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Coherent Effects in Multiple Scattering of Linearly Polarized Light

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Abstract—Comparing the stochastic Monte Carlo technique with the iteration procedure for solving the Bethe–Salpeter equation in the framework of numerical simulation, the time correlation function and the interference component of the coherent backscattering of a linearly polarized light wave in a multiply scattering medium are calculated. The results of the simulation agree well with theoretical results obtained by generalizing the Milne solution, as well as with experimental data.

INTRODUCTION

Coherent multiple scattering effects [1–5] are used more and more widely in studies of diverse colloid systems [6], including biological tissues [7–10].

It is usually assumed that, in the process of multiple scattering, light becomes completely depolarized and both the light transfer and the coherent effects can be treated in terms of the scalar field model. However, in the case of backscattering, the detected light contains scattering contributions of all multiplicities. The result of the contributions with lower multiplicity is that backscattered light is, in general, partially polarized. Experiments show a significant role of polarization [2, 5, 11] in backscattering.

A theory of coherent backscattering for an electromagnetic field was first considered in [12] for Rayleigh scattering taking into account the boundary effects using the mirror image technique. In [13–19], exact solutions for multiple Rayleigh scattering were obtained in the framework of generalization of the Milne approach to the case of an electromagnetic field. In [13, 14], the vector transfer equation was solved for scattering in the exactly backward direction taking into account the interference component; in [15, 17], the angular dependence of the backscattering peak taking into account the light polarization was calculated. The problem of the time correlation function, also for the case of Rayleigh scattering, was solved in [18].

In practically all applications, the size of the scatterer exceeds the wavelength and the scattering cross section is strongly anisotropic. However, the theory that simultaneously takes into account the electromagnetic nature of light and the anisotropy of the scattering cross section is rather complicated and cumbersome for specific applications.

Currently, extensive use is made of the Monte Carlo technique in the framework of stochastic simulation of the coherent multiple scattering effects in strongly inhomogeneous optical media [20–29].

In [30], a method was proposed for taking into account polarization effects by summing series in the scattering multiplicities. On the basis of this method, in [21], the Monte Carlo procedure was implemented for calculating the intensity of coherent backscattering including the coherent component. For isotropic Rayleigh scattering, the magnitude of the coherent backscattering peak proved to be much smaller than the result predicted by the exact solution [15, 19]. The backscattering of an electromagnetic field was also simulated in [31]. In this paper, the depolarization rate of linearly polarized light as a function of the number of scattering events was calculated. For Rayleigh scattering, the dependence thus obtained coincided with the prediction of the diffusion approximation. However, with increasing anisotropy of the single scattering cross section, the result of numerical simulation started to differ noticeably from the theory.

In this paper, by comparing the theoretical description of correlation transfer in the framework of the Bethe–Salpeter equation with the conventional scheme of radiation transfer in a strongly inhomogeneous medium in the framework of the Monte Carlo technique [32], we develop a method of stochastic simulation taking into account the electromagnetic field polarization. This method can be applied to different coherent effects within a unified approach. For Rayleigh scattering, the numerical values thus obtained agree with the theoretical results. This allows us to consider the data obtained for the general case of an anisotropic scattering cross section as reliable.

ELECTROMAGNETIC FIELD

The Bethe–Salpeter equation, which describes the transfer of pair correlations of an electromagnetic field in a random medium in the weak-scattering approximation, has the form

$$\hat{\mathbf{\Gamma}}(\mathbf{R}_2, \mathbf{R}_1, t|\mathbf{k}_s, \mathbf{k}_i) = k_0^4\tilde{G}(\mathbf{k}_s - \mathbf{k}_i, t)\delta(\mathbf{R}_2 - \mathbf{R}_1)\hat{I} + k_0^4\int d\mathbf{R}_3\tilde{G}(\mathbf{k}_s - \mathbf{k}_{23}, t)\hat{\Lambda}(R_{23})\hat{\Gamma}(\mathbf{R}_3, \mathbf{R}_1, t|\mathbf{k}_{23}, \mathbf{k}_i). \quad (1)$$

Here, the propagator $\Gamma_{\beta_1\beta_2\alpha_1\alpha_2}(\mathbf{R}_2, \mathbf{R}_1, t|\mathbf{k}_s, \mathbf{k}_i)$ is the fourth-rank tensor; it describes the transfer of two complex conjugate fields with the wave vector $\mathbf{k}_i$ arriving with the time delay t at the point $\mathbf{R}_1$ and coming out from the point $\mathbf{R}_2$ with the wave vector $\mathbf{k}_s$; $k_0 = 2\pi/\lambda$ is the wave number; λ is the wavelength; $k_s = k_i = k = nk_0$; and n is the refractive index of the medium ($n = n_1 + in_2$, where n_1 and n_2 are, respectively, the real and imaginary parts of n and $(2n_2k_0)^{-1} = l$). Due to the multiple scattering, the original polarization of the incident fields, specified by the Cartesian subscripts α_1 and α_2, is converted, in the process of this transfer, into a polarization with subscripts β_1 and β_2.

The fourth-rank tensor $\hat{\Lambda}(\mathbf{R})$

$$\Lambda_{\alpha\beta\mu\nu} = \left(\hat{I} - \frac{\mathbf{R}\otimes\mathbf{R}}{R^2}\right)_{\alpha\mu}\left(\hat{I} - \frac{\mathbf{R}\otimes\mathbf{R}}{R^2}\right)_{\beta\nu}\frac{\exp(-R/l)}{R^2} \quad (2)$$

is the direct product of the complex conjugate Green's functions of the Maxwell wave equation in the far zone. This tensor describes the transformation of a pair of fields with the polarizations μ and ν into a pair of fields with the polarizations α and β after a single scattering event. $\tilde{G}(\mathbf{q}, t)$ is the Fourier transform of the correlation function of fluctuations of permittivity,

$$\tilde{G}(\mathbf{q}, t) = \frac{1}{(4\pi)^2}\int d\mathbf{r}\langle\delta\varepsilon(0,0)\delta\varepsilon(\mathbf{r}, t)\rangle\exp(-i\mathbf{q}\mathbf{r}). \quad (3)$$

A crucial role in the problems of multiple scattering is played by the optical theorem, which connects the single scattering cross section with the scattering length. For an electromagnetic field, the optical theorem, in the weak-scattering or Born approximation [33], has the form

$$l_{\mathrm{sc}}^{-1} = \frac{1 + \overline{\cos^2\theta}}{2}k_0^4\int d\Omega_s\tilde{G}_0(\mathbf{k}_s - \mathbf{k}_i), \quad (4)$$

where $\tilde{G}_0(\mathbf{q}) = \tilde{G}(\mathbf{q}, t)$ is the Fourier transform of the static correlator of the permittivity fluctuations and

$$\overline{\cos^2\theta} = \frac{\int d\Omega_s\tilde{G}_0(\mathbf{k}_s - \mathbf{k}_i)\cos^2\theta_s}{\int d\Omega_s\tilde{G}_0(\mathbf{k}_s - \mathbf{k}_i)}$$

is the cosine squared of the angle between the wave vectors of the incident (k_i) and scattered (k_s) waves averaged over the single scattering cross section.

The mean free path of a photon l and the scattering length l_{sc} are connected through the relation

$$1/l = 1/l_{\mathrm{sc}} + 1/l_{\mathrm{a}}, \quad (5)$$

where l_{a} is the characteristic absorption length associated with inelastic scattering. For the media under consideration, $l_{\mathrm{a}} \gg l$ and the ratio l/l_{sc} is close to unity.

We define the time correlation function of the field at a large distance r from the scattering medium as

$$\hat{C}^{(E)}(t|\mathbf{k}_s, \mathbf{k}_i) = \hat{C}^{(L)}(t|\mathbf{k}_s, \mathbf{k}_i) + \hat{C}^{(V)}(t|\mathbf{k}_s, \mathbf{k}_i), \quad (6)$$

where, as in the case of scalar field, $\hat{C}^{(L)}(t|\mathbf{k}_s, \mathbf{k}_i)$ is the contribution of the ladder diagrams, related to the incoherent component, and $\hat{C}^{(L)}(t|\mathbf{k}_s, \mathbf{k}_i)$ is the interference component observed in backscattering.

The ladder and interference components of the coherence function have the form [30, 34]

$$C^{(L)}_{\beta_1\beta_2\alpha_1\alpha_2}(t|\mathbf{k}_s, \mathbf{k}_i) = \int d\mathbf{R}_1 d\mathbf{R}_2\Gamma_{\beta_1\beta_2\alpha_1\alpha_2}(\mathbf{R}_2, \mathbf{R}_1, t|\mathbf{k}_s, \mathbf{k}_i) \times\exp\left(-\frac{z_1}{l\cos\theta_i} - \frac{z_2}{l\cos\theta_s}\right), \quad (7)$$

$$C^{(V)}_{\beta_1\beta_2\alpha_1\alpha_2}(t|\mathbf{k}_s, \mathbf{k}_i) = \int d\mathbf{R}_1 d\mathbf{R}_2\left[\Gamma_{\beta_1\alpha_2\alpha_1\beta_2}\left(\mathbf{R}_2, \mathbf{R}_1, t\left|\frac{\mathbf{k}_s - \mathbf{k}_i}{2}, \frac{\mathbf{k}_i - \mathbf{k}_s}{2}\right.\right) - k_0^4\tilde{G}(\mathbf{k}_s - \mathbf{k}_i, t)\delta(\mathbf{R}_2 - \mathbf{R}_1)\delta_{\alpha_1\beta_1}\delta_{\alpha_2\beta_2}\right] \times\exp\left[-\frac{z_1 + z_2}{2l}\left(\frac{1}{\cos\theta_i} + \frac{1}{\cos\theta_s}\right) + in_1k_0(z_1 - z_2)(\cos\theta_i - \cos\theta_s) + in_1k_0(x_1 - x_2)(\sin\theta_i - \sin\theta_s)\right], \quad (8)$$

where θ_i is the angle of incidence and θ_s is the angle of scattering measured from the backward direction. The incident and scattered rays lie in the plane (x, z). It is easy to see that, in backscattering, only the polarized component of the interference contribution $\hat{C}^{(V)}(t|\mathbf{k}_s, \mathbf{k}_i)$ exactly coincides with the polarized component of the ladder contribution $\hat{C}^{(L)}(t|\mathbf{k}_s, \mathbf{k}_i)$ before subtraction of the single scattering contribution; the depolarized components do not coincide.

Using the optical theorem and the definition of the phase function

$$p_t(\mathbf{k}_i - \mathbf{k}_s) = \frac{\tilde{G}(\mathbf{k}_i - \mathbf{k}_s, t)}{\int\tilde{G}(\mathbf{k}_i - \mathbf{k}_s, 0)d\Omega_s} \quad (9)$$

we can represent the Fourier transform of the permittivity correlator in the form

$$k_0^4 \tilde{G}(\mathbf{k}_s - \mathbf{k}_i, t) = R l_{\mathrm{sc}}^{-1} p_t(\mathbf{k}_s - \mathbf{k}_i, t),$$

with the Rayleigh factor defined as

$$R = \frac{2}{1 + \cos^2\theta}.$$

As a result, the Bethe–Salpeter equation for an electromagnetic field can be written in the form

$$\hat{\Gamma}(\mathbf{R}_2, \mathbf{R}_1, t | \mathbf{k}_s, \mathbf{k}_i) = R l_{\mathrm{sc}}^{-1} p_t(\mathbf{k}_s - \mathbf{k}_i, t)\delta(\mathbf{R}_2 - \mathbf{R}_1)\hat{I} \tag{10}$$
$$+ R l_{\mathrm{sc}}^{-1} \int d\mathbf{R}_3 p_t(-\mathbf{k}_s + \mathbf{k}_{23}, t)\hat{\Lambda}(R_{23})\hat{\Gamma}(\mathbf{R}_3, \mathbf{R}_1, t | \mathbf{k}_{23}, \mathbf{k}_i).$$

The stochastic simulation, in fact, reproduces the solution of this equation represented in the form of an iteration series in the scattering multiplicities as a chain of a random number of successive scattering events. The main assumption of the Monte Carlo method postulates the distribution $f(s) = \mu\exp(-\mu s)$ [32] of a random quantity s (the photon free path), where $\mu = l^{-1}$ is the scattering coefficient. This law, the Poisson distribution law, follows from the form of the single scattering propagator (2). It follows from this distribution that $s = -l\ln\xi$, where ξ is the probability that the free path length is not smaller than s. The Monte Carlo technique implies choosing an arbitrary value of ξ using a random number generator within the interval [0, 1]. The change in the direction of propagation of the photon packet for each elastic scattering event is controlled by the scattering phase function.

As a result, we obtain a stochastic trajectory of a photon subjected to n collisions at points $\mathbf{R}_1, \ldots, \mathbf{R}_n$ and detected on the surface at point $\mathbf{R}_{n+1} = \mathbf{R}_D$, which modulates in a random way the nth-order term of the above iteration series. Thus, the integration over R_i in the Monte Carlo technique is replaced by a random choice of the quantity $|\mathbf{R}_i - \mathbf{R}_{i-1}| = s$. Unlike the case of a scalar field, for an electromagnetic wave one has to additionally follow the variation of the field polarization vector along the whole random path of the traveling photon. According to Eq. (2), one has to calculate for this purpose the effect of the chain of operators

$$\prod_{i=1}^{n}\left(\hat{I} - \frac{(\mathbf{R}_i - \mathbf{R}_{i-1}) \otimes (\mathbf{R}_i - \mathbf{R}_{i-1})}{|\mathbf{R}_i - \mathbf{R}_{i-1}|^2}\right) \tag{11}$$

on the incident field.

Let, as in the case of a scalar field, the weight of each incident photon (photon packet) be a unit. In an electromagnetic field, not only the weight but also the initial polarization of the photon field should be specified. Generally, this polarization is determined by three Cartesian components. Let the polarization of each incident photon be given by a set of three numbers $\mathbf{P}_x^{(\mathrm{in})} = (1, 0, 0)$. This initial polarization vector indicates that the incident field is polarized along the x axis.

In the process of scattering, the polarization of the field changes. The polarization vector of the field coming to the point of the first collision (which is found in a usual way in the framework of the stochastic Monte Carlo technique) is $\mathbf{P}_x^{(\mathrm{in})}$. At the point of the first collision, the polarization changes and becomes equal to

$$\mathbf{P}_2 = \left(\hat{I} - \frac{(\mathbf{R}_2 - \mathbf{R}_1) \otimes (\mathbf{R}_2 - \mathbf{R}_1)}{|\mathbf{R}_2 - \mathbf{R}_1|^2}\right)\mathbf{P}_x^{(\mathrm{in})}.$$

Thus, along with the conventional stochastic determination of the photon direction after the collision and the determination of the weight function according to the phase function, one should calculate, in each scattering event, a new polarization vector $\mathbf{P}_i$ based on the previous vector $\mathbf{P}_{i-1}$.

Let a photon experience, in total, n scattering events. Then, after the last, nth scattering event, the photon comes to the observation point $\mathbf{r}_D = \mathbf{r}_{n+1}$ with the polarization vector

$$\mathbf{P}_x^{(\mathrm{out})} = \prod_i\left(\hat{I} - \frac{(\mathbf{R}_{i+1} - \mathbf{R}_i) \otimes (\mathbf{R}_{i+1} - \mathbf{R}_i)}{|\mathbf{R}_{i+1} - \mathbf{R}_i|^2}\right)\mathbf{P}_x^{(\mathrm{in})}. \tag{12}$$

Three components of this vector can be combined to give three quantities $P_{xx}^{(\mathrm{out})}$, $P_{xy}^{(\mathrm{out})}$, and $P_{xz}^{(\mathrm{out})}$. Let W_i be the statistical weight of the ith photon arrived at point $\mathbf{r}_D$. Then, for the electromagnetic field, the statistical weight of the ith photon arrived at point $\mathbf{r}_D$ is determined by three numbers,

$$W_i(P_{i,xx}^{(\mathrm{out})2}, P_{i,xy}^{(\mathrm{out})2}, P_{i,xz}^{(\mathrm{out})2}).$$

These three numbers describe the intensity contributions from the field components of the ith photon with the initial polarization along the x axis and the polarization of the scattered field along the x, y, and z axes. After summation over all N_{ph} detected photons, we obtain for the polarized and unpolarized components (for brevity, the superscript out is omitted)

$$I_{\mathrm{pol}} = I_{XX} = \sum_{i=1}^{N_{\mathrm{ph}}} W_i P_{i,xx}^2 R^{n_i},$$
$$I_{\mathrm{depol}} = I_{XY} = \sum_{i=1}^{N_{\mathrm{ph}}} W_i P_{i,xy}^2 R^{n_i}, \tag{13}$$

with the summation performed over all detected photons. In the considered case of backscattering, the z component is absent.

Note that these formulas describe the incoherent contribution of the ladder diagrams, $I_{UV} = C_{UUVV}^{(L)}(0|\mathbf{k}_s, \mathbf{k}_i)$.

In the case of Rayleigh scattering, for normal incidence and scattering at an angle of 180°, the exact solution [18, 19], obtained by generalizing the Milne solution for an electromagnetic field, yields the ratio of the polarized and depolarized components of the incoherent contribution $(I_{pol}/I_{depol})_{Milne} \approx 1.92$. Using numerical simulation, we obtained $(I_{pol}/I_{depol})_{MC} \approx 1.94$. The ratio of the polarized component of the scattered light to the depolarized component allows one to find the residual polarization of the incoherent component of the backscattering. The generalized Milne solution yields [15, 18, 19] $(I_{pol} - I_{depol})/(I_{pol} + I_{depol}) \approx 0.31$. The result of numerical simulation is 0.326. A close value of 0.33 was obtained in [31].

Let us define the height of the peak of the polarized component of the coherent backscattering as

$$h_{CBS}^{pol} = \frac{2I_{pol} - I_{single}}{I_{pol}}.$$

The theoretical value is $h_{CBS}^{pol} \approx 1.75$ [15, 18, 19]. Analysis of the results of simulation in [21] yields the value $h_{CBS}^{pol} \approx 1.4$, strongly different from the theoretical prediction. The value $h_{CBS}^{pol} \approx 1.69$ is given in [35]. The result of our numerical simulation is $h_{CBS}^{pol} \approx 1.746$, which is in better agreement with the theory.

RESULTS OF SIMULATION AND DISCUSSION

Using numerical simulation, we calculated the time correlation function of the polarized and depolarized components of the backscattered light and the polarized and depolarized components of the coherent fraction of the backscattered light for systems with different values of the anisotropy parameter $\overline{\cos\theta}$ of the scattering cross section. We used the Henyey–Greenstein function as a phase function. In [21, 35], the Rayleigh–Gans function was used.

The dependence of the depolarization rate on the number of scattering events was analyzed in the framework of the diffusion approximation in [30]. For the case of an isotropic cross section of single scattering, the degree of residual polarization after n scattering events, according to [30], has the form

$$P(n) = \frac{I_{XX}(n) - I_{XY}(n)}{I_{XX}(n) + I_{XY}(n)} = \frac{3(0.7)^{n-1}}{2 + (0.7)^{n-1}}.$$

The number of scattering events is proportional to the path length: $n \sim s/l_s$. Figure 1 shows the results of the calculation of the quantity $P(n)$ as a function of the number of scattering events. As one can see, the depolarization does decrease exponentially with increasing optical path length, but the rate of this decrease differs from that predicted by the diffusion approximation. As

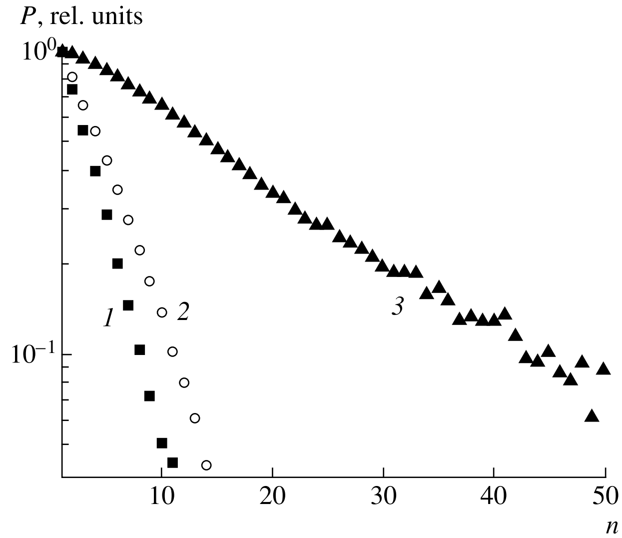

Fig. 1. The degree of depolarization $P(n)$ as a function of the number of scattering events n for $\overline{\cos\theta}$ = (*1*) 0, (*2*) 0.5, and (*3*) 0.9. Optical parameters of the medium: $l = l_{sc}$ = 33.33 μm and $n_1 = 1$. Semi-infinite layer.

the anisotropy increases, the characteristic depolarization length increases because, at larger values of $\overline{\cos\theta}$, the number of collisions that should be experienced by a photon to noticeably change its propagation direction (and, hence, its polarization) should be larger by a factor of $(1 - \overline{\cos\theta})^{-1}$ than in the isotropic case.

In the calculation of the time correlation functions of the electromagnetic field $g_{UV}^{(1)}(t) = C_{UUVV}^{(L)}(t|-\mathbf{k}_i, \mathbf{k}_i)$, we used the following formulas [28]:

$$g_{XX}^{(1)}(t) = \sum_{i=1}^{N_{ph}} W_i P_{i,xx}^2 R^{n_i} \exp\left(-2\frac{t}{\tau_0} n_i \left(1 - \frac{1}{n_i}\sum_l^{n_i} \cos\theta_l\right)\right),$$
$$g_{XY}^{(1)}(t) = \sum_{i=1}^{N_{ph}} W_i P_{i,xy}^2 R^{n_i} \exp\left(-2\frac{t}{\tau_0} n_i \left(1 - \frac{1}{n_i}\sum_l^{n_i} \cos\theta_l\right)\right), \quad (14)$$

where P_{UV} is the polarization vector with the initial and final polarizations U and V, respectively, arising as a result of successive action of n_i tensor operators of the form (11); τ_0 is the characteristic time of Brownian diffusion of the scatterer by the distance λ; and θ_l is the angle of scattering in the lth scattering event. The result remains practically the same upon the substitution

$$\frac{1}{n_i}\sum_l^{n_i} \cos\theta_l \longrightarrow \overline{\cos\theta}.$$

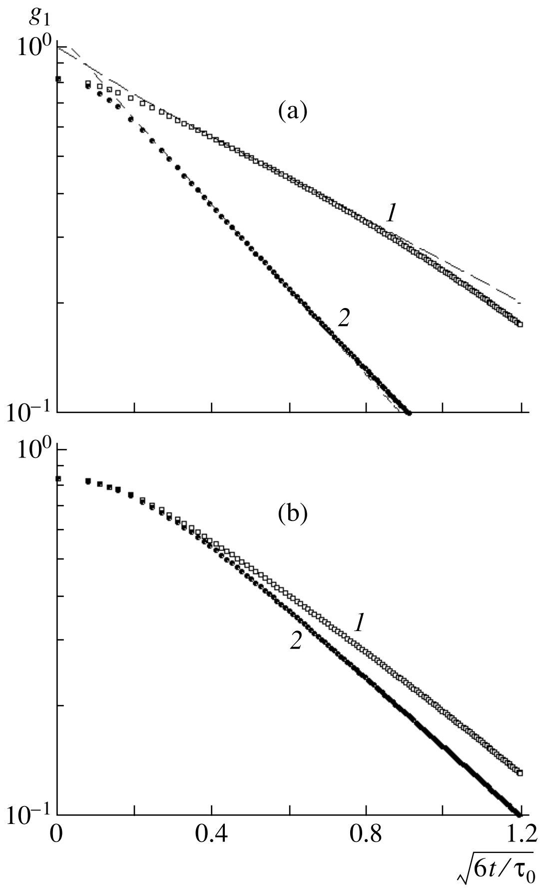

Fig. 2. The time correlation function of scattering of an electromagnetic field: (*1*) $g_{XX}^{(1)}(t)$ and (*2*) $g_{XY}^{(1)}(t)$. (a) Isotropic medium, $\overline{\cos\theta} = 0$; (b) anisotropic medium, $\overline{\cos\theta} = 0.9$. Dashed lines show the slopes $\gamma_{pol} \approx 1.42$ and $\gamma_{depol} \approx 2.68$ of curve *1* and curve *2*, respectively. The optical parameters of the medium are the same as in Fig. 1.

Figure 2 shows the results of simulation of the time correlation function for the field of polarized and depolarized components in the case of Rayleigh scattering, $\overline{\cos\theta} = 0$ (Fig. 2a), and in the case of a strongly anisotropic indicatrix, elongated in the forward direction, $\overline{\cos\theta} = 0.9$ (Fig. 2b).

The diffusion nature of the light propagation in the multiple scattering regime entails a linear decrease of the time correlation function of the field in the form $g_{UV}^{(1)}(t) \sim 1 - \gamma\sqrt{6t/\tau_0}$ [5], where γ is the slope controlling the rate of decrease of the time correlation function. Note that the initial small parameter in time, according to (14), is the quantity $(t/\tau_0)(1 - \overline{\cos\theta})$. For the case of a strongly anisotropic scattering cross section, this quantity remains small at fairly large values of the parameter t/τ_0, when the dependence $g_{UV}^{(1)}(t)$ on $\sqrt{t/\tau_0}$ may deviate strongly from a linear function.

For the case of Rayleigh scattering, the theory [15, 19] predicts the slope for the polarized component $\gamma_{pol} \approx 1.44$ and for the depolarized component $\gamma_{depol} \approx 2.75$. Analysis in the framework of the diffusion approximation [11] yields the values $\gamma_{pol} \approx 1.6$ and $\gamma_{depol} \approx 2.7$, close to the experimental data $\gamma_{pol} \approx 1.6 \pm 0.1$ and $\gamma_{depol} \approx 2.8 \pm 0.2$ obtained for a suspension of latex particles with the diameter $D = 0.091$ μm, much smaller than the wavelength (i.e., for almost Rayleigh scattering).

Note that, as in the real experiment [5], we did not manage to get rid of the nonlinear region at short times, related to the finiteness of the aperture and the finiteness of the number of scattering events taken into account; beginning from the values $\sqrt{t/\tau_0} = 0.15$, the slopes of the linear region predicted by the theory are $\gamma_{pol} \approx 1.42$ and $\gamma_{depol} \approx 2.68$. The dependence of the sum of the polarized and depolarized components, i.e., the time correlation function of unpolarized light, is close to the curve obtained by simulation of the time correlation of a scalar field. As the anisotropy parameter increases, the decrease of the correlation function with time becomes slower. Note that, in the case of an electromagnetic field, the slowing down of the decrease of the time correlations for systems with large anisotropy is more noticeable than in the scalar case.

We calculated the angular dependence of the coherent backscattering peak. The intensity of the polarized component $I_{XX}^{CBS} = C_{XXXX}^{(V)}(0|\mathbf{k}_s, \mathbf{k}_i)$ was calculated by the formula

$$I_{XX}^{CBS} = \sum_{i=1}^{N_{ph}} W_i P_{i,xx}^2 R^{n_i} \cos(kx_i \sin\theta_s) - I_{single},$$

where x_i is the projection of the distance from the point of entrance of the photon to the point of its exit onto the x axis. This axis specifies the plane of scanning in which the tilt angle θ_s is varied.

To calculate the peak of the depolarized component of the coherent backscattering, one has to trace, as follows from (8), the stochastic evolution of a pair of incident fields, namely, the fields polarized along the x and y axes, respectively. To calculate the depolarized component of the scattered light at the point of detection, one has to construct the product of the y component of the first field and the x component of the second field. Thus, one can calculate the result of action of the operator on two vectors with the initial values $\mathbf{P}_x^{(in)} = (1, 0, 0)$ and $\mathbf{P}_y^{(in)} = (0, 1, 0)$ along the trajectory of an individual photon. As a result, at the end of the trajectory, we obtain for these quantities the values $\mathbf{P}_x^{(out)}$ and

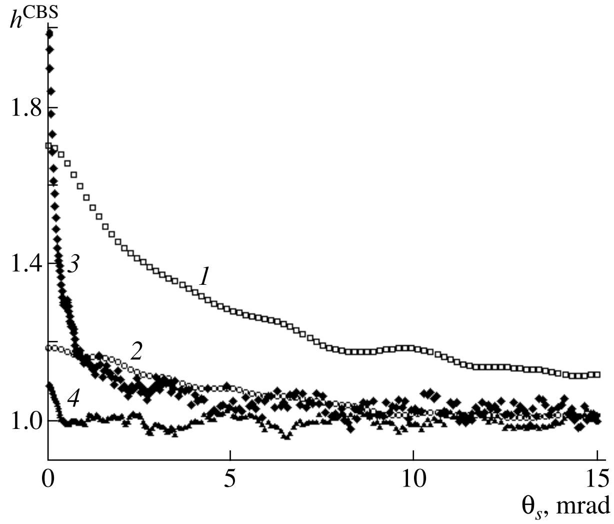

Fig. 3. The coherent backscattering peak h^{CBS} as a function of the angle θ_s: $g = 0$, (*1*) h_{XX}^{CBS}, (*2*) h_{XY}^{CBS}; $g = 0.9$, (*3*) h_{XX}^{CBS}, (*4*) h_{XY}^{CBS}. The optical parameters of the medium are the same as in Fig. 1.

$\mathbf{P}_y^{(out)}$. For each photon, we compose the product of the y component of the vector $\mathbf{P}_x^{(out)}$ and the x component of the vector $\mathbf{P}_y^{(out)}$. Then, the depolarized component of the coherent backscattering can be represented in the form

$$I_{XY}^{CBS} = \sum_{i=1}^{N_{ph}} W_i (\mathbf{P}_x^{(out)})_y (\mathbf{P}_y^{(out)})_x R^{n_i} \cos(kx_i \sin\theta_s).$$

Making in the formulas for I_{XX}^{CBS} and I_{XY}^{CBS} the substitution

$$\cos(kx_i \sin\theta_s) \longrightarrow \cos(ky_i \sin\theta_s),$$

we obtain the polarized and depolarized components of the coherent backscattering under scanning of the angle of scattering in the plane normal to the plane of polarization.

Figure 3 shows the peaks of the polarized and depolarized components of the coherent backscattering

$$(I_{XX} + I_{XX}^{CBS})/I_{XX},$$

$$(I_{XY} + I_{XY}^{CBS})/I_{XY},$$

where I_{XX} is the intensity of the backscattered polarized component,

$$I_{XX} = \sum_{i=1}^{N_{ph}} W_i P_{i,xx}^2 R^{n_i}, \quad I_{XY} = \sum_{i=1}^{N_{ph}} W_i P_{i,xy} P_{yx,i} R^{n_i}.$$

Simulation of the coherent components of the backscattered polarized light was also performed in [21]. For the Rayleigh scattering, the height of the peak of the polarized component was equal to 1.4, which substantially differed from the value of 1.75 predicted theoretically [15, 19]. Our value practically coincides with the theoretical result. The peak height of the depolarized component calculated by us exceeds the theoretical value of 1.15 [15] by approximately 5%, also for the case of Rayleigh scattering.

CONCLUSIONS

Multiple scattering with coherent effects is usually described in terms of a scalar field. As follows from this paper, taking into account the electromagnetic nature of light for backscattering leads to an essentially different quantitative description as compared with a scalar field because, in the case of backscattering, a considerable fraction of the scattered light consists of low-multiplicity contributions. In particular, the rates of decrease of the time correlation function for the polarized and depolarized components are, respectively, much smaller and much larger than the rate of decrease for the case of unpolarized light. Note that the latter of these two rates virtually coincides with the one for the scalar case. The polarized component of backscattered light exceeds by almost a factor of 2 the depolarized component.

The method of numerical simulation developed allows one to use the value of residual polarization to estimate the number of scattering events for light transmitted through a strongly inhomogeneous opaque medium. This is additional information as compared to that obtained by measuring the attenuation of unpolarized light, which allows one to find the value of the transport length.

The above results enable the area of applicability of methods based on coherent and correlation properties of diffusely scattered light to be extended.

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