Cranfield

College of Aeronautics Report No. 8905
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Second Quarterly Report on the Application of Modified Stepwise Regression for the Estimation of Aircraft Stability and Control Parameters.

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1.0 INTRODUCTION.

This report is intended to discuss the progress made during the past quarter, January - April, 1989 , in accordance with the terms of MOD Agreement No.2082/192, (REF.1). The research is concerned with the use of a Modified Stepwise Regression Method for estimating the stability and control derivatives of a B.Ae Hawk aircraft from data obtained by the use of a scaled model on a dynamic wind tunnel test rig.

At the last progress review meeting on the 26th January, 1989 a number of objectives were established for the present quarter. These being:

1. To produce a small perturbation simulation model of the basic aircraft using the Advanced Continuous Simulation Language (ACSL).

2. To set up a data acquisition system to recover and store data from the wind tunnel model in a usable form.

3. To carry out further development work on the Hawk model to install a sensor system for vertical height sensing and to calibrate various attitude and control surface angle sensors.

4. To extend the equations of motion of the aircraft to include kinematics appropriate to the experimental setup.

5. To further order the equations of motion into a format compatible with the regression method requirements.

Each of these objectives is discussed in the following sections.

2.0 ACSL SIMULATION.

The main purpose of the ACSL simulation is to model the basic aircraft equations of motion so that any response to inputs on the real aircraft can be reproduced by the simulation model. Thus any aircraft may be used initially to test the program. To simulate a particular aircraft it is simply a case of re-setting the numerical values of the stability and control derivatives in the program and defining the appropriate flight conditions. It was therefore decided to use Phantom (F4) data to test the simulation program since a design package on the BBC microcomputers had previously been used to produce graphs of the step and impulse responses of the F4. The response graphs from this work could be used to compare directly with the responses obtained using the ACSL simulation.

It was not thought necessary to consider the coupled responses of aircraft motion at this stage and so the simulation was split into two distinct programs which model the longitudinal and lateral equations of motion for small perturbations separately. When the dynamic model on the rig is considered some of the equations of motion will become redundant, for example there is no longitudinal translation of the model in the wind tunnel. However, it was considered appropriate to start with a full aircraft simulation, the required changes being made later.

2.1 LONGITUDINAL EQUATIONS.

The longitudinal equations of motion were established in the state variable form, $\dot{x} = Ax + Bu$,as shown below (REF 4):

$$\begin{pmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \\ \dot{h} \end{pmatrix} = \begin{pmatrix} x_u & x_w & (x_q - W_e) & -g \\ z_u & z_w & (U_e + z_q) & 0 \\ m_u & m_w & m_q & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & U_e \end{pmatrix} \begin{pmatrix} u \\ w \\ q \\ \theta \\ h \end{pmatrix} + \begin{pmatrix} x_\eta \\ z_\eta \\ m_\eta \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} \eta \end{pmatrix} \qquad (1)$$

where matrices A and B have already been pre-multiplied by the inverse mass matrix (M^{-1}) of the system and $x_u = (M^{-1})(\mathring{X}_u/m)$,etc.

For the F4 the numerical values for the A and B matrices were obtained from Reference 3 with the following flight conditions:

mass	m =	17671	kg
inertias	I_x =	34447	kgm^2
	I_y =	168351	kgm^2
	I_z =	192534	kgm^2
	I_{xz} =	3000	kgm^2
eqm.ht.	H =	0	m
Mach no.	M =	1.10	
eqm.speed	U =	377	ms^{-1}

$$A = \begin{pmatrix} -0.068 & 0.011 & 0.0 & -9.8 \\ 0.023 & -2.1 & 375 & 0.0 \\ 0.011 & -0.16 & -2.2 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & -1.0 & 0.0 & 377 \end{pmatrix} \qquad B = \begin{pmatrix} -0.41 \\ -77.0 \\ -61.0 \\ 0.0 \\ 0.0 \end{pmatrix}$$

The values of the derivatives are defined with respect to radians.

Taking the first line of equations (1), i.e.

$$\dot{u} = x_u.u + x_w.w + (x_q - W_e).q - g.\theta + x_\eta.\eta$$

this appears in the simulation program as :

UDOT = XU*U + XW*W + (XQ-WE)*Q - G*THETA + XETA*ETA

where the derivatives XU, XW, XQ are defined as constants at the beginning of the program and where θ and η are in radians.

A similar notation is adopted for the other lines of Eqns.(1) with $\dot{\theta}$ being called THETAD in the program due to an ACSL constraint which only allows six characters to specify a variable name.

To obtain a step input for the program ETA is set to 0.1745^c, ie.(1^o), at the beginning of the program for an arbitrary time (TIMEON) chosen to be 100s. This is far longer than the time needed record the short period step response of the aircraft. It is also greater than the time for which the program will actually be run. A comparison of the step responses for θ and q using the BBC package and the ACSL program are given in Figures A-1 and A-2 of Appendix A. On comparison of these figures, a very good correspondence between the graphs from the two different sources may be seen.

For the impulse input η is set to 1.0 at the start of the program and then reset to zero after a fraction of a second, typically about 0.01s. The shortest time that η can be set to 1.0 is with TIMEON equal to 10^{-10}s. This figure corresponds to the smallest integration time step possible in ACSL. However, in practice a variation in TIMEON between 10^{-10}s and 0.01s was found to make no difference to the quality of the results.

Figures A-3 and A-4 show the short period impulse responses obtained from the BBC package and ACSL. There is not such a good agreement in the responses using the ACSL program this time. This is thought to be due to the sudden change in η from 1.0 to 0.0 in one time step of the integration procedure. It is realised that it would be much better to model the impulse on η by changing some initial conditions and keeping ETA = 0 throughout the simulation. This is something which will be looked into during the next quarter to improve the simulation program. A brief description of the ACSL program as it stands at present is given in Appendix B.

2.2 LATERAL EQUATIONS.

The lateral equations of motion used in the simulation are of the form, $\dot{\underset{\sim}{x}} = A\underset{\sim}{x} + B\underset{\sim}{u}$,as shown below:

$$\begin{pmatrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} y_v & y_p & (y_r - U_e) & g \\ l_v & l_p & l_r & 0 \\ n_v & n_p & n_r & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} v \\ p \\ r \\ \phi \\ \psi \end{pmatrix} + \begin{pmatrix} y_\xi & y_\zeta \\ l_\xi & l_\zeta \\ n_\xi & n_\zeta \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \xi \\ \zeta \end{pmatrix} \qquad \text{Eqns.(2)}$$

where again the A and B matrices have been pre-multiplied by the inverse mass matrix (M^{-1}) of the system and $y_v = (M^{-1})(\mathring{Y}_v/m)$,etc.

For the F4 the numerical values for the A and B matrices for the flight conditions given above are as follows, (REF.3) :

$$A = \begin{pmatrix} -0.49 & 0.0 & -377 & -9.8 \\ -0.13 & -3.1 & 0.80 & 0.0 \\ 0.10 & 0.018 & -1.2 & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \end{pmatrix} \qquad B = \begin{pmatrix} 3.9 & 11.6 \\ -15.0 & 9.3 \\ -2.5 & -8.8 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \end{pmatrix}$$

It was decided to produce two lateral simulation programs, LATROL.CSL and LATYAW.CSL to model the roll response and yaw responses separately. This is not necessary as with a single ACSL program it is possible at run time to use two logical flags called ROLRES and YAWRES to specify whether an input to aileron or rudder is required. However, at present it is convenient to produce two lateral programs. These programs are identical in every respect apart from one having constants called XI and TIMEXI for aileron inputs and the other constants called ZETA and TIMEZT for rudder inputs. The lateral equations of motion are specified in a similar way to the longitudinal equations of motion.

For example, taking the first line of equations (2), i.e.

$$\dot{v} = y_v.v + y_p.p + (y_r - U_e).r + g.\phi + y_\xi.\xi + y_\zeta.\zeta$$

this appears in both the lateral simulation program as :

VDOT = YV*V + YP*P + (YR-UE)*R - G*PHI + YXI*XI + YZETA*ZETA

where the derivatives YV, YP, YR are defined at the beginning of the program and all angles are in radians.

A comparison of the step responses for ϕ and p using the BBC package and the LATROL.CSL program are given in Appendix A in Figures A-5 and A-6. A very good correspondence in the step responses from the two sources is obtained.

For the impulse response ξ is set to 1^0 at the start of the program and then reset to zero after 0.01s. There is quite good agreement between the impulse graphs shown in Figures A-7 and A-8 this time.

A comparison of the step responses for ψ and r using the BBC package and the LATROL.YAW program are given in Figures A-9 and A-10. Again good correspondence between the step responses using the two sources is obtained.

Finally, Figures A-11 and A-12 show the impulse responses for r and ψ respectively. The responses obtained from the BBC and ACSL program are also in very close agreement.

3.0 DATA ACQUISITION SYSTEM.

The electronic control unit for the dynamic wind tunnel test rig has a control panel which was originally designed with various accessible bus bar networks so that it is possible to output any combination of control surface inputs and attitude angles or rates, (REF.4). To obtain an output the appropriate parameter is linked by a 'free' wire link to one of eight ports. These ports have recently been permanently connected to an eight channel data cable during the course of a current MSc. project aimed at developing the test rig.

The data cable carries the outputs in the form of analogue voltages and these voltages need to be converted to digital form in order to be stored in files on a computer disk. To perform the ADC conversion it is proposed to record data from the model control unit using a Cambridge Electronics Design (CED) 1401 machine. The 1401 is operated by commands sent to it from the host IBM PC. A TURBO PASCAL program is used to send the instructions to record the output from the eight channel data cable and store the data in a buffer in the 1401's internal memory. Under software control the 1401 is told to transfer the data it holds to files opened on the IBM. At the end of the data acquisition procedure these files are closed and stored, they may then be read and analysed in as required. A diagram of the data acquisition system is shown in Figure 1.

To learn more about the 1401 and how to program it a one day course was attended in London in March. Further help has been obtained from a fellow PhD research student in the CoA. In particular he has written a PASCAL program which records data simultaneously from eight separate channels at a frequency which may be specified in the program to suit the users requirements. This program has been very useful and has only had to be modified slightly to make the 1401 record data from a single data cable through channels 8-15. Further, files opened to record information have been given suffixes to suggest what is being recorded through a defined channel. These suffixes are shown over leaf.

For example, a test run nominated TEST1 will record data from channel 8 in a file called TEST1.ELE, data from channel 9 in a file called TEST1.AIL, etc.

CHANNEL	PARAMETER	IBM FILE SUFFIX	
8	η	.ELE	(elevator)
9	ξ	.AIL	(aileron)
10	ζ	.RUD	(rudder)
11	θ or q	.PIT	(pitch angle/rate)
12	ϕ or p	.ROL	(roll angle/rate)
13	ψ or r	.YAW	(yaw angle/rate)
14		.SPA	(spare channel)
15		.INP	(model inputs)

Several sets of data files have been recorded to test out the program called REC8.PAS, with a signal generator being used to provide the necessary inputs. The data files created were then examined using a software package supplied with the CED 1401 machine called WATERFALL. This enabled a check to be made on what had been recorded and confirm that the program was working as expected. In future work it will be possible to analyse the spectral content of recorded data by using a fast fourier transform option within WATERFALL. It is also hoped that a smoothing program of some sort will help to improve the quality of the data recorded.

At present, the main difficulty in the data acquisition system lies with the fact that the data produced by the 1401 are recorded in a hexadecimal format on the IBM. Thus the next task will be to change this data into a form which can be read easily by the main Modified Stepwise Regression program which will be written in FORTRAN. A program to test whether or not FORTRAN can read the data as it stands through a free format READ statement will be tried. Alternatively the REC8.PAS program may be changed to output data as a 'FORTRAN readable' text file rather than in the present hexadecimal form.

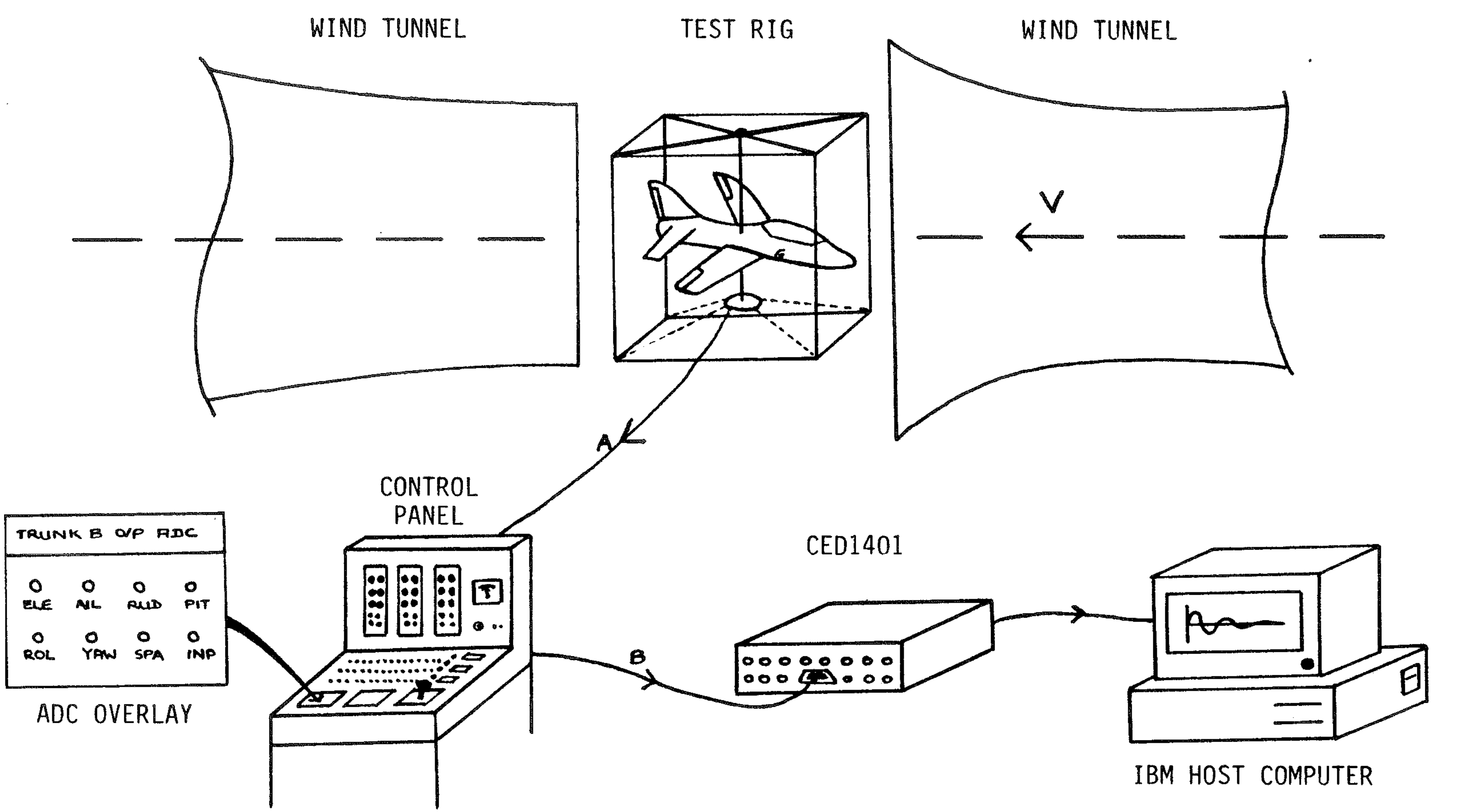


FIGURE 1: DATA ACQUISITION SYSTEM

4.0 MODEL STATUS.

The Hawk model is suspended in a large Dexion framework by means of a vertical rod through the model fuselage, as shown in Figure 2. The top of the rod has bearing mounting plates which are held in place by rigid wire bracing. An umbilical from the model connects its various actuators and servo mechanisms to the control panel. During the last quarter this Dexion assembly has been placed in the Weybridge wind tunnel at Cranfield and the Hawk model has been successfully flown on a number of occasions.

4.1 MODEL CALIBRATIONS.

Calibrations of the control surface angles have been carried out on the Hawk model. Figure 3 shows graphs of elevator, aileron and rudder angles versus input voltage. These graphs were produced by the MSc. project referred to above and it is encouraging to see that linear relationships are attained over almost all of the input voltage range.

Other work which still needs to be carried out in this area includes calibration of the attitude angles. The mass and inertias of the model also need to be measured, although this is best left until no further internal work is necessary on the model.

4.2 VERTICAL HEIGHT SYSTEM.

Work has been progressing well in the development of the vertical position sensor. Two strain gauge sensors have been tried out with the Hawk as it was found that the first gauge which was chosen was not robust enough and a stronger gauge was required. External to the model a servo controlled pulley system situated at the top of the Dexion framework has been installed. Additional circuitry for the control panel has also been designed and installed. Further work in this area will concentrate on checking the performance and accuracy of the various components in the system.

4.3 WIND TUNNEL AND MODEL REFURBISHMENT.

The Weybridge wind tunnel underwent a complete refurbishment earlier this year. Unfortunately, since this work was carried out there are several wind speeds which cause a large resonance in the wind tunnel. At a high wind speed these problems decrease. However it is then found that when the model is actually flying there are significant tunnel blockage effects which come into play. There is a noticeable decrease in wind speed when the model is taken off the bottom end stop to its 'flying position' which is higher up on the vertical rod.

Efforts are being made to minimise the resonance problems and the development of careful operating procedures has been shown to render the blockage effects insignificant providing that small perturbations about trim are adhered to.

FIGURE 2: MODEL SUSPENSION AND FRAMEWORK.

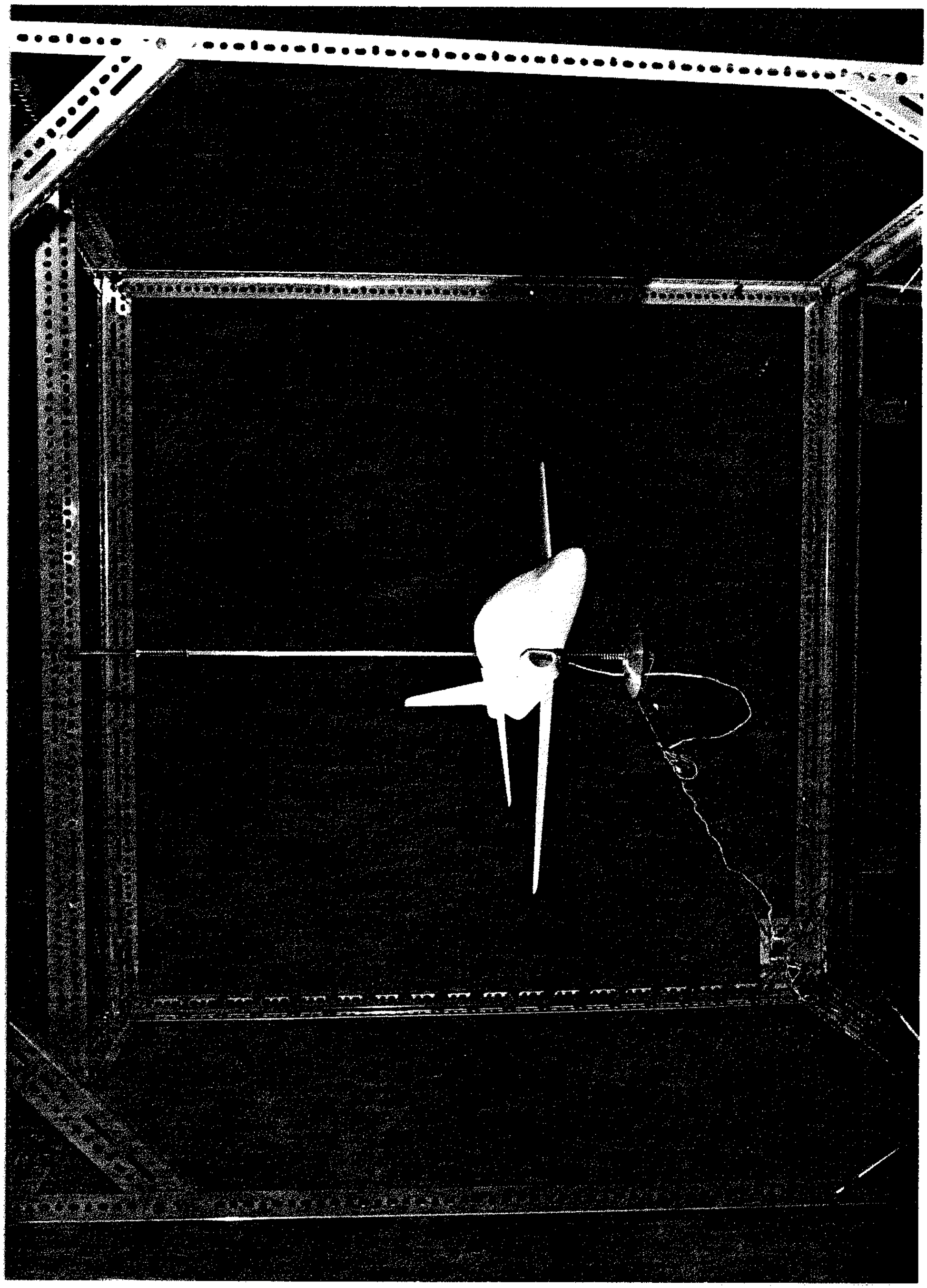
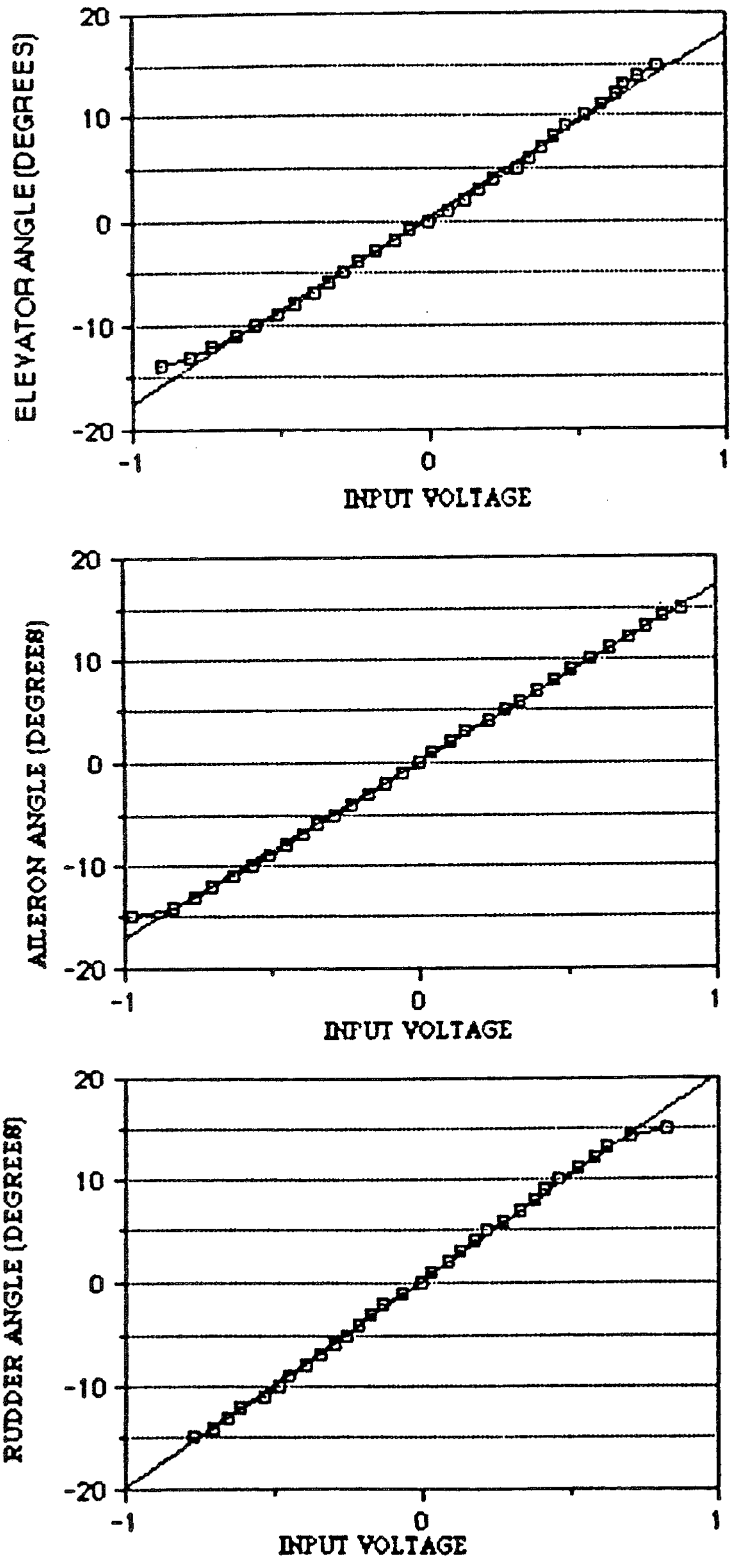


FIGURE 3: CALIBRATION OF THE MODEL CONTROL SURFACES.



5.0 KINEMATICS.

5.1 AXES SYSTEMS AND TRANSFORMATIONS.

It is convenient to define a set of axes $(Oxyz)_{wind}$ fixed in the aircraft such that the Ox axis is coincident with the resultant total velocity vector V in the plane of symmetry of the aircraft. This axis system is referred to as **wind** or **stability** axes and is equivalent to body axes rotated through the body incidence angle (α_e) about the Oy axis. Figure 4 shows the relationship between body and wind axes.

In a disturbance the attitude of the aircraft is defined by the orientation of the disturbed body axes (Oxyz) with respect to the steady state datum body axes ($OX_0Y_0Z_0$). The angular attitude of the aircraft may be established by considering the rotation, about each axis in turn, which is necessary to bring ($OX_0Y_0Z_0$) into coincidence with (Oxyz). Referring to Figure 5, the angles ψ, θ and ϕ define the aircraft attitude with respect to the datum and are called the **Euler Angles**.

In order to transform the linear quantities of displacement, velocity and acceleration or force it is usual to consider vector quantities $X_0Y_0Z_0$ in the first axes set ($OX_0Y_0Z_0$) and use Figure 5 to define their angular relationship with the transformed vector quantities xyz in the second axes set (Oxyz). Transforming $X_0Y_0Z_0$ by rotations through the yaw angle ψ, the pitch angle θ and then the roll angle ϕ leads to the following transformation relationship, (REF 7) :

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \underset{\sim}{A} \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix} \qquad \text{Eqn.(3)}$$

where

$$\underset{\sim}{A} = \begin{pmatrix} \text{Cos}\psi\text{Cos}\theta & \text{Sin}\psi\text{Cos}\theta & -\text{Sin}\theta \\ \text{Cos}\psi\text{Sin}\theta\text{Sin}\phi - \text{Sin}\psi\text{Cos}\phi & \text{Sin}\psi\text{Sin}\theta\text{Sin}\phi + \text{Cos}\psi\text{Cos}\phi & \text{Cos}\theta\text{Sin}\phi \\ \text{Cos}\psi\text{Sin}\theta\text{Cos}\phi + \text{Sin}\psi\text{Sin}\phi & \text{Sin}\psi\text{Sin}\theta\text{Cos}\phi - \text{Cos}\psi\text{Sin}\phi & \text{Cos}\theta\text{Cos}\phi \end{pmatrix}$$

The transformation matrix for angular perturbation quantities is that which relates attitude rates to body rates. If the angular velocities with respect to earth axes ($OX_0Y_0Z_0$) are $\dot{\phi}$, $\dot{\theta}$ and $\dot{\psi}$ and the angular velocities of the disturbed body fixed axes (Oxyz) are p, q and r , then the following linear relationships between the angular velocities in the two axes systems may be deduced from Figure 5 :

roll rate	$p = \dot{\phi} - \dot{\psi}\mathrm{Sin}\theta$
pitch rate	$q = \dot{\theta}\mathrm{Cos}\phi + \dot{\psi}\mathrm{Sin}\phi\mathrm{Cos}\theta$
yaw rate	$r = -\dot{\theta}\mathrm{Sin}\phi + \dot{\psi}\mathrm{Cos}\phi\mathrm{Cos}\theta$

i.e.

$$\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} 1 & 0 & -\mathrm{Sin}\theta \\ 0 & \mathrm{Cos}\phi & \mathrm{Sin}\phi\mathrm{Cos}\theta \\ 0 & -\mathrm{Sin}\phi & \mathrm{Cos}\phi\mathrm{Cos}\theta \end{pmatrix} \begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} \qquad \text{Eqns.(4)}$$

N.b. For small perturbations the first order approximations $p = \dot{\phi}$, $q = \dot{\theta}$ and $r = \dot{\psi}$ may be made.

FIGURE 4: AIRCRAFT AXES AND FLIGHT PATH ANGLE.

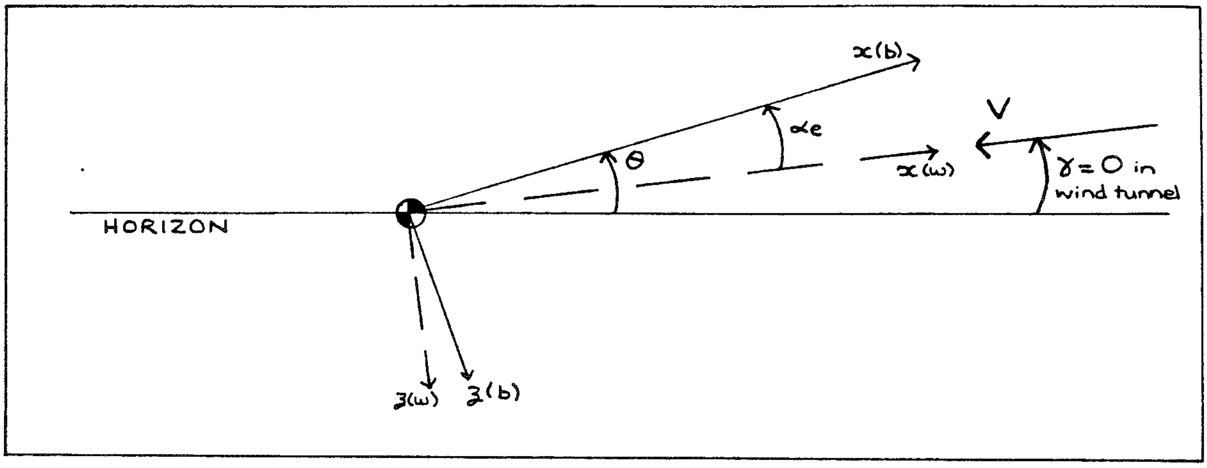
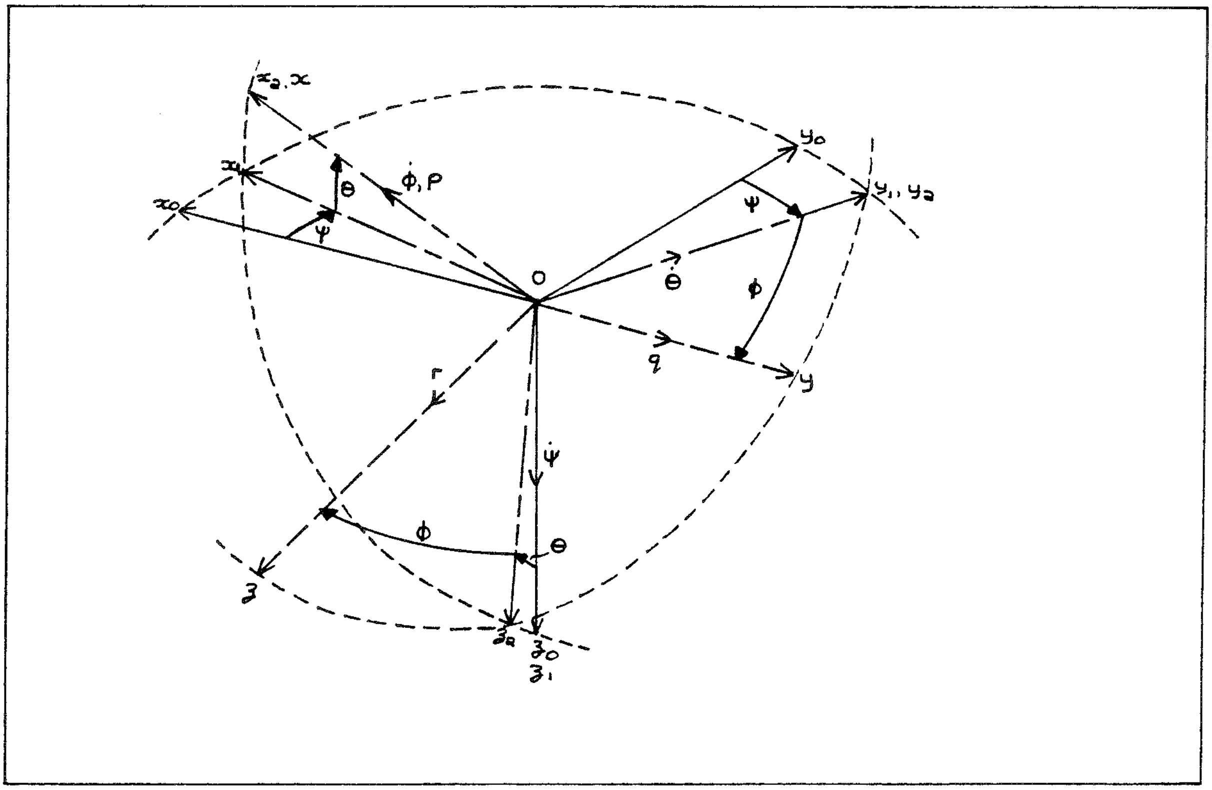


FIGURE 5: THE EULER ANGLES.



5.2 HAWK MODEL KINEMATICS

The Hawk model is suspended on a vertical rod in the wind tunnel by a gimbal Figure 6, the action of which is depicted in Figure 7. On examination it is found that the gimbal was designed so that any perturbed angles recorded by the yaw, pitch and roll potentiometers correspond directly to the Euler Angles defined in Figure 5. Thus the linear and rate transformations of Equations 3 and 4 may be applied directly with the required angles ψ, θ and ϕ being measured directly from the potentiometer readings.

The datum axes of the Hawk model are defined to be stability or wind axes $(Oxyz)_{wind}$. n the steady state these axes are coincident with the tunnel/earth axes as shown in Figure 5. Because the resultant aircraft velocity V is produced horizontally by the wind tunnel the flight path angle γ is zero.
This means that the angle of incidence of the aircraft is equal to its pitch angle θ. ie. $\alpha = \theta$. Also, further constraints on the lateral motion of the model means that the angle of sideslip β is equal to minus the yaw rate, ie. $\beta = -\psi$.

Because of model constraints in the wind tunnel the linear transformation of equations 3 will apply in the following form:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \underset{\sim}{A} \begin{pmatrix} 0 \\ 0 \\ Z_0 \end{pmatrix}$$ as there is no forward or lateral translation of the model allowed.

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \underset{\sim}{A} \begin{pmatrix} V \\ 0 \\ 0 \end{pmatrix}$$ where V is equal to the tunnel speed.

A problem which arises here is that the attitude rates $\dot{\phi}$, $\dot{\theta}$ and $\dot{\psi}$ are not immediately available and a suitable way to generate them will need to be found. Another practical constraint is that the model is free to rotate only $\pm 30^0$ in any direction. However, as we are dealing with small perturbations this should not present a problem.

FIGURE 6: MODEL GIMBAL.

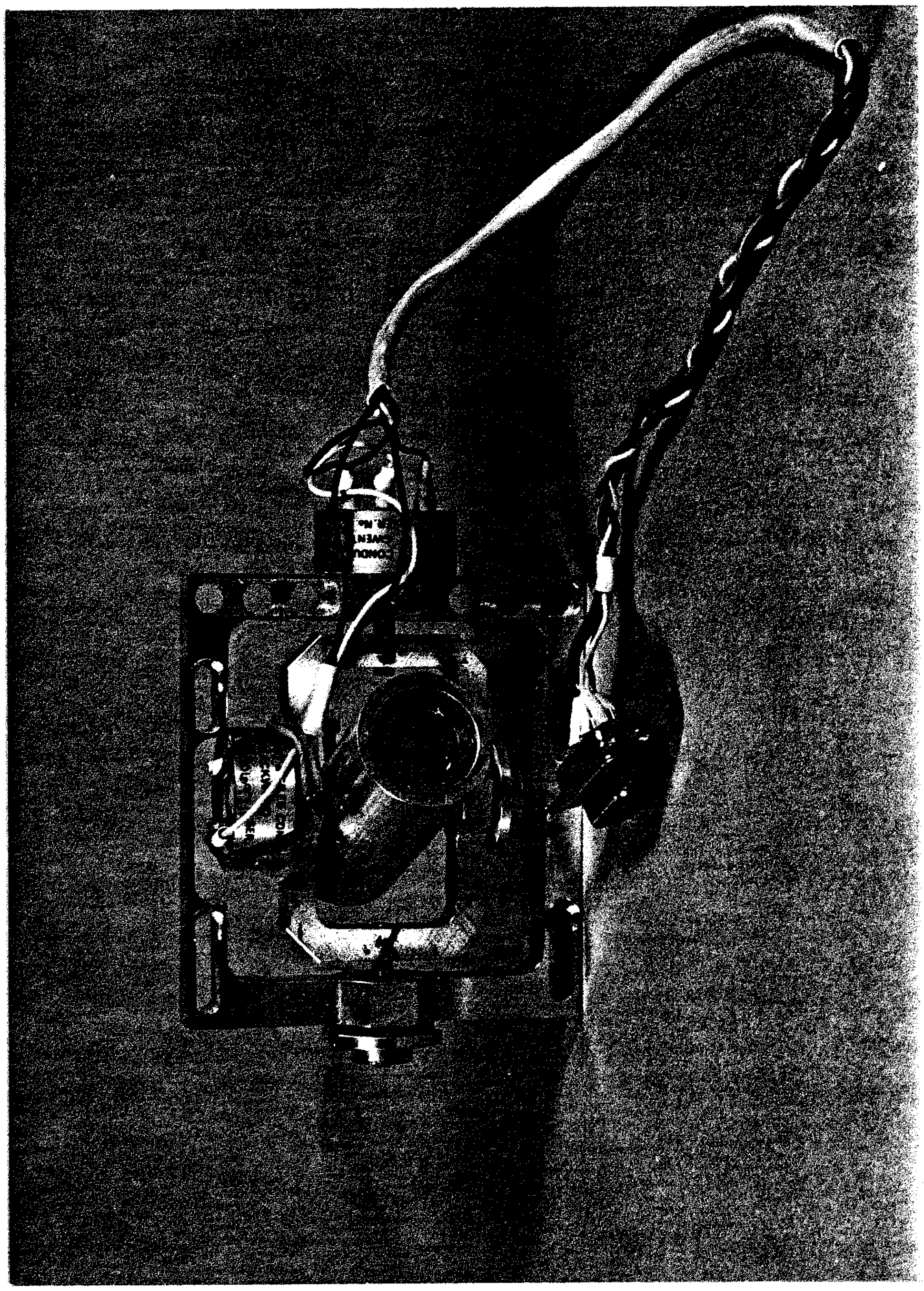
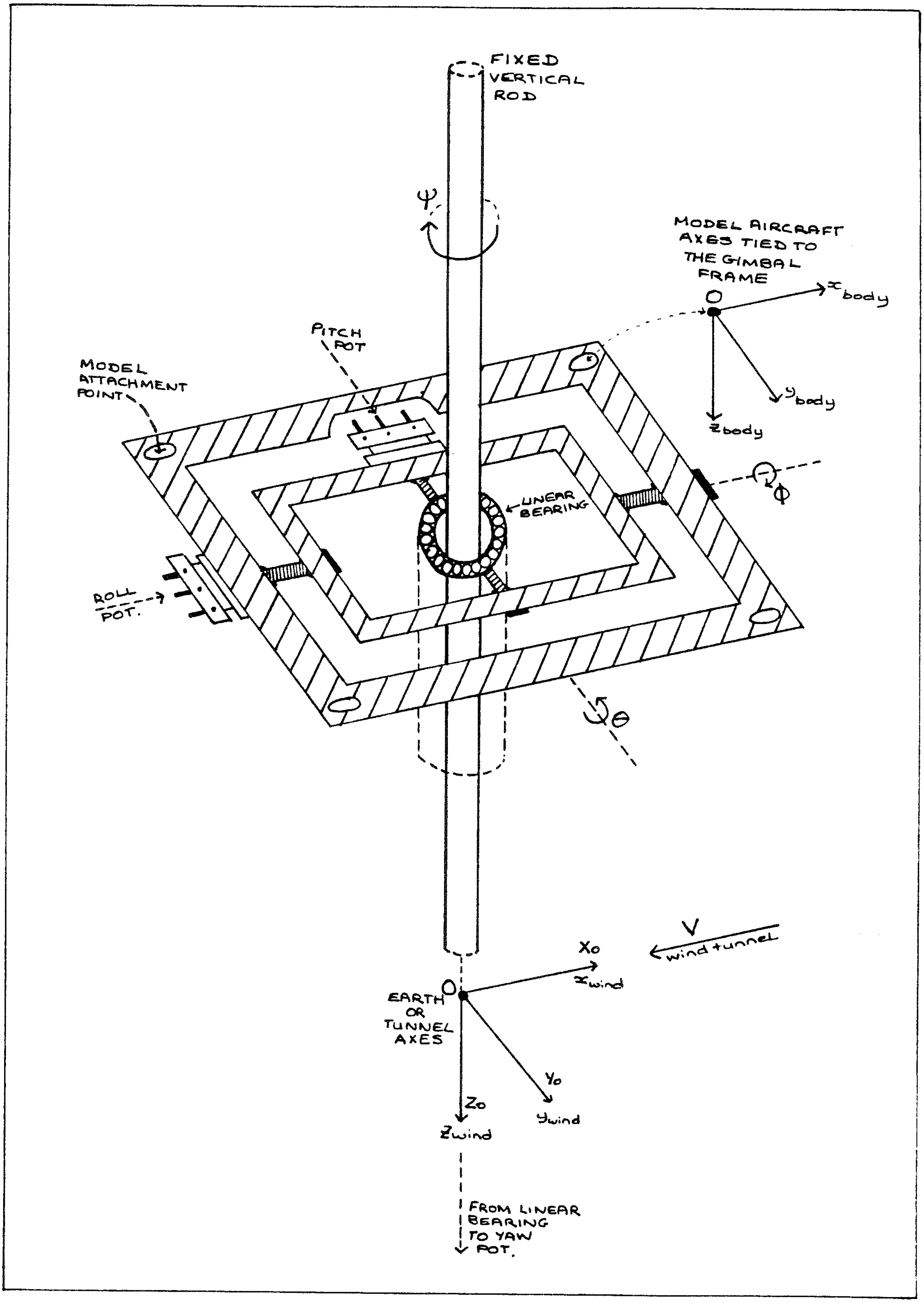


FIGURE 7: MODEL GIMBAL AND AXES.



6.0 MODIFIED STEPWISE REGRESSION REQUIREMENTS.

For the system identification of an aircraft operating at low angles of attack, the mathematical model structure for aerodynamic forces and moments is linear and may be written in the following form as described in Reference 5

$$y(t) = \theta_0 + \theta_1 x_1(t) + \theta_2 x_2(t) + \ldots + \theta_{n-1} x_{n-1}(t) \qquad \text{Eqns.(5)}$$

Where: $y(t)$ represents the resultant coefficient of aerodynamic force or moment $(C_X, C_Y, C_Z, C_l, C_m, C_n)$ at time t. These are the dependent variables.

$\theta_1, \theta_2, \ldots, \theta_{n-1}$ are the stability and control derivatives; and θ_0 is the value of any particular coefficient corresponding to the initial steady flight conditions.

$x_1, x_2, \ldots x_{n-1}$ are the independent aircraft state and control variables $(\alpha, q, \beta, p, r, \eta, \xi, \zeta)$ and may also include combinations of these variables at time t.

The aircraft equations of motion will now be looked at to express them in a form compatible with the regression requirements.

6.1 LONGITUDINAL REGRESSION.

In the first quarterly report (Ref.2), the standard state variable form of the equations of longitudinal motion for the model in the wind tunnel were derived and were as follows:

$$\begin{bmatrix} \dot{w} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} z_w & z_q & 0 \\ m_w & m_q & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} w \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} z_\eta \\ m_\eta \\ 0 \end{bmatrix} \begin{bmatrix} \eta \end{bmatrix} \qquad \text{Eqns.(6)}$$

where $z_w = \left[\dfrac{\breve{z}_w}{1-\mathring{z}_{\dot{w}}}\right]$; $\quad z_q = \left[\dfrac{u_e+\breve{z}_q}{1-\mathring{z}_{\dot{w}}}\right]$; $\quad z_\eta = \left[\dfrac{\breve{z}_\eta}{1-\mathring{z}_{\dot{w}}}\right]$

$$m_w = \left[\frac{\mathring{m}_{\dot{w}}\cdot\breve{z}_w}{(1-\breve{z}_{\dot{w}})} + \breve{m}_w\right]; \quad m_q = \left[\frac{(u_e+\breve{z}_q)\breve{m}_{\dot{w}}}{(1-\breve{z}_{\dot{w}})} + \breve{m}_q\right]; \quad m_\eta = \left[\frac{\mathring{m}_{\dot{w}}\breve{z}_\eta}{(1-\breve{z}_{\dot{w}})} + \breve{m}_\eta\right]$$

Equations 6 may be rearranged to correspond with the following form:

$$y(t) = \theta_0 + \theta_1.x_1(t) + \theta_2.x_2(t) + \ldots + \theta_{n-1}x_{n-1}(t)$$

thus, Equations 6 may be written:

$$\begin{aligned} \dot{w} &= Z_0 + z_w.w + z_q.q + 0.\theta + z_\eta.\eta \\ \dot{q} &= M_0 + m_w.w + m_q.q + 0.\theta + m_\eta.\eta \qquad \text{Eqns.(7)} \\ \dot{\theta} &= q \end{aligned}$$

These equations are now in the form required for the regression analysis. It is assumed that $\dot{w}$, $\dot{q}$, $\dot{\theta}$, w, q, θ and η can be measured or quantified satisfactorily.

6.2 LATERAL REGRESSION.

Also derived in the last report were the lateral equations of motion for the model in the wind tunnel:

$$\begin{bmatrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & W_e & -U_e & 0 & 0 \\ l_v & l_p & l_r & 0 & 0 \\ n_v & n_p & n_r & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v \\ p \\ r \\ \phi \\ \psi \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ l_\xi & l_\zeta \\ n_\xi & n_\zeta \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \xi \\ \zeta \end{bmatrix} \qquad \text{Eqns.(8)}$$

where $E_{xz} = 1 + e_x e_z$

$$l_v = \left\{ \frac{\mathring{l}_v}{E_{xz}} + \frac{e_x \mathring{n}_v}{E_{xz}} \right\} ; \quad l_p = \left\{ \frac{\mathring{l}_p}{E_{xz}} + \frac{e_x \mathring{l}_p}{E_{xz}} \right\} ; \quad l_r = \left\{ \frac{\mathring{l}_r}{E_{xz}} + \frac{e_x \mathring{l}_r}{E_{xz}} \right\} ;$$

$$n_v = \left\{ \frac{-e_z \mathring{l}_v}{E_{xz}} + \frac{\mathring{n}_v}{E_{xz}} \right\} ; \quad n_p = \left\{ \frac{-e_z \mathring{l}_p}{E_{xz}} + \frac{\mathring{n}_p}{E_{xz}} \right\} ; \quad n_r = \left\{ \frac{-e_z \mathring{l}_r}{E_{xz}} + \frac{\mathring{n}_r}{E_{xz}} \right\} ;$$

$$\text{and} \quad l_\xi = \left\{ \frac{\mathring{l}_\xi}{E_{xz}} + \frac{e_x \mathring{n}_\xi}{E_{xz}} \right\} ; \quad l_\zeta = \left\{ \frac{\mathring{l}_\zeta}{E_{xz}} + \frac{e_x \mathring{n}_\zeta}{E_{xz}} \right\}$$

$$n_\xi = \left\{ \frac{-e_z \mathring{l}_\xi}{E_{xz}} + \frac{\mathring{n}_\xi}{E_{xz}} \right\} ; \quad n_\zeta = \left\{ \frac{-e_z \mathring{l}_\zeta}{E_{xz}} + \frac{\mathring{n}_\zeta}{E_{xz}} \right\}$$

Upon rearrangement Equations 8 become

$$\dot{v} = Y_0 + 0.v + W_e.p + -U_e.r + 0.\phi + 0.\psi + 0.\xi + 0.\zeta$$

$$\dot{p} = L_0 + l_v.v + l_p.p + l_r.r + 0.\phi + 0.\psi + l_\xi.\xi + l_\zeta.\zeta$$

$$\dot{r} = N_0 + n_v.v + n_p.p + n_r.r + 0.\phi + 0.\psi + n_\xi.\xi + n_\zeta.\zeta$$

$$\dot{\phi} = p$$

$$\dot{\psi} = r$$

7.0 RESEARCH LITERATURE AND INFORMATION.

During the last quarter most work has been concentrated on literature already held. However a few new references have been obtained and are listed in the reference section at the end of this report.

One highlight of the last quarter was a visit to the College of Aeronautics by the NASA research scientist James Batterson. The Modified Stepwise Regression Method (REF.5), which was first introduced by J. Batterson and V.Klein forms the basis for the current research project discussed in this report.

Mr. Batterson was was shown the experimental dynamic rig and our research programme was outlined to him. During our talk the problem of interference from noise was mentioned. It is realised that in the MSR method noise is assumed to be implicit in the data analysis and it the best strategy is to reduce noise at source as much as possible. Various other topics were discussed and the visit proved to be very interesting and very valuable.

8.0 IMMEDIATE OBJECTIVES.

The broad aims for the next quarter are to continue development of the topics outlined in this report and may be summarised as follows:

1. The ACSL program will be looked at to try and improve the impulse input method.

2.0 A simulation using full scale Hawk derivative data may be run to obtain graphs of expected aircraft responses.

3.0 The data acquisition system will be tested further with particular emphasis on the way in which data is stored.

4.0 Further calibration work will be carried out on the wind tunnel model and the vertical position sensing system tested.

5.0 Work will be started on the FORTRAN program to model the MSR method.

9.0 CONCLUSION.

At the moment the time scale of work being carried out for the research programme is running as initially proposed in REF.6. At present it is expected that the overall objectives for the first year will be achieved.

LIST OF SYMBOLS

C_x	drag coefficient
C_y	sideforce coefficient
C_z	lift coefficient
C_l	rolling moment coefficient
C_m	pitching moment coefficient
C_n	yawing moment coefficient
I_x, I_y, I_z, I_{xz}	aircraft inertias
$\breve{L}_v, \mathring{L}_p, \breve{L}_r, \mathring{L}_\xi, \mathring{L}_\zeta$	dimensional rolling moment derivative due to sideslip, roll rate, yaw rate, etc.
$\breve{M}_u, \mathring{M}_w, \breve{M}_{\dot{w}}, \mathring{M}_q, \breve{M}_\eta$	dimensional pitching moment derivative due to forward velocity, side velocity, etc.
$\breve{N}_v, \breve{N}_p, \breve{N}_r, \breve{N}_\xi, \breve{N}_\zeta$	dimensional yawing moment derivative due to sideslip, roll rate, yaw rate, etc.
p, q, r	rate of roll pitch and yaw respectively
u, v, w	components of velocity
V	total velocity
$\breve{X}_u, \mathring{X}_w, \breve{X}_{\dot{w}}, \mathring{X}_q, \mathring{X}_\eta$	dimensional drag force derivative due to forward velocity, side velocity, etc.
$\mathring{Y}_v, \mathring{Y}_p, \mathring{Y}_r, \mathring{Y}_\xi, \mathring{Y}_\zeta$	dimensional sideforce derivative due to sideslip, roll rate, yaw rate, etc.
$\mathring{Z}_u, \mathring{Z}_w, \mathring{Z}_{\dot{w}}, \mathring{Z}_q, \breve{Z}_\eta$	dimensional lift force derivative due to forward velocity, side velocity, etc.
α	angle of attack, $\tan^{-1}(w/u)$
β	angle of sideslip, $\sin^{-1}(v/V)$
θ, ϕ, ψ	attitude in pitch, bank and azimuth
η, ξ, ζ	control surface angle of elevator, aileron and rudder respectively

Longitudinal Mass Matrices and Derivatives:

$$M = \begin{pmatrix} (1-\mathring{z}_{\dot{w}}) & 0 & 0 \\ -\mathring{m}_{\dot{w}} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}; \qquad M^{-1} = \begin{pmatrix} 1/(1-\mathring{z}_{\dot{w}}) & 0 & 0 \\ \mathring{m}_{\dot{w}}/(1-\mathring{z}_{\dot{w}}) & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathring{z}_{\dot{w}} = \mathring{Z}_{\dot{w}}/m \; ; \qquad \mathring{z}_{w} = \mathring{Z}_{w}/m \; ; \qquad \mathring{z}_{q} = \mathring{Z}_{q}/m \; ; \qquad \mathring{z}_{\eta} = \mathring{Z}_{\eta}/m \; .$$

$$\mathring{m}_{\dot{w}} = \mathring{M}_{\dot{w}}/I_y ; \qquad \mathring{m}_{w} = \mathring{M}_{w}/I_y ; \qquad \mathring{m}_{q} = \mathring{M}_{q}/I_y ; \qquad \mathring{m}_{\eta} = \mathring{M}_{\eta}/I_y .$$

$$z_w = \left(\frac{\mathring{z}_w}{1-\mathring{z}_{\dot{w}}}\right); \qquad z_q = \left(\frac{u_e+\mathring{z}_q}{1-\mathring{z}_{\dot{w}}}\right); \qquad z_\eta = \left(\frac{\mathring{z}_\eta}{1-\mathring{z}_{\dot{w}}}\right)$$

$$m_w = \left(\frac{\mathring{m}_{\dot{w}}\cdot\mathring{z}_w}{(1-\mathring{z}_{\dot{w}})} + \mathring{m}_w\right); \qquad m_q = \left(\frac{(u_e+\mathring{z}_q)\mathring{m}_{\dot{w}}}{(1-\mathring{z}_{\dot{w}})} + \mathring{m}_q\right); \qquad m_\eta = \left(\frac{\mathring{m}_{\dot{w}}\cdot\mathring{z}_\eta}{(1-\mathring{z}_{\dot{w}})} + \mathring{m}_\eta\right)$$

Lateral Mass Matrices and Derivatives:

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -e_x & 0 & 0 \\ 0 & e_x & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}; \qquad M^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1/(1+e_x e_z) & -e_z/(1+e_x e_z) & 0 & 0 \\ 0 & -e_x/(1+e_x e_z) & 1/(1+e_x e_z) & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$e_x = I_{xz}/I_x \; ; \qquad e_z = I_{xz}/I_z \; .$$

$$\mathring{l}_v = \mathring{L}_v/I_x ; \qquad \mathring{l}_p = \mathring{L}_p/I_x ; \qquad \mathring{l}_r = \mathring{L}_r/I_x ; \qquad \mathring{l}_\xi = \mathring{L}_\xi/I_x ; \qquad \mathring{l}_\zeta = \mathring{L}_\zeta/I_x .$$

$$n_v = \mathring{N}_v/I_z ; \qquad \mathring{n}_p = \mathring{N}_p/I_z ; \qquad \mathring{n}_r = \mathring{N}_r/I_z ; \qquad \mathring{n}_\xi = \mathring{N}_\xi/I_z ; \qquad \mathring{n}_\zeta = \mathring{N}_\zeta/I_z .$$

LIST OF SYMBOLS (continued)

$$E_{xz} = 1 + e_x e_z$$

$$l_v = \left\{ \frac{\mathring{l}_v}{E_{xz}} + \frac{e_x \mathring{n}_v}{E_{xz}} \right\} ; \qquad l_p = \left\{ \frac{\mathring{l}_p}{E_{xz}} + \frac{e_x \mathring{l}_p}{E_{xz}} \right\} ; \qquad l_r = \left\{ \frac{\mathring{l}_r}{E_{xz}} + \frac{e_x \mathring{l}_r}{E_{xz}} \right\} ;$$

$$n_v = \left\{ \frac{-e_z \mathring{l}_v}{E_{xz}} + \frac{\mathring{n}_v}{E_{xz}} \right\} ; \qquad n_p = \left\{ \frac{-e_z \mathring{l}_p}{E_{xz}} + \frac{\mathring{n}_p}{E_{xz}} \right\} ; \qquad n_r = \left\{ \frac{-e_z \mathring{l}_r}{E_{xz}} + \frac{\mathring{n}_r}{E_{xz}} \right\} ;$$

$$l_\xi = \left\{ \frac{\mathring{l}_\xi}{E_{xz}} + \frac{e_x \mathring{n}_\xi}{E_{xz}} \right\} ; \qquad l_\zeta = \left\{ \frac{\mathring{l}_\zeta}{E_{xz}} + \frac{e_x \mathring{n}_\zeta}{E_{xz}} \right\}$$

$$n_z = \left\{ \frac{-e_z \mathring{l}_\xi}{E_{xz}} + \frac{\mathring{n}_\xi}{E_{xz}} \right\} ; \qquad n_\zeta = \left\{ \frac{-e_z \mathring{l}_\zeta}{E_{xz}} + \frac{\mathring{n}_\zeta}{E_{xz}} \right\}$$

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2. COOK, M.V. and HINDS, H.A.: Initial Review of Research Into the Application of Modified Stepwise Regression for the Estimation of Aircraft Stability and Control Parameters. January 1989, Cranfield Institute of Technology.

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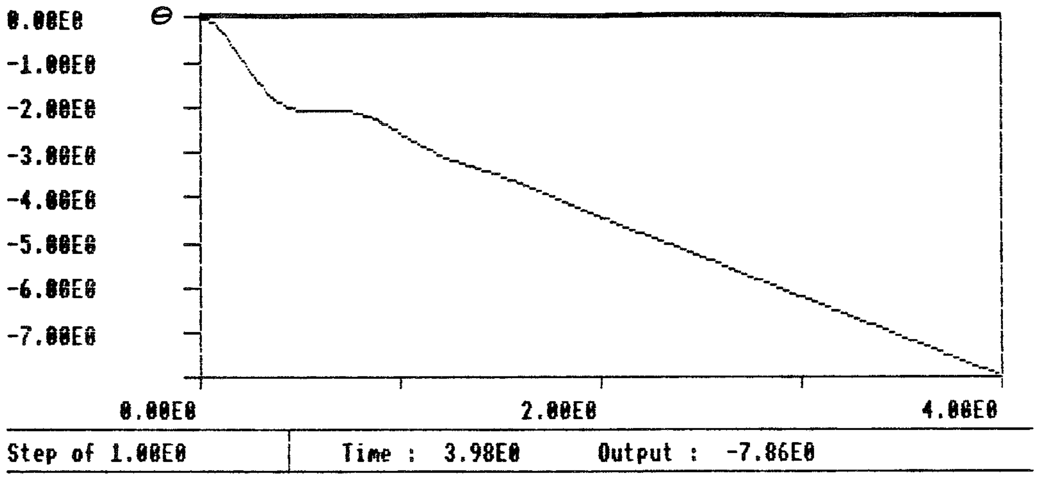
10. O'LEARY, C.O. and ROWTHORN, E.N.: Measurements of Oscillatory Lateral Derivatives of a High Incidence Research Model (HIRM 1) at speeds up to M = 0.8. Jn. of Aircraft, Jan. 1989.

APPENDIX A

AIRCRAFT RESPONSES OBTAINED USING THE BBC DESIGN PACKAGE AND THE ACSL SIMULATION PROGRAMS.

FIGURE A-1: PITCH ANGLE STEP RESPONSE TO 1 DEGREE OF ELEVATOR.

BBC



-41.4
+0.945*Exp(-2.15*T)*Sin(7.75*T+1.06)
-41.5*Exp(-3.51E-2*T)*Sin(4.14E-2*T-1.36)

ACSL

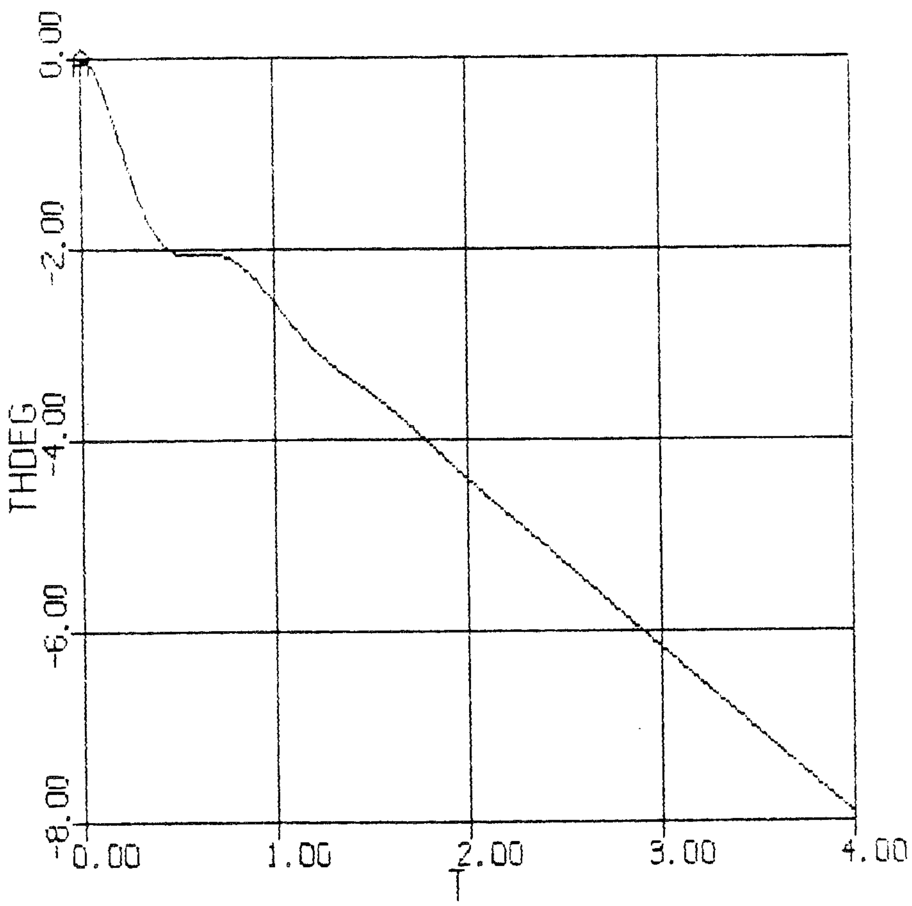
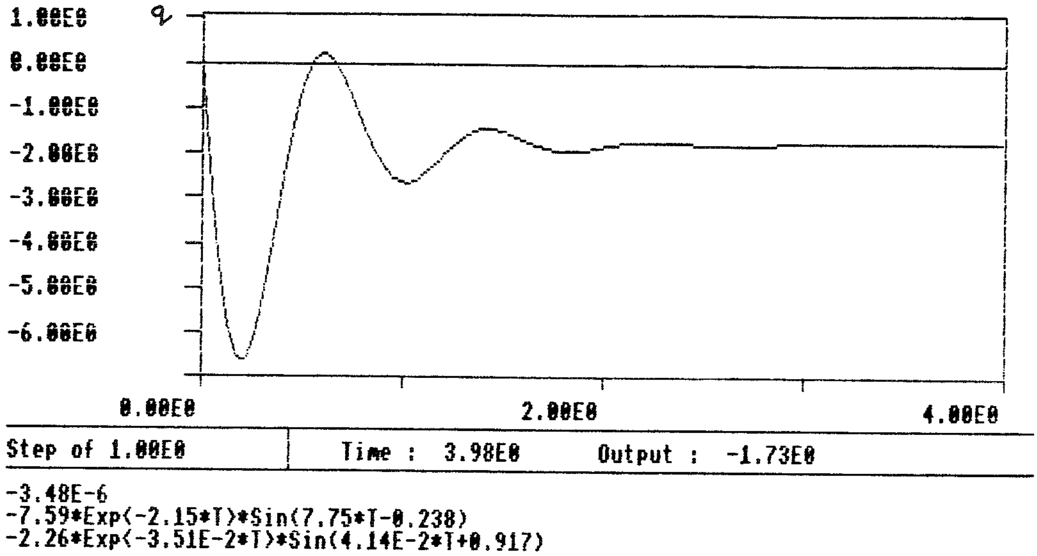


FIGURE A-2: PITCH RATE STEP RESPONSE TO 1 DEGREE OF ELEVATOR.

BBC



ACSL

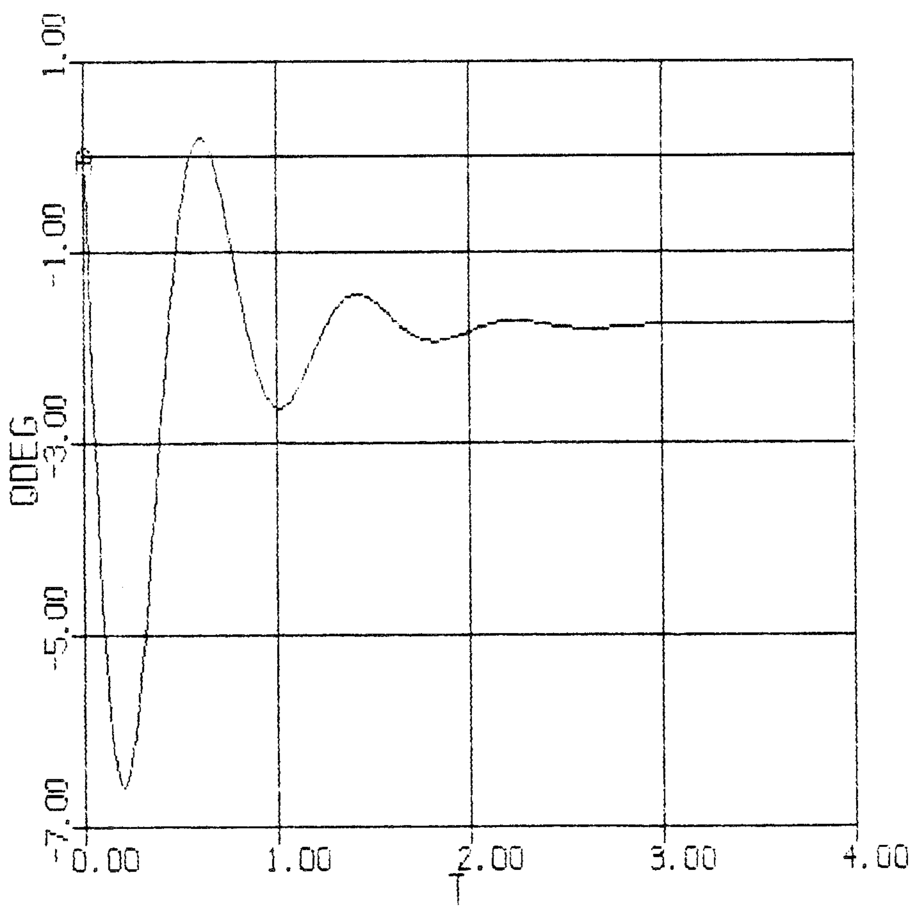
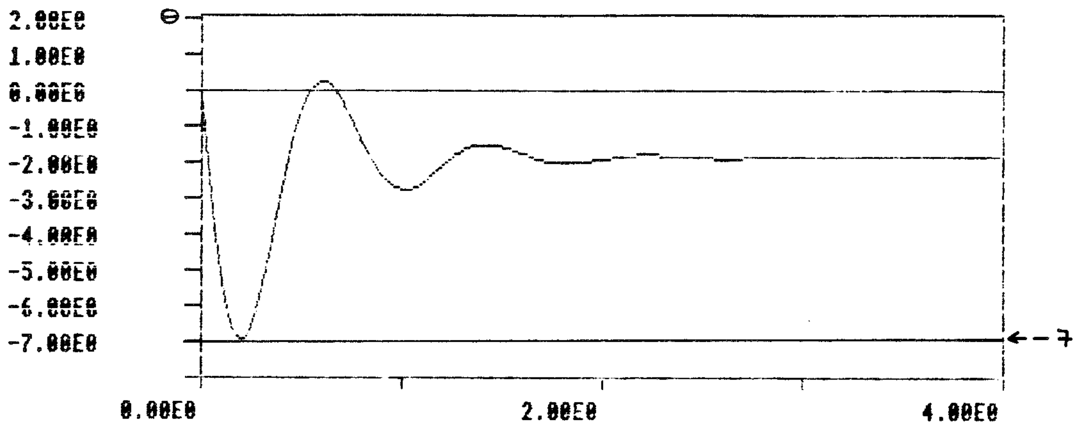


FIGURE A-3: PITCH ANGLE IMPULSE RESPONSE TO 1 DEGREE OF ELEVATOR.

BBC



Impulse of 1.00E0 | Time : 3.98E0 Output : -1.79E0

-7.96*Exp(-2.15*T)*Sin(7.75*T-0.227)
-2.4*Exp(-3.51E-2*T)*Sin(4.14E-2*T+0.864)

ACSL

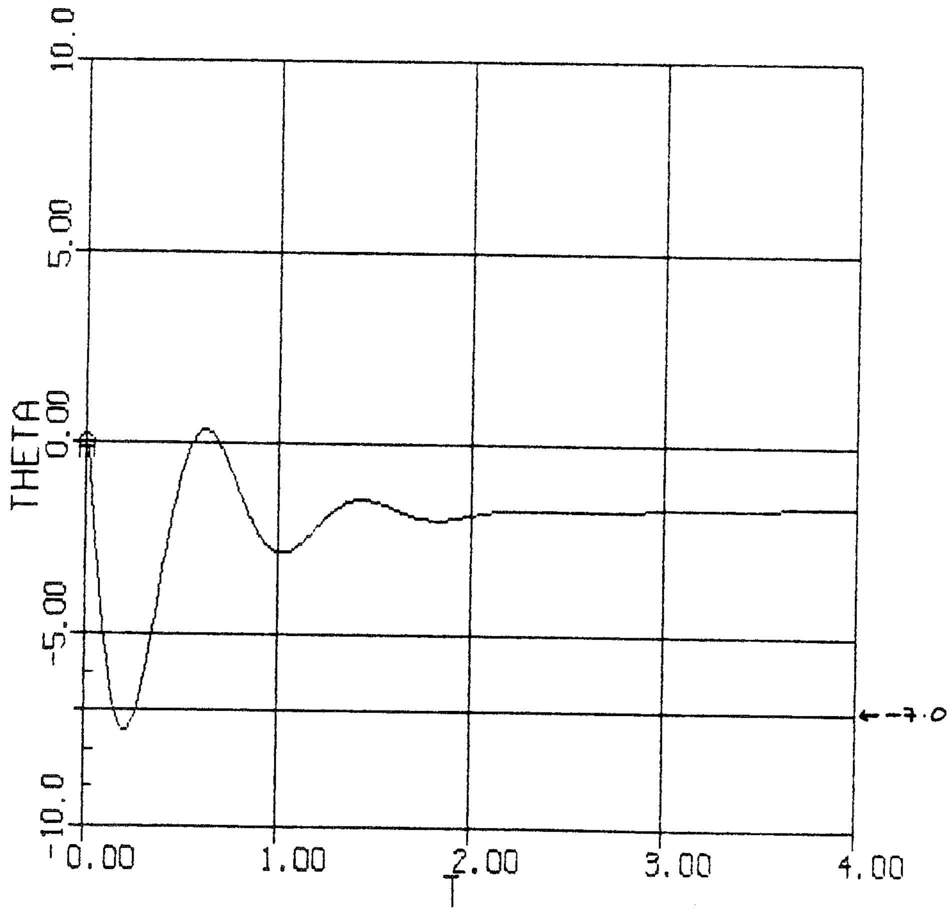
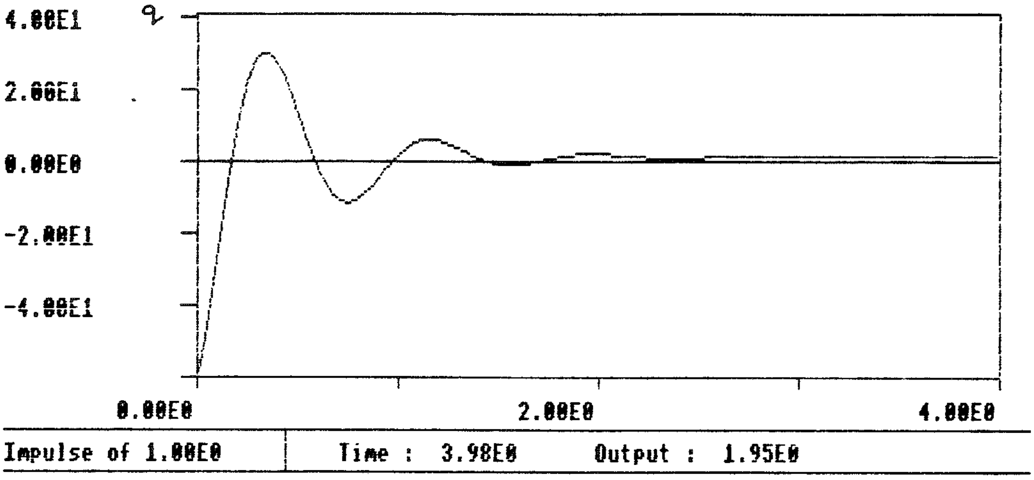


FIGURE A-4: PITCH RATE IMPULSE RESPONSE TO 1 DEGREE OF ELEVATOR.

BBC



+62.8*Exp(-2.15*T)*Sin(7.75*T-1.33)
+6.61*Exp(-3.51E-2*T)*Sin(4.14E-2*T+0.183)

ACSL

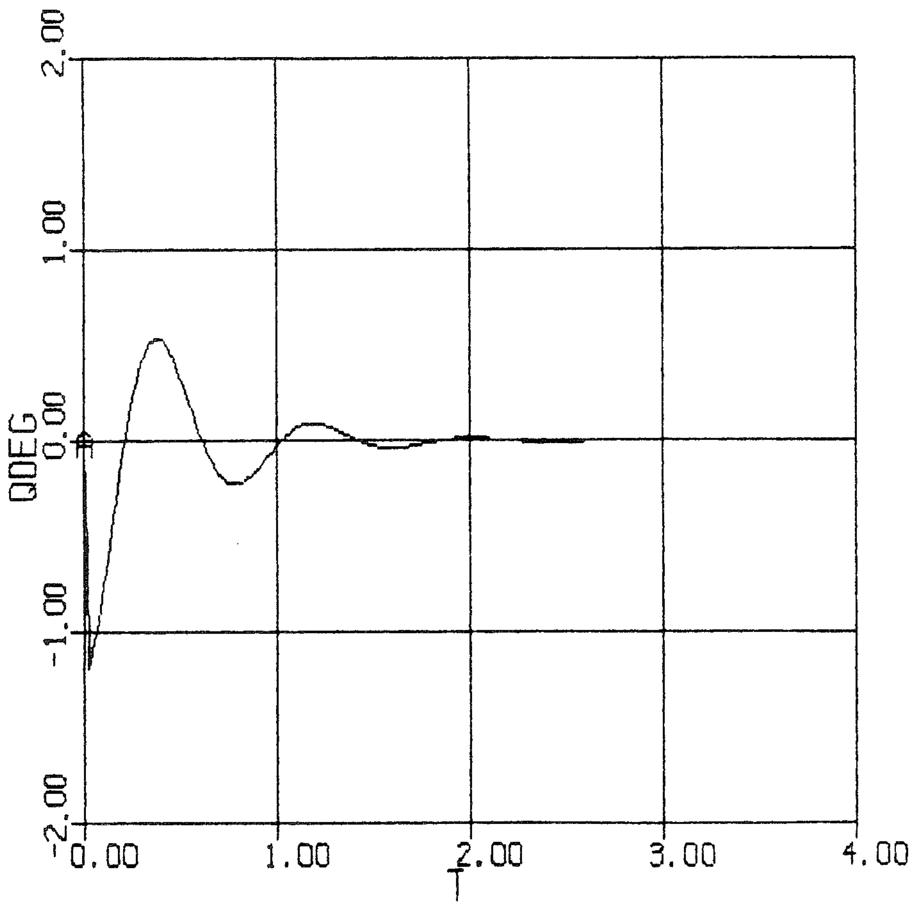
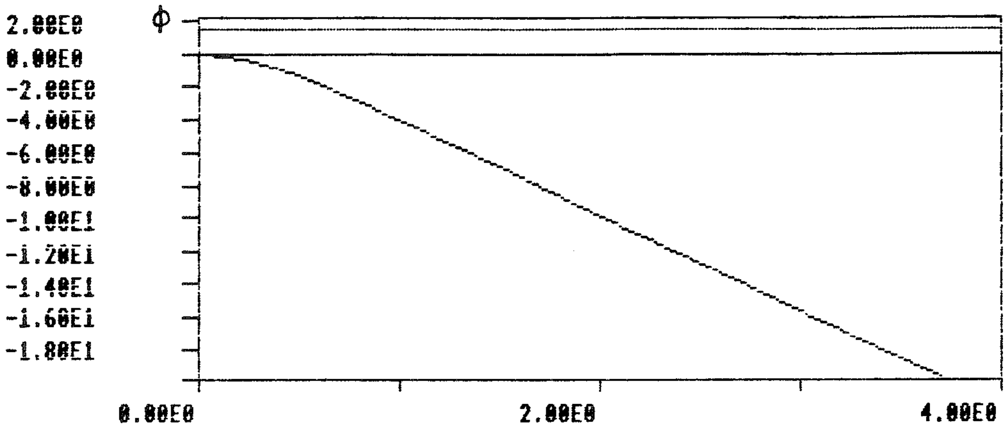


FIGURE A-5: ROLL ANGLE STEP RESPONSE TO 1 DEGREE OF AILERON.

BBC



Step of 1.00E0	Time : 3.98E0 Output : -2.13E1

+1.53
-938*Exp(6.22E-7*T)
+938*Exp(-6.28E-3*T)
-1.84*Exp(-3.1*T)
+8.02E-2*Exp(-0.84*T)*Sin(6.13*T-1.35)

ACSL

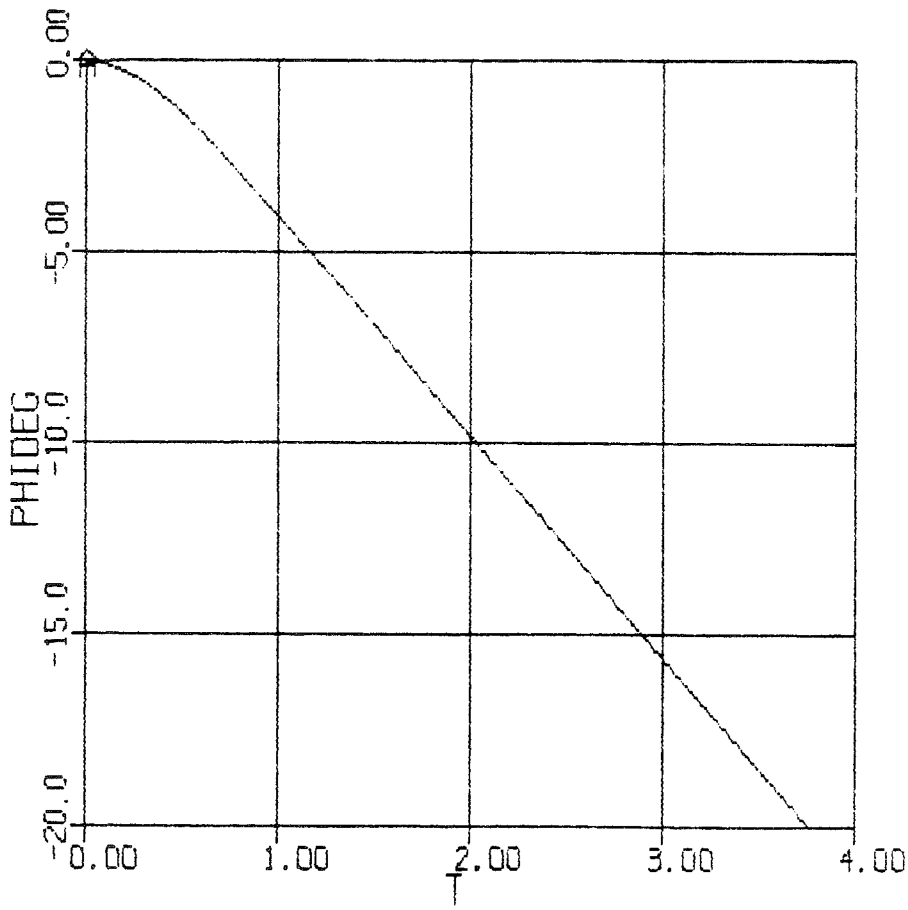
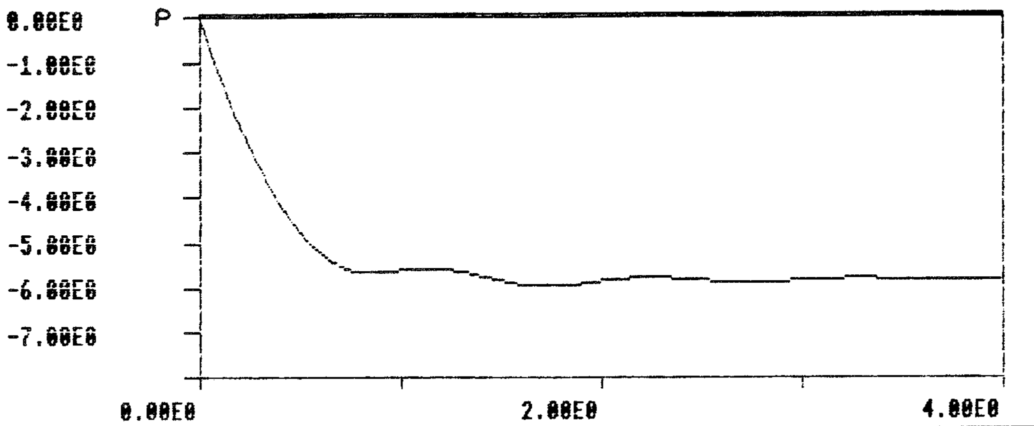


FIGURE A-6: ROLL RATE STEP RESPONSE TO 1 DEGREE OF AILERON.

BBC



Step of 1.00E0 | Time : 3.98E0 Output : -5.75E0

-10.7
+10.7*Exp(6.22E-7*T)
-5.89*Exp(-6.28E-3*T)
+5.72*Exp(-3.1*T)
+0.496*Exp(-0.84*T)*Sin(6.13*T+0.352)

ACSL

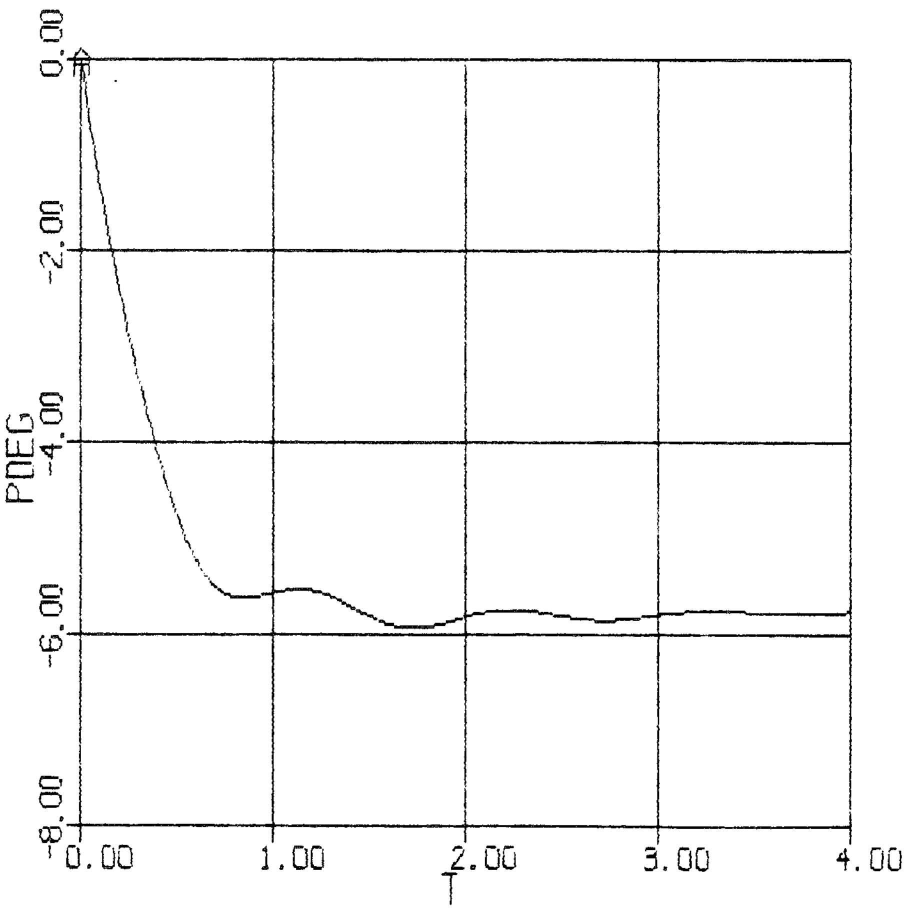
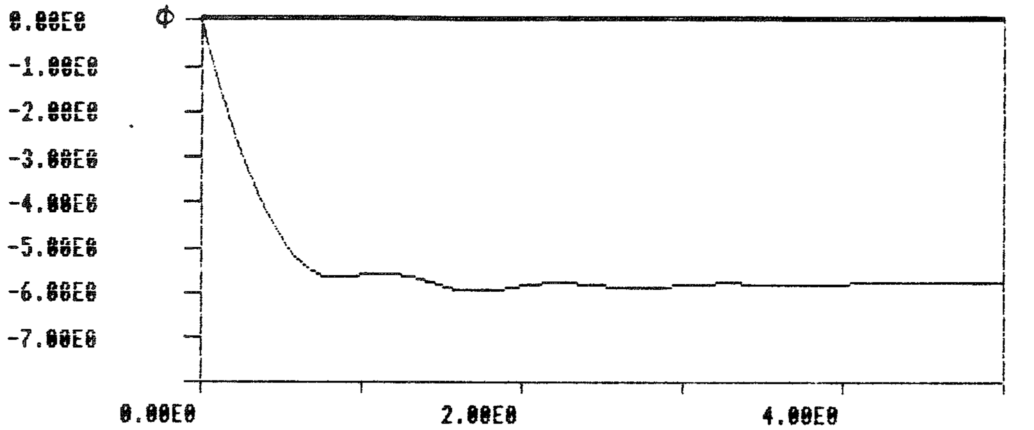


FIGURE A-7: ROLL ANGLE IMPULSE RESPONSE TO 1 DEGREE OF AILERON.

BBC



Impulse of 1.00E0 | Time : 5.00E0 Output : -5.71E0

-5.83E-4*Exp(6.22E-7*T)
-5.89*Exp(-6.28E-3*T)
+5.72*Exp(-3.1*T)
+0.496*Exp(-0.84*T)*Sin(6.13*T+0.352)

ACSL

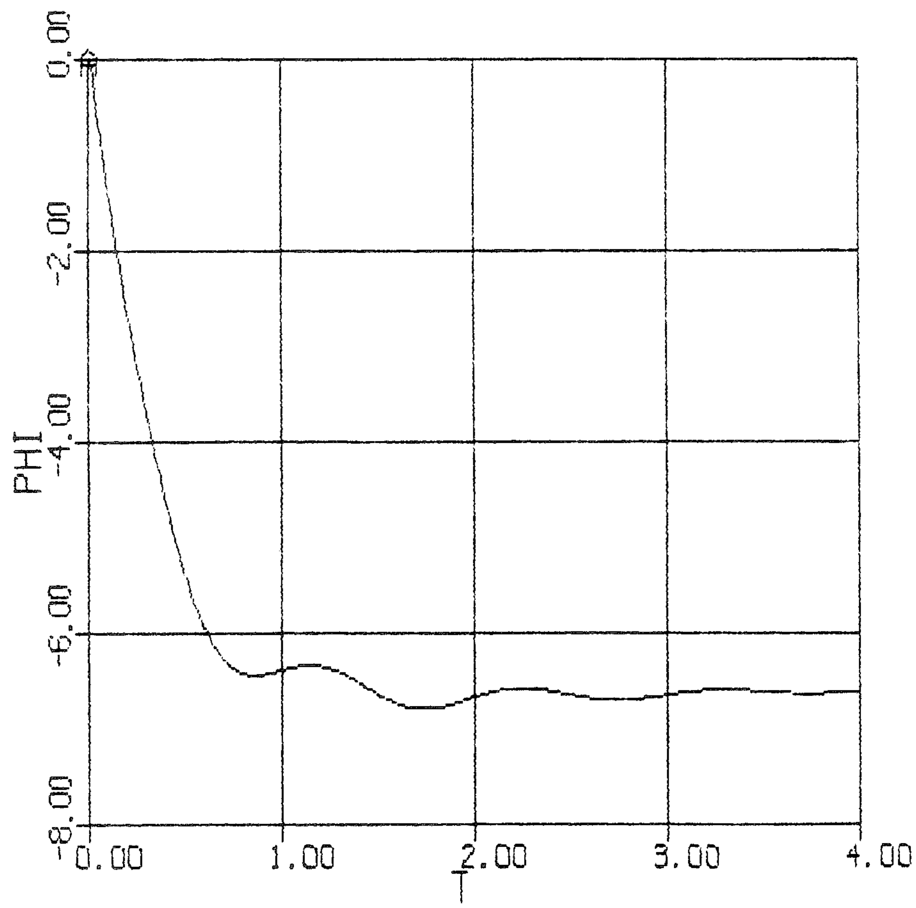
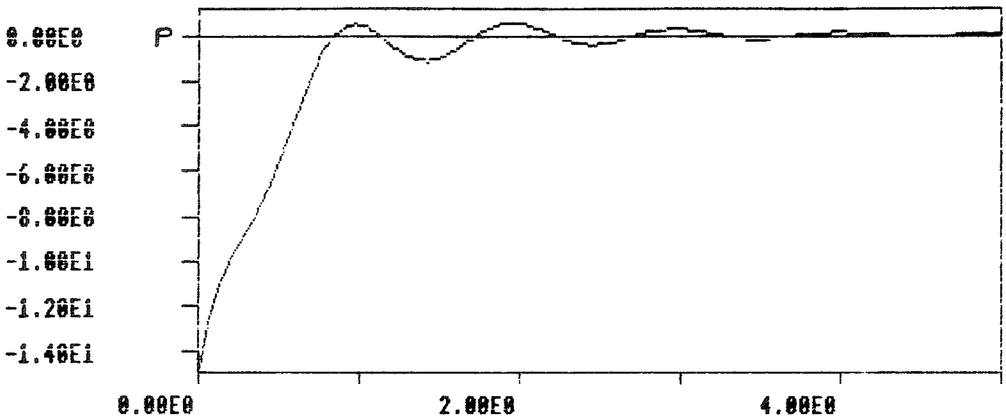


FIGURE A-8: ROLL RATE IMPULSE RESPONSE TO 1 DEGREE OF AILERON.

BBC



Impulse of 1.00E0	Time : 5.00E0 Output : 8.01E-2

+6.66E-6*Exp(6.22E-7*T)
+3.69E-2*Exp(-6.28E-3*T)
-17.7*Exp(-3.1*T)
-3.07*Exp(-0.84*T)*Sin(6.13*T-1.08)

ACSL

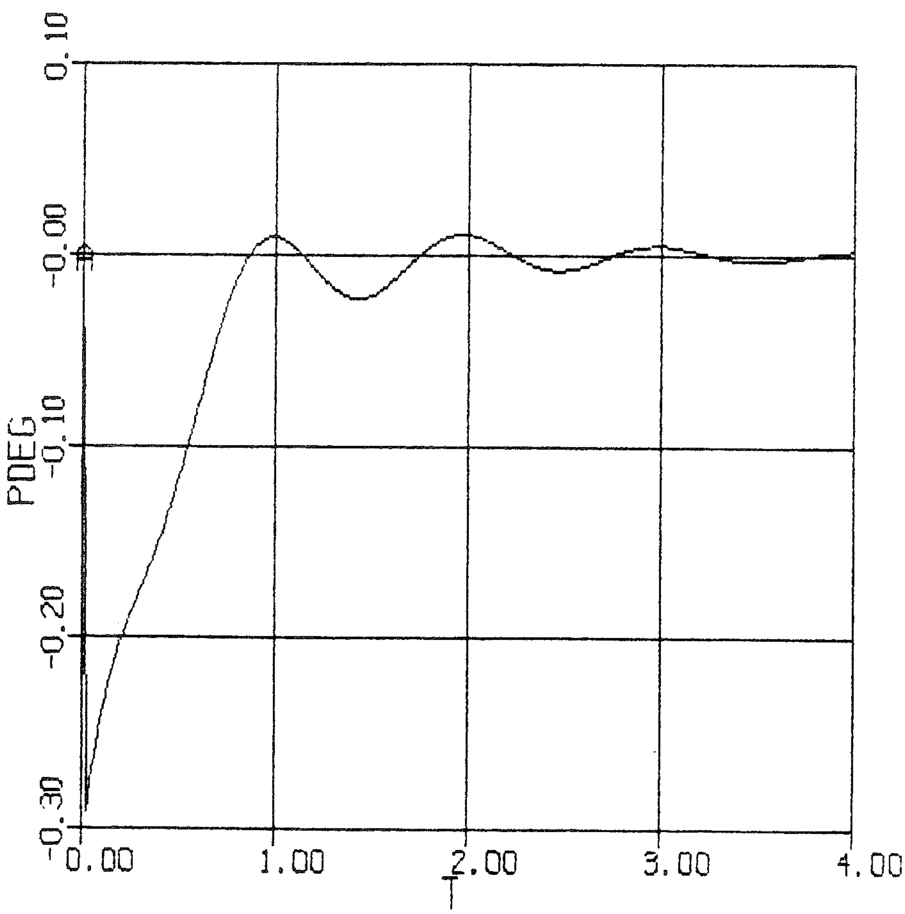
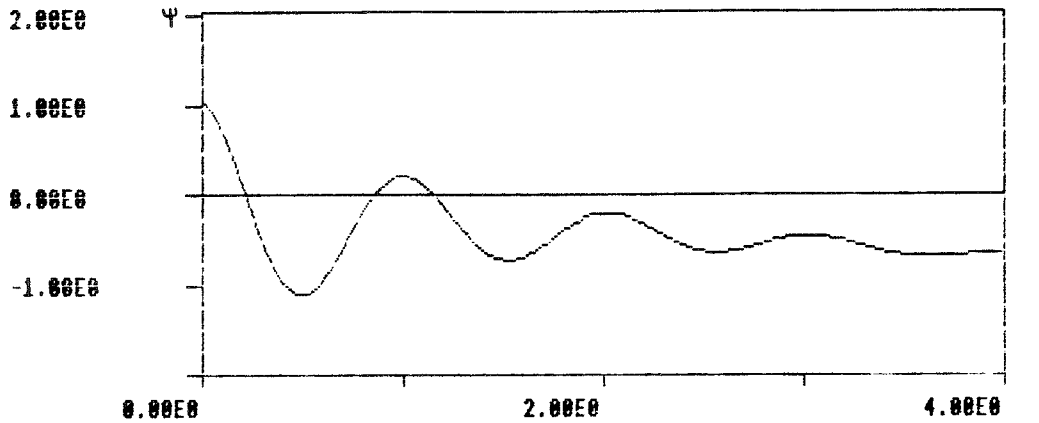


FIGURE A-9: YAW ANGLE STEP RESPONSE TO 1 DEGREE OF RUDDER.

BBC



Step of 1.00E0	Time : 3.98E0 Output : -6.22E-1

+4.53E6
-4.53E6*Exp(6.22E-7*T)
-437*Exp(-6.28E-3*T)
+2.52E-4*Exp(-3.1*T)
-1.26*Exp(-0.84*T)*Sin(6.13*T-1.57)

ACSL

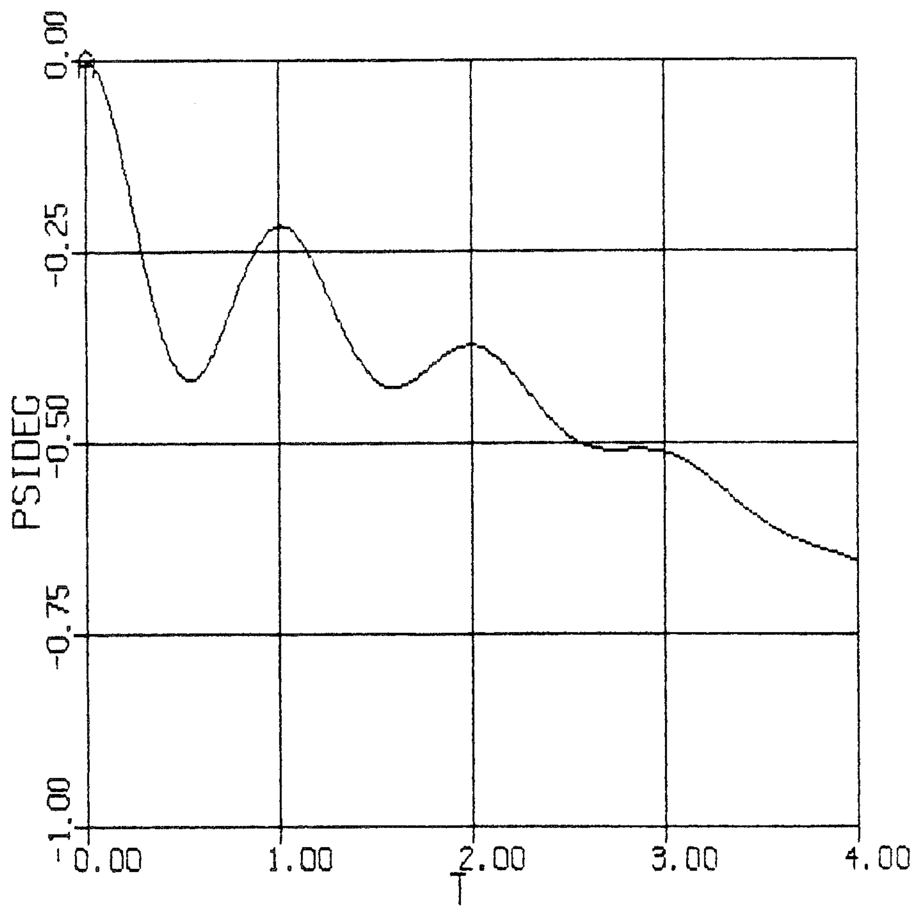
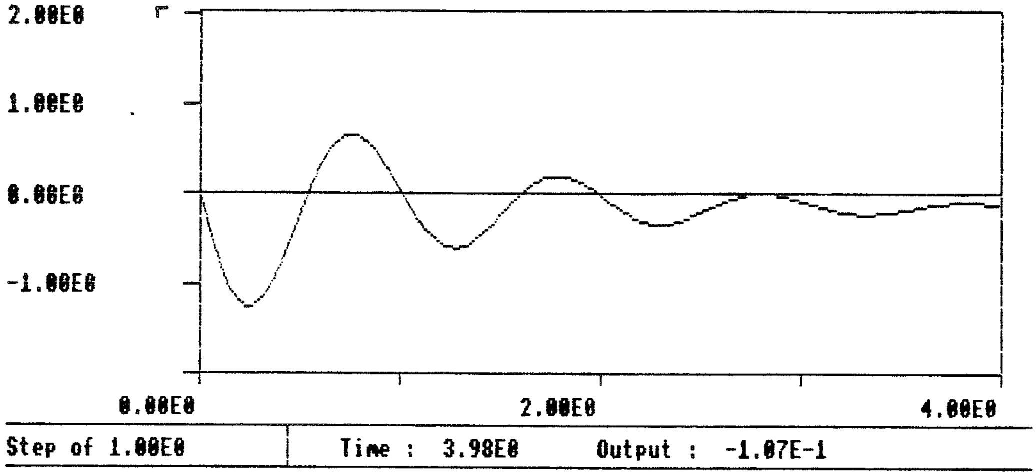


FIGURE A-10: YAW RATE STEP RESPONSE TO 1 DEGREE OF RUDDER.

BBC




```
+2.95
-5.77*Exp(6.22E-7*T)
+2.74*Exp(-6.28E-3*T)
-7.82E-4*Exp(-3.1*T)
-1.43*Exp(-0.84*T)*Sin(6.13*T-5.24E-2)
```

ACSL

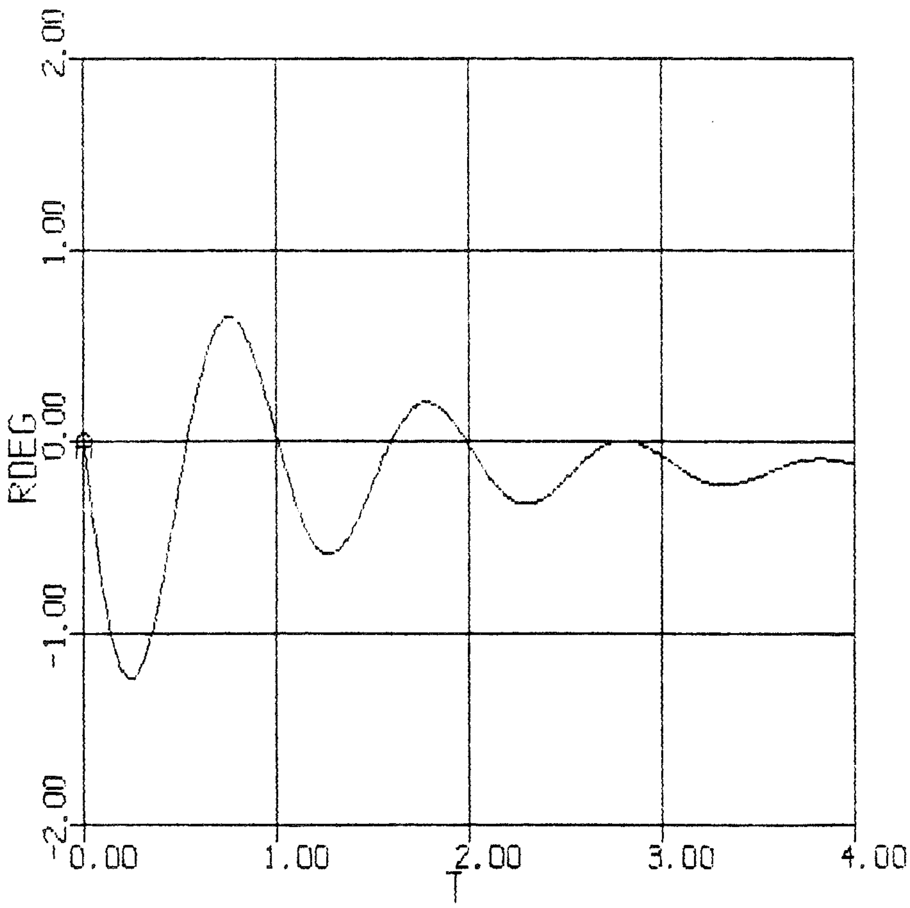
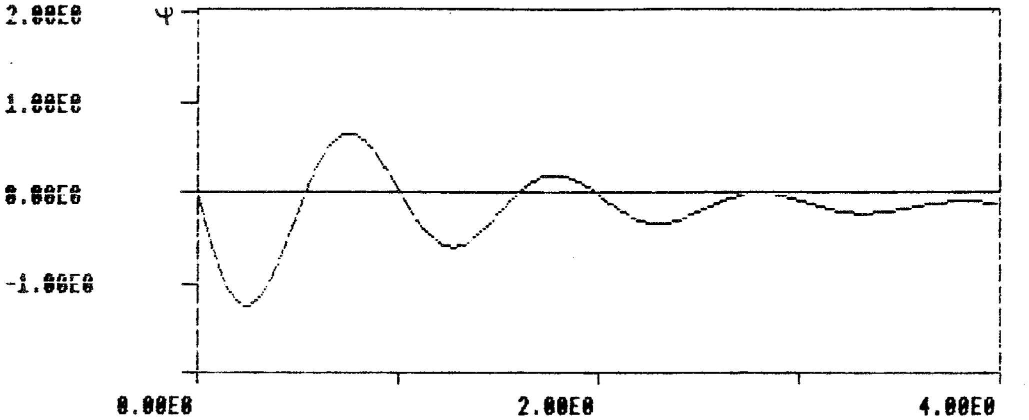


FIGURE A-11: YAW ANGLE IMPULSE RESPONSE TO 1 DEGREE OF RUDDER.

BBC



Impulse of 1.00E0 | Time : 3.98E0 Output : -1.07E-1

-2.82*Exp(6.22E-7*T)
+2.74*Exp(-6.28E-3*T)
-7.82E-4*Exp(-3.1*T)
-1.43*Exp(-0.84*T)*Sin(6.13*T-5.24E-2)

ACSL

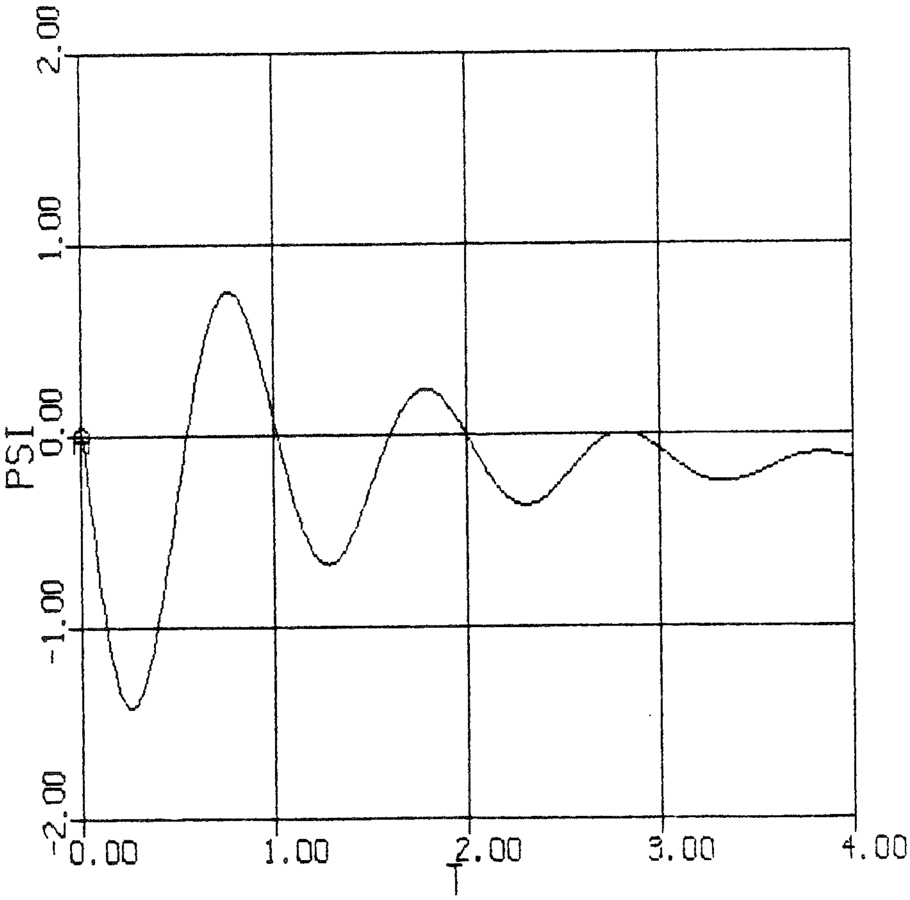
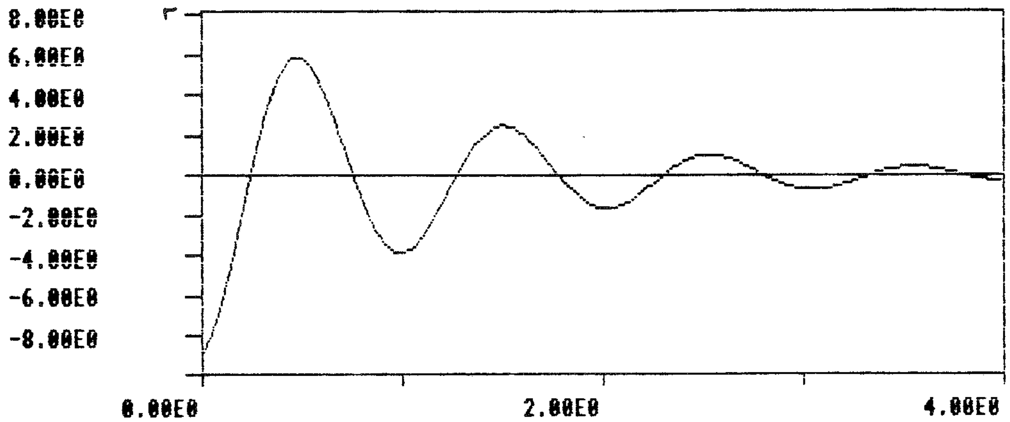


FIGURE A-12: YAW RATE IMPULSE RESPONSE TO 1 DEGREE OF RUDDER.

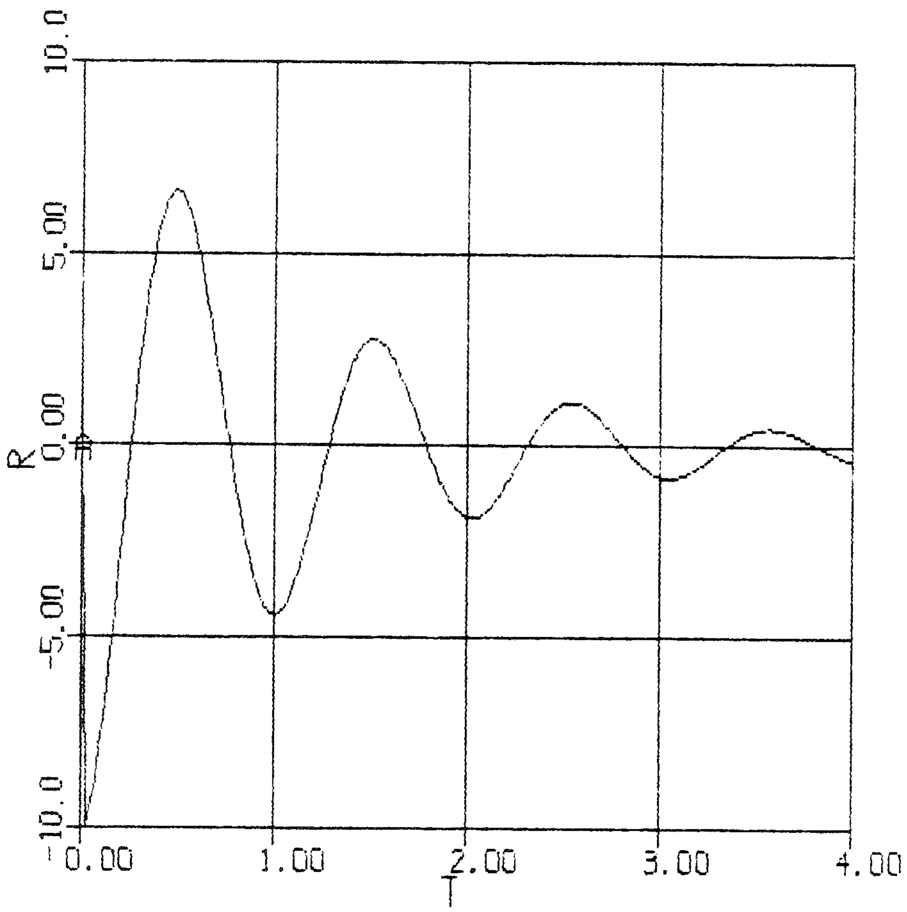
BBC



Impulse of 1.00E0 | Time : 3.98E0 Output : -2.68E-1

-3.58E-6*Exp(6.22E-7*T)
-1.72E-2*Exp(-6.28E-3*T)
+2.43E-3*Exp(-3.1*T)
+8.82*Exp(-0.84*T)*Sin(6.13*T-1.49)

ACSL



APPENDIX B

ACSL SIMULATION PROGRAMS.

ACSL SIMULATION PROGRAMS.

The three ACSL simulation programs LONG.CSL, LATROL.CSL and LATYAW.CSL all take the same basic form with each program being split up into the sections described below. The main differences between the programs lies with which equations of motion are used, (ie. longitudinal or lateral) and whether the elevator, aileron or rudder control surface is specified.

PROGRAM STRUCTURE.

```
PROGRAM TITLE

     INITIAL REGION
          specify the flight condition
          set values of stability and control derivatives
          set control surface angle and duration of deflection
     END OF INITIAL

     DYNAMIC REGION
          specify the time for the simulation to run
          specify the intervals at which data is to be saved
          check if control surface angle should be reset

               DERIVATIVE REGION
                    specify integration algorithm required
                    specify time step
                    define equations of motion
                    integrate states
                    perform any angular conversions necessary
               END OF DERIVATIVE

     END OF DYNAMIC

END OF PROGRAM
```