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THE LIFT INDUCED ON A CIRCULAR CYLINDER
BY A NARROW JET EMERGING AT AN ANGLE
TO THE UNIFORM STREAM

by

R. G. ANDERSON, B.A., B.Sc., and G. M. LILLEY, M.Sc., D.I.C.

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C R A N F I E L D

The lift induced on a circular cylinder by a narrow jet emerging at an angle to the uniform stream

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R.G. Anderson, B.A., B.Sc., and
G.M. Lilley, M.Sc., D.I.C., A.F.R.Ae.S., A.M.I.Mech.E.
of the Department of Aerodynamics



SUMMARY

A preliminary qualitative study has been made in a smoke tunnel of the flow around a stationary circular cylinder, having rear jet blowing, placed in a uniform stream. Photographs have been taken which show many interesting flow phenomena. A knowledge of these it is hoped will aid future quantitative studies.

It is found that when the jet is deflected downwards at an angle to the mainstream direction boundary layer separation is delayed around the cylinder and the cylinder develops a lift force in addition to its drag. The delay in boundary layer separation is suggested to be mainly the result of the 'sink effect' of the jet mixing. The lift, on the other hand, appears to be primarily the result of partial attainment of stagnation pressure on both sides of the jet at exit, and is therefore of similar origin to the 'Thwaites' flap. A smaller, but no less significant, contribution to the lift arises from the direct and indirect effects of the jet blowing.

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1. Introduction

The 'Thwaites' flap has been known for some time to produce circulation on a circular cylinder by fixing the rear stagnation point with a small flap and using suction to cause the boundary layer flow to adhere to the cylinder. By this means, lifts on the cylinder have been obtained of the order of those obtained from potential flow theory (see ref. 1).

It was suggested that a similar effect to the Thwaites flap could be obtained from a narrow jet emerging from the rear of a circular cylinder, since the rear stagnation point, at least in inviscid flow, would be fixed on the cylinder at the point of jet efflux. In order to investigate this phenomenon a preliminary qualitative study of the flow was made in a Lippisch type smoke tunnel at Cranfield.^{*} Although the work started is by no means complete, and no quantitative measurements are included in this report, it is felt desirable, in view of the great interest aroused on the question of jet blowing, to present these preliminary data in order to illustrate some important aspects of this phenomenon.

2. Apparatus

The smoke tunnel used was of the Lippisch type using paraffin smoke. The cylinder was fixed between the side walls of the tunnel. The cylinder was hollowed out, providing a chamber for the compressed air supply. The slot, of uniform width, was 0.2in. smaller in length than the width of the cylinder. The velocity of the jet and its angle of efflux relative to the mainstream direction could be fully varied. A diagram of the cylinder is shown in figure 1.

3. Smoke-tunnel photographs

Although a complete study of the flow was made with varying jet velocities and different angles of jet efflux, this short note presents only the photographs obtained for varying jet velocities when the angle of jet emergence was maintained constant at $\theta^\circ = 22\frac{1}{2}^\circ$.

In the discussion below the jet strength is denoted

/by the ...

* This work was completed in April, 1954.

by the jet momentum coefficient C_μ where

$$C_\mu = \frac{m v_b}{\frac{1}{2} \rho_o U_o^2 d}$$

where m is the rate of mass flow through the blowing slot per unit span

v_b is the jet velocity assuming isentropic expansion to freestream pressure p_o

$\frac{1}{2} \rho_o U_o^2$ is the freestream dynamic pressure

d is the cylinder diameter.

Although the flow is not wholly two-dimensional, the three-dimensional effects, arising from the small span/diameter ratio, will not be discussed.

The Reynolds number based on cylinder diameter is about 30,000.

3a. Group 1

Complete flow field

Figures 2,3,4,5 show the complete flow field around the cylinder as the jet velocity is increased. Values of C_μ have been added to each Figure but these should not be used for purposes other than comparison as only very rough estimates were made. The error may be 50 per cent.

Figure 2 $C_\mu = 0$. The Kármán vortices are clearly defined.

Figure 3 $C_\mu \approx 0.05$. The effect of the jet on the external stream is negligible.

Figure 4 $C_\mu \approx 0.60$. The whole of the boundary layer

separated flow on the underside of the cylinder is now suppressed. The separation on the upper surface is delayed together with the complete elimination of the Kármán vortex street. The boundaries of the jet are clearly defined up to 3 diameters from the jet exit. The 'sink effect' of the jet is apparent from the form the streamlines, from the undersurface, take on entering the 'jet boundary'. The lift on the cylinder can be deduced, qualitatively, from the pattern of streamlines and, in particular, from the movement downwards of the front stagnation point.

Figure 5 $C_{\mu} = 1.00$. The jet is of sufficient strength to move the front stagnation point to a position nearly symmetric with the position of the jet at the rear. The high lift on the cylinder is apparent from the streamline pattern and the movement downwards of the front stagnation point. The boundary layer separation on the upper surface is further delayed and it is noted that streamlines from both the upper and lower parts of the flow region enter the jet about symmetrically up to two diameters from the jet exit. Although it is not very clear in the photograph, it was observed that a stagnation point in the external flow existed on the upper surface of the jet boundary. On the lower part of the jet boundary it is not clear whether or not a stagnation point exists close to the jet exit. Certainly a very short distance from the jet exit the flow enters the 'jet' almost at right angles.

The lack of definition of the smoke filaments towards the left of the picture is mainly due to insufficient smoke filaments below the cylinder, and diffusion of those filaments which are present, owing to the long distances they have to travel before entering the jet mixing region.

It is worth noting the differences that exist between this flow and that shown in figure 13(b). There is little to distinguish these flows on the right hand side of the vertical centre line, but towards the left, especially in the undersurface flow, the differences are very significant.

Some further tests, not recorded here, show that further increases in the jet strength do not move the front stagnation point closer to the jet. The experiments indicated, at least qualitatively, that the approximate maximum movement of the front stagnation point was given by the position symmetric with the jet at the rear. This fact, although not in itself surprising in view of the analogy of the flow with the Thwaites flap, is probably more correctly explained in terms of the jet 'sink effect'. (See Appendix 1).

3b. Group 2

The flow in the neighbourhood of the jet

Figures 6,7,8,9,10,11,12, represent stages in the reattachment of the boundary layer at the rear of the cylinder with increasing jet strength.

Figure 6 $C_{\mu} = 0$. The commencement of the Kármán vortex street can clearly be seen.

Figure 7 $C_{\mu} = 0.07$. The closing up of the separated flow region behind the cylinder is seen in addition to a well defined 'stabilisation' of the flow outside the wake boundaries. The jet itself is barely visible. It can be seen, however, that part of the external flow is induced into the lower boundary of the jet but not close to the jet exit.

Figure 8 $C_{\mu} = 0.10$. The induced flow effect noted in figure 7 is now more marked and is present on both boundaries of the jet especially on the undersurface. The region of separated flow on the undersurface of the cylinder has now been decreased considerably in size and there is nearly complete suppression of the Kármán vortex street.

Figure 9 $C_{\mu} = 0.30$. The closing up of the separated flow regions is still continuing although the lower region is still much smaller than the upper. Some idea of the delay in boundary layer separation on the lower surface can be seen from the photograph. The 'sink effect' of the jet is now more pronounced and what appears to be a stagnation point exists on the lower boundary of the jet. Little movement, if any, of the front stagnation point has taken place.

Figure 10 $C_{\mu} = 0.50$. The reattachment of the undersurface flow is almost complete. The 'stagnation point' on the lower part of the jet boundary appears to divide the flow between that which flows downstream with the jet and that which forms the bounding streamline to the flow inside the separated region. There is a small downward movement of the front stagnation point indicating that the cylinder might experience a small lift force.

Figure 11 $C_{\mu} = 0.70$. The reattachment of the complete undersurface flow is now attained, at least as far as observation of smoke filaments can tell. The jet 'sink effect' is very noticeable. The front stagnation point has now moved downwards and it is significant that this has occurred more or less at the same time as the streamlines over the upper surface have taken on a curvature similar to that of the cylinder. In other words the lift on the cylinder appears to be more dependent on the suppression of separated flow on the upper surface than on the lower.

Figure 12 $C_{\mu} = 0.90$. It is shown that although the upper surface flow attachment is not complete the flow downstream of the jet exit is nearly symmetric. The front stagnation point is in a position approximately symmetric with that of the jet. The steadiness of the external flow is well demonstrated by the photographs in contrast with the flow at zero blow. In addition it should be apparent that the function of the jet in this experiment is not only one of boundary layer control but one of

circulation control which imposes larger changes on the flow external to the cylinder.

4. Discussion

The action of the jet on the flow described above around the cylinder can be summarised as follows.-

- (i) The jet strength must exceed a certain critical value before it can influence materially the control of boundary layer separation on the leeward side of the cylinder.
- (ii) The action of the jet mixing appears to be well represented by a distribution of sinks along the 'jet boundaries'. The effect is more marked on the undersurface of the cylinder than on the upper surface. (See figure 13(d) and (e)). It appears that the induced flow effects of jet mixing, more than any other cause, control and attempt to nullify regions of separated flow.
- (iii) Although in the Introduction the circular cylinder with a jet, without separated flow, was suggested as probably being analogous to the 'Thwaites' flap, one important distinction between them exists. In the latter case, with complete suppression of the boundary layer, say, by porous suction on the surface of the cylinder, the streamline pattern is exactly that obtained in Figure 13b. In the former case, due to the jet 'sink effect', the streamlines are either normal to or inclined towards the jet centre line. The 'sink effect' therefore modifies the pressure distribution around the cylinder from that obtained from the 'Thwaites' flap effect, and hence the lift will be smaller (see Appendix 1).
- (iv) An interesting feature of the flow is the formation of what appears to be a stagnation point on the jet boundaries. The flow in its vicinity is quite stable and contrasts strongly with the unstable flow found with the 'Thwaites' flap when the spoiler is retracted. A further important consideration is that the flow changes in step with the change in jet speed. A flow regime obtained at a given jet speed is not retained when the jet speed is decreased.
- (v) The direct effect of the jet blowing is twofold. Firstly the cylinder experiences the reaction of the jet thrust as a pressure force on the internal ducts.

Secondly in forming the jet boundaries it separates the two parts of the external flow downstream of the cylinder and thus allows a pressure difference to exist across these combined boundaries. Thus, neglecting the jet 'sink effect', the cylinder may be regarded as having a 'tail', formed by the vortex sheets of the jet. It is obvious that the pressure distribution, and therefore the lift, on the cylinder with 'tail' should be different from that obtained on the cylinder alone. From the flow pictures it appears that, in this experiment, the lift on the cylinder due to jet blowing is not much greater than that obtained from the 'Thwaites' flap effect and indeed may well be ^{less} due to the reduction in lift by the 'sink effect'. However, it is not yet possible to give a quantitative estimate of the gains or losses in lift due to the various causes enumerated above.

- (vi) The overall effects of an external source (sink) or vortex distribution on the pressure distribution and forces on the cylinder cannot be completely determined from a study of the flow photographs. In order to confirm some of the previous qualitative conclusions the action of a sink and vortex placed downstream of a lifting circular cylinder in a uniform flow will be briefly discussed (see Appendix 1).

The effect of an external source (or sink)₂ on a non-lifting circular cylinder has been discussed by Lamb². The result is that the cylinder experiences a force in the direction of the source (or sink). In Appendix 1 it is shown that the action of a sink, placed downstream in the vicinity of a lifting cylinder, is to increase the drag and to decrease the lift on the cylinder. Similarly the lift is increased and the drag is reduced when a vortex, having the same sign as the circulation around the cylinder, is close to the cylinder.

Although the 'sink effect' causes, in general, a reduction in lift it also modifies the pressures distribution around the cylinder, especially on the leeward side, to such an extent that the region of adverse pressure gradient is reduced.

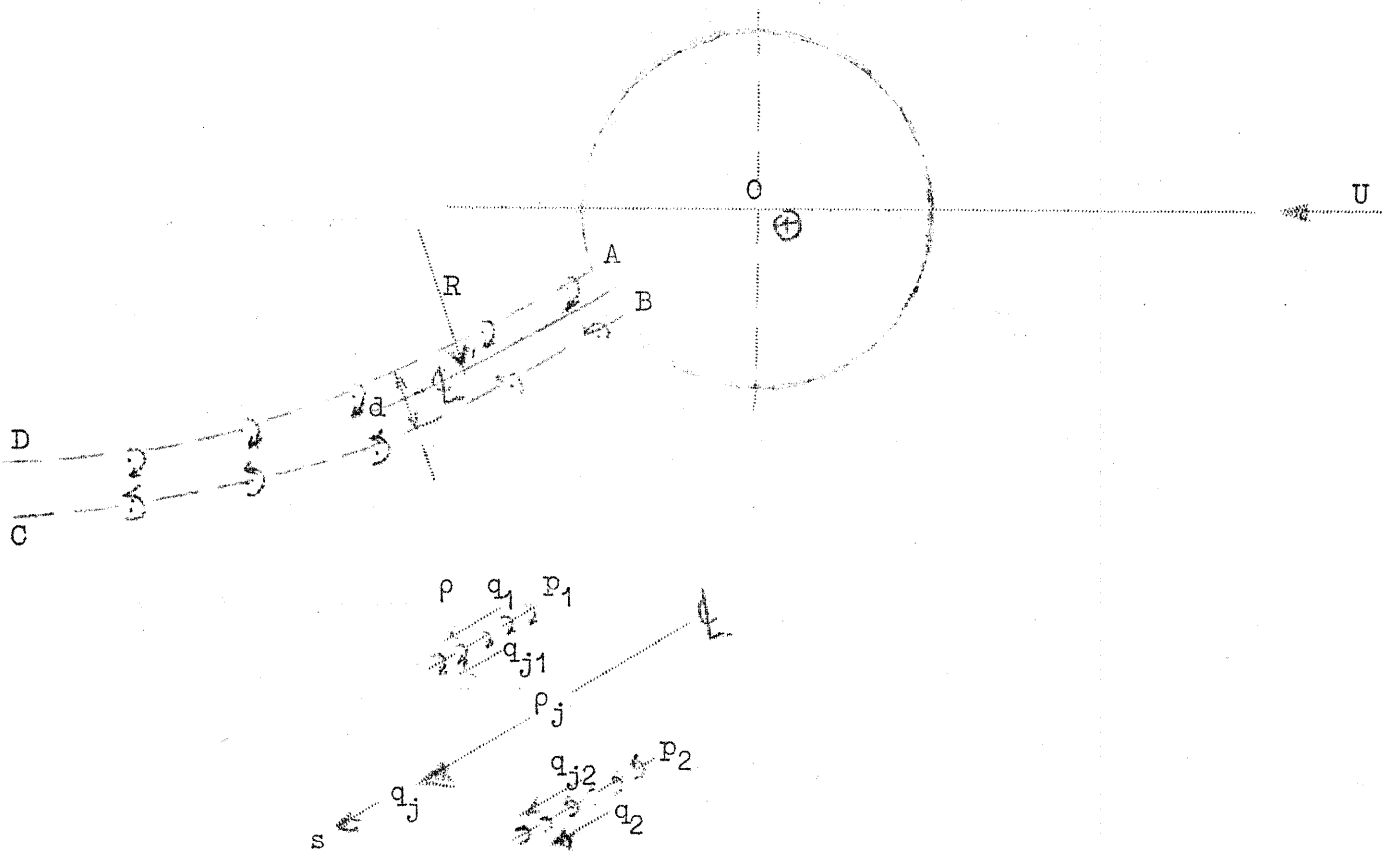
These remarks relate only to a single source or vortex placed in the lee of a cylinder. It is not expected that these conclusions will be invalidated when the single source or vortex is replaced by a suitable distribution of singularities.

5. Mathematical model

From the analysis of the flow photographs arise certain suggestions for the calculation of the flow field around a circular cylinder with a jet. The suggested model for these calculations is discussed below.

(a) The 'sink effect' neglected

The fluid inside and outside the cylinder is inviscid and incompressible and both flows are irrotational although the total head of the internal flow is assumed larger than the external total head.



(i) Internal flow

The internal flow consists of a source placed at the origin and a corresponding sink for downstream between the vortex sheets AD and BC . The flow from the source issues through the

slit AB in the cylinder. The boundary condition to be applied inside the cylinder is zero normal velocity at the walls. The pressure distribution and total force on the internal walls is required.

AD and BC are vortex sheets springing from the lips A and B of the slit in the cylinder. The pressure across any normal cross-section of the jet is not necessarily constant but the boundary conditions, that the pressure on each side of each vortex sheet is the same, and no flow across them must be applied.

Since the flow is irrotational the total head everywhere in the internal and external flow is constant. Hence if q_1 and q_2 are the external velocities on the jet boundaries and q_{j1} and q_{j2} are the corresponding internal jet velocities

$$H = p_1 + \frac{1}{2}\rho q_1^2 = p_2 + \frac{1}{2}\rho q_2^2 \quad \dots\dots\dots(1)$$

and

$$H_j = p_1 + \frac{1}{2}\rho_j q_{j1}^2 = p_2 + \frac{1}{2}\rho_j q_{j2}^2 \quad \dots\dots\dots(2)$$

where H, p, and ρ with the appropriate suffix, denote the total head, static pressure and density respectively.

If R is the radius of curvature of the jet where its width is d then, on the assumption that the jet is thin, the balance between the centrifugal force and the pressure on the jet faces, over a length ds, gives

$$p_2 - p_1 = \rho_j d \frac{q_j^2}{R} \quad \dots\dots\dots(3)$$

where q_j is the mean jet velocity = $\frac{q_{j1} + q_{j2}}{2}$.

But from equations (1) and (2)

$$p_2 - p_1 = \frac{1}{2}\rho (q_1^2 - q_2^2) = \frac{1}{2}\rho_j (q_{j1}^2 - q_{j2}^2) \quad \dots\dots(4)$$

which together with (3), leads to

$$\left(\frac{q_{j1} - q_{j2}}{q_j} \right) = \frac{d}{R} \quad \dots\dots\dots(5)$$

On the assumption that the jet is thin the rate of volume flow

through the jet is given by

$$q_j d = Q_j = \text{const.} \dots\dots\dots(6)$$

where Q_j is the strength of the source.

The strengths of the vortex sheets AD and BC, therefore, become respectively

$$\text{and } \left. \begin{aligned} \gamma_1 &= - (q_{j1} - q_1) \\ \gamma_2 &= (q_{j2} - q_2) \end{aligned} \right\} \dots\dots\dots(7)$$

and from equations (7), together with (5) the net strength

$$\begin{aligned} \gamma &= \gamma_2 \left(1 + \frac{d}{2R}\right) + \gamma_1 \left(1 - \frac{d}{2R}\right) \\ &= - (q_2 - q_1) - \frac{(q_1 + q_2)d}{2R} \dots\dots\dots(8) \end{aligned}$$

From equation (4) we obtain, if $\Delta p = p_2 - p_1$,

$$q_{j1} - q_{j2} = \frac{\Delta p}{\rho_j q_j} \quad \text{and} \quad q_1 - q_2 = \frac{2\Delta p}{\rho(q_1 + q_2)} \dots\dots(9)$$

Therefore on substitution of equation (9) into (8) we get for the net vortex strength

$$\gamma = \frac{2\Delta p}{\rho(q_1 + q_2)} - \frac{(q_1 + q_2)d}{2R} \dots\dots\dots(10)$$

But from equation (3) $\Delta p = \frac{\rho_j d q_j^2}{R}$, and with (6), equation (10) becomes

$$\gamma = \frac{Q_j}{R} \left(\frac{2 \rho_j q_j}{\rho(q_1 + q_2)} - \frac{(q_1 + q_2)}{2q_j} \right) \dots\dots\dots(11)$$

The width of the jet far downstream is given by

$$d_\infty = \frac{Q_j}{q_{j\infty}}$$

and since the pressure will then be constant over the whole jet

width and outside we find that

$$\Delta H = H_j - H = \frac{1}{2} \left(\rho_j q_{j\infty}^2 - \rho U^2 \right) \dots\dots\dots(12)$$

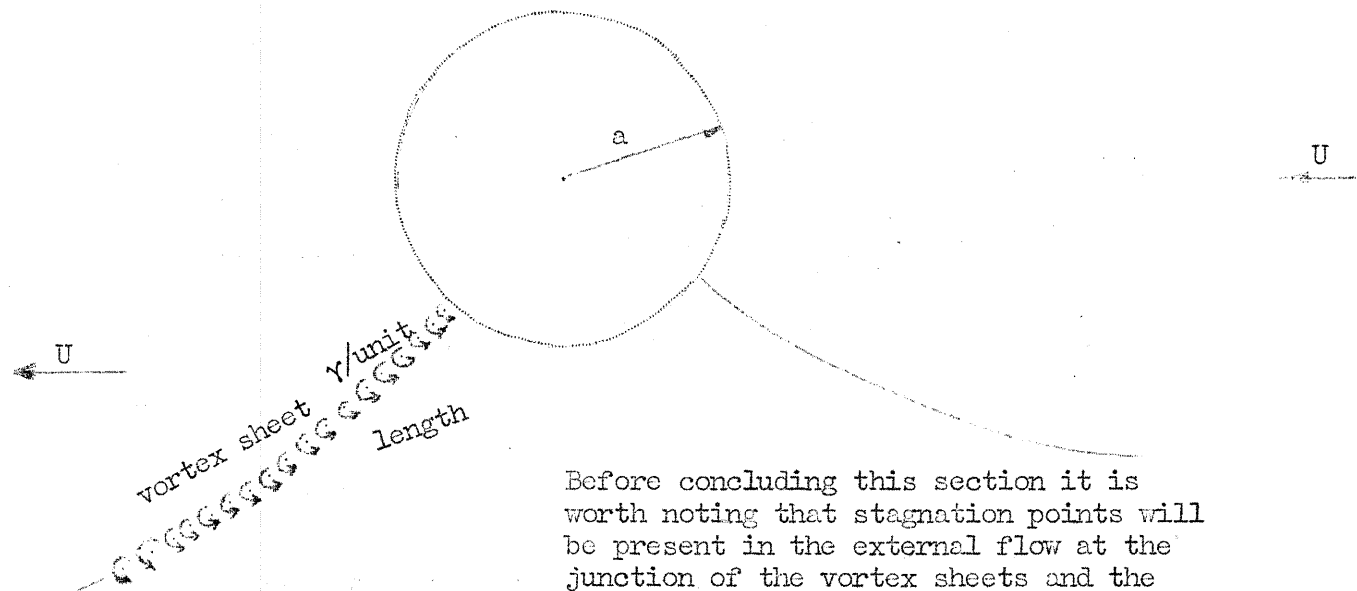
which together with equation (6) gives

$$\frac{\Delta H}{\frac{1}{2}\rho U^2} = \left(\frac{\rho_j Q_j^2}{\rho d_{\infty}^2 U^2} - 1 \right) \dots\dots\dots(13)$$

(ii) External Flow

If the jet is thin we can replace it by a distribution of vortices of strength γ per unit length placed along its centre line. The local strength, γ , can be obtained from equation (11).

The problem is to determine the flow around the circular cylinder with the trailing vortex sheet where the shape of the latter is only found at the end of the calculation. There are a number of suitable methods, mainly iterative, available for the solution of such a problem. These will not be enumerated here.



Before concluding this section it is worth noting that stagnation points will be present in the external flow at the junction of the vortex sheets and the cylinder. Thus the pressure across the jet will be constant, the radius of curvature will be infinite and the vortex strength will be zero. In addition a first approximation to the shape of the vortex sheet is given by the rear stagnation streamline for the flow without the vortex sheet but with a circulation about the cylinder adjusted so that the rear stagnation point on

the cylinder coincides with the position of the jet. In this approximation the strength of the vortex sheet can be found from equation (11) with the radius of curvature, R , and the velocity $q = q_1 + q_2/2$, found from their values on the rear stagnation streamline described above.

(b) With 'sink effect'

The strength of the sinks must be determined empirically from the existing experiments on turbulent jet mixing. The sinks can then be superposed on the line of vortices and their influence on the flow determined.

6. Conclusions

(1) A jet emerging from the rear of a cylinder provides a strong, natural form of boundary layer control for that body. Indeed the 'sink effect', due to the jet mixing with the external flow, provides suction effects over the complete length of the jet which are additional to the direct effect of jet blowing.

(2) With a jet issuing at an angle to the mainstream direction the flow around the cylinder is analogous to the flow obtained with a Thwaites flap, placed at the position of the jet at exit from the cylinder. Differences between the two flows arise due to the action of the 'sink effect' and the action of the jet in 'straightening out' the equivalent rear stagnation streamline.

7. Acknowledgement

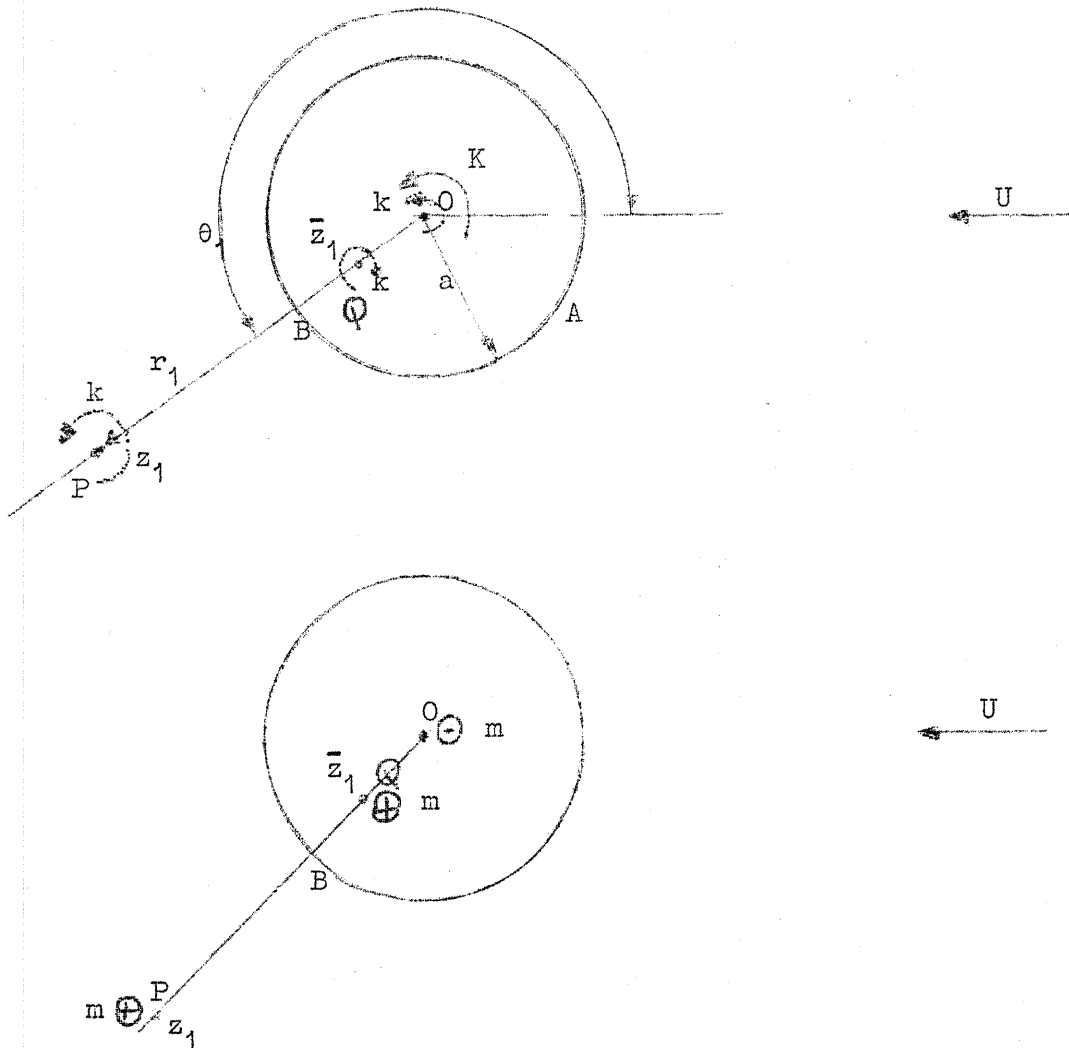
One of the authors (R.G. Anderson) was holder of the English Electric Company's Fellowship at the College of Aeronautics when this paper was written. The authors wish to thank Dr. Kitchemann for very helpful discussions.

8. References

<u>No.</u>	<u>Author</u>	<u>Title, Date, etc.</u>
1.	Pankhurst, R.C., and Thwaites, B.	Experiments on the flow past a porous circular cylinder fitted with a Thwaites Flap. A.R.C. R. and M. 2787 (1950).
2.	Lamb, H.	Hydrodynamics. C.U.P. (1932), p.93.

APPENDIX 1

The force on a lifting cylinder placed in a uniform stream due to the presence of an external source (sink) and vortex.



Consider a cylinder of radius, a , placed in a uniform stream of velocity $-U$ in the direction of the x -axis. A circulation, K , acts on the cylinder and is of strength such that with no external singularities stagnation points exist at $A(a, \theta_2)$ and $B(a, \theta_1)$, where $\theta_1 + \theta_2 = 3\pi$, i.e. A and B are symmetric points with respect to the y -axis.

An external vortex of strength, k , acts at $P(z_1)$ on OB together with a source of strength, m . Image sources of strength

$+m$, and $-m$ act at the inverse point, \bar{z}_1 with respect to z_1 , and the origin respectively. Similarly image vortices of strength, $-k$, and $+k$ act at the inverse point \bar{z}_1 and the origin respectively.

If the potential function $w = \phi + i\psi$, is defined so that $dw/dz = qe^{-i\theta}$ then w has the following components:-

- (i) $w_1 = -Uz$ uniform flow with velocity U parallel to the negative x direction,
- (ii) $w_2 = -\frac{Ua^2}{z}$ doublet of strength, $2\pi U$, at the origin,
- (iii) $w_3 = -\frac{iK}{2\pi} \log z$ vortex of strength, K , at the origin.
- (iv) $w_4 = -\frac{ik}{2\pi} \log(z-z_1)$ vortex of strength, k , at $P(z_1)$.
- (v) $w_5 = \frac{ik}{2\pi} \log(z-\bar{z}_1)$ ' ' ' $-k$, at $Q(\bar{z}_1)$.
- (vi) $w_6 = -\frac{ik}{2\pi} \log z$ ' ' ' $+k$, at the Origin.
- (vii) $w_7 = \frac{m}{2\pi} \log(z-z_1)$ source of strength, m , at $P(z_1)$
- (viii) $w_8 = \frac{m}{2\pi} \log(z-\bar{z}_1)$ ' ' ' m , at $Q(\bar{z}_1)$
- (ix) $w_9 = -\frac{m}{2\pi} \log z$ ' ' ' $-m$, at the origin.

The sum of these components can be written

$$w = -U \left(z + \frac{a^2}{z} \right) - \frac{(i(K+k)+m)}{2\pi} \log z + \frac{m-ik}{2\pi} \log(z-z_1) + \frac{(m+ik)}{2\pi} \log(z-\bar{z}_1) \dots (1)$$

$$= -U \left(z + \frac{a^2}{z} \right) + A \log z + B \log(z-z_1) + C \log(z-\bar{z}_1) \dots (2)$$

where $2\pi A = -(m+i(K+k))$; $2\pi B = (m-ik)$; $2\pi C = (m+ik)$ and hence

$$\frac{dw}{dz} = -U \left(1 - \frac{a^2}{z^2} \right) + \frac{A}{z} + \frac{B}{(z-z_1)} + \frac{C}{(z-\bar{z}_1)} \dots (3)$$

The components (X,Y) of the resultant force acting on the cylinder are given by

$$X - iY = \frac{1}{2}\rho i \int_C \left(\frac{dw}{dz} \right)^2 dz \dots\dots\dots(4)$$

where the curve C may be taken as the circle of radius a with its centre at the origin since no singularities exist on the surface of the circular cylinder. After substitution of equation (3) into (4) and integrating it can be shown that

$$\frac{X-iY}{\frac{1}{2}\rho} = + 4\pi UA + 4\pi UC + \frac{4\pi Ua^2 B}{z_1^2} + \frac{4\pi AB}{z_1} + \frac{4\pi BC}{(z_1 - \bar{z}_1)} \dots(5)$$

or alternatively after some rearrangement

$$\frac{X-iY}{\frac{1}{2}\rho} = 4\pi(A+C)U + 4\pi \left(\frac{Ua^2}{z_1^2} + \frac{A}{z_1} + \frac{C}{(z_1 - \bar{z}_1)} \right) B \dots\dots(6)$$

or,

$$X-iY = - i\rho UK + \frac{\rho(m-ik)}{2\pi} \left[\frac{2\pi Ua^2}{z_1^2} - \frac{(m+i(K+k))}{z_1} + \frac{(m+ik)}{z_1 - \bar{z}_1} \right] \dots\dots\dots(7)$$

Since $z_1 = r_1 e^{i\theta_1}$ and $\bar{z}_1 = \frac{a^2}{r_1} e^{-i\theta_1}$, we have on

separating out the real and imaginary parts from equation (7), the force components

$$X = \frac{\rho U a^2}{r_1^2} (m \cos 2\theta_1 - k \sin 2\theta_1) - \frac{\rho \cos \theta_1}{2\pi r_1}$$

$$\left(k(K+k) - \frac{k^2 + \frac{m^2 a^2}{r_1^2}}{1 - \frac{a^2}{r_1^2}} \right) - \frac{\rho \sin \theta_1}{2\pi r_1} m K \dots\dots(8)$$

and

$$Y = \rho UK + \frac{\rho U a^2}{r_1^2} (m \sin 2\theta_1 + k \cos 2\theta_1) - \frac{\rho \sin \theta_1}{2\pi r_1} \left(k(K+k) - \frac{k^2 + \frac{m^2 a^2}{r_1^2}}{1 - \frac{a^2}{r_1^2}} \right) + \frac{\rho \cos \theta_1}{2\pi r_1} m K \dots\dots\dots(9)$$

When the strengths of the external source and vortex are zero i.e. $m = k = 0$ we have

$$Y = \rho UK \quad \text{and} \quad X = 0 \dots\dots\dots(10)$$

In the general case of equation (3) the condition for a stagnation point at B (a, θ_1) is found to be

$$K = -4\pi U a \sin \theta_1 + \frac{2 a - k}{r_1 - a} \dots\dots\dots(11)$$

When $k = 0$, $m = 0$ the stagnation points will be in symmetrical positions about the vertical through O, but in general this will not be true and the position of the front stagnation point will be a function of the values of m and k .

Putting in the result of equation (11) in order to keep the rear stagnation point fixed, we obtain in slightly different notation,

$$X' = \frac{X}{\frac{\pi}{2} \rho U^2 a} = \bar{a}^2 (m' \cos 2\theta_1 - k' \sin 2\theta_1) + \frac{\bar{a}}{4} \cos \theta_1 \left\{ 8k' \sin \theta_1 - k'^2 \left(\frac{1+a^2}{1-a} \right) + \left(\frac{k'^2 + m'^2 \bar{a}^2}{1 - \bar{a}^2} \right) \right\} + \frac{\bar{a} \sin \theta_1}{4} m' \left(8 \sin \theta_1 - \frac{2\bar{a} k'}{1 - \bar{a}} \right) \dots\dots\dots(12)$$

$$Y' = \frac{Y}{\frac{\pi}{2} \rho U^2 a} = - \left(1 + \frac{\bar{a} m' \cos \theta_1}{4} \right) \left(8 \sin \theta_1 - \frac{2\bar{a} k'}{1 - \bar{a}} \right) + \bar{a}^2 (m' \sin 2\theta_1 + k' \cos 2\theta_1) + \frac{\bar{a} \sin \theta_1}{4} \left(8k' \sin \theta_1 - k'^2 \frac{(1+\bar{a})}{(1-\bar{a})} + \frac{k'^2 + m'^2 \bar{a}^2}{1 - \bar{a}^2} \right)$$

where $m' = \frac{2m}{\pi Ua}$; $k' = \frac{2k}{\pi Ua}$; and $\bar{a} = \frac{a}{r_1}$.

If the terms in m' and k' are collected together equations (12) and (13) become

$$X' = m' \bar{a}^{-2} \left(1 + 2 \sin^2 \beta \left(\frac{1-\bar{a}}{\bar{a}} \right) - \frac{m' \bar{a} \cos \beta}{4(1-\bar{a}^2)} \right) + k' \bar{a} \left((1-\bar{a}) \sin 2\beta + \frac{k'(2+\bar{a})}{(1-\bar{a}^2)} \frac{\bar{a} \cos \beta}{4} \right) + \frac{\bar{a}^2 k' \sin \beta m'}{2(1-\bar{a})} \dots (14)$$

$$Y' = + 8 \sin \beta$$

$$- m' \bar{a}^{-2} \left(\sin 2\beta \left(\frac{1-\bar{a}}{\bar{a}} \right) + \frac{m' \bar{a} \sin \beta}{4(1-\bar{a}^2)} \right) + k' \bar{a} \left(+ \frac{(2-\bar{a})(1+\bar{a})}{(1-\bar{a})} + (1-\bar{a}) 2 \sin^2 \beta + \frac{k'(2+\bar{a})}{1-\bar{a}^2} \frac{\bar{a} \sin \beta}{4} \right) - \frac{m' \bar{a}^{-2} k' \cos \beta}{2(1-\bar{a})} \dots (15)$$

where $\beta = \theta_1 - \pi$, with $\beta > 0$, and $\sin \beta$, $\cos \beta$ and $\sin 2\beta$ are all positive.

Examples

Case (a) $k' = 0$; $m' < 0$.

From equations (14) and (15)

$$X' = m' \bar{a}^{-2} \left(1 + 2 \sin^2 \beta \left(\frac{1-\bar{a}}{\bar{a}} \right) - \frac{\bar{a} m' \cos \beta}{4(1-\bar{a}^2)} \right) \dots (16)$$

and

$$\Delta Y' = Y' - 8 \sin \beta = - m' \bar{a}^{-2} \sin \beta \left(2 \left(\frac{1-\bar{a}}{\bar{a}} \right) \cos \beta + \frac{\bar{a} m'}{4(1-\bar{a}^2)} \right) (17)$$

Equation (16) shows that when $m' < 0$ (sink) a positive drag ($X' < 0$) acts on the cylinder. Similarly, from equation (17), the increment $\Delta Y'$ above the 'Thwaites' flap lift is positive for a sink when

$$2 \left(\frac{1-\bar{a}}{\bar{a}} \right) \cos \beta > \frac{\bar{a} |m'|}{4(1-\bar{a}^2)} .$$

However when \bar{a} is nearly unity this condition will not be satisfied and negative lift will be obtained. It seems almost certain that the sink effect in practice will decrease lift since the main effect is near the nozzle (i.e. $\bar{a} \approx 1$).

Case (b) $m' = 0$; $k' > 0$.

From equations (14) and (15)

$$X' = k' \bar{a} \left((1-\bar{a}) \sin 2\beta + \frac{k'(2+\bar{a})}{1-\bar{a}^2} \frac{\bar{a} \cos \beta}{4} \right) \dots\dots\dots(18)$$

and

$$\Delta Y' = + k' \bar{a} \left[\frac{(2-\bar{a})(1+\bar{a})}{(1-\bar{a})} + (1-\bar{a}) 2 \sin^2 \beta + \frac{k'(2+\bar{a})}{1-\bar{a}^2} \frac{\bar{a} \sin \beta}{4} \right] \dots\dots\dots(19)$$

Equation (18) shows that the effect of the vortex is to produce a thrust when $0 < \beta < \pi/2$, whilst the lift increment, $\Delta Y'$ from equation (19) is always positive when $0 < \beta < \pi$.

A further problem, of interest in the interpretation of the smoke-tunnel photographs, is the motion of the forward stagnation point which is situated at (a, θ_2) in the 'Thwaites' flap type flow. This motion will be determined by finding the velocity at $A(a, \theta_2)$ in the general flow field. The relative position of the stagnation point may then be inferred.

The velocity at $A(a, \theta_2)$ is given by substituting $z = ae^{-i\beta}$ into equation (3), where $\theta_2 = -\beta$, then

$$\begin{aligned} \left(\frac{dw}{dz} \right)_A &= -U(1 - e^{2i\beta}) + \frac{e^{i\beta}}{2\pi a} [-m - i(K+k)] \\ &+ \frac{m - ik}{2\pi(ae^{-i\beta} + r_1 e^{i\beta})} + \frac{m + ik}{2\pi \left(ae^{-i\beta} + \frac{a^2}{r_1} e^{i\beta} \right)} \dots\dots\dots(20) \end{aligned}$$

Substituting $K+k = + 4\pi U a \sin \beta + \frac{k(r_1 + a)}{r_1 - a}$ in (20),

$$\left(\frac{dw}{dz}\right)_A = + \frac{m e^{i\beta}}{2\pi a} \left[-1 + \frac{1}{1 + \frac{r_1}{a} e^{2i\beta}} + \frac{1}{1 + \frac{a}{r_1} e^{2i\beta}} \right] - \frac{i k e^{i\beta}}{2\pi a} \left[\frac{r_1 + a}{r_1 - a} + \frac{1}{1 + \frac{r_1}{a} e^{2i\beta}} - \frac{1}{1 + \frac{a}{r_1} e^{2i\beta}} \right], \dots\dots\dots(21)$$

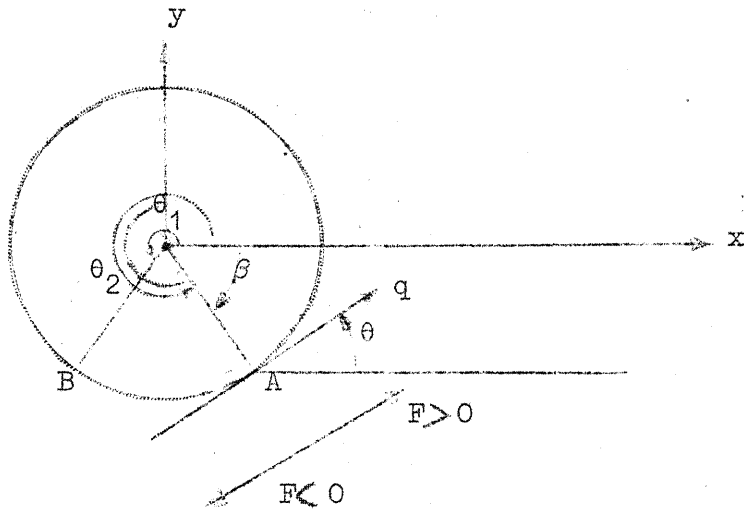
and thus

$$\left(\frac{dw}{dz}\right)_A = - \frac{i 2\pi U a \cos \beta e^{i\beta} F}{\pi(a^2 + r_1^2 + 2ar_1 \cos 2\beta)} = q e^{-i\theta} \dots\dots(22)$$

where θ now denotes the direction of the velocity at A,

and $F = m \sin \beta + k \cos \beta \frac{(r_1 + a)}{(r_1 - a)}$ and hence when $F > 0$ the

stagnation point is to the left of A but when $F < 0$ the stagnation point is to the right of A. It will be noticed that the value of m must be negative for A to move to the right.



The term in k in equation (22) contains the factor $(r_1 - a)$ in the denominator and hence this term will be large at

points near the cylinder. It will also be recalled that the sink effect (i.e. $m < 0$) should be large for points near the cylinder. It is not, therefore, surprising that little movement of the stagnation point to the left of A can be found for high jet momentum coefficients, since the two terms in F may well be of the same order numerically but of opposite sign.

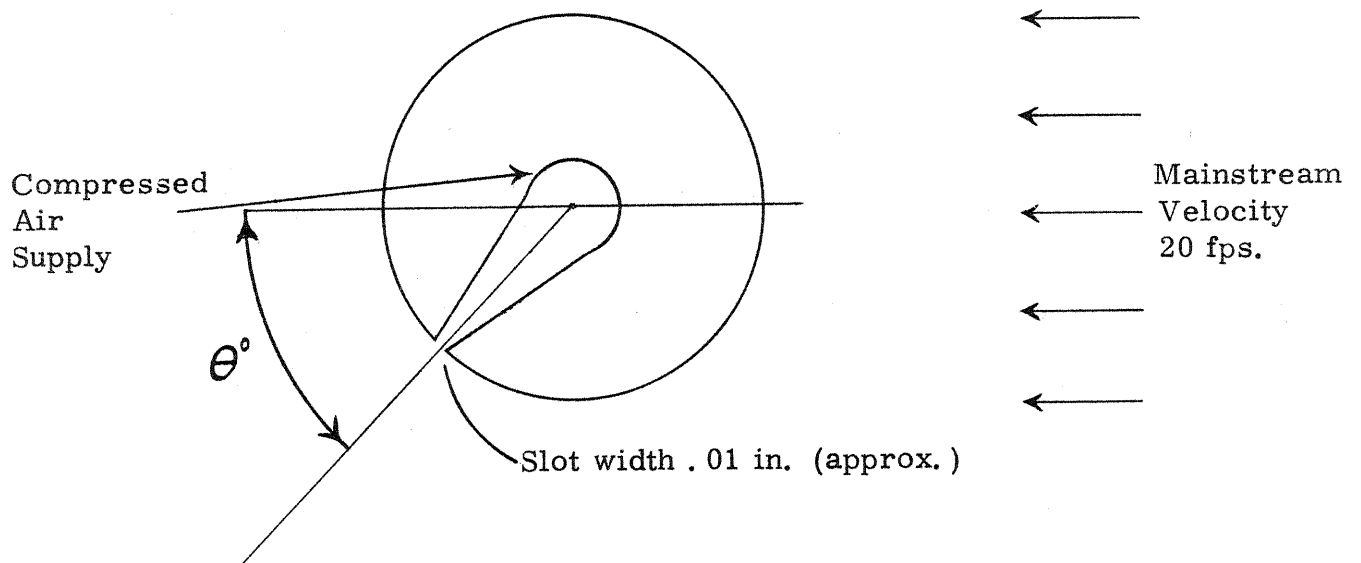


FIGURE 1. DIAGRAM OF THE CYLINDER 3" IN DIAMETER.

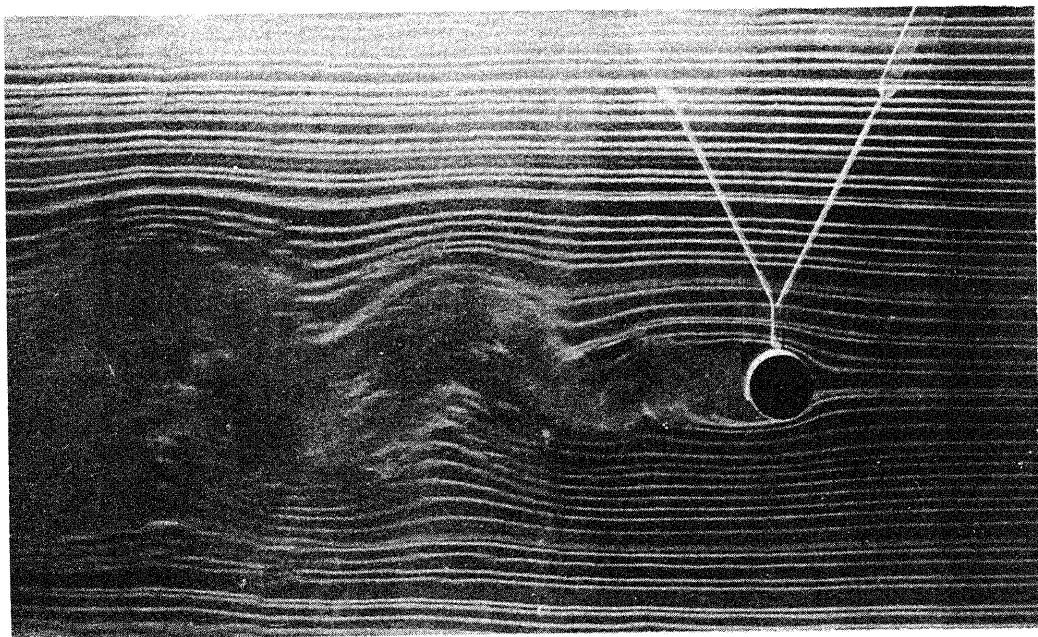


FIGURE 2. FLOW PAST A CYLINDER WITH A JET
($\theta = 22\frac{1}{2}^\circ$ $C_p = 0$)

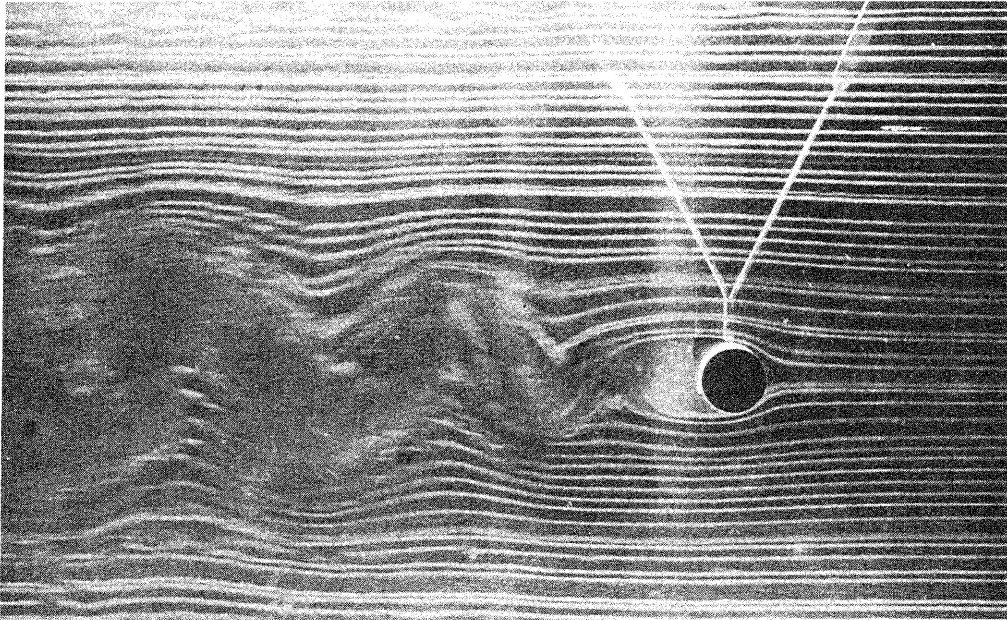


FIGURE 3. FLOW PAST A CYLINDER WITH A JET
 ($\theta = 22\frac{1}{2}^\circ$ $C_p \approx .05$)

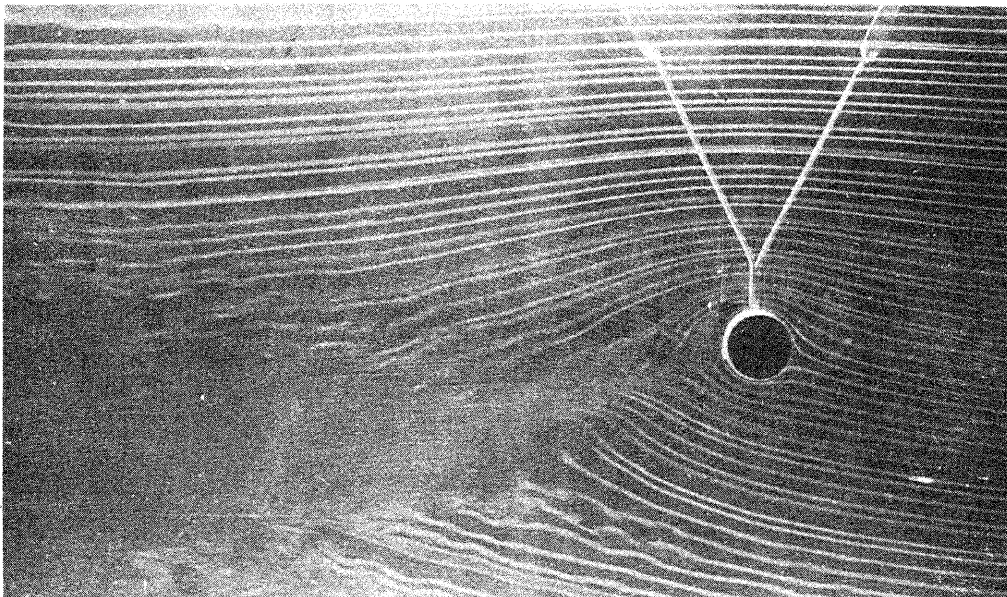


FIGURE 4. FLOW PAST A CYLINDER WITH A JET
 ($\theta = 22\frac{1}{2}^\circ$ $C_p \approx .6$)

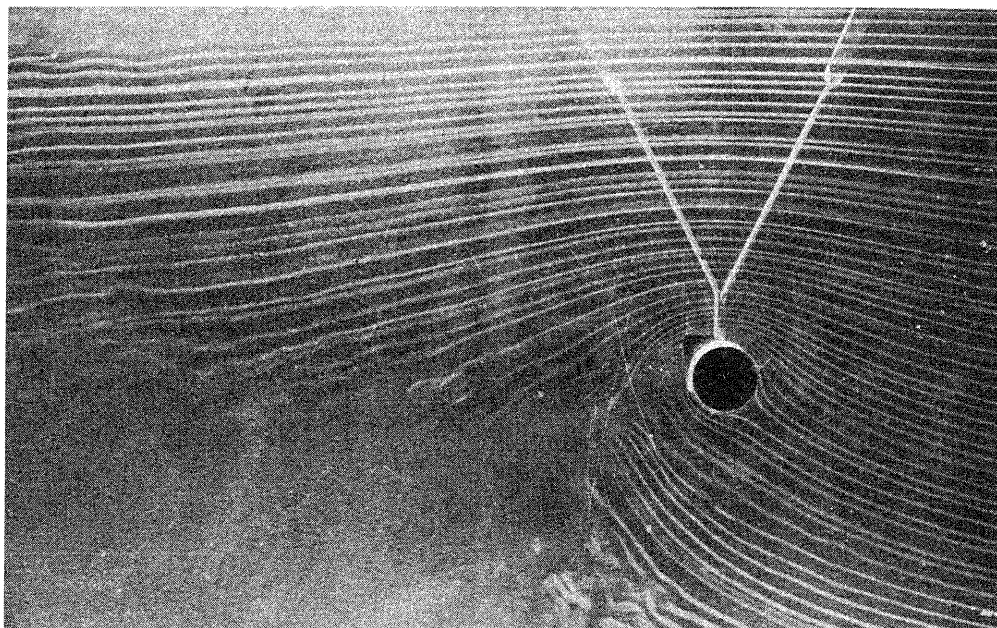


FIGURE 5. FLOW PAST A CYLINDER WITH A JET
 $(\theta = 22\frac{1}{2}^\circ \quad C_{\mu} \approx 1.0)$

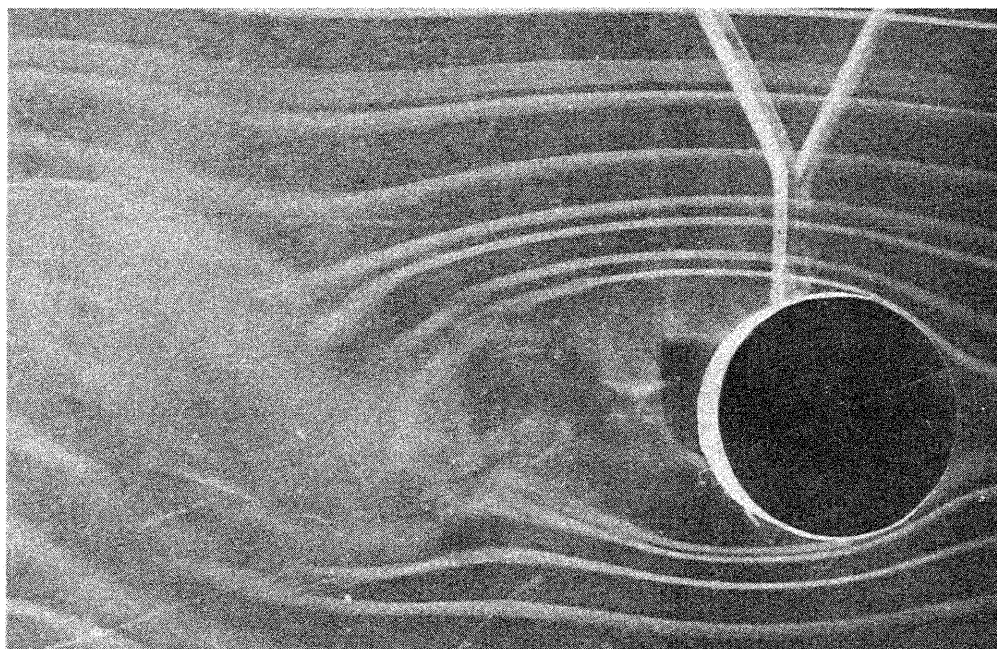


FIGURE 6. FLOW IN THE NEIGHBOURHOOD OF A CIRCULAR
CYLINDER WITH A JET $(\theta = 22\frac{1}{2}^\circ \quad C_{\mu} = 0)$

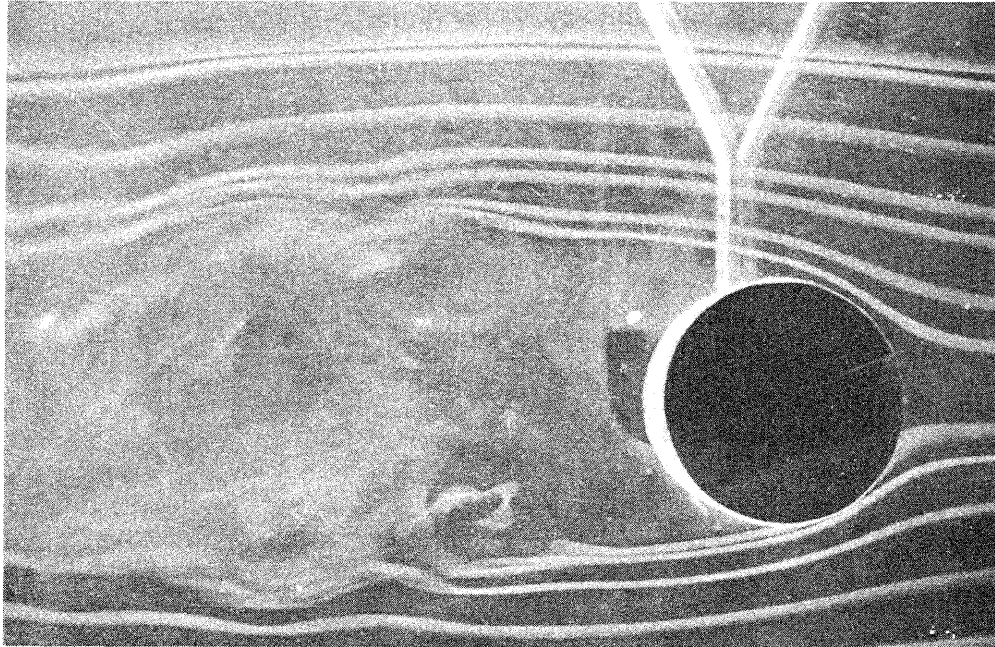


FIGURE 7. FLOW IN THE NEIGHBOURHOOD OF A CIRCULAR
CYLINDER WITH A JET ($\theta = 22\frac{1}{2}^\circ$ $C_p \approx .07$)

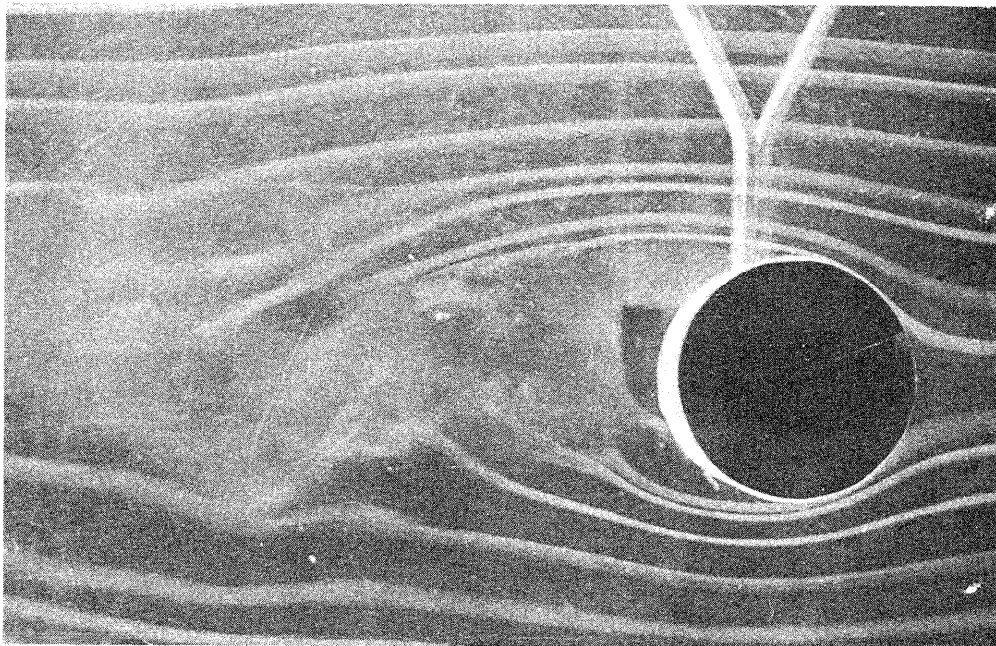


FIGURE 8. FLOW IN THE NEIGHBOURHOOD OF A CIRCULAR
CYLINDER WITH A JET ($\theta = 22\frac{1}{2}^\circ$ $C_p = .1$)

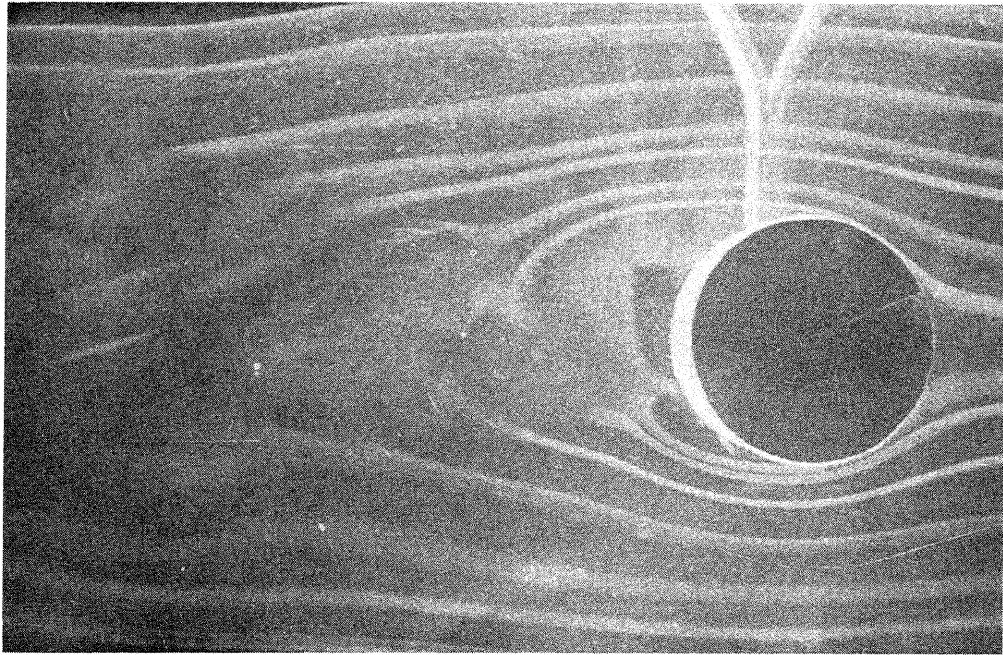


FIGURE 9. FLOW IN THE NEIGHBOURHOOD OF A CIRCULAR
CYLINDER WITH A JET ($\theta = 22\frac{1}{2}^\circ$ $C_p = .3$)

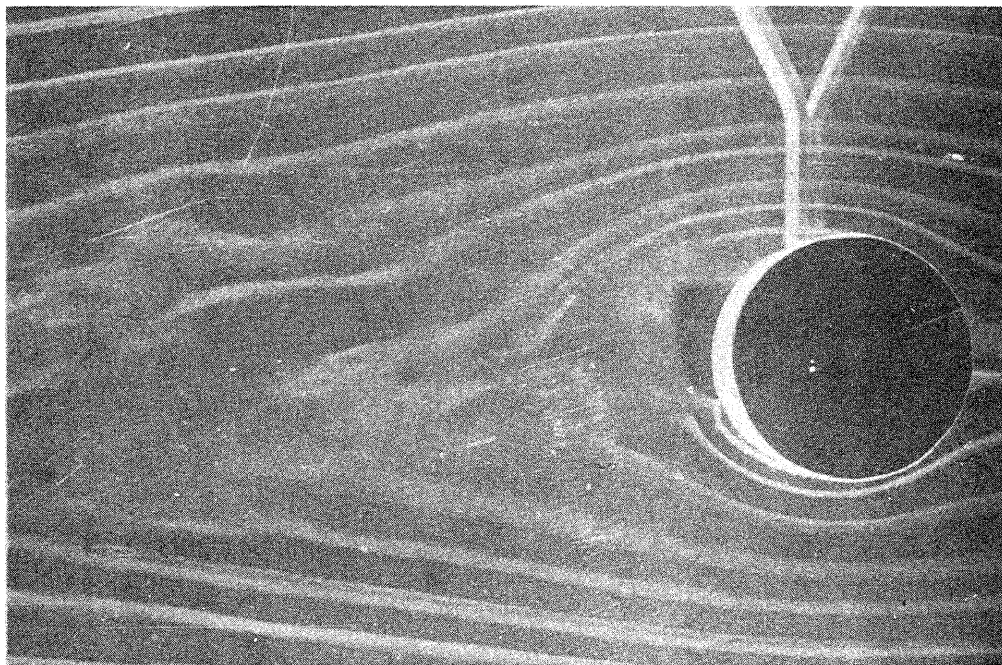


FIGURE 10. THE FLOW IN THE NEIGHBOURHOOD OF A
CIRCULAR CYLINDER WITH A JET.
 ($\theta = 22\frac{1}{2}^\circ$ $C_p \approx .5$)

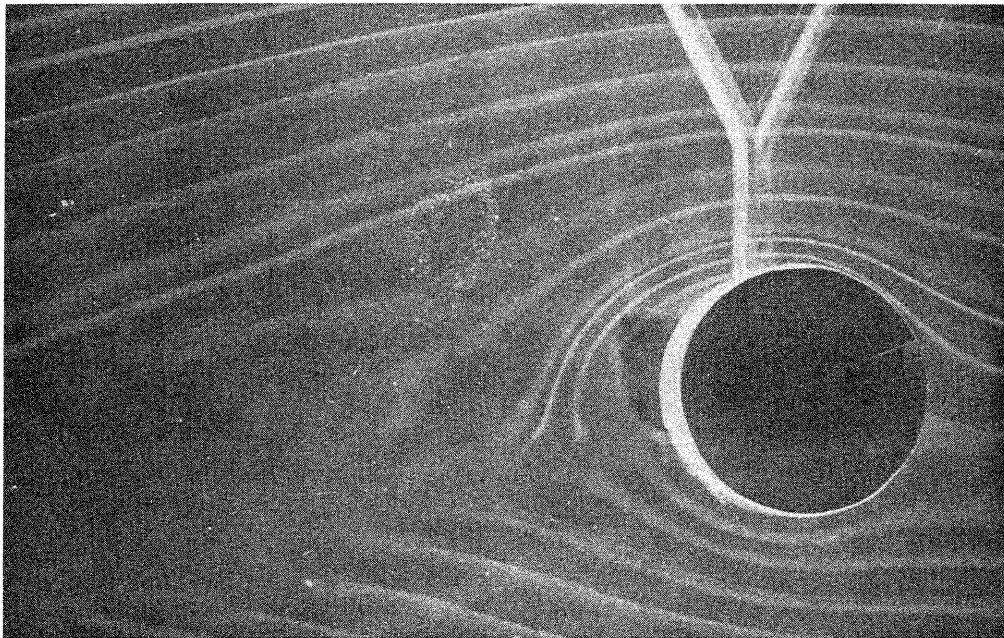


FIGURE 11. THE FLOW IN THE NEIGHBOURHOOD OF A
CIRCULAR CYLINDER WITH A JET.
 ($\theta = 22\frac{1}{2}^\circ$ $C_p \approx .7$)

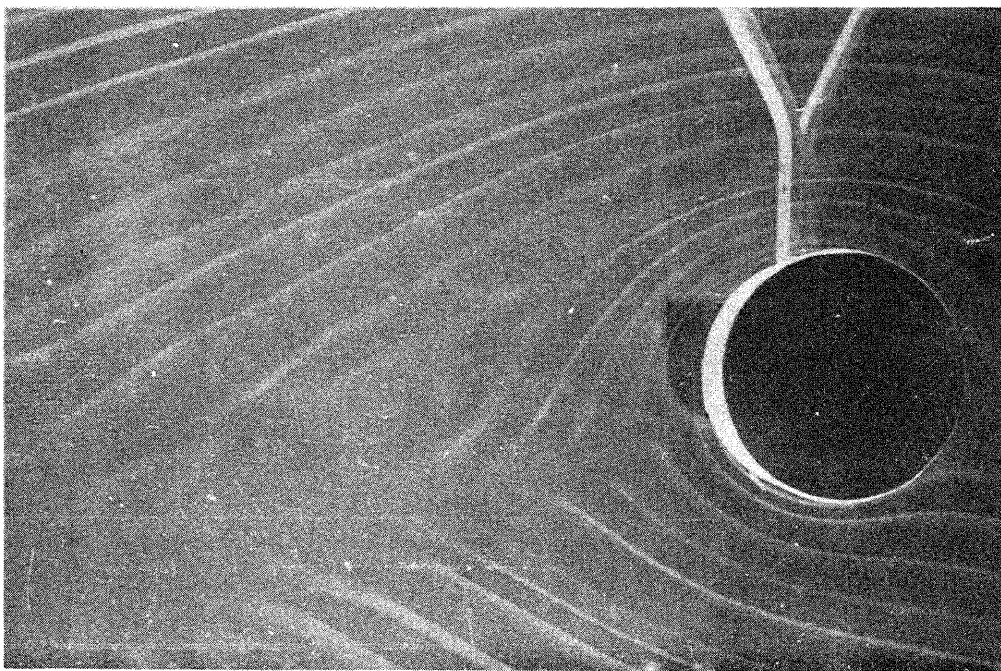


FIGURE 12. THE FLOW IN THE NEIGHBOURHOOD OF A
CIRCULAR CYLINDER WITH A JET
 ($\theta = 22\frac{1}{2}^\circ$ $C_p \approx .9$)

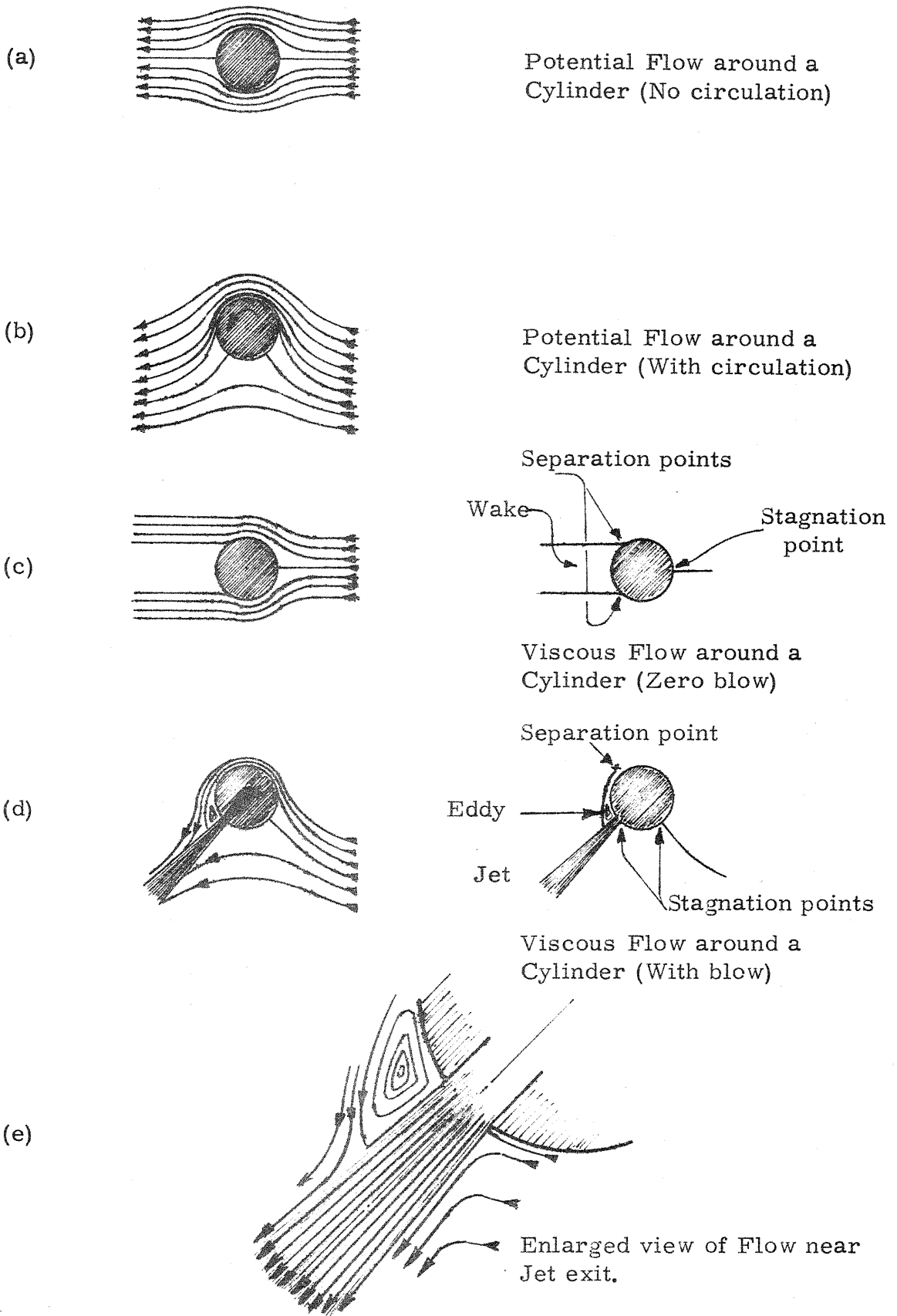


FIGURE 13. VARIOUS STATES OF FLOW AROUND A CIRCULAR CYLINDER.