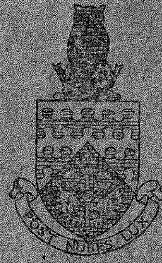




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ON THE STRESS DISTRIBUTION IN PRESSURISED
CYLINDERS

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C R A N F I E L D

The Influence of Frame Pitch and Stiffness on the
Stress Distribution in Pressurised Cylinders.

- by -

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SUMMARY

An analysis is made of the stresses occurring in stringer reinforced cylinders due to the restraining action of the frames. Graphs are presented showing the effect of variation in frame pitch and stiffness, on the bending moment and shear force in the skins, and the hoop stress in the skins between frames. The results are used to show how the optimum structural geometry can be chosen for any given stress ratios.

NOTATION

$x, y, z.$	are co-ordinates: 'x' is measured along the length of the element, 'y' along the arc and 'z' perpendicular to the arc.
$u, v, w.$	Displacements in x, y, z, directions
p	Pressure differential
R	Radius of cylinder
A_f	Cross sectional area of frame
A_s	Cross sectional area of stringer
t	Skin thickness
$t_x = \frac{A_s}{b}$	Equivalent thickness of stringers
$t_y = \frac{A_f}{\ell_f}$	Equivalent thickness of frames
ℓ_f	Frame pitch
b	Stringer pitch
I	Moment of Inertia of stringer
E	Young's Modulus
ν	Poisson's Ratio
ϵ_x, ϵ_y	Strains in directions of x and y.
σ_1, σ_2	Longitudinal and hoop stress in skin
σ_x, σ_y	Stringer and frame stress
G	Bending moment/unit width
N	Shear force/unit width parallel to 'z' axis
T_1	Longitudinal load/unit width
T_2	Circumferential load/unit length
μ^4	$\frac{t^4 b}{4IR^2}$
ϕ	$1 - \frac{\nu t}{2 [t_x (1 - \nu^2) + t]}$

1. Introduction

It is well known that in aircraft pressure cabins, the presence of structural discontinuities such as cutouts or reinforcements, tend to disrupt the simple membrane theory, and substantially affect the local stress distribution.

In this note, the case of a circular cylinder having longitudinal and transverse stiffeners in the form of stringers and frames is considered, and the effects of varying the frame geometry on the stress distribution are shown.

Considering the effect of crack propagation in pressure cabins, the present trend is to relate the critical crack length to the maximum hoop stress in the skin, and to design the frame pitch to be less than the critical crack length. No evidence is available to show the effect of a non-uniform stress distribution such as experienced in an aircraft pressure cabin, but it would seem reasonable to expect the crack propagation rate to reduce in the region of the frames, since here the hoop stress will be substantially less than the maximum hoop stress. For this reason results are presented showing the influence of frame geometry on this hoop stress ratio, in addition to the shear and bending stresses occurring between frames, since only by including these latter effects can a correct analysis of the crack behaviour be carried out.

An assessment is made in which the frame geometry is varied in such a way that the condition of constant frame weight is maintained. The effect of having light frames of approximately half the frame pitch of present pressurised aircraft, is to substantially reduce the maximum hoop stress in the skin, giving a critical crack length of two or three times the frame pitch. The influence of frame geometry on the frame stress is considered.

2. The Stress Distribution in a Pressurised Circular Cylinder.

Considering the element shown in Fig. 1, the following equations result from equilibrium conditions

$$\left. \begin{aligned} \frac{dN}{dx} - \frac{T_2}{R} + p &= 0, \\ \frac{dG}{dx} + N &= 0, \\ \text{and } T_1 &= \frac{pR}{2}. \end{aligned} \right\} \quad (1.01)$$

(T_1 is assumed independent of 'x').

$$G = \frac{EI}{b} \times \frac{d^2 w}{dx^2}. \quad (1.02)$$

Expressing T_1 and T_2 in terms of strain components,

$$\left. \begin{aligned} T_1 &= t_x E \epsilon_x + \frac{Et}{1-\nu^2} (\epsilon_x + \nu \epsilon_y), \\ \text{and } T_2 &= \frac{Et}{1-\nu^2} (\epsilon_y + \nu \epsilon_x). \end{aligned} \right\} \quad (1.03)$$

Where $\epsilon_x = \frac{du}{dx}$,

and $\epsilon_y = \frac{w}{R}$. (tangential displacement is zero by symmetry).

As $T_1 = \frac{pR}{2}$,

then $\epsilon_x = \frac{\frac{pR}{2E} (1 - \nu^2) - \nu t \epsilon_y}{t_x (1 - \nu^2) + t}$.

and T_2 can be expressed as :-

$$T_2 = E \epsilon_y t' + pR (1 - \phi), \quad (1.04)$$

where
$$t' = \frac{t}{1 - v^2} - \frac{v^2 t^2}{t_x (1 - v^2)^2 + t(1 - v^2)},$$

and
$$\phi = 1 - \frac{v t}{2 [t_x (1 - v^2) + t]}.$$

From equation (1.01), putting $4\mu^4 = \frac{t'b}{IR^2}$,

$$\frac{d^4 w}{dx^4} + 4\mu^4 w = \frac{pb}{EI} \cdot \phi. \quad (1.05)$$

The solution is

$$w = C_1 \cos \mu x \cosh \mu x + C_2 \sin \mu x \sinh \mu x + C_3 \cos \mu x \sinh \mu x + C_4 \sin \mu x \cosh \mu x + w_0, \quad (1.06)$$

where
$$w_0 = \frac{pR^2}{Et'} \cdot \phi,$$

which reduces to

$$w = C_1 \cos \mu x \cosh \mu x + C_2 \sin \mu x \sinh \mu x + w_0, \quad (1.07)$$

(since w is symmetric in x),

and
$$\frac{dw}{dx} = \mu(C_2 - C_1) \sin \mu x \cosh \mu x + \mu(C_1 + C_2) \cos \mu x \sinh \mu x. \quad (1.08)$$

Using the condition of zero slope at the frames

$$\left(\frac{dw}{dx}\right)_{x = \pm \frac{l_f}{2}} = 0,$$

and putting $\frac{\mu l_f}{2} = \theta$,

$$\text{then } C_2 = \frac{\tan \theta - \tanh \theta}{\tan \theta + \tanh \theta} C_1 = \rho C_1, \quad (1.09)$$

At the frame, from conditions of equilibrium,

$$\frac{EI}{b} \frac{d^3 w}{dx^3} = \frac{EA_f}{2R^2} w,$$

or $\left(\frac{d^3 w}{dx^3}\right)_{x = \pm \frac{l_f}{2}} = \frac{A_f b}{2IR^2} (w)_{x = \pm \frac{l_f}{2}} \quad (1.10)$

As $\frac{d^2 w}{dx^2} = 2 \mu^2 (C_2 \cos \mu x \cosh \mu x - C_1 \sin \mu x \sinh \mu x),$

and $\frac{d^3 w}{dx^3} = 2 \mu^3 [(C_2 - C_1) \cos \mu x \sinh \mu x - (C_1 + C_2) \sin \mu x \cosh \mu x],$

then using the conditions in (1.10) gives

$$2 \mu^3 [(C_2 - C_1) \cos \theta \sinh \theta - (C_1 + C_2) \sin \theta \cosh \theta]$$

$$= \frac{A_f b}{2IR^2} [C_1 \cos \theta \cosh \theta + C_2 \sin \theta \sinh \theta + w_0],$$

and substitution of equation (1.09) gives

$$C_1 = \frac{-w_0}{\cos \theta \cosh \theta + \rho \sin \theta \sinh \theta + \frac{t'}{\mu A_f} [(1 + \rho) \sin \theta \cosh \theta + (1 - \rho) \cos \theta \sinh \theta]}.$$

By substituting

$$\frac{2 \tan \theta}{\tan \theta + \tanh \theta} \quad \text{for } 1 + \rho,$$

and $\frac{2 \tanh \theta}{\tan \theta + \tanh \theta} \quad \text{for } 1 - \rho,$

the equations for C_1 and C_2 can be more conveniently expressed as

$$C_1 = - \frac{w_0 (\sin \theta \cosh \theta + \cos \theta \sinh \theta)}{\frac{1}{2} (\sin 2\theta + \sinh 2\theta) + \frac{2t'}{\mu A_f} (\sin^2 \theta + \sinh^2 \theta)}$$

$$\text{Or } C_1 = -w_0 \gamma_1, \quad (1.11)$$

$$\text{and } C_2 = -w_0 \gamma_2, \quad (1.12)$$

$$\text{where } \frac{\gamma_2}{\gamma_1} = \frac{\sin \theta \cosh \theta - \cos \theta \sinh \theta}{\sin \theta \cosh \theta + \cos \theta \sinh \theta} = \rho.$$

Finally the bending moment, shear force and hoop force equations can be expressed as :-

(1) The Bending Moment/unit width :

$$G = \frac{EI}{b} \frac{d^2 w}{dx^2} = 2\mu \frac{EI}{b} (C_2 \cos \mu x \cosh \mu x - C_1 \sin \mu x \sinh \mu x),$$

$$\text{or } \frac{2\mu^2 G}{\phi p} = G' = \gamma_1 \sin \mu x \sinh \mu x - \gamma_2 \cos \mu x \cosh \mu x. \quad (1.13)$$

(2) The Shear Force/unit width :

$$N_s = \frac{-EI}{b} \frac{d^3 w}{dx^3} = -2\mu^3 \frac{EI}{b} \left[(C_2 - C_1) \cos \mu x \sinh \mu x - (C_1 + C_2) \frac{\sin \mu x}{\cosh \mu x} \right],$$

$$\text{or } \frac{2\mu N}{\phi p} = N' = (\gamma_2 - \gamma_1) \cos \mu x \sinh \mu x - (\gamma_1 + \gamma_2) \frac{\sin \mu x}{\cosh \mu x}. \quad (1.14)$$

(3) The Hoop Force/unit length:

As $T_2 = \frac{Ewt'}{R} + pR(1 - \phi),$

then $\frac{T_2}{pR} = 1 - \phi (\gamma_1 \cos \mu x \cosh \mu x + \gamma_2 \sin \mu x \sinh \mu x),$

or $\frac{1}{\phi} (1 - \frac{T_2}{pR}) = T'_2 = \gamma_1 \cos \mu x \cosh \mu x + \gamma_2 \sin \mu x \sinh \mu x. (1.15)$

3. Discussion of Results.

Using equations 1.13, 1.14 and 1.15, Fig. 3 shows the values of the bending moment, shear force and hoop force parameters G' , N' and T'_2 occurring at the frame positions for a practical range of frame pitch and stiffness. Figs. 4, 5 and 6 show the same parameters at positions between frames. In this case, the frame stiffness parameter $\frac{2t'}{\mu A_f}$ has been varied between 0.5 and 20 and the frame pitch parameter between .60 and 3.0. As an aid to simplifying this work Fig. 2 was constructed to show the variation of the parameter ϕ with the skin thickness (t) and the ratio of stringer area to stringer pitch ($\frac{t_x}{x}$). It was found that for many aircraft pressure cabins having typical structural geometry, a constant value of $\phi = .90$ could be used with reasonable accuracy.

Using this approximate value for ϕ , Fig. (7a) shows the ratio of maximum hoop stress in the skin to the nominal hoop stress (as predicted by simple 'boiler theory'), against the frame pitch parameter $\frac{\mu l_f}{2}$ and the frame stiffness parameter $\frac{2t'}{\mu A_f}$. The maximum hoop stress is seen to equal the nominal hoop stress at a value of $\frac{\mu l_f}{2} = 2.36$ for all values of $\frac{2t'}{\mu A_f}$. This means that at this unique value of $\frac{\mu l_f}{2}$, the frame size ceases to influence the maximum hoop stress in the skins.

The reason for this is seen by examination of equations 1.15 and 1.11. Since the maximum hoop stress occurs midway between frames (when $x = 0$), then equation 1.15 reduces to $\frac{1}{\phi} (1 - \frac{2}{pR}) = \gamma_1$. When $\frac{2}{\sigma} \frac{2t'}{\mu A_f}$ (that is $\frac{T_2}{pR}$) approaches 1.0, then γ_1 approaches zero giving a solution to equation (1.11) which is independent of the parameter $\frac{2t'}{\mu A_f}$ of $\frac{\mu l_f}{2} = 2.36$. In a similar manner Fig. 7b shows the hoop stress in the skin at the position of the frames.

Fig. (8) shows the ratio of the direct stress in the frames σ_y to the nominal stress. The parameter σ'_y is given as $\frac{pR'}{t}$ where R' is the mean radius of the frame. For relatively shallow frames this can be approximated by the nominal hoop stress in the skin $\frac{pR}{t}$.

The ratio of maximum hoop stress in the skin to the frame direct stress is given in Fig. 9(a) for various geometries. In Fig. 9(b) this ratio is plotted against the frame pitch parameter $\frac{\mu \ell_f}{2}$ and the parameter $\frac{t'}{t_y}$. If the stringer area remains constant, which is reasonable since the stringer geometry is determined by considerations other than pressure, then for any given skin thickness t , the lines of constant $\frac{t'}{t_y}$ are lines of constant weight.

The use of these results can best be illustrated by examples.

CASE 1.

(a)	Skin thickness	t	=	.048 in.
	Diameter	$2R$	=	10 ft.
	Frame pitch	ℓ_f	=	27 in.
	Frame area	A_f	=	.345 in. ²
	Stringer pitch	b	=	6.25 in.
	Stringer area	A_s	=	.129 in. ²
		$\frac{t'}{t}$	=	1.029
		I_s	=	.0439 in. ⁴
		μ	=	.1485
		$\frac{\mu \ell_f}{2}$	=	2.0
		$\frac{2t'}{\mu A_f}$	=	1.925
		$\frac{t'}{t_y}$	=	3.875

From Fig. 7a	From Fig. 7b	From Fig. 8	From Fig. 9
$\frac{\sigma_2}{\frac{pR}{t}} = .953$	$\frac{\sigma_2(\text{frame})}{\frac{pR}{t}} = .651$	$\frac{\sigma_y}{\sigma'_y} = .54$	$\frac{\sigma_2}{\sigma_y} = 1.765$

(b)

Reducing the frame pitch to 13.5 in and the frame area to .1725 in.², that is keeping the ratio $\frac{t'}{t_y}$ constant gives:-

$\frac{\sigma_2}{\frac{pR}{t}} = .81$	$\frac{\sigma_2(\text{frame})}{\frac{pR}{t}} = .77$	$\frac{\sigma_y}{\sigma'_y} = .615$	$\frac{\sigma_2}{\sigma_y} = 1.30$
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CASE 2.

(a)

The geometry is identical with case 1 except that the frame pitch is 20 ins.

Then $\frac{\mu l_f}{2} = 1.48$

$$\frac{2t'}{\mu A_f} = 1.925$$

$$\frac{t'}{t_y} = 2.86$$

$\frac{\sigma_2}{\frac{pR}{t}} = .842$	$\frac{\sigma_2(\text{frame})}{\frac{pR}{t}} = .70$	$\frac{\sigma_y}{\sigma'_y} = .55$	$\frac{\sigma_2}{\sigma_y} = 1.530$
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(b) Reducing the pitch to 10" and the frame area to .1725 in² again keeping the ratio $\frac{t'}{t}$ constant gives :-

$\frac{\sigma_2}{\frac{pR}{t}} = .74$	$\frac{\sigma_2(\text{frame})}{\frac{pR}{t}} = .73$	$\frac{\sigma_y}{\sigma'_y} = .575$	$\frac{\sigma_2}{\sigma_y} = 1.285$
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Case 1 shows that by reducing the frame pitch from 27 in. to 13.5 in., at the same time keeping the structure weight constant, gives a substantial reduction in maximum hoop stress in the skin, but increases the frame stress and the hoop stress in the skin at the frame positions. The ratio of maximum hoop stress in the skin to frame stress reduces. Case 2 shows the same trend.

The effect of using the frames as a crack stopper.

Using the results of cases 1 and 2, and bearing in mind the recent work on crack propagation, one wonders which is the best structural configuration in the design of aircraft pressure cabins.

Having widely spaced frames means high maximum hoop stress, but the stresses in the region of the frames are relatively low, and hence such frames become effective crack stoppers, providing of course that the frame pitch is less than the critical crack length.

If the structure weight is kept constant and the frame pitch is decreased, a reduction of 15% and 12% occurs in the maximum hoop stress for cases 1 and 2 respectively.

This reduction in maximum hoop stress means a substantial increase in the critical crack length. Case 2b shows that by reducing the frame pitch from 20 ins to 10 ins, only gives a slight increase in the stresses at the frames. This means that although such frames are less effective as crack stoppers, the crack would have to penetrate probably two frames before the critical crack length is reached.

4. Conclusions.

Results are presented giving the stress ratios in a reinforced circular cylinder. These results show the effect of structural variation on these stress ratios.

Some advantage seems to be gained by using relatively closely spaced frames in pressure cabin design, since such frames serve to substantially reduce the maximum hoop stress in the skins, which could be used either to reduce the skin thickness or to increase the critical crack length.

5. Acknowledgement:

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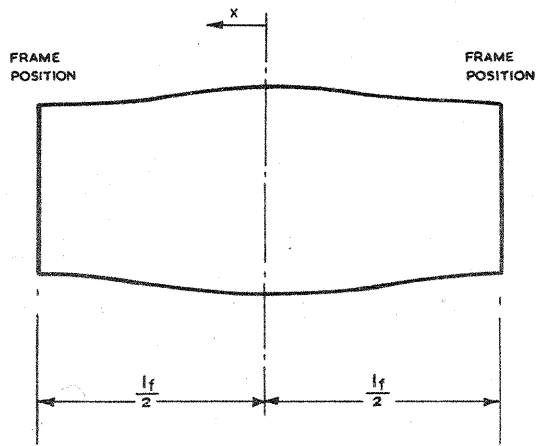
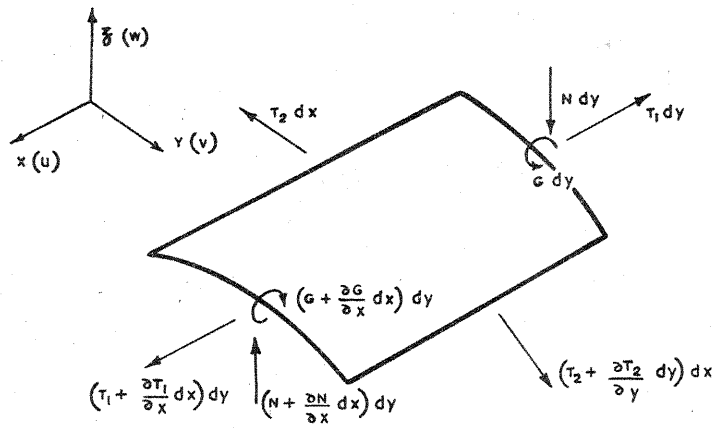


FIG. 1.

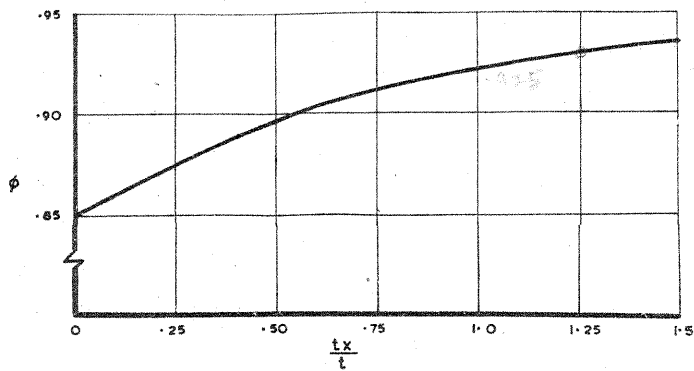


FIG. 2. THE PARAMETER ϕ AGAINST THE THICKNESS RATIO $\frac{tx}{t}$

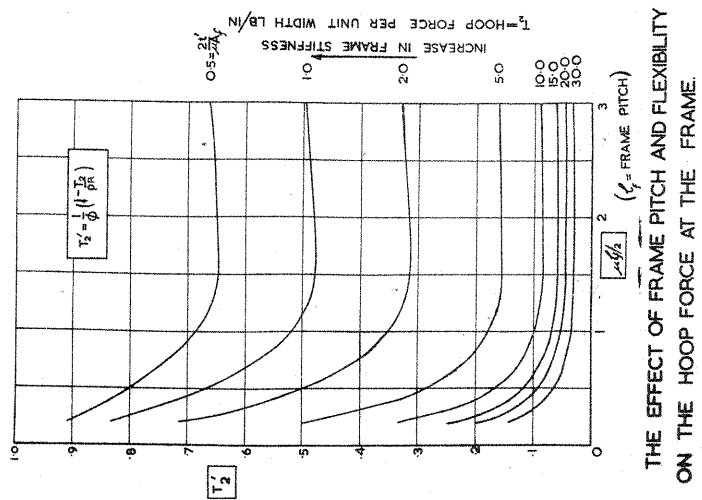
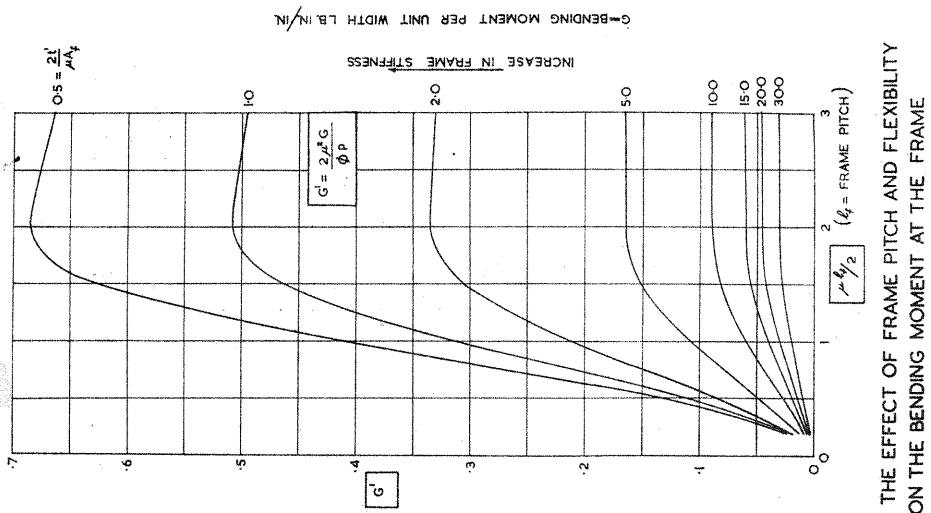
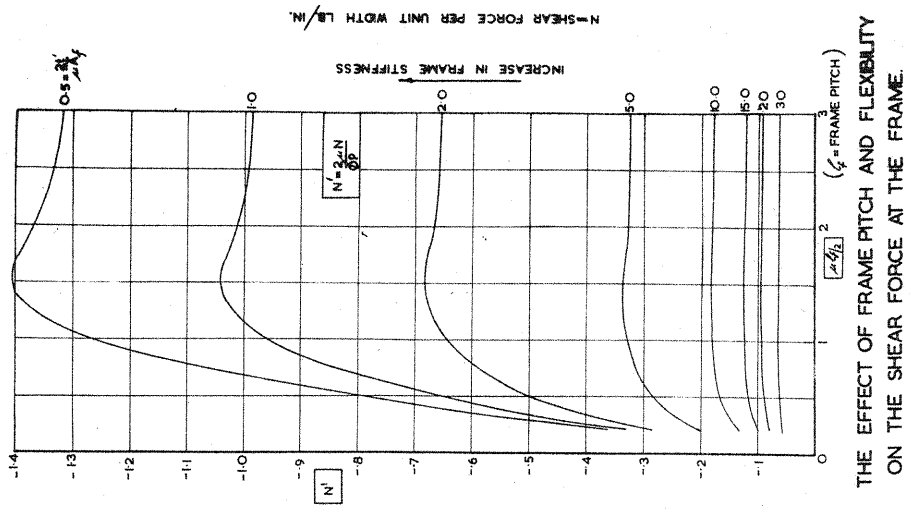


FIG. 3.

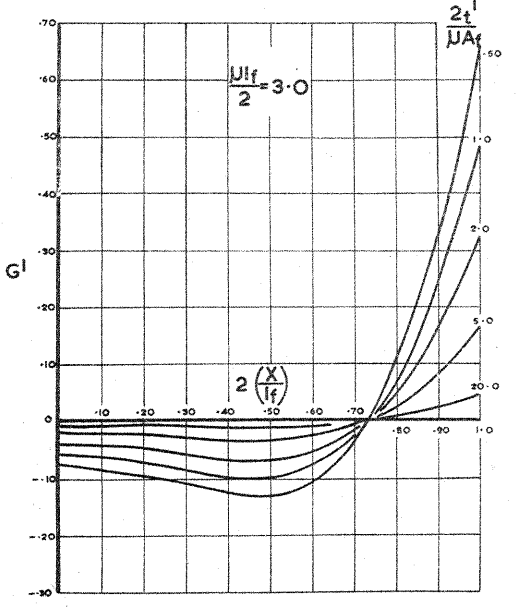
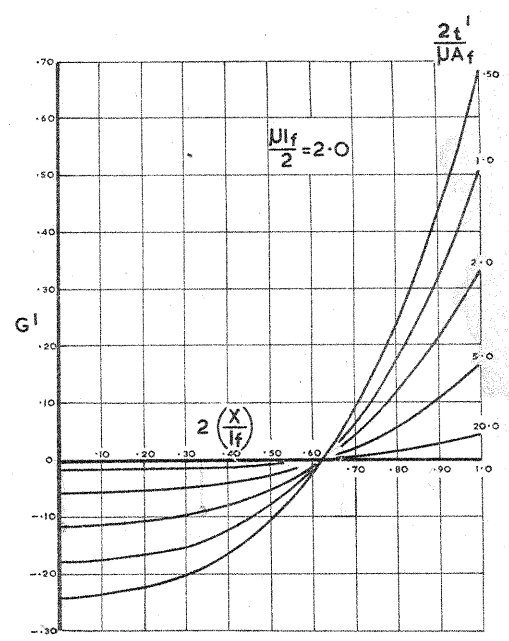
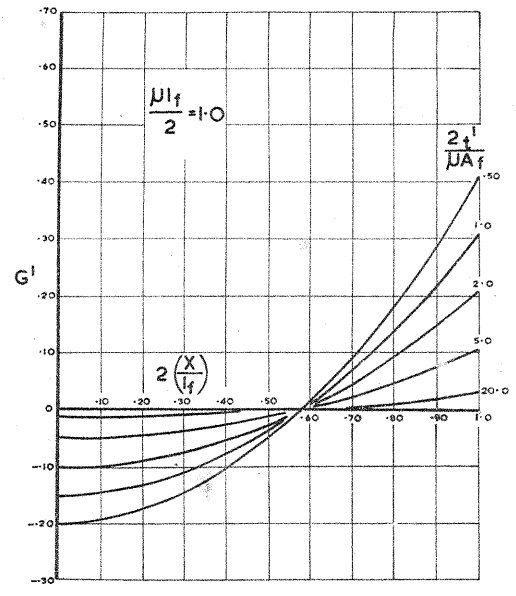
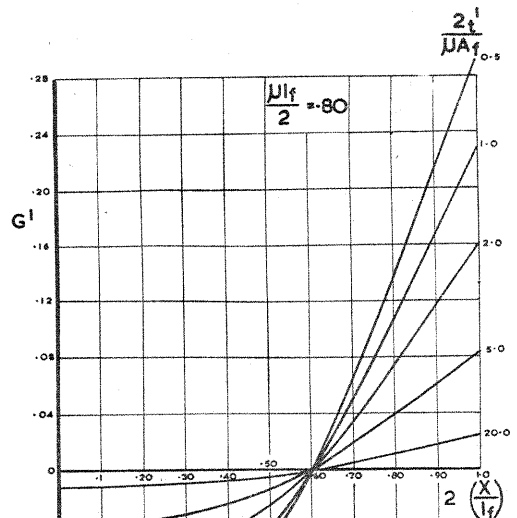
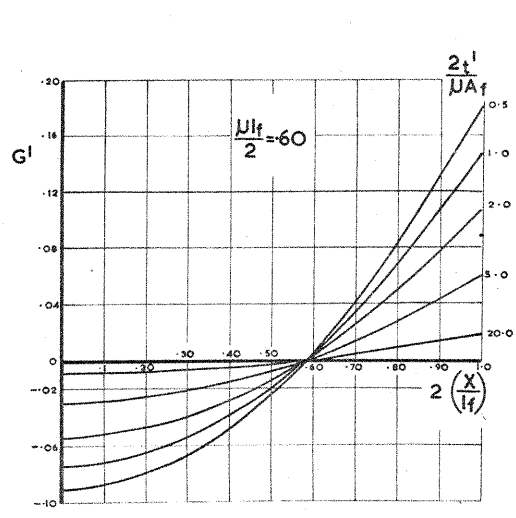


FIG. 4. THE EFFECT OF FRAME PITCH AND FLEXIBILITY ON THE BENDING MOMENT AT POSITIONS BETWEEN FRAMES

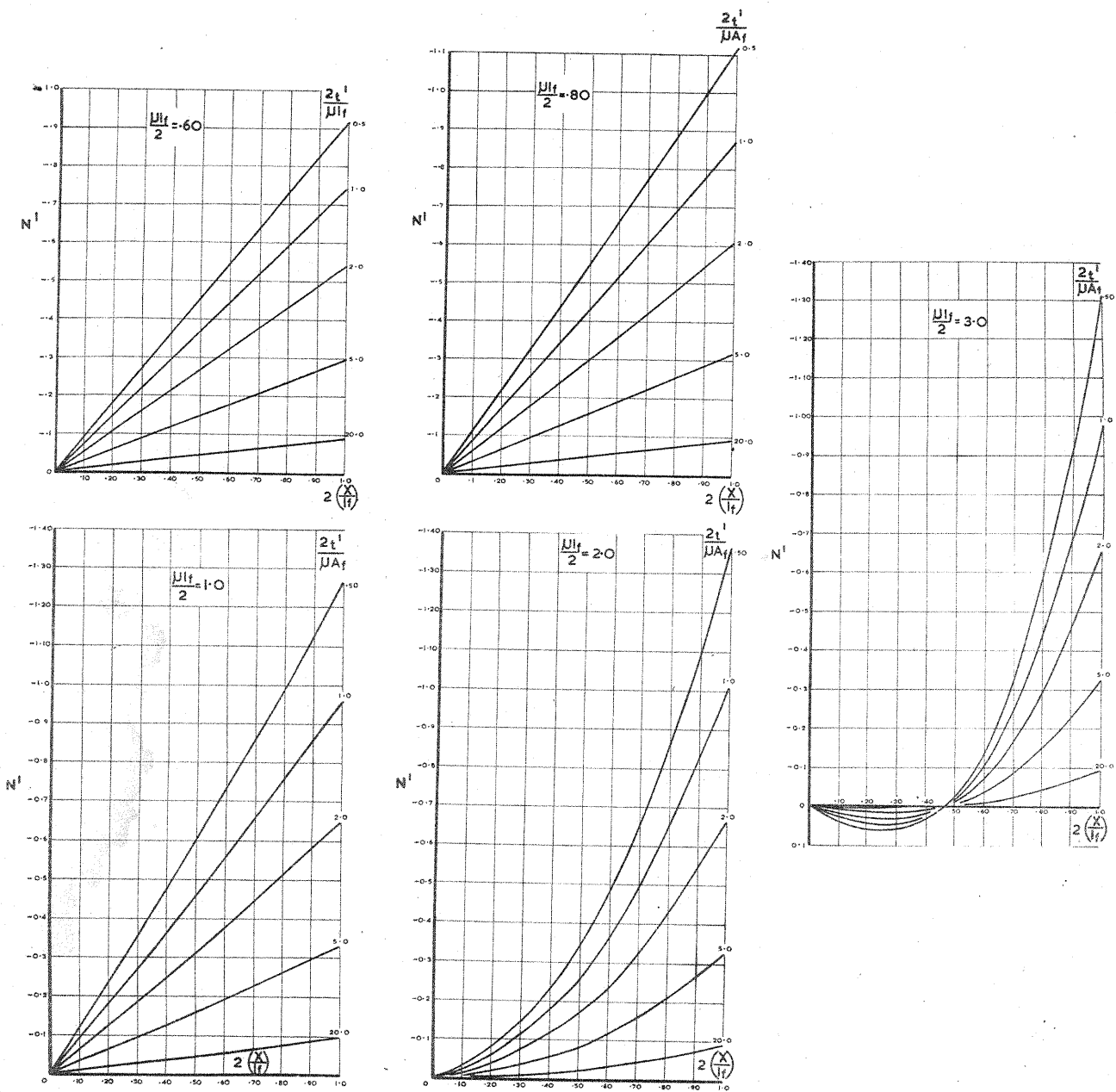


FIG. 5. THE EFFECT OF FRAME PITCH AND FLEXIBILITY ON THE SHEAR FORCE AT POSITIONS BETWEEN FRAMES

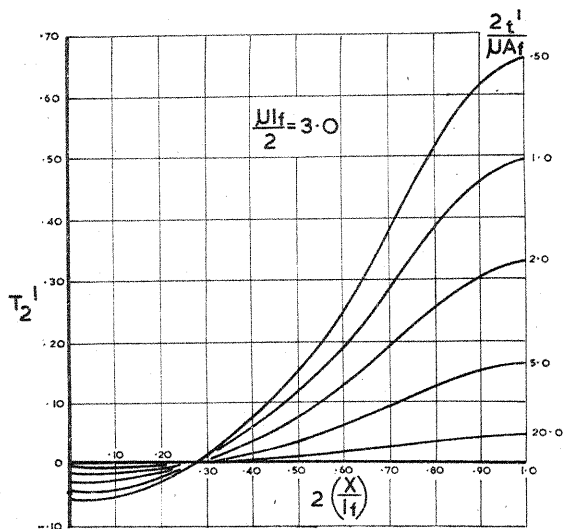
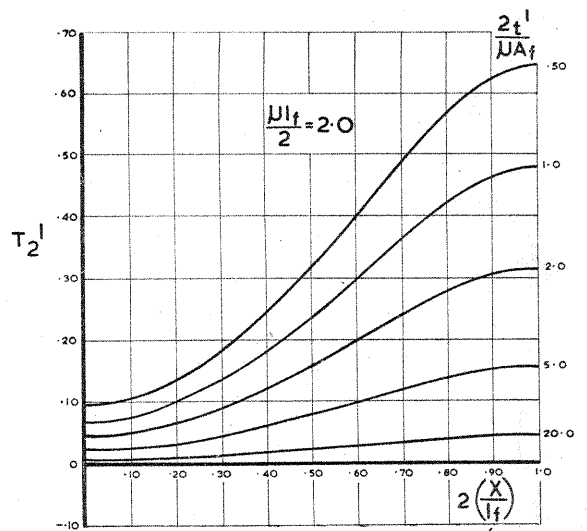
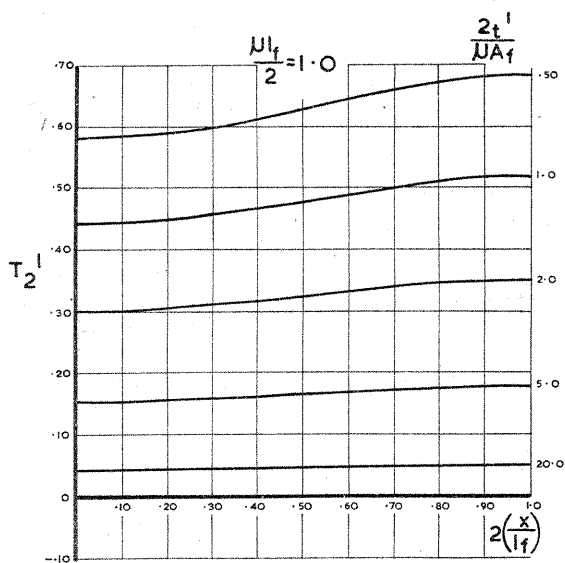
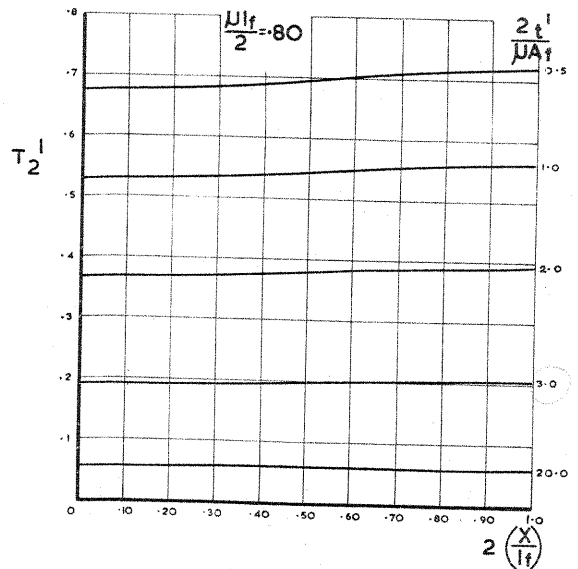
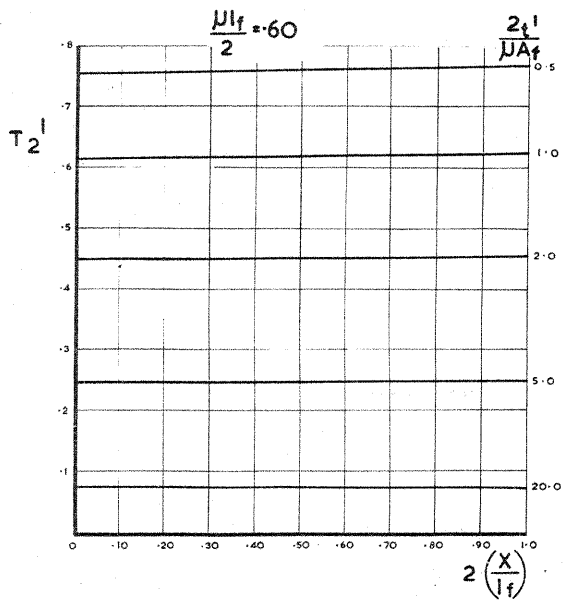


FIG. 6. THE EFFECT OF FRAME PITCH AND FLEXIBILITY ON THE HOOP FORCE AT POSITIONS BETWEEN FRAMES

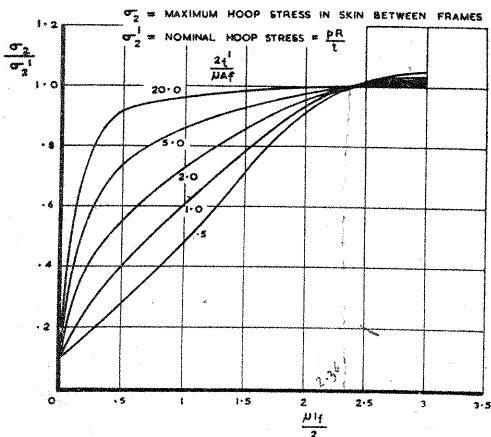


FIG. 7A. EFFECT OF FRAME PITCH & STIFFNESS ON THE MAXIMUM HOOP STRESS IN THE SKIN BETWEEN FRAMES

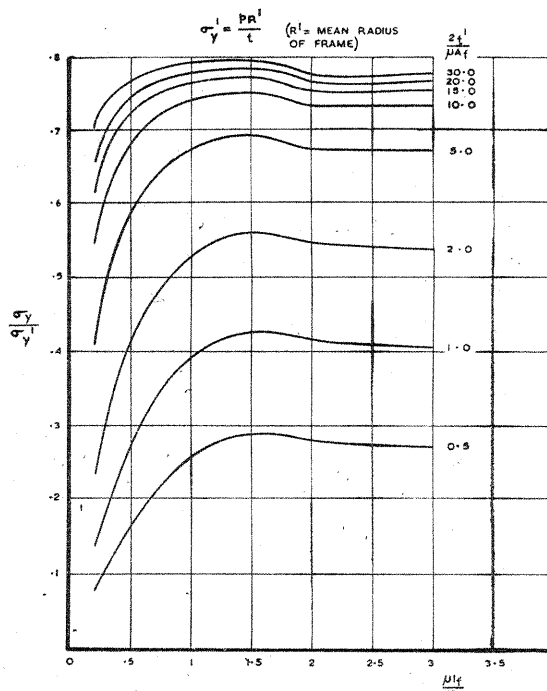


FIG. 8. THE EFFECT OF FRAME PITCH & STIFFNESS ON THE FRAME STRESS

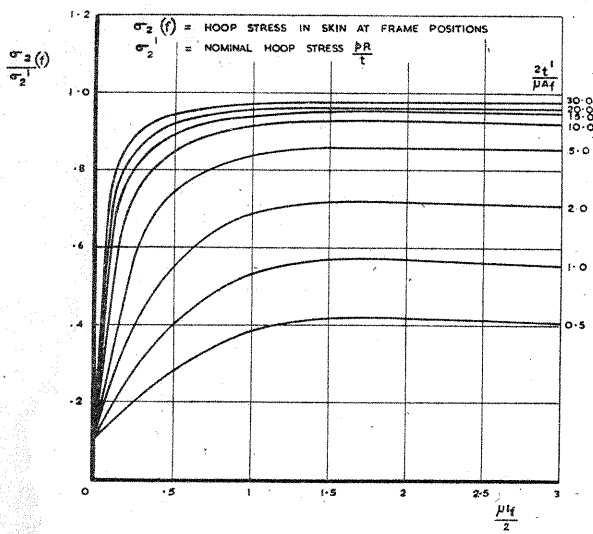


FIG. 7B. THE EFFECT OF FRAME PITCH AND STIFFNESS ON THE HOOP STRESS IN THE SKIN IN THE REGION OF THE FRAMES

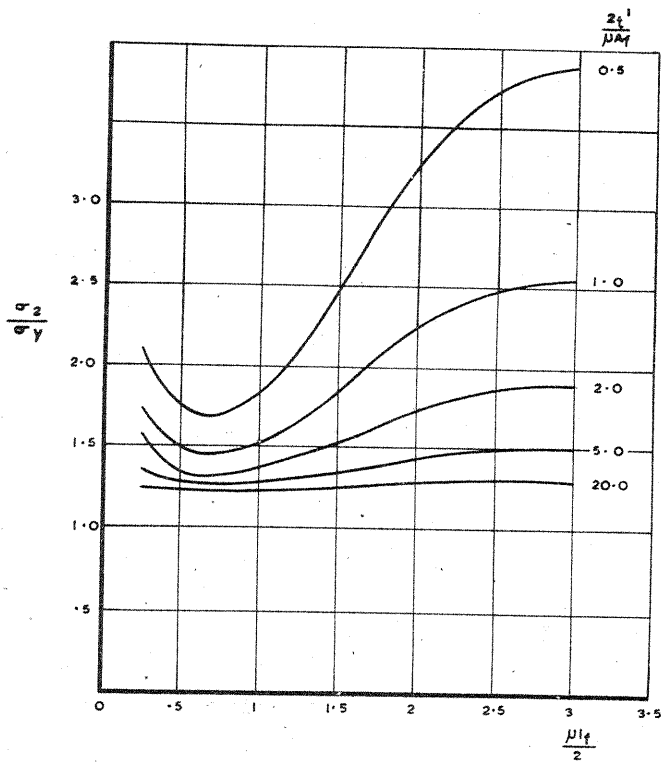


FIG. 9A.

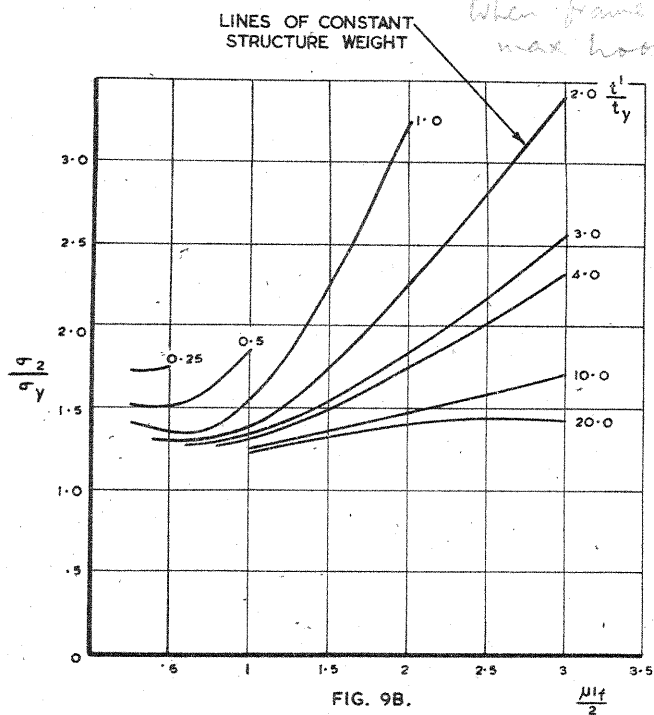


FIG. 9B.

THE RATIO OF MAXIMUM HOOP STRESS IN SKIN TO FRAME STRESS FOR VARIOUS STRUCTURAL GEOMETRIES