Robust Aircraft Conceptual Design using AD in Matlab

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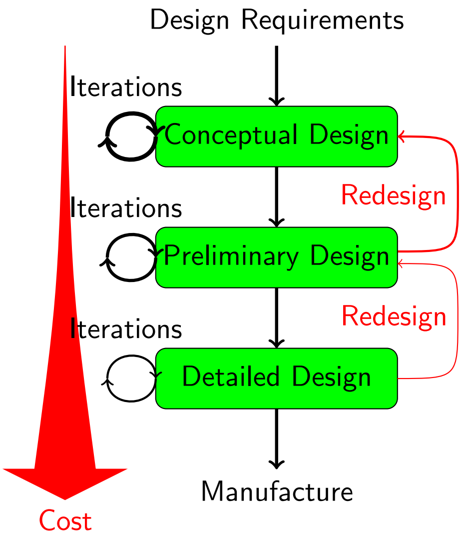
Outline

Aeronautical Design



- Design Requirements - e.g., business jet, passengers 20, range 2000 km.
- Conceptual Design - rapid evaluation and optimisation of low fidelity models.
- Preliminary Design - define and validate design using CAD, CFD, FEA etc.
- Detailed Design - engineering data for tooling and manufacture.

Conceptual Design

- Design parameters e.g.,
  - `Awing` - wing area
  - `tuc` - wing thickness-to-chord ratio.
  - `Npax` - number of passengers
- Multidisciplinary, interacting components.
- e.g., Fuel capacity,

```
Fuel= (A * Awing*tuc + B ) * Awing*tuc;
```

  where `A` and `B` are propriety constants.
- Several hundred other algebraically simple relationships linking e.g., range to drag and fuel; lift to cruise altitude, speed, wing area etc.

Robust Design

- Seek designs with performance relatively insensitive to conceptual design.
- Reduces likelihood of expensive, large-scale design changes downstream and cycling between design phases [CA96].
- Robust design models the design parameters as taken from statistical distributions so increasing modelling complexity.

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Robust Design (ctd.)

If the design task is to minimise some deterministic objective (e.g., fuel consumption) subject to multiple deterministic constraints (e.g., maximum wing span, range, ...) then we:

1. Assume design parameters are of known, independent, statistical distributions.
2. Estimate means and variances of the objective and constraints.
3. Form robust objective and robust constraints favouring designs with:
   - Low objective value and standard deviation.
   - High likelihood of satisfying deterministic constraints.
4. Solve numerically the robust optimisation problem [PLLH06].

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Deterministic Design Optimization

Conceptual design analyses performed for:

- Objective functions $f(\mathbf{x})$.
- Constraint functions $g_i(\mathbf{x}),\ i = 1, 2, \ldots, r$

with $\mathbf{x} \in \mathbb{R}^n$ the vector of the design variables.

Deterministic Design Optimization Problem

$$\begin{aligned} \text{Find } \mathbf{x} \text{ to minimise} \quad & y = f(\mathbf{x}) \\ \text{such that} \quad & g_i(\mathbf{x}) \leqslant 0, i = 1, 2, \ldots, r,\ \ \mathbf{x}_L \leqslant \mathbf{x} \leqslant \mathbf{x}_U. \end{aligned} \tag{1}$$

Robust Design Optimisation

- $\mathbf{x}$'s components assumed stochastic and of known independent probability distributions with mean $\mathbf{E}_x$ and variance $\mathbf{V}_x$.
- Simultaneously minimize objective's variance V_f and its expectation E_f via some robust objective $F(E_f(\mathbf{x}), V_f(\mathbf{x}))$.
- Ensure constraints, distributed with mean E_{g_i} and variance V_{g_i}, satisfied to some probabilistic satisfaction (e.g., 99% certainty) via robust constraint functions $G_i(E_{g_i}(\mathbf{x}), V_{g_i}(\mathbf{x}))$.

Robust Design Optimization Problem

$$\min_{\mathbf{E}_x} F(E_f(\mathbf{x}), V_f(\mathbf{x})), \text{ such that:}$$

$$G_i(E_{g_i}(\mathbf{x}), V_{g_i}(\mathbf{x})) \leqslant 0\,, i = 1, 2, \ldots, r,$$

$$P(\mathbf{x}_L \leqslant \mathbf{E}_x \leqslant \mathbf{x}_U) \geq \mathbf{p}_{bounds},$$

where $\mathbf{p}_{bounds}$ is the prescribed probability with which the mean of the design variables belongs to the original deterministic range.

Robust Design Optimisation (ctd.)

- If all variables are continuous,

$$
\begin{aligned}
E_f(\mathbf{x}) &= \int_{-\infty}^{+\infty} f(\mathbf{t}) p_{\mathbf{x}}(\mathbf{t}) d\mathbf{t} \\
V_f(\mathbf{x}) &= \int_{-\infty}^{+\infty} [f(\mathbf{t}) - E_f(\mathbf{x})]^2 p_{\mathbf{x}}(\mathbf{t}) d\mathbf{t},
\end{aligned}
$$

  in which $p_{\mathbf{x}}$ is $\mathbf{x}$'s joint probability density function.
- Closed-form expression for integrals of practical interest rarely exist.
- Numerical approximation techniques, termed uncertainty propagation, involve trade-offs between cost and accuracy.
  - Monte Carlo methods.
  - Taylor-based method of moments.
  - Quadrature-based techniques[PCG07].
  - Polynomial chaos expansions.

Method of Moments

- Statistical moments estimated from the moments of a truncated Taylor series expansion of y about the mean of $\mathbf{x}$.
- Third order approximation (IIIMM)

$$
\begin{aligned}
f(\mathbf{x}) = \\
& f(\mathbf{E}_x) + \sum_{p=1}^{n} \left(\frac{\partial f}{\partial x_p}\right)(x_p - E_{x_p}) \\
& + \frac{1}{2} \sum_{p=1}^{n} \sum_{q=1}^{n} \left(\frac{\partial^2 f}{\partial x_p \partial x_q}\right)(x_p - E_{x_p})(x_q - E_{x_q}) \\
& + \frac{1}{3!} \sum_{p=1}^{n} \sum_{q=1}^{n} \sum_{r=1}^{n} \left(\frac{\partial^3 f}{\partial x_p \partial x_q \partial x_r}\right)(x_p - E_{x_p})(x_q - E_{x_q})(x_r - E_{x_r}) \\
& + \dots
\end{aligned}
$$

Method of Moments - Mean Estimation

- Assume:
  - The components of $\mathbf{x}$ are independently distributed,

$$E\left((x_p - E_{x_p})(x_q - E_{x_q})\right) = 0 \text{ for } p \neq q,$$

  - Each component x_p is symmetrically distributed about its mean E_{x_p} (zero skewness).
- Taking expectations and using $E(x_p) = E_{x_p}$, we obtain third order Taylor estimate of mean $E_{f_{\mathrm{IIIMM}}}$

$$E_{f_{\mathrm{IIIMM}}} = \overbrace{f(\mathbf{E}_x)}^{m_1} + \overbrace{\frac{1}{2}\sum_{p=1}^{n}\left(\frac{\partial^2 f}{\partial x_p^2}\right) V_{x_p}}^{m_2} + O(\mathbf{V}_x^2), \tag{2}$$

- Retaining just terms m_1 would give 1^{st} order $E_{f_{\mathrm{IMM}}}$ - 1^{st} order accurate in variance.
- For symmetric distributions - no third order term for mean estimation.

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Method of Moments - Variance Estimation

Similar approach for variance gives, with K_{x_p} the kurtosis of x_p.

$$\begin{aligned} V_{f_{\mathrm{IIIMM}}} = & \overbrace{\sum_{p=1}^{n} \left(\frac{\partial f}{\partial x_p} \right)^2 V_{x_p}}^{v_1} \\ & + \sum_{p=1}^{n} \left[\left(\frac{\partial^3 f}{\partial x_p^3} \right) \left(\frac{\partial f}{\partial x_p} \right) \frac{K_{x_p}}{3} + \left(\frac{\partial^2 f}{\partial x_p^2} \right)^2 \frac{K_{x_p} - 1}{4} \right] V_{x_p}^2 \\ & + \sum_{p=1}^{n} \sum_{\substack{q=1 \\ q \neq p}}^{n} \left[\left(\frac{\partial^3 f}{\partial x_p^2 \partial x_q} \right) \left(\frac{\partial f}{\partial x_q} \right) \frac{1}{2} \left(\frac{\partial^2 f}{\partial x_p \partial x_q} \right)^2 \right] V_{x_p} V_{x_q}, \end{aligned} \quad (3)$$

which is second order accurate in variance.
Term v_1 alone gives first order approximation $V_{f_{\mathrm{IMM}}}$.

Sigma-Point (SP) Technique [PCG07]

- Based on numerical quadrature,

$$
\begin{aligned}
E_{f_{\mathrm{SP}}} &= W_0 f(\mathbf{x}_0) + \sum_{p=1}^{n} W_p[f(\mathbf{x}_{p^+}) + f(\mathbf{x}_{p^-})], && (4) \\
V_{f_{\mathrm{SP}}} &= \frac{1}{2}\sum_{p=1}^{n}\left\{W_p\left[f(\mathbf{x}_{p^+}) - f(\mathbf{x}_{p^-})\right]^2\right. \\
&\quad \left. +(W_p - 2W_p^2)\left[f(\mathbf{x}_{p^+}) + f(\mathbf{x}_{p^-}) - 2f(\mathbf{x}_0)\right]^2\right\}. && (5)
\end{aligned}
$$

- Sampling points $\mathbf{x}_0 = \mathbf{E}_x$ and $\mathbf{x}_{p^\pm} = \mathbf{E}_x \pm \sqrt{V_{x_p} K_{x_p}}\mathbf{e}_p$.
- Weights are $W_0 = 1 - \sum_{p=1}^{n} \frac{1}{K_{x_p}}$ and $W_p = \frac{1}{2K_{x_p}}$.

Sigma-Point (SP) Technique (ctd.)

The SP technique:

- has a higher accuracy for the mean than IMM [PCG07].
- requires $2n + 1$ function evaluations for each analysis.
- Robust objective/constraints do not require derivatives of deterministic objective/constraints.
- Gradient of SP objective/constraints are weighted combination of deterministic objective/constraints.

AD of the Conceptual Design Package

- Aircraft conceptual design package was coded in Matlab so MAD[For06] package adopted.
- Forward mode AD:
  - Forward mode `fmad` class objects posses `value` and `deriv` components.
  - Overloading used to propagate derivatives, e.g., for `z = x.*y`,

    ```
    z.value = x.value.*y.value;
    z.deriv = x.value.*y.deriv + y.value.*x.deriv;
    ```
  - For single directional derivative all components are of Matlab's intrinsic class `double`.
- For multiple directional derivatives the highly optimised `derivvec` class is used to store and manipulate derivatives.

Calculating Higher Derivatives

- Rendered MAD's `fmad` and `derivvec` classes self-differentiable for forward-over-forward(-over-forward. . .) differentiation.
- Objects of `derivvec` class contain derivative components that themselves must be differentiated - add source code line,

  ```
  superiorto('fmad')
  ```

  to the `derivvec` class constructor.
- Ensures differentiation applied to components of `derivvec` objects.
- Calculate 2^{nd} derivatives of `f(x)` using:

  ```
  xfmad = fmad(x,eye(length(x)));
  xfmad2 = fmad(xfmad,eye(length(x)));
  yfmad2 = f(xfmad2);
  y = getvalue(getvalue(yfmad2));
  Dy = getinternalderivs(getvalue(yfmad2));
  D2y=getinternalderivs(getinternalderivs(yfmad2));
  ```

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Differentiating the Nonlinear Solve of `fsolve`

- Within our design model, some variables $\mathbf{w} \in \mathbb{R}^p$ are found in terms of some predecessors $\mathbf{v} \in \mathbb{R}^p$, as the solution of,

$$\mathbf{h}(\mathbf{w}, \mathbf{v}) = 0, \tag{6}$$

  with nonlinear function $\mathbf{h} \in \mathbb{R}^p \times \mathbb{R}^p \to \mathbb{R}^p$ for some $p > 1$.
- Matlab's trust-region `fsolve` used to solve (6) using $\mathbf{h}$'s Jacobian.
- Our strategy to calculate derivatives of order d:
  1. Solve (6) for the value component of $\mathbf{w}$ using a call to `fsolve` making use of only $\mathbf{v}$'s value
  2. Perform d Newton iterations,

$$\mathbf{w} \leftarrow \mathbf{w} - \left(\frac{\partial \mathbf{h}}{\partial \mathbf{w}}(\mathrm{value}(\mathbf{w}), \mathrm{value}(\mathbf{v})) \right)^{-1} \mathbf{h}(\mathbf{w}, \mathbf{v}), \tag{7}$$

     with $\mathbf{w}$ and $\mathbf{v}$ manipulated as nested `fmad` objects storing derivatives up to and including those of order d.
- Jacobian matrix $\partial \mathbf{h} / \partial \mathbf{w}$ needed but not its higher derivatives.
- More efficient strategies not investigated ($p = 2$ in our test case).

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Aircraft Sizing Test Case

- Robust optimization of a Matlab-implemented, industrially relevant, conceptual design test case.
- Determination of performance and sizing of a short-to-medium range commercial passenger aircraft.
- 96 sub-models and 126 variables.

Deterministic Problem

- Objective: Minimize Maximum Take-Off Weight $MTOW$ with respect to the design variables $\mathbf{x}$ (next slide).
- Constraints:
  1. Approach speed: $v_{app} < 120$ Kts $\Rightarrow$ $g_1 = v_{app} - 120$;
  2. Take-off field length: $TOFL < 2000$ m $\Rightarrow$ $g_2 = TOFL - 2000$;
  3. Percentage of total fuel stored in wing tanks: $K_F > 0.75$ $\Rightarrow$ $g_3 = 0.75 - K_F$;
  4. Percentage of sea-level thrust available during cruise: $K_T < 1$ $\Rightarrow$ $g_4 = K_T - 1$;
  5. Climb speed: $v_{zclimb} > 500$ ft/min $\Rightarrow$ $g_5 = 500 - v_{zclimb}$;
  6. Range: $R > 5800$ Km $\Rightarrow$ $g_6 = 5800 - R$.

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Design Variables

Design Variable	Definition [units]	Bounds [min,max]
S	Wing area [m^2]	[140, 180]
BPR	Engine bypass ratio [ ]	[5, 9]
b	Wing span [m]	[30, 40]
Λ	Wing sweep [deg]	[20, 30]
t/c	Wing thickness to chord ratio [ ]	[0.07, 0.12]
$T_{e_{SL}}$	Engine sea level thrust [kN]	[100, 150]
FW	Fuel weight [Kg]	[12000, 20000]

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Fixed Parameters

Parameter	Value
Number of passengers	150
Number of engines	2
Cruise Mach number	0.75
altitude [ft]	31000

Robust Optimisation

- Assume $\mathbf{x}$ are independent Gaussian variables with $\mathbf{V}_{x_i}^{1/2} = 0.07\mathbf{E}_{x_i}$.
- Robust objective is taken as

$$MTOW_{\mathrm{rob}} = E_{MTOW} + V_{MTOW}^{1/2}.$$

- Constraints take the form,

$$G_i(\mathbf{x}) = E_{g_i}(\mathbf{x}) + kV_{g_i}^{1/2}(\mathbf{x}) \leq 0,$$

and

$$\mathbf{x}_L + k\mathbf{V}_x^{1/2} \leqslant \mathbf{E}_x \leqslant \mathbf{x}_U - k\mathbf{V}_x^{1/2}.$$

Coefficient $k = 1$ enforces constraint satisfaction with probability of about 84.1%.

Robust Optimisation (ctd.)

Two different techniques for mean and variance

1. First-order method-of-moments (IMM):
   - MAD used to calculate first derivatives of deterministic objective and constraint to approximate E_{f,g_i} and V_{f,g_i}.
   - MAD used to calculate second derivatives to form gradients $\nabla E_{f,g_i}$ and $\nabla V_{f,g_i}$ for optimisation.
2. Sigma-point (SP) Technique:
   - MAD is used to calculate the gradient of objectives $E_{f_{\mathrm{SP}},g_{i\mathrm{SP}}}$ and $V_{f_{\mathrm{SP}},g_{i\mathrm{SP}}}$ constraints for optimisation.

Both optimised using Matlab's gradient-based constrained `fmincon`.

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Results - Design Variables

Design variable	I MM	SP
E_S $[m^2]$	160.843	162.558
E_{BPR} [ ]	8.580	8.580
E_b [m]	37.753	37.753
E_Λ [deg]	21.531	21.531
$E_{t/c}$ [ ]	0.095	0.094
$E_{T_{eSL}}$ [kN]	122.553	123.224
E_{FW} [Kg]	18084.940	18171.282

Results - Objective and Constraints

		I MM	SP
Objective			
Max. take-off weight	$MTOW_{\mathrm{rob}}$ [Kg]	86023.272	86207.016
Constraints			
Approach speed	G_1 [Kts]	0.000	0.000
Take-off field length	G_2 [m]	−161.568	−151.806
% fuel in wings	G_3 [ ]	0.000	0.000
% thrust for cruise	G_4 [ ]	−0.114	−0.107
Climb speed	G_5 [ft/min]	0.000	0.000
Range	G_6 [Km]	0.000	0.000

Post-Optimality Analysis

Relative error compared to Monte Carlo estimates of mean and variance at each optima.

	% Error Mean		% Error Variance	
	IMM	SP	IMM	SP
Objective				
$MTOW_{rob}$	-0.83×10^{-4}	0.27×10^{-6}	0.41	0.39
Constraint				
G_1	-0.12	0.25×10^{-3}	-0.71	0.24
G_2	-0.88	0.17×10^{-2}	-1.89	-0.31
G_3	-0.86	0.17×10^{-1}	-1.27	-0.48
G_4	-0.87	0.12×10^{-2}	-2.18	0.17
G_5	1.15	0.10×10^{-3}	0.23	0.73
G_6	0.23	-0.11×10^{-2}	0.56	0.01

Post-Optimality Analysis (ctd.)

- SP method attains increased accuracy in the mean estimate compared to IMM benefiting design optimisation (lower truncation error).
- Results from IIIMM analysis at the optimal design points are comparable to those obtained by the SP method (cross-derivatives are negligible [PCG07]).
- IIIMM more accurate than other methods and can be adopted to reduce cost of the post-optimality analysis:
  - IIIMM c.p.u. time $\approx$ 8 s.
  - MCS c.p.u. time $\approx$ 133 s.

Effect of using Finite-Differencing for Gradients

	SP		IMM	
	Iterations	**c.p.u. time [s]**	**Iterations**	**c.p.u. time [s]**
AD	10	351	10	45
FD	10	708	24	1212

Conclusions

- Performed robust optimisations of an industrially relevant, Matlab-implemented aircraft sizing problem using the AD tool MAD.
- Two robust design strategies:
  - First Order Method of Moments (IMM) - robust objective and constraints use AD-obtained first order derivatives; second order derivatives used for gradients.
  - Sigma Point Method (SP) - reduced quadrature for robust objective and constraints; AD for their gradients.
- For test case considered, a Monte-Carlo post-optimality analysis indicates that SP more accurate for estimation of the mean but IMM more efficient (with AD gradients).
- In both cases AD gradients significantly reduced optimisation c.p.u. time compared to finite-differencing.
- AD may benefit robust optimisation for aircraft conceptual design.

Further Work

- To date all Method of Moments mean and variance estimates built up from AD gradients by hand-coding - Marina Menshikova (poster) is developing an easy to use interface for arbitrary order Taylor estimates of arbitrary statistical moments.
- Closer coupling of the optimization algorithms with the uncertainty propagation methods.
- Integration with suitable visualization techniques.

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