = fixed costs per unit of product comprising one hole and one shaft.

It is assumed that 5% of items are inspected at the component stage for control purposes even if the hundred percent inspection is more economic at the assembly stage.

$$Q = w_1 + x_1 + y_1 + \left\{ A(w_2 + x_2)/1 - P \right\} + A\,y_2$$

$$Q_1 = w_1 + x_1 + y_1 + z_1 + \left\{ (w_2 + .05\,x_2 + y_2 + z_2)/1 - P_1 \right\}$$

$$Q \gtrless Q_1 \text{ if:}$$

$$\left(\frac{1}{1 - P} - \frac{1}{1 - P_1}\right) w_2 + \left(\frac{1}{1 - P} - \frac{.05}{1 - P_1}\right) x_2 - \frac{P_1}{1 - P_1} y_2 - \frac{z_1}{A} - \frac{z_2}{1 - P_1} \gtrless 0$$

We can transform P_1 into P by the use of Fig. 10. However for rectangular component distribution $P_1 = P^2$.

Thus if the component distributions are rectangular we have that

$Q \gtrless Q_1$ if:

$$P\,w_2 + (.95 + P)x_2 - P^2 y_2 - \frac{1 - P^2}{A}\,z_1 - z_2 \gtrless 0$$

The above function has many variables and remains quite involved. If we assume that only the costs, w_2 and y_2, are significant, we have that:

$Q \gtrless Q_1$ if:

$$w_2 P - y_2 P^2 \gtrless 0$$

i.e. if

$$w_2 \gtrless y_2 P$$

Thus if unit costs of transportation are very high relative to component raw material and manufacturing costs it can be more economic to reject at the component stage of manufacture.

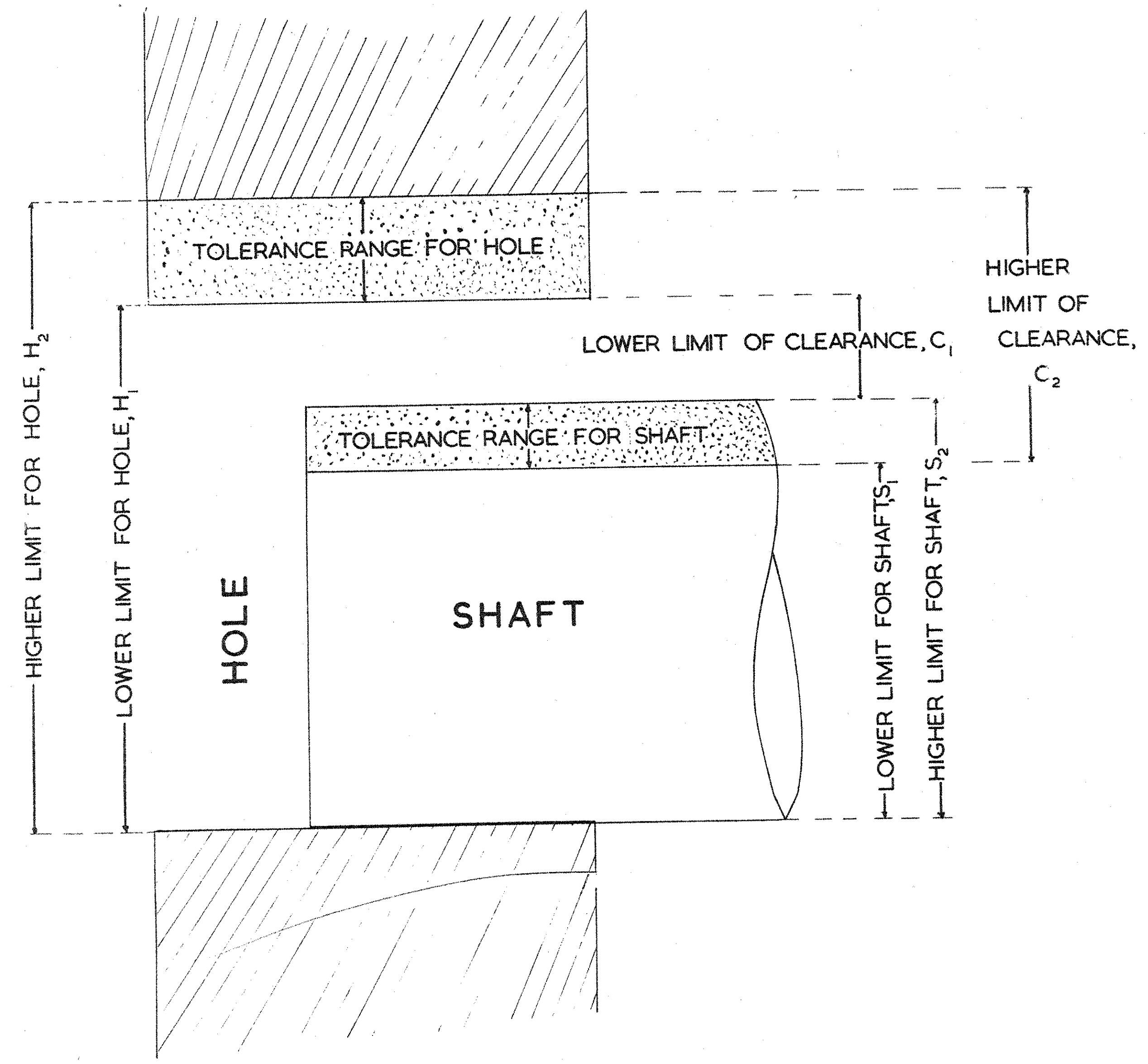

FIG. 1. The relationship between the tolerances and limits on parts and their combined tolerance and limits.

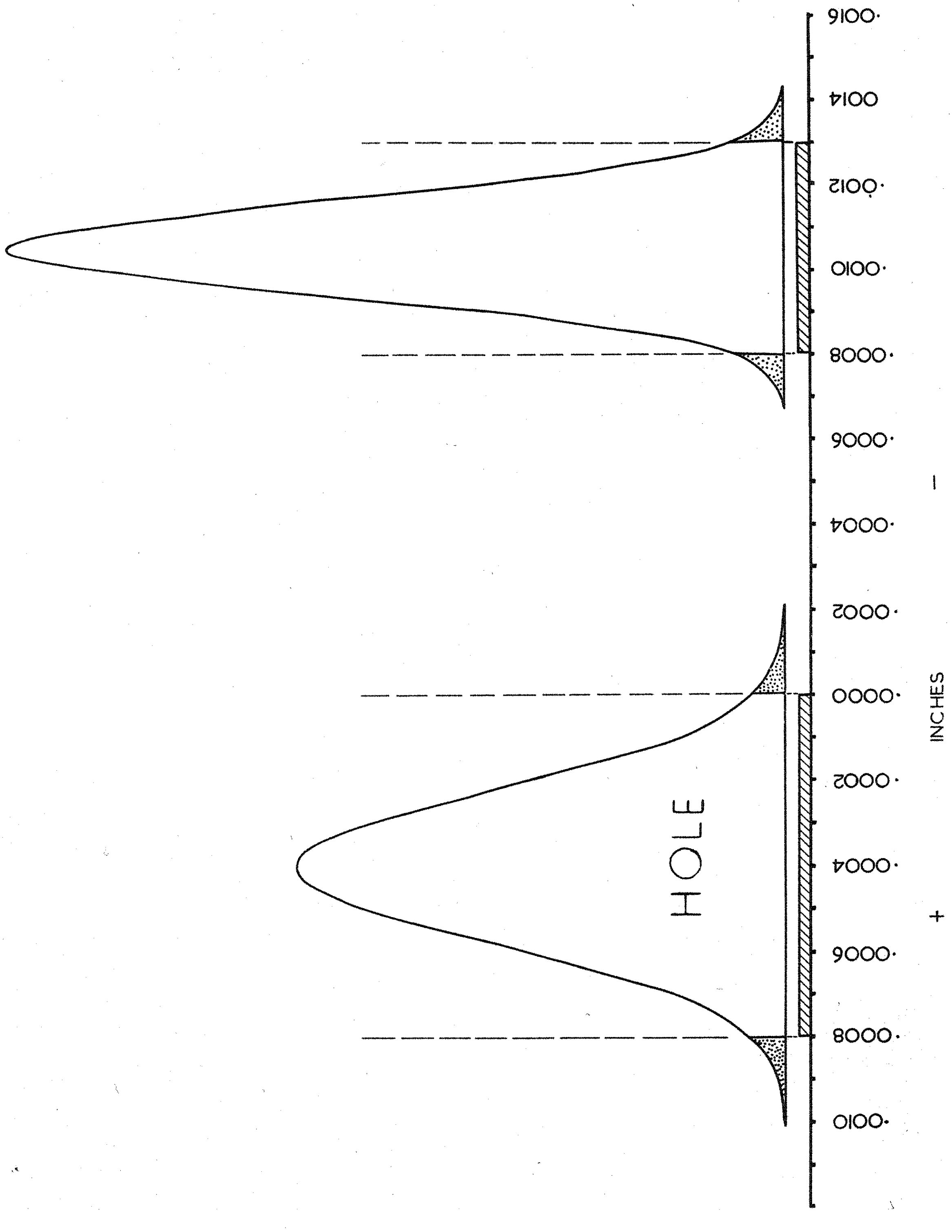

FIG. 2. Gaussian size distributions of equal numbers of holes and shafts with 2% of parts outside tolerances in each case; 1% of parts are too small and 1% too large.

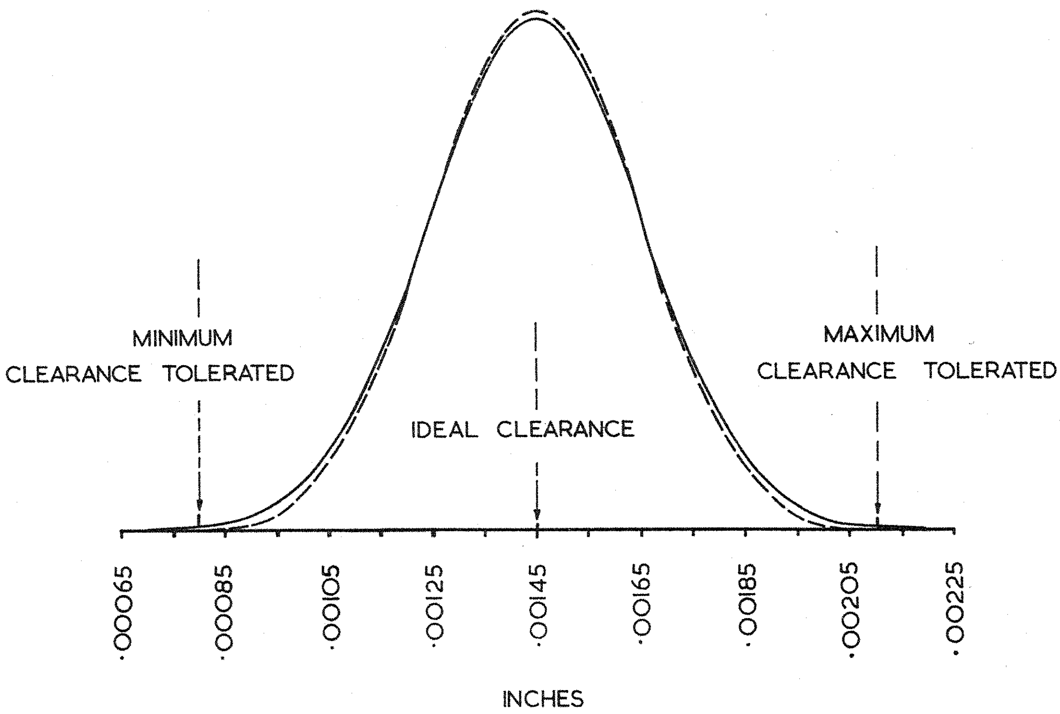

FIG. 3. The distribution of assembly clearances for the components of Fig. 2. The continuous line represents the random assembly of all components, including those beyond individual tolerances, whilst the broken line excludes those beyond individual tolerances.

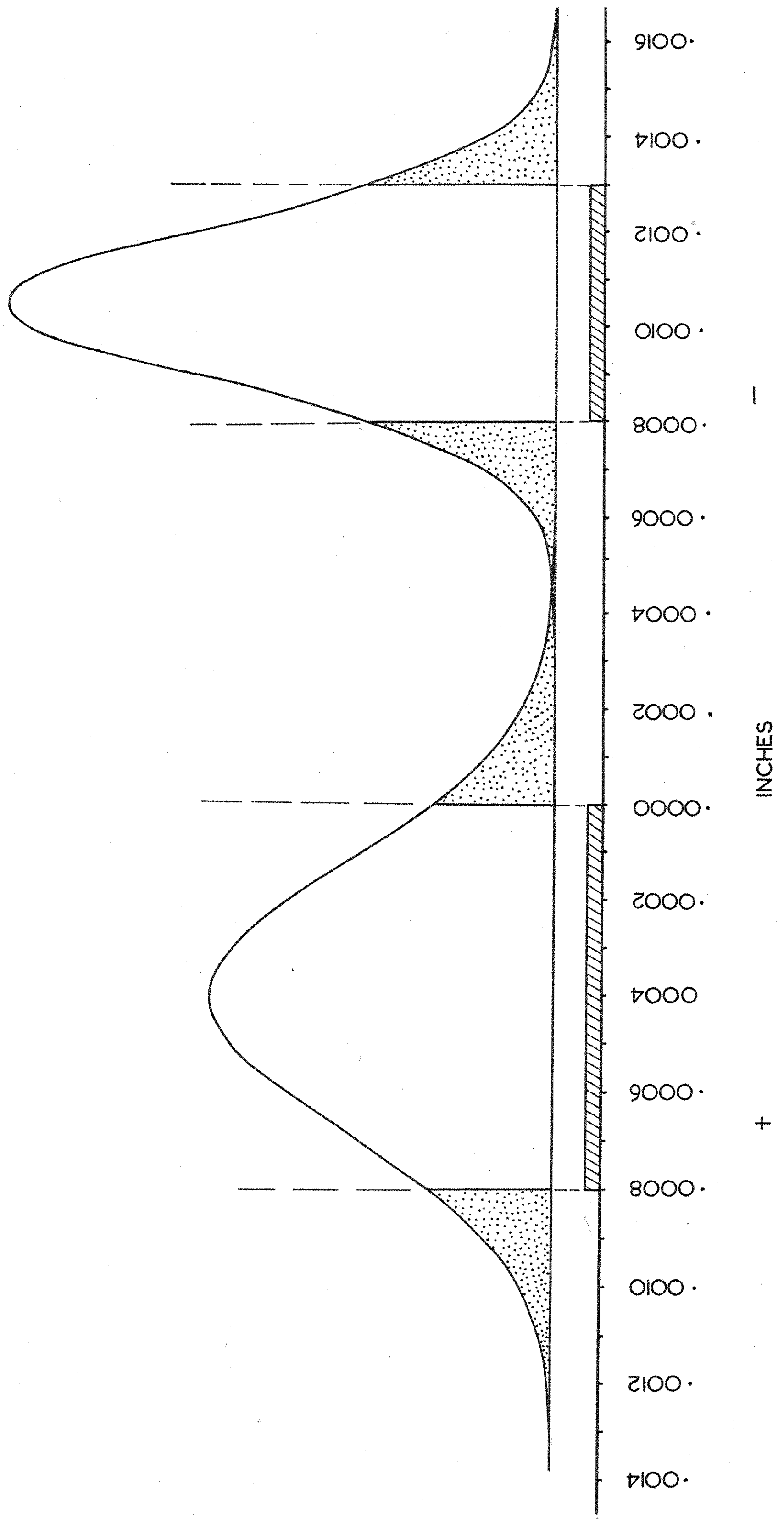

FIG. 4. Gaussian size distributions of equal numbers of holes and shafts with 15% of parts outside tolerances in each case; 7.5% of parts are too small and 7.5% too large.

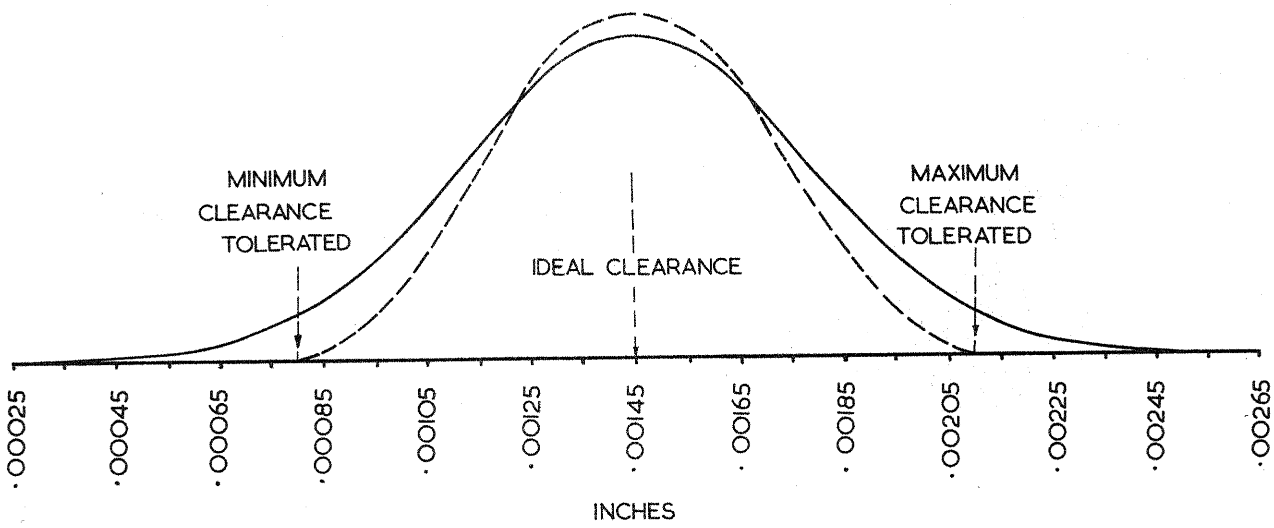

FIG. 5. The distribution of assembly clearances for the components of Fig. 4. The continuous line represents the random assembly of all components, including those beyond individual tolerances, whilst the broken line excludes those beyond individual tolerances.

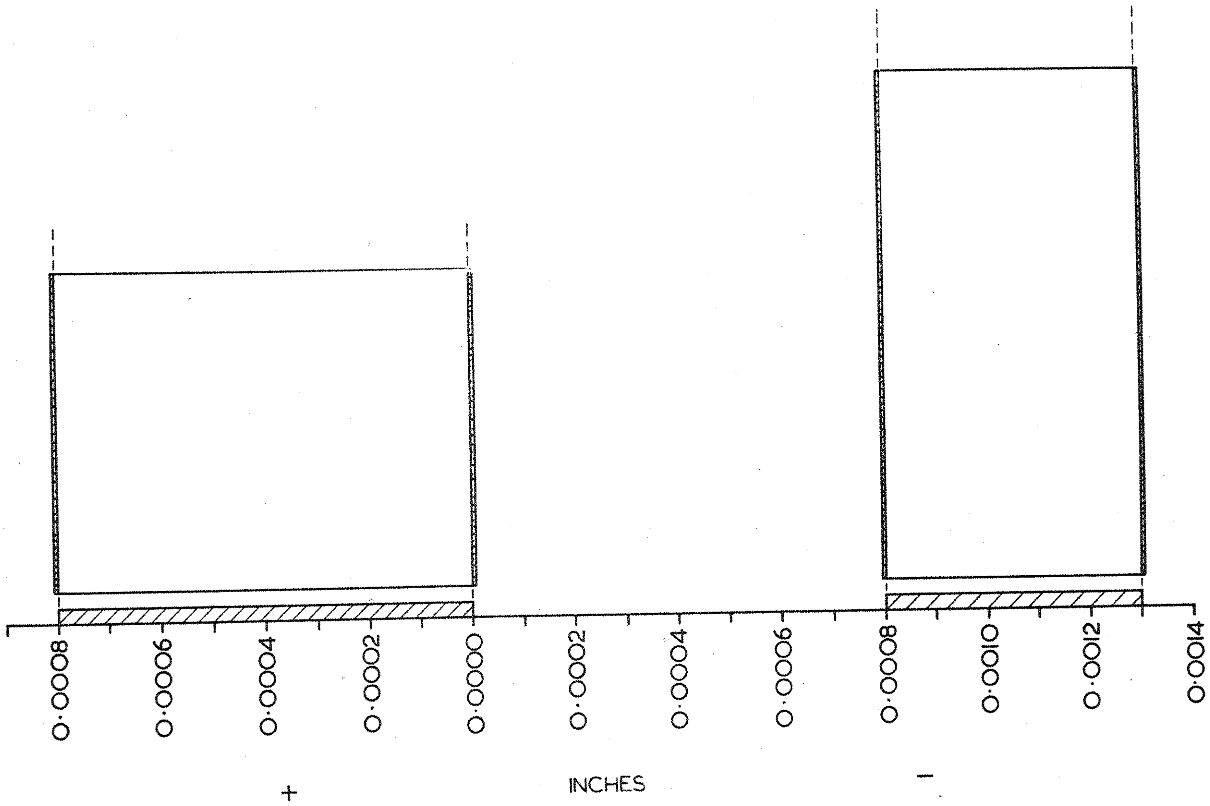

FIG. 6. Rectangular size distributions of equal numbers of holes and shafts with 2% of parts outside tolerances in each case; 1% of parts are too small and 1% too large.

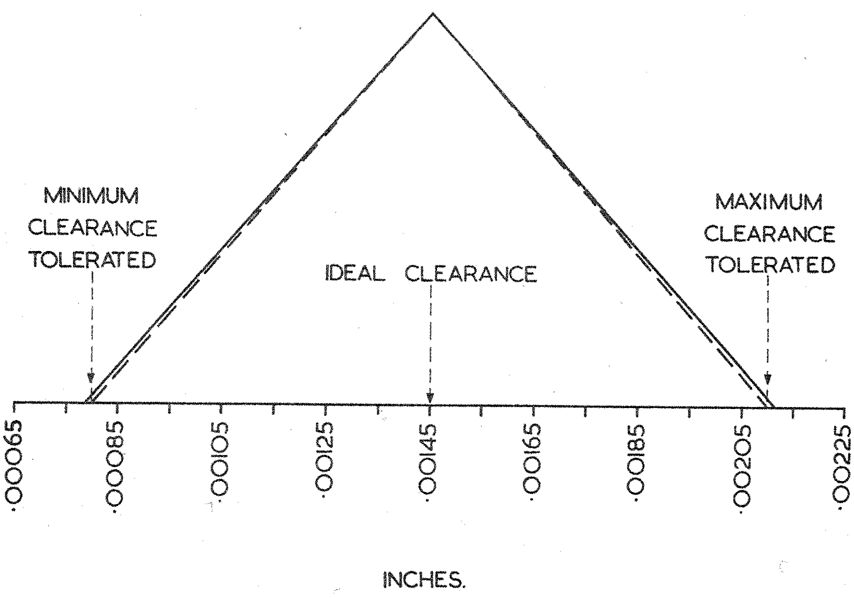

FIG. 7. The distribution of assembly clearances for the components of Fig. 6. The continuous line represents the random assembly of all parts, including those beyond individual tolerances, whilst the broken line excludes those beyond individual tolerances.

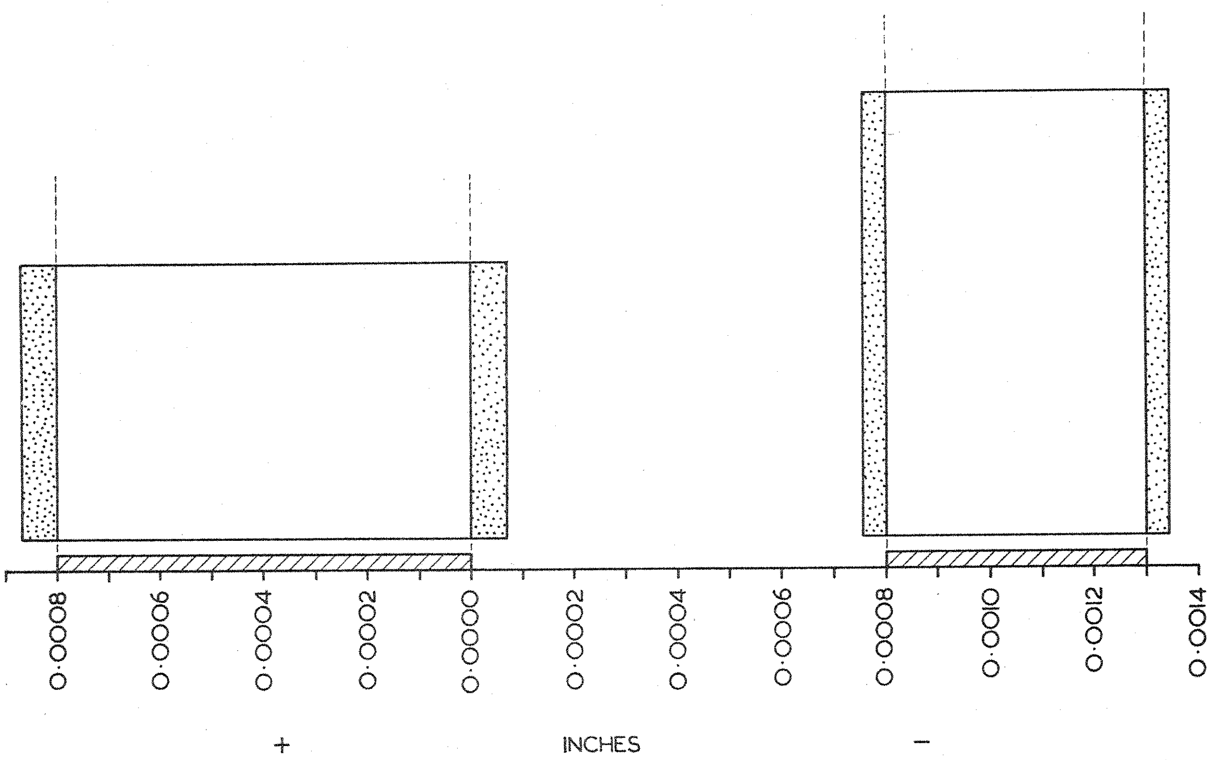

FIG. 8. Rectangular size distributions of equal numbers of holes and shafts with 15% of parts outside tolerances in each case; 7.5% of parts are too small and 7.5% too large.

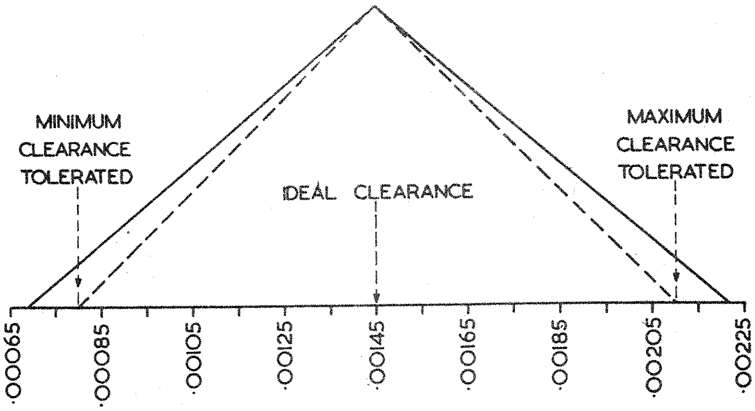

FIG. 9. The distribution of assembly clearances for the components of Fig. 8. The continuous line represents the random assembly of all parts, including those beyond individual tolerances, whilst the broken line excludes those beyond individual tolerances.

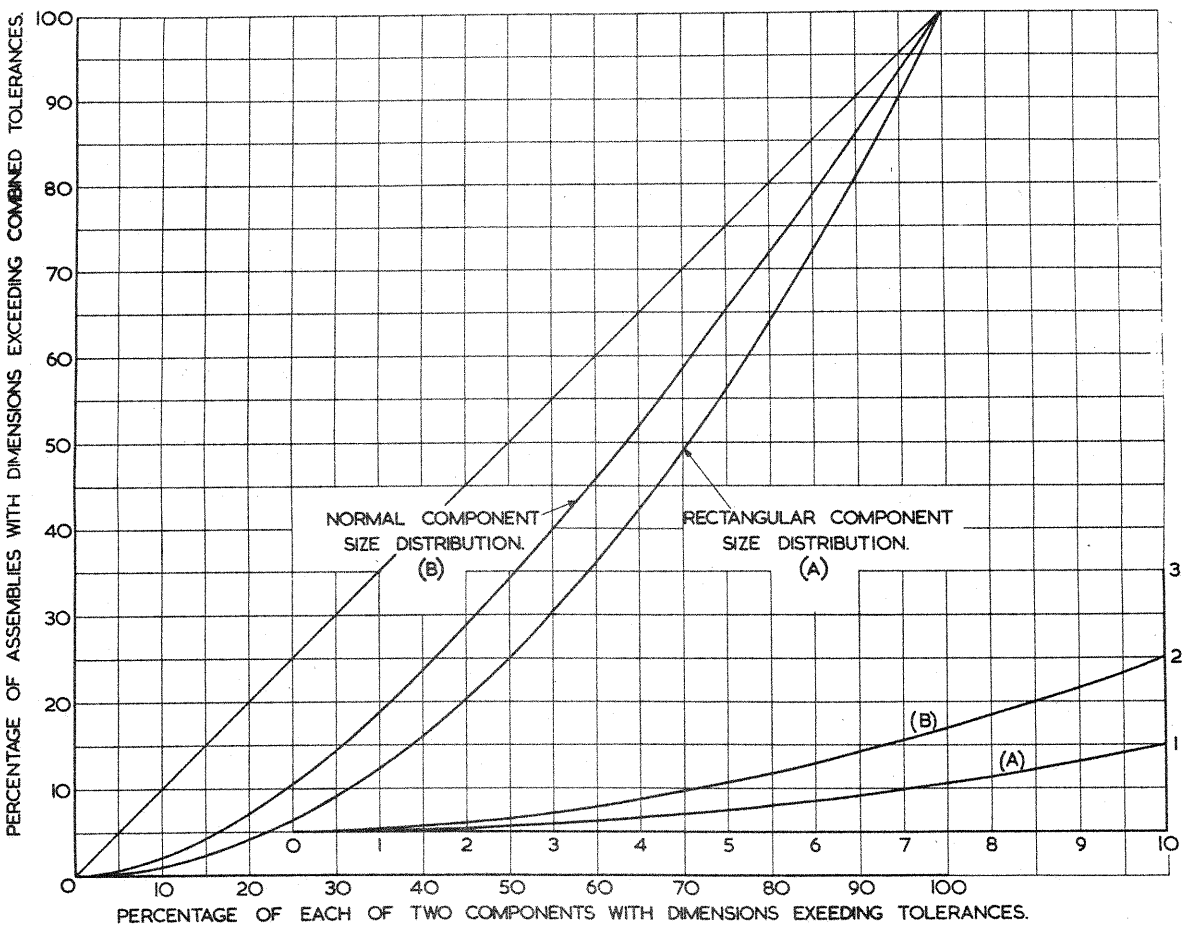

FIG.10. Relationship between the proportion of each of two components with dimensions exceeding individual tolerances and the proportion of assemblies with dimensions exceeding the combined tolerances. The proportion of components beyond individual limits is assumed to be symmetrically situated and the assembly of parts is assumed to be random.

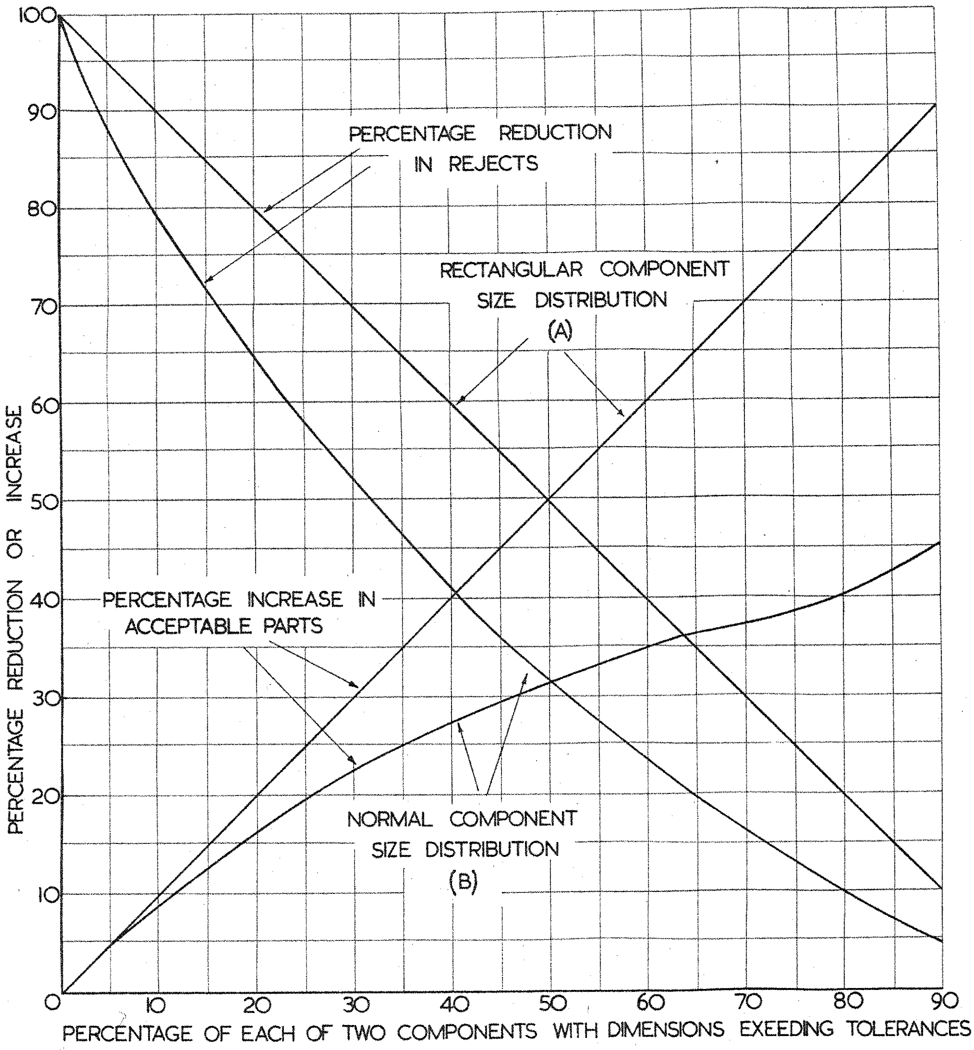

FIG.11. The decrease in the proportion of rejected parts and the increase in the proportion of accepted parts, resulting from rejection on combined limits as against rejection on individual component limits, for variations in the proportion of components beyond individual limits. The proportion of components beyond individual limits is assumed to be symmetrically situated and the assembly of parts is assumed to be random.

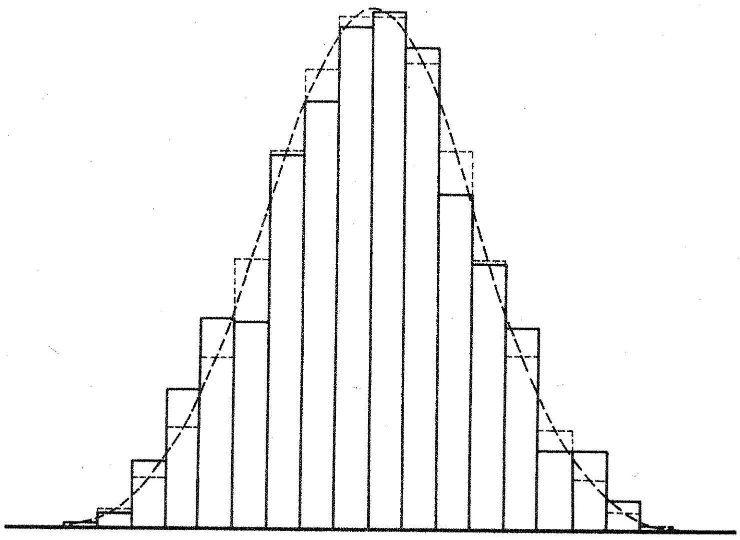

FIG. 12. Mathematically derived (broken lines) and numerically derived (continuous lines) distribution of clearances for random assembly of components of Fig. 2 excluding those beyond individual tolerances.

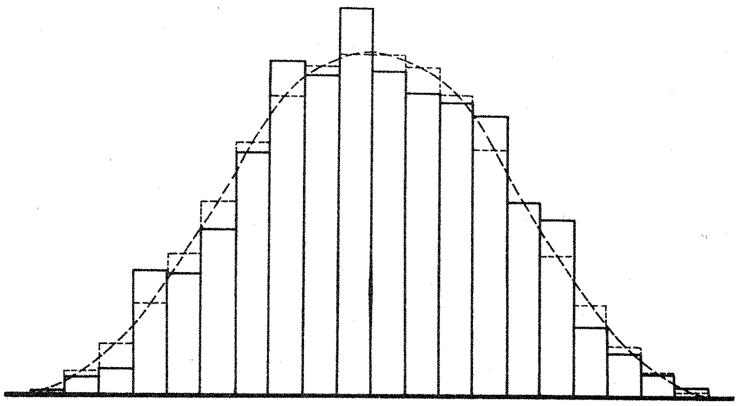

FIG. 13. Mathematically derived (broken lines) and numerically derived (continuous lines) distribution of clearances for the random assembly of components of Fig. 4 excluding those beyond individual tolerances.